Assignment 1

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12.13.1.12 Question : Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
(i) the youngest is a girl (ii) at least one is a girl?

Answer : (i)$\frac{1}{2}$ (ii)$\frac{1}{3}$

Solution : Lets us random variables X_i where $i \in \{1, 2\}$ as follows

$X_i = 1$	ith born child is a boy	$\Pr(X_i = 1) = 0.50$
$X_i = 0$	ith born child is a girl	$\Pr(X_i = 0) = 0.50$

Part (i) : The required probability is the conditional probability that both the children are girls given that the youngest is a girl i.e,

$$\begin{aligned}
&\Pr((X_1 = 0, X_2 = 0)|X_2 = 0) \\
&= \frac{\Pr(X_1 = 0, X_2 = 0)}{\Pr(X_2 = 0)} \\
&= \frac{\Pr(X_1 = 0) \,.\, \Pr(X_2 = 0)}{\Pr(X_2 = 0)} \\
&= \Pr(X_1 = 0) \\
&= \frac{1}{2}
\end{aligned}$$

Part (ii) : The required probability is the conditional probability that both the children are girls given that at least one a girl i.e,

$$\begin{aligned}
&\Pr(X_1 = 0, X_2 = 0|(X_1 + X_2) < 2) \\
&= \frac{\Pr(X_1 = 0, X_2 = 0)}{1 - \Pr((X_1 + X_2) = 2)} \\
&= \frac{\Pr(X_1 = 0, X_2 = 0)}{1 - \Pr(X_1 = 1, X_2 = 1)} \\
&= \frac{\Pr(X_1 = 0) \,.\, \Pr(X_2 = 0)}{1 - \Pr(X_1 = 1) \,.\, \Pr(X_2 = 1)} \\
&= \frac{\frac{1}{2} \cdot \frac{1}{2}}{1 - \frac{1}{2} \cdot \frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned}$$