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Standard Form:

total profit: $\max z = 300000x_1 + 100000x_2 \Rightarrow$ maximize $z = 300000x_1 + 100000x_2 - Ma_1$

total quantity: $x_1 + x_2 \geq 10$

upper limit: $100x_1 + 200x_2 \leq 1600$

$400x_1 + 200x_2 \leq 3200$

non-negative: $x_1, x_2 \geq 0$

Subject to
$$\begin{cases} x_1 + x_2 - s_1 + a_1 = 10 \\ 100x_1 + 200x_2 + s_2 = 1600 \\ 400x_1 + 200x_2 + s_3 = 3200 \\ x_1, x_2, s_1, s_2, s_3, a_1 \geq 0 \end{cases}$$

Simplex Table:

Cj	Basic Variable	300000 x_1 ②	100000 x_2	0 s_1	0 s_2	0 s_3	-M a_1	bi	bi/aij
-M	a_1	1	1	-1	0	0	1	10	10
0	s_2	100	200	0	1	0	0	1600	16
0	s_3	400	200	0	0	1	0	3200	8 ②
	zj	-M	-M	M			-M		
	Cj-zj	300000+M ①	100000+M	-M			0		

①: $300000+M > 100000+M \Rightarrow$ select x_1 column

② $8 < 10 < 16 \Rightarrow$ select s_3 row ③ x_1 joins the basic variable

Cj	Basic Variable	300000 x_1	100000 x_2 ③	0 s_1	0 s_2	0 s_3	-M a_1	bi	bi/aij
-M	a_1	0	0.5	-1	0	-1/400	1	2	4 ②
0	s_2	0	150	0	1	-1/4	0	800	16/3
300000	x_1	1	0.5	0	0	1/400	0	8	16
	zj		150000-0.5M	M		M/400 + 750			
	Cj-zj		0.5M-50000 ①	-M		-M/400 - 750			

select a_1 row

①: select x_2 column ③ x_2 joins the basic variable

Cj	Basic Variable	300000 x_1	100000 x_2	0 s_1	0 s_2	0 s_3	-M a_1	bi	bi/aij
100000	x_2	0	1	-2	0	-1/200	2	4	
0	s_2	0	0	300	1	1/2	-300	200	
300000	x_1	1	0	1	0	1/200	-1	6	
	zj			100000		1000	-100000	300000×6 + 100000×4 = 2200000	
	Cj-zj			-100000		-1000	100000-M		

$-100000 \leq 0$, $-1000 \leq 0$, $100000 - M \leq 0$

$\Rightarrow$ we can get the optimal solution

$(z, x_1, x_2, s_1, s_2, s_3, a_1) = (2200000, 6, 4, 0, 200, 0, 0)$