Assignment 2

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12.13.6.10 Question : How many times must a man toss a fair coin so that the probability of having at least one head is more than 90?
Answer : 4
Solution : Lets X,Y,n,p be defined as follows Since each toss either results in a heads or a tails

Parameter	Value	Description
X	$0 \leq X \leq n$	Heads count in n tosses
Y	$0 \leq Y \leq n$	Tails count in n tosses
n	$n \in N$	Number of tosses
p	p = 0.5	Probability of a tail

TABLE I
Random variable definitions.

$$X + Y = n \tag{1}$$

Required n such that $X \geq 1$

$$\implies Y \leq n - 1 \tag{2}$$

Also Y is a binomial random variable

$$Y = Bin(n, p) \tag{3}$$

$$\Pr(Y = k) = \binom{n}{r} p^k (1 - p)^{n-k} \tag{4}$$

Consider the cumulative distribution function of Y

$$F_Y(k) = \sum_{r=0}^{r=k} \Pr(Y = r) \tag{5}$$

$$= \sum_{r=0}^{r=k} \binom{n}{r} p^r (1 - p)^{n-r} \tag{6}$$

Required n such that

$$F_Y(n - 1) \geq 0.9 \tag{7}$$

$$\sum_{r=0}^{r=n-1} \binom{n}{r} \frac{1}{2}^n \geq 0.9 \tag{8}$$

$$\sum_{r=0}^{r=n} \binom{n}{r} \frac{1}{2}^n - \frac{1}{2}^n \geq 0.9 \tag{9}$$

$$1 - \frac{1}{2}^n \geq 0.9 \tag{10}$$

$$\frac{1}{2}^n \leq 0.1 \tag{11}$$

$$2^n \geq 10 \tag{12}$$

$$n \geq \log_2 10 \tag{13}$$

$$n \geq 3.3219 \tag{14}$$

$$n \geq 4 \tag{15}$$

Least value of n such that the probability of having at least one head while tossing a coin n times more than 90 is 4