Assignment 1

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12.13.1.12 Question : Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
(i) the youngest is a girl (ii) at least one is a girl?

Answer : (i)$\frac{1}{2}$ (ii)$\frac{1}{3}$

Solution : Lets us define events B_i and G_i where $i \in 1, 2$ as follows

A_i	ith child is a boy	$\Pr(B_i)$=0.50
B_i	ith child is a girl	$\Pr(G_i)$=0.50

Part (i) : The required probability is the conditional probability that both the children are girls given that the youngest is a girl i.e, $\Pr((G_1.G_2)|G_2)$

$$\begin{aligned}
&\Pr((G_1.G_2)|G_2) \\
&= \frac{\Pr(G_1.G_2.G_2)}{prG_2} \\
&= \frac{\Pr(G_1.G_2)}{prG_2} \\
&= \frac{\Pr(G_1).\Pr(G_2)}{\Pr(G_2)} \\
&= \Pr(G_1) \\
&= \frac{1}{2}
\end{aligned}$$

Part (ii) : The required probability is the conditional probability that both the children are girls given that at least one a girl i.e, $\Pr(G_1.G_2|(G_1 + G_2))$

$$\begin{aligned}
&\Pr(G_1.G_2|(G_1 + G_2)) \\
&= \frac{\Pr(G_1.G_2)}{\Pr(G_1) + \Pr(G_2) - \Pr(G_1.G_2)} \\
&= \frac{\Pr(G_1).\Pr(G_2)}{\Pr(G_1) + \Pr(G_2) - \Pr(G_1)\Pr(G_2)} \\
&= \frac{\frac{1}{2}.\frac{1}{2}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{2}.\frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned}$$