Assignment 1

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CS22BTECH11026

12.13.1.12 Question : Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1) The youngest is a girl
2) At least one is a girl

Answer : (i)$\frac{1}{2}$ (ii)$\frac{1}{3}$

Solution : Lets us random variables X_i where $i \in \{1, 2\}$ as follows

Variable	Description	Probability
$X_i = 1$	ith born child is a boy	$\Pr(X_i = 1) = 0.50$
$X_i = 0$	ith born child is a girl	$\Pr(X_i = 0) = 0.50$

TABLE I

RANDOM VARIABLE DEFINITIONS.

1) The required probability is the conditional probability that both the children are girls given that the youngest is a girl i.e,

$$\Pr\left((X_1 + X_2)'|X_2'\right) = \frac{\Pr\left((X_1'X_2')X_2'\right)}{\Pr\left(X_2'\right)}$$

$$= \frac{\Pr\left(X_1'\right)\Pr\left(X_2'\right)}{\Pr\left(X_2'\right)}$$

$$= \Pr(X_1')$$

$$= \frac{1}{2}$$

2) The required probability is the conditional probability that both the children are girls given that at least one a girl i.e,

$$\Pr\left((X_1 + X_2)'|(X_1X_2)'\right) = \frac{\Pr\left((X_1'X_2')(X_1' + X_2')\right)}{1 - \Pr(X_1X_2)}$$

$$= \frac{\Pr\left(X_1'X_2'\right)}{1 - \Pr(X_1X_2)}$$

$$= \frac{\Pr\left(X_1'\right)\Pr\left(X_2'\right)}{1 - \Pr(X_1)\Pr(X_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{1 - (\frac{1}{2} \times \frac{1}{2})} = \frac{1}{3}$$