Standard Form:

total profit : $\max z = 300000 X_1 + 100000 X_2 \Rightarrow$ maximize $z = 300000 X_1 + 100000 X_2 - Ma_1$

total quantity: $X_1 + X_2 \geq 10$

upper limit : $100 X_1 + 200 X_2 \leq 1600$

$400 X_1 + 200 X_2 \leq 3200$

non-negative : $X_1, X_2 \geq 0$

Subject to
$$\begin{cases} X_1 + X_2 - S_1 + a_1 = 10 \\ 100 X_1 + 200 X_2 + S_2 = 1600 \\ 400 X_1 + 200 X_2 + S_3 = 3200 \\ X_1, X_2, S_1, S_2, S_3, a_1 \geq 0 \end{cases}$$

Simplex Table:

C_j	Basic Variable	300000 X_1	100000 X_2	0 S_1	0 S_2	0 S_3	-M a_1	$\bar{b}$	$\bar{b}_i/a_{ij}$
-M	a_1	1	1	-1	0	0	1	10	10
0	S_2	100	200	0	1	0	0	1600	16
0	S_3	400	200	0	0	1	0	3200	8
	Z_j	-M	-M	M			-M		
	$C_j - Z_j$	300000+M	100000+M	-M			0		

① $300000 + M > 100000 + M \Rightarrow$ select X_1 column

② $8 < 10 < 16 \Rightarrow$ select S_3 row   ③ X_1 joins the basic variable

Iteration 1: select a_1, s_2, s_3 as basic variables , set 1 to pivot , set 0 to all cells of the other rows of pivot column

Solution: $(z, x_1, x_2, s_1, s_2, s_3) = (0, 0, 0, 10, 1600, 3200)$

$c_j - z_j \geq 0$, the coefficients of x_1, x_2 are positive => not the optimal solution => repeat above steps

C_j	Basic Variable	300000 X_1	100000 X_2	0 S_1	0 S_2	0 S_3	-M a_1	$\bar{b}$	$\bar{b}_i/a_{ij}$
-M	a_1	0	0.5	-1	0	-1/400	1	2	4
0	S_2	0	150	0	1	-1/4	0	800	$\frac{16}{3}$
300000	X_1	1	0.5	0	0	1/400	0	8	16
	Z_j		150000-0.5M	M		$\frac{M}{400}+750$	-M		
	$C_j - Z_j$		0.5M-50000	-M		$-\frac{M}{400}-750$			

① Select X_2 column   ② Select a_1 row   ③ X_2 joins the basic variable

Iteration 2: replace s_3 with x_1 as basic variables, set 1 to pivot , set 0 to all cells of the other rows of pivot column

$c_j - z_j = 0.5M - 50000 \geq 0$ => not the optimal solution => repeat above steps

C_j	Basic Variable	300000 X_1	100000 X_2	0 S_1	0 S_2	0 S_3	-M a_1	$\bar{b}$	$\bar{b}_i/a_{ij}$
100000	X_2	0	1	-2	0	-1/200	2	4	
0	S_2	0	0	300	1	1/2	-300	200	
300000	X_1	1	0	1	0	1/200	-1	6	
	Z_j			100000		1000	-100000	300000×6 + 100000×4 = 2200000	
	$C_j - Z_j$			-100000		-1000	100000-M		

Iteration 3: replace a_1 with x_2 as basic variables, set 1 to pivot , set 0 to all cells of the other rows of pivot column

all $c_j - z_j \leq 0$ => find the optimal solution $(a_1 = 0)$

The optimal solution : $(z, x_1, x_2, s_1, s_2, s_3, a_1) = (2200000, 6, 4, 0, 200, 0, 0)$

The optimal value $z = 2200000$

Thus, to get the maximum profit, we should produce 6 tons of ProdA and 4 tons of ProdB. The total profit is 2200000 ¥.

Need quantity of material2 : $400x_1 + 200x_2 = 3200kg$ (reach the upper limit)

The shadow price of material2 is 1000 ¥/kg, which means 1kg material2 is bought, 1000yen optimal value will be increased.

But the purchase price of material2 is 2000 ¥/kg, which means we will loss 1000 ¥ profit for every 1kg material2 purchased.

Thus we should not consider purchase additional material2.