Closed Form linear regression

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h(\theta)-y)^2 = \frac{1}{2m}\sum_{i=1}^{m}(\theta^T x - y)^2$$

In Matrix Notation.

$$J(\theta) = \frac{1}{2}(x\theta - y)^T(x\theta - y)$$

$$J(\theta) = \frac{1}{2}(\theta^T x^T - y^T)(x\theta - y)$$

$$= \frac{1}{2}(\theta^T x^T x\theta - \theta^T x^T y - y^T x\theta + y^T y)$$

The goal is to find theta (θ) values where loss function (Convex function) is zero (Minimum) ~~&~~

~~To fix~~ Gradient tells us the direction of function.

① If Gradient is >0 It is going upward ↑
② If Gradient is $=0$ (Global Minima)
③ If Gradient is ~~$<$~~0 (~~Globa~~ going downward ↓)

$\therefore$ We want $\nabla J(\theta) = 0 \longrightarrow$ global Minima.

$$\frac{dJ(\theta)}{d\theta} = \frac{1}{2}\frac{d}{d\theta}(\theta^T x^T x\theta - \theta^T x^T y - y^T x\theta + y^T y)$$

$$= \frac{1}{2}(2(x^T x\theta) - x^T y - y^T x + 0)$$

$$\Rightarrow \frac{1}{2}(2(x^T x\theta - 2x^T y) = 0$$

$$\Rightarrow x^T x\theta - x^T y = 0 \rightarrow x^T x\theta = x^T y$$

$$\theta = \frac{x^T y}{x x^T}$$

$x x^T$ is Square matrix $\rightarrow \frac{1}{\text{Square Matrix}} = (\text{Square Matri})^{-1}$

$$\theta^* = (x x^T)^{-1} \circ x^T \circ y$$

$\therefore$ Optmised θ Values are $(x x^T)^{-1} \circ x^T \circ y$