"Lights out" Game Report

There are two typical features for of this game which make it easier for the solution to be described in terms of linear algebra. Firstly, each button of the 25*25 matrix just needs to be pushed once, in other words no more than once because double push equals no push. Secondly, the order of pushing is not significant because the state of a button depends only on how many times it is clicked on. Therefore, with these two features, we can say that the solution of the lights matrix can be given by a vector. That means, it is the problem of Ax = b, where A is the game system which is always constant for 25*25 size of this game, x is the strategy vector, b is the initial configuration of the game system. That is to say, starting with empty configuration and applying the strategy vector x will lead us to the vector b, which obviously means if we start from initial status b we will end with turned-off lights because every light has only two possible status ('1' or 'on' and '0' or 'off').

The system Ax=b is a system of 25 linearly independent equation. In fact, there are many possible ways to solve this system (find x) in linear algebra. One effective approach is to use the Gauss-jordan elimination (RREF ([A | b])) in order to get the reduced row echelon form matrix which gives us the following information:

Rank(A)=Col(A)=23 => Number of free variables=25-23=2

This means that this system has either an infinitely number of solutions or no solution. More importantly, not every initial configuration b has a solution. This implies that in order to get the solution, first of all there should be a validation step that checks if the vector b belongs to the column space of A. This could be done by simply checking if b is perpendicular the row space of A (or column space of A since it is symmetric). Hence, taking the dot product of b with the free variables of A, which constitutes the basis of the null space of A (N(A)) is sufficient to know if any b is winnable or not.

By supposing that b is winnable, the infinite number of solutions in this case is reduced to only four solutions since this system, as mentioned previously, is a binary system:

X= Xp+{a1*Xs1+a2*Xs2}, where a1&a2 belongs to modulus 2 numbers (0,1) and Xp Xs1, Xs2 are the particular solution, the first special solution, and the second special solution respectively. ➔

X1=Xp, X2=Xp+1*Xs1, X3=Xp+1*Xs2, X4=Xp+1*Xs1+Xs2

Thus, finding the best solution (minimum number of clicks) is a straightforward matter. All of the previously described techniques are implemented using a MATLAB program and the final result is the Lights out game with the best solution for every possible winnable configuration.

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