Exercise 3: Convex function optimization

$$f(\boldsymbol{x}) = 2x_1^2 + x_1x_2 + x_2^2 - 5x_1 - 3x_2 + 4$$ $f(\boldsymbol{x})$ is convex

1. f の勾配 ∇f を求めよ
2. $(0,0),(1,2),(1,0.5),(1,1)$ における f の勾配を求めよ
3. f を最小にする $\boldsymbol{x}$ とその時の $f(\boldsymbol{x})$ を求めよ

1. Find the gradient ∇f of f
2. Find the gradient of f at (0, 0), (1, 2), (1, 0.5), (1, 1)
3. Find $\boldsymbol{x}$ that minimizes f and $f(\boldsymbol{x})$ at that time

1. $\frac{\partial f}{\partial x_1} = 4x_1 + x_2 - 5$, $\frac{\partial f}{\partial x_2} = x_1 + 2x_2 - 3$

$\nabla f = \begin{bmatrix} 4x_1 + x_2 - 5 \\ x_1 + 2x_2 - 3 \end{bmatrix}$

2. $\nabla f_{(0,0)} = \begin{bmatrix} -5 \\ -3 \end{bmatrix}$, $\nabla f_{(1,2)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$,

$\nabla f_{(1,0.5)} = \begin{bmatrix} -0.5 \\ -1 \end{bmatrix}$, $\nabla f_{(1,1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

3. let $\nabla f = 0$

$\Rightarrow \begin{cases} 4x_1 + x_2 - 5 = 0 \\ x_1 + 2x_2 - 3 = 0 \end{cases} \Rightarrow (x_1, x_2) = (1, 1)$

$f(1,1) = 2 + 1 + 1 - 5 - 3 + 4 = 0$

thus. when $x_1 = 1$, $x_2 = 1$, f minimize at that time, minimum is 0.