SMART
INDUSTRY
LABORATORY

Scheduling Algorithms (4)

- Production Planning Algorithm (1) -

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What is Production Planning?

Production Planning :

Evaluate the values of productivity, total profits and/or the other measurements, subject to the constraints of material quantities, facility abilities, man power, budget and/or the other factors.

Production Planning is to make a total balanced plan for given time period

Example of Production Planning

- Make two products called ProA and ProB
- Resources to make products, quantities of those for each product and the upper limits for those are given in the following table.

	ProA	ProB	Upper Limit
Material（kg）	4	11	440
Man Power（man-hour）	5	7	350
processing time（hour）	7	6	420

Profit for unit quantity of ProA is 40k¥.

Profit for unit quantity of ProB is 90k¥.

Decide the optimal quantities to get the largest total profit.

Solving with figure (2 dimensions)

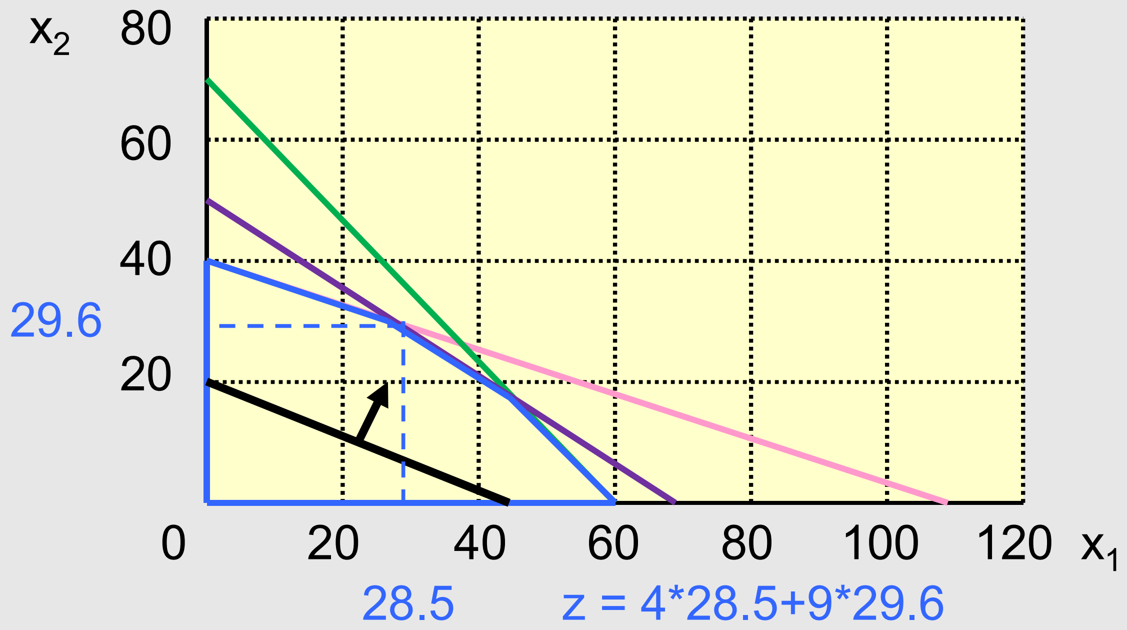
Objective total benefit : max $z = 4x_1 + 9x_2$

subject to material constraint : $4x_1 + 11x_2 <= 440$ (a)

man power constraint : $5x_1 + 7x_2 <= 350$ (b)

facility constraint : $7x_1 + 6x_2 <= 420$ (c)

non-negative constraint : $x_1, x_2 >= 0$



In the case of more than 3 variables ??

Terms for Linear Programming

Mathematical Programming:

Method to give the optimal solution that maximize or minimize the objective function, subject to constraints

Linear Programming:

One of Mathematical Programming

Objective function and constrains are given as linear real functions.

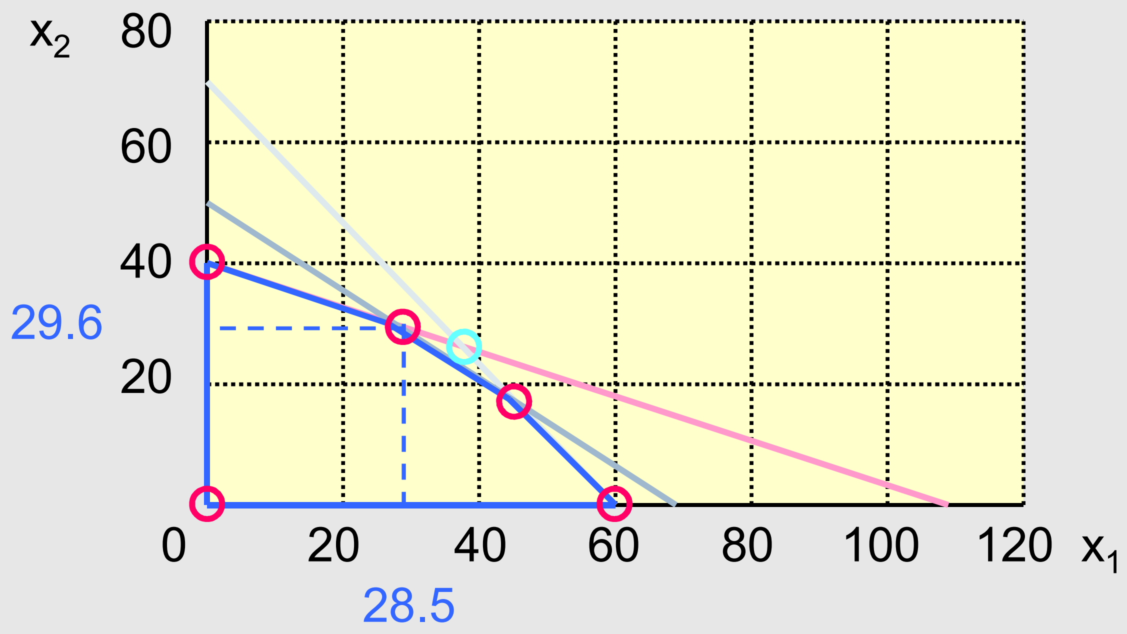
（In the graph, line for 2 dimensions
plain for 3 dimensions, ...)

$f(x_1, x_2, \ldots) = ax_1 + bx_2 + \ldots$ （a, b, ... is fixed））

LP: Linear Programming

- Feasible Solution:
  Solution that satisfies constraints
- Feasible Region:
  Variable region including feasible solutions
- Optimal Solution :
  the most optimized (maximized or minimized) solution of feasible solutions.
- Optimal Value:
  the value of objective function at the optimal solution

Where is the optimal solution in the feasible region?



extreme point is：
Intersection point of the boundaries of constraints
2 variables:
intersection point of 2 boundary lines
More than 3 variables:
intersection point of boundary surfaces
（Extreme points are located at the convex surface of boundaries）

Extreme points of feasible region
= part of all the intersection points of constraint equations

Search extreme points and confirm if it is optimal solution.

Optimal Solution in the feasible region

- One optimal solution ⇒ at the extreme point.
- Many optimal solutions

  ⇒ on the boundary surface

  Coefficients of constraint and objective function are same.
- No optimal solution ⇒ No feasible region
- No optimal solution ⇒ Feasible region is too wide.

  The value of the objective function is infinitely large.

Standard Form

$$\text{maximize } z = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } \sum_{j=1}^{n} a_{ij} x_j = b_i \quad (i = 1, 2, \ldots, m)$$

$$x_j >= 0 \qquad (j = 1, 2, \ldots, n)$$

a_{ij}, c_j : fixed values,

b_i : fixed values（non-negative）,

x_j : variables

m : num of equations, n : num of variables

Transformation to Standard form

Objective	total benefit	: max $z = 4x_1 + 9x_2$
subject to	material constraint	: $4x_1 + 11x_2 <= 440$
	man power constraint	: $5x_1 + 7x_2 <= 350$
	facility constraint	: $7x_1 + 6x_2 <= 420$
	non-negative constraint	: $x_1, x_2 >= 0$

$$\begin{array}{ll} \text{maximize} & z = 4x_1 + 9x_2 \\ \text{subject to} & 4x_1 + 11x_2 + s_1 = 440 \\ & 5x_1 + 7x_2 + s_2 = 350 \\ & 7x_1 + 6x_2 + s_3 = 420 \\ & x_1, x_2, s_1, s_2, s_3 >= 0 \end{array}$$

Transformation to Standard form

- Constraint equation is expressed by inequality.
  - =< : use slack variable ⇒ use equal
  - => : use surplus variable ⇒ use equal
- Right side fixed value of constraint equation is negative.
  - Times -1 for both sides
- Free variable that is not defined as non-negative exists.
  - Divide the variable to two non-negative variables (positive part and negative part).

Constraint equation is expressed by inequality.

Change to equal equation using not negative additional variable.

- (the left side) $=<$ b

  (the left side) + s = b

  s => 0 slack variable

  ex : $4x_1 - 7x_2 =< 12 \Rightarrow 4x_1 - 7x_2 + s = 12$

- (the left side) $=>$ b

  (the left side) - t = b

  t => 0 surplus variable

  ex : $4x_1 - 7x_2 => 12 \Rightarrow 4x_1 - 7x_2 - t = 12$

Free variable that is not defined as non-negative exists.

Free variable is replaced with the difference between two non-negative variables x^+ and x^-.

Free variable x ⇒ $x = x^+ - x^-$, $x^+ \geqq 0$, $x^- \geqq 0$

$4x_1 - 7x_2 =< 6$, $x_1 => 0$

$4x_1 - 7\ (x_2^+ - x_2^-) =< 6$,

$x_1 => 0$, $x_2^+ >= 0$, $x_2^- >= 0$

Solution of simultaneous equations

maximize $z = 4x_1 + 9x_2$

subject to

$$
\begin{aligned}
4x_1 + 11x_2 + s_1 &= 440 \\
5x_1 + 7x_2 + s_2 &= 350 \\
7x_1 + 6x_2 + s_3 &= 420
\end{aligned}
$$

$x_1, x_2, s_1, s_2, s_3 >= 0$

Select x_1, x_2 as independent variables,
substitute 0 for these variables.
Values of s_1, s_2, s_3 are decided.

Relationship between the number of variables and the number of equations

m: number of variables,

n: number of equations

- $m =< n$:
  Solution is decided uniquely or
  is not decided uniquely
  is not decided
- $m > n$:
  $m – n$ variables are not decided uniquely
  (these are called independent variables)
  ⇔ If the values of $m - n$ independent variables are given,
  the values of the remained variables are decided.

To find the extreme points

1 Transform constraints to Standard form

2 Select the independent variables and set 0 to decide the values of the other variables.

The solution of the simultaneous equations

= basic solution

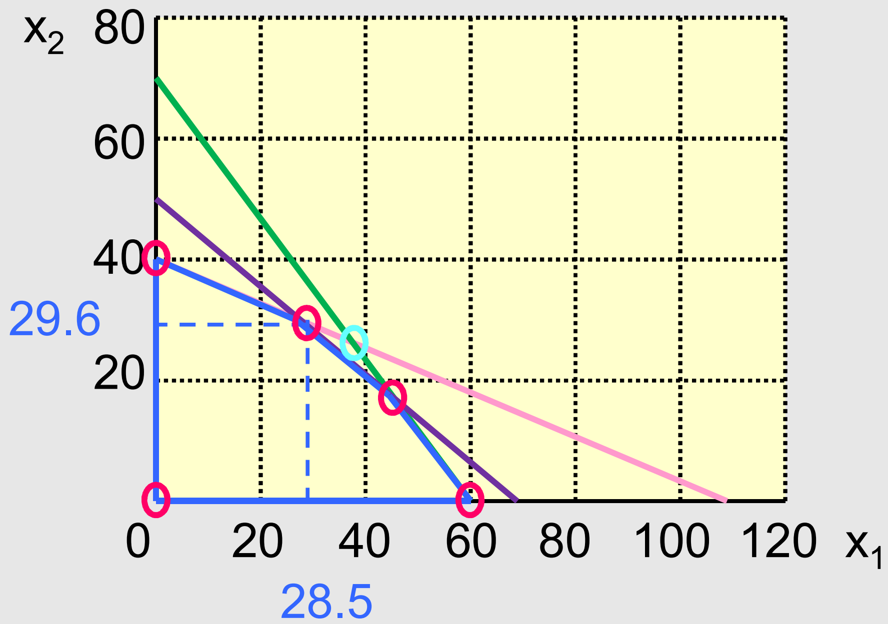
variables to find values = basic variable

⇔variables to be set 0

(independent variables) = non-basic variable

3 When all of values of the basic solution are non-negative, this is one of the extreme points.

Extreme Points in the all of the intersection point of constraint equations



x_1	x_2	s_1	s_2	s_3
0	0	440	350	420
0	40	0	70	180
0	50	-110	0	120
0	70	-330	-140	0
110	0	0	-200	-350
70	0	160	0	-70
60	0	200	50	0
28.5	29.6	0	0	42.9
37.4	26.4	0	-21.8	0
44.2	18.4	60.8	0	0

In the next lecture,

Simplex method

Basic Method to derive the optimal solution for Linear Programming problem,

finding only the extreme points increasing the value of the objective function.

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Thank you