Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7,2/7,-3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that $x^2+y^2+z^2=1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

Answer:

Let C1 be [2/7,3/7,6/7], C2 be [6/7, 2/7, -3/7] and C3 be [x, y, z]

The dot product of any two columns must be zero.

C1.C2 =
$$(2/7 * 6/7) + (3/7 * 2/7) + (6/7 * -3/7) = 0$$

C2.C3 = $(6/7 * x) + (2/7 * y) + (-3/7 * z) = 0 \rightarrow 6x + 2y - 3z = 0 - Eq 1$
C3.C1 = $(x * 2/7) + (y * 3/7) + (z * 6/7) = 0 \rightarrow 2x + 3y + 6z = 0 - Eq 2$
 $2 * Eq 1 + Eq 2 \rightarrow 12x + 4y - 6z + 2x + 3y + 6z = 0 \rightarrow 14x + 7y = 0 \rightarrow y = -2x$
 $3 * Eq 2 - Eq 1 \rightarrow 6x + 9y + 18z - 6x - 2y + 3z = 0 \rightarrow 7y + 21z = 0 \rightarrow y = -3z$

x: y: z = -2: 1: -3

Question 2: Find the eigenvalues and eigenvectors of the following matrix:



You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

Answer:

Let the given matrix be A =

```
23
1
and the eigen vector be of the form
3 10
е
231
1
* = \lambda * \rightarrow 2 + 3e = \lambda and 3 + 10e = \lambdae \rightarrow 3 + 10e = (2 + 3e)e
3 10 e
е
2
3e - 8e + 3 = 0 \rightarrow e = 3, -1/3
1
1
The eigen vectors are and
-1/3
3
The eigen values are 2 + 3e = \lambda \rightarrow \lambda = 2 + 3*3 = 11 and \lambda = 2 + 3*(-1/3) =
1
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Question 3: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

Answer:

Given the eigen vector of some matrix be M = [1,3,4,5,7]

To get the unit eigen vector of given matrix, we need to divide each component by

square root of sum of squares in the same direction.

Sum of squares = 1 2 + 3 2 + 4 2 + 5 2 + 7 2 = 100 and its square root is 10

Unit Eigen Vector = [1/10,3/10,4/10,5/10,7/10]

Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

Answer:

The given three points in a 2- D space are (1,1), (2,2), and (3,4).

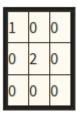
We should construct a matrix whose rows correspond to points and columns correspond

to dimensions of the space.

Mt M = 14 17

17 21

Question 5: Consider the diagonal matrix M =



Compute its Moore-Penrose pseudoinverse.

Moore-Penrose pseudoinverse means the matrix having diagonal elements replaced by 1

and divided by corresponding elements of given matrix and the other elements will be

0

0

zero. Moore-Penrose pseudoinverse of given matrix is 0 1/2 0

0

0

0

Question 6: When we perform a CUR dcomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

Answer:

Probability with which we choose now =
sum of squares of elements in the rows
sum of squares of elements in the matrix
Sum of squares of elements in the matrix = 12*13*25/6 = 3900/6 = 650
P(R1) =
P(R3) =

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