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Q1 Convolutional Neural Network Problem

(a)

- **Conv Layer 1** (Kernel 4x4, 8 filters):
  - The input has 3 channels.
  - **Weights**: $(4 \times 4 \times 3) \times 8 = 384$
  - **Biases**: 1 per filter = 8
- **Conv Layer 2** (Kernel 8x8, 16 filters):
  - The input has 8 channels from the previous layer.
  - **Weights**: $(8 \times 8 \times 8) \times 16 = 8192$
  - **Biases**: 1 per filter = 16
- **Max Pooling Layer**: This layer has no trainable weights or biases.
- **Fully Connected Layer 1** (64 neurons):
  - **Weights**: input size × number of neurons = 1,024 × 64 = 65536
  - **Biases**: 1 per neuron = 64
- **Fully Connected Layer 2** (8 neurons):
  - The input size is 64 from the previous layer.
  - **Weights**: 64 × 8 = 512
  - **Biases**: 1 per neuron = 8
- **Softmax Layer**: This layer has no trainable weights or biases.

Total Parameters:

- **Total Weights**: 384 + 8,192 + 65,536 + 512 = 74624
- **Total Biases**: 8 + 16 + 64 + 8 = 96

Layer	Weights	Biases
Conv 1	384	8
Conv2	8192	16
Max Pooling	0	0
FC1	65536	64
FC2	512	8
Total	74624	96

(b)

Layer	Output Shape
Input	84 × 84 × 3
Conv1	84 × 84 × 8
Conv2	26 × 26 × 16
MaxPooling	8 × 8 × 16
FC1	64
FC2	8
Softmax	8

(c)

To compute the receptive field:

Formula: RF_{out} = RF_{in} + $(Kernel_{size} - 1)$ × J_{in}, where J is the cumulative stride.

1. Conv1: Kernel: 4, stride: 1

> RF = 1+(4−1)×1=4
>
> Cumulative Stride (J_{out}) = 1×1=1

2. Conv2: Kernel: 8, stride: 3

> RF = 4+(8−1)×1=11
>
> Cumulative Stride (J_{out}) = 1×3=3

3. MaxPooling: Kernel: 3, stride: 3

> RF = 11+(3−1)×3=11+6=17

The receptive field size of the neurons after the max pooling layer is 17x17

Q2 1-D Convolution Problem

Input:[1, 2, 6, 7, 8, 9, 1]

- Kernel size = 3, stride = 1, padding = valid
- Output length = 7 - 3 + 1 = 5

Layer 1 Output: The first convolutional layer has 4 filters with weights [0,1,0], [0,0,1], [1,0,0], and [1,0,1]

- **Output of Filter 1 ([0,1,0])**: [2, 6, 7, 8, 9]
- **Output of Filter 2 ([0,0,1])**: [6, 7, 8, 9, 1]

- **Output of Filter 3 ([1,0,0])**: [1, 2, 6, 7, 8]
- **Output of Filter 4 ([1,0,1])**: [7, 9, 14, 16, 9]

Output after Conv1: shape = (5×4) =

$$\begin{bmatrix} 2 & 6 & 1 & 7 \\ 6 & 7 & 2 & 9 \\ 7 & 8 & 6 & 14 \\ 8 & 9 & 7 & 16 \\ 9 & 1 & 8 & 9 \end{bmatrix}$$

Layer 2 Output: The second layer has 1 filter with a kernel size of 3 and all weights equal to 1. Its depth is 4 to match the previous layer's output channels. It takes the four feature maps from layer 1 as input.

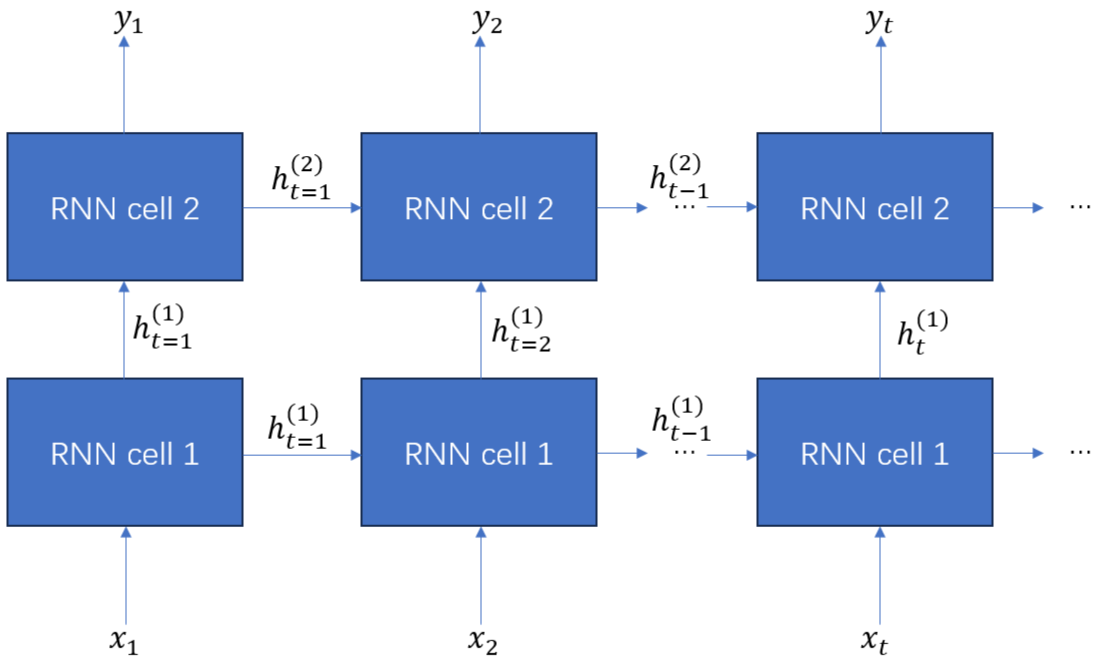
- Input: 5×4 (5 time steps, 4 channels)
- Kernel size = 3, stride = 1, valid padding
- Filter weights: all 1s → 3×4 = 12 weights

So at each step, take 3 rows of shape (3×4) and flatten → dot with all-ones = sum of all 12 values.

- Step 1: sum of first 3 rows = `[2+6+1+7 + 6+7+2+9 + 7+8+6+14] = 75`
- Step 2: sum of rows 2-4 = `[6+7+2+9 + 7+8+6+14 + 8+9+7+16] = 99`
- Step 3: sum of rows 3-5 = `[7+8+6+14 + 8+9+7+16 + 9+1+8+9] = 102`
- **Output of the filter in Layer 2**: [75, 99, 102]

Q3 RNN Problem

Each row represents a different layer of RNN cells. The input at each time step x_t feeds into the first RNN cell of that column. The output of RNN Cell i at time t becomes the input to RNN Cell $i+1$ at the same time t. Crucially, the hidden state $h_i, t-1$ from RNN Cell i at time $t-1$ is passed as input to RNN Cell i at time t. The final output y_t is typically generated from the last RNN cell at time t.



Programming Part

In this part, I choose CNN to solve the problem.

To improve the performance of my CNN on the MNIST dataset, I tried the following techniques:

1. **Added Dropout layers** to reduce overfitting (e.g., after each fully connected layer, with dropout rate 0.5).
2. **Added Batch Normalization** after each convolutional layer to stabilize and accelerate training.

Baseline CNN vs. Improved CNN

```
=== Baseline CNN ===
Baseline Epoch 1: Train Acc=0.9805, Val Acc=0.9800
Baseline Epoch 2: Train Acc=0.9891, Val Acc=0.9859
Baseline Epoch 3: Train Acc=0.9932, Val Acc=0.9881
Baseline Epoch 4: Train Acc=0.9930, Val Acc=0.9873
Baseline Epoch 5: Train Acc=0.9945, Val Acc=0.9880
Baseline Epoch 6: Train Acc=0.9947, Val Acc=0.9873
Baseline Epoch 7: Train Acc=0.9968, Val Acc=0.9898
Baseline Epoch 8: Train Acc=0.9975, Val Acc=0.9895
Baseline Epoch 9: Train Acc=0.9980, Val Acc=0.9890
Baseline Epoch 10: Train Acc=0.9972, Val Acc=0.9887

=== Improved CNN (BatchNorm + Dropout) ===
Improved Epoch 1: Train Acc=0.9844, Val Acc=0.9832
Improved Epoch 2: Train Acc=0.9848, Val Acc=0.9822
Improved Epoch 3: Train Acc=0.9916, Val Acc=0.9887
Improved Epoch 4: Train Acc=0.9907, Val Acc=0.9877
Improved Epoch 5: Train Acc=0.9925, Val Acc=0.9886
Improved Epoch 6: Train Acc=0.9942, Val Acc=0.9901
Improved Epoch 7: Train Acc=0.9959, Val Acc=0.9905
Improved Epoch 8: Train Acc=0.9960, Val Acc=0.9908
Improved Epoch 9: Train Acc=0.9968, Val Acc=0.9910
Improved Epoch 10: Train Acc=0.9972, Val Acc=0.9917

Baseline CNN Test Accuracy: 0.9896
Improved CNN Test Accuracy: 0.9929
```