Advanced Image Analysis
Lab 1 Report: Image Analysis in Frequency Domain

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1 Introduction

In this lab experiment, the idea of frequency domain analysis for digital images has been studied in details. The effect of filtering the image in the frequency domain and its correspondence to the spatial domain have been inspected carefully by implementing a number of experiments in Matlab program.

2 Phase and magnitude manipulation

The following figure shows two images, 'Lena' and 'circuit', and their corresponding phase images.

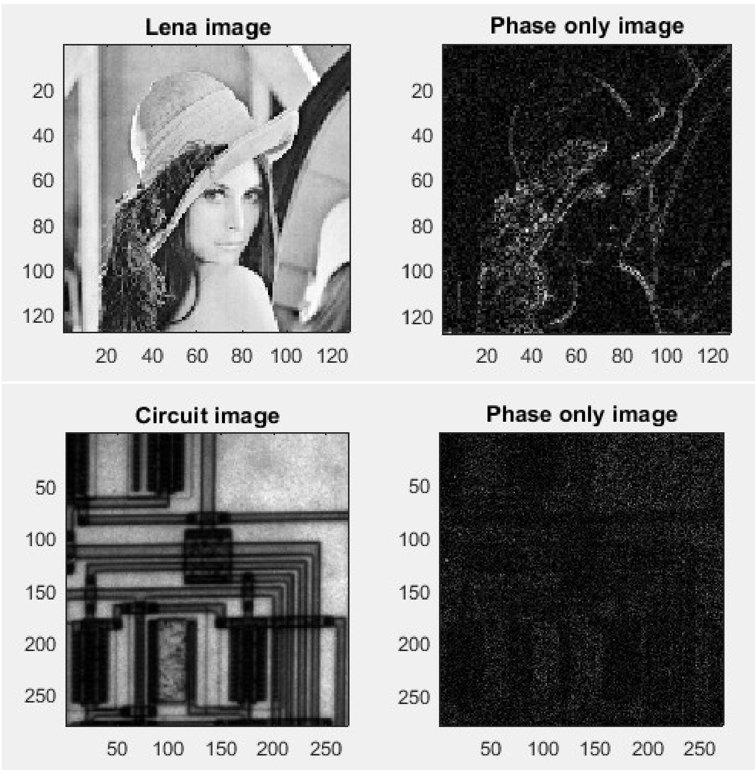

Figure 1: phase only images

As the figure demonstrates, transforming an image to the frequency domain and then using the only phase to transformed it back to the spatial domain gives an image with very close structure to the original one. In fact, the phase is a measure of displacement of the various sinusoids with respect to their origin. Thus, while the magnitude of the 2-D DFT is an array whose components determine the intensities in the image, the corresponding phase is an array of angles that carry much of the information about where discern able objects are located in the image.

In the next figure, the effect of reconstructing images from noisy phase information has been demonstrated. viewing those images in more details reveals that randomizing the phase has severe repercussion on the reconstructed image since, as stated previously, phase information plays a significant role in the reconstruction procedure of the original image.

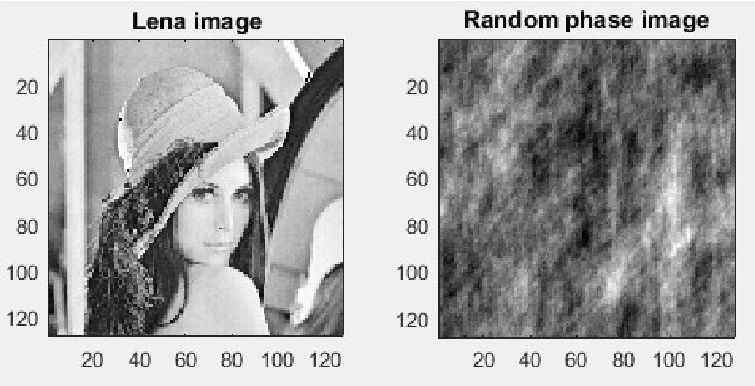

Figure 2: Random-phase Lena image

Looking at the sonar and random-phase image shows how both of them are very similar. However, this is because the sonar image is basically a noisy one. Consequently, randomizing the phase still gives a noisy version, which appears to be similar to the original one.

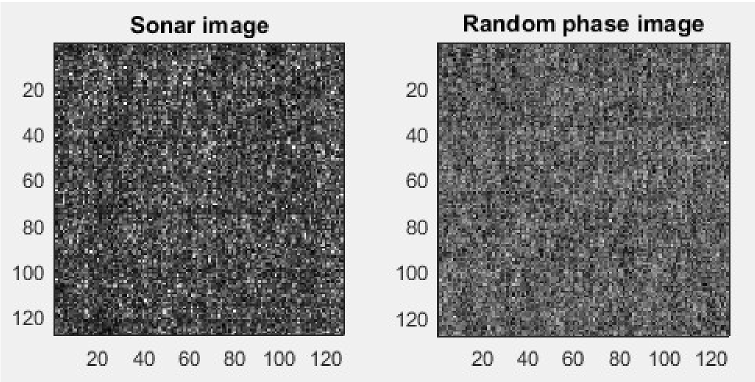

Figure 3: Random-phase sonar image

In the second part of this experiment, the ramifications of randomizing the magnitude spectrum of digital images have been studied, where figure 4 presents such an example in which the random-magnitude image is similar to the phase-only image shown before as a result of changing only the magnitudes of the different exponential terms but preserving their location with respect to the origin. This explains why phase information is more crucial in reconstructing the original image than the magnitude information. Repeating the same test in figure 5 on the SAR image gives also similar image to the spatial one; but here one should not forget that the SAR image is basically a noisy one.

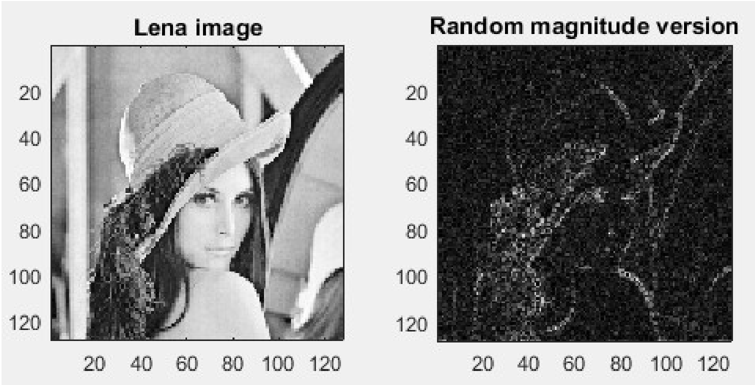

Figure 4: Random-magnitude Lena image

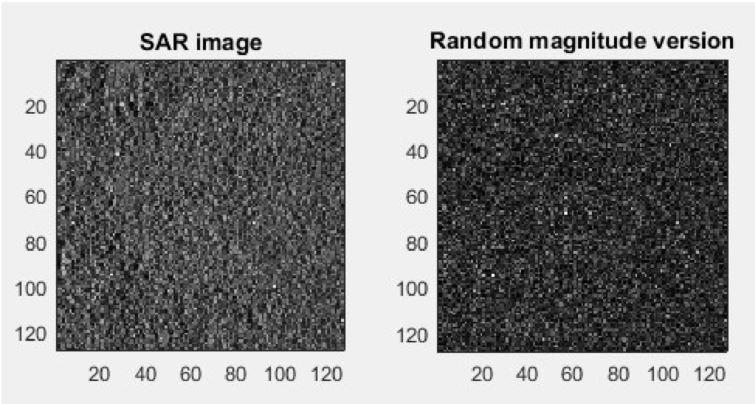

Figure 5: Random-magnitude SAR image

Overall, the phase spectrum of an image relates closely to its shape characteristics, and consequently it has more effect on reconstructing the original image. Nevertheless, magnitude spectrum is more visually meaningful when looking at it in the frequency domain, which means one can expect some of the features of the spatial domain image by looking at its magnitude spectrum.

3 Simple Fourier Filtering

In this section, an analysis of frequency domain filtering of digital images is presented by the mean of applying low-pass and high-pass filters with different cut-off frequencies on two different images. The next two figures show lena image filtered with a low pass filter with two dissimilar cut-off frequencies, 0.2 and 0.6 respectively.

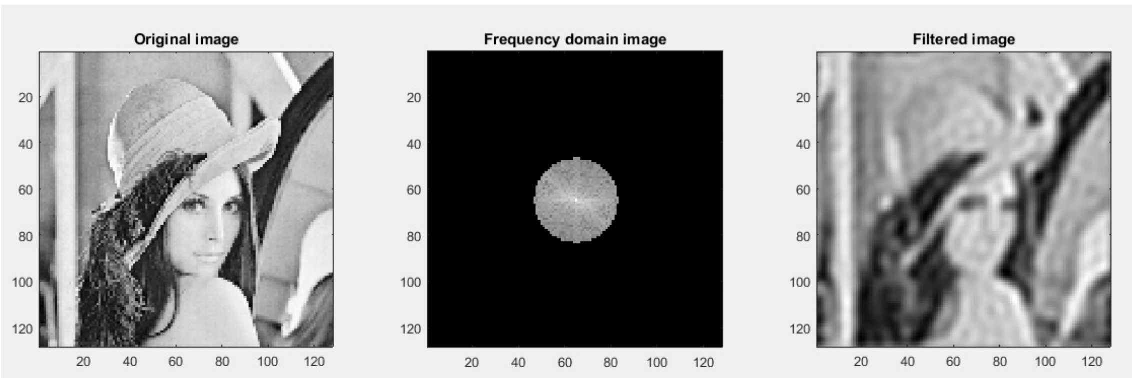

Figure 6: low-pass filtering with cut-off frequency of of 0.2

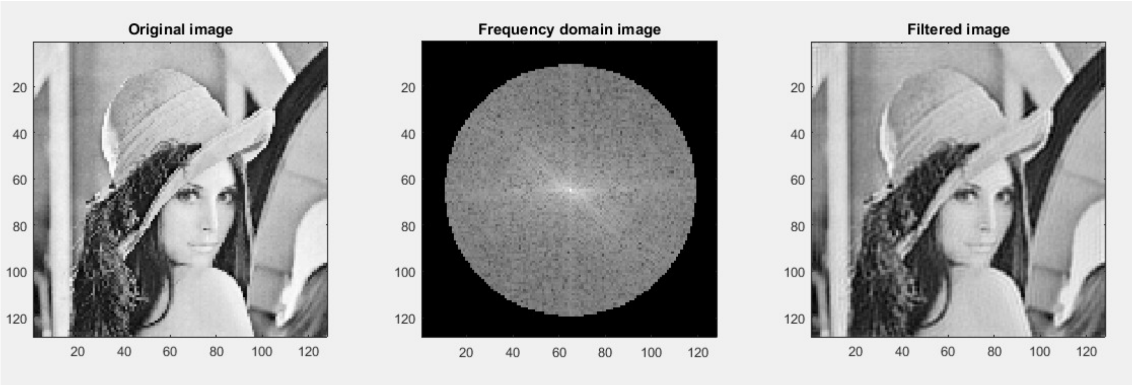

Figure 7: low-pass filtering with cut-off frequency of of 0.6

Inspecting those images in more details proves the smoothing effect of low pass filtering on images, where the lower the cut-off frequency is the higher the smoothing effect. On the contrary, high pass filtering sharpens the images to an extent that is dependent on the cut-off frequency, the higher the more. This is demonstrated in figures 8 and 9 with 0.2 and 0.6 cut-off frequencies.

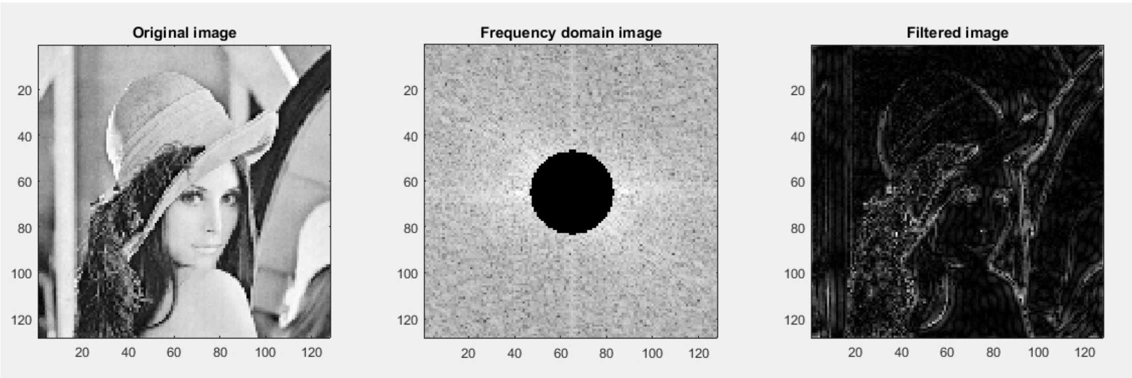

Figure 8: high-pass filtering with cut-off frequency of of 0.2

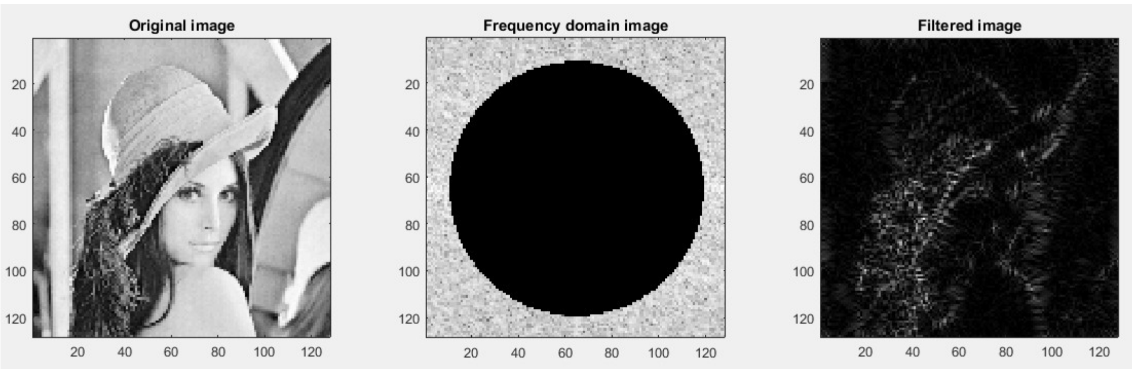

Figure 9: high-pass filtering with cut-off frequency of of 0.6

4 Developing Fast-Fourier Transform

This section is mainly about the implementation of 2-D Fast Fourier Transform algorithm. In order to filter an image in the frequency domain, the following steps should be taken into account:

1. Read the original image.
2. Multiply the input image by $(-1)^{x+y}$ for centring the transform for filtering.

```
im=imread('images/lena.gif');
[i,j]=meshgrid((1:size(im,1)),(1:size(im,2)));
shift=(-1).^(i+j);
im=double(im);
im_shifted=shift.*im;
```

3. Compute the 2-D DFT and the spectrum.

```
[u,x]=meshgrid(0:size(im,1)-1);
w1=exp(-1i*2/size(im,1)*pi*u.*x);
[v,y]=meshgrid(0:size(im,2)-1);
w2=exp(-1i*2/size(im,2)*pi*v.*y);
fft_im=w1*im_shifted*w2;
mag=abs(fft_im);
subplot(2,3,1);imagesc(im);
title('Original image');
colormap(gray);
subplot(2,3,2);imagesc(log(1+mag));
title('Magnitude spectrum');
colormap(gray);
subplot(2,3,3);imagesc(angle(fft_im));
title('Phase spectrum');
colormap(gray);
```

4. Multiply the resulting (complex) array by a real filter function (in the sense that the the real coefficients multiply both the real and imaginary parts of the transforms) and compute the spectrum..

```
% create a low-pass filter, which is of a Butterworth type
for i=1:size(im,1)
    for j=1:size(im,2)
        D(i,j)=sqrt((i-1-size(im,1)/2)^2+(j-1-size(im,2)/2).^2);
    end
end
cutoff=20; %an arbitrary cut-off frequency, just as an example
n=5;
filter=1./(1+(D/cutoff).^(2*n));
filtered_im=fft_im.*filter;
subplot(2,3,4);imagesc(log(1+filter));
title('Low pass filter');
colormap(gray);
mag_filtered=abs(filtered_im);
subplot(2,3,5);imagesc(log(1+mag_filtered));
title('Magnitude specturm of the filtered image');
colormap(gray);
```

5. Compute the inverse 2-D Fourier transform.
6. Multiply the result by $(-1)^{x+y}$ and take the real part.

```
orig_im=w1.^(-1)*filtered_im*w2.^(-1);
orig_im=shift.*orig_im;
figure;imagesc(real(orig_im));
title('filtered image');
colormap(gray);
subplot(2,3,6);imagesc(im);
title('Original image');
colormap(gray);
```

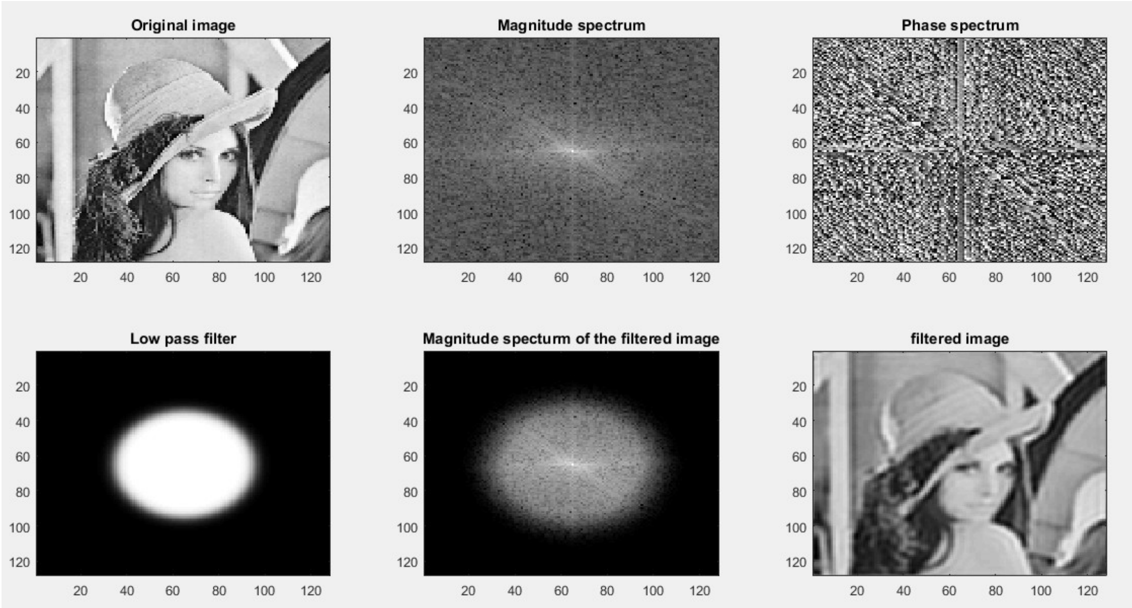

Figure 10: 2-D FFT with filtering by a low pass filter

The filter used in this implementation is the Butterworth filter. As opposed to the ideal filters, the Butterworth filter is a continuous filter of the form:

$$\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

Where n is the order of the filter, D0 is the filter radius (cut-off point) from the centre of the filter and D(U,V) is the filter characteristic describing the shape of the filter; circular, ellipse, etc.

5 Fourier Spectrum and Average Value

One of the Fourier transform properties is that the average of the original signal is equivalent to its Fourier transform evaluated at zero. For digital images, this corresponds to take the pixel value at the center of the magnitude spectrum image, since a shift step takes place while doing the transformation, and then divides this value by the total size of the image (number of rows*number of columns). It is noteworthy that the average value of an image represents the pixel with the highest intensity in the corresponding magnitude spectrum image. The following figure shows the magnitude spectrum of an image, before and after shifting it.

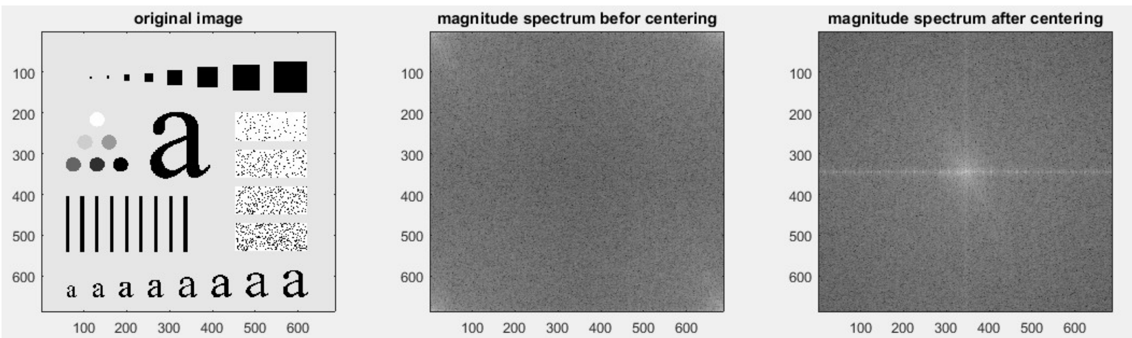

Figure 11: Magnitude spectrum of an image before and after centring it

As it is shown in the centred magnitude spectrum image, the center pixel is the one with the highest intensity value, which represents the average value of the original image after dividing it by the total size of the image. Thus, for the previously shown image the average value is 207.3147.

6 Compression and DCT

In this section, light has been shed on the repercussions of quantizing the magnitude and phase spectrum of an image, and as a result a conclusion has been drawn on which one is more sensitive to quantization. The next figures show the results of using the provided *quant_fft* quantization function with different parameters.

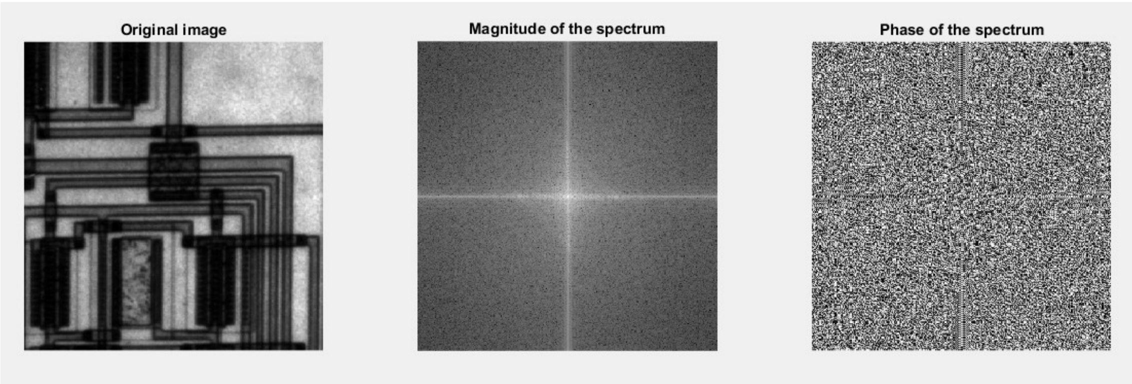

Figure 12: Circuit image with its magnitude and phase spectrum

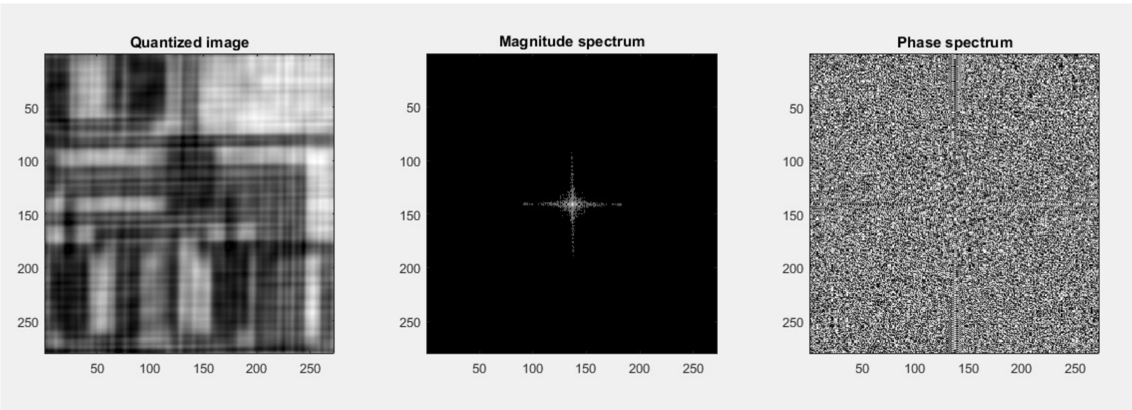

Figure 13: Quantized image with max_level=1000,nb_levels_mag=128,nb_levels_phi=1000

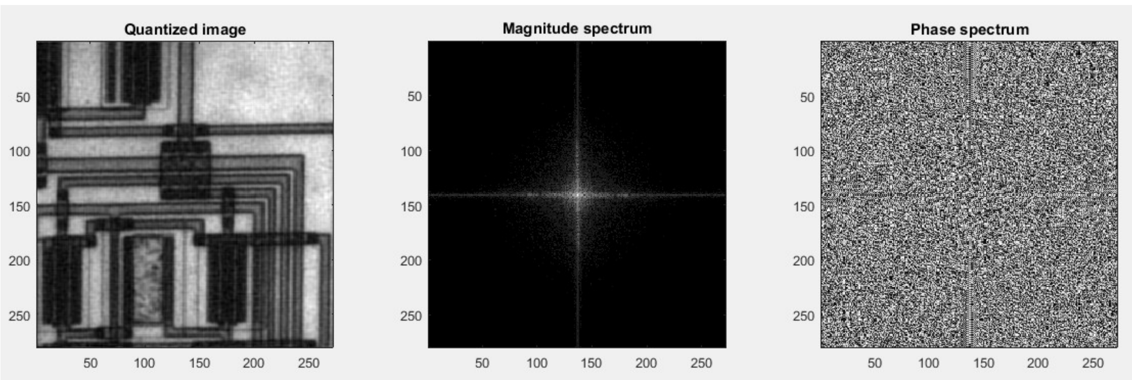

Figure 14: Quantized image with max_level=1000,nb_levels_mag=1000,nb_levels_phi=128

Looking at the previous figures, it is evident that the magnitude is more sensitive to quantization than the phase. For example, less quantization levels in the magnitude spectrum produces a more blurred image compared to the phase spectrum quantization case with the same number of levels.

7 Conclusion

During this lab session, the properties of the frequency domain analysis and its relation to the spatial one in case of digital images have been comprehended more by implementing a number of experiments in Matlab program. Moreover, a number of comparisons have been drawn between the effect of randomizing either the magnitude or phase spectrum of an image on the reconstruction of the original one. Then, a 2-D FFT algorithm has been implemented and used to filter the image in the frequency domain. Finally, phase spectrum has proven to be less sensitive to quantization than the magnitude spectrum.