#### **CONTROL OF ROBOTIC MANIPULATORS**

A Project Report submitted to NIT Durgapur in partial fulfillment of the requirement for the

Bachelor Degree

In

Electrical Engineering

By

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Under the Supervision of Dr. Jayati Dey



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Koushik Sen Roll Number 12/EE/26

## **CERTIFICATE**

This is to certify that **Koushik Sen** with **Roll No 12/EE/26** has worked of "CONTROL OF ROBOT **MANIPULATORS**" under my supervision and has fulfilled all the requirements for regulations of the institute relating to nature and period of work.

.....

Name of Faculty Designation Department of Electrical Engineering National Institute of Technology Durgapur-713209

## **ABSTRACT**

Manipulators are composed of an assembly of links and joints. Links are defined as the rigid sections that make up the mechanism and joints are defined as the connection between two links. The device attached to the manipulator which interacts with its environment to perform tasks is called the endeffector.

A robot is required to carry out a specific task by moving its end-effector accurately and repeatedly with minimum overshoot. In the case of open loop control (no feedback loop), the actuator torque can be directly computed from the dynamic model of the manipulator. This model does not include the effects of friction, backlash and so on, at the joints and external disturbances or noise. These effects are highly non-linear and hence difficult to model.

These limitations are overcome by using a closed loop scheme. Here, at every instant of time the actual joint positions and velocities are compared with the reference to compute the error between the desired and actual positions and velocities.

PID tuning method is used in the closed loop control scheme to achieve the desired performance. P, I and D are adjusted to minimize the overshoot, settling time, etc. The performance of the PID controller in presence of external disturbances is also examined.

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### **INTRODUCTION**

A ROBOT MANIPULATOR IS THE MAIN BODY OF THE ROBOT LINKS, THE JOINTS, AND WHICH CONSISTS OF THE **OTHER** THE ROBOT.WITHOUT STRUCTURAL ELEMENTS OF **OTHER** ELEMENTS, THE MANIPULATOR ALONE IS NOT A ROBOT. THIS PAPER PRESENTS THE ANALYSIS AND CONTROL OF 2 LINK AND 3 ROBOT MANIPULATORS.A POSITION AND LINK VELOCITY FEEDBACK PID CONTROL SYSTEM IS DESIGNED USING MATLAB SIMULINK MODEL FOR 2 LINK ROBOT MANIPULATOR AS WELL AS 3 LINK ROBOT MANIPULATOR .PID TUNING METHOD IS USED TO TUNE THE P,I,D VALUES OF THE SYTEM IN ORDER TO MINIMIZE THE OVERSHOOT WITHIN SOME SPECIFIED LIMIT KEEPING IN MIND THE OTHER FACTORS LIKE RISE TIME, SETTLING TIME, GAIN MARGIN AND PHASE MARGIN VALUES, ETC. ACTUATORS ARE REQUIRED TO GENERATE THE REQUIRED TORQUE OBTAINED FROM THE PID DESIGN OUTPUT. THEY ARE THE "MUSCLES" OF THE MANIPULATORS.SENSORS ARE REQUIRED TO COLLECT INFORMATION ABOUT THE INTERNAL STATE OF THE ROBOT AND SEND THOSE SIGNALS AS FEEDBACK IN A CLOSED LOOP CONTROL OF ROBOT MANIPULATORS.

#### **KEY TERMS**

#### DIRECT KINEMATICS

• IT REFERS TO THE CALCULATION OF END EFFECTOR POSITION, ORIENTATION, VELOCITY AND ACCELERATION WHEN CORRESPONDING JOINT VALUES ARE GIVEN.

#### **INVERSE KINEMATICS**

• THE REQUIRED JOINT VALUES ARE CALCULATED FOR GIVEN END EFFECTOR VALUES. AS DONE IN PATH PLANNING.

#### **DIRECT DYNAMICS**

• IT REFERS TO THE CALCULATION OF ACCELERATION IN THE ROBOT ONCE THE APPLIED FORCES ARE KNOWN.

#### **INVERSE DYNAMICS**

• IT IS THE CALCULATION OF ACTUATOR FORCES NECESSARY TO CREATE A PRESCRIBED END EFFECTOR ACCELERATION.

#### DEGREES OF FREEDOM

THE NUMBER OF DEGREES OF FREEDOM OF A MECHANISM ARE DEFINED
AS THE NUMBER OF INDEPENDENT VARIABLES THAT ARE REQUIRED TO
COMPLETELY IDENTIFY ITS CONFIGURATION IN SPACE.

#### **MANIPULATION**

- ROBOTS NEED TO MANIPULATE OBJECTS: PICKUP, MODIFY, DESTROY OR OTHERWISE HAVE AN EFFECT
- GENERAL PURPOSE MANIPULATOR HUMAN HAND

### **NEWTONIAN VERSUS LAGRANGIAN MECHANICS**

IN NEWTONIAN MECHANICS THE EQUATIONS OF MOTION ARE GIVEN BY NEWTON'S LAWS. THE SECOND LAW "FORCE EQUAL TO MASS TIMES ACCELERATION" APPLIES TO EACH PARTICLE. INSTEAD OF FORCES LAGRANGIAN MECHANICS USES THE ENERGIES IN THE SYSTEM.

#### ADVANTAGES OF LAGRANGIAN MECHANICS

- 1. LAGRANGIAN MECHANICS IS ACTUALLY IS LESS GENERAL THAN NEWTONIAN MECHANICS.
- 2. IT HELPS IN DEEP UNDERSTANDING OF PHYSICS.
- 3. It is used in optimization problem of dynamic system.
- 4. It is ideal for conservative forces and are used for bypassing the constrained forces and some non conservative forces.

CONSTRAINED FORCES ARE THE FORCES THAT THE CONSTRAINING OBJECT EXERTS ON THE OBJECT TO MAKE IT FOLLOW THE MOTIONAL CONSTRAINTS.

CONSTRAINED MOTION RESULTS WHEN AN OBJECT IS FORCED TO MOVE IN A RESTRICTED WAY.

CONSTRAINED ADJUST THEMSELVES ACCORDING TO NEWTONS 2ND LAW SO THAT THE ACCELERATION OF AN OBJECT IS JUST THE RIGHT VALUE FOR THE OBJECT TO FOLLOW THE MOTION REQUIRED BY THE PARTICULAR CONSTRAINT.

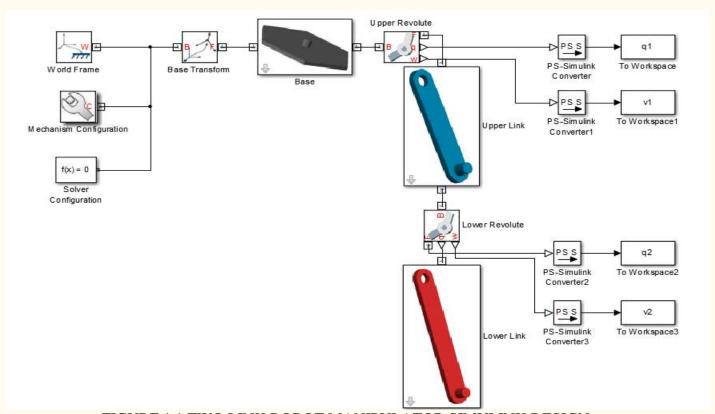


FIGURE 1.1 TWO LINK ROBOT MANIPULATOR SIMULINK DESIGN

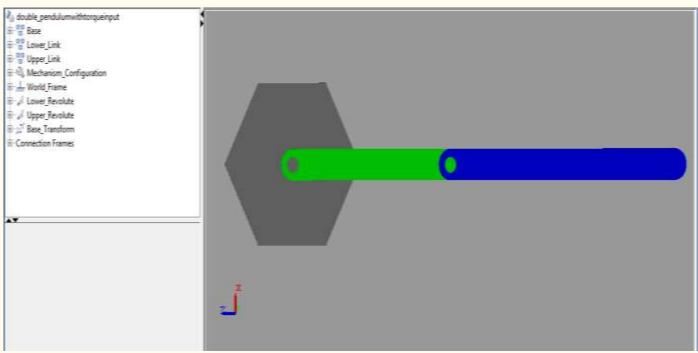
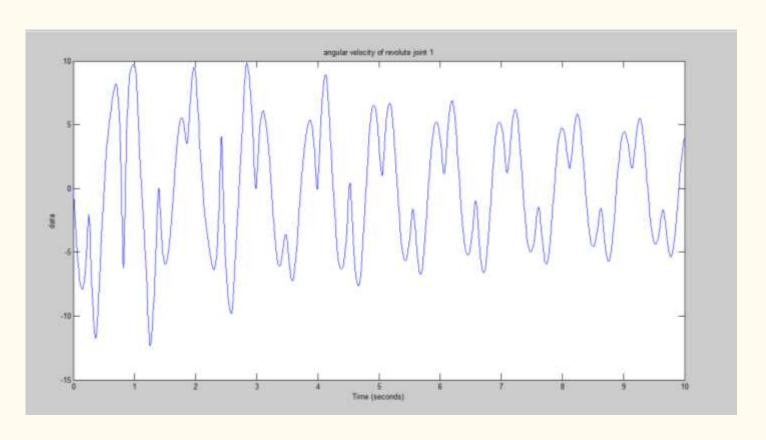


FIGURE 1.2 TWO LINK ROBOT MANIPULATOR MECHANICS EXPLORER



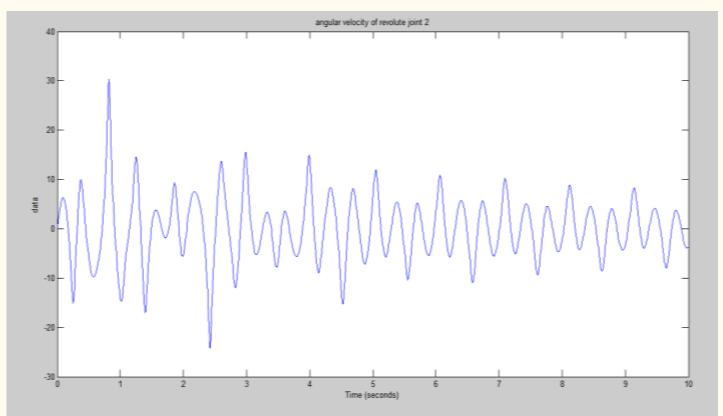
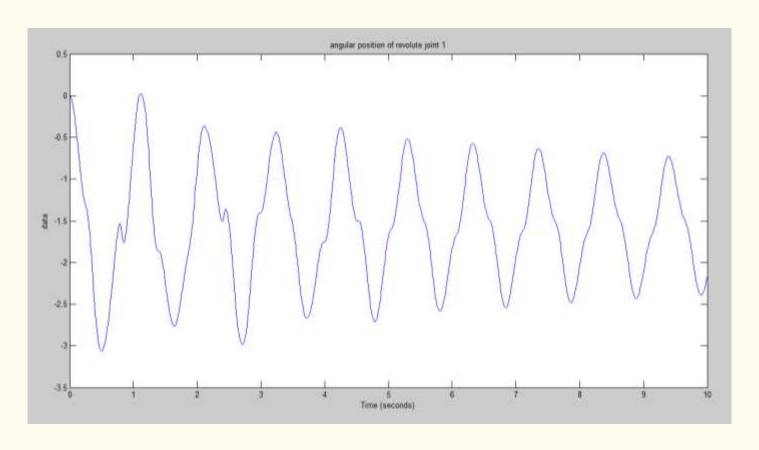


FIGURE 1.3 ANGULAR VELOCITY OF 2 LINKS WITHOUT ANY ACTUATION



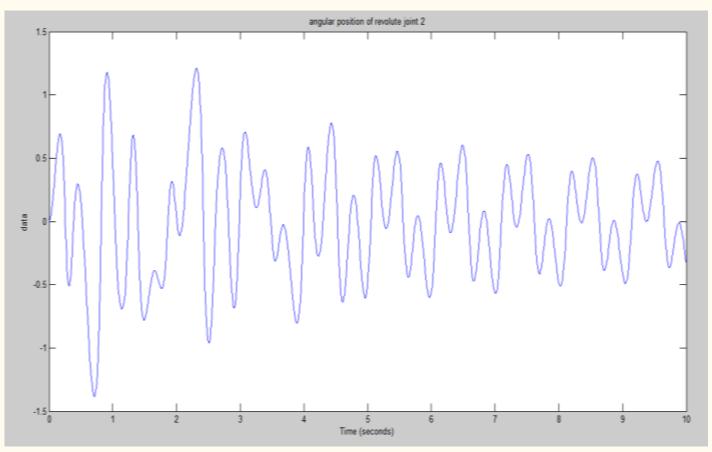


FIGURE 1.4 ANGULAR POSITION OF 2 LINKS WITHOUT ANY ACTUATION

### Kinematics of a 2 DOF Robot Manipulator:-

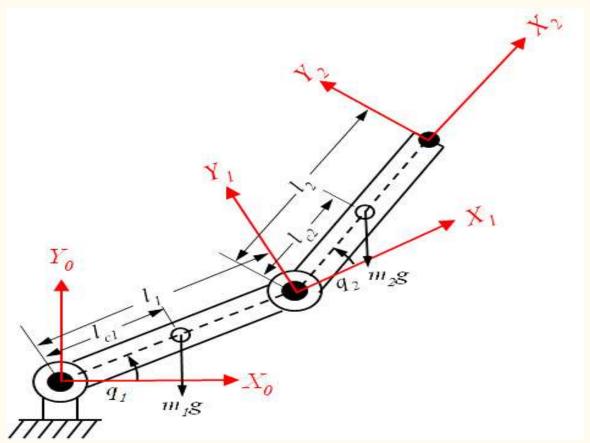


FIGURE 1.5 TWO LINK MANIPULATOR

$$\mathbf{x} = \mathbf{l}_1 \cos \mathbf{\theta}_1 + \mathbf{l}_2 \cos(\mathbf{\theta}_1 + \mathbf{\theta}_2) \qquad (i)$$

$$y = l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \qquad \dots (ii)$$

So, the joint variables are known, using forward kinematic equations, the location of the links can be calculated.

Let 
$$\cos \theta_1 = c_1$$
  
 $\cos(\theta_1 + \theta_2) = c_{1+2}$   
 $\sin \theta_1 = s_1$   
 $\sin(\theta_1 + \theta_2) = s_{1+2}$ 

#### Inverse Kinematics of a 2 DOF Robot Manipulator:-

$$\begin{split} x^2 &= I_1{}^2\cos^2\theta_1 + I_2{}^2\cos^2(\theta_1 + \theta_2) + 2.l_1.l_2.\cos\theta_1.\cos(\theta_1 + \theta_2) \\ y^2 &= I_1{}^2\sin^2\theta_1 + I_2{}^2\sin^2(\theta_1 + \theta_2) + 2.l_1.l_2.\sin\theta_1.\sin(\theta_1 + \theta_2) \\ &\dots \text{from equation (i) \& (ii)} \\ &\text{Adding the above equations;} \\ x^2 + y^2 &= I_1{}^2 + I_2{}^2 + 2.l_1.l_2.\cos\theta_1.\cos(\theta_1 + \theta_2) + 2.l_1.l_2.\sin\theta_1.\sin(\theta_1 + \theta_2) \\ &\text{or,} \quad x^2 + y^2 &= I_1{}^2 + I_2{}^2 + 2.l_1.l_2.\cos\theta_2 \\ &\text{or,} \quad \theta_2 &= \cos^{-1}\left[\{(x^2 + y^2) - (I_1{}^2 + I_2{}^2)\}/2.I_1.I_2\right] \qquad \dots (iii) \end{split}$$

Now, 
$$x = l_1 \cos\theta_1 + l_2 \cos\theta_2 \cdot \cos\theta_1 - l_2 \sin\theta_1 \cdot \sin\theta_2$$

or, 
$$x/\cos\theta_1 = (l_1 + l_2 \cos\theta_2) - l_2 \tan\theta_1 .\sin\theta_2$$
 .....(a)

$$y = l_1 \sin\theta_1 + l_2 \sin\theta_1.\cos\theta_2 - l_2 \sin\theta_2.\cos\theta_1$$

or, 
$$y/\cos\theta_1 = \tan\theta_1 \cdot (l_1 + l_2 \cos\theta_2) - l_2 \sin\theta_2$$
 (b)

$$\Rightarrow y \{ l_1 + l_2 . \cos \theta_2 - l_2 . \tan \theta_1 . \sin \theta_2 \} = x \{ \tan \theta_1 (l_1 + l_2 . \cos \theta_2) + l_2 . \sin \theta_2 \}$$

or, 
$$\theta_1 = \tan^{-1} \left[ \left\{ y \left( l_1 + l_2 \cos \theta_2 \right) - x. l_2. \sin \theta_2 \right\} / \left\{ x \left( l_1 + l_2 \cos \theta_2 \right) + y. l_2. \sin \theta_2 \right\} \right]$$
 .....(iv)

#### TO TRACK A CIRCULAR TRACK BY USING A 2 DOF ROBOT ARM

```
clear;
clc;
close all
P = [4 5];
r = 1;
L1 = 5;
L2 = 4;
xp = P(1);
yp = P(2);
Ri = sqrt(xp^2+yp^2);
Rf = Ri + r;
if Rf > (L1+L2)
fprintf('Some of the points on the circle are
nonreachable\
n');
end
alpha = 0:2*pi/20:2*pi;
x = xp + r*cos(alpha);
y = yp + r*sin(alpha);
for i = 1:1:length(x)
R(i) = sqrt(x(i)^2+y(i)^2);
th2(i) = acos((R(i)^2 - L1^2 - L2^2)/(2*L1*L2));
th1(i) = atan(((y(i)*(L1+L2*cos(th2(i))))-
x(i) *L2*sin(th2(i)))/((x(i) *(L1+L2*cos(th2(i))))+y(i)*L
2*sin(th2(i)));
link1x(i,2) = L1*cos(th1(i));
link1y(i,2) = L1*sin(th1(i));
link2x(i,1) = L1*cos(th1(i));
link2y(i,1) = L1*sin(th1(i));
link2x(i,2) = L2*cos(th1(i)+th2(i))+L1*cos(th1(i));
link2v(i,2) = L2*sin(th1(i)+th2(i))+L1*sin(th1(i));
plot(link1x(i,:),link1y(i,:),'linewidth',2);hold
on; plot (link2x(i,:), link2y(i,:), 'r', 'linewidth', 2)
plot(x,y,'black','linewidth',5);grid
```

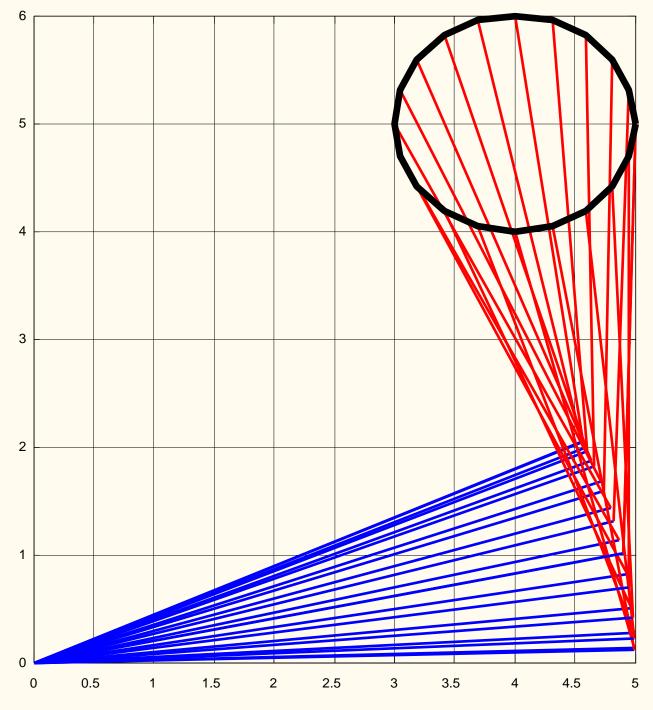


FIGURE 1.6 CIRCULAR TRACK TRACKING RESULT

#### TO TRACK ANY GIVEN POINT BY USING A 2 DOF ROBOT ARM

```
clc;
close all
x=7, y=5;
L1 = 5;
L2 = 4;
z = sart(x^2+y^2);
if z > (L1+L2)
fprintf('POINT OUT OF RANGE OF THE MANIPULATOR');
else
for i = 1:1:length(x)
R(i) = sqrt(x(i)^2+y(i)^2);
th2(i) = acos((R(i)^2 - L1^2 - L2^2)/(2*L1*L2));
th1(i) = atan(((y(i)*(L1+L2*cos(th2(i))))-
x(i) L2*sin(th2(i)))/((x(i) *(L1+L2*cos(th2(i))))+y(i)*L2*sin(th2(i)))
(i))));
link1x(i,2) = L1*cos(th1(i));
linkly(i,2) = L1*sin(th1(i));
link2x(i,1) = L1*cos(th1(i));
link2y(i,1) = L1*sin(th1(i));
link2x(i,2) = L2*cos(th1(i)+th2(i))+L1*cos(th1(i));
link2y(i,2) = L2*sin(th1(i)+th2(i))+L1*sin(th1(i));
plot(link1x(i,:),link1y(i,:),'linewidth',2);hold
on; plot(link2x(i,:),link2v(i,:),'r','linewidth',2)
end
plot(x,y,'black','linewidth',5);grid
for i = 1:1:length(x)
R(i) = sgrt(x(i)^2+y(i)^2);
th2(i) = -1*acos((R(i)^2 - L1^2 - L2^2)/(2*L1*L2));
th1(i) = atan(((y(i)*(L1+L2*cos(th2(i))))-
x(i) L2*sin(th2(i)))/((x(i) L1+L2*cos(th2(i))))+y(i)*L2*sin(th2(i)))
(i))));
link1x(i,2) = L1*cos(th1(i));
link1y(i,2) = L1*sin(th1(i));
link2x(i,1) = L1*cos(th1(i));
link2y(i,1) = L1*sin(th1(i));
link2x(i,2) = L2*cos(th1(i)+th2(i))+L1*cos(th1(i));
link2y(i,2) = L2*sin(th1(i)+th2(i))+L1*sin(th1(i));
plot(link1x(i,:),link1y(i,:),'linewidth',2);hold
on;plot(link2x(i,:),link2y(i,:),'r','linewidth',2)
end
plot(x,y,'black','linewidth',5);grid
end
```

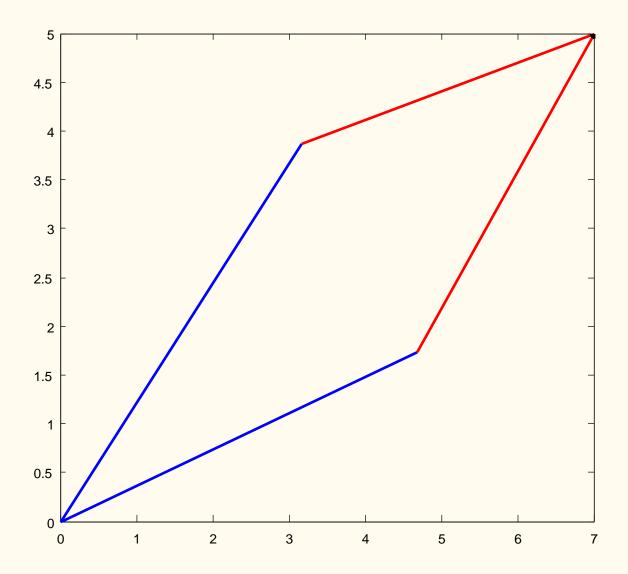


FIGURE 1.7 A SINGLE POINT TRACKING RESULT

## Lagrangian Mechanics:-

It is based on the differentiation of the energy terms with respect to the system's variables and time.

Lagrangian is defined as: L = K - P;

where K = Kinetic energy of the system

P = Potential energy of the system

It is based on following two generalised equations;

$$F_{i} = \frac{\partial}{\partial t} (\partial L / \partial x_{i}^{*}) - \partial L / \partial x_{i} \qquad \text{(for linear motion)}$$

$$T_{i} = \frac{\partial}{\partial t} (\partial L / \partial \theta_{i}^{*}) - \partial L / \partial \theta_{i} \qquad \text{(for rotational motion)}$$

 $F_i$  = Summation of all external forces for a linear motion.

 $T_i$  = Summation of all external forces for a rotational motion.

 $\theta_i$  &  $x_i$  are system variables. ( $\theta_i$ ` &  $x_i$ ` is  $1^{st}$  differential of  $\theta_i$  &  $x_i$  respectively)

## Equation of Motion for the 2 DOF Robot Manipulator Using Lagrangian Method:-

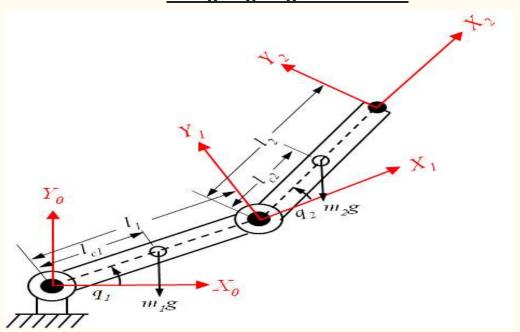


FIGURE 2.1 TWO LINK MANIPULATOR

$$\begin{split} X_D &= I_1 c_1 + 0.5 \ I_1 c_{12} \\ Y_D &= I_1 s_1 + 0.5 \ I_2 s_{12} \dots & (i) \\ X_{D'} &= -I_1 s_1 \theta_1' - 0.5 \ I_2 s_{12} \left(\theta_1' + \theta_2'\right) \\ Y_{D'} &= I_1 c_1 \ \theta_1' - 0.5 \ I_2 c_{12} \left(\theta_1' + \theta_2'\right) \dots & (ii) \end{split}$$

#### Total Velocity of COM of link 2 (Point D);

$$V_D = X_{D'}^2 + Y_{D'}^2 = \theta_{1'}^2 (I_{1}^2 + 0.25 I_{2}^2 + I_{1}I_{2} (s_{1}s_{12} + c_{1}c_{12})) + \theta_{2'}^2 (0.25 I_{2}^2) + \theta_{1'}^2 \theta_{2'}^2 (0.5 I_{2}^2 + I_{1}I_{2}^2 c_{2})$$

#### The Total K.E. in the sum of K.E. of links 1 and link 2;

$$K = K_1 + K_2$$

$$= (0.5 \text{ I}_A \theta_1^{"2}) + (0.5 \text{ I}_D(\theta_1' + \theta_2')^2 + 0.5 \text{ m}_2\text{V}_D^2)$$
(Purely rotational (Rotational K.E. (Translational About A) About B) About D)

 $I_A$  = Moment of Inertia about endpoint = 1/3 \*  $MI^2$ 

 $I_D$  = Moment of Inertia about middle point = 1/12 \*  $Ml^2$ 

K.E. = 
$$(0.5 (1/3 * Ml^2) \theta_1'^2) + (0.5(1/12 * Ml^2).(\theta_1' + \theta_2') + 0.5 m_2 V_D^2)$$

 $K = \theta_1'^2 (1/6 \text{ m}_1 \text{l}_1^2 + 1/6 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2 + 0.5 \text{m}_2 \text{l}_1 \text{c}_1 \text{c}_2) + \theta_2'^2 (1/6 \text{ m}_2 \text{l}_2^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2 \text{c}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2 \text{c}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2 \text{c}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2^2) + (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2^2) + (1/3 \text{ m}_2^2 + 0.5 \text{m}$ 

0.5m<sub>2</sub>l<sub>1</sub>c<sub>1</sub>c<sub>2</sub>)

$$P = m_1 g I_1 s_1 / 2 + m_2 g (I_1 s_1 + I_2 s_{12} / 2)$$

So, Lagrangian for the 2 link robot arm;

$$L = K - P$$

$$L = \theta_1'^2 (1/6 \text{ m}_1 \text{l}_1^2 + 1/6 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1^2 + 0.5 \text{m}_2 \text{l}_1 \text{c}_1 \text{c}_2) + \theta_2'^2 (1/6 \text{ m}_2 \text{l}_2^2)$$

$$+ \theta_1^2 \theta_2^2 (1/3 \text{ m}_2 \text{l}_2^2 + 0.5 \text{m}_2 \text{l}_1 \text{c}_1 \text{c}_2) - \text{m}_1 \text{gl}_1 \text{s}_1 / 2 + \text{m}_2 \text{g}(\text{l}_1 \text{s}_1 + \text{l}_2 \text{s}_{12} / 2)$$

Taking the Derivatives of the Lagrangian and substituting in

$$T_1 = \partial/\partial t(\partial L/\partial \theta_1) - \partial L/\partial \theta_1$$

$$T_2 = \partial/\partial t(\partial L/\partial \theta_2) - \partial L/\partial \theta_2$$

+ 
$$\begin{pmatrix} 0 & -0.5 \text{ m}_2 \text{l}_1 \text{l}_2 \text{s}_2 \\ 0.5 \text{ m}_2 \text{l}_1 \text{l}_2 \text{s}_2 \end{pmatrix}$$
  $\begin{pmatrix} \theta_1'^2 \\ \theta_2'^2 \end{pmatrix}$  +  $\begin{pmatrix} -\text{m}_2 \text{l}_1 \text{l}_2 \text{s}_2 \\ 0 \end{pmatrix}$   $\begin{pmatrix} \theta_1' & \theta_2' \\ \theta_2' & \theta_1' \end{pmatrix}$ 

+ 
$$(0.5 \text{ m}_1 + \text{m}_2) \text{ gl}_1\text{c}_1 + 0.5\text{m}_2\text{gl}_2\text{c}_{12}$$
  
 $0.5 \text{ m}_2\text{gl}_2\text{c}_{12}$ 

- θ" terms related to angular acceleration
- $\theta^{2}$  terms related to centrifugal acceleration
- θ<sub>1</sub>' θ<sub>2</sub>' terms are Coriolis acceleration.(present in link 1 only)
- In this first link acts as a rotating frame for link 2

# Matlab Simulation to Study the Torque Orientation by Application of Dynaminc Model Equations:-

The following data's are assumed;

$$m_1 = m_2 = 6kg$$
  $l_1 = l_2 = 1m$ 

So, the equations of motion becomes;

$$\tau_1 = (10 + 6c_2) \theta_1^{"} + (2 + 2c_2) \theta_2^{"} - 3s_2 \theta_2^{"} - 6s_2 \theta_1^{"}.$$
  
$$\theta_2^{"} + (2g.c_{12} + 9g.c_1)$$

and,

$$\tau_2 = (2 + 2c_2) \theta_1^{"} + 2.\theta_2^{"} + 3s_2.\theta_1^{"2} + 3g.c_{12}$$

The velocity, position and acceleration profiles are applied as desired to calculate the required external torque profile to be applied in the joints for desired motion.

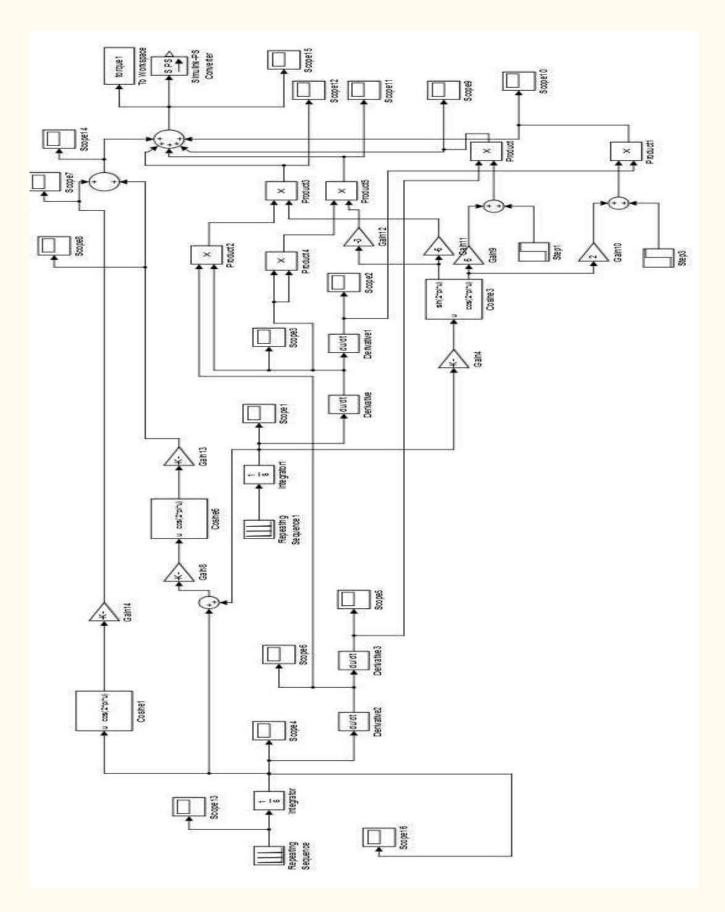


FIGURE 2.2 TORQUE 1 GENERATION FOR UPPER JOINT

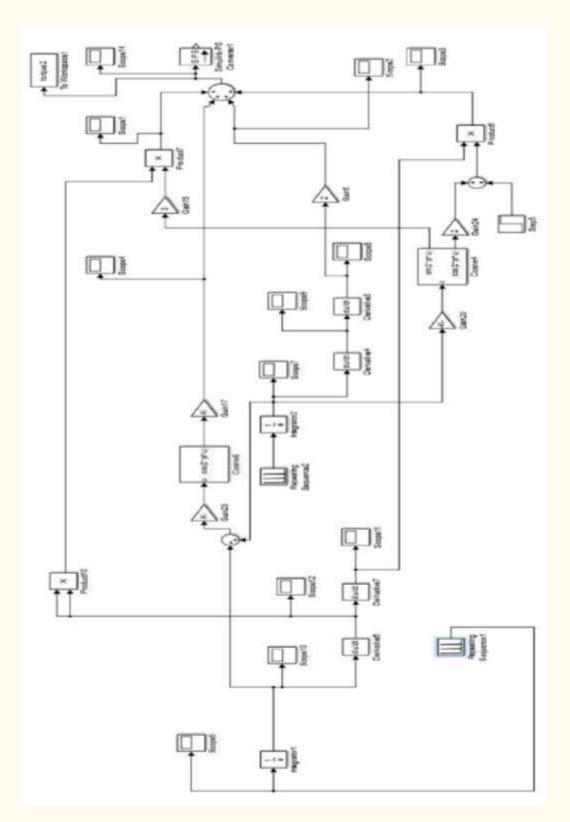
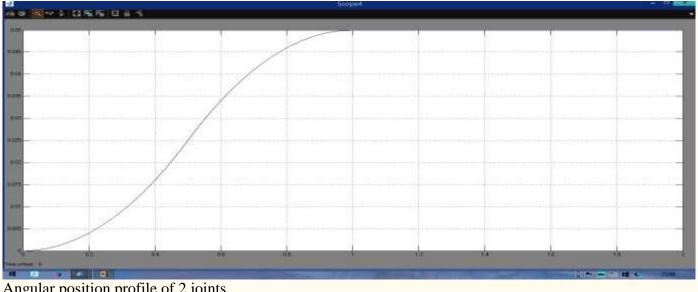
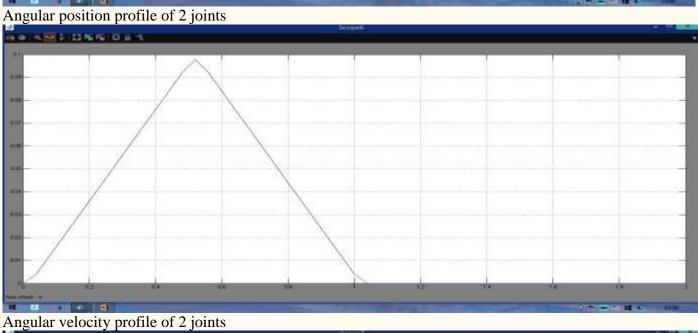
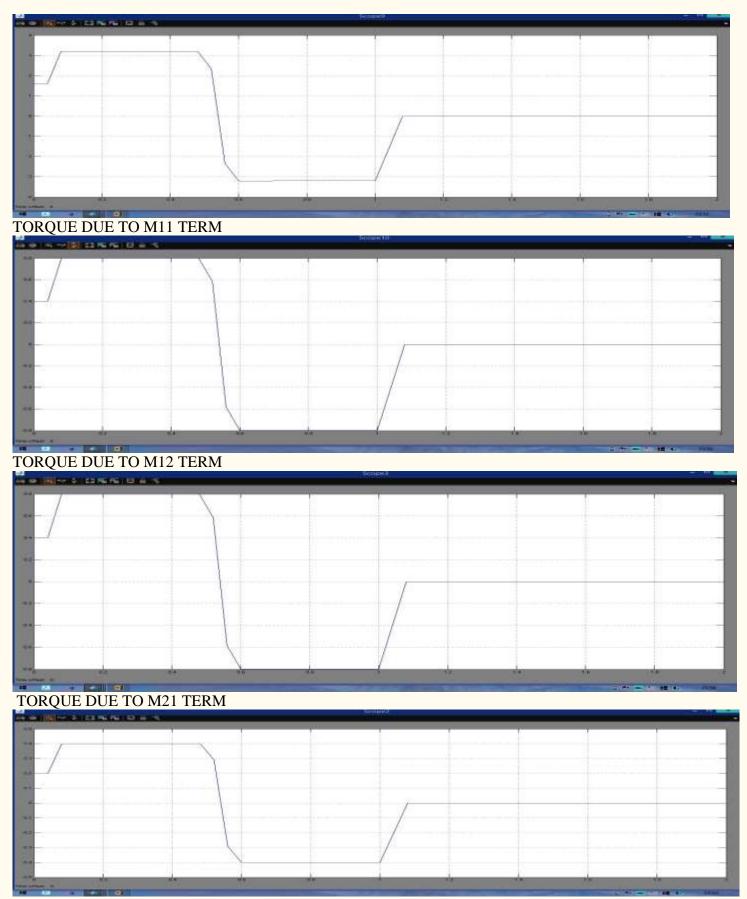


FIGURE 2.3 TORQUE 2 GENERATION FOR LOWER JOINT

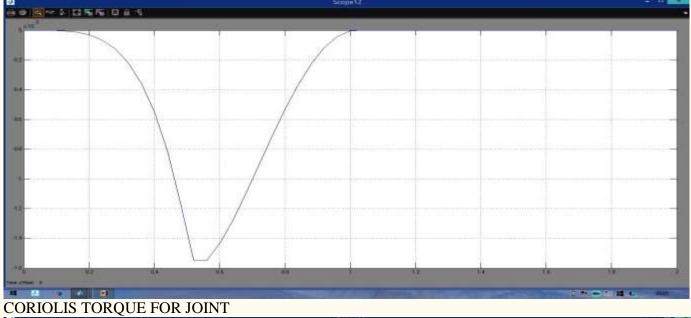


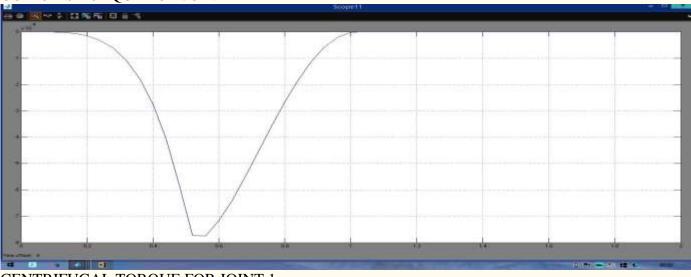




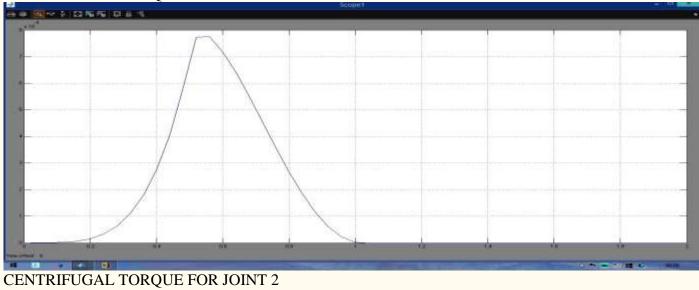


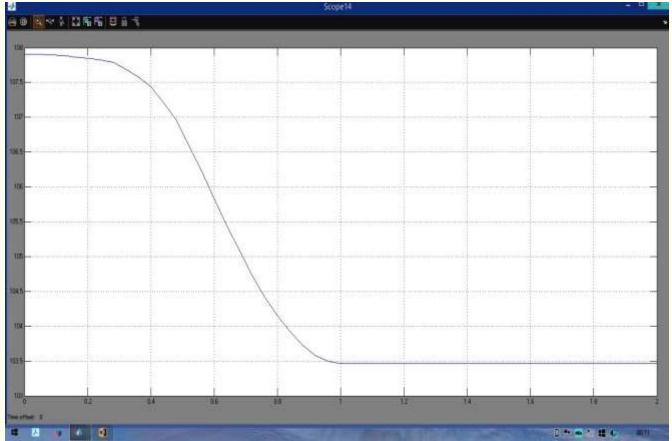
TORQUE DUE TO M22 TERM

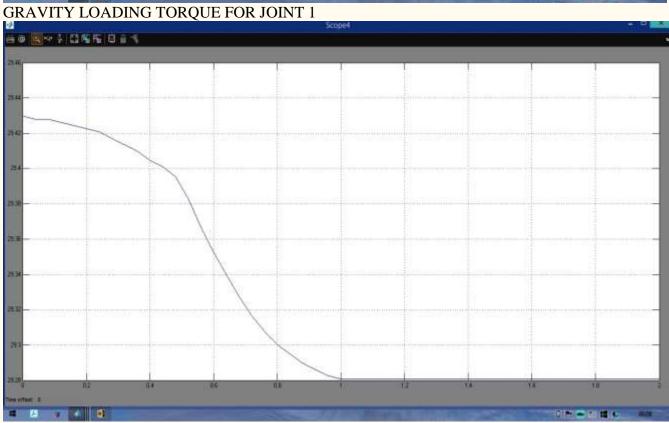




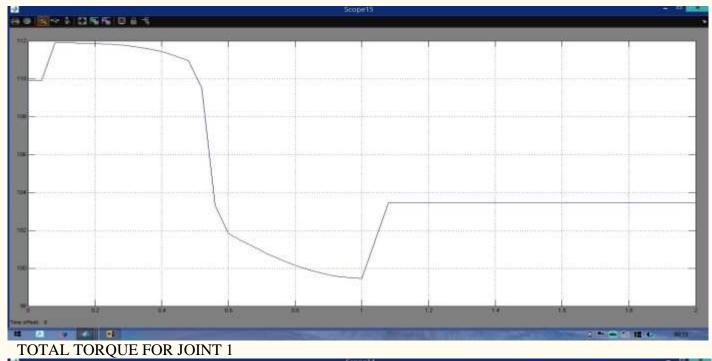
CENTRIFUGAL TORQUE FOR JOINT 1

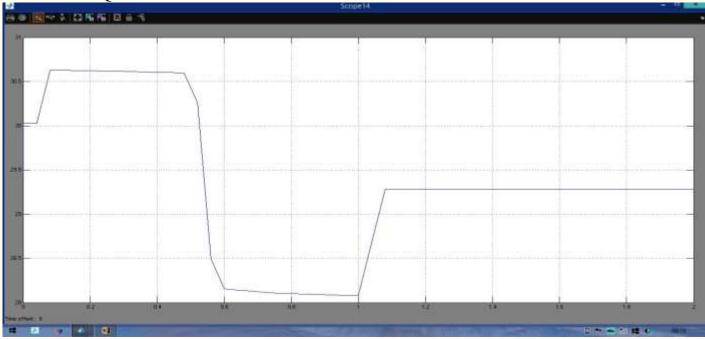






**GRAVITY LOADING TORQUE FOR JOINT 2** 





TOTAL TORQUE FOR JOINT 2

FIGURE 2.4 COMPONENTS OF THE TORQUE AND THE FINAL TORQUE
FOR JOINT 1 and 2

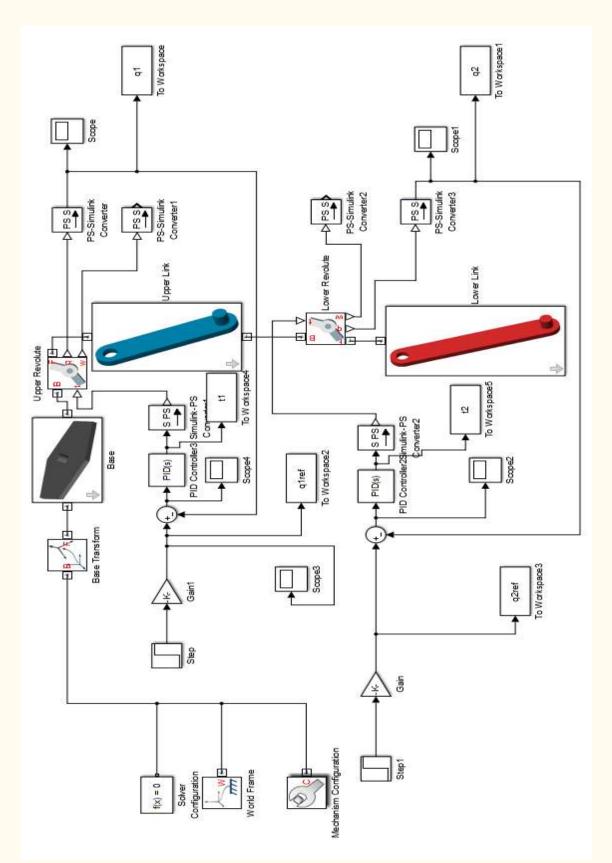
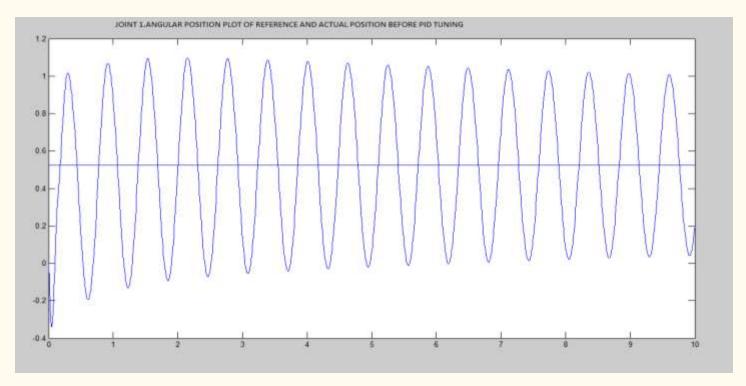


FIGURE 3.1 POSITION FEEDBACK CONTROL OF 2 LINK ROBOT

MANIPULATOR

#### **DAMPING COEFFICIENT 0.1 (N-m/(rad/sec))**



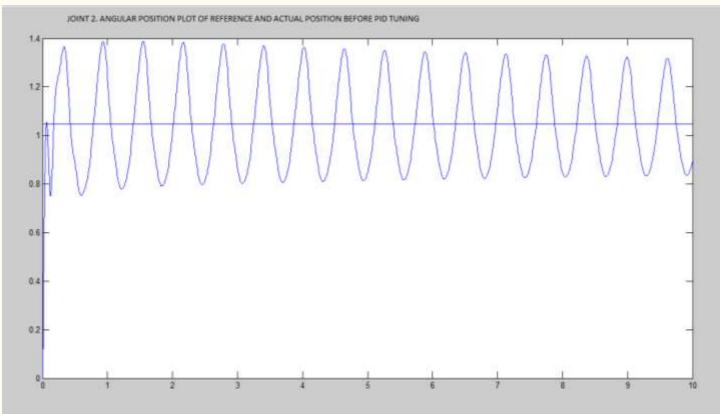
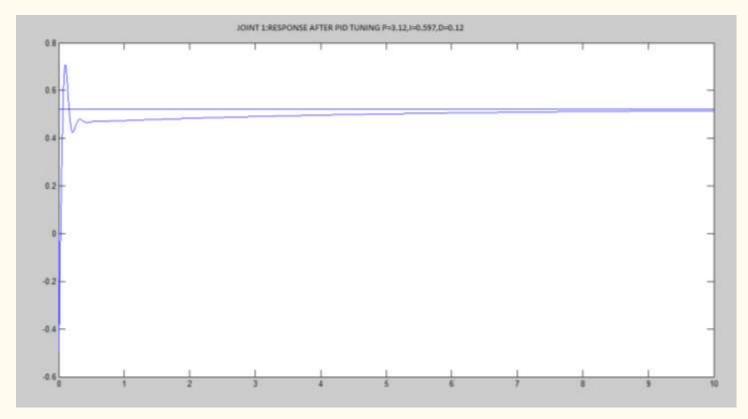


FIGURE 3.2 ANGULAR POSITION BEFORE PID TUNING IS DONE

#### **DAMPING COEFFICIENT 0.1 (N-m/(rad/sec))**



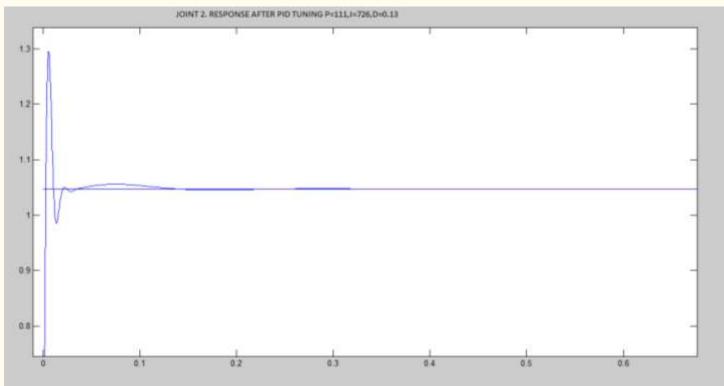


FIGURE 3.3 ANGULAR POSITION AFTER PID TUNING IS EXECUTED

#### DAMPING COEFFICIENT 1 (N-m/(rad/sec))

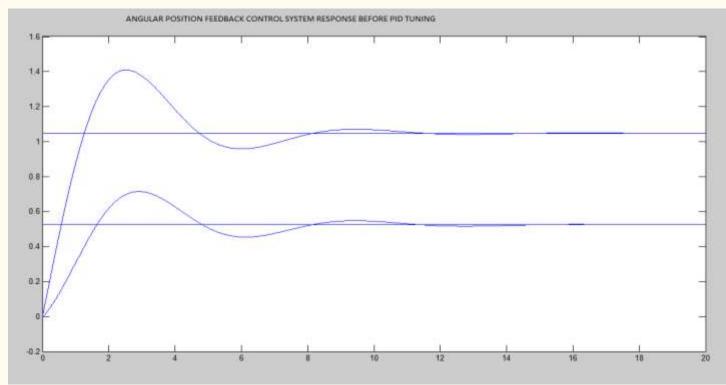


FIGURE3.4 ANGULAR POSITION BEFORE PID TUNING IS DONE

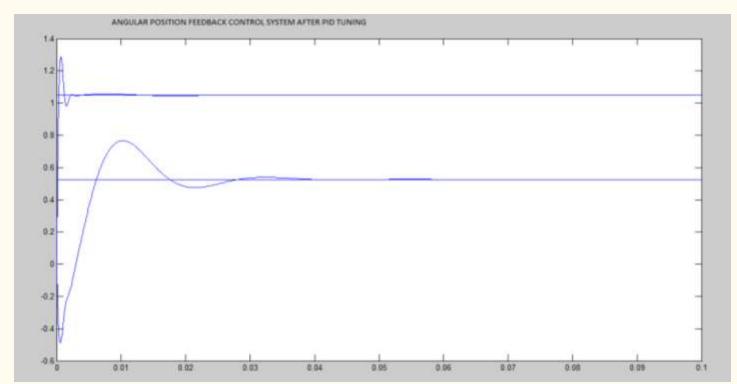


FIGURE 3.5 ANGULAR POSITION AFTER PID TUNING OF ONE JOINT ONLY

#### DAMPING COEFFICIENT 1 (N-m/(rad/sec))

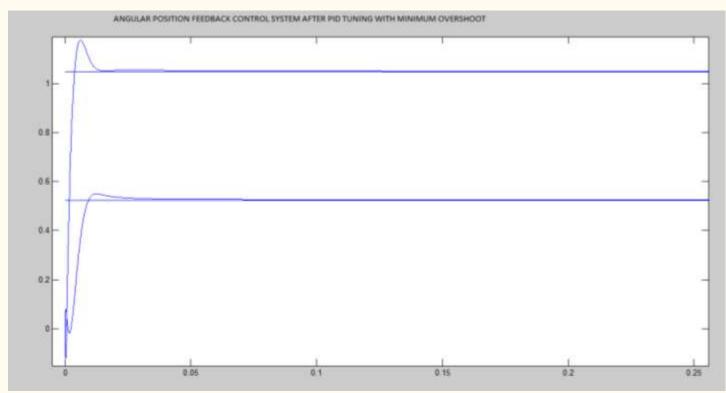


FIGURE 3.6 ANGULAR POSITION AFTER PID TUNING OF BOTH JOINTS IS DONE

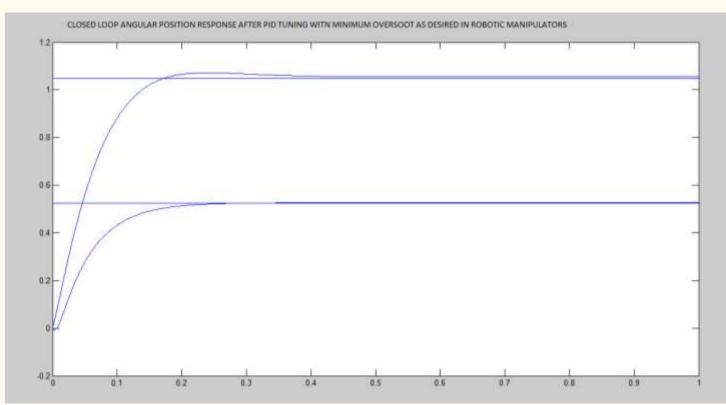


FIGURE 3.7 ANGULAR POSITION AFTER PID TUNING OF JOINTS (MANUAL OVERSHOOT ADJUSTMENT IS DONE)

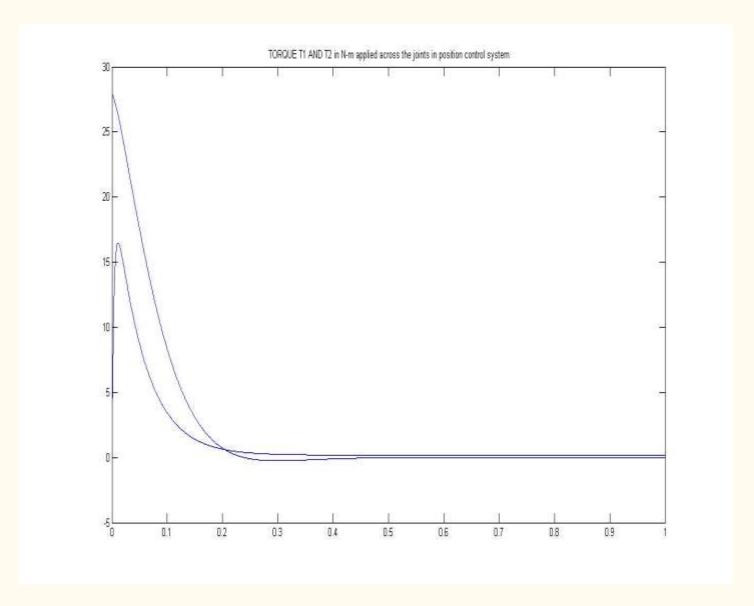


FIGURE 3.8 APPLIED TORQUES IN THE POSITION CONTROL SYSTEM

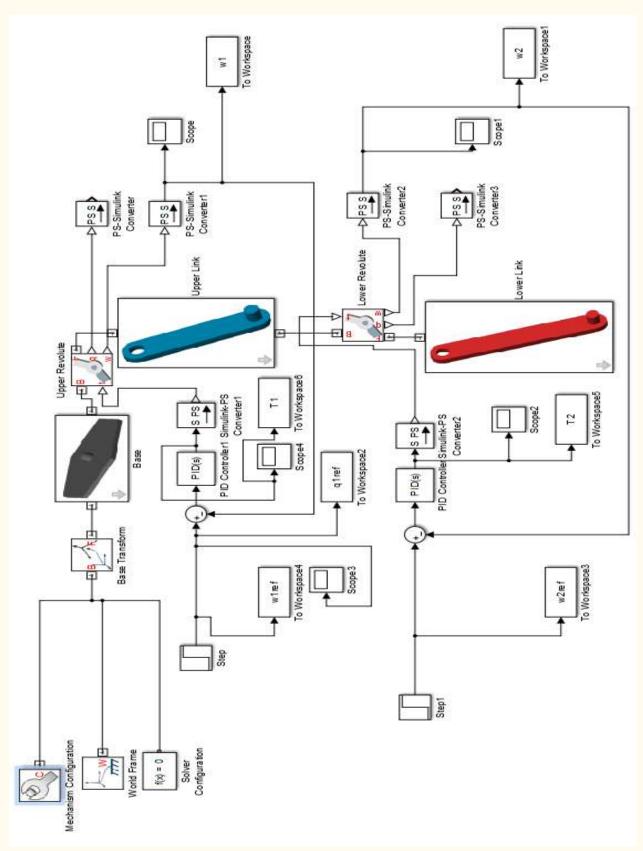


FIGURE 3.9 VELOCITY FEEDBACK CONTROL OF 2 LINK ROBOT MANIPULATOR

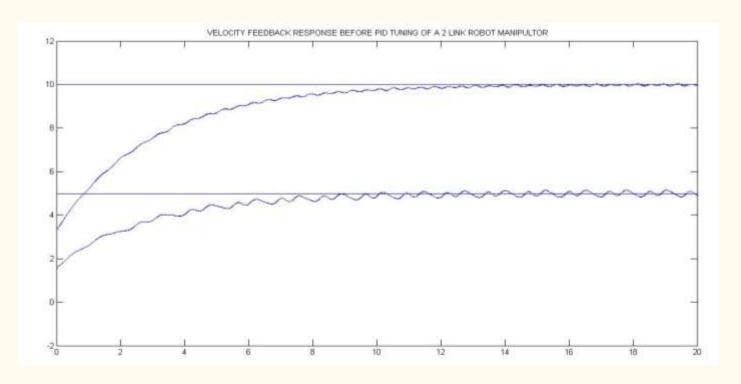


FIGURE 3.10 VELOCITY FEEDBACK RESPONSE BEFORE PID TUNING

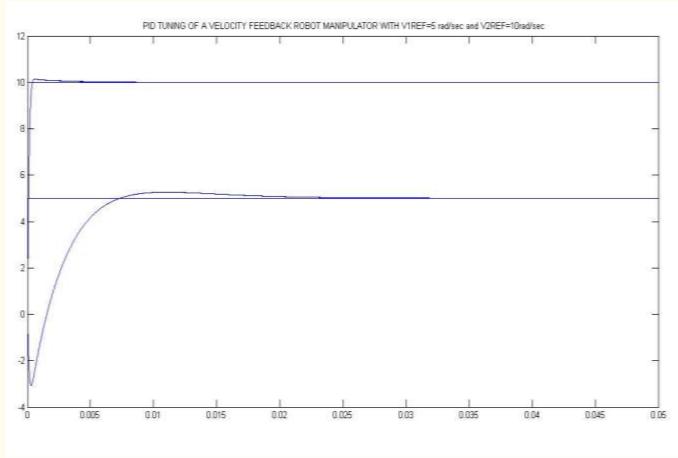


FIGURE 3.11 VELOCITY FEEDBACK RESPONSE AFTER PID TUNING

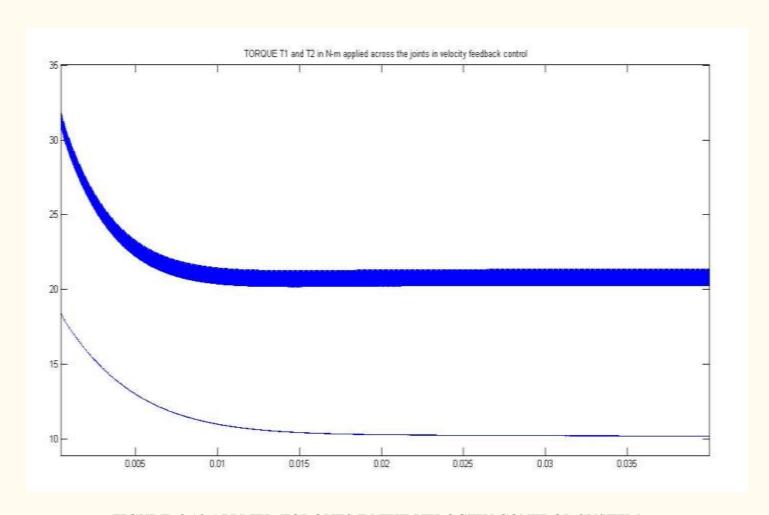


FIGURE 3.12 APPLIED TORQUES IN THE VELOCITY CONTROL SYSTEM

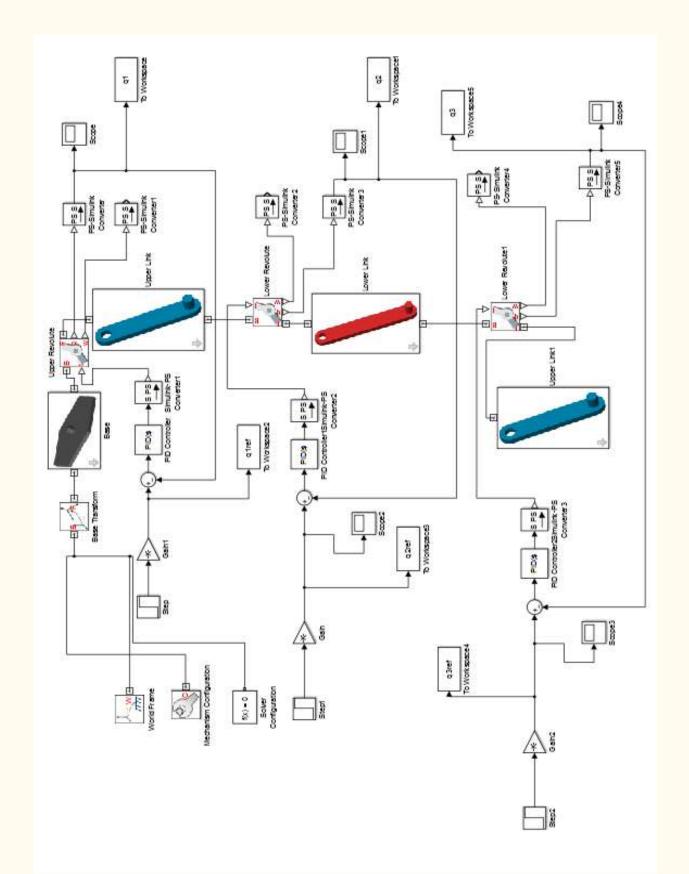


FIGURE 3.13 POSITION FEEDBACK CONTROL OF 3 LINK ROBOT

MANIPULATOR

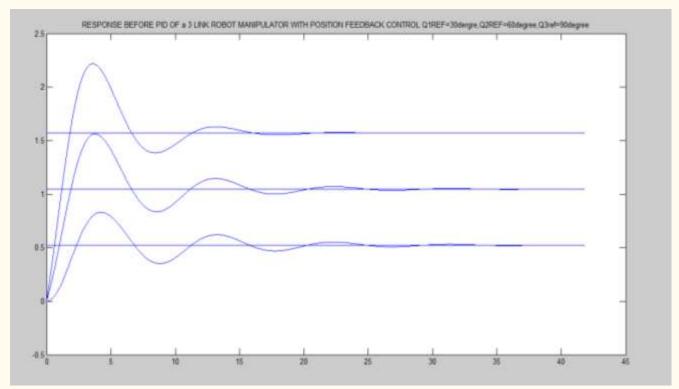


FIGURE 3.14 POSITION RESPONSE BEFORE PID TUNING

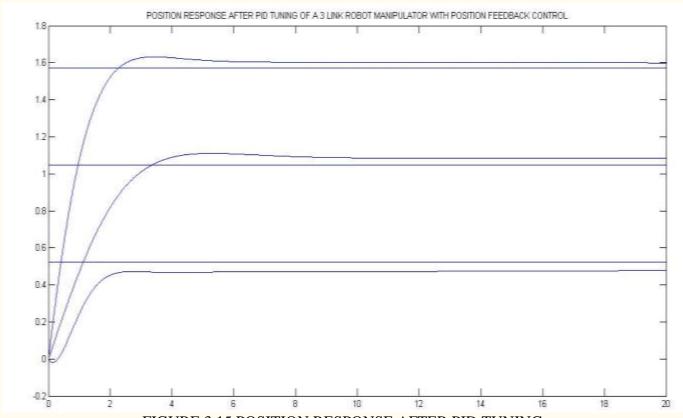


FIGURE 3.15 POSITION RESPONSE AFTER PID TUNING

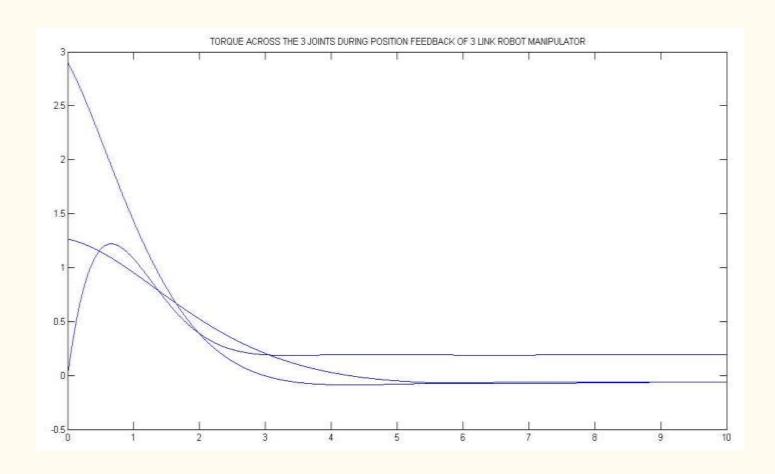


FIGURE 3.16 APPLIED TORQUES IN THE POSITION CONTROL SYSTEM

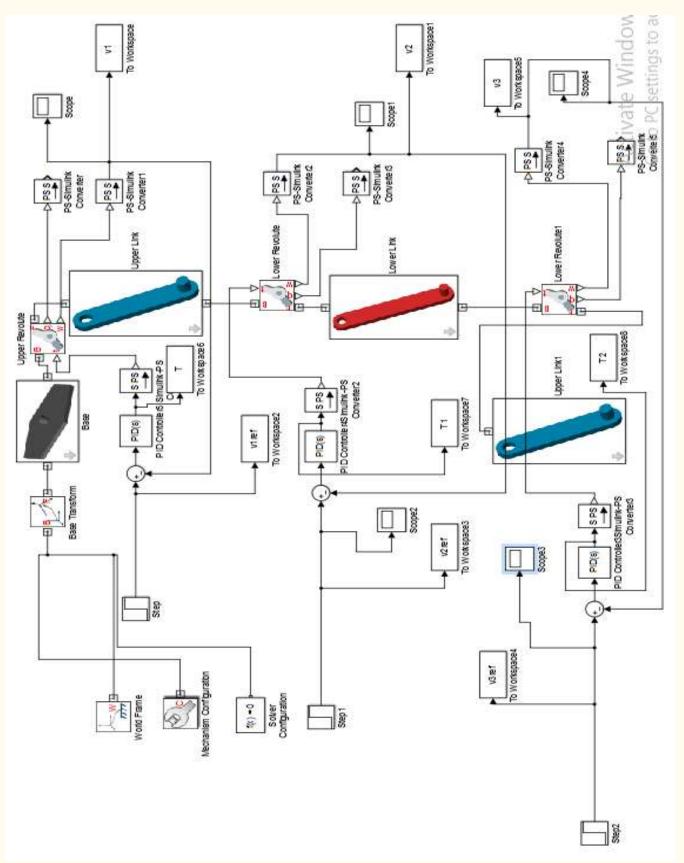


FIGURE 3.17 VELOCITY FEEDBACK CONTROL
OF 3 LINK ROBOT MANIPULATOR

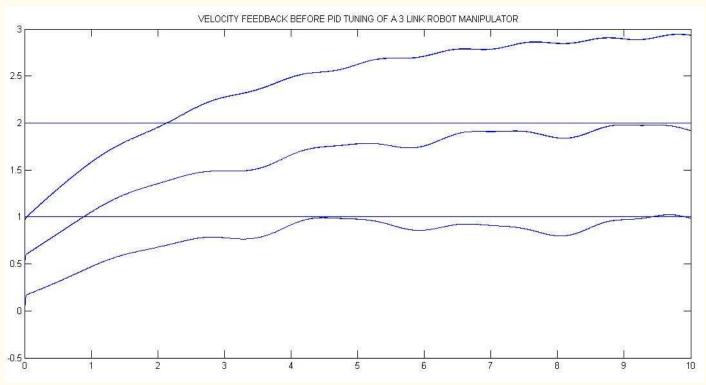


FIGURE 3.18 VELOCITY FEEDBACK RESPONSE BEFORE PID TUNING

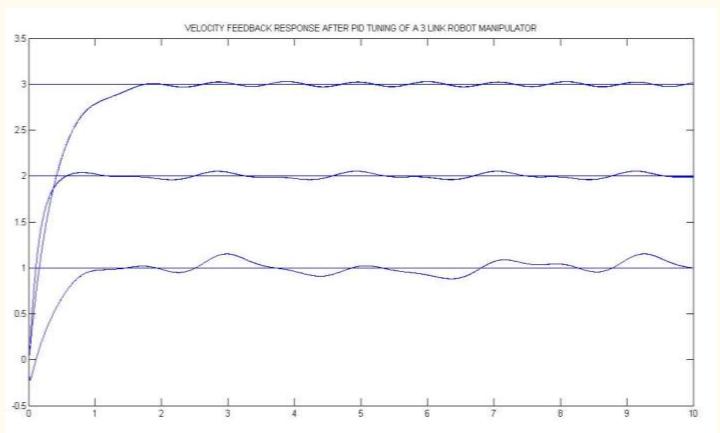


FIGURE 3.19 VELOCITY FEEDBACK RESPONSE AFTER PID TUNING

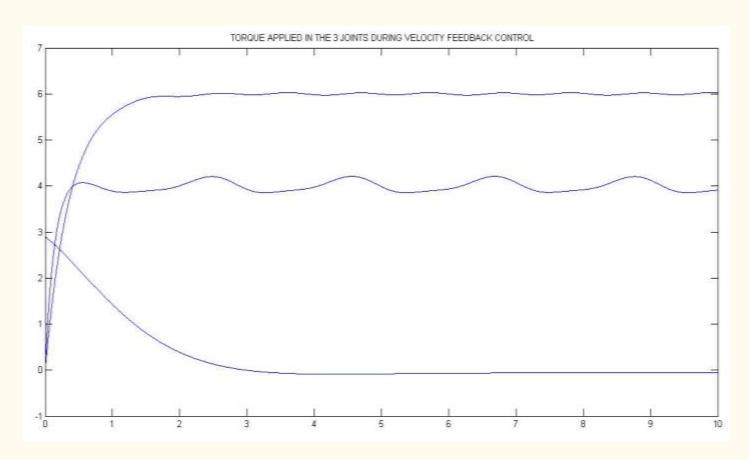


FIGURE 3.20 APPLIED TORQUES IN THE VELOCTY CONTROL SYSTEM

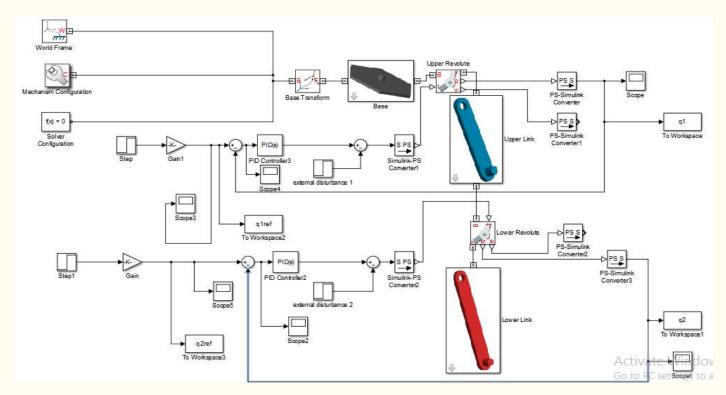


FIGURE 3.21 POSITION FEEDBACK CONTROL OF 2 LINK ROBOT MANIPULATOR WITH EXTERNAL DISTURBANCES

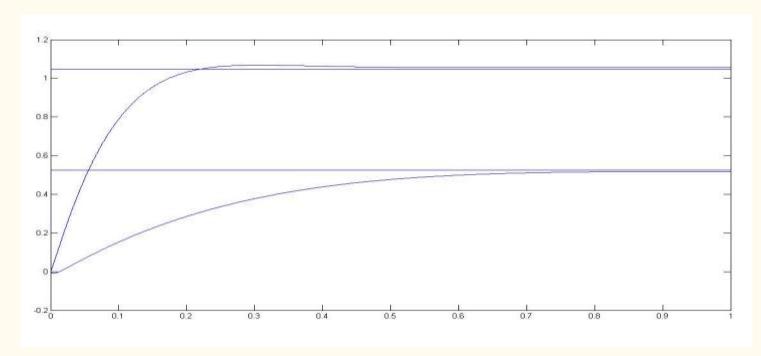


FIGURE 3.22 POSITION FEEDBACK PID TUNED RESPONNSE OF 2

LINK ROBOT MANIPULATOR WITH EXTERNAL DISTURBANCE

TORQUE

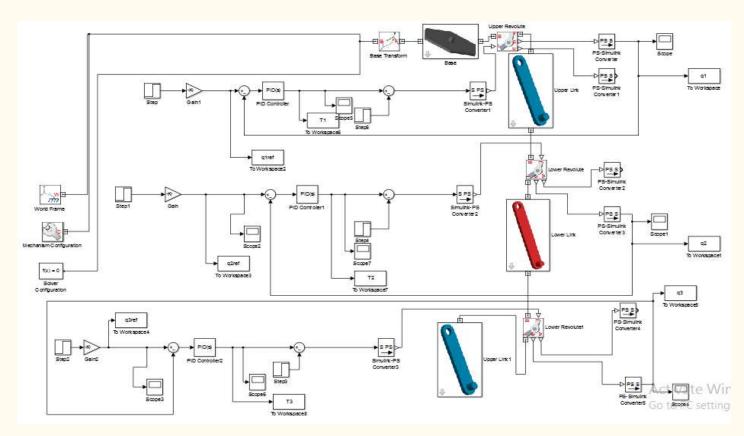


FIGURE 3.23 POSITION FEEDBACK CONTROL OF 3 LINK ROBOT MANIPULATOR WITH EXTERNAL DISTURBANCES

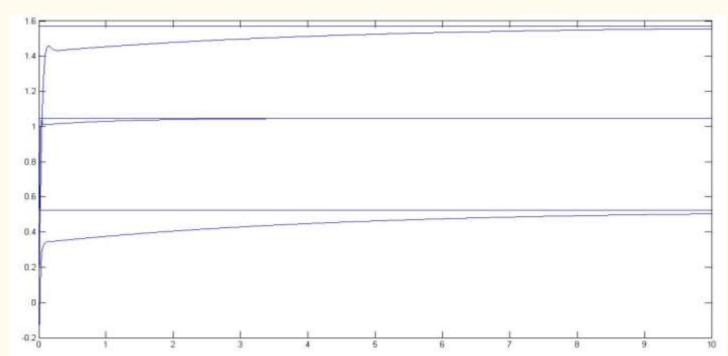


FIGURE 3.24 POSITION FEEDBACK PID TUNED RESPONNSE OF 3LINK
ROBOT MANIPULATOR WITH EXTERNAL DISTURBANCE TOROUE

## **CONCLUSION**

In general, techniques such as Newtonian mechanics can be used to find dynamic equations for robot manipulators. However, it is very difficult to use Newtonian mechanics. So, we opt for Lagrangian mechanics which is based on energy terms only, and therefore, in many cases easier to use.

The Robot manipulator design and simulation in MATLAB is essential to design a robot. It helps us to control the robot arms accurately as desired. The PID tuning method is required to minimize the overshoot of the robotic manipulator. The PID tuner linearizes one loop at a time. So, tuning one loop will affect the other loops. Hence, repeated tuning is required to reduce the above problem. Sometimes, solution may not exist. In those we need to linearize the system about a small error point. So, we can conclude that cross coupling is present between the loops.

The disturbance rejection performance of a PID controller is better than a PD controller. The presence of steady state error in a PD scheme means that the end-effector will not reach the target point and will always be slightly away from it. But, in a PID controller the manipulator can reach the desired target point even in presence of a constant disturbance.

The future work is to design a robot manipulator with minimum energy requirement to reach a certain target with minimum overshoot.

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