Admittance Smith Chart

Characteristics Admittance $Y_0 = 1/Z_0$

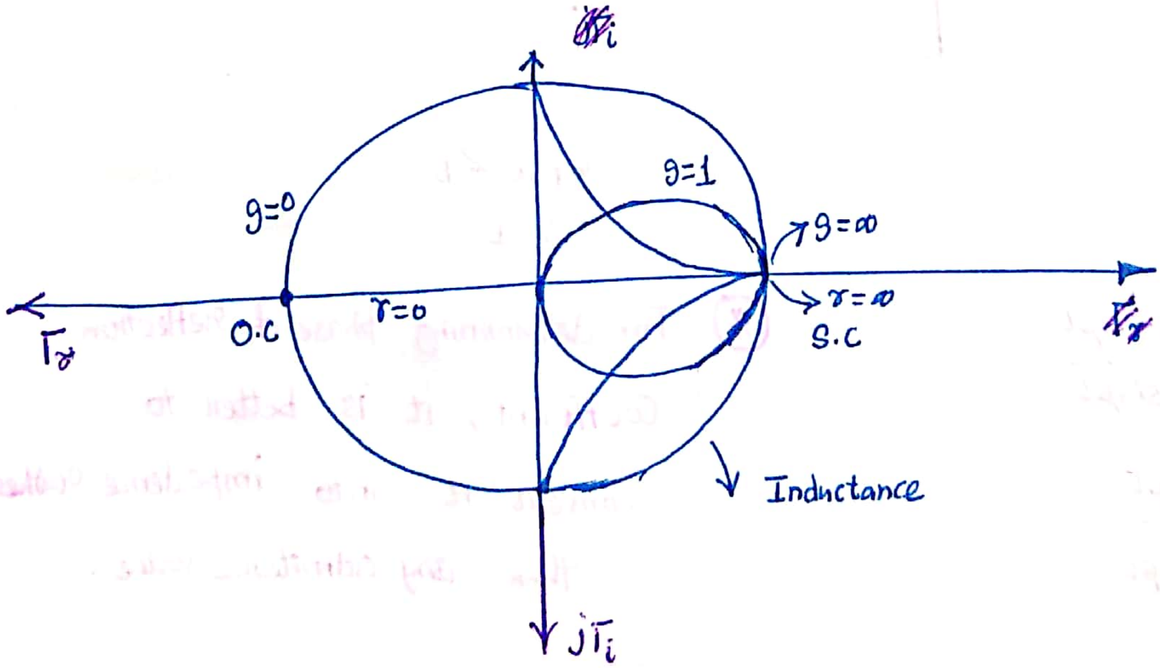
Normalized Admittance $\bar{y} = \frac{y}{Y_0}$, $\bar{y}_L = \frac{y_L}{y_0}$

Reflection Coefficient $\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0}$

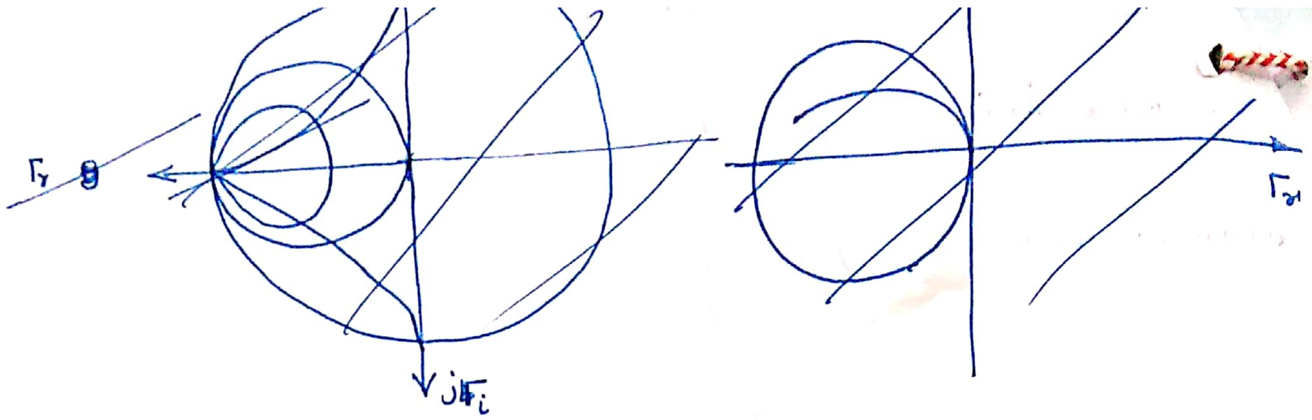
$$= \frac{1/y(l) - 1/y_0}{1/y(l) + 1/y_0} = \frac{Y_0 - y(l)}{Y_0 + y(l)}$$

$$= \frac{1 - \bar{y}(l)}{1 + \bar{y}(l)} = \frac{\bar{y}(l) - 1}{1 + \bar{y}(l)} e^{j\pi}$$

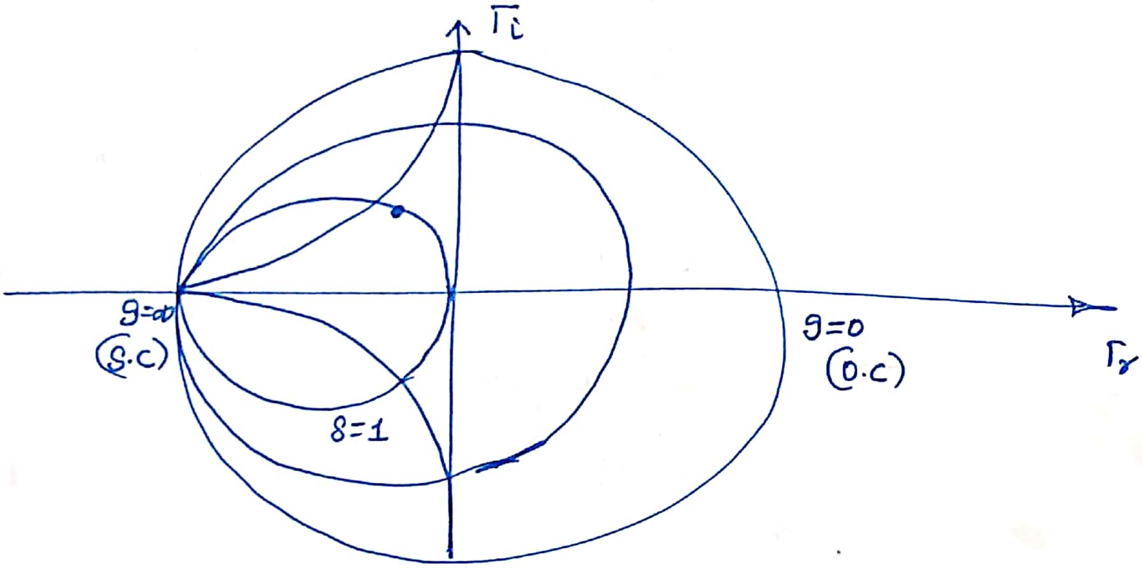
$$\Gamma(l) = \frac{\bar{y}(l) - 1}{\bar{y}(l) + 1} e^{j\pi}$$

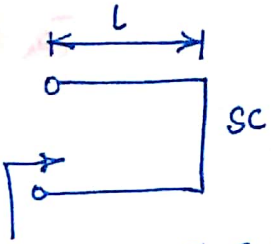


$$\bar{y} = g + jb = \frac{G + jB}{Y_0}$$



'Γ' plane is rotated by 180°





~~g+jb~~ ~~-jb~~

~~g=1~~

(*) For determining phase of reflection coefficient, it is better to convert it into impedence rather than using admittance value.

$$Z(l) = Z_0 \frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}$$

$$= Z_0 \frac{\bar{Z}_L \cos\beta l + j\sin\beta l}{\cos\beta l + j\bar{Z}_L \sin\beta l}$$

if $Z_L = 0$

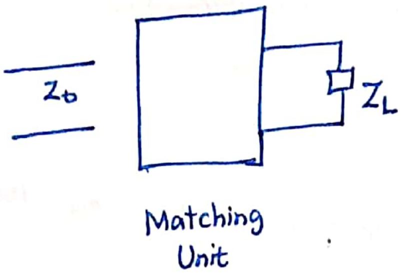
$$Z(L) = Z_0 j\tan\beta l$$

if $Z_L = \infty$

$$Z(L) = Z_0 \frac{\cos\beta l}{j\sin\beta l} = -jZ_0 \cot\beta l$$

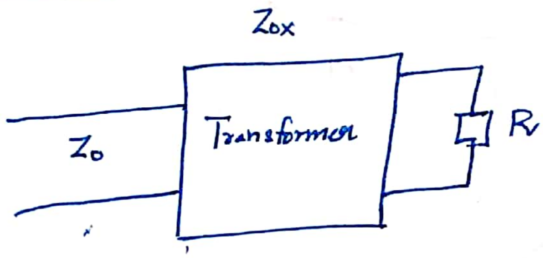
Pure reactance along the length of line

Impedance Matching

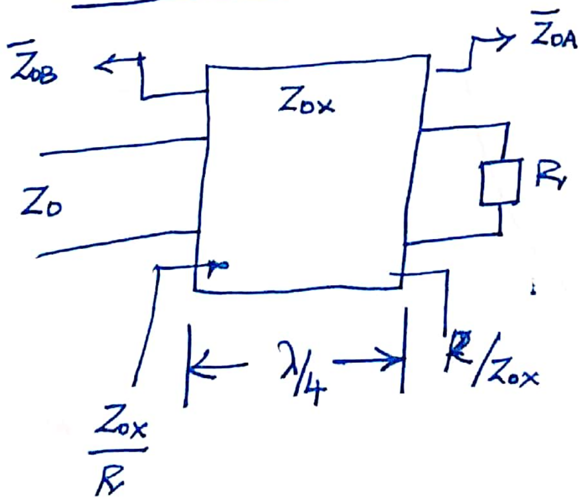


Ideally matching unit should be lossless.

Match a real load impedance to a real char. impedance.



λ/4 transformer technique



$$\bar{Z} = \frac{Z_{ox}}{R}$$

$$Z = \frac{Z_{ox}^2}{R}$$

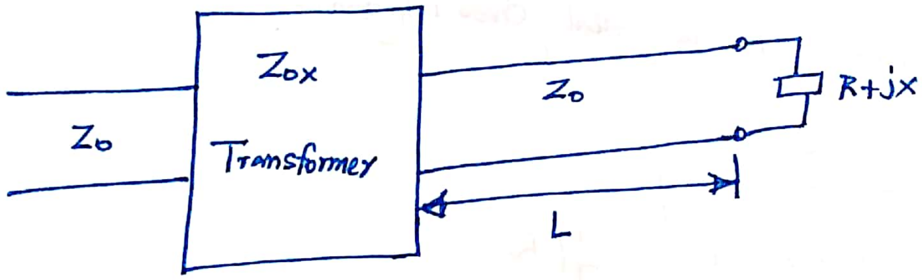
$$Z_o = \frac{Z_{ox}^2}{R}$$

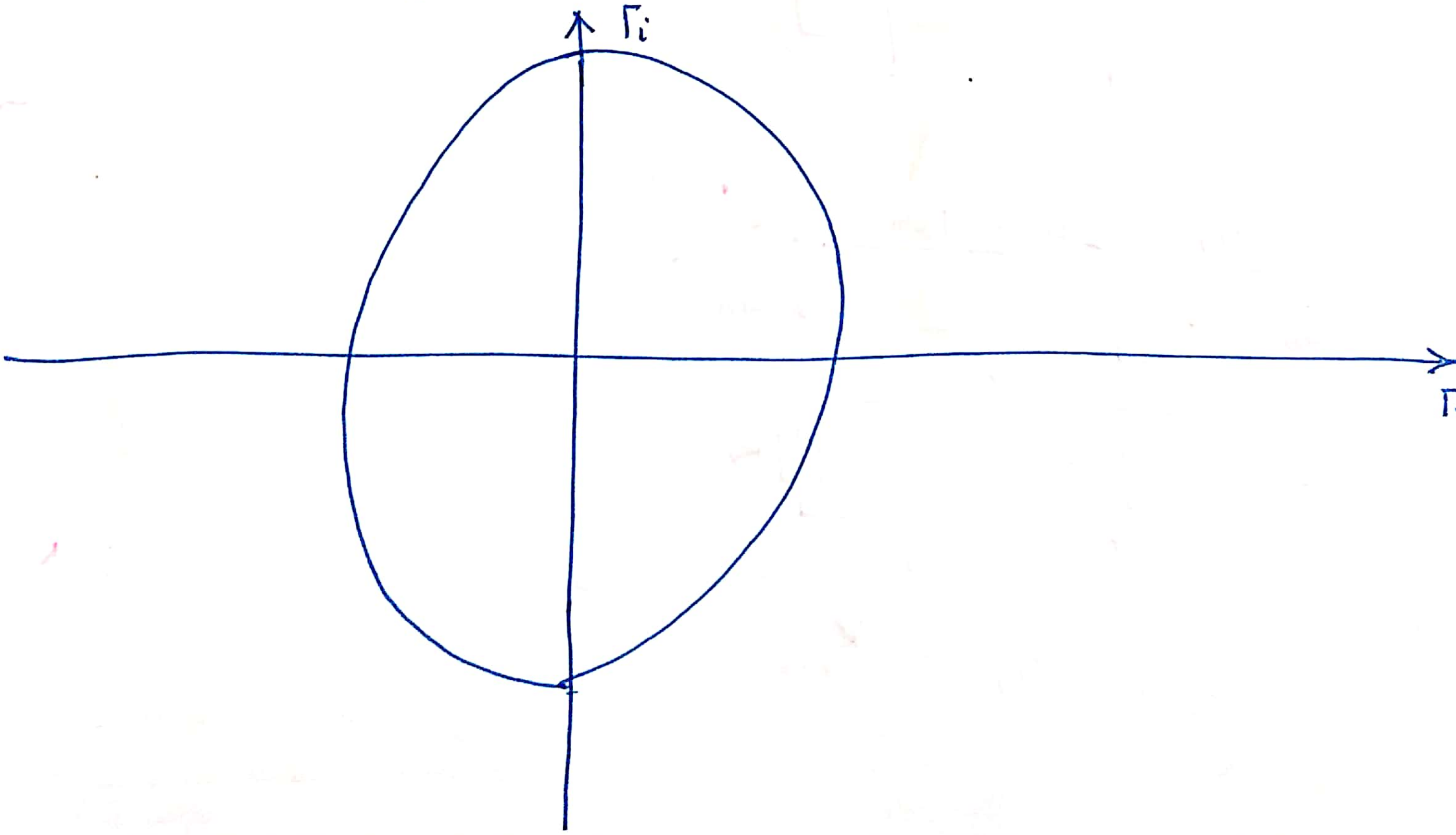
$$\boxed{Z_{ox} = \sqrt{Z_o R}}$$

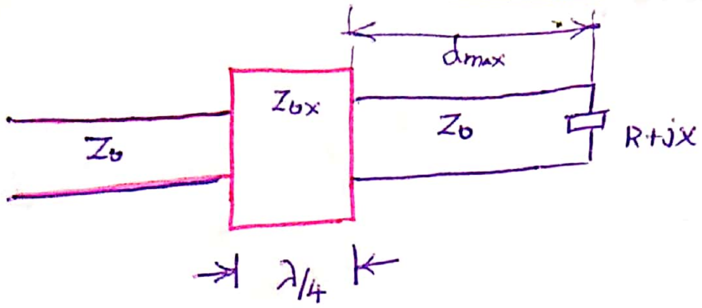
→ On Either side of box, impedance is equal to Z_0 & R respectively.

→ $\lambda/4$ technique is possible only for purely real impedance.

However, it is possible to match Complex impedance using $\lambda/4$ transformer technique.



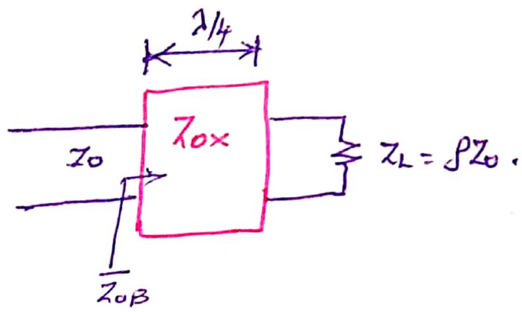




At d_{max}

$$\bar{Z}_{OA} = \rho$$

$$Z_{OA} = \rho \cdot Z_0$$



$$\bar{Z}'_{OA} = \frac{\rho Z_0}{Z_{ox}}$$

$$\bar{Z}_{OB} = \frac{Z_{ox}}{\rho Z_0}$$

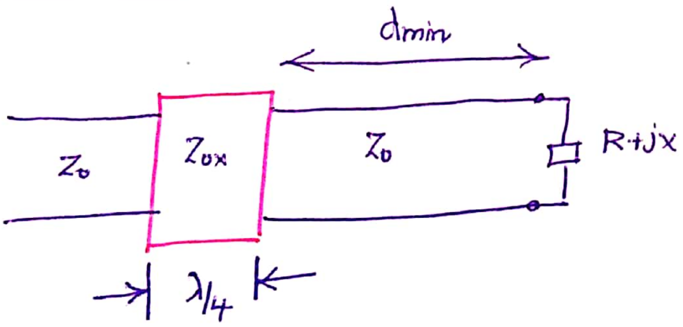
$$Z_{OB} = \frac{Z_{ox}^2}{\rho Z_0}$$

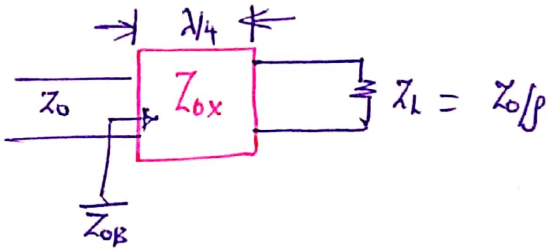
Z_{OB} Should be equal to Z_0.

$$Z_0 = \frac{Z_{ox}^2}{\rho Z_0}$$

$$Z_{ox}^2 = \rho Z_0^2$$

$$\boxed{Z_{ox} = \sqrt{\rho}\, Z_0}$$





At d_{min}

$$\bar{Z}_{OA} = 1/\rho$$

$$Z_{OA} = 1/\rho \cdot Z_0$$

$$\bar{Z}'_{OA} = \cancel{\rho/Z_0}\ Z_0/\rho \cdot 1/Z_{ox}$$

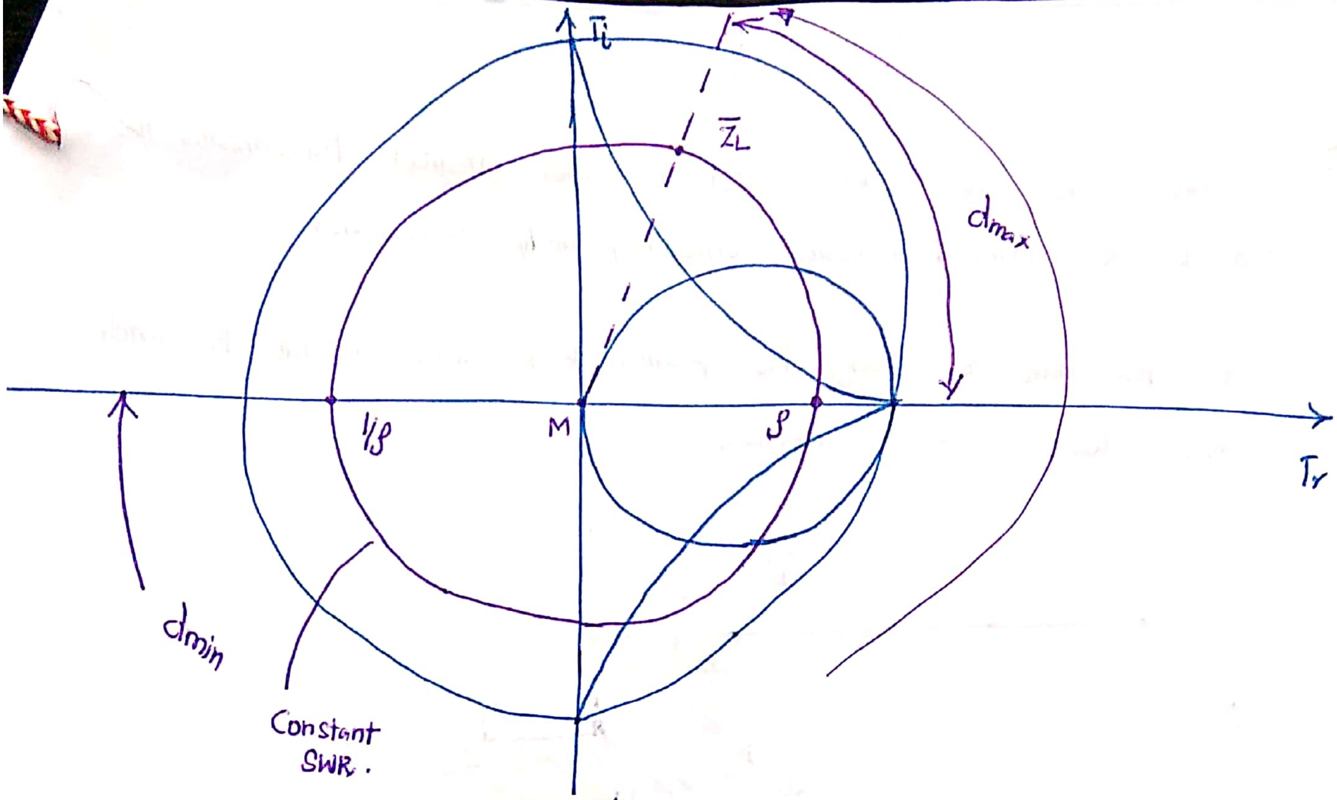
$$\bar{Z}_{OB} = \frac{Z_{ox}\rho}{Z_0}$$

$$Z_{OB} = \frac{Z_{ox}\rho \cdot Z_{ox}}{Z_0}$$

Z_{OB} Should be equal to Z_0

$$Z_0 = \frac{Z_{ox}\rho \cdot Z_{ox}}{Z_0}$$

$$\frac{Z_0^2}{\rho} = Z_{ox}^2 \qquad \boxed{Z_{ox} = Z_0/\sqrt{\rho}}$$



After a distance of d_{max} $\bar{Z}_{OA} = \rho$ $\quad \bar{Z}_{OB} = Z_0 = \frac{1}{\rho} \cdot Z_{ox}$

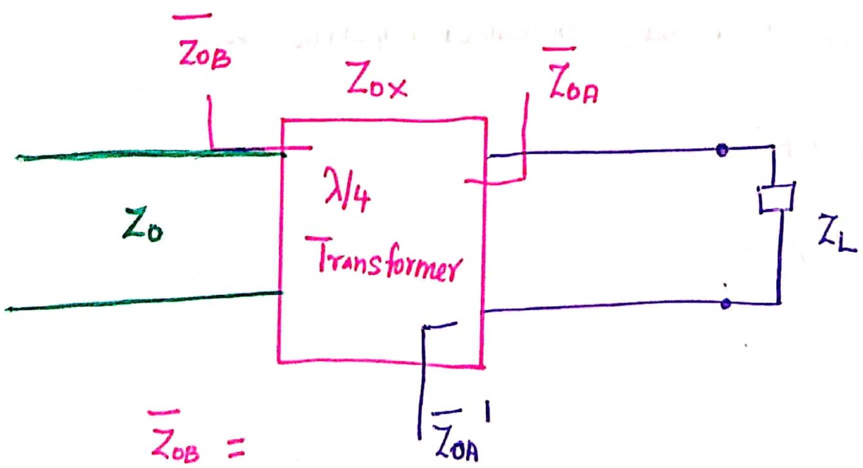
$\Rightarrow \quad Z_{ox} = \rho \cdot Z_0$

After a distance of d_{min} $\bar{Z}_{OA} = 1/\rho$ $\quad \bar{Z}_{OB} = Z_0 = \rho \cdot Z_{ox} \Rightarrow Z_{ox} = Z_0/\rho$

→ This technique has a serious drawback, we need an unique value of char. impedance (Z_{ox}) to match the load impedance.

→ So it is highly difficult to perform the imp. matching for a very random load.

→ Realizing any random value of char impedance is difficult.



$\bar{Z}_{OB} =$

At d_{max}

$\bar{Z}_{OA} = \rho$

$Z_{OA} = \rho Z_0.$

$\bar{Z}_{OA}' = \frac{\rho Z_0}{Z_{ox}}$

$\bar{Z}_{OB} = \frac{Z_{ox}}{\rho Z_0}$

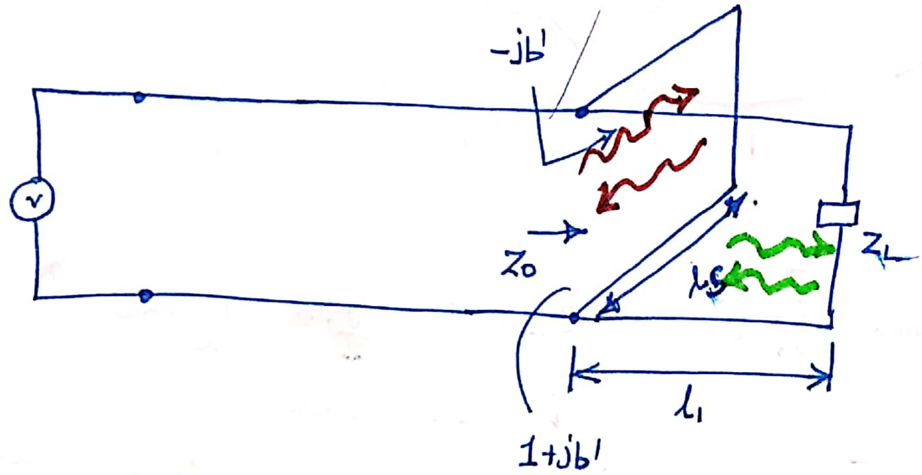
$Z_{OB} = \frac{Z_{ox}^2}{\rho Z_0}$

$Z_0^2 \cdot \rho = Z_{ox}^2$

Stub Matching Tech.

Stubs are either Short Circuited or open circuited transmission line which is Connected to the Main transmission either in parallel or in Series

(*) You have to Change the position & the length of Stub to match the arbitary load with Char. impedance.



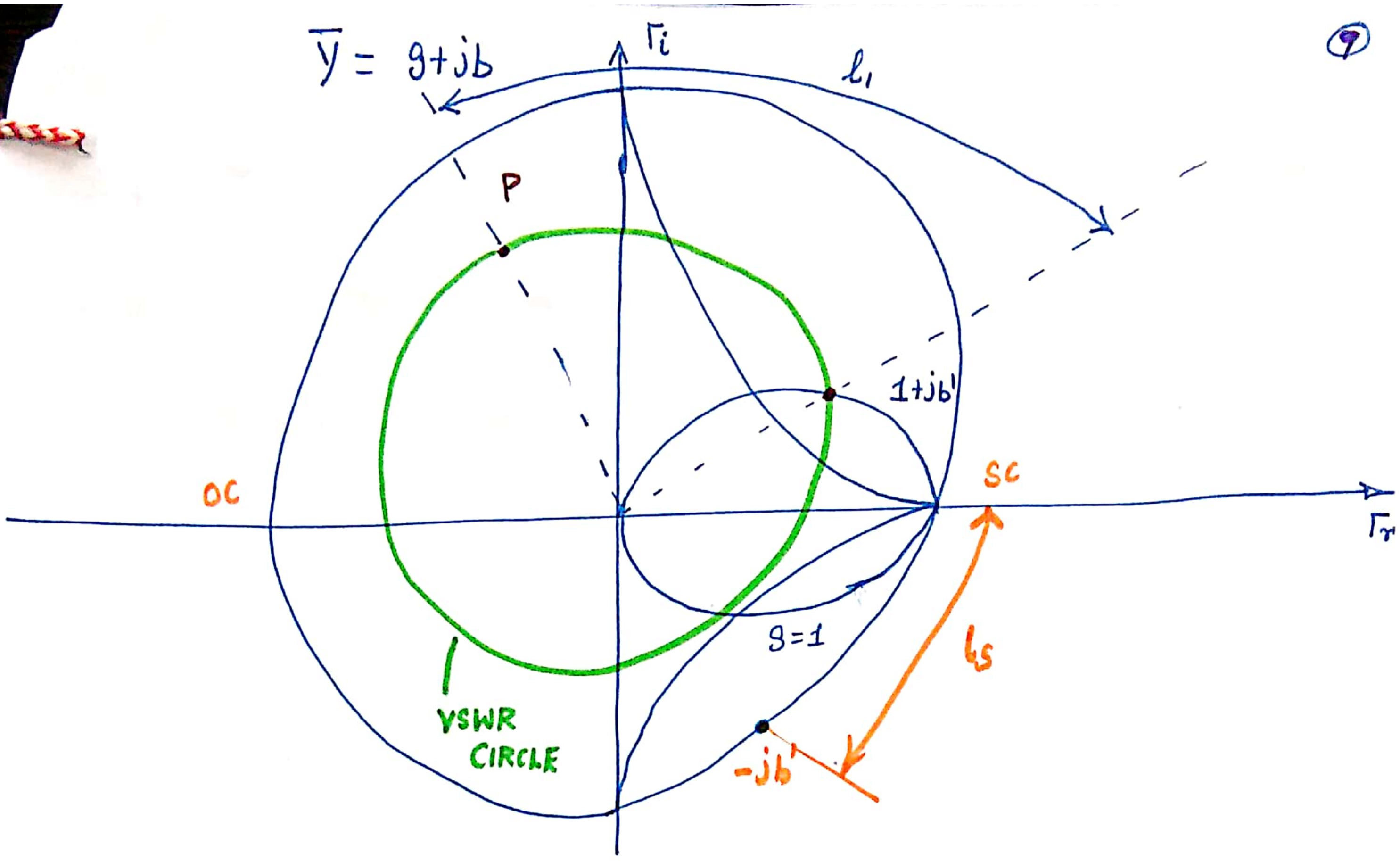
Voltage wave travelling from Source sees two path, towards Stub and one towards load. If we make reflected wave from load and the Short circuit is of equal amplitude & ~~phase~~ 180 ~~we~~ and 180° out of phase we can cancel this reflected wave reaching towards load.

'l_1' location of Stub, l_s length of Stub.

~~After the~~ At the load End, normalized impedance is

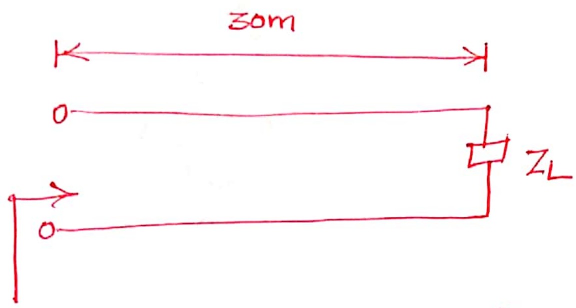
$$\bar{Z}_L = r + jx$$

$$\bar{Y}_L = g + jb$$



→ For Open Circuit Stub, we have to choose the ~~dis~~ length of Stub from Open Circuit point.

The input admittance of 50Ω lossless transmission line is $0.041 - j0.0023$. Transmission line is of length 30m and operates at 2MHz. Velocity of signal in tx line is 0.6c. Determine Voltage reflection Coefficient, VSWR & load impedance at load point



$$Y_{in} = 0.041 - j0.0023 \ \boxed{S}$$

$$\bar{Y}_{in} = \frac{Y_{in}}{Y_0} = Y_{in} * Z_0 = 2.05 - j0.115$$

$$v = 0.6 \times 3\times10^8 \qquad \lambda = v/f = 90 \text{ m}.$$

$$30\text{m} \Rightarrow 30 \cdot \frac{\lambda}{90} = \lambda/3 \text{ m}$$

or We can say load is at the distance of $\lambda/3$ from input.

$$\begin{array}{r} \cancel{0.254\lambda} \\ \cancel{0.300} \\ \hline \cancel{84} \end{array} \qquad \begin{array}{r} \cancel{0.046\lambda} \\ \hline \end{array} \qquad \begin{array}{r} 0.496\lambda \\ 0.333 \\ \hline 0.829 \\ 0.500 \\ \hline 0.329 \end{array} \rightarrow \text{I/p Imp. position}$$

$$\begin{array}{r} \cancel{0.246\lambda} \\ \cancel{0.3} \\ \hline \cancel{0.546\lambda} \end{array} \qquad \begin{array}{r} \cancel{0.496\lambda} \\ \cancel{0.5\lambda} \\ \hline \cancel{0.796\lambda} \end{array}$$

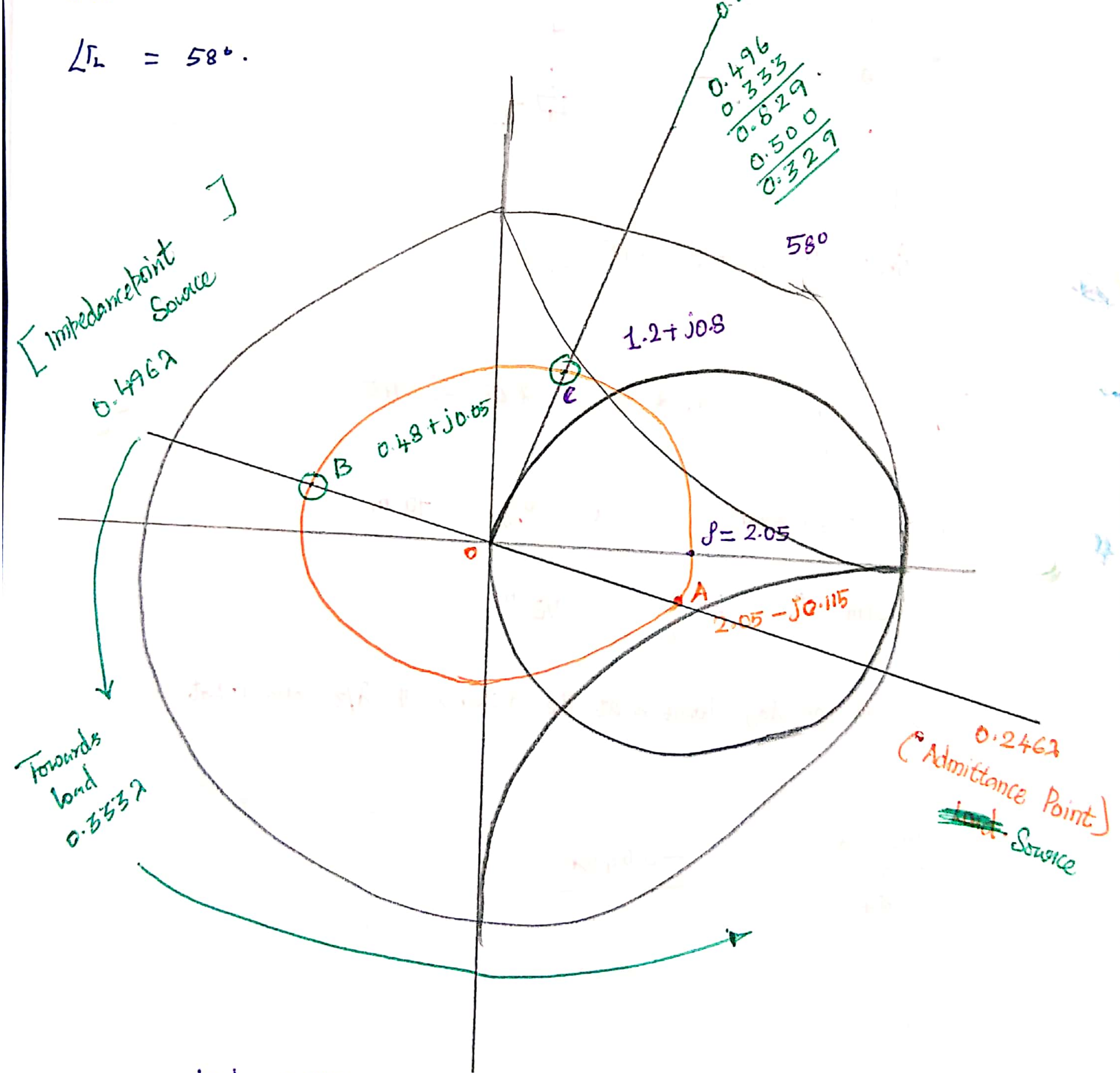
$\bar{Z}_{in} = 0.48 + j0.05$ (at the position 0.496λ)

$\bar{Z}_L = 1.2 + j0.8$ (at the position 0.329λ)

$S = 2.05$

$|\Gamma_L| = 0.35$

$\angle\Gamma_L = 58°$.



$|\Gamma_L| = 0.35$

$\angle\phi_L = 58°$

The Complete Smith Chart

Black Magic Design

$\bar{Z}_{in} = 0.48 + j0.05$

Input Impedance

0.496λ

Towards load

$\bar{Z}_L = 1.2 + j0.8$

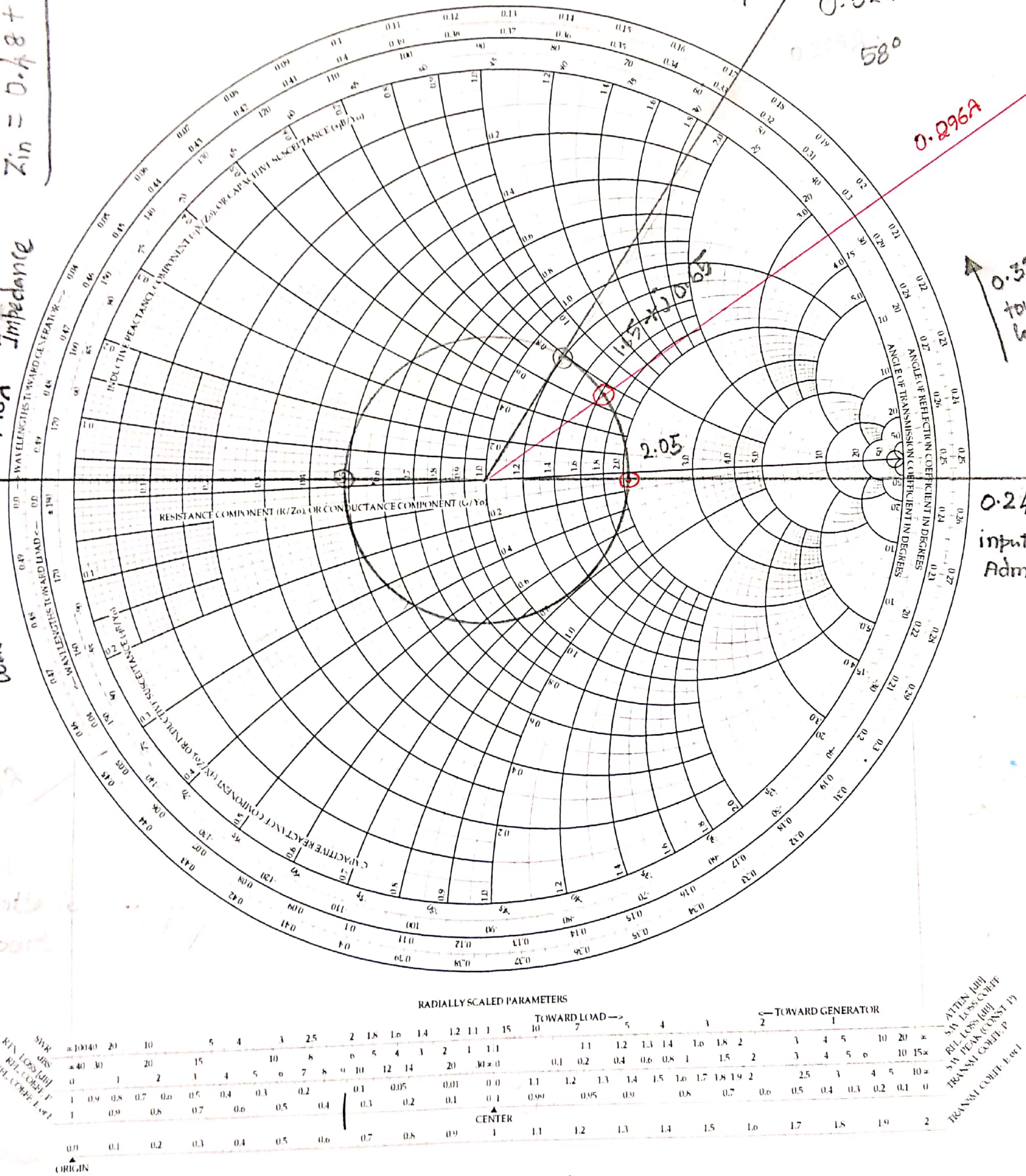
0.3292

58°

0.296λ

0.323λ toward load

0.246λ input Admittance



$|\Gamma_L| = 0.35 \quad \angle\Gamma_L = 58°$

(*) $Z_L = 60 + j40 \ \Omega$ $\qquad \bar{Z}_L = \frac{Z_L}{50} = 1.2 + j0.8$

$$\bar{y}_L = y_L / y_0 = \frac{50}{(60+j40)} = \frac{50\,(60 - j40)}{(60)^2 + (40)^2}$$

$$= \frac{3000 - j\,2000}{3600 + 1600} = 0.5770 - j\,0.3846$$

$\bar{y}_L = 0.5770 - j0.3846$

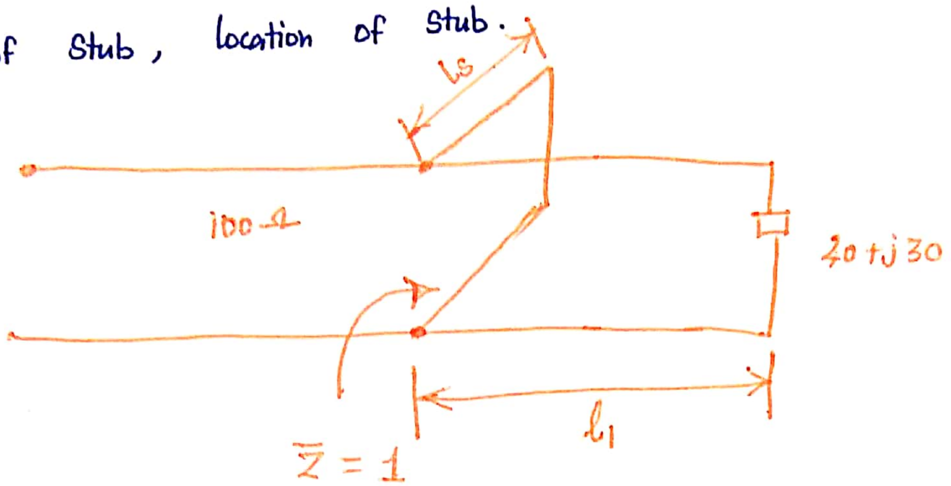
$Z_{in} =$

$\lambda/3 =$

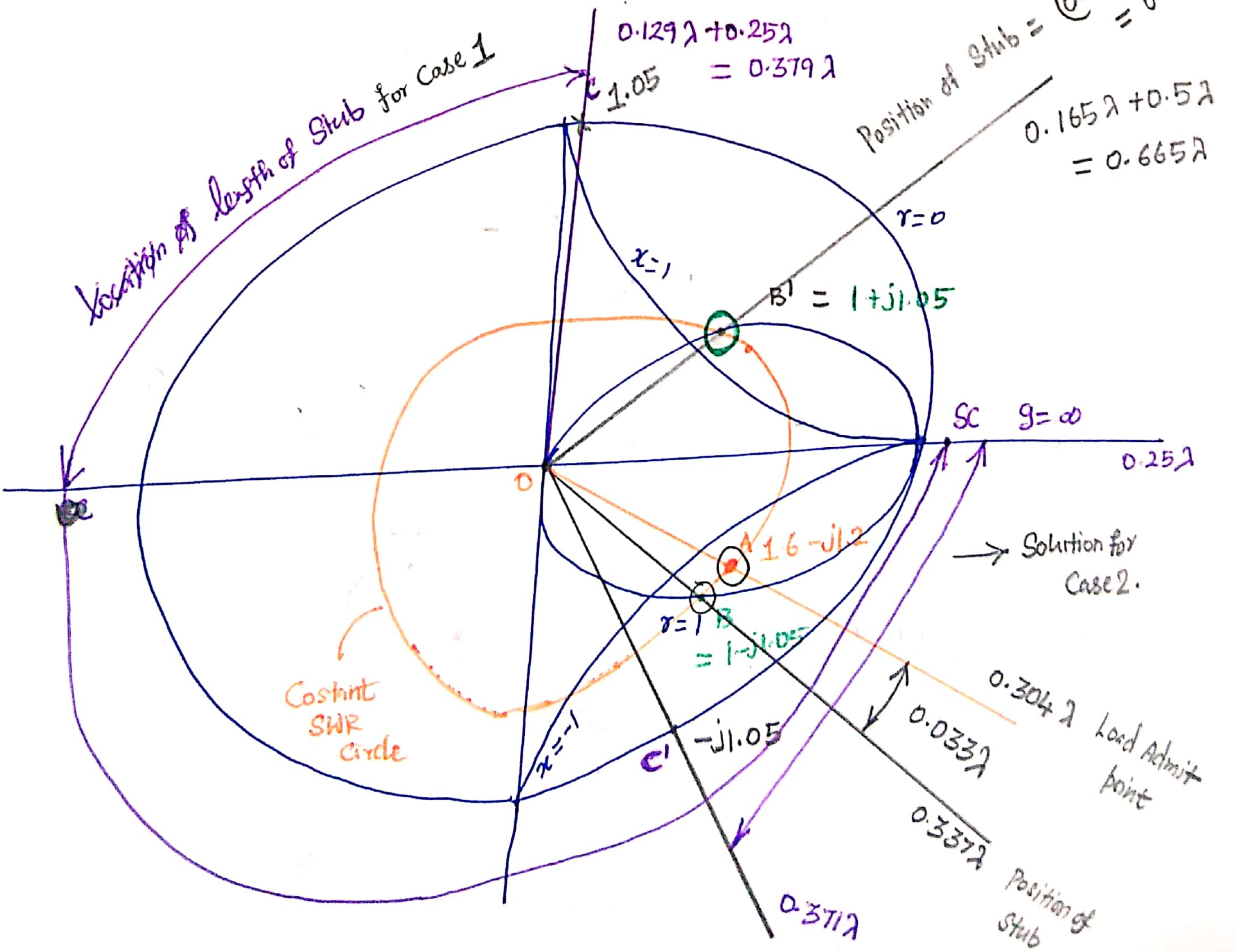
$$\begin{array}{r} 0.173\,\lambda \\ 0.300\,\lambda \\ \hline 0.473 \end{array}$$

Single Stub Matching Problem

The load of $Z_L = 40 + j30\ \Omega$ is to be matched to a $100\ \Omega$ tx. line using a Shorted Stub Connected in parallel. Determine the ~~position~~ length of Stub, location of Stub.

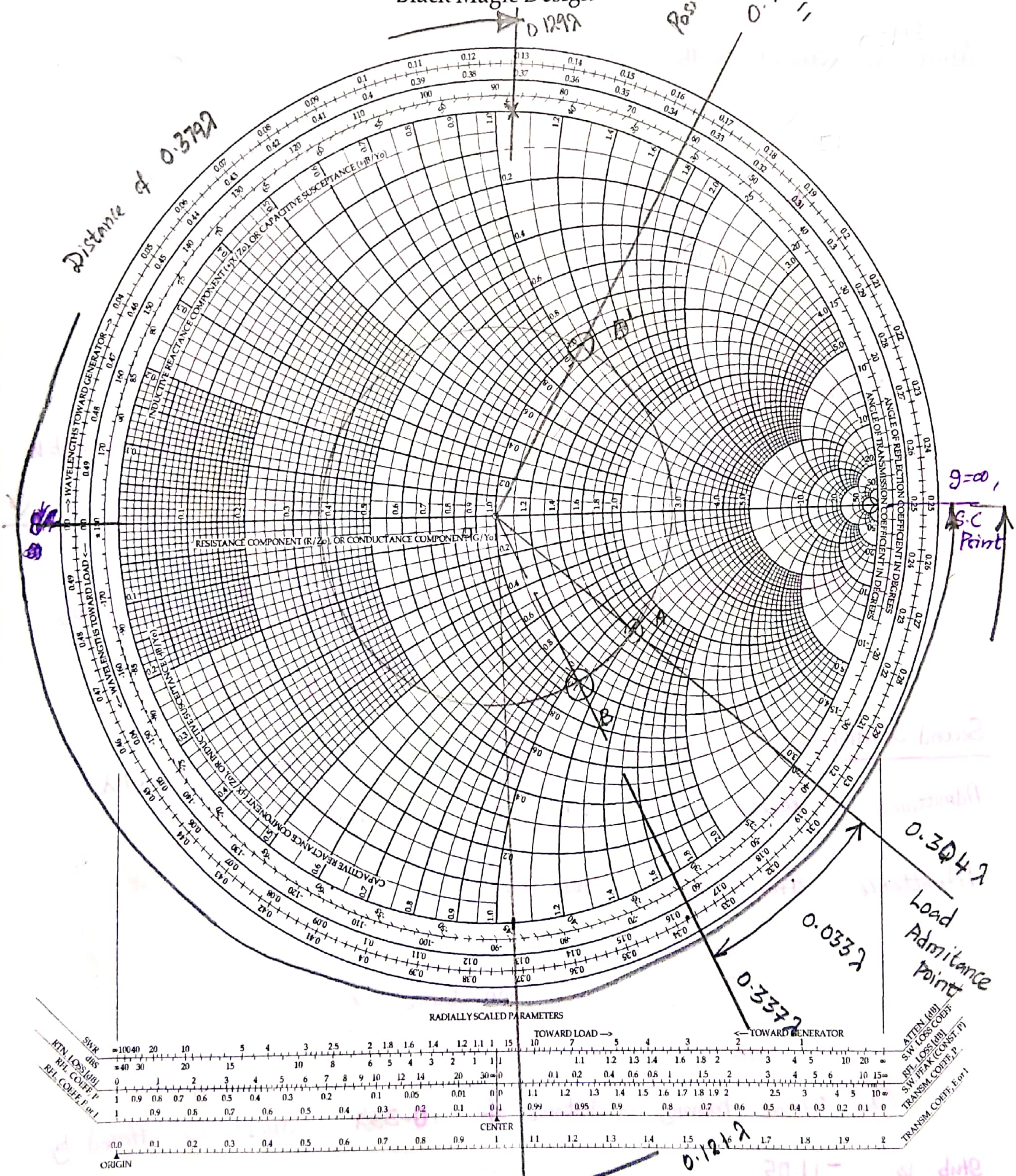


$$\bar{Z}_L = \frac{40 + j30}{Z_0} \qquad \bar{Y}_L = \frac{Z_0}{40 + j30} = \frac{100}{(40 + j100)} = 1.6 - j1.2$$



The Complete Smith Chart

Black Magic Design



$B = 1.0 - j1.05$

$-j\,1.05$

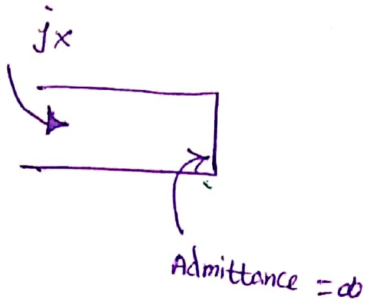
$B' = 1.0 + j1.05$

$(0.371)\lambda$

Location of Stub l_1 = 0.033λ

Admittance at the location of Stub [B] = $1 - j1.05$

Admittance offered by the Short Stub is



$$\begin{array}{r} 0.129 \\ .25 \\ \hline 0.379 \end{array}$$

(*) After moving a distance of $0.129\lambda + 0.25\lambda$ Admittance offered by Stub is $j1.05$

$= 0.379\lambda$.

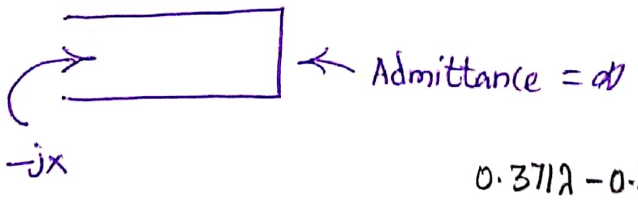
(*) Total admittance at point A A' is $1 - j1.05 + j1.05$

$= 1$.

Second Solution :- Location of Stub $l_1 = 0.665\lambda - 0.304\lambda = 0.361\lambda$

Admittance at location of Stub [B'] = $1 + j1.05$

Admittance offered by the Short Stub is



$0.371\lambda - 0.25\lambda = 0.121\lambda$

(*) After moving a distance of ~~0.33λ~~ admittance offered by Stub is $-j1.05$

~~0.33λ~~

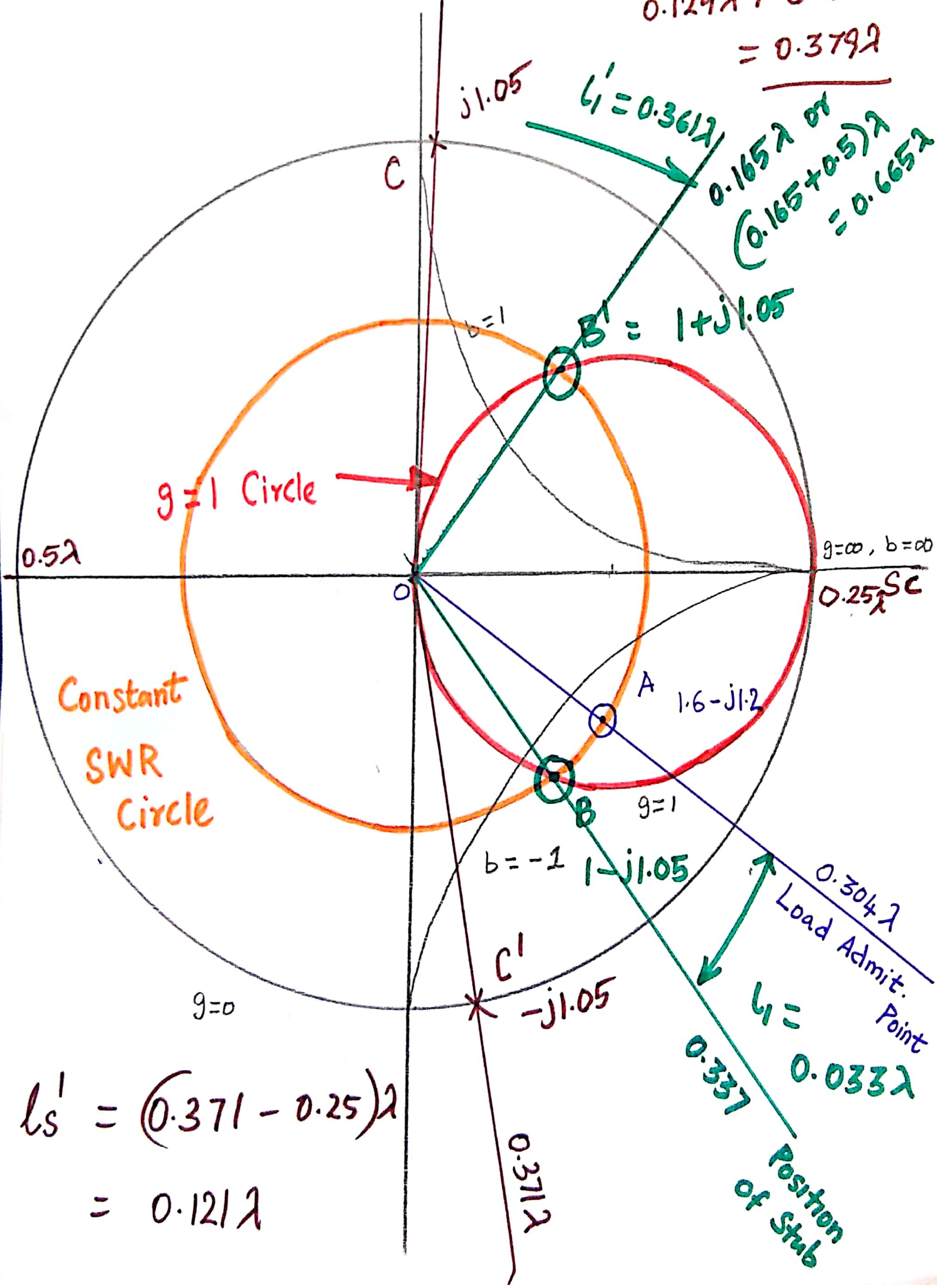
(*) Total admittance at point AA' is $1 + j1.05 - j1.05$

$= 1.0$

$$l_1' = (0.665 - 0.304)\lambda = 0.361\lambda$$

Length of Stub =

$$0.129\lambda + 0.25\lambda = 0.379\lambda$$



$$l_s' = (0.371 - 0.25)\lambda = 0.121\lambda$$