Ηλεκτρομαγνητικά Πεδία Α

2η εργασία

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Άσκηση 9:

(α) Έστω σ_1 η επιφανειακή πυκνότητα στο $y=a+h$ και σ_2 η επιφανειακή πυκνότητα στο $y=a$.

Ο αγωγός είναι αφόρτιστος, οπότε $\boxed{\sigma_1+\sigma_2=0}$ (1)

Όπως το Η.Π. είναι μηδενικό στο εσωτερικό του αγωγού, άρα:

$$\vec{E}=0 \Rightarrow \hat{i}_y\left[\frac{\sigma_0}{2\varepsilon_0}(-1)+\frac{\sigma_1}{2\varepsilon_0}(-1)+\frac{\sigma_2}{2\varepsilon_0}+\int_{-\infty}^{0}\frac{\rho(y)}{2\varepsilon_0}dy\right]=0$$

$$\int_{-\infty}^{0}\frac{\rho(y)}{2\varepsilon_0}dy=\frac{\rho_0}{2\varepsilon_0}\int_{-\infty}^{0}e^{-\frac{|y|}{d}}dy \overset{y<0}{=} \frac{\rho_0}{2\varepsilon_0}\int_{-\infty}^{0}e^{\frac{y}{d}}dy=\frac{\rho_0}{2\varepsilon_0}d$$

άρα $-\sigma_0-\sigma_1+\sigma_2+\rho_0 d=0 \overset{(1)}{\Rightarrow} \boxed{\sigma_1=\frac{\rho_0 d-\sigma_0}{2}}$ $\boxed{\sigma_2=\frac{\sigma_0-\rho_0 d}{2}}$

(β) $y>b$: $\vec{E}=\hat{i}_y\left[+\frac{\sigma_0}{2\varepsilon_0}+\frac{\sigma_1}{2\varepsilon_0}+\frac{\sigma_2}{2\varepsilon_0}+\frac{\rho_0 d}{2\varepsilon_0}\right]=\boxed{\hat{i}_y\left[\frac{\sigma_0}{2\varepsilon_0}+\frac{\rho_0 d}{2\varepsilon_0}\right]}$

$a+h<y<b$: $\boxed{\vec{E}=\hat{i}_y\left[-\frac{\sigma_0}{2\varepsilon_0}+\frac{\rho_0 d}{2\varepsilon_0}\right]}$

$a<y<a+h$: $\boxed{\vec{E}=0}$

$0<y<a$: $\boxed{\vec{E}=\hat{i}_y\left[-\frac{\sigma_0}{2\varepsilon_0}+\frac{\rho_0 d}{2\varepsilon_0}\right]}$

$0>y$: $\vec{E}=\hat{i}_y\left[-\frac{\sigma_0}{2\varepsilon_0}+\int_{y}^{0}\frac{\rho_0}{2\varepsilon_0}e^{\frac{y'}{d}}dy'(-1)+\int_{-\infty}^{y}\frac{\rho_0}{2\varepsilon_0}e^{\frac{y'}{d}}dy'(+1)\right]$

$$\frac{\rho_0}{2\varepsilon_0}\int_{y}^{0}e^{\frac{y'}{d}}dy'(-1)=\frac{\rho_0}{2\varepsilon_0}\cdot d\cdot(1-e^{\frac{y}{d}}),\quad \frac{\rho_0}{2\varepsilon_0}\int_{-\infty}^{y}e^{\frac{y'}{d}}dy'=\frac{\rho_0}{2\varepsilon_0}de^{\frac{y}{d}}$$

άρα $\boxed{\vec{E}=\hat{i}_y\left[-\frac{\sigma_0}{2\varepsilon_0}-\frac{\rho_0 d}{2\varepsilon_0}+2\rho_0 d e^{\frac{y}{d}}\right]}$

(γ) $\rho_0 = \frac{1nC}{cm^3} = 10^{-3} \frac{C}{m^3}$, $d = 10cm = 10^{-1} m$, $\sigma_0 = \frac{1nC}{cm^2} = 10^{-5} \frac{C}{m^2}$

$a = 5cm = 5 \cdot 10^{-2} m$, $h = 2cm = 2 \cdot 10^{-2} m$, $b = 12cm = 1.2 \cdot 10^{-1} m$

(Graph 1)

$\vec{E} = 0$, για $0 < y < b$ άρα $\frac{\sigma_0}{2\varepsilon_0} = \frac{\rho_0 d}{2\varepsilon_0} \Rightarrow \sigma_0 = \rho_0 d \Rightarrow$

$\Rightarrow \boxed{\sigma_0 = 10^{-4} \frac{C}{m^2}} = \sigma_2$

(Graph 2)

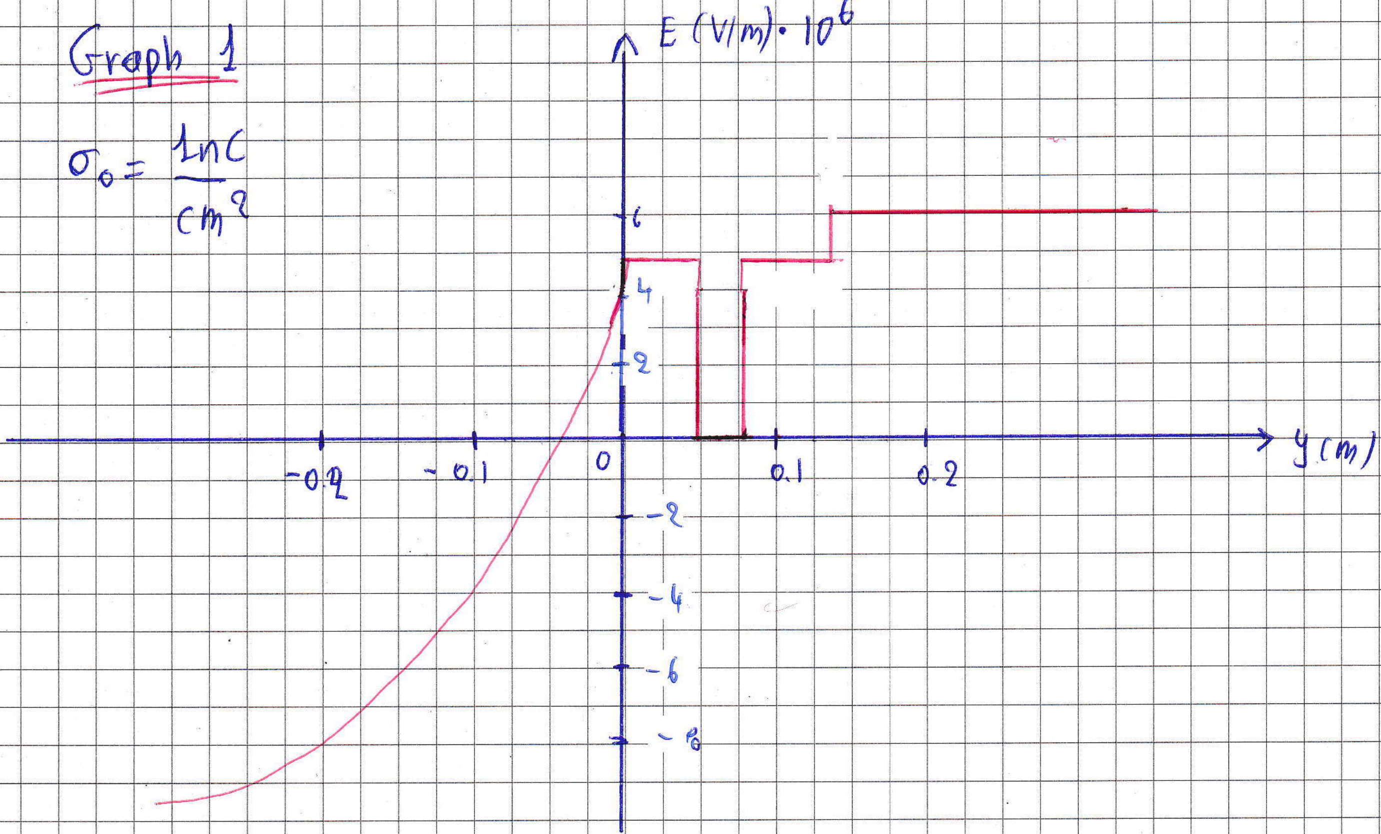
(Graph 3) για $\sigma_0 = 2\sigma_2$

Σε σχέση με το Graph 2, παρατηρούμε ότι

για $0 < y < 0.12m$ $E(y) < 0$, (αν $y \notin (5cm, 7cm)$)

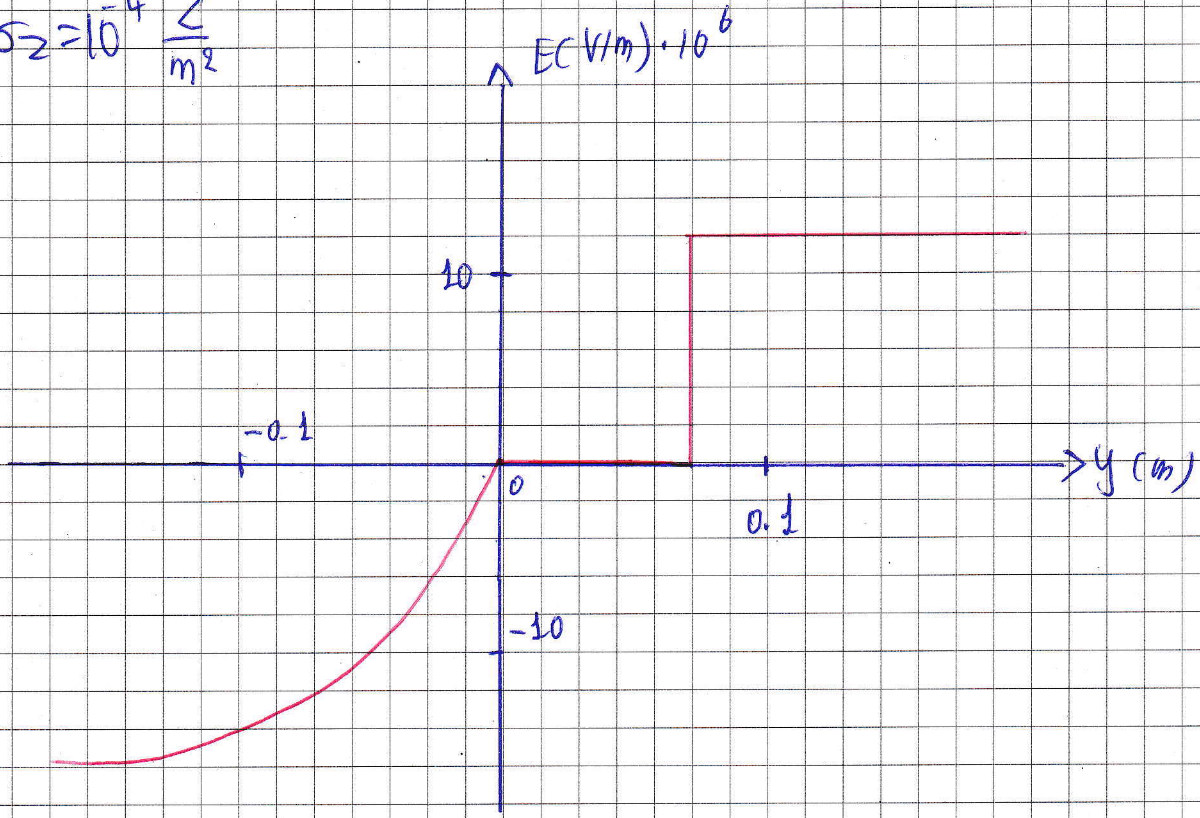
Graph 1

$\sigma_0 = \dfrac{1\,nC}{cm^2}$



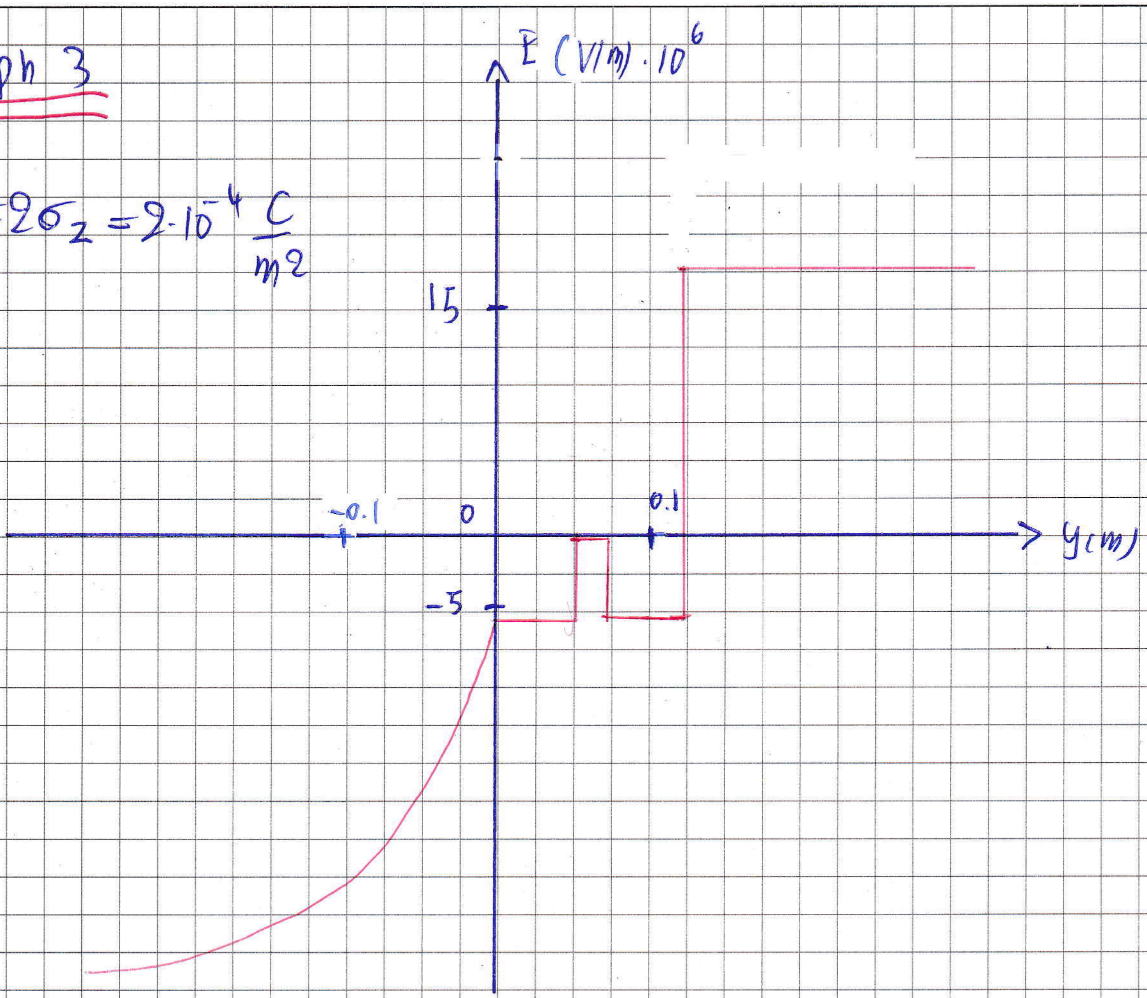
Graph 2

$\sigma_0 = \sigma_2 = 10^{-4}\ \dfrac{C}{m^2}$



Graph 3

$\sigma_0 = 2\sigma_2 = 2\cdot 10^{-4}\ \frac{C}{m^2}$



Άσκηση 10:

(α) ΝΔΦ διαφορική σχέση: $\vec{\nabla}\cdot\vec{J}=0 \overset{J_{r_T}=0}{\underset{J_z=0}{\Rightarrow}} \dfrac{1}{r_T}\dfrac{\partial J_\varphi}{\partial\varphi}=0 \Rightarrow$

$J(r_T,\varphi)=C_0+f(r_T)$ ①

$J(r_T,0)=C_1$

$J(r_T,\pi)=C_2$

Οριακές συνθήκες: $\hat{i}_n(\vec{J}^+-\vec{J}^-)=-\vec{\nabla}_2\vec{K}\Rightarrow$

$\hat{i}_y J(r_T,\varphi=0)\hat{i}_y=-\dfrac{dK}{dx} \Rightarrow C_1=J(r_T,0)=+\dfrac{K_0}{a}e^{-\frac{r_T}{a}}$, $|x|=x=r_T$, αφού $\varphi=0$

$\hat{i}_n(\vec{J}^+-\vec{J}^-)=-\vec{\nabla}_2\vec{K}\Rightarrow \hat{i}_y J(r_T,\varphi=\pi)(-\hat{i}_y)=-\dfrac{dK}{dx}=-\dfrac{K_0}{a}e^{-\frac{r_T}{a}}$

αφού $|x|=-x=r_T$ οπότε $J(r_T,\pi)=\dfrac{K_0}{a}e^{-\frac{r_T}{a}}=C_2$

Οπότε ① $\rightarrow \begin{cases} f(r_T)=\dfrac{1}{a}K_0e^{-\frac{r_T}{a}} \\ C_0=0 \end{cases}$

άρα $\boxed{\vec{J}(r_T,\varphi)=\dfrac{K_0}{a}e^{-\frac{r_T}{a}}\hat{i}_\varphi}$

(β) $\vec{J}(x,y)=\dfrac{K_0}{a}e^{-\frac{\sqrt{x^2+y^2}}{a}}\left(-\dfrac{y}{\sqrt{x^2+y^2}}\hat{i}_x+\dfrac{x}{\sqrt{x^2+y^2}}\hat{i}_y\right)$

(γ) Έχουμε ότι $\frac{\partial}{\partial z}=0$ και $I_z=K_z=J_z=0$ άρα $\vec{H}_{\perp z}=0$ άρα

$\vec{H}=\hat{i}_z H(r_T,\varphi)$

Έστω βρόχος C κάθετος στο $\hat{i}_x$, τότε για $y>0$

$\oint_C \vec{H}\cdot d\vec{l}=\int_S \vec{J}\cdot d\vec{S}=l\cdot K(x)\Rightarrow l\cdot H(r_T,\varphi)=K(x)l \Rightarrow$

$H(r_T,\varphi)=K_0 e^{-\frac{r_T}{a}}$, $r_T\geq 0$, $0\leq\varphi\leq\pi$ $(y>0)$

Αν $y<0$: για $r_T\to\infty$ έχουμε ότι $H(r_T,\varphi)=0$

άρα: $\boxed{\vec{H}(r_T,\varphi)=\begin{cases}0, & y<0,\ \pi\leq\varphi\leq 2\pi,\ r_T\geq 0\\ \hat{i}_z K_0 e^{-\frac{r_T}{a}}, & y>0,\ 0\leq\varphi\leq\pi,\ r_T\geq 0\end{cases}}$

• $\vec{J}=\vec{\nabla}\times\vec{H}\Rightarrow\begin{cases}J_T(r_T,\varphi)=\frac{1}{r_T}\frac{\partial H_z}{\partial\varphi}-\frac{\partial H_\varphi}{\partial z}=0\\ J_z(r_T,\varphi)=\frac{1}{r_T}\frac{\partial(r_T H_\varphi)}{\partial r_T}-\frac{1}{r_T}\frac{\partial H_{r_T}}{\partial\varphi}=0\\ J_\varphi(r_T,\varphi)=\frac{\partial H_{r_T}}{\partial z}-\frac{\partial H_z}{\partial r_T}=\frac{K_0}{a}e^{-\frac{r_T}{a}}\end{cases}$

άρα $\vec{J}=\hat{i}_\varphi\frac{K_0}{a}e^{-\frac{r_T}{a}}$

$\vec{K}=\hat{i}_n\times(\vec{H}^+-\vec{H}^-)=\hat{i}_y\times\left(\hat{i}_z K_0 e^{-\frac{r_T}{a}}\right)=\hat{i}_x K_0 e^{-\frac{r_T}{a}}=\hat{i}_x K_0 e^{-|x|/a}$

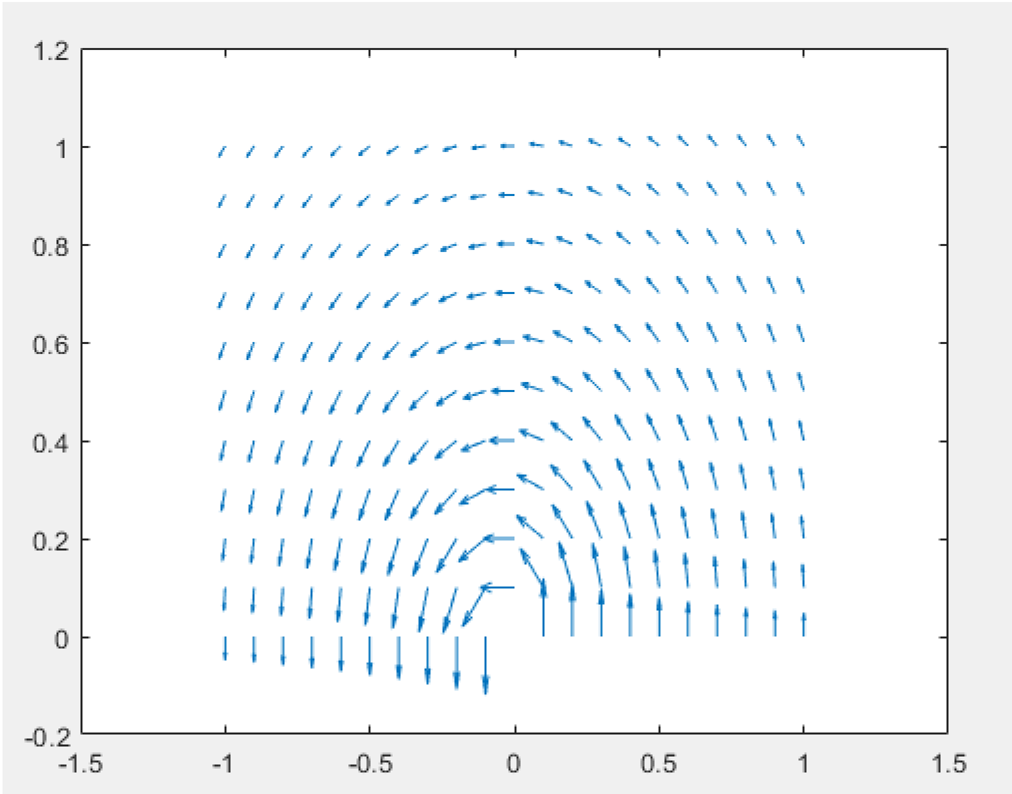
(δ) $H(x,y)=\begin{cases}0, & y<0,\ x\in\mathbb{R}\\ K_0 e^{-\frac{\sqrt{x^2+y^2}}{a}}, & y>0,\ x\in\mathbb{R}\end{cases}$

Για τα ερωτήματα β) και δ) της άσκησης 10:

Ο κώδικας:

```
%b
[x,y] = meshgrid(-1:0.1:1,0:0.1:1);

J0=exp(-sqrt(x.^2+y.^2))./(sqrt(x.^2+y.^2));
Jx=-J0.*y;
Jy=J0.*x;

figure(1)
quiver(x,y,Jx,Jy);

figure(2)
streamslice(x,y,Jx,Jy);

figure(3)
streamline(x,y,Jx,Jy,x,y);

%d
H=exp(-sqrt(x.^2+y.^2));

figure(4)
surface(x,y,H);
shading interp
```

```
figure(1)
```

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figure(2)
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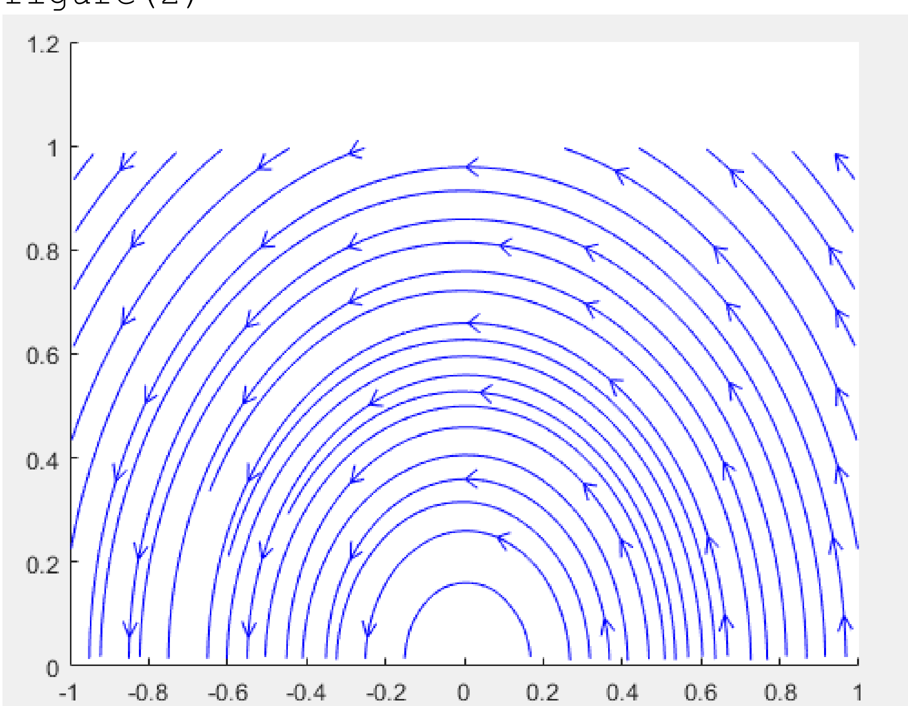
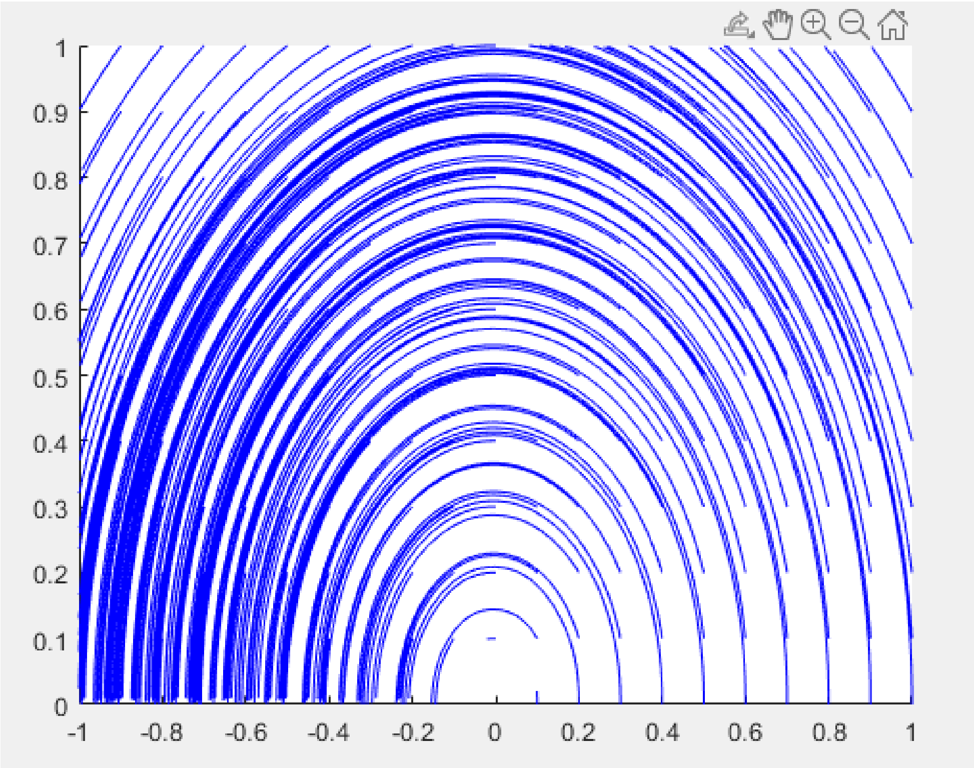
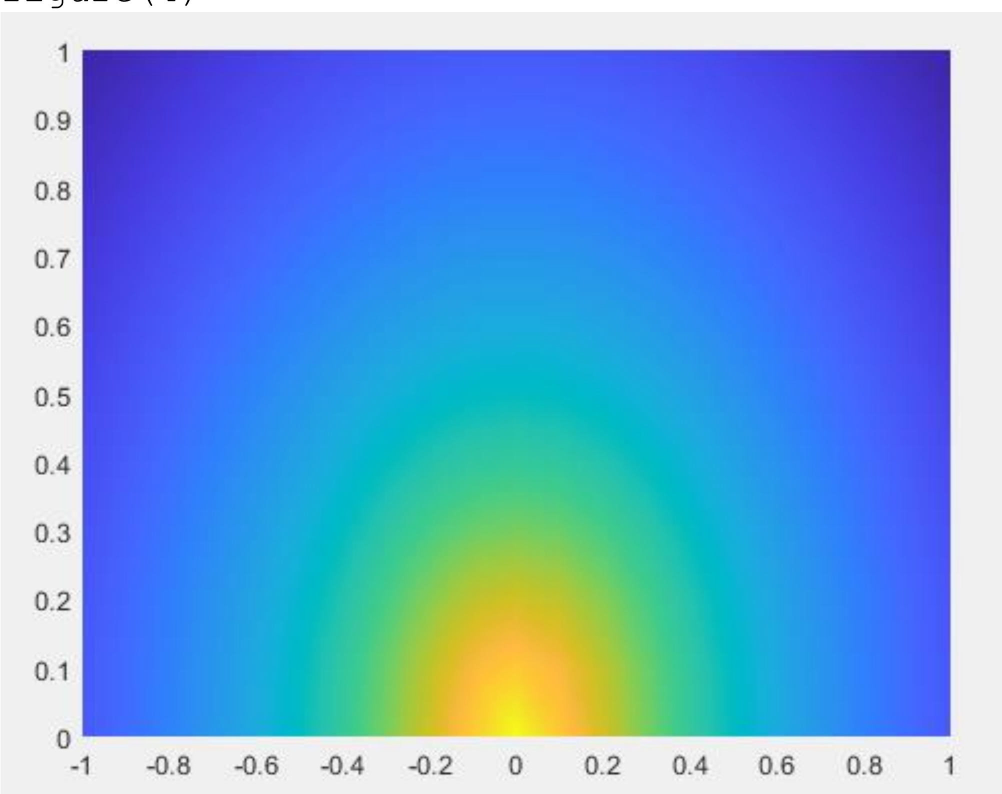


figure(3)



figure(4)



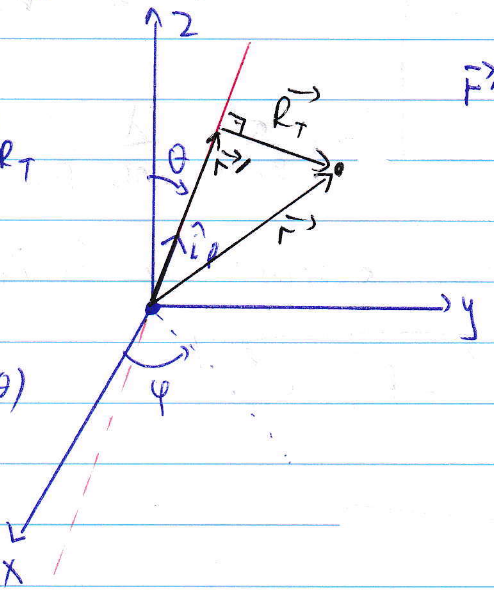
Άσκηση 11:

(α) Το πρόβλημα ανάγεται στην επαλληλία 2 προβλημάτων:

A Για την άπειρη επίπεδη κατανομή ηλ. φορτίου με πυκνότητα σ, έχουμε:

- αν $z<c$: $\vec{E}_1(x,y,z) = \hat{i}_z\left(-\frac{\sigma}{2\varepsilon_0}\right)$
- αν $z>c$: $\vec{E}_1(x,y,z) = \hat{i}_z\left(+\frac{\sigma}{2\varepsilon_0}\right)$

B Για την άπειρη γραμμική κατανομή ηλ. φορτίου με πυκνότητα λ, έχουμε



$\vec{E}_2(x,y,z) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_T}\hat{i}_{R_T}$

$\vec{r'} = t\cdot\hat{i}_p$

$\theta = 45^o,\ \varphi = 60^o$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\sin\varphi = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\cos\varphi = \frac{1}{2}$

$\vec{r} = (x, y, z)$

$\hat{i}_p = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$\vec{R_T} = \vec{r} - t^*\hat{i}_p$

$\vec{R_T}\cdot\hat{i}_p = 0 \Rightarrow t^* = \vec{r}\cdot\hat{i}_p = x\sin\theta\cos\varphi + y\sin\theta\sin\varphi + z\cos\theta$

$\vec{R_T} = \left(x - t^*\sin\theta\cos\varphi,\ y - t^*\sin\theta\sin\varphi,\ z - t^*\cos\theta\right)$

άρα $R_T = |\vec{R_T}|$, $\hat{i}_{R_T} = \frac{\vec{R_T}}{R_T}$

Για ευκολία αντικαθιστούμε με $\varphi=60^\circ$, $\Theta=45^\circ$:

$$t^* = \frac{\sqrt{2}}{4}x + \frac{\sqrt{6}}{4}y + \frac{\sqrt{2}}{2}z$$

$$\vec{R_T} = \hat{i}_x\left(\frac{7x}{8} + \left(1-\frac{\sqrt{3}}{8}\right)y + \frac{7z}{4}\right) + \hat{i}_y\left(\left(1-\frac{\sqrt{3}}{8}\right)x + \frac{3y}{8} + \left(1-\frac{\sqrt{3}}{4}\right)z\right)$$
$$+ \hat{i}_z\left(\frac{7x}{4} + \left(1-\frac{\sqrt{3}}{4}\right)y + \frac{1}{2}z\right)$$

$R_T = |\vec{R_T}|$ και $\hat{i}_{R_T} = \dfrac{\vec{R_T}}{R_T}$

Άρα $$\vec{E}(x,y,z) = \begin{cases} \hat{i}_z\left(-\frac{\sigma}{2\varepsilon_0}\right) + \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_T}\hat{i}_{R_T}, & z<c \\ \\ \hat{i}_z\left(\frac{\sigma}{2\varepsilon_0}\right) + \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_T}\hat{i}_{R_T}, & z>c \end{cases}$$

Το $\hat{i}_{R_T}$ γράφεται συναρτήσει των $\hat{i}_x$, $\hat{i}_y$, $\hat{i}_z$ παραπάνω.
και:
$$R_T = \sqrt{R_{Tx}^2 + R_{Ty}^2 + R_{Tz}^2}$$

(β) Προβολή του $\vec{E}(x,y,z)$ στο επίπεδο xy, για $z=0$:

$$\vec{E}(x,y,0) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_{T\beta}}\hat{i}_{R_{T\beta}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_{T\beta}^2}\vec{R_T}$$

όπου $\vec{R_{T\beta}} = \hat{i}_x\left(\frac{7x}{8} + \left(1-\frac{\sqrt{3}}{8}y\right)\right) + \hat{i}_y\left(\left(1-\frac{\sqrt{3}}{8}\right)x + \frac{3y}{8}\right)$

(γ) Προβολή του $\vec{E}(x,y,z)$ στο επίπεδο xz, για
$y = y_0 = 0.5m$

$$\vec{E}(x,y_0,z) = \begin{cases} \hat{i}_z\left(-\frac{\sigma}{2\varepsilon_0}\right) + \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_{T\gamma}}\hat{i}_{R_T}, & z<c \\ \hat{i}_z\left(\frac{\sigma}{2\varepsilon_0}\right) + \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{R_{T\gamma}}\hat{i}_{R_{T\gamma}}, & z>c \end{cases}$$

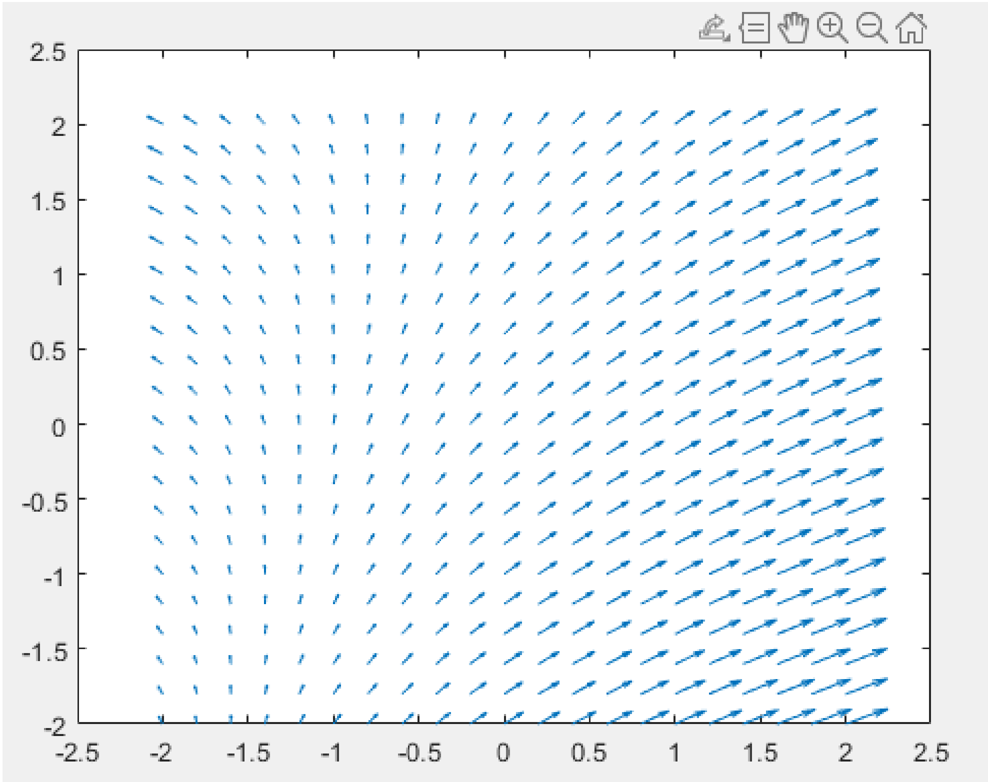
$$\vec{R_{T\gamma}} = \hat{i}_x\left(\frac{7x}{8} + \left(\frac{1}{2}-\frac{\sqrt{3}}{16}\right) + \frac{7z}{4}\right) + \hat{i}_z\left(\frac{7x}{4} + \left(\frac{1}{2}-\frac{\sqrt{3}}{8}\right) + \frac{1}{2}z\right)$$

Για τα ερωτήματα β) και γ) της άσκησης 11:

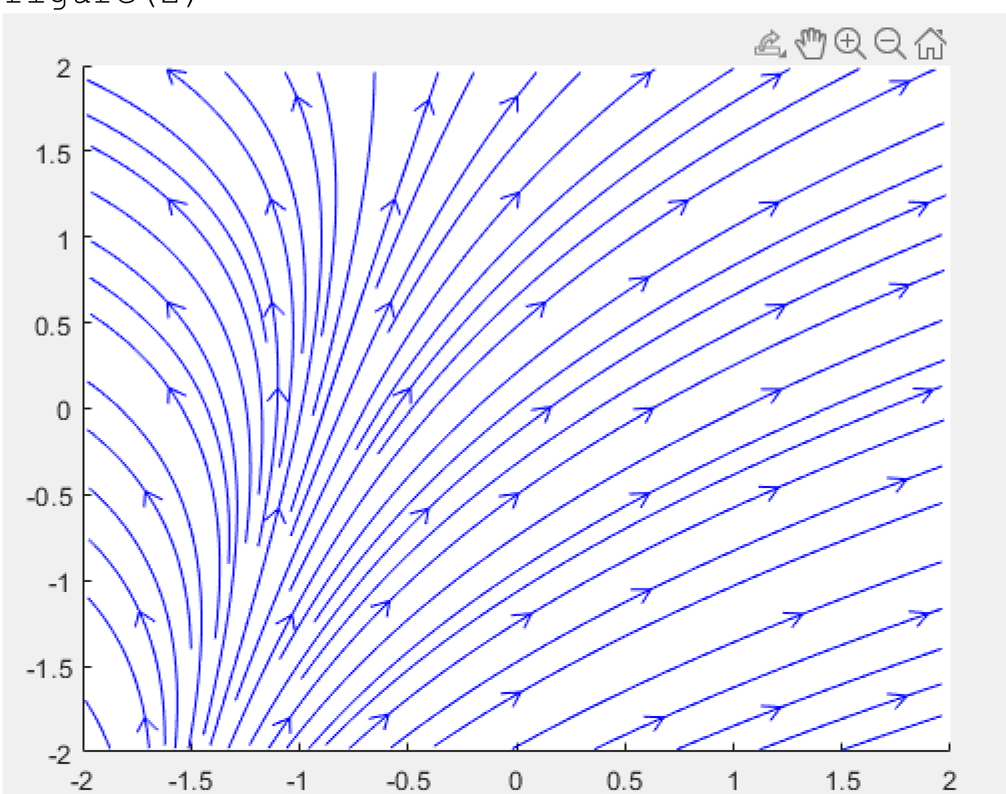
Ο κώδικας:

```
%b
[x,y] = meshgrid(-2:0.2:2,-2:0.2:2);

Rtx_b=7/8*x+(1-(sqrt(3)/8)*y);
Rty_b=(1-(sqrt(3)/8)*x)+3/8*x;

Rt_magn_b=norm([Rtx_b Rty_b]);

E0_b=1.5/(2*pi*Rt_magn_b.^2);

Ex_b=E0_b.*Rtx_b;
Ey_b=E0_b.*Rty_b;

figure(1)
quiver(x,y,Ex_b,Ey_b, 1);

figure(2)
streamslice(x,y,Ex_b,Ey_b);

%c
[x,z] = meshgrid(-2:0.1:2,-2:0.1:2);

Rtx_c=(7/8*x)+0.5-(sqrt(3)/16)+(7*z/4);
Rtz_c=(7/4*x)+0.5-(sqrt(3)/8)+z./2;

Rt_magn_c=norm([Rtx_c Rtz_c]);

E0_c=1.5/(2*pi*Rt_magn_c.^2);

Ex_c=E0_c.*Rtx_c;

%z<1
Ez_c1=-0.5+E0_c.*Rtz_c;

%z>1
Ez_c2=0.5+E0_c.*Rtz_c;

Ez_c=Ez_c1.*(x<1)+Ez_c2.*(x>1);

figure(3)
quiver(x,z,Ex_c,Ez_c, 1);

figure(4)
streamslice(x,z,Ex_c,Ez_c);
```

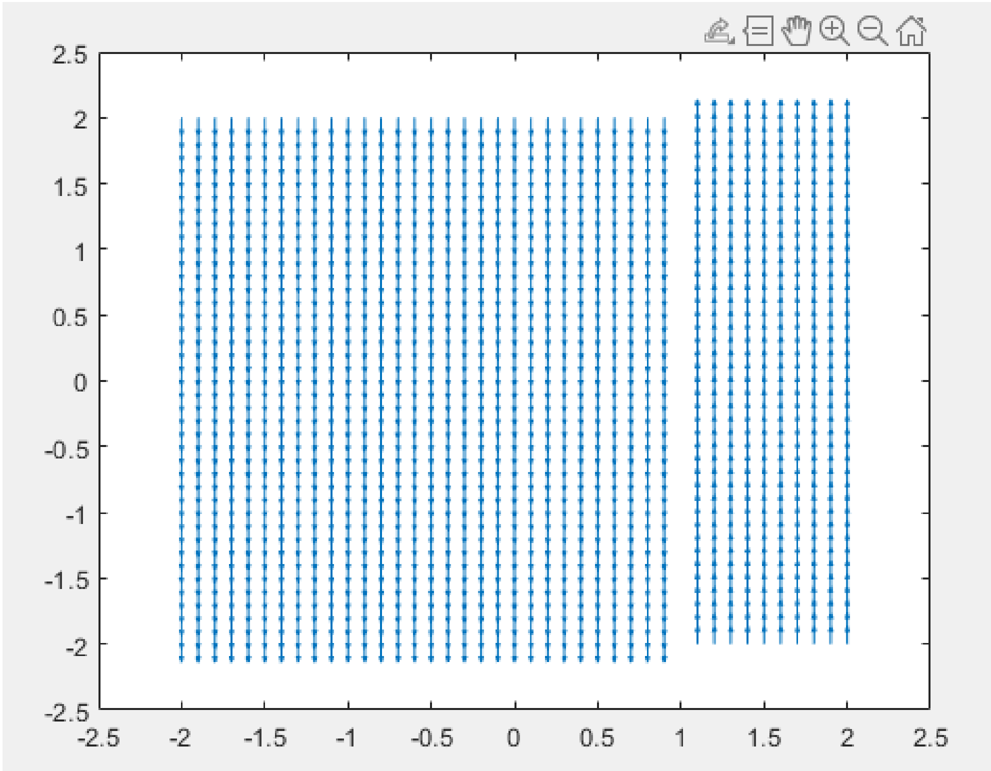
figure(1)



figure(2)



figure(3)



figure(4)