

B31XM – Advanced Image Analysis

Lab 3: Total Variation image processing

Total Variation flow

show your solution either to Ahmed Karam (TA) or Alex Belyaev (instructor)

Matlab script tv2.m implements a simple gradient descent flow for a total variation energy

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(\frac{\nabla u}{|\nabla u|_\varepsilon} \right) + \lambda(f - u) \quad \text{where} \quad |\nabla u|_\varepsilon = \sqrt{\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \varepsilon}.$$

So we can write

$$\frac{\partial u}{\partial t} = \frac{u_{xx}(u_y^2 + \varepsilon) - 2u_x u_y u_{xy} + u_{yy}(u_x^2 + \varepsilon)}{(u_x^2 + u_y^2 + \varepsilon)^{3/2}} + \lambda(f - u)$$

One possible extension of the total variation approach consists of applying it the first-order derivatives of a given image and then reconstructing a smoothed image from smoothed derivatives (a good matlab-based reconstruction-from-gradient code can be found at <http://web.media.mit.edu/~raskar/photo/code.pdf>).

Your task is to implement such an extension and test whether it is useful for image denosing purposes.