Exercise on Analysis of Algorithms

Show the upper-bound or lower-bound of the following functions as specified.

You need to show two constants c and n0 shown p.18, Ch. 4 exist. Page limit is one page in PDF.

(1) $5\log n + 2\sqrt{n}$ (upper-bound)

(2) $2n^2 + 3n \log n$ (upper-bound)

(3) $2^n + 3^{n/2}$ (lower-bound)

(1) Let $f(n) = 5\log n + 2\sqrt{n}$

because $\log n < \sqrt{n}$

we can conclude that: $5\log n + 2\sqrt{n} < 5\sqrt{n} + 2\sqrt{n} = 7\sqrt{n} = c \cdot \sqrt{n}$ $(n \geq 2)$

thus, $f(n) = 5\log n + 2\sqrt{n} < c\sqrt{n}$ (let $c=7$, $n_0=2$)

the upper bound of $f(n)$ is $O(\sqrt{n})$

(2) Let $f(n) = 2n^2 + 3n\log n$

for $n \geq 2$, we can conclude that $\log n \leq n$

thus, $f(n) = 2n^2 + 3n\log n \leq 2n^2 + 3n^2 = 5n^2$, $(n \geq 2)$

Let $c=5$, $n_0=2$

therefore, the upper bound of $f(n)$ is $O(n^2)$

(3) $f(n) = 2^n + 3^{n/2} \geq 2^n$ (for all $n \geq 1$)

Let $c=1$, $n_0=1$

thus, the lower bound of $f(n)$ is $\Omega(2^n)$