Assignment4 Huang Jiahui 44251017

Problem Definition of FSSP

Minimize C_{max}

> We aim to minimize the makespan, which is the total time required to complete all jobs.

Constraint

Subject to

$$C_{max} \geq s_{j,m} + p_{j,m} \ \ (j = 1, 2, \ldots, J)$$
$$s_{j,k+1} \geq s_{j,k} + p_{j,k} \ \ (j = 1, 2, \ldots, J; k = 1, 2, \ldots, m-1)$$
$$s_{j,k} \geq s_{l,k} + p_{l,k} - M(1 - x_{j,l}^{(k)})$$
$$s_{l,k} \geq s_{j,k} + p_{j,k} - Mx_{j,l}^{(k)} \ \ (j \neq l; j, l = 1, 2, \ldots, J; k = 1, 2, \ldots, m)$$

Step 1: Makespan Constraint

$C_{max} \geq s_{j,m} + p_{j,m} \ \ (j = 1, 2, \ldots, J)$

> The makespan must be greater than or equal to the finish time of the last operation of every job

Step 2: Job Precedence Constraint (Intra-job ordering)

$s_{j,k+1} \geq s_{j,k} + p_{j,k}$

> A job's next operation on machine M_{k+1} can only start after it finishes on machine M_k

Step 3: Machine Conflict Constraints (Inter-job sequencing)

$s_{j,k} \geq s_{l,k} + p_{l,k} - M(1 - x_{j,l}^{(k)})$
$s_{l,k} \geq s_{j,k} + p_{j,k} - Mx_{j,l}^{(k)} \ \ (j \neq l; j, l = 1, 2, \ldots, J; k = 1, 2, \ldots, m)$

> Jobs cannot overlap on the same machine. Either job j goes before job l, or vice versa.
>
> Binary variable $x_{j,l}^{(k)}$ determines order. Big-M logic ensures at least one ordering is enforced.

Fixed values

J : number of jobs

m : number of machines

$p_{j,k}$: process time of job j on machine M_k

Decision Variables

C_{max} : makespan

$s_{j,k}$: start time of job j on machine M_k

$x_{j,l}^{(k)} \in \{0, 1\}$: 1 if job j is before job l on machine M_k