$44251017\ HuangJiahui$

Condition

	M1	M2	M3
Pro A	a_1	a_2	a_3
Pro B	b_1	b_2	b_3

$max\{M2\ processtime\} \leq min\{M1, M3\ process\ time\}$

Proof

Show that under the above condition, we can apply Johnson's algorithm by converting the 3-machine problem into a 2-machine problem using pseudo-machines:

- **F1 = M1 + M2**
- **F2 = M2 + M3**

Total lead time when scheduling A $\rightarrow$ B

$T_{A->B} = a_1 + b_3 + max\{(b_1 + b_2), (a_2 + b_2), (a_2 + a_3)\}$

Since:

$max\{M2\ process\ time\} \leq min\{M1, M3\ process\ time\}$

$\Rightarrow min\{(b_1 + b_2), (a_2 + b_2), (a_2 + a_3)\} = (a_2 + b_2)$

$\Rightarrow T_{A->B} = a_1 + b_3 + max\{(b_1 + b_2), (a_2 + a_3)\}$

Assume $T_{A->B}$ is shorter:

Then $T_{A->B} \leq T_{B->A}$

$\Rightarrow a1 + b3 + max\{(b1 + b2), (a2 + a3)\} \leqq b1 + a3 + max\{(a1 + a2), (b2 + b3)\}$

$\Rightarrow max\{-(b1 + b2), -(a2 + a3)\} \leqq max\{-(a1 + a2), -(b2 + b3)\}$

$\Rightarrow -min\{(b1 + b2), (a2 + a3)\} \leqq -min\{(a1 + a2), (b2 + b3)\}$

$\Rightarrow -min\{(b1 + b2), (a2 + a3)\} \leqq -min\{(a1 + a2), (b2 + b3)\}$

$\Rightarrow min\{(b1 + b2), (a2 + a3)\} \geqq min\{(a1 + a2), (b2 + b3)\}$

$\Longleftrightarrow Process\ time\ of\ (a1 + a2)\ or\ (b2 + b3)\ is\ shortest$

Thus, by Johnson's algorithm:

If the front (F1 = M1+M2) is smaller → place early

If the back (F2 = M2+M3) is smaller → place later

the 3-machine problem can be reduced to a 2-machine pseudo problem