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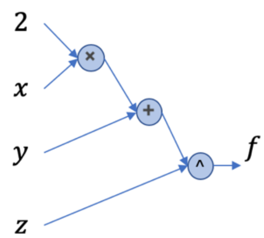
11040A Neural Networks

Assignment 3
Deadline: May 29, 2025 24:00:00

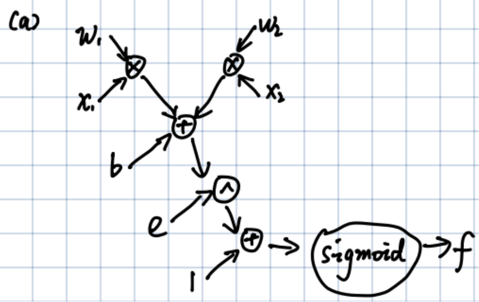
1. Composite functions can be depicted as computational graphs.
   For example, the figure below shows the corresponding computational graph of function $f(x, y, z) = z^{2x+y}$.



   Draw the computational graphs for the following functions and calculate the partial derivatives of all the functions with respect to x_1.

   (a) $f(x_1, x_2) = (1 + e^{w_1x_1+w_2x_2+b})^{-1}$

(a)

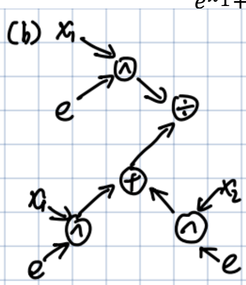


Let $f(x_1,x_2)=\sigma(z)$, $z=w_1x_1+w_2x_2+b$

$$\frac{\partial f(x_1,x_2)}{\partial x_1} = \sigma(z)(1-\sigma(z))\cdot w_1$$

   (b) $f(x_1, x_2) = \frac{e^{x_1}}{e^{x_1}+e^{x_2}}$

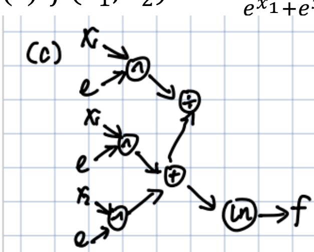
(b)



$$\frac{\partial f}{\partial x_1} = \frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2}$$

   (c) $f(x_1, x_2) = \ln\frac{e^{x_1}}{e^{x_1}+e^{x_2}}$

(c)

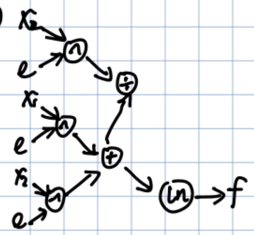


$f(x_1,x_2) = x_1 - \ln(e^{x_1}+e^{x_2})$

$$\Rightarrow \frac{\partial f(x_1,x_2)}{\partial x_1} = 1 - \frac{e^{x_1}}{e^{x_1}+e^{x_2}} = \frac{e^{x_2}}{e^{x_1}+e^{x_2}}$$

(d) $f(x_1, x_2) = \ln\frac{e^{x_2}}{e^{x_1}+e^{x_2}}$

(d)

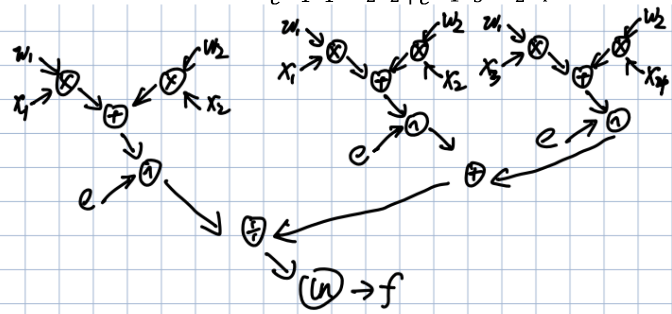


$f(x_1,x_2) = x_2 - \ln(e^{x_1}+e^{x_2})$

$\Rightarrow \frac{\partial f(x_1,x_2)}{\partial x_1} = -\frac{e^{x_1}}{e^{x_1}+e^{x_2}}$

(e) $f(x_1, x_2, x_3, x_4) = \ln\frac{e^{w_1x_1+w_2x_2}}{e^{w_1x_1+w_2x_2}+e^{w_1x_3+w_2x_4}}$

(e)

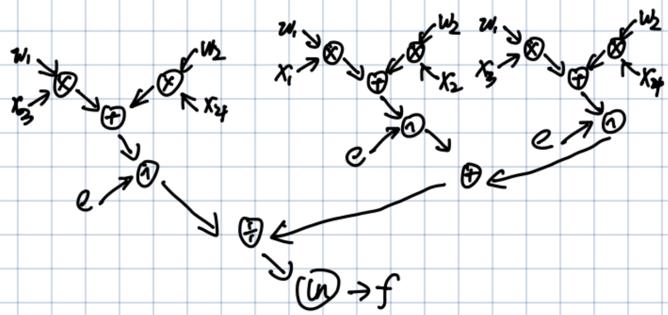


Let $A = w_1x_1+w_2x_2$, $B = w_1x_3+w_2x_4$

$f = \ln\left(\frac{e^A}{e^A+e^B}\right) = A - \ln(e^A+e^B) \Rightarrow \frac{\partial f}{\partial x_1} = w_1 \cdot \frac{e^B}{e^A+e^B}$

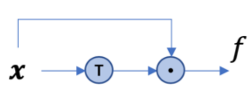
(f) $f(x_1, x_2, x_3, x_4) = \ln\frac{e^{w_1x_3+w_2x_4}}{e^{w_1x_1+w_2x_2}+e^{w_1x_3+w_2x_4}}$

(f)



Same as (e). $f = \ln\left(\frac{e^B}{e^A+e^B}\right) = B - \ln(e^A+e^B) \Rightarrow \frac{\partial f}{\partial x_1} = -w_1 \cdot \frac{e^A}{e^A+e^B}$

2. For simplicity, a node can also represent a vector or a matrix in the computational graph. For example, function $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{x}$, where $\boldsymbol{x} = [x_1,\ x_2]^T$ can be described as the figure below.



Draw the computational graphs for the following functions and calculate the gradients of all the functions with respect to $\boldsymbol{x}$.

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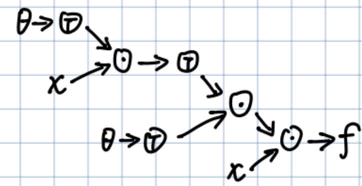
(a) $f(\boldsymbol{x}) = (\boldsymbol{\theta}^T\boldsymbol{x})^T\boldsymbol{\theta}^T\boldsymbol{x}$, where $\boldsymbol{\theta} \in \mathbb{R}^{d\times h}$ and $\boldsymbol{x} \in \mathbb{R}^{d\times 1}$.

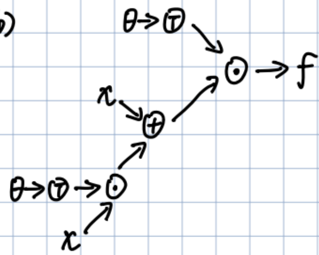
(a)



$f(x) = \|\theta^T x\|^2 \Rightarrow \nabla_x f = 2\theta\theta^T x$

(b) $f(\boldsymbol{x}) = \boldsymbol{\theta}^T(\boldsymbol{x} + \boldsymbol{\theta}^T\boldsymbol{x})$, where $\boldsymbol{\theta} \in \mathbb{R}^{d\times d}$ and $\boldsymbol{x} \in \mathbb{R}^{d\times 1}$.

(b)

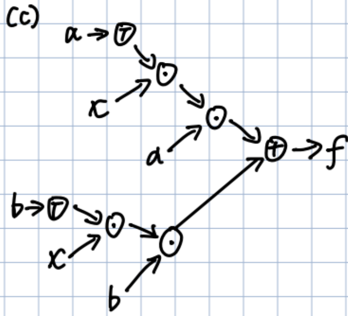


$f(x) = \theta^T(x+\theta^T x) = \theta^T x + \theta^T\theta^T x$

$\Rightarrow \nabla_x f = \theta + \theta\theta$

(c) $f(\boldsymbol{x}) = (\boldsymbol{a}^T\boldsymbol{x})\boldsymbol{a} + (\boldsymbol{b}^T\boldsymbol{x})\boldsymbol{b}$, where $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{x} \in \mathbb{R}^{d\times 1}$.

(c)



$f(x) = (a^T x)a + (b^T x)b$

$\Rightarrow \nabla_x f = aa^T + bb^T$

Notice that matrix calculus has largely two consistent layout conventions: *Numerator layout* and *Denominator layout*. Therefore, sometimes, the question may have more than one answer. (You may refer to the "Matrix calculus" entry on Wikipedia for more information.)