Assignment 1

Gunnam Sri Satya Koushik

CS22BTECH11026

12.13.1.12 Question : Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1) The youngest is a girl
2) At least one is a girl

Answer : (i)$\frac{1}{2}$ (ii)$\frac{1}{3}$

Solution : Lets us random variables X_i where $i \in \{1, 2\}$ as follows

$X_i = 1$	ith born child is a boy	$\Pr(X_i = 1) = 0.50$
$X_i = 0$	ith born child is a girl	$\Pr(X_i = 0) = 0.50$

TABLE I

RANDOM VARIABLE DEFINITIONS.

1) The required probability is the conditional probability that both the children are girls given that the youngest is a girl i.e,

$$
\begin{aligned}
\Pr((X_1 = 0, X_2 = 0)|X_2 = 0) &= \frac{\Pr(X_1 = 0, X_2 = 0)}{\Pr(X_2 = 0)} \\
&= \frac{\Pr(X_1 = 0)\Pr(X_2 = 0)}{\Pr(X_2 = 0)} \\
&= \Pr(X_1 = 0) \\
&= \frac{1}{2}
\end{aligned}
$$

2) The required probability is the conditional probability that both the children are girls given that at least one a girl i.e,

$$
\begin{aligned}
\Pr((X_1 = 0, X_2 = 0)|(X_1 + X_2) < 2) &= \frac{\Pr(X_1 = 0, X_2 = 0)}{1 - \Pr((X_1 + X_2) = 2)} \\
&= \frac{\Pr(X_1 = 0, X_2 = 0)}{1 - \Pr(X_1 = 1, X_2 = 1)} \\
&= \frac{\Pr(X_1 = 0)\Pr(X_2 = 0)}{1 - \Pr(X_1 = 1)\Pr(X_2 = 1)} \\
&= \frac{\frac{1}{2} \times \frac{1}{2}}{1 - (\frac{1}{2} \times \frac{1}{2})} = \frac{1}{3}
\end{aligned}
$$