ME 231- Fluid Mechanics

Term Paper- Application of Fluid Mechanics in Ink-Jet Printers

Presented by:

S.V.Koushik (11941070)

S.Yaswanth (11941180)

❖ Introduction:

Fluid Mechanics is one of the most important branches of engineering mechanics which deals with the study of fluids which include liquids, gases, and even plasmas. Fluid Mechanics has many vibrant applications in the field of engineering which include power generation in hydropower plants, irrigational applications like making pumps, compressors, etc, jet propulsion, maintaining air traffic, controlling water supply, and many more. One such most important application of fluid mechanics is the **Ink-Jet Printing** mechanism.

The concept of Ink-Jet printers was initially discovered in the 1950s by **Ichiro Endo** accidentally but later developed and brought into commercial purpose in the 1970s by various companies. Two important technologies use the concept behind inkjet printers. They are:

i. **Continuous Inkjet Printers (CIJ):**
   The continuous ink-jets commercially called CIJs are mostly used for marking different codes on the products in manufacturing industries. These codes include price, manufacturing and expiry dates, etc. Since the products need to manufacture continuously, the ink-jet ejects the ink continuously on the products.

ii. **Drop on demand Inkjet Printers (DOD):**
   The drop on demand ink-jet printers uses ink only when it is needed but not continuous. We have two types of these printers based on types of vibrating heads:
   a. **Thermal Ink-Jet Technology (TIJ)** which is mostly used for basic purposes
   b. **Piezo Printhead Technology** which is commonly used for industrial purposes.

Although there are many types of ink-jet printers that are classified based on the purpose, type of vibrating tool, and printheads, the basic concept of using **Fluid Mechanics** to produce the required print remains the same.

❖ Application of Fluid Mechanics in Ink-Jet Printers:

The usage of precisely formed drops has many applications in our daily life. One such example is **Ink-Jet Printers.**

When we have a flow of liquid through a hole of a cup, we just see a flow as in ***fig 1.1***. The flow just as a continuous steady flow.

But when this is observed with very slow speed one could observe the wobbling and the formation of bubbles as shown in ***fig 1.2***

fig 1.1

fig 1.2

The droplets which are almost spherical in shape are formed because of the liquid tendency to minimize its surface area. But, this forms after a fixed distance due to the fact that the liquid experiences the inertial force in the beginning. After a fixed distance the inertial force is dominated by the liquid tendency to obtain minimum surface area. So, the droplets are formed.

This cup when tapped continuously forms the drop well before travelling a certain distance due to the fact that we reduced the inertial force. This phenomenon is used to obtain the drops of our desired shape and desired location in ink-jet printers.

Basic Structure of Ink-Jet Printers:

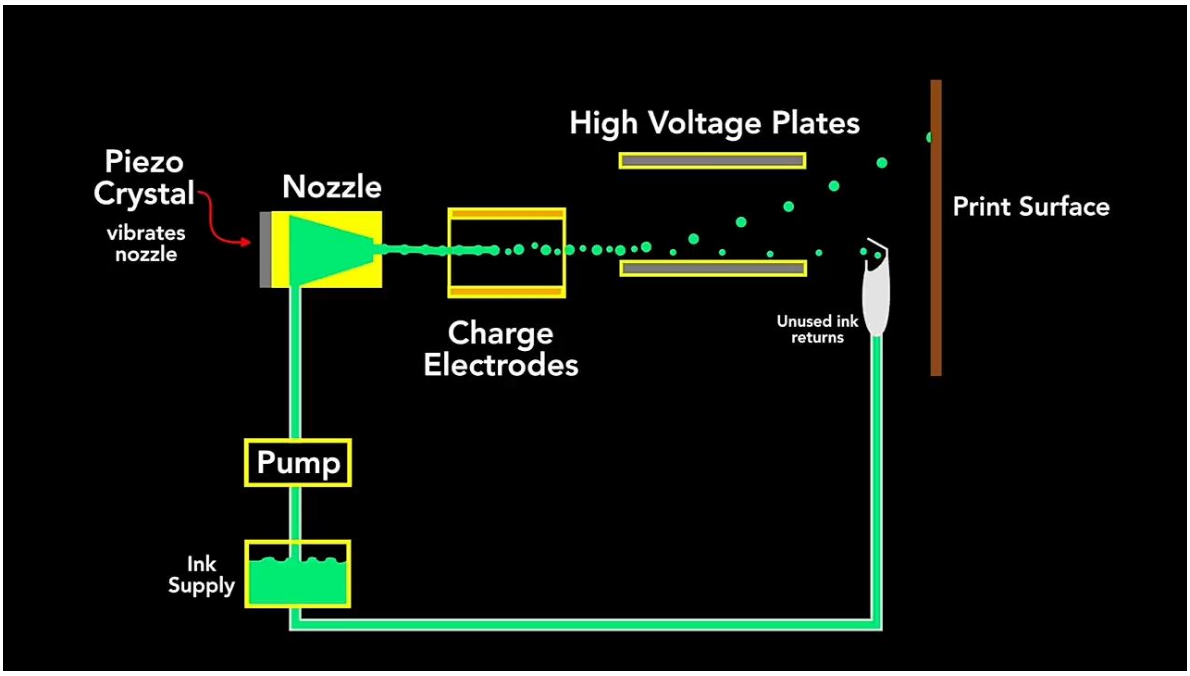


fig 1.3

In the ***fig 1.3*** there is ink supply from where the ink is supplied to the nozzle through the pump. The nozzle consists a **piezo-electric** material which produces pressure waves. These pressure waves help as a tapping force and helps in the formation of drop along with some extremely small sized particles called **satellite droplets**. These satellites are not desired as the it interrupts the quality of the print.

To get rid of these satellites from the desired sized droplets we charge the droplets and passed through a high voltage across the plates. This leads to the deflection of desired sized droplets. The satellites go undeflected into an ink collector and it returns back to the ink supply. The desired droplets hit the substrate and helps us to get the print.

The piezo-electric material vibrates according to the type of pulse given like square wave, sinusoidal wave etc. These are the materials which deforms when voltage is applied making it to produce pressure waves.

These ink-jet printers are majorly used in the industries to print different codes as you could see in ***fig 1.4***

fig 1.4

❖ Real Life Application of Ink-Jet Printers:

Problem Statement:

You are an engineer who checks the performance of ink-jet printers (DOD) in your company. The printers are printing using Silver neo-decanoate dissolved in toluene which is assumed to be a Newtonian Fluid and has 1000 nozzles in each printer. You are given these measurements to check the performance:

- **6mm radius of capillary tube inside the nozzle**
- **3mm radius of hole through which ink ejects from the nozzle**
- **Viscosity of ink is 4 mPa/sec**
- **Length of capillary tube is 0.02m**
- **The pressure wave applied by the piezo-electric transducer is a discontinuous square wave with wave width 5×10^{-5} sec and the pressure gradient factor (K) is 0.05. It is the factor which alters the speed of the droplet based on the pressure gradient applied**
- **The forcing pressure which includes all the pressures (pressure generated by piezo-electric transducer, pressure due to ink reservoir, pressure due to capillary force, pressure due to gravitational force due to droplets falling) coming from inside the printer is 1.5atm**
- **The resistant pressure which includes all the pressures (atmospheric pressure, pressure due to surface tension, momentum force pressure caused by an ink droplet) coming from outside the printer is 1.475atm**

Apart from these it is found that, 44% of the ink evaporated when dried on the substrate and 25% of the ink ejected from the nozzle are satellite droplets.

According to the product analysis the product is considered to be effective if the volume of ink on the substrate after 10sec is in between 35mL and 40mL. As an engineer estimate the total volume of ink on the substrate after 10sec and also decide whether the product is effective or not.

Problem Solution:

The given information in question is

No. of nozzles in the printer = 1000.

Radius of capillary tube inside the nozzle (R) = 6mm

Radius of ~~capillary~~ hole through which ink ejects from nozzle (r_0) = 3mm.

Viscosity of ink = 4mPa/sec

Length of capillary tube = 0.02m

The width of discontinuous square wave caused by pressure due to piezo-electric transducer (T) = 5×10^{-5} sec.

pressure gradient factor (K) = 0.05.

Forcing pressure (P_1) = 1.5 atm.

Resisting pressure (P_2) = 1.475 atm.

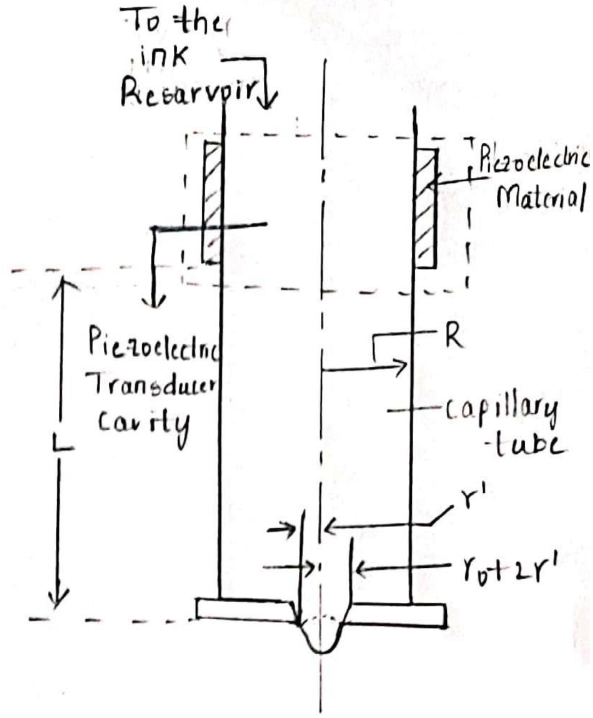
Note: 44% of the ink is evaporated when dried on the substrate.

25% of ink ejected from nozzle are satellite droplets.

Req: Total volume of ink on the substrate after 10sec.

Note: The product is considered to be effective if the volume of ink on the substrate after 10sec is 35mL to 40mL.

In this process an axial flow will commence initially and gradually approaches the steady Poiseuillie flow.



By considering the Navier-Stokes equation in cylindrical co-ordinates (r,θ,z).

$$\frac{1}{r}\left(\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r}\frac{\partial(u_\theta)}{\partial\theta}+\frac{\partial u_z}{\partial z}=\Delta U$$

$\vec{U}$ – is velocity field in (r,θ,z)

Since the flow is along the z-direction, the Navier-Stokes equation will become

$$\rho\left[\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial\theta}+u_z\frac{\partial u_z}{\partial z}\right]$$
$$=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]$$

By considering the assumptions

* Incompressible $(\Delta U=0)$
* steady flow — a flow in which velocity of fluid at a particular fixed point does not change with time.
* negligible radial velocity — [since the flow is along z-direction remaining factors can be negligible]

So, the Navier-Stokes equation will become.

$$\boxed{\mu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right]=-\frac{\partial p}{\partial z}} \quad \text{— (1)}$$

, u – is longitudinal velocity.

consider P_1 as summation of all the pressures formed due to forcing (or) forcing preasure.

P_2 as summation of all the pressures formed due to resisting (or) resisting pressure.

L — length of capillary

Then $\frac{\partial P}{\partial z}$ becomes $\frac{(P_2 - P_1)}{L}$

Now the Navior-Stokes equation will becomes

$$\boxed{\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \frac{P_1 - P_2}{\mu L} = 0 \quad -②}$$

, where μ is the viscosity of fluid.

As the above equation is of the form of a second-Order differential equation.

After solving, the equation becomes,

$$\boxed{u(r) = (k)\left(\frac{P_1 - P_2}{4\mu L}\right)(R^2 - r^2) \quad — ③}$$

where (k) is the pressure gradiant factor which retians after the flow converts due to the change in pressure into Poiseuillie flow from axial flow.

Now,

The rate flow is the quantity of liquid passing through any cross section per unit time, is given by

$$Q = \int_0^{2\pi}\int_0^R u(r)\, dr\, d\theta$$

By considering $y = r/R$ and $a = r_0/R$

$$Q = 2\pi R^2 \int_0^a K\left(\frac{P_1 - P_2}{4\mu L}\right) R^2(1 - y^2)\, y\, dy$$

$$Q = 2\pi R^4 (k) \left(\frac{P_1 - P_2}{4\mu L}\right) \int_0^a (y - y^3)\, dy$$

$$Q = \frac{P_1 - P_2}{8\mu L} K\pi R^2 r_0^2 \left[1 - \left(\frac{1}{2}\right)\left(\frac{r_0}{R}\right)^2\right]$$

By considering the real-time steady flow rate, the flow rate will become proportional to the applied jet frequency (f).

$$Q_r = fTQ$$

Then

$$\boxed{Q_r = fT \frac{\pi R^2 r_0^2 K}{8\mu L}\left[1 - \left(\frac{1}{2}\right)\left(\frac{r_0}{R}\right)^2\right](P_1 - P_2)} \quad -④$$

And the Total volume of ink on substrate after a certain time t (sec) is

$$\boxed{V(cm^3) = Q(m^3/s) \times t(s) \times 10^6 (cm^3/m^3) \times \left(\begin{array}{c}\text{evapouration}\\ \text{factor}\end{array}\right) \times \left(\begin{array}{l}\text{ink remaining}\\ \text{after the formation}\\ \text{of satellite droplets.}\end{array}\right)} \quad -⑤$$

According to the question

No. of nozzles in each printer though which the ink ejects per unit second is frequency (f) = 1000 s^{-1}

$r_0 = 3mm$

$R = 6mm$

viscosity of ink $(\mu) = 4 mPa/sec$

$L = 0.02m$

P_1 - forcing pressure = 1.5atm

P_2 = resistant pressure = 1.475atm

$T = 5\times10^{-5}$ sec

$K = 0.05$

evapouration factor $= (100-44) = 56\%$.

Ink formed over the substrate after formation of satellite droplets $= (100-25) = 75\%$.

Now the net rate of flow according to equation (4) is

$$Q_r = fT \frac{\pi R^2 r_0^2 K}{8 \mu L}\left[1-\left(\frac{1}{2}\right)\left(\frac{r_0}{R}\right)^2\right](P_1-P_2)$$

$$= (1000)(5\times10^{-5}) \frac{\pi (6\times10^{-3})^2 (3\times10^{-3})^2 (0.05)}{(8)(4\times10^{-3})(0.02)}\left[1-\left(\frac{1}{2}\right)\left[\frac{3\times10^{-3}}{6\times10^{-3}}\right]^2\right](0.025) \times (101325)$$

$[\because 1 \text{ atm} = 101325 \text{ pa}]$

$$\Rightarrow \frac{(5\times10^{-2})\,\pi\,(18\times10^{-6})^2 (5\times10^{-2})}{(8)(4\times10^{-3})(2\times10^{-2})}\left[1-\left(\frac{1}{2}\right)\left[\frac{1}{2}\right]^2\right](25\times10^{-3})(101325)$$

$$\Rightarrow \frac{(5\times10^{-2})\,\pi\,(18)^2\times10^{-12}\times5}{8\times(4\times10^{-3})\times(2)}\left[7/8\right][25\times10^{-3}](101325)$$

$$= \boxed{8.866\times10^{-6} \ m^3/s}$$

Now, the volume of ~~substrate~~ ink on substrate after 10sec is

$$V(cm^3) = Q \times (10sec) \times \left(\begin{matrix}\text{evapouration}\\ \text{factor}\end{matrix}\right) \times \left(\begin{matrix}\text{ink on substrate after}\\ \text{removal of satellite}\\ \text{droplets}\end{matrix}\right) \times (10^6 \ cm^3/m^3)$$

$$= 8.866 \times 10^{-6} \times 10 \times \left(\frac{56}{100}\right)\times\left(\frac{75}{100}\right)\times10^6$$

$$= 37.23 \ cm^3 \simeq \boxed{37 ml.}$$

<u>Note</u>: As per the question product is considered to be effective if volume of ink on substrate after 10sec is inbetween 35ml to 40ml.

As we got volume of ink as <u>37ml</u> the product is <u>effective</u>.

❖ Conclusion:

The ink-jet printers are one of the most useful applications which uses the basic principles of fluid mechanics like Navier-Stokes equation, pressure wave concepts and serves good purpose in our daily life.

❖ References Used:

- Journal published by Butterworth Publishers
- **Explainthatstuff** website (*www.explainthatstuff.com*)
- **Imaging** website (*www.imaging.org*)