問題1

(b) $s(t) = x(t-2)$

(c) $u(t) = \frac{3}{2}x(t)$

(d) $v(t) = x(2t)$

(e) $w(t) = -x(t)+2$

(f) $y(t) = x(-t)$

(g) $z(t) = 2x(-t-2)-1$

問題2 (1)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

$$z(t) = \sum_{n=-\infty}^{\infty} e_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} (2c_n - 3d_n) e^{jn\omega_0 t}$$

$$= 2\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} - 3\sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

$$= 2x(t) - 3y(t)$$

(2)

$$w(t) = \sum_{n=-\infty}^{\infty} f_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0(-t)}$$

$$= x(-t)$$

(3)

$$\frac{d}{dt}x(t) = \sum_{n=-\infty}^{\infty} jn\omega_0 c_n e^{jn\omega_0 t}$$

$$\therefore d_n = jn\omega_0 c_n$$

$$|c_n| = \sqrt{c_n c_n^*}$$

$$|d_n| = \sqrt{(jn\omega_0)^2 c_n c_n^*}$$

$$= jn\omega_0 \sqrt{c_n c_n^*}$$

$$|d_n| = jn\omega_0 |c_n|$$

問題3

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \sum_{n=-\infty}^{\infty} c_n c_n^*$$

$$= \sum_{n=-\infty}^{\infty} c_n \left(\frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt \right)^*$$

$$= \frac{1}{T}\int_0^1 x(t)^* \left(\sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_0 t} \right) dt$$

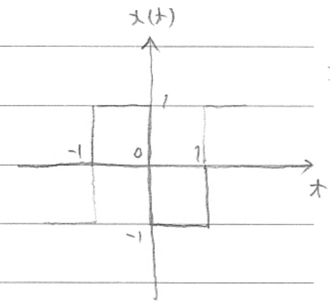
$$= \frac{1}{T}\int_0^1 x(t)^* x(t)\, dt$$

ここで $x(t)$ は実数なので、

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{T}\int_0^1 x(t)^2 dt$$

問題4 (1) (a)

$$x(t) = \begin{cases} 1 & (-1 < t \leqq 0) \\ -1 & (0 < t \leqq 1) \end{cases}$$



$$x(t+2) = x(t)$$

周期 $T=2$ の周期関数

$$\therefore \omega_0 = \frac{2\pi}{T} = \pi$$

$$c_n = \frac{1}{2}\int_{-1}^{1} x(t) e^{-jn\pi t} dt$$

$$= \frac{1}{2}\int_{-1}^{1} x(t) \{\cos(n\pi t) - j\sin(n\pi t)\} dt$$

$$= \frac{1}{2}\int_{-1}^{1} x(t) \{-j\sin(n\pi t)\} dt$$

$$= j\int_0^1 (-1)\{-\sin(n\pi t)\} dt$$

$$= j\int_0^1 \sin(n\pi t)\, dt$$

$$= j\left[-\frac{\cos(n\pi t)}{n\pi} \right]_0^1$$

$$= \frac{j}{n\pi}(1-(-1)^n)$$

$$c_1 = \frac{2j}{\pi}$$

$$|c_1| = \sqrt{c_1 c_1^*}$$

$$= \frac{2}{\pi}$$

$$|c_n| = \sqrt{c_n c_n^*}$$

$$= \frac{1}{n\pi}(1-(-1)^n)$$

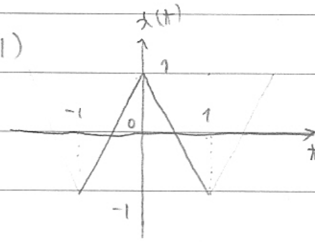
$$= \frac{1}{2n}(1-(-1)^n)|c_1|$$

$$\therefore \frac{1}{2n}(1-(-1)^n) \text{ 倍}$$

問題4 (1) (b)

$x(t) = 1 - |2t| \quad (-1 < t \leqq 1)$

$x(t+2) = x(t)$



周期T=2の周期関数.

$\therefore \omega_0 = \frac{2\pi}{T} = \pi$

$$C_n = \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-jn\pi t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} x(t)\left(\cos(n\pi t) - j\sin(n\pi t)\right) dt$$

$$= \int_0^1 (1-2t)\cos(n\pi t)\, dt$$

$$= \int_0^1 (1-2t)\left(\frac{\sin(n\pi t)}{n\pi}\right)' dt$$

$$= \left[(1-2t)\left(\frac{\sin(n\pi t)}{n\pi}\right)\right]_0^1 + \frac{2}{n\pi}\int_0^1 \sin(n\pi t)\, dt$$

$$= \frac{2}{(n\pi)^2}\left[-\cos(n\pi t)\right]_0^1$$

$$= \frac{2}{(n\pi)^2}\left(1-(-1)^n\right)$$

$C_1 = \frac{8}{\pi^2}$

$|C_1| = \frac{8}{\pi^2}$

$|C_n| = \frac{2}{(n\pi)^2}\left(1-(-1)^n\right)$

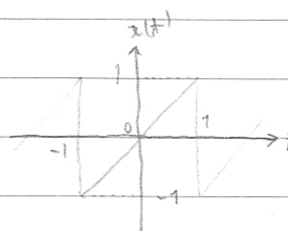
$= \frac{1}{4n^2}\left(1-(-1)^n\right)|C_1|$

$\therefore \frac{1}{4n^2}(1-(-1)^n$ 倍

(c)

$x(t) = t \quad (-1 < t \leqq 1)$

$x(t+2) = x(t)$



周期T=2の周期関数

$\therefore \omega_0 = \frac{2\pi}{T} = \pi$

$$C_n = \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-jn\pi t}$$

$$= \frac{1}{2}\int_{-1}^{1} x(t)\left(\cos(n\pi t) - j\sin(n\pi t)\right) dt$$

$$= \frac{1}{2}\int_{-1}^{1} x(t)\left(-j\sin(n\pi t)\right) dt$$

$$= j\int_0^1 t\left(-\sin(n\pi t)\right) dt$$

$$= j\int_0^1 t\left(\frac{\cos(n\pi t)}{n\pi}\right)' dt$$

$$C_n = j\left\{\left[t\cdot\frac{\cos(n\pi t)}{n\pi}\right]_0^1 - \frac{1}{n\pi}\int_0^1 \cos(n\pi t)\, dt\right\}$$

$$= j\left\{\frac{\cos(n\pi)}{n\pi} - \frac{1}{(n\pi)^2}\left[\sin(n\pi t)\right]_0^1\right\}$$

$$= \frac{j}{n\pi}(-1)^n$$

$C_1 = -\frac{j}{\pi}$

$|C_1| = \frac{1}{\pi}$

$|C_n| = \frac{1}{n\pi}$

$= \frac{1}{n}|C_1| \quad \cdots \quad (4.1)$

問題4 (2) (c)

$E_{all} = \sum_{n=-\infty}^{\infty} |C_n|^2$ パーシバルの定

$= \frac{1}{2}\int_{-1}^{1} t^2\, dt$

$= \int_0^1 t^2\, dt$

$= \left[\frac{1}{3}t^3\right]_0^1$

$= \frac{1}{3}$

$E_{基} = E_1 + E_{-1}$

$= |C_1|^2 + |C_{-1}|^2$

$= \frac{2}{\pi^2}$

$\frac{E_{基}}{E_{all}} = \frac{\frac{2}{\pi^2}}{\frac{1}{3}}$

$= \frac{6}{\pi^2} = 0.608$

$\therefore$ 61 %

(3)

$E_{all} = \sum_{n=-\infty}^{\infty} |C_n|^2$ (4.1)より

$= \sum_{n=-\infty}^{\infty} \frac{1}{n^2}|C_1|^2$

$= |C_1|^2 \sum_{n=-\infty}^{\infty} \frac{1}{n^2}$

$= \frac{1}{\pi^2}\cdot 2 \cdot \frac{\pi^2}{6}$

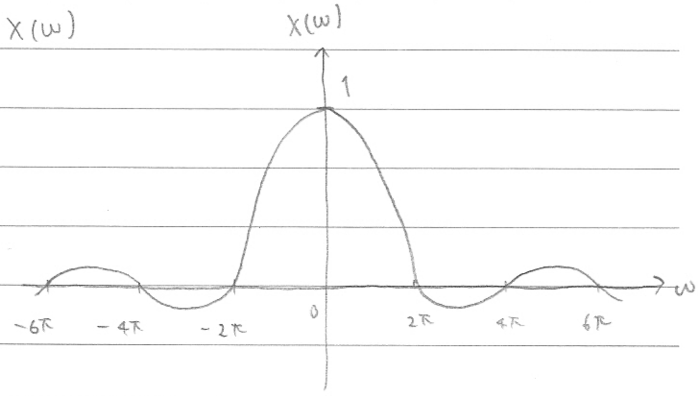
$= \frac{1}{3}$

$\therefore$ (2)の E_{all} と等しくなった.

問題5 (1)

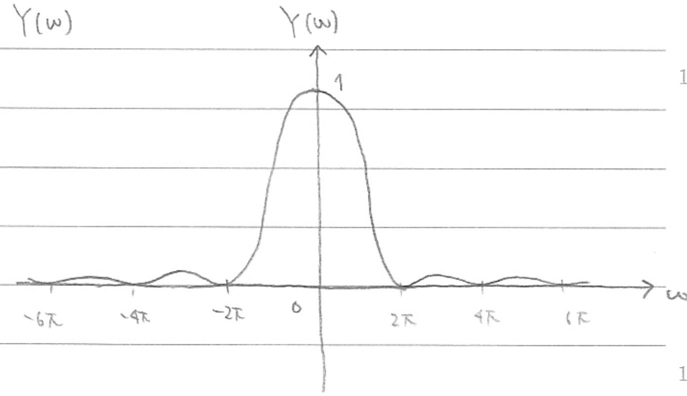
$$x(t) = \begin{cases} 1 & \left(-\frac{1}{2} < t \leqq \frac{1}{2}\right) \\ 0 & \text{otherwize.} \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$
$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j\omega t}\, dt$$
$$= 2\int_0^{\frac{1}{2}} \cos\omega t\, dt$$
$$= 2\left[\frac{\sin\omega t}{\omega}\right]_0^{\frac{1}{2}}$$
$$= \frac{2}{\omega}\sin\frac{\omega}{2}$$



$$y(t) = \begin{cases} 1-|t| & (-1 < t \leqq 1) \\ 0 & \text{otherwize} \end{cases}$$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$
$$= 2\int_0^1 (1-t)\cos\omega t\, dt$$
$$= 2\int_0^1 (1-t)\left(\frac{\sin\omega t}{\omega}\right)' dt$$
$$= 2\left\{\left[(1-t)\left(\frac{\sin\omega t}{\omega}\right)\right]_0^1 + \frac{1}{\omega}\int_0^1 \sin\omega t\, dt\right\}$$
$$= \frac{2}{\omega}\left[-\frac{\cos\omega t}{\omega}\right]_0^1$$
$$= \frac{2}{\omega^2}(1-\cos\omega)$$



(2)

$$Y(\omega) = \frac{4}{\omega^2}\cdot\frac{1-\cos\omega}{2}$$
$$= \frac{4}{\omega^2}\sin^2\frac{\omega}{2}$$
$$= \left(\frac{2}{\omega}\sin\frac{\omega}{2}\right)^2$$
$$= X(\omega)\cdot X(\omega)$$

$y(t)$ は $x(t)$ と $h(t)$ の畳み込みであるので

$$y(t) = x(t) * h(t)$$
$$Y(\omega) = X(\omega)H(\omega)$$
$$\therefore\ H(\omega) = X(\omega).$$
$$h(t) = x(t)$$
$$h(t) = \begin{cases} 1 & \left(-\frac{1}{2} < t \leqq \frac{1}{2}\right) \\ 0 & \text{otherwize} \end{cases}$$