Scheduling Algorithms

Assignment 1

Please proof the applicability of Johnson Algorithm to the special case for 3 operations

- Applied Condition:
  max{M2 process time}≦min{M1 and M3 process time}
- Applied Method:
  Transfer to the 2 pseudo composed facilities,
  and apply to Johnson Algorithm

	M1	M2	M3
Pro A	a1	a2	a3
Pro B	b1	b2	b3

→

	M1+M2	M2+M3
Pro A	a1+a2	a2+a3
Pro B	b1+b2	b2+b3

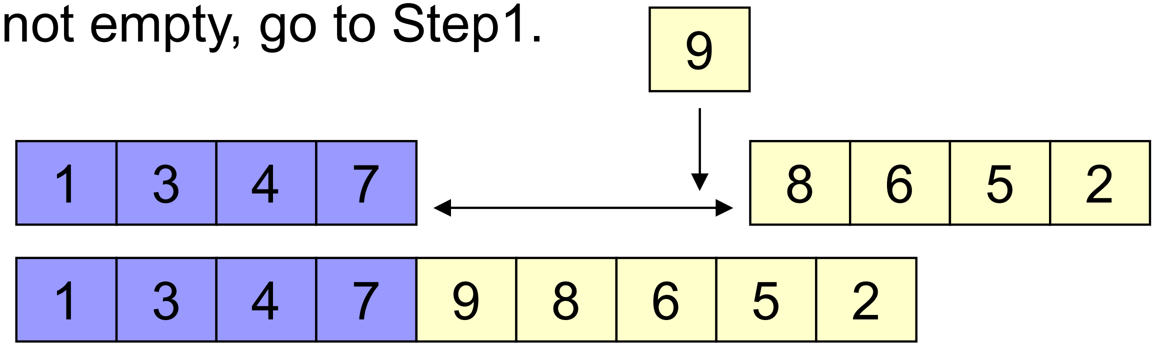
Reference1: Johnson Algorithm

This is applied for flow shop type jobs that have 2 operations each.

Step1: Pick the operation which processing time is the shortest
in the list of remaining operations.
(If some operations have the same shortest time, it is not matter which operation is picked.)

Step2: In the case of the picked operation is the first (or second) operation in the job, the job is added at the end of the earlier item sequence (or at the first of the later item sequence).

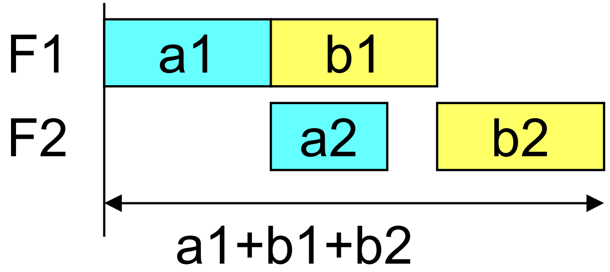
Step3: If the list of remaining operations is empty,
concatenate the earlier item sequence and the later item sequence.
If it is not empty, go to Step1.



Reference2:
Proof of Johnson Algorithm

job	F1	F2
A	a1	a2
B	b1	b2

b1>a2



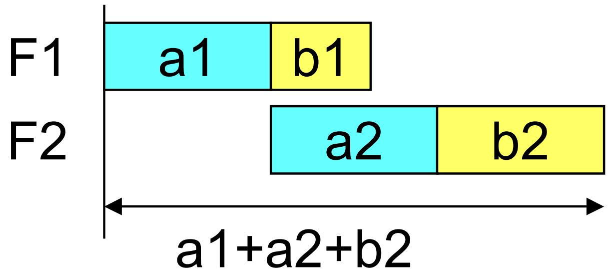
Total Lead Time for A→B

$= a1+b1+b2 \quad (b1>a2)$

$\quad a1+a2+b2 \quad (b1<=a2)$

$= a1+b2+\max\{b1,a2\}$

b1<=a2



Reference 2:
Proof of Johnson Algorithm

(Total Lead Time for A→B)=$a1+b2+\max\{a2,b1\}$
(Total Lead Time for B→A)=$b1+a2+\max\{b2,a1\}$

Assume that total Lead Time for A→B is shorter.

$$a1+b2+\max\{a2, b1\} \leqq b1+a2+\max\{b2, a1\}$$
$$\max\{-b1,-a2\} \leqq \max\{-b2,-a1\}$$
$$-\min\{b1,a2\} \leqq -\min\{b2,a1\}$$
$$\min\{b1,a2\} \geqq \min\{b2,a1\}$$

⇔ Process Time of a1 or b2 is shortest.

According to Johnson Algorithm, all pairs of direct neighboring jobs satisfy the above-mentioned condition

∴ Johnson Algorithm derives the minimum make span solution.