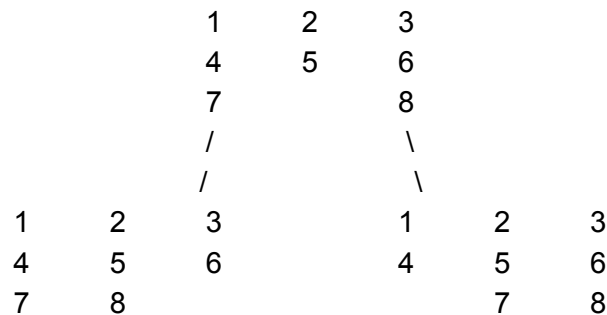


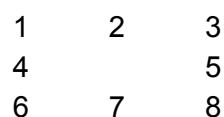
(a) 1

example: the state like the second line state. The left bottom one only has 1 successor, that '6' moves down.



(b) 4

example: the blank is at the center of the puzzle. This state could have 4 successors: '2' moves down, '4' moves right, '5' moves left and '7' moves up.



(c) answer: the minimum number of moves needed to reach the goal state is 4.

$g(n)$ = steps moved

$h(n)$ = number of tiles in wrong position

$f(n) = g(n) + h(n)$

Prove:

Since you can only move one tile one step, in other words, only one tile can be fixed in one step, the h value only can be reduced by 1 with one move.

As the search tree below, when I found the goal, its f value is 4. The smallest f value besides 4 is 5. For example, $f(n) = 1 + 4 = 5$, that $g(n) = 1$, $f(n) = 4$, means there are 4 tiles in wrong position. Even in the best condition, it can make all 4 tiles in the right position. It still needs 4 steps. At that time $g(n) = 1 + 4 = 5$, $f(n) = 5 + 0 = 5$. So we can get that $f(n)$ would not be reduced with the node expansion. Hence, no state can be found whose f value is less than 4, which means no solution can be found that is less than 4 steps.

Therefore, the minimum number of moves needed to reach the goal state is 4.

	1	2	3
	4		5
	6	7	8

$$f(n) = 1 + 3 = 4 \quad f(n) = 1 + 3 = 4 \quad f(n) = 1 + 4 = 5 \quad f(n) = 1 + 4 = 5$$

left			right			up			down				
	1	3		1	3			2	3		1	2	3
4	2	5	4	2	5		1	4	5		6	4	5
6	7	8	6	7	8		6	7	8			7	8

$$f(n) = 2 + 2 = 4 \quad f(n) = 2 + 4 = 6 \quad f(n) = 2 + 3 = 5 \quad f(n) = 2 + 4 = 6$$

		down	
4	1	3	
	2	5	
6	7	8	

$$f(n) = 3 + 1 = 4$$

right			down		
4	1	3	4	1	3
2		5	6	2	5
6	7	8		7	8

$$f(n) = 4 + 0 = 4 \quad f(n) = 4 + 2 = 6$$