Exercise 1:
Minimize the mean square error of single regression by the steepest descent method

$$E(w) = \frac{1}{N} \sum_{i=1}^{N} (t_i - (wx_i + b))^2$$

1. Find the derivative of the mean squared error $E(w)$ with parameters w and b.

2. Find the update equation for Step 2 of the Steepest Descent Method. Let the initial parameters be (w_0, b_0), the t–th update parameters be (w_t, b_t), and the step size parameter be η.

1.

$$
\begin{aligned}
\frac{\partial E}{\partial w} &= \frac{\partial}{\partial w} \frac{1}{N} \sum_{i=1}^{N} (t_i - (wx_i + b))^2 & (1) \\
&= \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial w} (t_i - (wx_i + b))^2 & (2) \\
&= \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial z_i} z_i^2 \frac{\partial}{\partial w} z_i \text{ where } z_i = t_i - (wx_i + b) & (3) \\
&= \frac{2}{N} \sum_{i=1}^{N} (t_i - (wx_i + b))(-x_i) & (4) \\
&= \frac{2}{N} \sum_{i=1}^{N} x_i((wx_i + b) - t_i) & (5)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E}{\partial b} &= \frac{\partial}{\partial b} \frac{1}{N} \sum_{i=1}^{N} (t_i - (wx_i + b))^2 & (6) \\
&= \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial b} (t_i - (wx_i + b))^2 & (7) \\
&= \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial z_i} z_i^2 \frac{\partial}{\partial b} z_i \text{ where } z_i = t_i - (wx_i + b) & (8) \\
&= \frac{2}{N} \sum_{i=1}^{N} ((wx_i + b) - t_i) & (9)
\end{aligned}
$$

2.

$$w^{t+1} \quad \leftarrow \quad w^t - \eta \frac{2}{N} \sum_{i=1}^{N} x_i((w^t x_i + b^t) - t_i) \tag{10}$$

$$b^{t+1} \quad \leftarrow \quad b^t - \eta \frac{2}{N} \sum_{i=1}^{N} ((w^t x_i + b^t) - t_i) \tag{11}$$