Assignment 1

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12.13.1.12 Question : Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
(i) the youngest is a girl (ii) at least one is a girl?

Answer : (i)$\frac{1}{2}$ (ii)$\frac{1}{3}$

Solution : Lets us define events B_i and G_i where $i \in \{1, 2\}$ as follows

B_i	ith born child is a boy	$\Pr(B_i)=0.50$
G_i	ith born child is a girl	$\Pr(G_i)=0.50$

Part (i) : The required probability is the conditional probability that both the children are girls given that the youngest is a girl i.e, $\Pr((G_1.G_2)|G_2)$

$$
\begin{aligned}
\Pr((G_1.G_2)|G_2) \\
&= \frac{\Pr(G_1.G_2.G_2)}{prG_2} \\
&= \frac{\Pr(G_1.G_2)}{prG_2} \\
&= \frac{\Pr(G_1).\Pr(G_2)}{\Pr(G_2)} \\
&= \Pr(G_1) \\
&= \frac{1}{2}
\end{aligned}
$$

Part (ii) : The required probability is the conditional probability that both the children are girls given that at least one a girl i.e, $\Pr(G_1.G_2|(G_1 + G_2))$

$$
\begin{aligned}
\Pr(G_1.G_2|(G_1 + G_2)) \\
&= \frac{\Pr(G_1.G_2)}{\Pr(G_1) + \Pr(G_2) - \Pr(G_1.G_2)} \\
&= \frac{\Pr(G_1).\Pr(G_2)}{\Pr(G_1) + \Pr(G_2) - \Pr(G_1)\Pr(G_2)} \\
&= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned}
$$