SMART
INDUSTRY
LABORATORY

Scheduling Algorithms (3)

- How to Solve Scheduling Problems -

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Production and Systems

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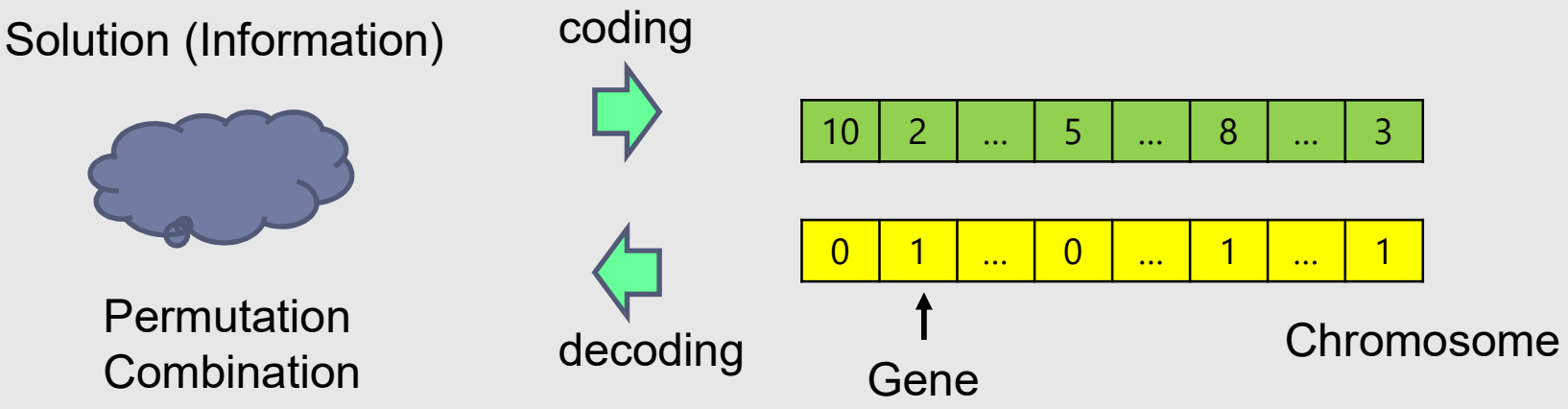
Applying Meta Heuristic Methods

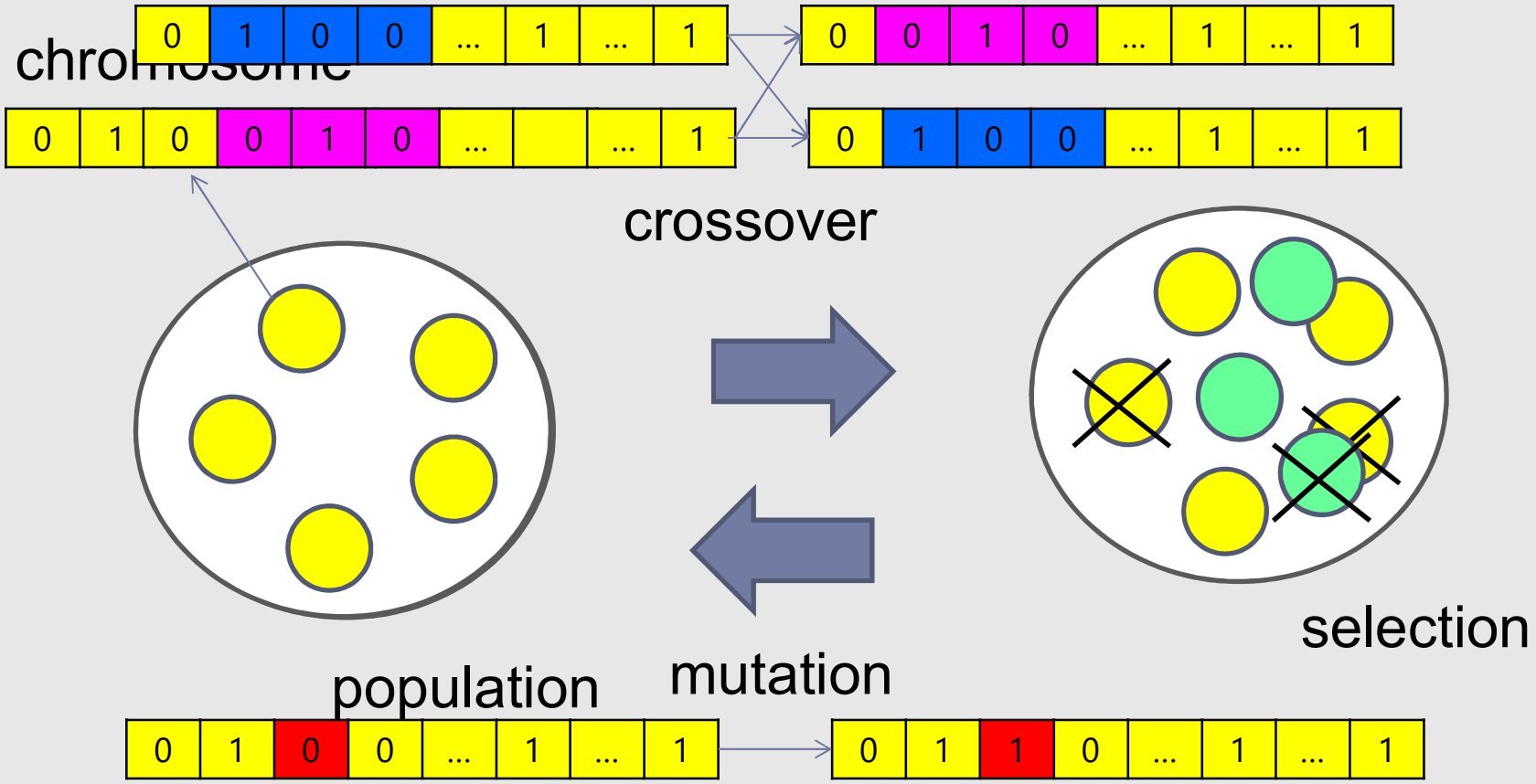
Genetic Algorithms:***GA*** (1975 Holland) is one of Meta Heuristic Methods.

Meta Heuristic Methods can be applied for Many kinds of Optimization Problems.



Genetic Algorithms

Evolutional operations (making offsprings)



Effective for every kind of problems?

No!

Look for the best way

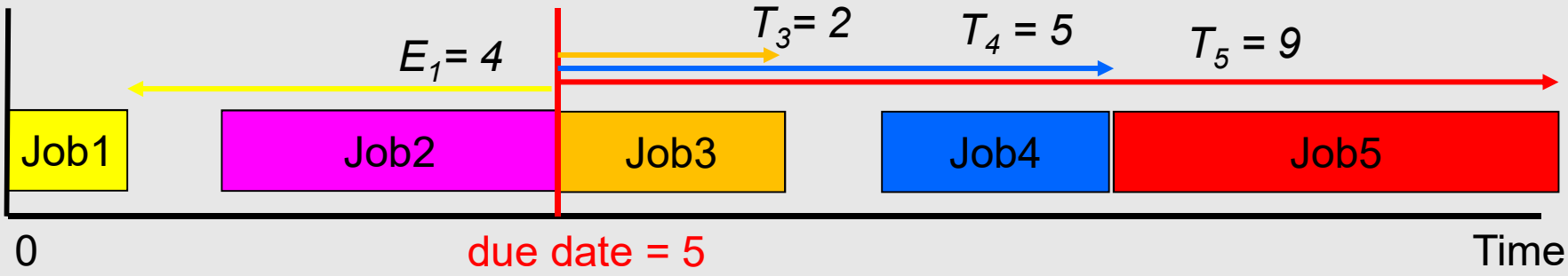
to solve scheduling problems
in consideration of problem characteristics!

Single Machine Scheduling Problem to minimize Earliness and Tardiness Penalties for a Common Due Date

Job Index	Job	Processing Time(p_j)	Earliness Penalty(α_j)	Tardiness Penalty(β_j)
1	1	1	1	3
2	2	3	1	5
3	3	2	2	1
4	4	2	1	4
5	5	4	3	3

Penalty Function

$f(S) = 4 * 1 + 2 * 1 + 5 * 4 + 9 * 3$



Background

- Most manufacturers are facing with the problem of earliness and tardiness in job completion, because earliness may raise inventory cost, while tardiness breaches the deadlines of customer contracts.
- How to get a minimum of earliness penalty and tardiness penalty is the core of many scheduling systems focusing on the concept of Just-In-Time.

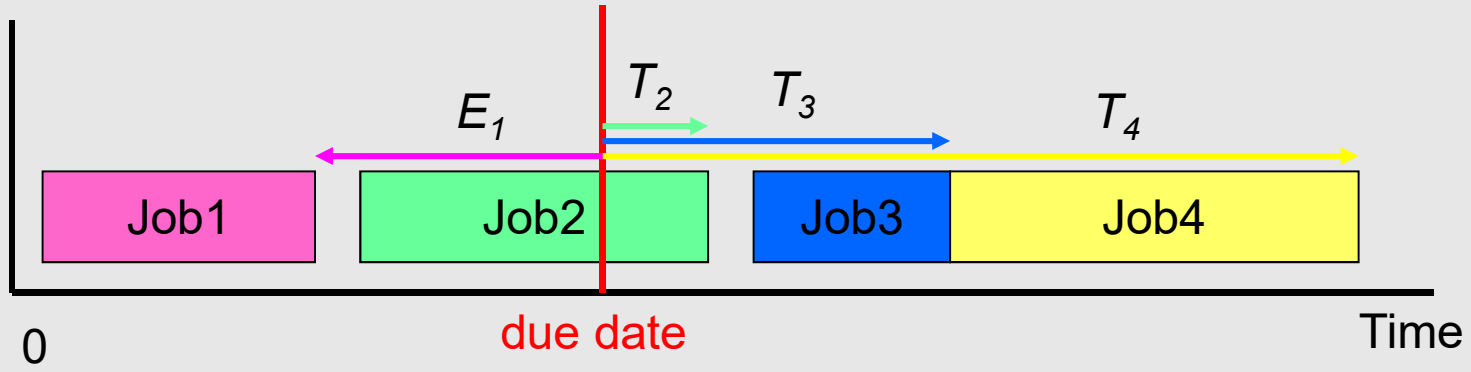
Single Machine Scheduling Problem to minimize Earliness and Tardiness Penalties for a Common Due Date

Penalty Function

$$f(S)=\sum_{j=1}^{n}\left(\alpha_j E_j+\beta_j T_j\right)$$

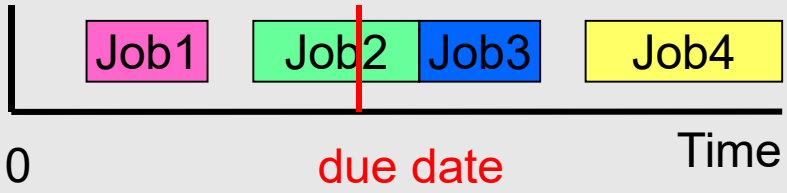
$$E_j=\max\left(0,d-C_j\right)$$

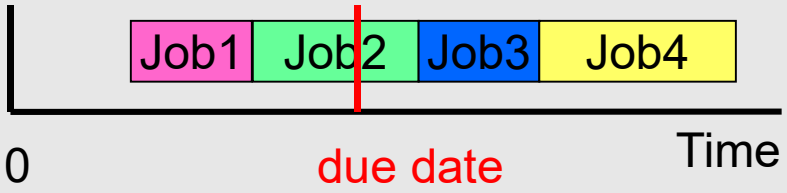
$$T_j=\max\left(0,C_j-d\right)$$



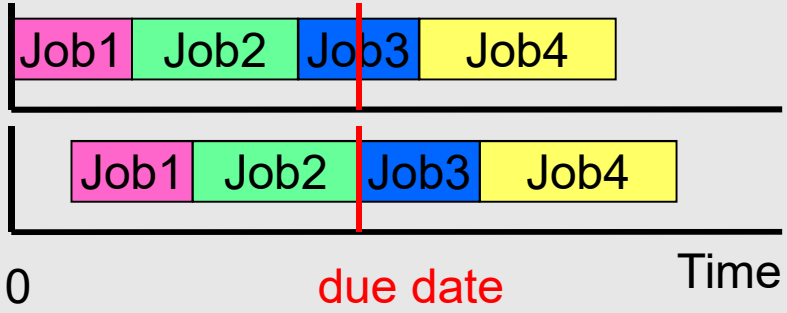
Characteristics (1)

1) No idle times are inserted between consecutive jobs.





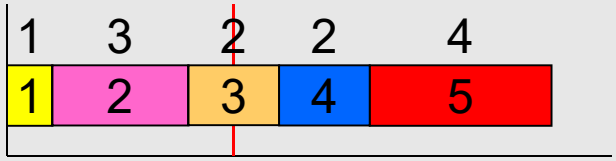
2) There is an Optimal schedule in which either the processing of the first job starts at time zero or job is completed at the due date.



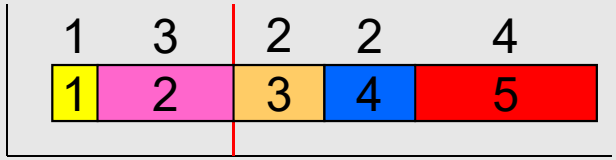
One Job is straddling the due date.

Let's try to find the best solution!

Job Index	Job	Processing Time(p_j)	Earliness Penalty(α_j)	Tardiness Penalty(β_j)
1	1	1	1	3
2	2	3	1	5
3	3	2	2	1
4	4	2	1	4
5	5	4	3	3



$F(S_1)$ = 1*4 + 1*1 + 1*1 + 4*3 + 3*7
= 39



$F(S_2)$ = 1*3 + 1*0 + 1*2 + 4*4 + 3*8
= 45

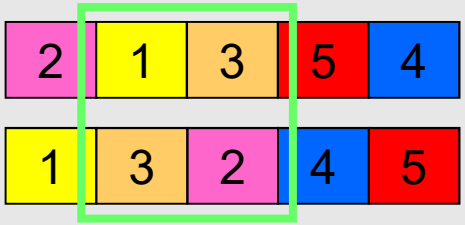
If You want to solve with GA, …(1)

1) Design Genetic Representation

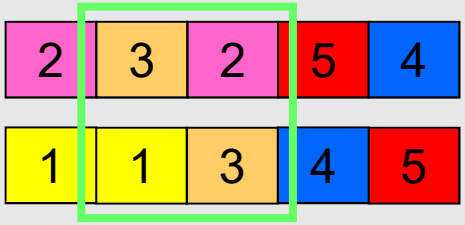
A Chromosome has genes for number of jobs.
A Gene is expressed by job index.

2) Design Crossover Operation

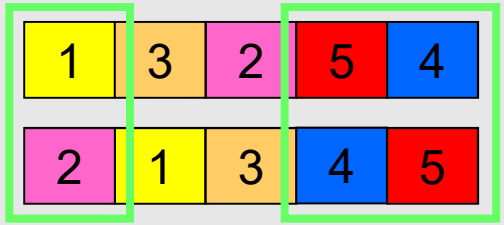
Step1: select items at random



Step2: exchange items



Step3: regulate the outside of selected items



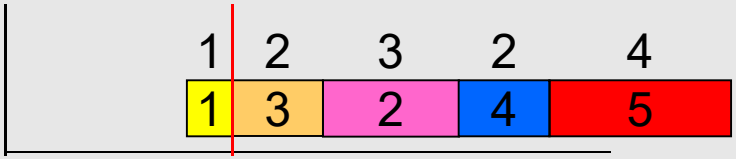
If You want to solve with GA, …(1)

3) Design Mutation Operation

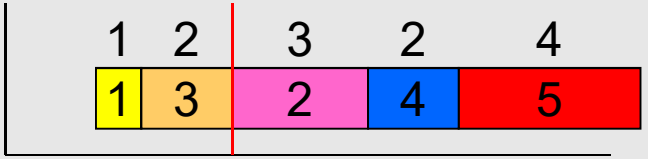
Step1: select 2 jobs and exchange

4) Define Evaluation Method

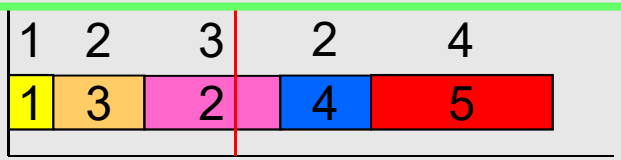
Step1: evaluate all cases
Step2: select the best one
and set penalty value



$F(S_1) = 1*0 + 1*2 + 5*5 + 4*7 + 3*11 = 89$



$F(S_2) = 1*2 + 2*0 + 5*3 + 4*5 + 3*9 = 64$



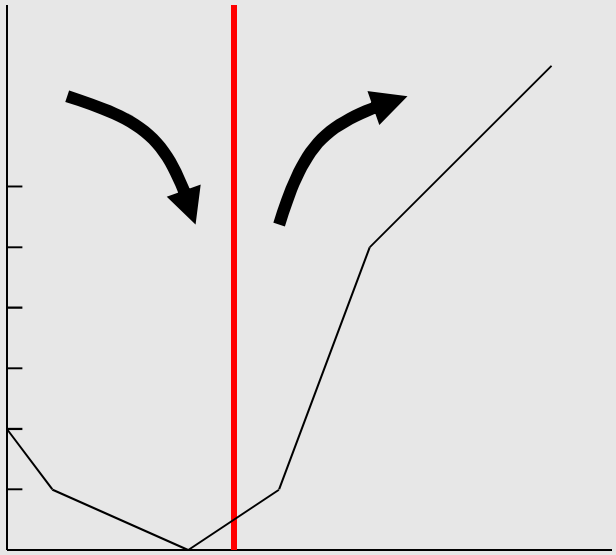
$F(S_3) = 1*4 + 2*2 + 5*1 + 4*3 + 3*7 = 46$

Characteristics (2)

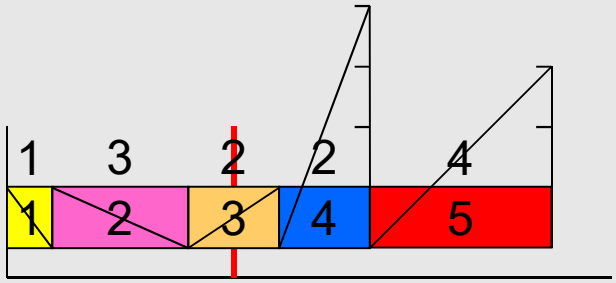
- An optimal schedule has the V-shape property. This means that the jobs belonging to the earliness group should be ordered according to increasing ratios α_j/p_j, and the jobs belonging to the tardiness group should be ordered according to decreasing ratios β_j/p_j.

An Example of V-Shape

Job Index	Job	Processing Time(p_j)	Earliness Penalty(α_j)	α_j/p_j	Tardiness Penalty(β_j)	β_j/p_j
1	1	1	1	1.0	3	3.0
2	2	3	1	0.33	5	1.67
3	3	2	2	1.0	1	0.5
4	4	2	1	0.5	4	2.0
5	5	4	3	0.75	3	0.75



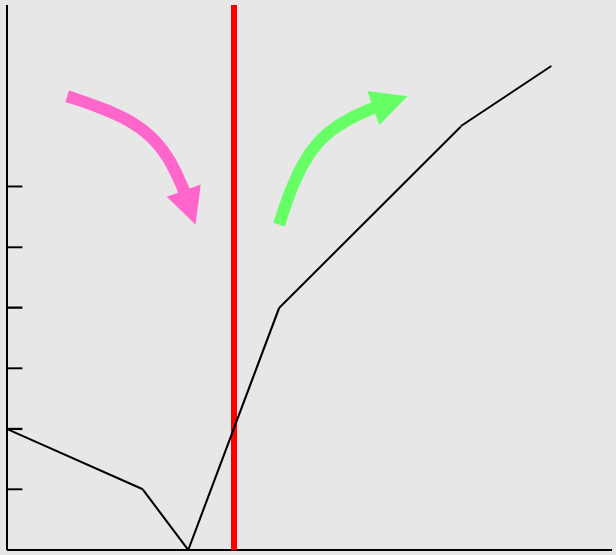
- Before V-shape:
- increasing sort α_j/p_j ×
- decreasing sort β_j/p_j ×



F(S) = 1*4 + 1*1 + 1*1 + 4*3 + 3*7
= 39

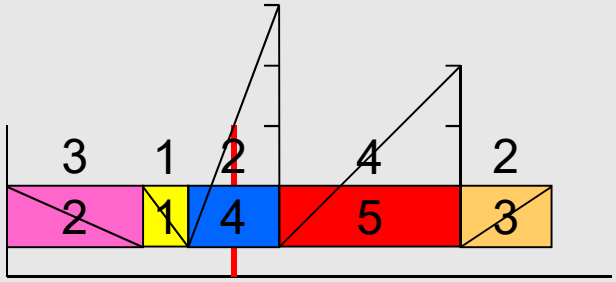
An Example of V-Shape

Job Index	Job	Processing Time(p_j)	Earliness Penalty(α_j)	α_j/p_j	Tardiness Penalty(β_j)	β_j/p_j
1	1	1	1	1.0	3	3.0
2	2	3	1	0.33	5	1.67
3	3	2	2	1.0	1	0.5
4	4	2	1	0.5	4	2.0
5	5	4	3	0.75	3	0.75



After V-shape:

- increasing sort α_j/p_j
- decreasing sort β_j/p_j



F(S) = 1*2 + 1*1 + 4*1 + 3*5 + 1*7
= 29

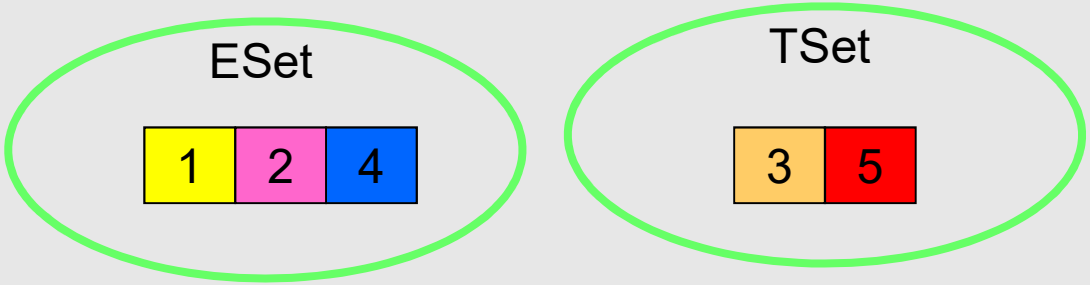
If You want to solve with GA, …(2)

1) Design Genetic Representation

Job Index	1	2	3	4	5
	0	0	1	0	1

A Chromosome has genes for number of jobs.
A Gene is expressed by 0 or 1 for each job index.

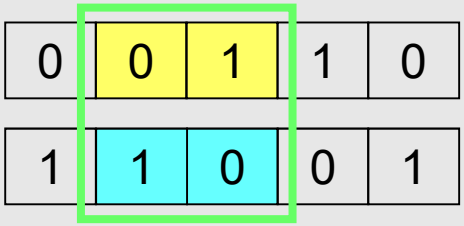
If the value of gene for job index is 0,
the job is in the set of finished before due date
(Earliness Job Set: ESet).
If the value of gene for job index is 1,
the job is in the set of finished at or after due date
(Tardiness Job Set: TSet).



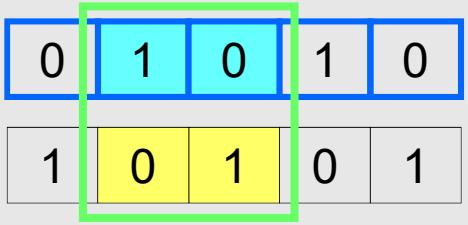
If You want to solve with GA, …(2)

2) Design Crossover Operation

Step1: select items at random

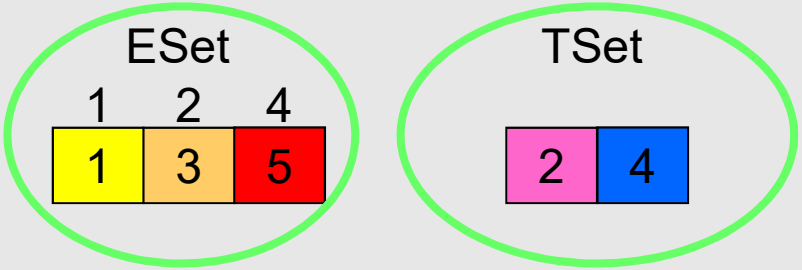


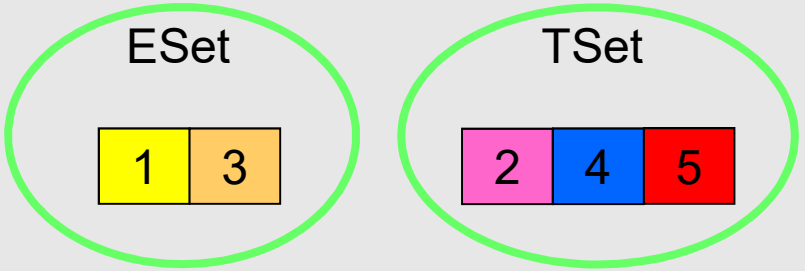
Step2: exchange items



Step3: regulate ESet

If the total processing time of jobs in ESet is longer than term to due date, select one item randomly and move from ESet to TSet until the total time is less than it.

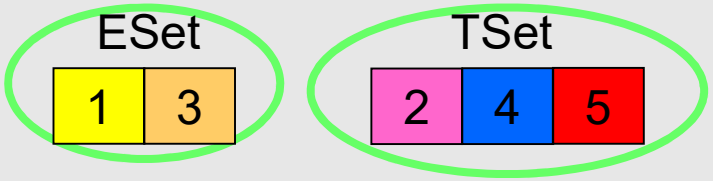


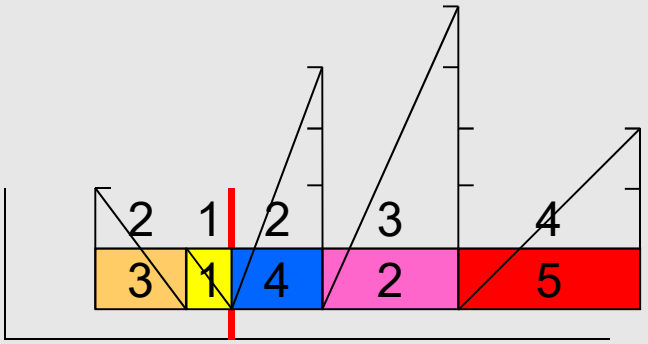


If You want to solve with GA, …(2)

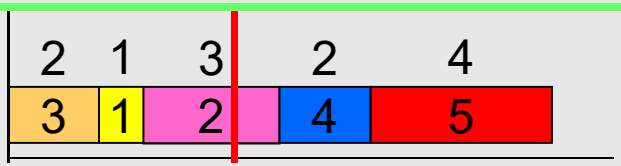
3) Design Mutation Operation
   - Step1: select items at random
   - Step2: change the value
   - Step3: regulate ESet

4) Define Evaluation Method

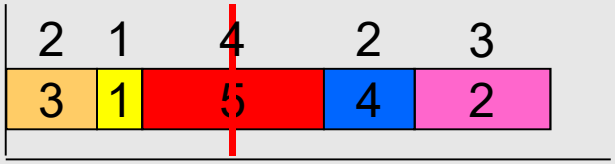




$F(S_1)$ = 1*2 + 1*0 + 4*2 + 5*5 + 3*9 = 62



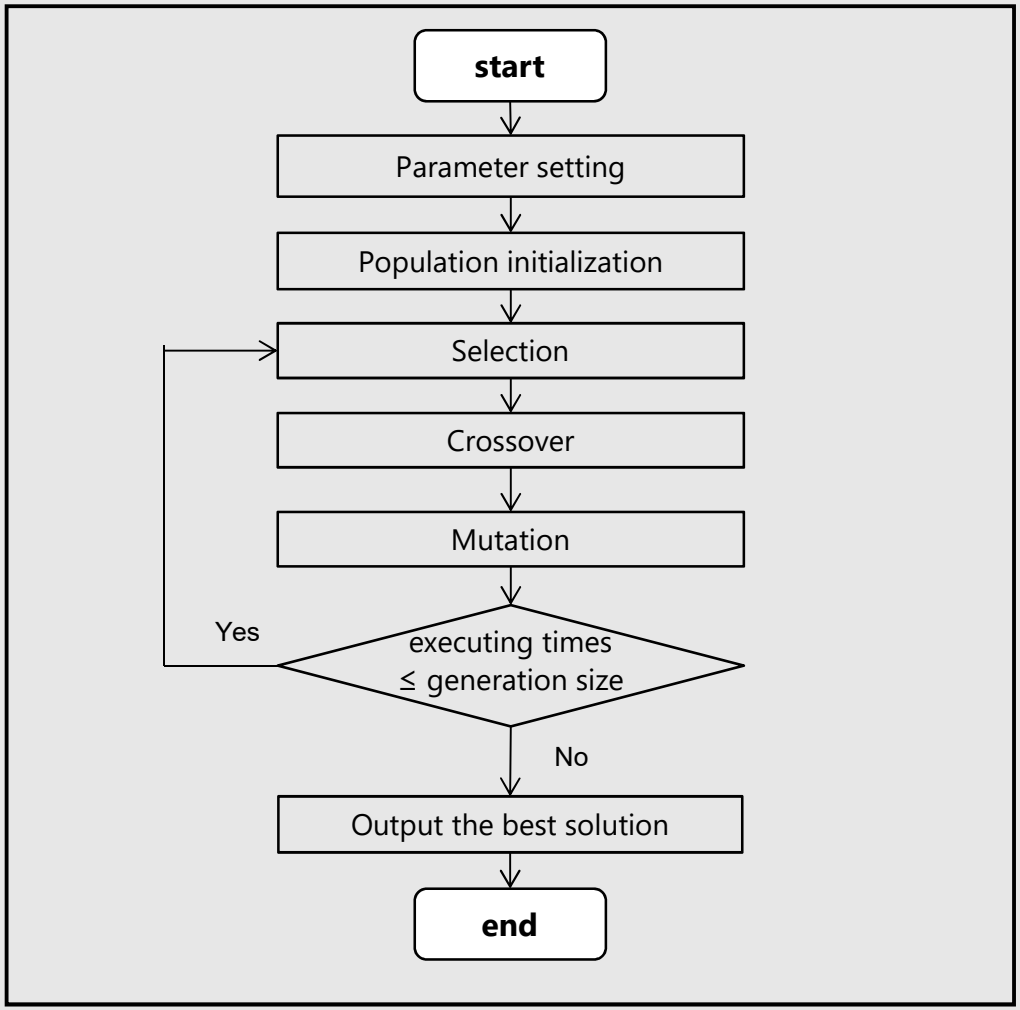
$F(S_2)$ = 1*3 + 1*2 + 5*1 + 4*3 + 3*7 = 43



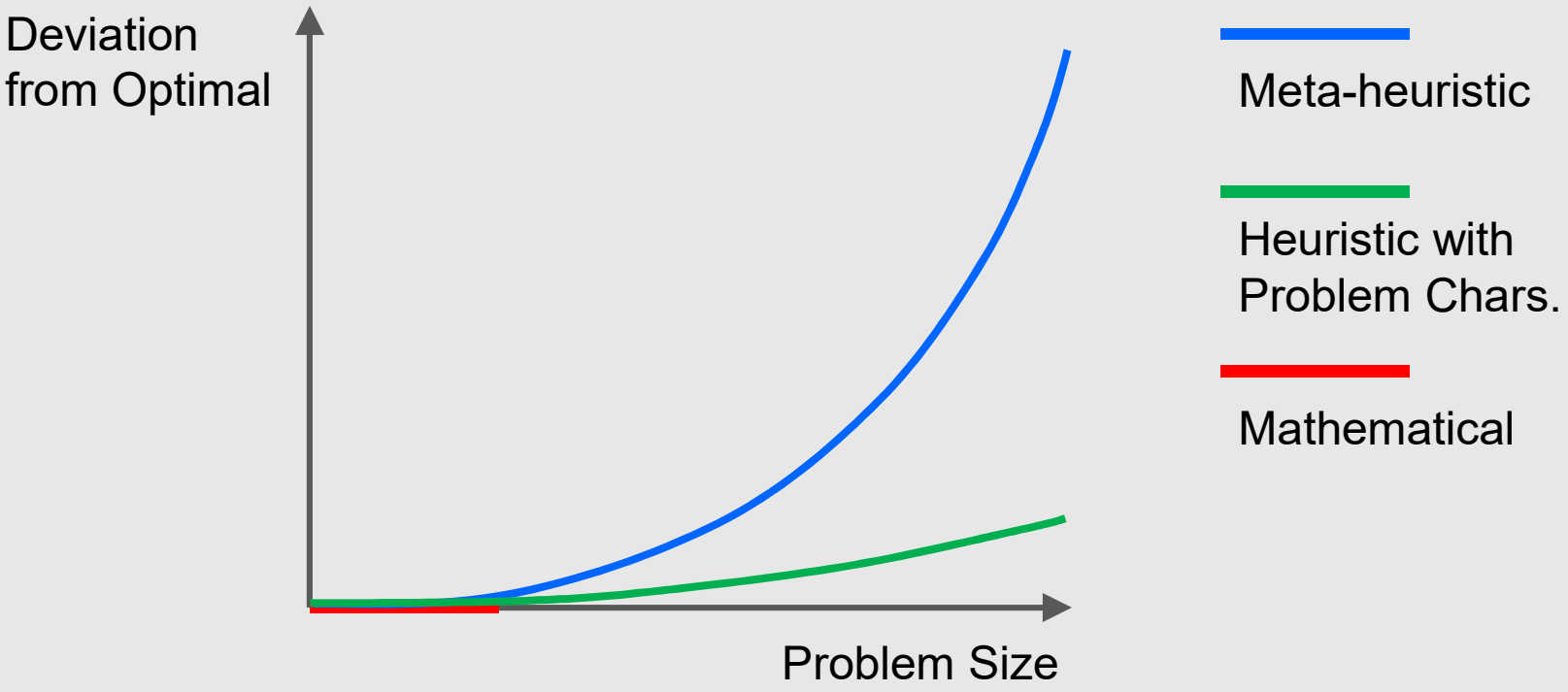
$F(S_3)$ = 1*3 + 1*2 + 3*2 + 4*4 + 5*7 = 62

Step1: sequence jobs for each set
Step2: evaluate a case that the last job of ESet completes at due date
Step3: evaluate a case that the first job of ESet starts at 0
If a job in TSet completes at or before due date, change the sequence of jobs in TSet so that all jobs in TSet complete after due date and evaluate.
Step4: select the best one and set penalty value

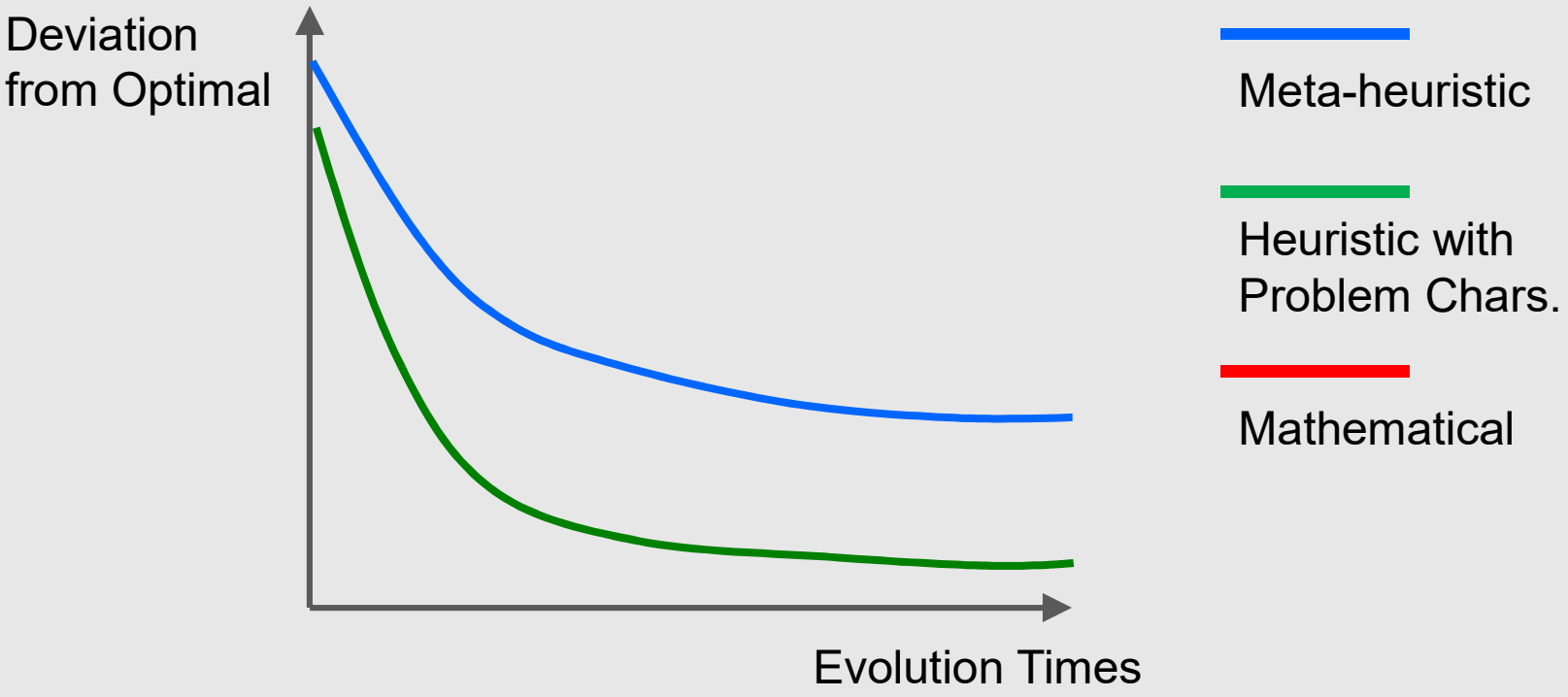
Overall Procedure of GA



Comparison


Deviation
from Optimal
Problem Size
Meta-heuristic
Heuristic with
Problem Chars.
Mathematical

Comparison


Deviation
from Optimal
Evolution Times
Meta-heuristic
Heuristic with
Problem Chars.
Mathematical

Summary

To solve production scheduling problems,
FIND THE BEST WAY !

Don’t apply GA as Simple Meta Heuristic Approach !

Gain your insight to see the essential characteristics of problems !

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Thank you