(a) 1

example: the state like the second line state. The left bottom one only has 1 successor, that ‘6’ moves down.

```
                    1    2    3
                    4    5    6
                    7         8
                    /          \
                   /            \
          1    2    3          1    2    3
          4    5    6          4    5    6
          7    8                    7    8
```

(b) 4

example: the blank is at the center of the puzzle. This state could have 4 successors: ‘2’ moves down, ‘4’ moves right, ‘5’ moves left and ‘7’ moves up.

```
1    2    3
4         5
6    7    8
```

(c) answer: the minimum number of moves needed to reach the goal state is 4.

g(n) = steps moved
h(n) = number of tiles in wrong position
f(n) = g(n) + h(n)

Prove:
Since you can only move one tile one step, in other words, only one tile can be fixed in one step, the h value only can be reduced by 1 with one move.

As the search tree below, when I found the goal, its f value is 4. The smallest f value besides 4 is 5. For example, f(n) = 1 + 4 = 5, that g(n) = 1, f(n) = 4, means there are 4 tiles in wrong position. Even in the best condition, it can make all 4 tiles in the right position. It still needs 4 steps. At that time g(n) = 1 + 4 = 5, f(n) = 5 + 0 = 5. So we can get that f(n) would not be reduced with the node expansion. Hence, no state can be found whose f value is less than 4, which means no solution can be found that is less than 4 steps.

Therefore, the minimum number of moves needed to reach the goal state is 4.

1	2	3
4		5
6	7	8

up

1		3
4	2	5
6	7	8

$f(n)=1+3=4$

left

1	2	3
	4	5
6	7	8

$f(n)=1+3=4$

right

1	2	3
4	5	
6	7	8

$f(n)=1+4=5$

down

1	2	3
4	7	5
6		8

$f(n)=1+4=5$

left

	1	3
4	2	5
6	7	8

$f(n)=2+2=4$

right

1	3	
4	2	5
6	7	8

$f(n)=2+4=6$

up

	2	3
1	4	5
6	7	8

$f(n)=2+3=5$

down

1	2	3
6	4	5
	7	8

$f(n)=2+4=6$

down

4	1	3
	2	5
6	7	8

$f(n)=3+1=4$

right

4	1	3
2		5
6	7	8

$f(n)=4+0=4$

down

4	1	3
6	2	5
	7	8

$f(n)=4+2=6$