Ηλεκτρομαγνητικά Πεδία Α

1η εργασία

el20116

Κουρής Γεώργιος

Μάρτιος-Απρίλιος 2022

Κουρής Γεώργιος el20116

Άσκηση 6:

- $\vec{J} = \hat{i}_{r_T} J_T(r_T,\varphi) + \hat{i}_\varphi J_0 (a/r_T) \sin^2\varphi$, $b < r_T < a$, $0 < \varphi < \pi/2$

- $\vec{K_1} = \hat{i}_{r_T} K_1(r_T)$, $b < r_T < a$, $\varphi = 0$
  $\vec{K_4} = \hat{i}_{r_T} K_4(r_T)$, $b < r_T < a$, $\varphi = \frac{\pi}{2}$

- $\vec{K_2} = \hat{i}_\varphi K_2(\varphi)$, $r_T = a$, $0 < \varphi < \frac{\pi}{2}$
  $\vec{K_3} = \hat{i}_\varphi K_0 \sin^2\varphi$, $r_T = b$, $0 < \varphi < \frac{\pi}{2}$

(α) ΝΔΦ σε διαφορική του μορφή:

$$\vec{\nabla}\cdot\vec{J} = 0 \Rightarrow \frac{1}{r_T}\frac{\partial}{\partial r_T}(r_T J_T) + \frac{1}{r_T}\frac{\partial J_\varphi}{\partial \varphi} = 0 \Rightarrow \frac{\partial (r_T J_T)}{\partial r_T} = -J_0 \frac{a}{r_T} 2\cos\varphi\sin\varphi \Rightarrow$$

$$r_T J_T = -J_0 a 2\cos\varphi\sin\varphi \ln(r_T) + f(\varphi) \Rightarrow J_T = -J_0 \frac{a}{r_T} 2\cos\varphi\sin\varphi \ln(r_T) + \frac{f(\varphi)}{r_T}$$

Για $r_T = b$: $\hat{i}_{r_T}(\vec{J}_+ - \vec{J}_-) = -\vec{\nabla}_s\cdot\vec{K_3} - \cancel{\frac{\partial\sigma}{\partial t}}^{\,0} \Rightarrow J_T(b,\varphi) = -\frac{1}{b}\frac{\partial K_3}{\partial\varphi} \Rightarrow$

$$\Rightarrow J_T(b,\varphi) = -\frac{1}{b} K_0 2\cos\varphi\sin\varphi \Rightarrow -J_0 a 2\cos\varphi\sin\varphi \ln b + f(\varphi) = -K_0 2\cos\varphi\sin\varphi$$

$$\Rightarrow f(\varphi) = -2K_0\cos\varphi\sin\varphi + 2aJ_0\cos\varphi\sin\varphi\ln b$$

$$J_T(r_T,\varphi) = -J_0\frac{2a}{r_T}\cos\varphi\sin\varphi\ln(r_T) - \frac{2}{r_T}K_0\cos\varphi\sin\varphi + \frac{2}{r_T}aJ_0\cos\varphi\sin\varphi\ln b$$

$$= \frac{2}{r_T}\cos\varphi\sin\varphi\left[-aJ_0\ln(r_T) - K_0 + aJ_0\ln b\right]$$

$$\left(\begin{array}{l} \hat{i}_x \cdot \hat{i}_{r_T}\Big|_{\varphi=\frac{\pi}{2}} = \cos\varphi\Big|_{\varphi=\frac{\pi}{2}} = 0 \\ \hat{i}_x \cdot \hat{i}_\varphi\Big|_{\varphi=\frac{\pi}{2}} = -\sin\varphi\Big|_{\varphi=\frac{\pi}{2}} = -1 \end{array} \right)$$

Για το $\vec{K_4}$:

$$\hat{i}_x \cdot [\vec{J}^+ - \cancel{\vec{J}^-}^{\,0}] = -\frac{dK_4}{dy} \Rightarrow J_T(\hat{i}_x \cdot \hat{i}_{r_T}) + J_\varphi(\hat{i}_x \cdot \hat{i}_\varphi)\Big|_{\varphi=\frac{\pi}{2}} = -\frac{dK_4}{dy} \quad (1)$$

$J_\varphi(r_T, \frac{\pi}{2}) = J_0 \frac{a}{r_T}$ άρα $(1) \rightarrow -\frac{dK_4}{dy} = -J_0 \frac{a}{r_T} \Rightarrow$

$$\frac{dK_4}{dy} = J_0 \frac{a}{r_T} = J_0 \frac{a}{y} \Rightarrow K_4(y) = J_0 a \ln y + c$$

$$K_4(y=b) - K_3(\varphi=\tfrac{\pi}{2}) = 0 \Rightarrow J_0 a \ln b + c - K_0 = 0 \Rightarrow$$

$c = -J_0 a \ln b + K_0$ άρα $\boxed{K_4(y) = J_0 a \ln y - J_0 a \ln b + K_0}$

$\varphi = \frac{\pi}{2}, \quad b \leq y \leq a$

Για το $\vec{K_1}$:

$$\hat{i}_y [\vec{J}_+ - \cancel{\vec{J}_-}^{\,0}] = -\frac{dK_1}{dx} \Rightarrow J_T(\hat{i}_y \cdot \hat{i}_{r_T}) + J_\varphi(\hat{i}_y \cdot \hat{i}_\varphi)\Big|_{\varphi=0} = -\frac{dK_1}{dx}$$

όμως $\hat{i}_y \cdot \hat{i}_{r_T}\Big|_{\varphi=0} = \sin\varphi = 0$ και $\hat{i}_y \cdot \hat{i}_\varphi\Big|_{\varphi=0} = \cos\varphi = 1$ και

$J_\varphi\Big|_{\varphi=0} = 0$ οπότε $\frac{dK_1}{dx} = 0 \Rightarrow K_1 = c'$

$$K_1(x=b) + K_3(\varphi=0) = 0 \Rightarrow c' + 0 = 0 \Rightarrow c' = 0$$

οπότε $\boxed{K_1(x) = 0}$ $b < x < a$ και $\varphi = 0$

Για το $\vec{K_2}$:

$$\hat{i}_{r_T}(\cancel{\vec{J}^1}^{\,0} - \vec{J}^2) = -\vec{\nabla}_2 \vec{K_2} \Rightarrow J_T|_{r_T=a} = \frac{1}{a}\frac{\partial K_2}{\partial \varphi} \Rightarrow$$

$$2\cos\varphi\sin\varphi(-J_0\ln a + J_0\ln b - K_0) = \frac{1}{\alpha}\frac{\partial K_2}{\partial\varphi} \Rightarrow$$

$$K_2(\varphi) = \alpha(-J_0\ln a + J_0\ln b - K_0)\sin^2\varphi + c''$$

$$+\ K_2(\varphi=0) - K_1(x=a) = 0 \Rightarrow c''=0$$

$$\boxed{K_2(\varphi) = a(-J_0\ln a + J_0\ln b - K_0)\sin^2\varphi}, \quad 0<\varphi<\frac{\pi}{2},\ r_T = a$$

(β)-(γ)

Κάνουμε τις αντικαταστάσεις:

- $\vec{J} = \hat{i}_{r_T}\cdot J_T(r_T,\varphi) + \hat{i}_\varphi J_\varphi(r_T,\varphi) \quad 1<r_T<2,\ 0<\varphi<\frac{\pi}{2}$

$$\left(J_T(r_T,\varphi) = \frac{2}{r_T}\cos\varphi\sin\varphi\left(-2\ln(r_T) - 1\right) \text{ και } J_\varphi(r_T,\varphi) = \frac{2}{r_T}\sin^2\varphi \right)$$

$$J_T(x,y) = \frac{-2xy}{(x+y)^{\frac{3}{2}}}\left(\ln(x^2+y^2)+1\right), \quad J_\varphi(x,y) = \frac{2y^2}{(x+y)^{\frac{3}{2}}}$$

$$\vec{J} = \left[J_T(x,y)\cos\varphi - J_\varphi(x,y)\sin\varphi\right]\hat{i}_x + \left[J_T(x,y)\sin\varphi + J_\varphi(x,y)\cos\varphi\right]\hat{i}_y \Rightarrow$$

$$\vec{J} = \left[\frac{-2x^2y\left(\ln(x^2+y^2)+1\right)}{(x+y)^2} - \frac{2y^3}{(x+y)^2}\right]\hat{i}_x + \left[\frac{-2xy^2}{(x+y)^2}\left(\ln(x^2+y^2)+1\right) + \frac{2xy^2}{(x+y)^2}\right]\hat{i}_y$$

$$|\vec{J}| = \sqrt{J_x^2 + J_y^2} = \frac{1}{(x+y)^2}\sqrt{\left[2x^2y\left(\ln(x^2+y^2)+1\right)+2y^3\right]^2 + \left(-2xy^2\left(\ln(x^2+y^2)+1\right)+2xy^2\right)^2}$$

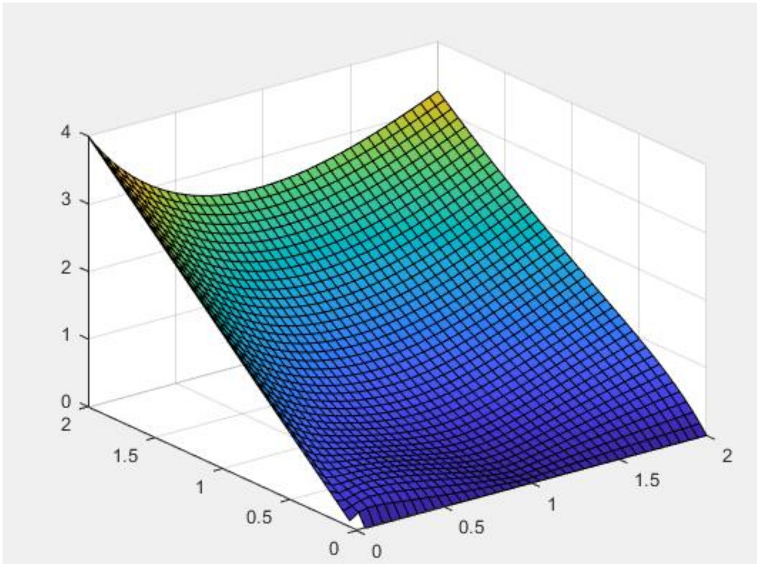
Για τα ερωτήματα (β) και (γ) της άσκησης 6

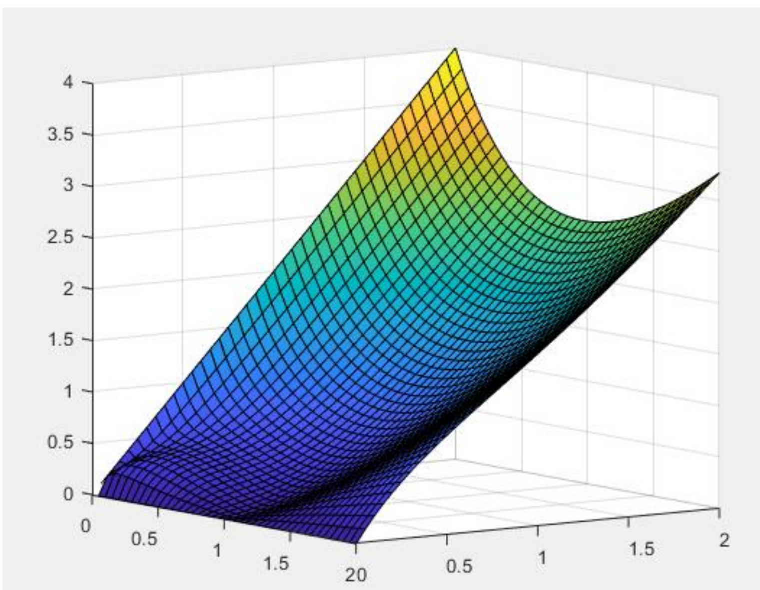
Ο κώδικας:

```
%b
[x,y] = meshgrid(0:0.05:2,0:0.05:2);

Jx=(1./(x+y).^2).*(-2*x.^2.*y.*log(x.^2+y.^2)-2.*y.^3);
Jy=(1./(x+y).^2).*(-2*y.^2.*x.*log(x.^2+y.^2)+2.*y.^2.*x);
J_magn=(Jx.^2 + Jy.^2).^0.5;

figure(1)
surf(x,y,J_magn);

figure(2)
contour(x,y,J_magn,[0:0.05:1]);

%c
figure(3)
quiver(x,y,Jx,Jy);

figure(4)
streamslice(x,y,Jx,Jy);

figure(5)
streamline(x,y,Jx,Jy,x,y);
```

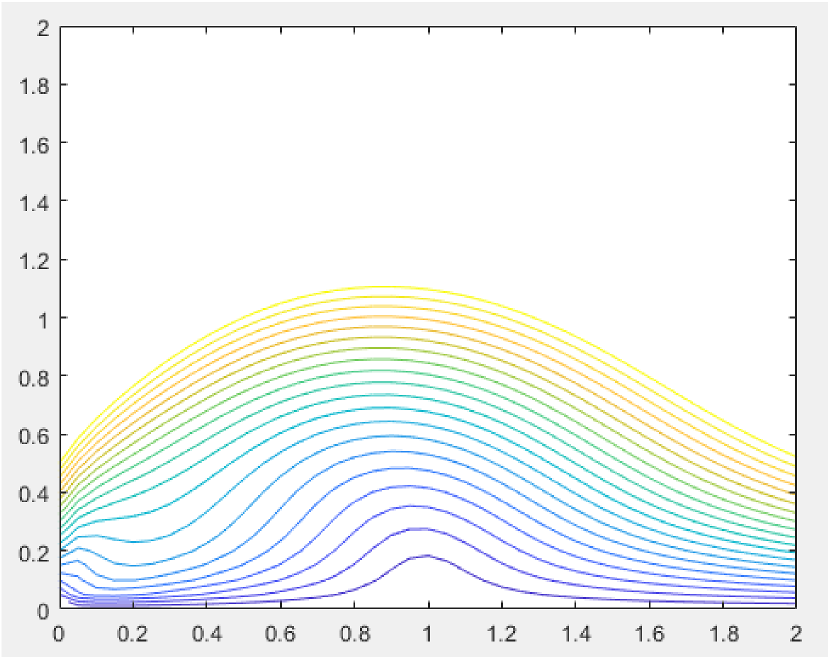
Οι γραφικές παραστάσεις:

➔ surf(x,y,J_magn);

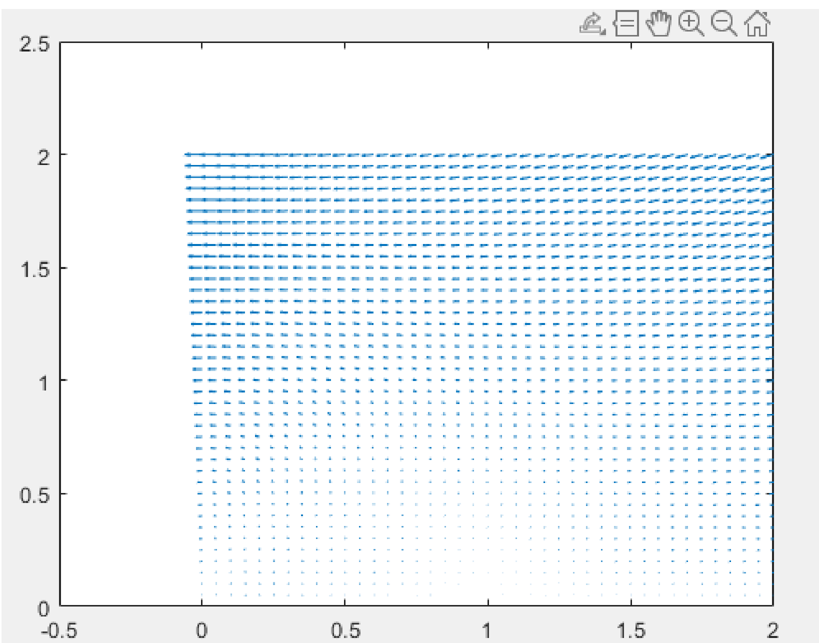




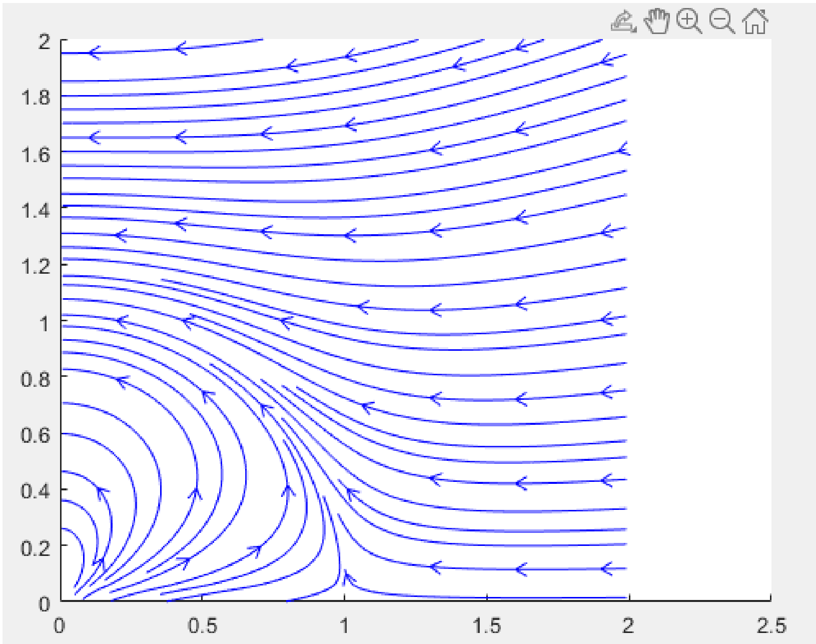
➔ contour(x,y,J_magn,[0:0.05:1]);



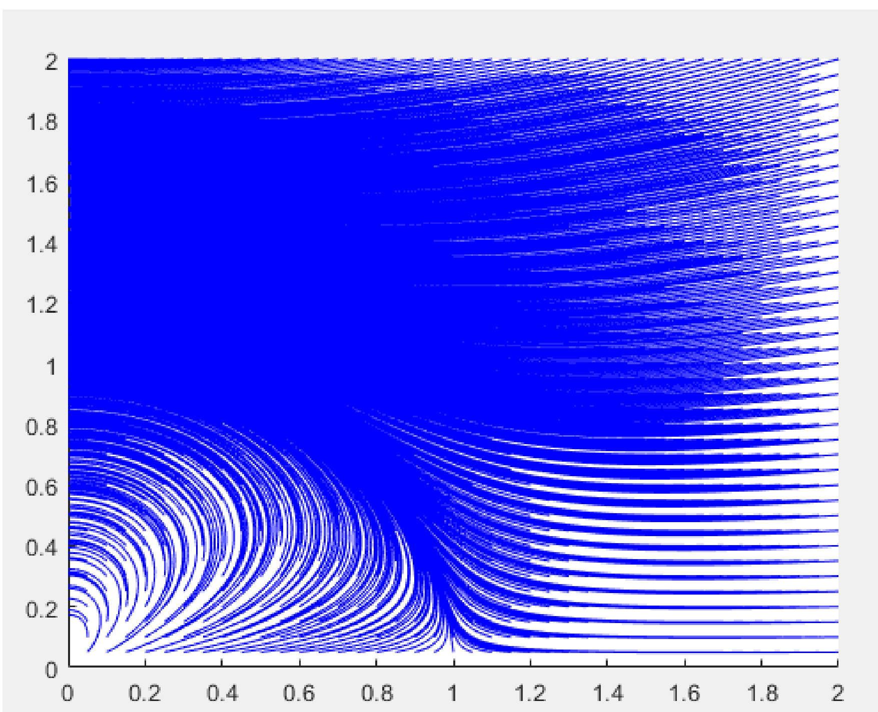
➔ quiver(x,y,Jx,Jy);



→ streamslice(x,y,Jx,Jy);



→ streamline(x,y,Jx,Jy,x,y);



Άσκηση 7:

$$\vec{J} = J_T(r_T,\varphi)\cdot \hat{i}_{r_T} + J_0 \cos\varphi\, \hat{i}_\varphi, \quad r_T < a, \quad 0 < \varphi < \frac{2\pi}{3}$$

$$\vec{K_1} = K_1(r_T)\, \hat{i}_{r_T}, \quad r_T < a, \quad \varphi = 0$$

$$\vec{K_2} = K_2(r_T)\, \hat{i}_{r_T}, \quad r_T < a, \quad \varphi = \frac{2\pi}{3}$$

(a) ΝΔΦ στη διαφορική του μορφή:

$$\vec{\nabla}\cdot\vec{J} = 0 \Rightarrow \frac{1}{r_T}\frac{\partial}{\partial r_T}(J_T r_T) + \frac{1}{r_T}\frac{\partial J_\varphi}{\partial \varphi} = 0 \Rightarrow$$

$$\Rightarrow \frac{\partial}{\partial r_T}(r_T J_T) - J_0 \sin\varphi = 0 \Rightarrow r_T J_T = J_0 \sin\varphi\, r_T + f(\varphi) \Rightarrow$$

$$J_T(r_T,\varphi) = J_0 \sin\varphi + \frac{f(\varphi)}{r_T}$$

Για $r_T = a$: $\hat{i}_{r_T}(\cancel{\vec{J}^+}^{\,0} - \vec{J}^-)\Big|_{r_T=a} = -\vec{\nabla}_2\cdot \cancel{\vec{K}_{\epsilon\xi}}^{\,0} = 0 \Rightarrow$

$$J_T(a,\varphi) = 0 \Rightarrow J_0 \sin\varphi + \frac{f(\varphi)}{a} = 0 \Rightarrow f(\varphi) = -a J_0 \sin\varphi$$

Άρα: $\boxed{J_T(r_T,\varphi) = J_0 \sin\varphi - \frac{a J_0 \sin\varphi}{r_T}}$ $\quad r_T < a, \quad 0 < \varphi < \frac{2\pi}{3}$

Για το $\vec{K_1}$, για $\varphi = 0$:

$$\hat{i}_\varphi(\vec{J}_+ - \cancel{\vec{J}_-}^{\,0})\Big|_{\varphi=0} = -\vec{\nabla}_2 \vec{K_1} \Rightarrow J_0 \cos 0 = -\frac{dK_1}{dr_T} \Rightarrow$$

$K_1(r_T) = -J_0 r_T + c$ και $K_1(a) = 0$ άρα $c = J_0 a$

$$\boxed{K_1(r_T) = J_0(a - r_T)}$$

Για το $\vec{K_2}$, για $\varphi = \frac{2\pi}{3}$:

$$\hat{i}_\varphi (\cancel{\vec{J}_+}^{\,\to 0} - \vec{J}_-)\Big|_{\varphi=\frac{2\pi}{3}} = -\vec{\nabla}_2 \cdot \vec{K_2} \Rightarrow -J_0\left(-\frac{1}{2}\right) = -\frac{dK_2}{dr_T} \Rightarrow$$

$$\Rightarrow K_2(r_T) = -\frac{J_0 r_T}{2} + c'. \text{ Όμως } K_2(a) = 0 \Rightarrow c' = \frac{J_0}{2} a$$

άρα $\boxed{K_2(r_T) = \frac{J_0}{2}(a - r_T)}$

(β)-(γ)

- $\vec{J} = \hat{i}_{r_T} \cdot J_T(r_T, \varphi) + \hat{i}_\varphi J_\varphi(r_T, \varphi)$, $r_T < a$, $0 < \varphi < \frac{2\pi}{3}$

$\left(J_T(r,\varphi) = J_0 \sin\varphi - \frac{a J_0 \sin\varphi}{r_T},\ J_\varphi(r_T,\varphi) = J_0 \cos\varphi\right)$, $J_0 = 1A/m^2$, $a = 1m$

$$J_T(x,y) = \frac{y}{\sqrt{x^2+y^2}}\left(1 - \frac{1}{\sqrt{x^2+y^2}}\right), \quad J_\varphi(x,y) = \frac{x}{\sqrt{x^2+y^2}}$$

$$\vec{J} = \left(J_T(x,y)\cos\varphi - J_\varphi(x,y)\sin\varphi\right)\hat{i}_x + \left(J_T(x,y)\sin\varphi + J_\varphi(x,y)\cos\varphi\right)\hat{i}_y$$

$$\vec{J} = \left(\frac{xy}{x^2+y^2}\left(1 - \frac{1}{\sqrt{x^2+y^2}}\right) - \frac{xy}{x^2+y^2}\right)\hat{i}_x + \left(\frac{y^2}{x^2+y^2}\left(1 - \frac{1}{\sqrt{x^2+y^2}}\right) + \frac{x^2}{x^2+y^2}\right)\hat{i}_y$$

$$\Rightarrow \vec{J} = \left(\frac{xy}{(x^2+y^2)^{\frac{3}{2}}}\right)\hat{i}_x + \left(\frac{y^2}{x^2+y^2}\left(1 - \frac{1}{\sqrt{x^2+y^2}}\right) + \frac{x^2}{x^2+y^2}\right)\hat{i}_y$$

$$|\vec{J}| = \sqrt{J_x^2 + J_y^2}$$

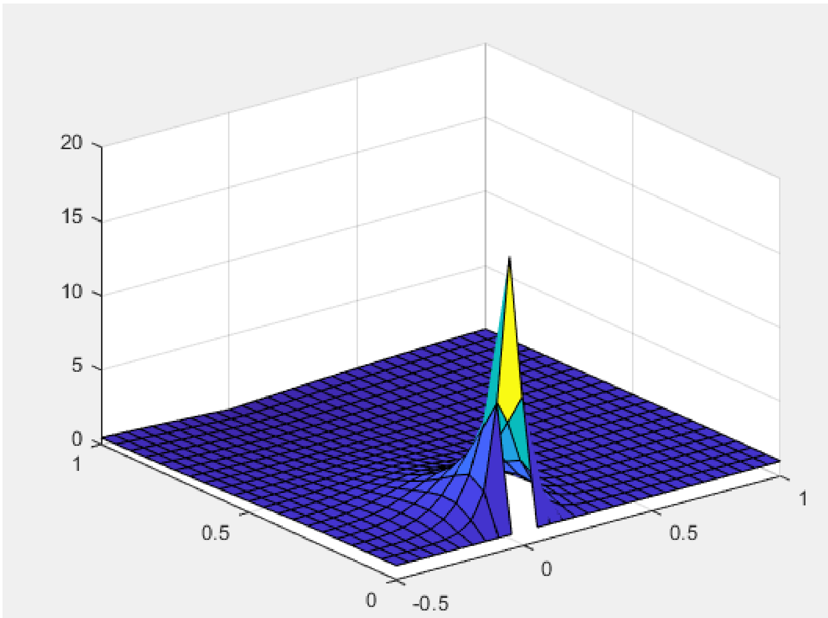
Για τα ερωτήματα (β) και (γ) της άσκησης 7

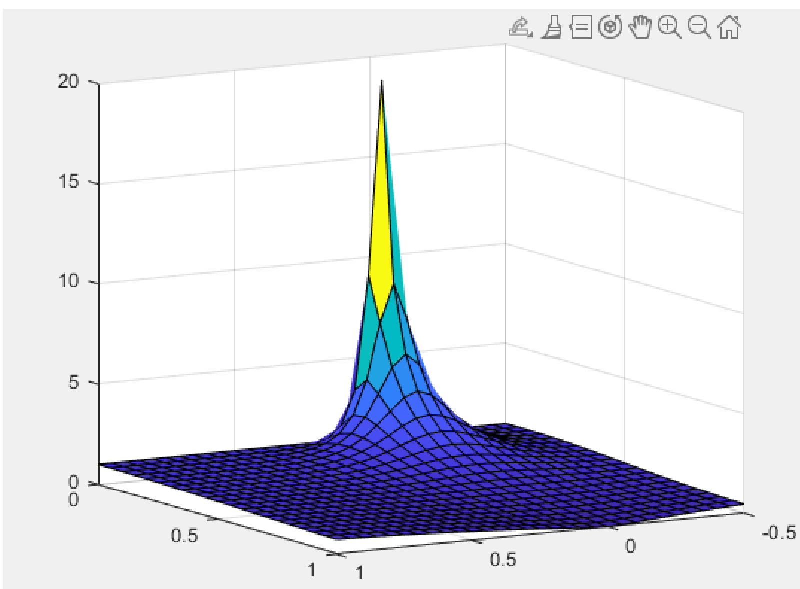
Ο κώδικας:

```
%b
[x,y] = meshgrid(-0.5:0.05:1,0:0.05:1);

Jx=(x.*y./(x.^2+y.^2).^(3/2));
Jy=(1./(x.^2+y.^2)).*((y.^2).*(1-(1./(x.^2+y.^2).^(1/2)))+x.^2);
J_magn=(Jx.^2 + Jy.^2).^0.5;

figure(1)
surf(x,y,J_magn);

figure(2)
contour(x,y,J_magn,[0.25:0.25:5]);

%c
figure(3)
quiver(x,y,Jx,Jy);

figure(4)
streamslice(x,y,Jx,Jy);

figure(5)
streamline(x,y,Jx,Jy,x,y);
```

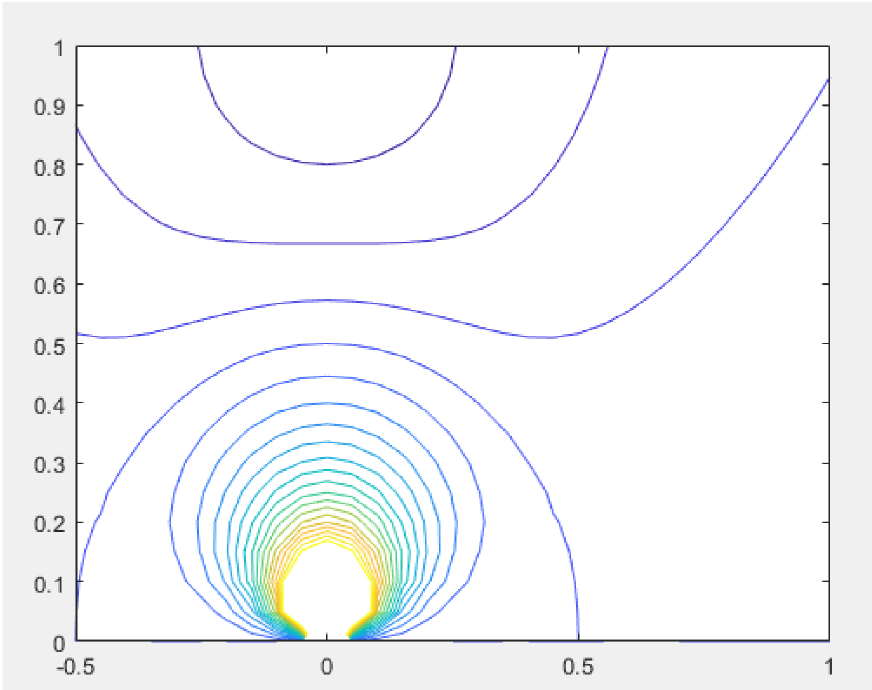
Οι γραφικές παραστάσεις:

➔ surf(x,y,J_magn);

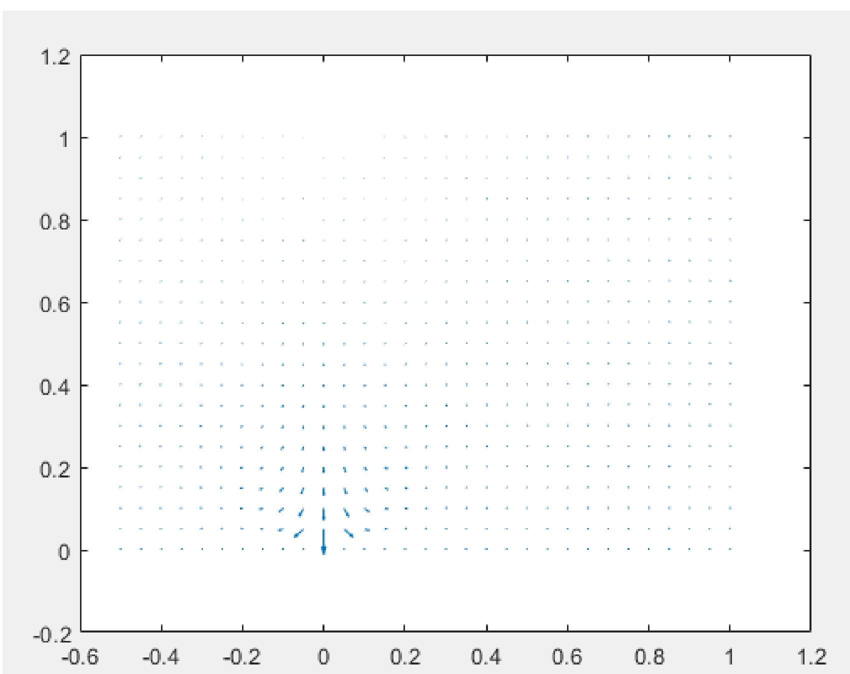




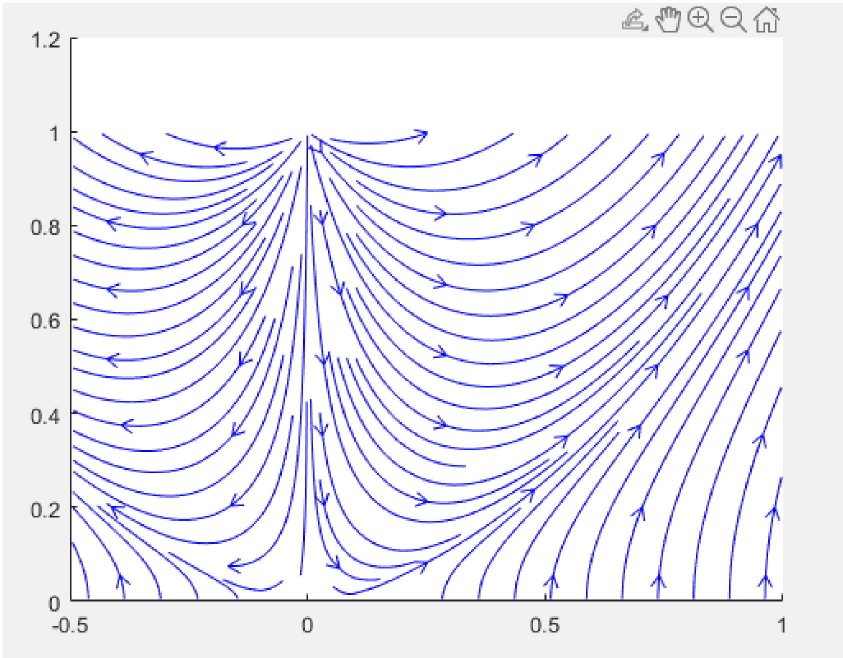
➔ contour(x,y,J_magn,[0.25:0.25:5]);



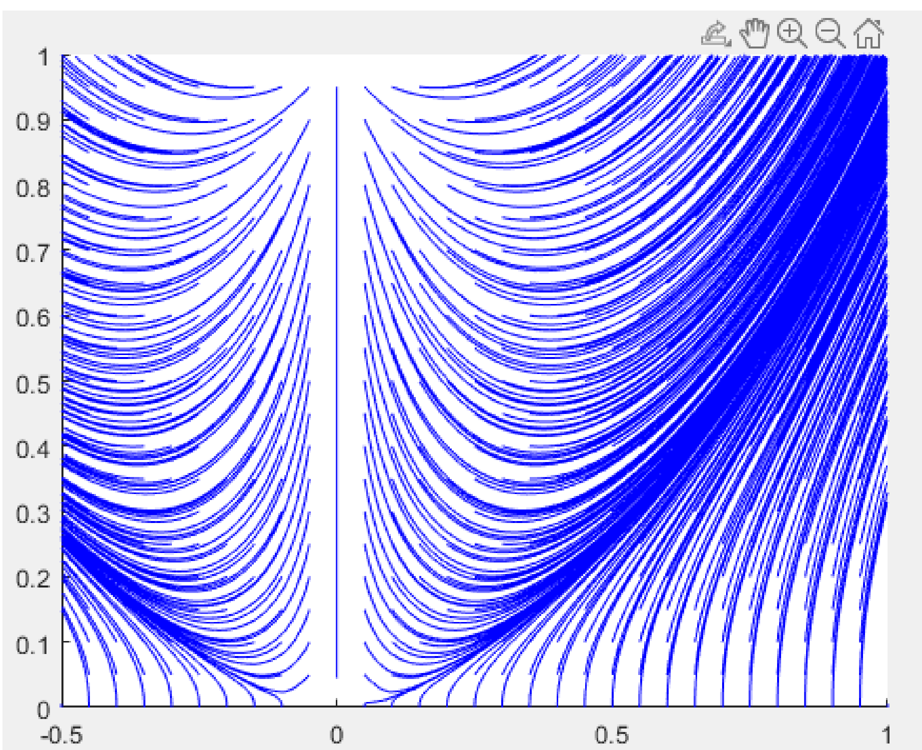
➔ quiver(x,y,Jx,Jy);



→ streamslice(x,y,Jx,Jy);



→ streamline(x,y,Jx,Jy,x,y);



Άσκηση 8 :

$$\vec{J} = \hat{i}_x \, J_0\left(1 + \frac{y}{b}\right), \quad \begin{array}{l} 0<x<a \\ 0<y<b \end{array}$$

$$\vec{K_1} = \hat{i}_y \, K_1(y), \quad \begin{array}{l} 0<y<b \\ x=0 \end{array}$$

$$\vec{K_2} = \hat{i}_x \, K_2(x), \quad \begin{array}{l} 0<x<a \\ y=0 \end{array}$$

$$\vec{K_3} = \hat{i}_y \, K_3(y), \quad \begin{array}{l} 0<y<b \\ x=a \end{array}$$

(α) Για το $K_1(y)$:

$$\hat{i}_x(\vec{J}_+ - \cancel{\vec{J}_-}^{\,0}) = -\frac{dK_1}{dy} \Rightarrow \frac{dK_1}{dy} = -J_0 - \frac{J_0}{b}\cdot y \Rightarrow$$

$K_1(y) = -J_0 y - \dfrac{J_0}{2b}y^2 + c$ και $K_1(b)=0$ άρα:

$c = b\,J_0 + J_0\dfrac{b}{2} = J_0\dfrac{3b}{2}$ άρα: $\boxed{K_1(y) = J_0\left[\frac{3b}{2} - y - \frac{y^2}{2b}\right]}$

$0<y<b$, $x=0$

Για το $K_2(x)$:

$$\hat{i}_y(\vec{J}_+ - \cancel{\vec{J}_-}^{\,0}) = -\frac{dK_2}{dx} \Rightarrow \frac{dK_2}{dx} = 0 \Rightarrow K_2 = c'$$

και $K_1(0) + K_2(0) = 0 \Rightarrow c' = -J_0\dfrac{3b}{2}$ άρα $\boxed{K_2(x) = -J_0\frac{3b}{2}}$

$0<x<a$, $y=0$

Για το $K_3(y)$:

$$\hat{i}_x(\cancel{\vec{J}_+}^{\,0} - \vec{J}_-) = -\frac{dK_3}{dy} \Rightarrow \frac{dK_3}{dy} = -K_1 \Rightarrow K_3(y) = -K_1(y) + c''$$

όμως $K_3(0) = K_1(0) = 0 \Rightarrow c'' = 0$ άρα:

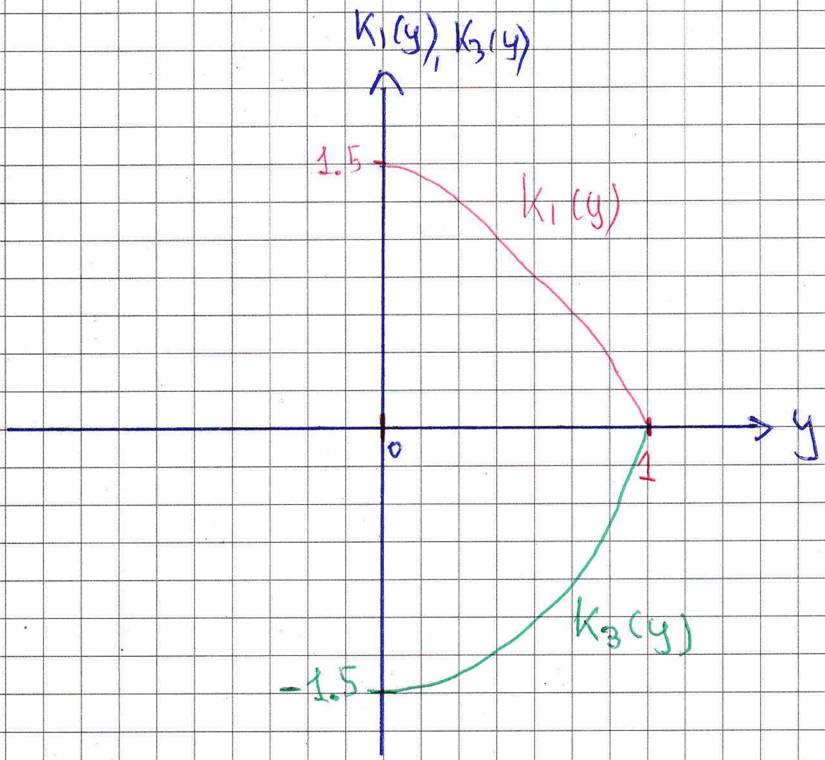
$$\boxed{K_3(y) = -K_1(y) = J_0\left[-\frac{3b}{2} + y + \frac{y^2}{2b}\right]} \quad 0<y<b, \; x=a$$

(β) $K_1(y) = \frac{3}{2} - y - \frac{y^2}{2}$, $0<y<1$, $x=0$

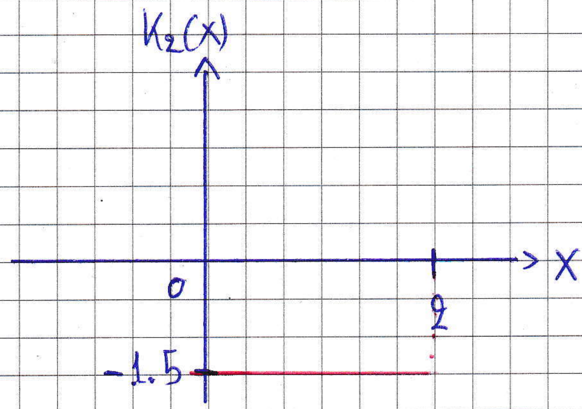
$K_2(x) = -\frac{3}{2}$, $0<x<2$, $y=0$

$K_3(y) = \frac{y^2}{2} + y - \frac{3}{2}$, $0<y<1$, $x=2$

(γ)-(δ) $\vec{J} = (y+1)\hat{i}_x$, $0<x<2$, $0<y<1$



K1(y), K3(y)
1.5
K1 (y)
y
0
1
K3 (y)
-1.5



K2(x)
x
0
2
-1.5

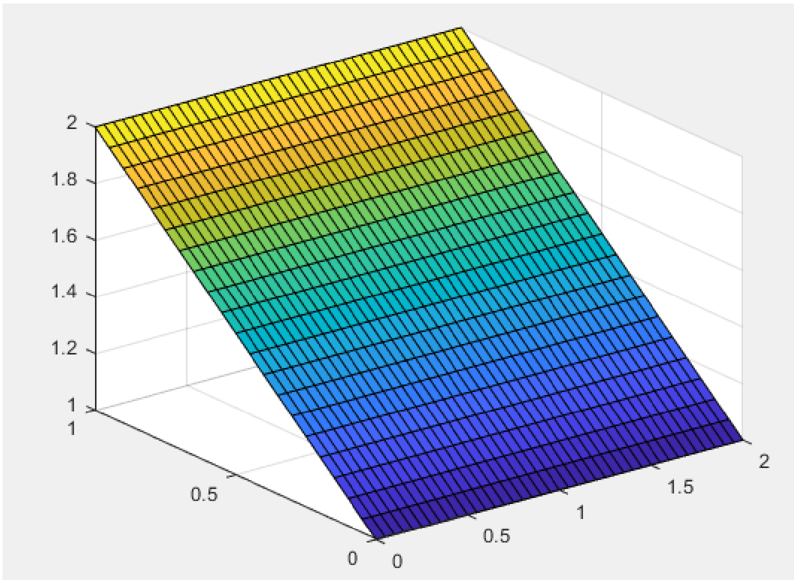
Για τα ερωτήματα (γ) και (δ) της άσκησης 8

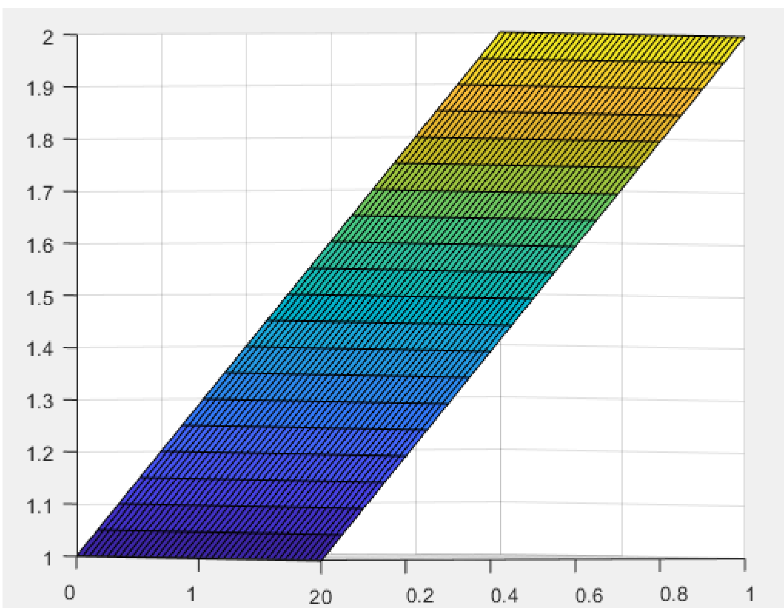
Ο κώδικας:

```
%c
[x,y] = meshgrid(0:0.05:2,0:0.05:1);

Jx=(y+1);
Jy=zeros(21,41);
J_magn=Jx;

figure(1)
surf(x,y,J_magn);

figure(2)
Jmin=1;
Jmax=2;
contour(x,y,J_magn,[Jmin:0.2:Jmax]);

%d
figure(3)
quiver(x,y,Jx,Jy);

figure(4)
streamslice(x,y,Jx,Jy);
```

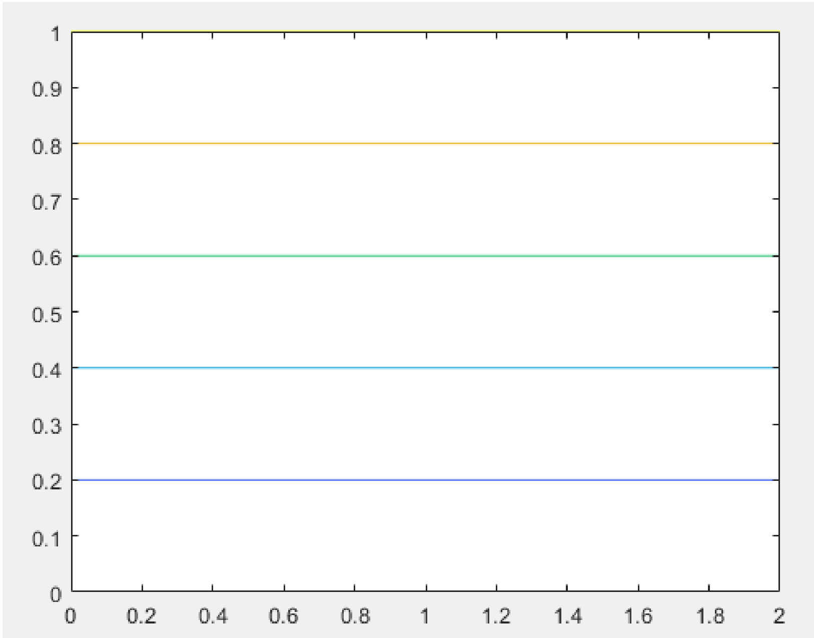
Οι γραφικές παραστάσεις:

➔ surf(x,y,J_magn);

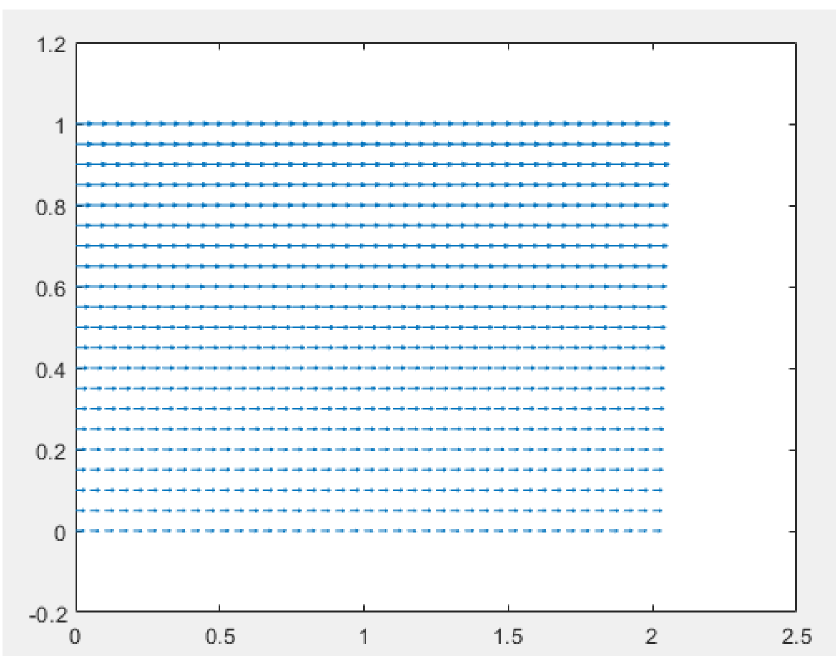




➔ contour(x,y,J_magn,[Jmin:0.2:Jmax]);



➔ quiver(x,y,Jx,Jy);



➔ streamslice(x,y,Jx,Jy);