問題1

(b) $s(t) = x(t-2)$

(c) $u(t) = \dfrac{3}{2} x(t)$

(d) $v(t) = x(2t)$

(e) $w(t) = -x(t) + 2$

(f) $y(t) = x(-t)$

(g) $z(t) = 2x(-t-2) - 1$

問題2 (1)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

$$z(t) = \sum_{n=-\infty}^{\infty} e_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} (2c_n - 3d_n) e^{jn\omega_0 t}$$

$$= 2\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} - 3\sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

$$= 2x(t) - 3y(t)$$

(2)

$$u(t) = \sum_{n=-\infty}^{\infty} f_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 (-t)}$$

$$= x(-t)$$

(3)

$$\frac{d}{dt} x(t) = \sum_{n=-\infty}^{\infty} jn\omega_0 c_n e^{jn\omega_0 t}$$

$$\therefore d_n = jn\omega_0 c_n$$

$$|c_n| = \sqrt{c_n c_n^*}$$

$$|d_n| = \sqrt{(jn\omega_0)^2 c_n c_n^*}$$

$$= jn\omega_0 \sqrt{c_n c_n^*}$$

$$|d_n| = jn\omega_0 |c_n|$$

問題3

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \sum_{n=-\infty}^{\infty} c_n c_n^*$$

$$= \sum_{n=-\infty}^{\infty} c_n \left(\frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} \, dt \right)^*$$

$$= \frac{1}{T} \int_0^1 x(t)^* \left(\sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_0 t} \right) dt$$

$$= \frac{1}{T} \int_0^1 x(t)^* \, x(t) \, dt$$

ここで $x(t)$ は実数なので、

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{T} \int_0^1 x(t)^2 \, dt$$

問題4 (1) (a)

$$x(t) = \begin{cases} 1 & (-1 < t \leq 0) \\ -1 & (0 < t \leq 1) \end{cases}$$

$$x(t+2) = x(t)$$

周期 $T=2$ の周期関数

$$\therefore \omega_0 = \frac{2\pi}{T} = \pi$$

$$c_n = \frac{1}{2} \int_{-1}^{1} x(t) e^{-jn\pi t} \, dt$$

$$= \frac{1}{2} \int_{-1}^{1} x(t) \{\cos(n\pi t) - j\sin(n\pi t)\} \, dt$$

$$= \frac{1}{2} \int_{-1}^{1} x(t) \{ -j\sin(n\pi t) \} \, dt$$

$$= j \int_0^1 (-1) \{ -\sin(n\pi t) \} \, dt$$

$$= j \int_0^1 \sin(n\pi t) \, dt$$

$$= j \left[-\frac{\cos(n\pi t)}{n\pi} \right]_0^1$$

$$= \frac{j}{n\pi} (1 - (-1)^n)$$

$$c_1 = \frac{2j}{\pi}$$
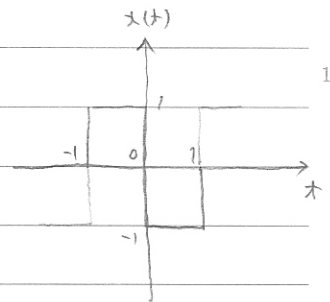
$$|c_1| = \sqrt{c_1 c_1^*}$$

$$= \frac{2}{\pi}$$

$$|c_n| = \sqrt{c_n c_n^*}$$

$$= \frac{1}{n\pi} (1 - (-1)^n)$$

$$= \frac{1}{2n} (1 - (-1)^n) |c_1|$$

$$\therefore \frac{1}{2n} (1 - (-1)^n) \ 倍$$

問題 4 (1) (b)

$x(t) = 1 - |2t| \quad (-1 < t \leq 1)$

$x(t+2) = x(t)$

周期 $T=2$ の 周期関数.

$\therefore \omega_0 = \frac{2\pi}{T} = \pi$

$C_n = \frac{1}{2} \int_{-1}^{1} x(t) e^{-jn\pi t} dt$

$= \frac{1}{2} \int_{-1}^{1} x(t) (\cos(n\pi t) - j\sin(n\pi t)) dt$

$= \int_{0}^{1} (1-2t) \cos(n\pi t) dt$

$= \int_{0}^{1} (1-2t) \left(\frac{\sin(n\pi t)}{n\pi} \right)' dt$

$= \left[(1-2t) \left(\frac{\sin(n\pi t)}{n\pi} \right) \right]_{0}^{1} + \frac{2}{n\pi} \int_{0}^{1} \sin(n\pi t) dt$

$= \frac{2}{(n\pi)^2} \left[-\cos(n\pi t) \right]_{0}^{1}$

$= \frac{2}{(n\pi)^2} (1 - (-1)^n)$

$C_1 = \frac{8}{\pi^2}$

$|C_1| = \frac{8}{\pi^2}$

$|C_n| = \frac{2}{(n\pi)^2} (1-(-1)^n)$

$= \frac{1}{4n^2} (1-(-1)^n) |C_1|$

$\frac{1}{4n^2} (1-(-1)^n)$ 倍

(c)

$x(t) = t \quad (-1 < t \leq 1)$

$x(t+2) = x(t)$

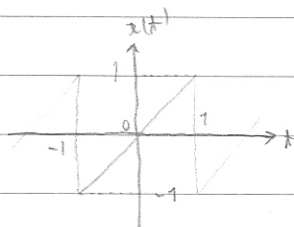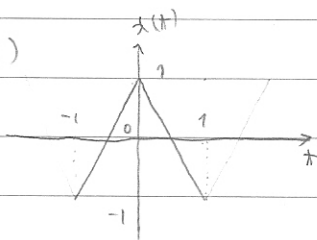周期 $T=2$ の 周期関数

$\therefore \omega_0 = \frac{2\pi}{T} = \pi$

$C_n = \frac{1}{2} \int_{-1}^{1} x(t) e^{-jn\pi t}$

$= \frac{1}{2} \int_{-1}^{1} x(t) (\cos(n\pi t) - j\sin(n\pi t)) dt$

$= \frac{1}{2} \int_{-1}^{1} x(t) (-j\sin(n\pi t)) dt$

$= j \int_{0}^{1} t (-\sin(n\pi t)) dt$

$= j \int_{0}^{1} t \left(\frac{\cos(n\pi t)}{n\pi} \right)' dt$

$C_n = j \left\{ \left[t \cdot \frac{\cos(n\pi t)}{n\pi} \right]_{0}^{1} - \frac{1}{n\pi} \int_{0}^{1} \cos(n\pi t) dt \right\}$

$= j \left\{ \frac{\cos(n\pi)}{n\pi} - \frac{1}{(n\pi)^2} \left[\sin(n\pi t) \right]_{0}^{1} \right\}$

$= \frac{j}{n\pi} (-1)^n$

$C_1 = -\frac{j}{\pi}$

$|C_1| = \frac{1}{\pi}$

$|C_n| = \frac{1}{n\pi}$

$= \frac{1}{n} |C_1| \quad \cdots (4.1)$

問題 4 (2) (c)

$E_{all} = \sum_{n=-\infty}^{\infty} |C_n|^2$ $\left. \right\}$ パーシバルの定理

$= \frac{1}{2} \int_{-1}^{1} t^2 dt$

$= \int_{0}^{1} t^2 dt$

$= \left[\frac{1}{3} t^3 \right]_{0}^{1}$

$= \frac{1}{3}$

$E_基 = E_1 + E_{-1}$

$= |C_1|^2 + |C_{-1}|^2$

$= \frac{2}{\pi^2}$

$\dfrac{E_基}{E_{all}} = \dfrac{\frac{2}{\pi^2}}{\frac{1}{3}}$

$= \frac{6}{\pi^2} = 0.608$

$\therefore 61\%$

(3)

$E_{all} = \sum_{n=-\infty}^{\infty} |C_n|^2$ $\left. \right\}$ (4.1) より

$= \sum_{n=-\infty}^{\infty} \frac{1}{n^2} |C_1|^2$

$= |C_1|^2 \sum_{n=-\infty}^{\infty} \frac{1}{n^2}$

$= \frac{1}{\pi^2} \cdot 2 \cdot \frac{\pi^2}{6}$

$= \frac{1}{3}$

$\therefore$ (2) の E_{all} と 等しく なった.

問題5 (1)

$$x(t) = \begin{cases} 1 & (-\frac{1}{2} < t \le \frac{1}{2}) \\ 0 & \text{otherwize.} \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j\omega t} dt$$

$$= 2 \int_{0}^{\frac{1}{2}} \cos \omega t \, dt$$

$$= 2 \left[\frac{\sin \omega t}{\omega} \right]_{0}^{\frac{1}{2}}$$

$$= \frac{2}{\omega} \sin \frac{\omega}{2}$$

$$y(t) = \begin{cases} 1 - |t| & (-1 < t \le 1) \\ 0 & \text{otherwize.} \end{cases}$$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

$$= 2 \int_{0}^{1} (1-t) \cos \omega t \, dt$$

$$= 2 \int_{0}^{1} (1-t) \left(\frac{\sin \omega t}{\omega} \right)' dt$$

$$= 2 \left\{ \left[(1-t) \left(\frac{\sin \omega t}{\omega} \right) \right]_{0}^{1} + \frac{1}{\omega} \int_{0}^{1} \sin \omega t \, dt \right\}$$

$$= \frac{2}{\omega} \left[-\frac{\cos \omega t}{\omega} \right]_{0}^{1}$$

$$= \frac{2}{\omega^2} (1 - \cos \omega t)$$

(2)

$$Y(\omega) = \frac{4}{\omega^2} \cdot \frac{1 - \cos \omega t}{2}$$

$$= \frac{4}{\omega^2} \sin^2 \frac{\omega}{2}$$

$$= \left(\frac{2}{\omega} \sin \frac{\omega}{2} \right)^2$$

$$= X(\omega) \cdot X(\omega)$$

$y(t)$ は $x(t)$ と $h(t)$ の 畳み込みであるので

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) H(\omega)$$

$$\therefore H(\omega) = X(\omega).$$

$$h(t) = x(t)$$

$$h(t) = \begin{cases} 1 & (-\frac{1}{2} < t \le \frac{1}{2}) \\ 0 & \text{otherwize} \end{cases}$$