Bode plots :-

w.r.to frequency response will about

① To analyse the freq response of Open loop Tr function (loop gain - GH)

② To analyse CL systems stability

③ To compute Gain margin & Phase Margin

Procedure: to draw Bode plot :

① Replace $s = j\omega$ in GH function

2) find the magnitude(dB) & phase of GH

$$M_{dB} = 20\log|G(j\omega)H(j\omega)|$$

$$\phi = \tan^{-1}\left(\frac{\text{Img. part}}{\text{Real part}}\right)$$

$\omega_c = 25$

ω_s

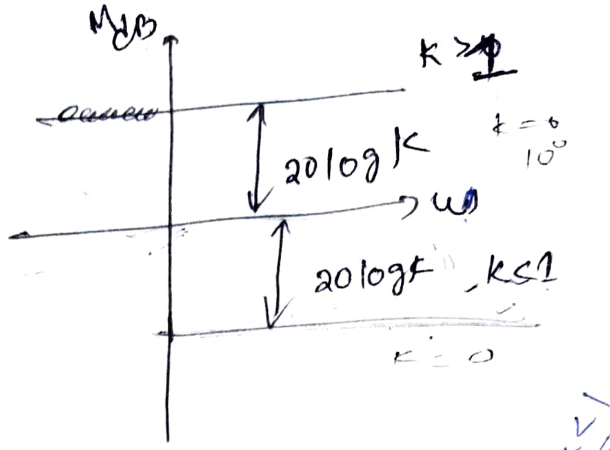
(3) Draw M_{dB} & ϕ w.r.to ω

Ex:- $GH = K$

$G(j\omega)H(j\omega) = K$

$M_{dB} = 20\log K$

$\phi = 0^\circ$

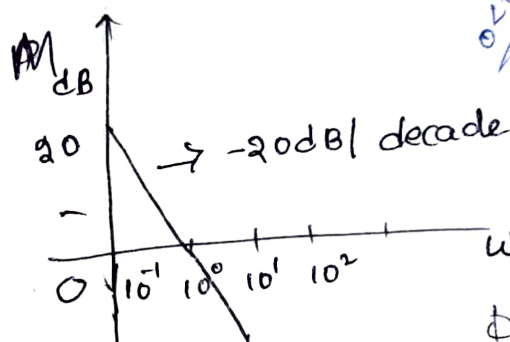


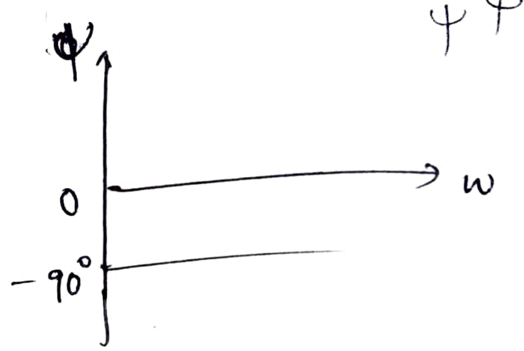
Ex: $GH = \frac{1}{s}$

$G(j\omega)H(j\omega) = \frac{1}{j\omega}$

$M_{dB} = -20\log \omega$

$\phi = -90^\circ$



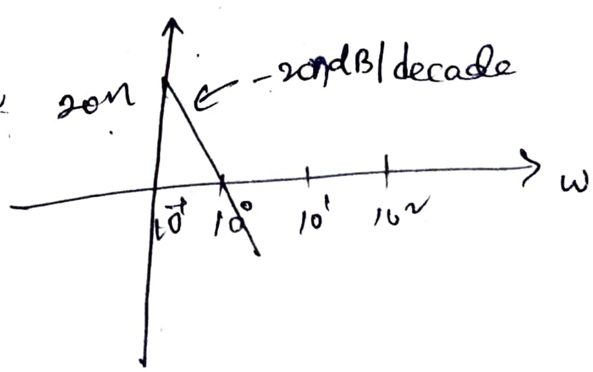


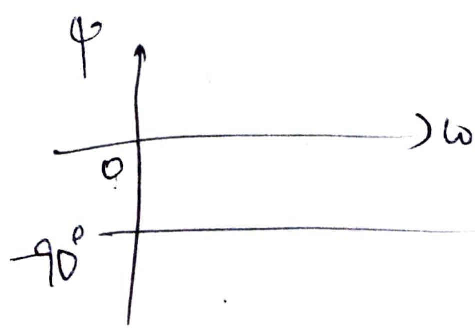
$GH = \frac{1}{s^n}$

$G(j\omega)H(j\omega) = \frac{1}{(j\omega)^n}$

$M_{dB} = -20n\log\omega$

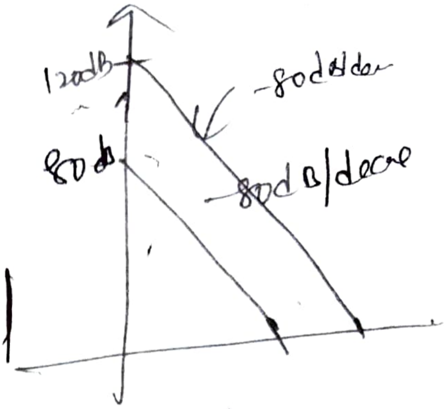
$\varphi = -90^\circ n$





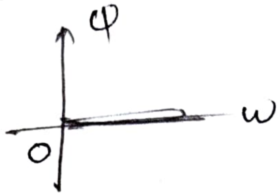
Ex: $GH = \dfrac{100}{s^4}$

$G(j\omega)H(j\omega) = \dfrac{100}{(j\omega)^4}$



$M_{dB} = 100 \times (-20) \log \left| \dfrac{100}{\omega^4} \right|$

$M_{dB} = -80 \log \omega$



$100 \Rightarrow 20\log(100) = 40\,dB$

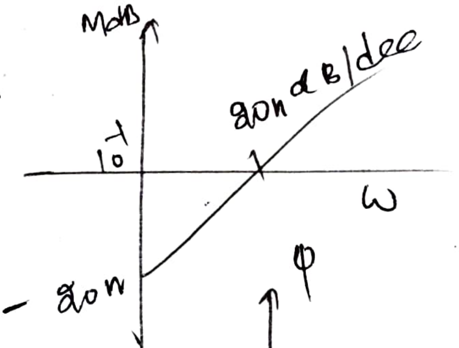
$80 + 40 = 120\,dB$ (shifted by 40dB)

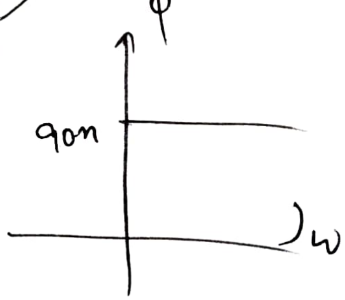
Ex:- $GH = s^n$

$G(j\omega)H(j\omega) = (j\omega)^n$

$M_{dB} = 20n \log \omega$

$\varphi = 90n$





Ex: $GH = \dfrac{1}{1+sT}$

$G(j\omega)H(j\omega) = \dfrac{1}{1+j\omega T}$

$M_{dB} = -20 \log \sqrt{1+\omega^2 T^2}$

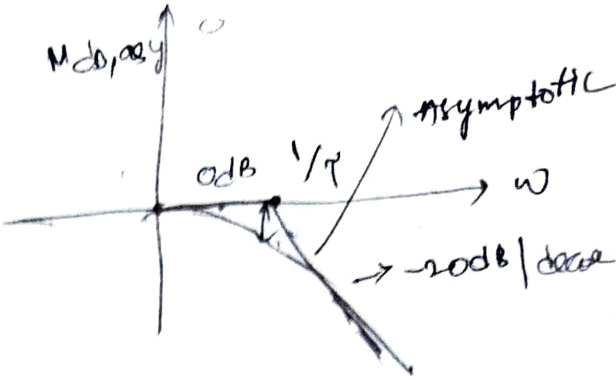
$\varphi = \tan^{-1}(\omega T)$

Asymptotic / Approximation plot

case 1) $\omega\tau << 1$

$M_{dB, asy} = 0dB$

$\phi_{asy} = 0^\circ$



case 2) $\omega\tau >> 1$

$M_{dB, asy} = -20\log\omega\tau$

$\phi_{asy} = -90^\circ$

$\omega = \frac{1}{\tau}$ = corner frequency or break frequency

$$\boxed{E = M_{act} - M_{asy}}$$

$E \to$ Error

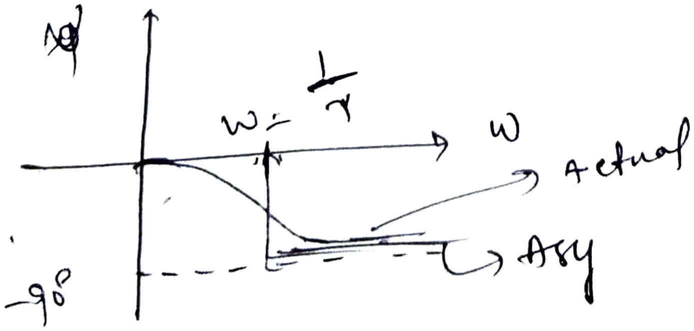
$M_{act}\Big|_{\omega = \frac{1}{\tau}} = -3dB$

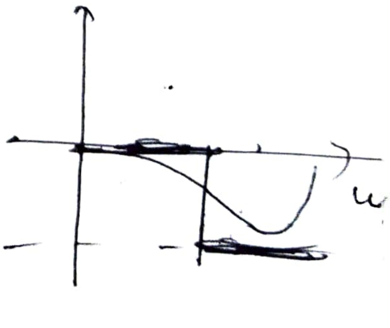
Max. error at Corner frequency

$M_{asy} = 0dB$

$E\Big|_{\omega=\frac{1}{\tau}} = -3dB - 0dB = -3dB$

$E\Big|_{\omega=\frac{0.5}{\tau}} = -0.96dB - 0 = -0.96dB$





$\phi\Big|_{\omega=\frac{1}{T}} = -45 - 0 = -45^\circ$

$\phi\Big|_{\omega=\frac{0.5}{T}} = -26 - 0 = -26^\circ$

$E\Big|_{\omega=\frac{2}{T}}$

$= -63 - (-90)$

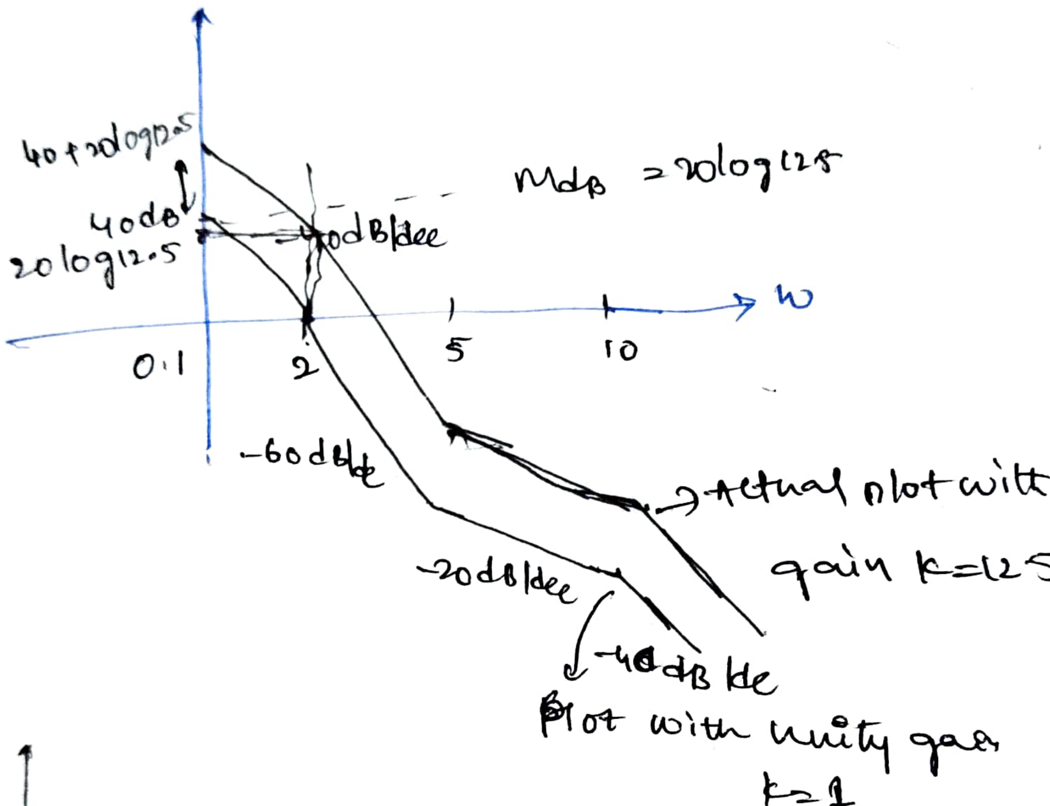
$= 27^\circ$

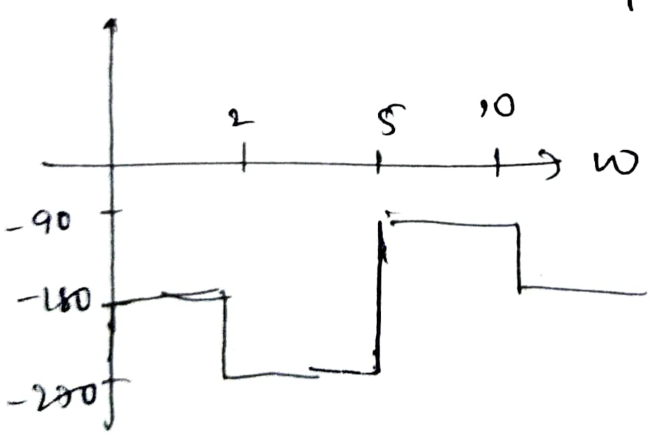
Q: $G(s)H(s) = \frac{10(s+5)^2}{s^2(s+2)(s+10)} = \frac{K(1+sz_1)(1+sz_2)}{(1+sP_1)(1+sP_2)}$

$= \frac{125\left(1+\frac{s}{5}\right)^2}{s^2\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$

Pole = -20 dB

zero = $+20$ dB

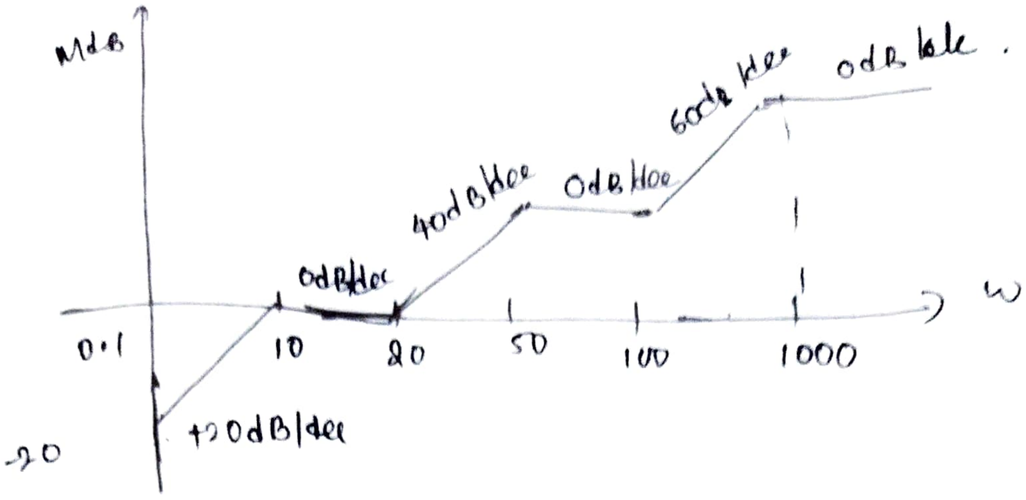




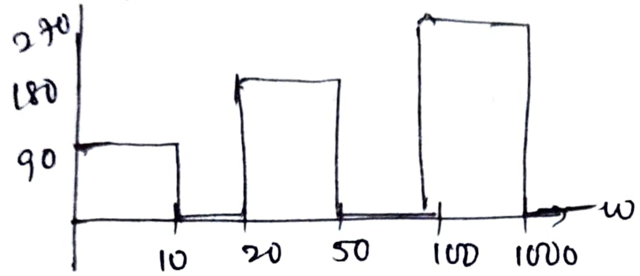
Q:

$$G(s)H(s) = \frac{s\left(1+\frac{s}{20}\right)^2\left(1+\frac{s}{100}\right)^3}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{50}\right)^2\left(1+\frac{s}{1000}\right)^3}$$

Sol:- corner frequencies 0.1, 20, 100, 10, 50, 1000



Unity gain k = 1



Ex:- $G(s)H(s) = \dfrac{\left(1+\frac{s}{2}\right)^2 \left(1+\frac{s}{50}\right)^3}{\left(1+\frac{s}{0.2}\right)^1 \left(1+\frac{s}{10}\right)^4}$

-270 + 3×90

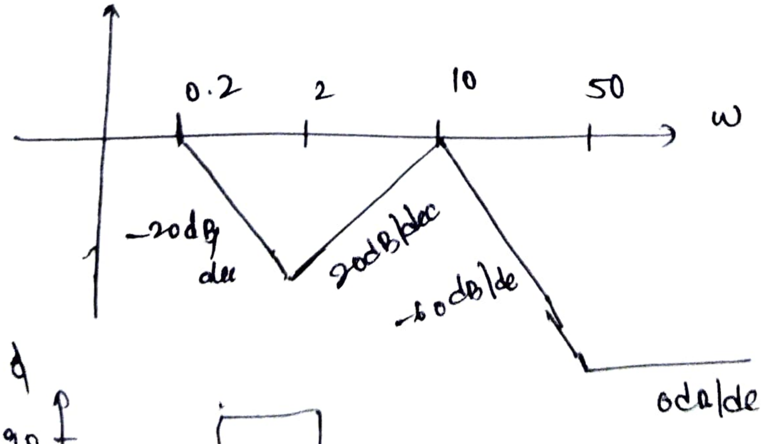
-90 + 180

-90 +

4× -90

-360 +90

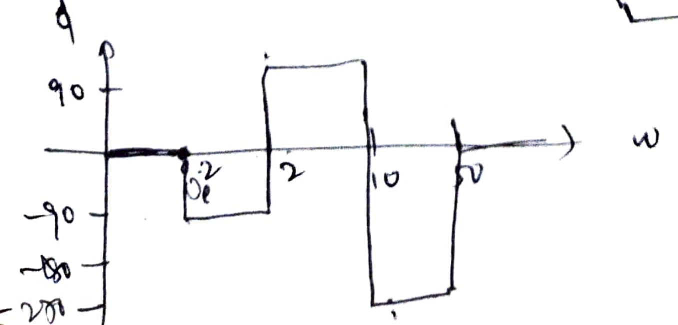
Sol:- corner frequencies 0.2, 2, 10, 50.

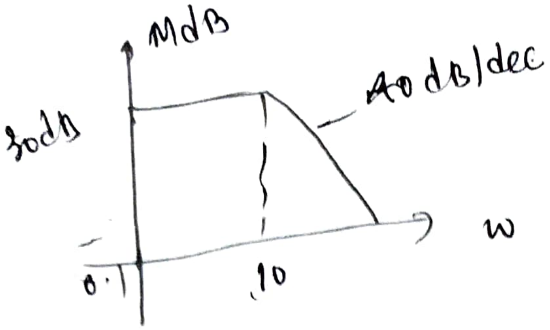


$(1+s)^2$

$\frac{1}{(j)^3}$ -270

$j = 90 \times 2$





GH = ?

loop gain k = ?

Sol: $GH = \dfrac{k}{\left(1+\frac{s}{10}\right)^2}$

$M_{dB}'\big|_{0.1} = 30\,dB$

$G(j\omega)H(j\omega) = \dfrac{k}{\left(1+\frac{j\omega}{10}\right)^2}$

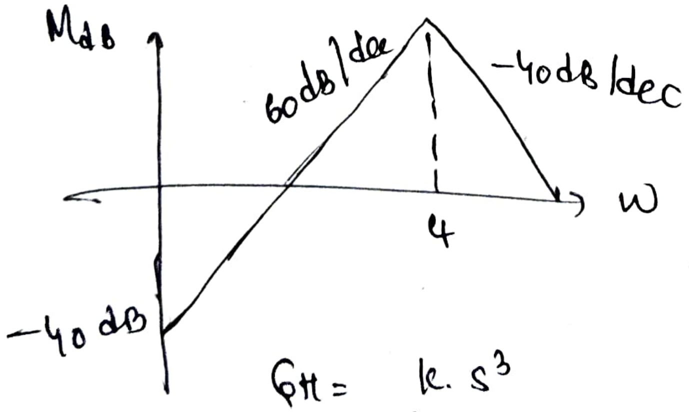
$20\log|G(j\omega)H(j\omega)| = 20\log k - 20\log\left(1+\frac{\omega^2}{100}\right)$

$30 = 20\log k - 20\log\left(1+\frac{0.01}{100}\right)$ ≈ 0

$20\log k = 30$

$k = 10^{1.5} = 31.62$

$GH = \dfrac{31.62}{\left(1+\frac{s}{10}\right)^2}$



$|1+j\omega|$

$\sqrt{1^2+j^2}$

$GH = \dfrac{k.s^3}{\left(1+\frac{s}{4}\right)^5}$

$M_{dB}\Big|_{0.1} = -40$

$20\log k + 60\log(0.1) - 20\log\left(\cancel{\sqrt{1+\frac{(0.1)^{5}}{4^5}}}\right) = -40$

$20\log k + 60\log(0.1) = -40$

$20\log k - 60 = -40$

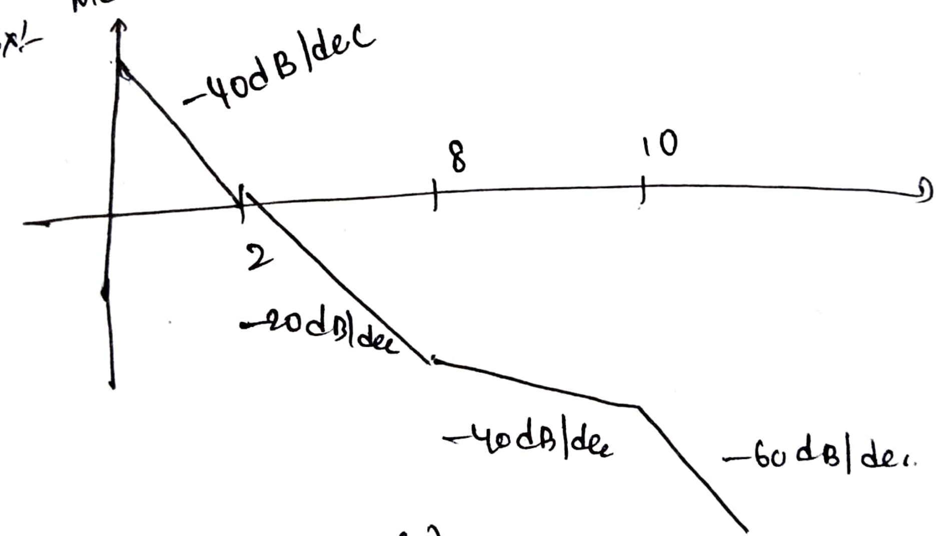
$20\log k = 20$

$\boxed{k = 10}$

$M_{dB} = 20\log k + 20\times 3\log(\omega) - 20\times 5\log\left(\sqrt{1+\frac{\omega^2}{16}}\right)\Bigg|_{\omega=0.1} = -40$

$GH = \dfrac{10s^3}{\left(1+\frac{s}{4}\right)^5}$

Ex:-



Sol:- $GH = \dfrac{k\left(1+\frac{s}{2}\right)}{s^2\left(1+\frac{s}{8}\right)\left(1+\frac{s}{10}\right)}$

$M_{dB}\Big|_{\omega=2} = 0$

$20\log k + 20\log\left(\sqrt{1+\frac{\omega^2}{4}}\right) - 40\log\omega - 0 - 0 = 0$

$$20\log k + 20\log(\sqrt{2}) - 40\log 2 = 0$$

$\cancel{3.01}$

$$20\log k = 9.03$$

10

3.01

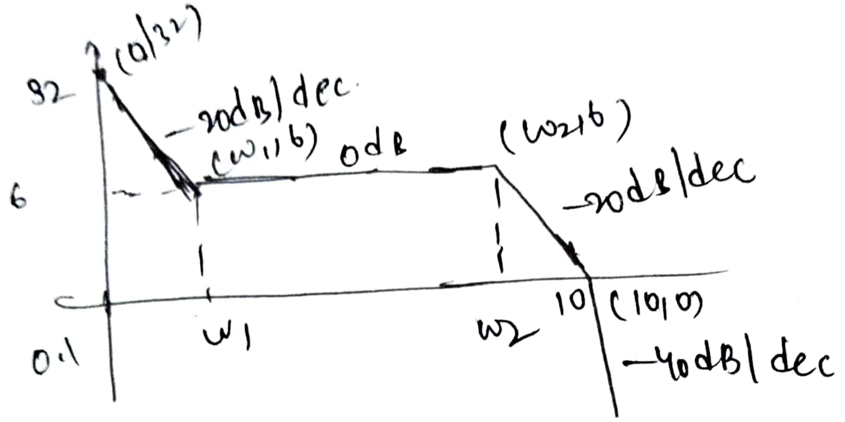
12.04

$$\log k = 0.4515$$

$$k = 2.828$$

$$k \simeq 3$$

Ex]



$$G_H = \frac{k\left(1+\frac{s}{\omega_1}\right)}{s\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{10}\right)}$$

$$-20 = \frac{6-32}{\log\omega_1 - \log 0.1}$$

$$+20 = \frac{+26}{\log\omega_1 + 1}$$

$$\log\omega_1 + 1 = 1.3$$

$$\log\omega_1 = 0.3$$

$$\omega_1 = 1.995 \simeq 2 \text{ rad/sec}$$

$$-20 = \frac{6-0}{\log\omega_2 - \log 10}$$

$$-20 = \frac{6}{\log\omega_2 - 1}$$

$$1 - \log\omega_2 = 0.3$$

$$\log\omega_2 = 0.7$$

$$GH = \frac{K\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)}$$

$$M_{dB}\Big|_{\omega=0.1} = 32\,dB$$

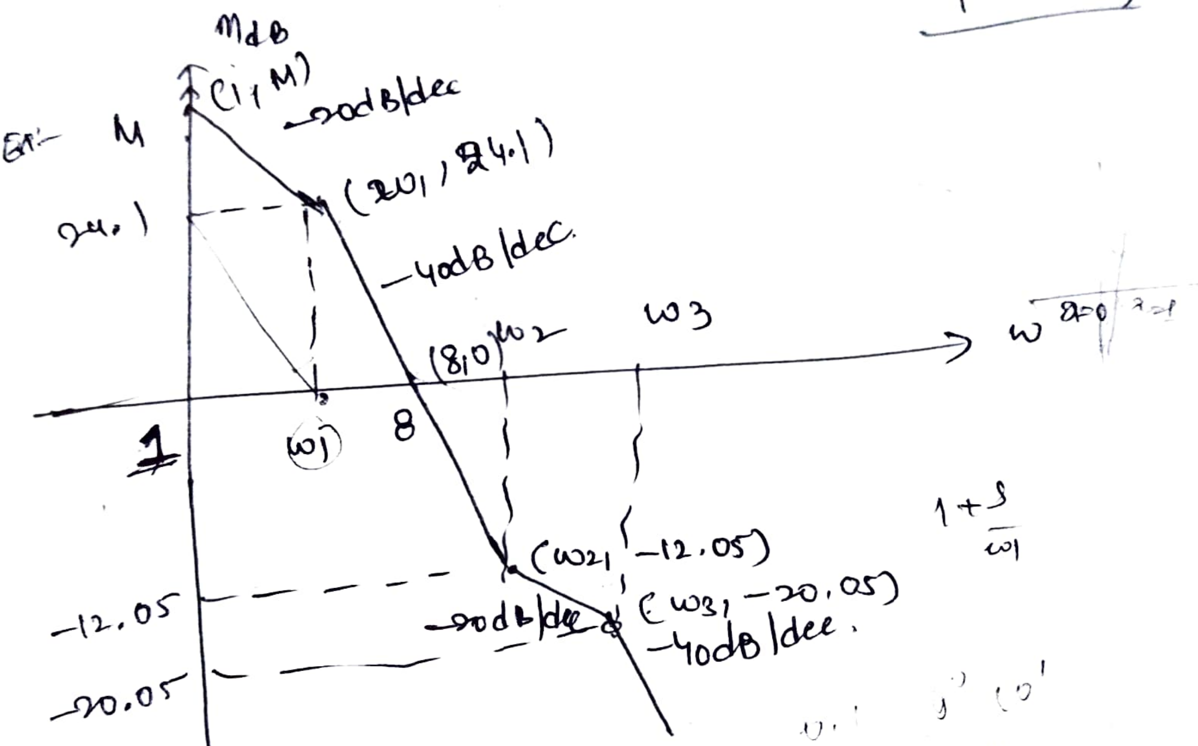
$$20\log K + 20\log\left(\sqrt{1+\frac{\omega^2}{4}}\right) - 20\log\omega = 32$$

$$20\log K + 0 - 20\log 0.1 = 32$$

$$20\log K + 20 = 32$$

$$20\log K = 12 \qquad \log K = 0.6 \Rightarrow K \simeq 3.98$$

$$\boxed{K \simeq 4}$$



$$GH = \frac{K\left(1+\frac{s}{\omega_2}\right)}{(\cancel{s+8})\, s\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_3}\right)}$$

$1\ 10^0$

$j\omega$

$$M_{dB}\ -40 = \frac{24.1 - 0}{\log\omega_1 - \log 8} \qquad -20 = \frac{M - 24.1}{\log 1 - \log\omega_1}$$

$$\omega_1 = 2\ rad/sec \qquad -20 = \frac{M - 24.1}{0 - \log 2}$$

$$\boxed{M = 30.12\,dB}$$

$$-40 = \frac{-12.05 - 0}{\log \omega_2 - \log 8}$$

$$-20 = \frac{-20.05 + 12.05}{\log \omega_3 - \log \omega_2}$$

$\log \omega_2 = 1.204$

$\omega_2 \simeq 16$ rad/sec

$\log \omega_3 = 1.604$

$\omega_3 \simeq 40$ rad/sec

$$GH = \frac{K\left(1+\frac{s}{16}\right)}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{40}\right)}$$

$$M_{dB}\Big|_{\omega=8} = 0$$

$$20\log K + 0 - 20\log \omega - 20\log\left(\sqrt{1+\frac{\omega^2}{4}}\right) + 0 = 0$$

$$20\log K - 20\log 8 - 20\log\left(\sqrt{17}\right) = 0$$

$$20\log K = 30.37$$

$$\log K = 1.52$$

$$K \simeq 33.11 \qquad \boxed{K = 32}$$

Minimum phase & Non-Minimum Phase Systems

T.F having neither poles nor zeroes on RH of s-plane called "minimum phase system".

T.F having poles / or zeroes of RH of s-plane called Non-minimum Phase systems

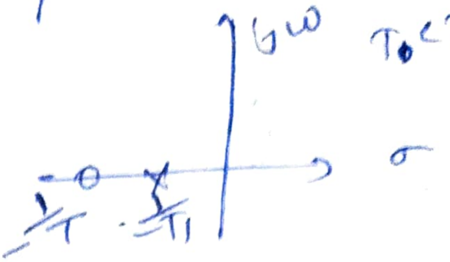
Allpass system:-

the system in which zeroes lies in RH of s-plane & Poles lies on LH of s-plane & the locus of pole-zero pair is symm. about Img axis The system is called Allpass system

All pass System.

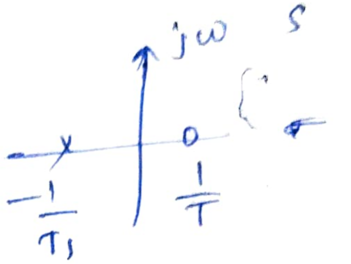
$T_0 < T_1$

$$G_1(s) = \frac{1+Ts}{1+T_1 s}$$



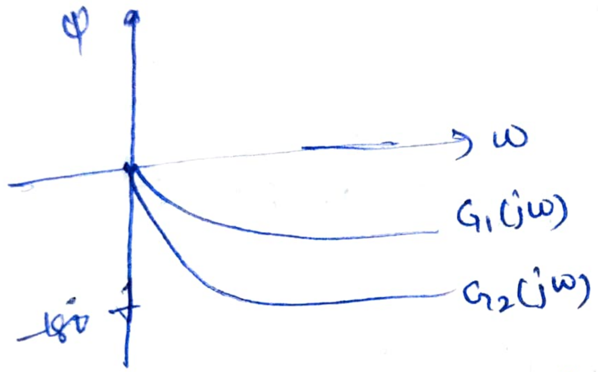
Minimum phase system

$$G_2(s) = \frac{1-Ts}{1+T_1 s}$$



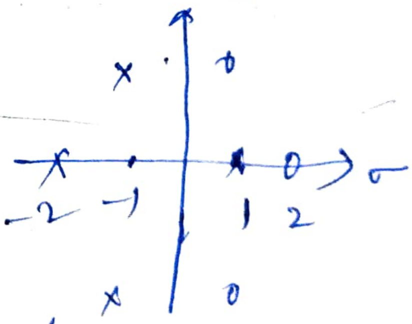
Non-minimum phase system

$\left(\frac{1}{T} - s\right)$



$$\angle G_2(j\omega) = 3\pi/2 + \tan^{-1}(T) - \tan^{-1}(T_1)$$

$$G_3(s) = \frac{(s-2)(s^2-2s+2)}{(s+2)(s^2+2s+2)}$$

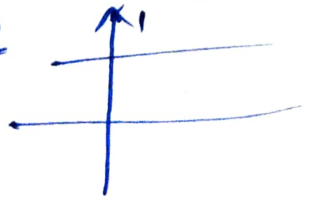


All pass system (APS)

magnitude is 1 for APS

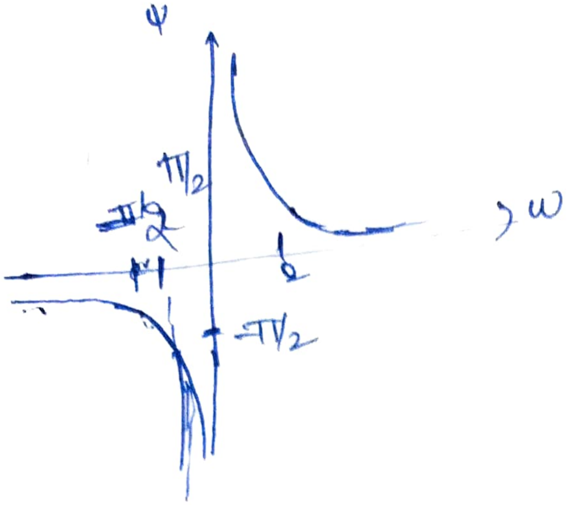
$|G(j\omega)|$

$|G_3(j\omega)| = 1$



$$\angle G_3(j\omega) = \left(\pi - \tan^{-1}\left(\frac{\omega}{2}\right)\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= \pi - 2\tan^{-1}\left(\frac{\omega}{2}\right)$$



Ex! $G(s) = \dfrac{(s-1)(s+4)}{(s+2)(s+3)}$ → Non minimum phase system NMPS

$$G(s) = \frac{(s-1)(s+4)(s+1)}{(s+2)(s+3)(s+1)}$$

$$= \frac{(s+4)(s+1)}{(s+2)(s+3)} \times \frac{(s-1)}{(s+1)}$$ → All pass system

↳ Minimum phase system MPS

N.M.P.S = M.P.S * A.P.S

Gain Margin & Phase Margin:

Gain Margin (GM) = $-20\log M\big|_{\omega pc}$, $M = |GH|$, $M = \cancel{20\log}|GH|$

dB

Phase Margin (PM) = $180 + \phi\big|_{\omega gc}$, ϕ = Phase angle of GH

ωpc = Phase crossover frequency → $\phi = -180°$

ωgc = gain crossover frequency → $|M| = 1$

$|M| = 0$ dB

$\omega_{pc} \rightarrow$ frequency at which phase of GH is $-180°$

$\omega_{gc} \rightarrow$ frequency at which gain is 1 or 0 dB

→ stability → GM, PM – positive

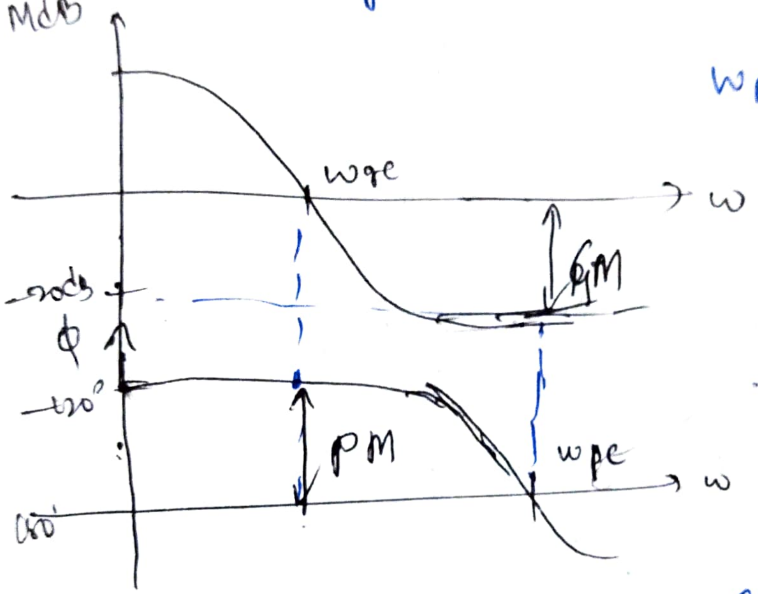
To obtain this $\omega_{pc} > \omega_{gc}$

→ System is Marginally stable GM = 0 dB, PM = 0°

$\omega_{pc} = \omega_{gc}$

→ system is Unstable GM, PM – negative

$\omega_{pc} < \omega_{gc}$



$\omega_{pc} > \omega_{gc}$

$20\log M\Big|_{\omega_{pc}} = -20\,dB$

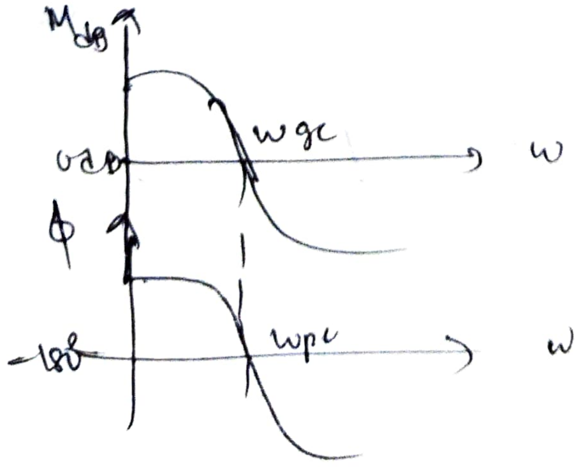
$-GM = -20\,dB$

$\boxed{GM = 20\,dB}$

GM, PM ⇒ +ve

$\phi\Big|_{\omega_{gc}} = -120°$

$180 + \phi\Big|_{\omega_{gc}} = 180 - 120 = 60$

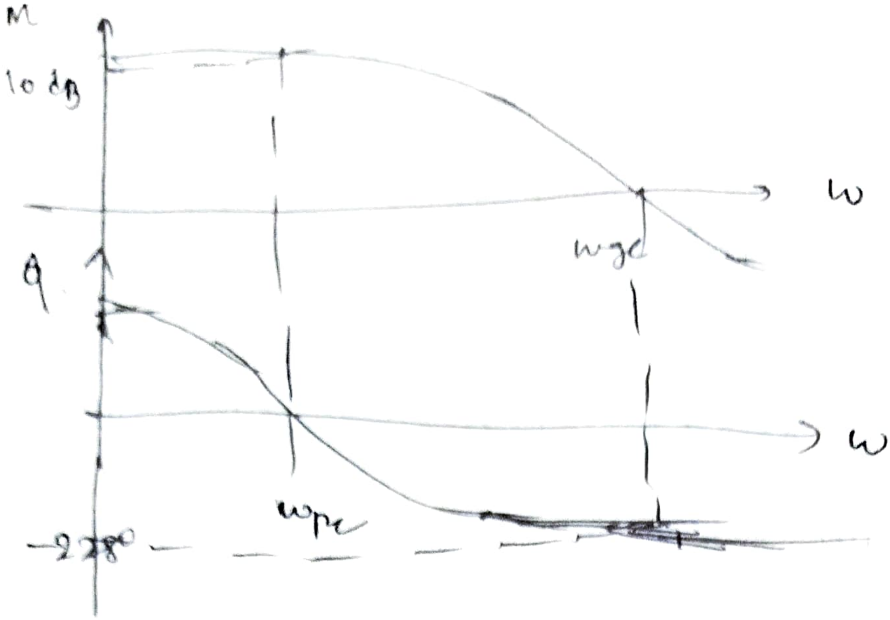


$\omega_{gc} = \omega_{pc}$

$GM\Big|_{\omega_{pc}} = 0\,dB$

$\phi\Big|_{\omega_{gc}} = -180°$

$PM = 180 - 180 = 0°$



GM = -10 dB

PM = 180 + (-228)

= -48°

Unstable

GM, PM both -ve

Compute GM & PM of $GH = \frac{1}{s(s+5)(s+10)}$

phase cross over frequency

$\angle GH |_{\omega = \omega_{pc}} = -180°$

$-90 - \tan^{-1}\left(\frac{\omega_{pc}}{5}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{10}\right) = -180$

$90 = \tan^{-1}\left[\frac{\frac{\omega_{pc}}{5} + \frac{\omega_{pc}}{10}}{1 - \frac{\omega_{pc}^2}{50}}\right]$

$\tan^{-1}(A) + \tan^{-1}(B) = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$1 - \frac{\omega_{pc}^2}{50} = 0$

$\omega_{pc} = \sqrt{50}$ rad/sec

Gain crossover frequency

$K|M|_{w=w_{gc}} = 1$ or $20 \log M = 0\,dB$

$|GH|_{w=w_{gc}} = 1$

$$\frac{1}{w_{gc}\sqrt{w_{gc}^2+25}\sqrt{w_{gc}^2+100}} = 1$$

$$w_{gc}^2 (w_{gc}^2+25)(w_{gc}^2+100) = 1$$

$$w_{gc}^5 + w_{gc}^3 \times 125 + w_{gc} \times 2500 = 1$$

The open loop Transfer function of system is

$$GH = \frac{K}{s(s+2)(s+4)}$$

Det. value of K such that phase Margin $= 60 dB$

$180 + \phi|_{w=w_{gc}} = 60$

$|GH|_{w_{gc}} = 1$

$\boxed{w_{gc} = 0.72 \text{ rad/sec}}$

$\phi|_{w_{gc}} = -120$

$$\frac{K}{w\sqrt{w_{gc}^2+4}\sqrt{w_{gc}^2+16}} = 1$$

$\angle GH|_{w=w_{gc}} = -120$

$w\sqrt{w_{gc}^2+4}\sqrt{w_{gc}^2+16} = K$ — (1)

$-90 - \tan^{-1}\left(\frac{w_{gc}}{2}\right) - \tan^{-1}\left(\frac{w_{gc}}{4}\right) = -120$

$$30 = \tan^{-1}\left(\frac{\frac{\omega_{gc}}{2} + \frac{\omega_{gc}}{4}}{1 - \frac{\omega_{gc}^2}{8}}\right) \Rightarrow \frac{1}{\sqrt{3}} = \tan \frac{\frac{3\omega_{gc}}{4}}{\frac{8 - \omega_{gc}^2}{8}} \cdot \frac{6\omega_{gc}}{8 - \omega_{gc}^2}$$

$\Rightarrow \omega_{gc} = 0.72$ rad/sec.

$$(0.72)(\sqrt{(0.72)^2 + 4})(\sqrt{0.72^2 + 16}) = K$$

$$\boxed{k = 6.22}$$

Rules for plotting polar plot :-

step 1: sub. $s = j\omega$ in GH function

2: find the $|GH|_{\omega=0}$ & $\angle GH|_{\omega=0}$

Starting Mag & Phase.

3: find the $|GH|_{\omega=\infty}$ & $\angle GH|_{\omega=\infty}$

ending Mag & phase

4: Identify the Starting direction

→ If finite poles is located near imag. axis then (of GH)
the starting direcn is clockwise

→ if finite zero of GH is located near imag axis then starting direcn is Anti-clockwise.

5: Ending direction :-

$$\phi = \phi_1 - \phi_2$$

$\phi \rightarrow +ve \rightarrow$ ED(is clockwise)

$\phi \rightarrow -ve \rightarrow$ ED(is Anticlockwise

Ex: $GH = \dfrac{1}{1+sT}$

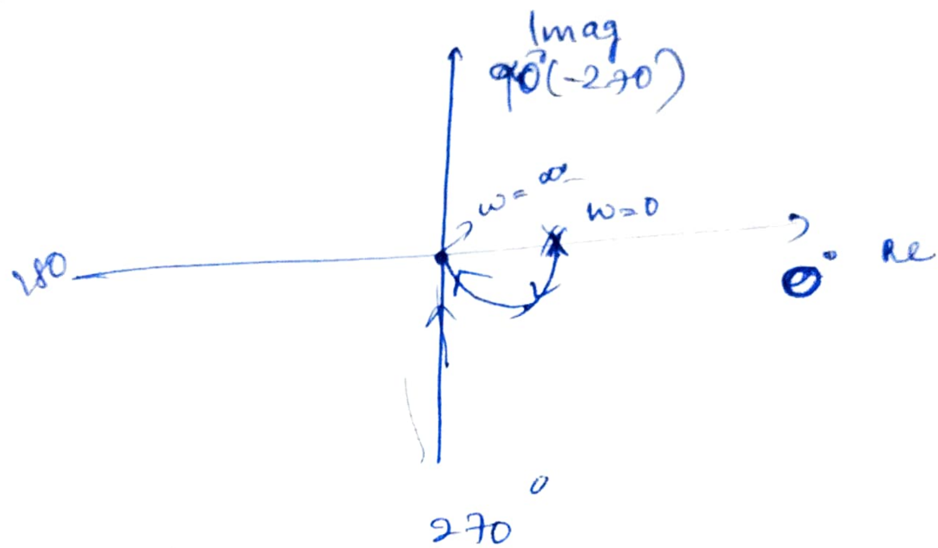
$G(j\omega)H(j\omega) = \dfrac{1}{1+j\omega T}$

$|GH|_{\omega=0} = 1 \qquad \angle GH|_{\omega=0} = \angle 0^\circ \Rightarrow \omega=0 \quad GH = 1\angle 0^\circ$

$\omega = \infty \rightarrow 0\angle -90^\circ$

→ SD ⇒ clockwise

→ ED ⇒ $\phi = \phi_1 - \phi_2 = (+ve)$ → clockwise



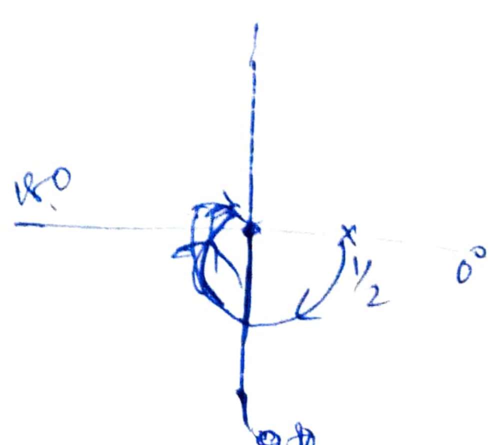
$\omega = \dfrac{1}{T} = 0.707\angle -45^\circ$

Ex $GH = \dfrac{1}{(s+1)(s+2)}$

$G(j\omega)H(j\omega) = \dfrac{1}{(j\omega+1)(j\omega+2)}$

$\omega = 0 \qquad |GH| = 1/2 \Rightarrow 1/2\angle 0^\circ$

$\omega = \infty \qquad 0\angle -180$



S.D = Clockwise

ED = $0-(-180) \Rightarrow +ve$ clockwise

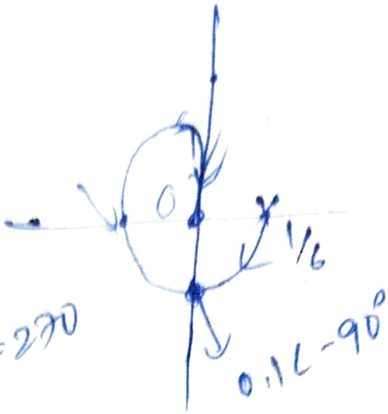
$$GH = \frac{1}{(s+1)(s+2)(s+3)}$$

$\omega = 0,\ GH = \frac{1}{6}\angle 0^\circ$

$\omega = \infty,\ GH = 0\angle -270^\circ$

SD = clkw ED = $\phi_1 - \phi_2$ = +ve

clw $\angle\phi = 270$



$$\phi = \tan^{-1}(w) - \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}\left(\frac{w}{3}\right) = -90^\circ$$

$$= \tan^{-1}(w) + \tan^{-1}\left(\frac{\frac{w}{2} + \frac{w}{3}}{1 - \frac{w^2}{6}}\right) = -90$$

$$= \tan^{-1}\left(\frac{w + \left(\frac{\frac{w}{2} + \frac{w}{3}}{1 - \frac{w^2}{6}}\right)}{1 - w\left(\frac{\frac{w}{2} + \frac{w}{3}}{1 - \frac{w^2}{6}}\right)}\right) = -90^\circ$$

$$\frac{\frac{5w}{\cancel{6}}}{\frac{6 - w^2}{\cancel{6}}}$$

$$\frac{5w}{6 - w^2} + w$$

$$\frac{5w + 6w - w^3}{1 - w\left(\frac{5w}{6 - w^2}\right)}$$

$$= \tan^{-1}\left(\frac{(5w + 6w - w^3)(6 - w^2)}{6 - w^2 - 5w^2}\right) = \frac{1}{0}$$

$$= 6 - 6w^2 = 0$$

$w^2 = 1$

$w = \pm 1$ rad/sec

$$|GH|_{w=1} = \frac{1}{\sqrt{2}\sqrt{5}\sqrt{10}} = \frac{1}{10} = 0.1$$

Touching real axis $\phi = 180°$

$$\tan^{-1}\left(\frac{(5\omega + 6\omega - \omega^3)(6-\omega^2)}{6-\omega^2-5\omega^2}\right) = 180$$

$$(5\omega + 6\omega - \omega^3)(6-\omega^2)$$

$\omega = \sqrt{11}$ rad/sec

Ex

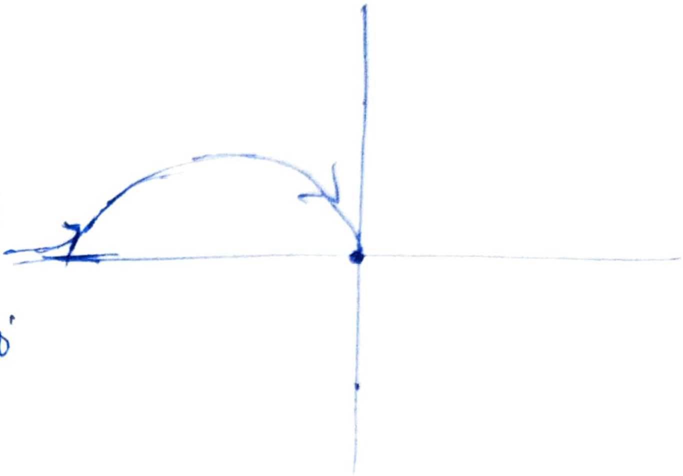
$$GH = \frac{1}{s^2(s+1)}$$

$\omega = 0, \quad \infty \angle -180°$

$\omega = \infty, \quad 0 \angle -270°$

SD $\Rightarrow$ CW

ED $\rightarrow$ CW



$0° \quad = 0$

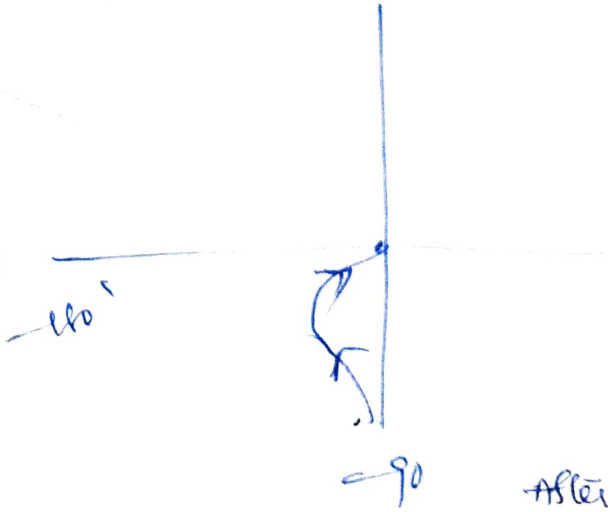
$90°$

Ex

$$GH = \frac{1}{s(s+1)}$$

$\omega = 0, \quad \infty \angle -90°$

$\omega = \infty, \quad 0 \angle -180°$



SD $\Rightarrow$ CW

ED $\rightarrow$ CW

After 3 p

$$GH = \frac{s+1}{s^3}$$ polar plot.

Poles 0,0,0

zeros -1

$$GH(j\omega) = \frac{1+j\omega}{-j\omega^2}$$

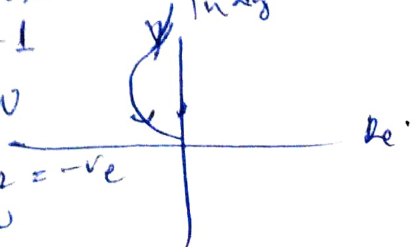
S.D = ACW

ED = $\phi_1 - \phi_2 = -ve$

ACW

$\omega = 0, \quad -\infty \angle -270°$

$\omega = \infty, \quad 0 \angle -180$



Nyquist Plot:-

Rules for drawing Nyquist Plot:-

R1:- Draw the polar plot

R2: Draw the mirror image of the polar plot

R3: the no. of infinite radius half circles will be equal to the no. of poles or zeroes at origin

→ the infinite radius half circle will start at the point where mirror image of polar plot ends

→ The infinite radius half circle will end at the point where the polar plot starts

Stability analysis using the Nyquist plot:

→ Transfer function (OL system) $GH = \dfrac{K N(s)}{D(s)}$ — ①

→ Closedloop system Transfer function, $H(s) = 1$ $\dfrac{C(s)}{R(s)} = \dfrac{G}{1+G}$

$$\frac{C(s)}{R(s)} = \frac{G(s)\,D(s)}{D(s) + K N(s)} \quad — ②$$

Characteristic equation : $1+G$

$$q(s) = 1+G$$

$$= 1 + \frac{K N(s)}{D(s)}$$

$$= \frac{D(s) + K N(s)}{D(s)}$$

- Poles of CE are same as poles of OL T.F
- zeroes of CE are same as poles of CL TF

Poles of CE = poles of OL TF

zeroes of CE = poles of CL TF

$1 + GH = 0$

$D(1 + \frac{GH}{D})$

Nyquist stability criteria

$\boxed{N = P - Z}$

N = no. of encirclements of the points $-1+j0$

P = poles of CE (= poles of OL T/F) that lies in the RH of s-plane

Z = zeroes of the CE (= Poles of CL T/F) that lies on the RH of the s-plane.

① The open loop system is said to be stable when $\boxed{N = -Z}$

② The closed loop system is said to be stable when $\boxed{N = P}$

$$GH = \frac{10}{s+5}$$

$GH = \frac{10}{(s+5)}$ (No poles or zeroes at origin)

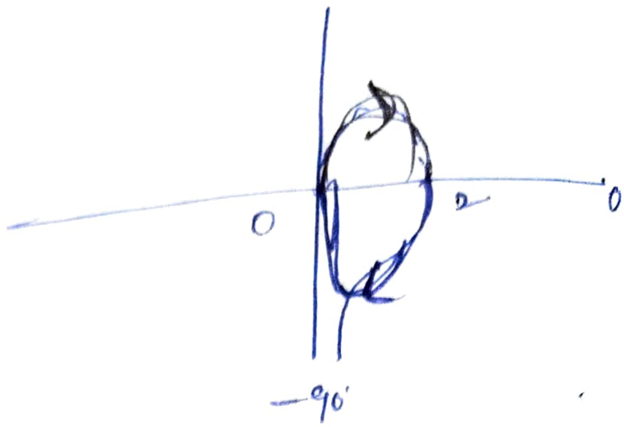
$\phi = -\tan^{-1}\left(\frac{w}{5}\right)$

$w=0,\ 2\angle 0^\circ$

$w=\infty,\ 0\angle -90^\circ$

SD = CW

ED ⇒ CW



Stable condition

OL system

$N = -Z$

$N=0$

$Z=0$

stable

CL system

$N=P$

$N=0$

$P=0$

stable.

~~continuation~~

$GH = \frac{K(s+3)}{s(s-1)}$

$= \left|\frac{K(j\sqrt{3}+3)}{(j\sqrt{3})(j\sqrt{3}-1)}\right|_{s=j\sqrt{3}}$

$\tan^{-1}\left(\frac{w+\frac{w}{3}}{1-\frac{w^2}{3}}\right) = \tan^{-1}(90)$

$= \infty$

$= \frac{K\sqrt{9+3}}{\sqrt{3}\sqrt{3+1}} = \frac{K\ 2\sqrt{3}}{\sqrt{3}\ 2}$

$= K$

$\frac{\frac{3w+w}{3}}{\frac{3-w^2}{3}} = \infty$

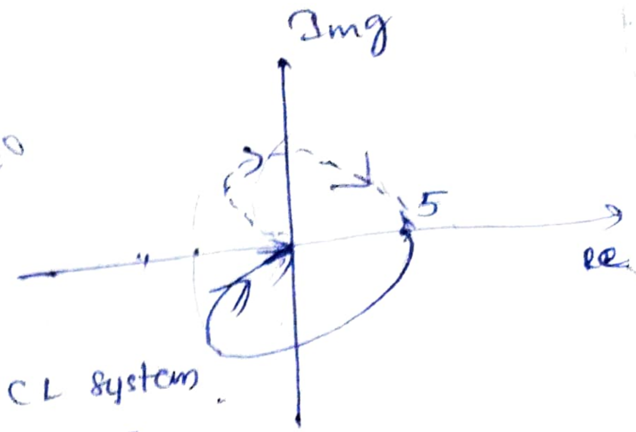
$3-w^2 = \infty$

$w = \pm\sqrt{3}$

$w = \sqrt{3}$

$$GH = \frac{10}{(s+1)(s+2)}$$

$1+GH=0$



$\omega = 0 \quad |GH| = 5\angle 0^\circ$

$\omega = \infty, \quad GH = 0\angle -180^\circ$

O.L system

$N = -Z$

$N = 0$

$Z = 0$ $\therefore$ stable

CL system

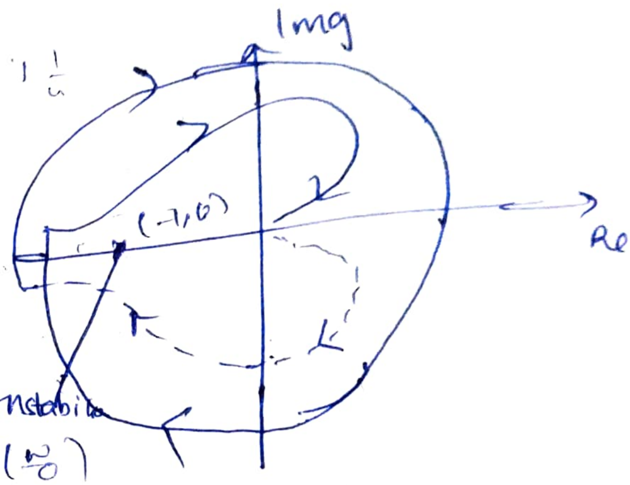
$N = P$

$P = 0$ stable

$N = 0$

$$\rightarrow GH = \frac{10}{s^2(s+1)(s+2)}$$

$j\omega+1 \quad \infty \quad j\omega+2 \quad \frac{1}{\omega}$



$\omega = 0, \quad GH = \infty \angle -180^\circ$

$\omega = \infty, \quad GH = 0\angle -360^\circ$

S.D → CW

E.D → CW

OL system stability $N = -Z$

~~N = 0~~

$Z = 0$

pole of CL

ACW ⇒ $N = -ve$

CL stability

$N = P$

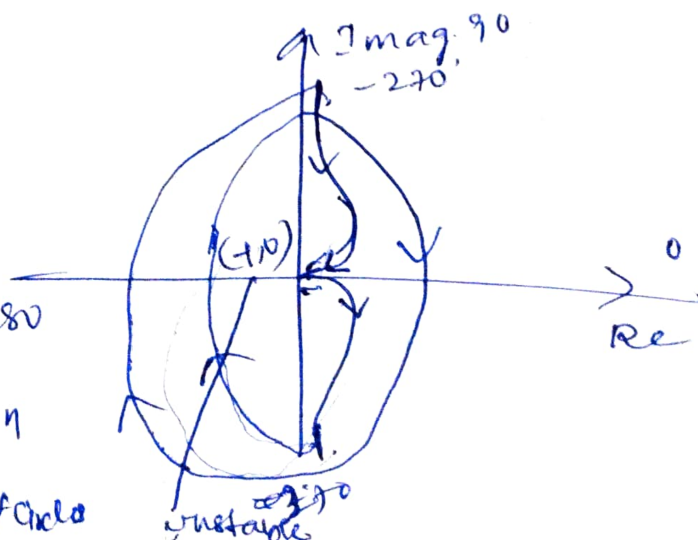
$P = 0$

~~N = 0~~

poles of OL

unstable

$$\rightarrow GH = \frac{1}{s^3(s+1)}$$



$\omega = 0, \quad \infty \angle -270^\circ$

$\omega = \infty, \quad 0\angle -360^\circ$

SD → CW

ED → CW

No. of poles at origin = No. of half circle

CL system stability

$N = P - Z$

$-2 = 0 - Z$

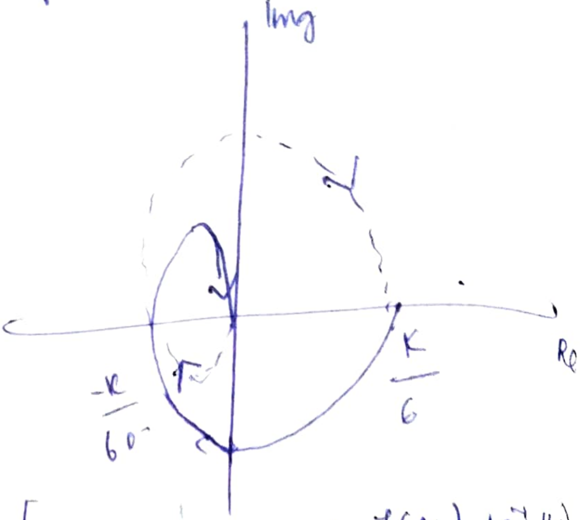
$\boxed{Z = 2}$ 2 CL poles lies in the RHS of s-plane

$N = 2$ Unstable

$P = 0$

$N = -2$ ↳ Negative bcz of cw direction

$$GH = \frac{K}{(s+1)(s+2)(s+3)}$$



$\omega = 0, \ \frac{K}{6} \angle 0°$

$\omega = \infty, \ 0 \angle -270°$

CL system stability

$N = P$

$N = 0, \ \frac{-K}{60} > -1$

$K < 60$

↳ $-\tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) = -180°$

ω

$\omega = \frac{-K}{60}$

GH

$P = 0$

$\frac{K}{6} > -1 \quad K > -6$

K range for CL stability

$\boxed{-6 < K < 60}$

tan

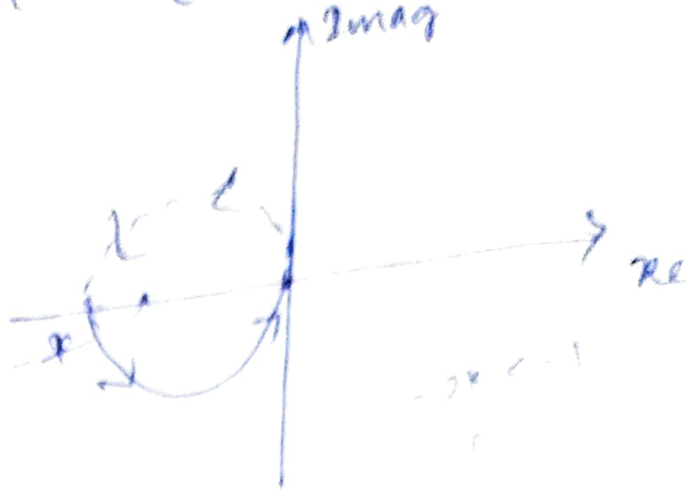
$j\omega + 2$

Ex:- $$GH = \frac{K(s+2)}{(s+1)(s-1)}$$

$\omega = 0, \ -2K \angle -180$

$\omega = \infty, \ 0 \angle -90°$

$\phi = + \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}(w) - (180 - \tan^{-1} w)$



SP → Not required

ED → $\phi_1 - \phi_2 = -ve$

ACW

closed L system

$N = P$

$P = 1 \quad N = 1$ ie $-2K < -1$

$\boxed{K > 1/2}$

$GH = \frac{K(s+3)}{s(s-1)}$

$\phi = -\tan^{-1}(\infty) - \tan^{-1}(-w) + \tan^{-1}$

$\phi = -90 - (180 - \tan^{-1} w) + \tan^{-1}\left(\frac{w}{3}\right)$

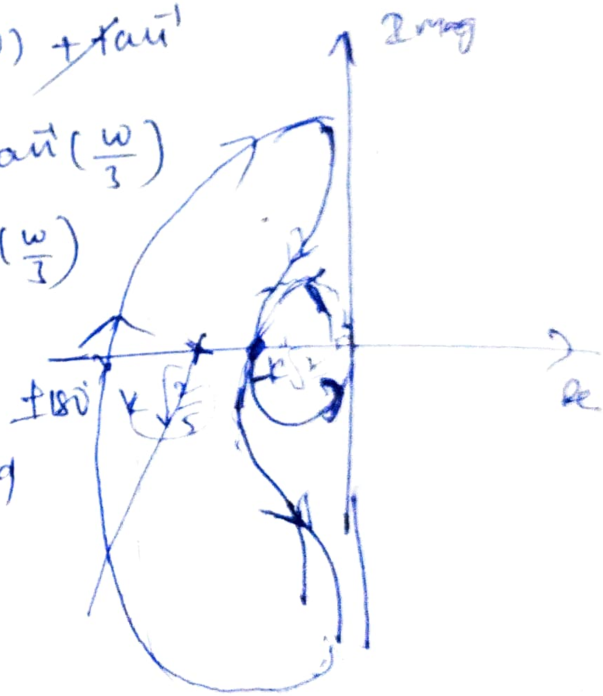
$-180 = -270 + \tan^{-1} w + \tan^{-1}\left(\frac{w}{3}\right)$

$w = 0, \ \infty \angle -270°$ $\quad -270150$

$w = \infty, \ 0 \angle -90°$ $\quad \phi_1 - \phi_2 =$

SP ⇒ not required

ED ⇒ ACW



cl system

$N = P$

$-K < -1$

$\boxed{K > 1}$

-180 $N = P$

$P = 1$

$K\sqrt{\frac{2}{5}}$

$K\sqrt{\frac{5}{2}}$

Polar plot examples :-

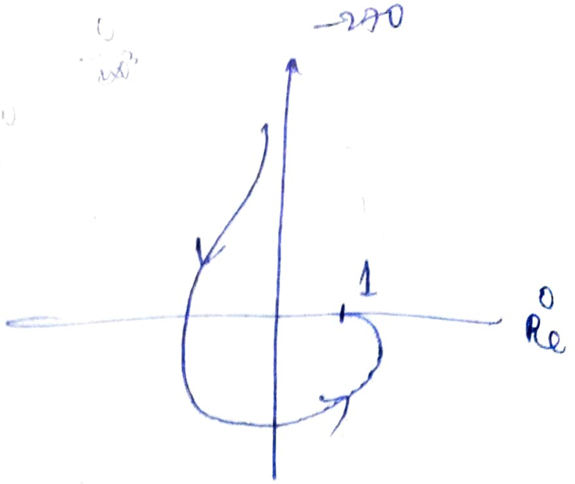
$GH = \frac{(s+1)(s+2)(s+3)}{s^3}$

$\omega = 0, \ \infty \angle -270^\circ$

$\omega = \infty, \ 1 \angle 0^\circ$

SD → ACW

ED → $-270 - 0 = -270$ = ACW



$GH = \frac{1}{s(s-1)}$

$-90 \quad -(180 - \tan^{-1}(\omega))$

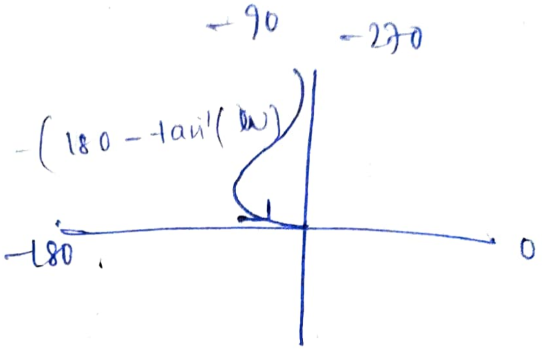
$\omega = 0 \quad \infty \angle -270^\circ$

$\omega = \infty, \quad 0 \angle -180$

SD → not considered when pole lie on RH of s-plane. i.e

ED → ACW

except



$GH = \frac{1}{s(-s-1)}$ $\quad -90 - (180 + \tan^{-1}(\omega))$

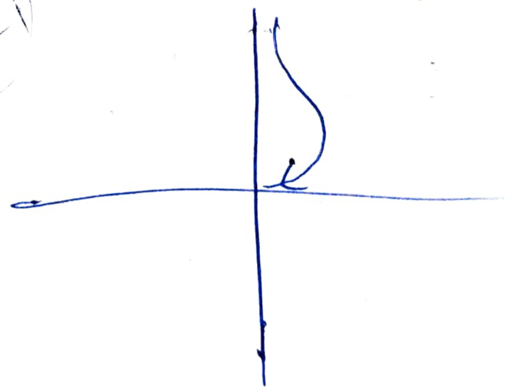
Both +ve
s+1

$\omega = 0, \quad \infty \angle -270$

$\omega = \infty, \quad 0 \angle -360^\circ$

SD → Not required

ED → CW



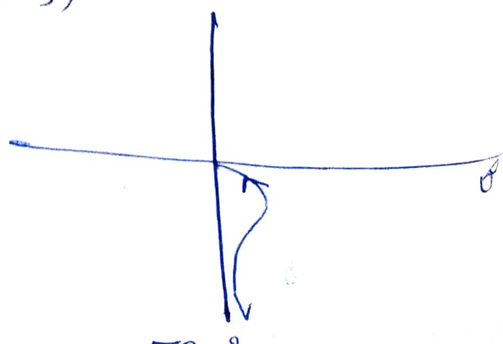
$GH = \frac{1}{s(-s+1)}$ $\quad -90 - (-\tan^{-1}(\omega))$

$\omega = 0, \quad \infty \angle -90$

$\omega = \infty, \quad 0 \angle 0^\circ$

S.D → Not required

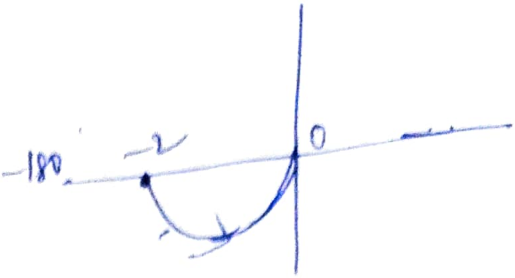
ED → ACW



$$GH = \frac{s+2}{(s+1)(s-1)} \quad \phi = -\tan^{-1}(\omega) - \left(180 - \tan^{-1}(\omega)\right) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

$\omega = 0, \; -2 \angle -180^\circ$

$\omega = \infty, \; 0 \angle -90$



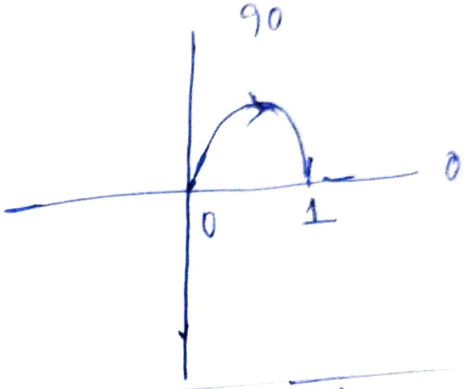
SD ⇒ Not required P > 2

ED ⇒ ACW

$$GH = \frac{s}{s+1} \quad \phi = 90 - \tan^{-1}(\omega)$$

$\omega = 0, \; 0 \angle 90^\circ$

$\omega = \infty, \; 1 \angle 0^\circ$



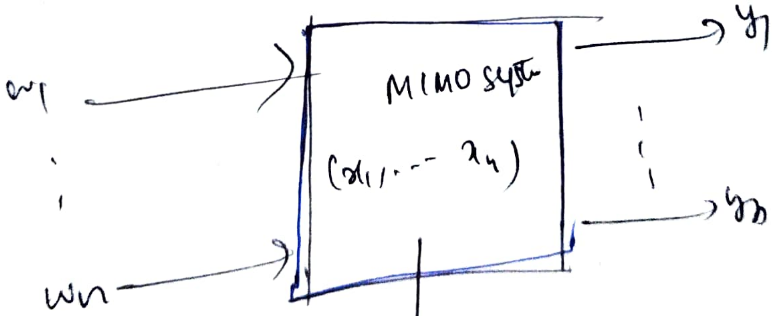
SD ⇒ CW

ED ⇒ CW

15/11/2022

$$\to GH = \frac{k(s+3)}{s(s-1)}$$

State space analysis :



state variables

Standard form of state Model :

$$\dot{X} = AX + BU$$

$n\times n \; n\times 1$; $n\times m \cdot m\times 1$; $(n\times 1)$

state equation or Diff. state vector

A → state matrix

X → state vector

B → I/P matrix

U → I/p vector

$\underset{p\times 1}{Y} = \underset{p\times n}{C}X + DU$

o/p vector, o/p matrix, transition matrix

$$Y_{p\times 1} = C_{p\times n}\, X_{n\times 1} + D_{p\times m}\, U_{m\times 1}$$

Ex: $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 5y = 10u(t)$

one - i/p $\Rightarrow m = 1$

one - o/p $\rightarrow p = 1$

i/p $\xrightarrow{(m\times 1)}$ $\boxed{\text{MIMO } (n\times 1)}$ $\rightarrow$ o/p $(p\times 1)$

$$U = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

No. of state variables = order of DE

$n = 2$

$$\underset{2\times 1}{\dot{X}} = \underset{2\times 2}{A}\,\underset{2\times 1}{X} + \underset{2\times 1}{B}\,\underset{1\times 1}{U}$$

$u(t) \rightarrow$ i/p

$y(t) \rightarrow$ o/p

$$\underset{1\times 1}{y} = \underset{1\times 2}{C}\,\underset{2\times 1}{X} + D_{1\times 1}U_{1\times 1}$$

Ex: $y''' + 2y'' + 3y' + y = u$

$\Rightarrow \dot{x}_3 + 2\dot{x}_2 + 3\dot{x}_1 + x_1 = u \Rightarrow \dot{x}_3 = -2x_2 - 3x_1 - x_1 + u$

State variables = 3 $\{x_1, x_2, x_3\}$

Assume $x_1 = y$

i/p $m = 1$

o/p $p = 1$

$\dot{x}_1 = \frac{dy}{dt} = \dot{y} = x_2$

$\dot{x}_2 = \ddot{y} = x_3$

$\dot{x}_3 = \dddot{y}$

$$\dot{x}_{n\times 1} = Ax + Bu_{m\times 1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix}_{3\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times 1} [u]_{1\times 1}$$

$$\dot{x}_3 = -2\dot{x}_2 - 3\dot{x}_1 - x_1 + u$$

$$= -2x_3 - 3x_2 - x_1 + u$$

$$y''' + \cancel{2y''} + 3y' + \cancel{y} + u$$

$$-2x_3 - 3x_2 \cancel{-x_1} + u + x_3 + \cancel{x_2} + \cancel{x_1} + u$$

$$\cancel{Y = CX + DU}$$

$$\cancel{= [0\ -2\ -1]_{1\times 3} \{x_1\ x_2\ x_3\}_{3\times 1} + [2]_{1\times 1}} \{U\}_{1\times 1}$$

$$Y = CX + DU \quad \Rightarrow Y = x_1$$

$$Y = [1\ 0\ 0]_{1\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} + [0]_{1\times 1} [U]_{1\times 1}$$

Ex:

$$y'''' + 10y''' - 6y'' + 7y' + 5y = 10u(t)$$

$$\dot{x}_4 + 10\dot{x}_3 + 6\dot{x}_2 + 7\dot{x}_1 + 5x_1 = 10U$$

$m = 1$ $\quad x_1 = y$ $\quad x_3 = \ddot{y} = \dot{x}_2$

$p = 1$ $\quad x_2 = \dot{y} = \dot{x}_1$ $\quad x_4 = \dddot{y} = \dot{x}_3$

$x_5 = \ddddot{y} = \dot{x}_4$

$n = 4$

$$\dot{x} = Ax \cancel{+ BU}$$

$$\dot{x}_4 = -10\dot{x}_3 + 6\dot{x}_2 - 7\dot{x}_1 - 5x_1 + 10U$$

$$= -10x_4 + 6x_3 - 7x_2 - 5x_1 + 10U$$

$$\underset{n\times1}{\dot{X}} = \underset{n\times n}{A}\ \underset{n\times1}{X} + \underset{n\times m}{B}\ \underset{m\times1}{U}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \underset{4\times4}{\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-5&-7&+6&-10\end{bmatrix}}\underset{4\times1}{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}} + \underset{4\times1}{\begin{bmatrix}0\\0\\0\\10\end{bmatrix}}\underset{1\times1}{[U]}$$

$$\underset{p\times1}{Y} = \underset{p\times n}{C}\ \underset{n\times1}{X} + \underset{p\times m}{D}\ \underset{m\times1}{U}$$

$$\underset{1\times1}{[y]} = \underset{1\times4}{[1\ 0\ 0\ 0]}\underset{4\times1}{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}} + \underset{1\times1}{[0]}\underset{1\times1}{[U]}$$

$$\frac{Y(s)}{U(s)} = \frac{10s+5}{s^3+6s^2+7s+8}$$

$\dot{x}_n = s^n$

$x_2 = \dot{x}_1 = s$

$x_3 = \dot{x}_2 = s^2$

$\dot{x}_3 = s^3$

s power = no. of variables.

$U(s) = s^3+6s^2+7s+8$

$= \dot{x}_3 + 6\dot{x}_2 + 7\dot{x}_1 + 8x_1$

$= \dot{x}_3 + 6x_3 + 7x_2 + 8x_1$

$\dot{x}_3 = -8x_1 - 7x_2 - 6x_3 + U(s)$

$m = 1$

$p = 1$

$n = 3$

$Y(s) = 10s+5$

$= 10\dot{x}_1 + 5x_1$

$= 10x_2 + 5x_1$

$$\underset{3\times1}{\dot{X}} = \underset{3\times3}{A}\cdot\underset{3\times1}{X} + \underset{3\times1}{B}\,\underset{1\times1}{U}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \cancel{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -7 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underset{3\times1}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} [U]$$

$$\underset{1\times1}{y} = \underset{p\times3}{C}\,\underset{3\times1}{X} + \underset{1\times1}{D}\,\underset{1\times1}{U}$$

$$y = [5 \;\; 10 \;\; 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0][U]$$

$$\rightarrow \frac{Y(s)}{U(s)} = \frac{s^2 + 5s + 10}{s^4 + 3s^3 + 6s^2 + 5}$$

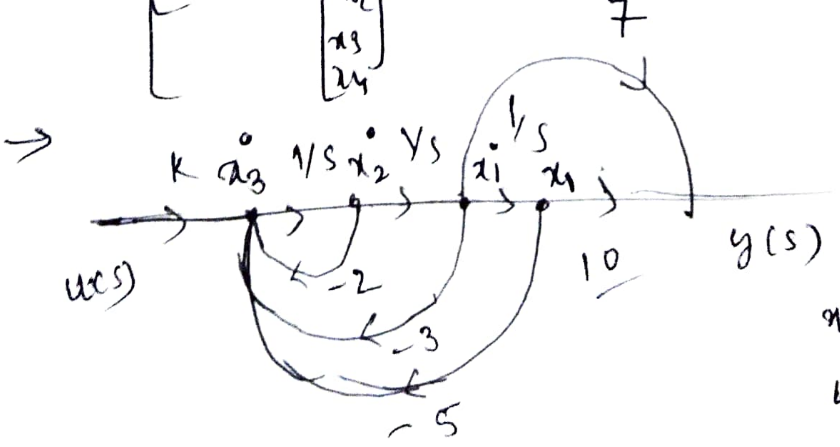
$$\underset{4\times1}{\dot{X}} = \underset{4\times4}{A}\,\underset{4\times1}{x} + \underset{4\times1}{B_n}\,\underset{1\times1}{U}$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [U]$$

$$\underset{1\times1}{Y} = \underset{1\times4}{C}\,\underset{4\times1}{X} + \underset{1\times1}{D}\,\underset{4\times1}{U}$$

$$= [10 \;\; 5 \;\; 10] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0](U)$$

$\rightarrow$



3 → state variable
no of $(\frac{1}{s}) = 3$

$x_1 = \dot{x}_1 \cdot \frac{1}{s}$

Laplace tx

$= x_1 \cancel{s} \cdot \frac{1}{\cancel{s}}$

$= X_1$

$x_1 = x_1$

$\dot{x}_1 = x_2$

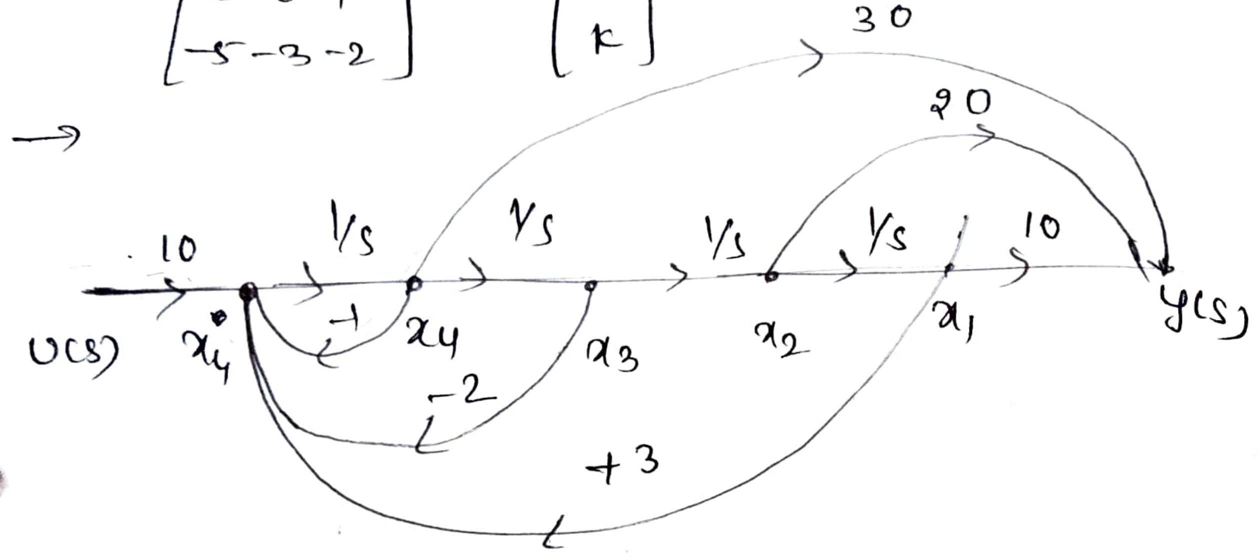
$\dot{x}_2 = x_3$

$\dot{x}_3 = -5x_1 - 3x_2 - 2x_3 + K u(s)$

$y(s) = 10x_1 + 7x_2$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} \quad C = \begin{bmatrix} 10 & 7 & 0 \end{bmatrix} \quad D = [0]$$

→



$\dot{x}_4 = 3x_1 - 2x_3 - x_4 + 10U(s)$

$y(s) = 10x_1 + 20x_2 + 30x_4$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & -2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \quad C = \begin{bmatrix} 10 & 20 & 0 & 30 \end{bmatrix}$$

$D = [0]$

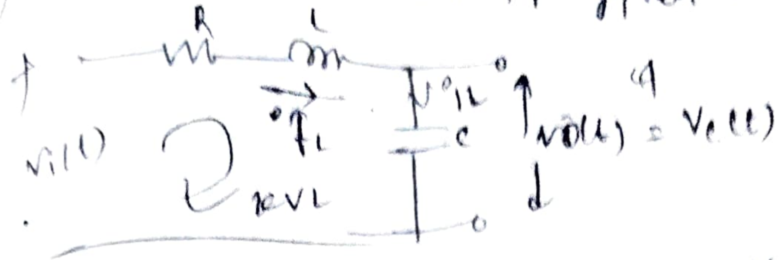
Procedure for obtaining the state eqn for Electrical N/w:-

Step 1:- Select the state variables such that voltage across capacitors & current through the inductors.

2) write independent KCL & KVL eqns for Elec. N/W

3) Resultant eqn must consist of state variables & diff. state variables, i/p & o/p variables.

Q) Obtain the state model for given Elec- N/W



S.V = $v_c(t)$, $i_L(t)$

$i_L = C\frac{dv_c}{dt}$ → ① $\dot{v}_c$

$v = L\frac{di}{dt}$ $\dot{i}_L$

$I = \frac{1}{C}\int v$

$v_i(t) = i_L(t) \times R + L\frac{di_L}{dt} + v_c(t)$

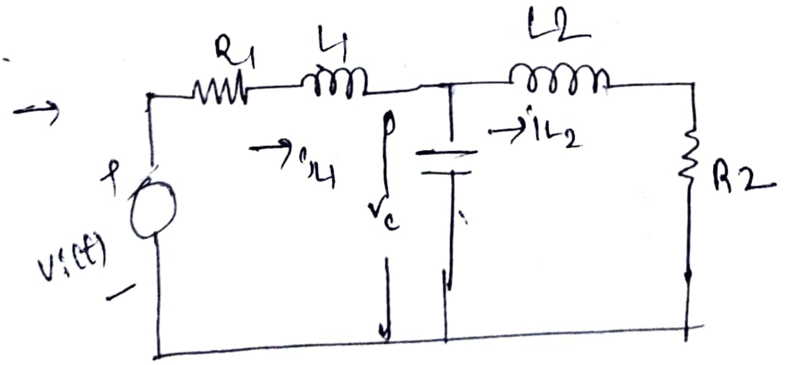
$\dot{v}_c = \frac{i_L}{C}$, $\dot{i}_L = \frac{v_i(t) - i_L(t) \times R - v_c(t)}{L}$

$$\begin{bmatrix} \dot{v}_c(t) \\ \dot{i}_L(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} [v_i(t)]$$

↳ i/p

$$v_0(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + (0)\,[v_i(t)]$$

→



$i_{L1} = i_{L2} + C\frac{dv_c}{dt}$

$\dot{v}_c = \frac{1}{C}[i_{L1} - i_{L2}]$

~~$v_i(t) = i_{L1}[R1 + L1$~~

$$v_i(t) = i_{L_1}R_1 + L_1\frac{di_{L_1}}{dt} + V_C$$

$$\dot{i}_{L_1} = \frac{1}{L_1}\left[v_i(t) - V_C - i_{L_1}R_1\right]$$

$$V_C(t) = L_2\frac{di_{L_2}}{dt} + R_2 \, i_{L_2}$$

$$\dot{i}_{L_2} = \frac{1}{L_2}\left[V_C(t) - i_{L_2}R_2\right]$$

$$\begin{bmatrix} \dot{V}_C \\ \dot{i}_{L_1} \\ \dot{i}_{L_2} \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L_1 & -R_1/L_1 & 0 \\ 1/L_2 & 0 & -R_2/L_2 \end{bmatrix} \begin{bmatrix} V_C \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_1 \\ 0 \end{bmatrix} v_i(t)$$

$\rightarrow$

$$\dot{X} = AX + BU$$

$$sX(s) = AX(s) + BU(s)$$

$$X(s) = (sI - A)^{-1} BU(s) \quad \text{——— (1)}$$

$$Y = CX + DU$$

$$= C(sI - A)^{-1} BU(s) + DU(s)$$

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$\rightarrow$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\overset{A}{\begin{bmatrix} 2 & 3 \\ -4 & -2 \end{bmatrix}}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} [u]$$

$$[Y] = [1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad D$$

1Q! Nature of system, stability, Tr function

$$T_r = \frac{Y(s)}{U(s)} = [1 \quad 1]\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -4 & -2 \end{bmatrix}\right]^{-1}\begin{bmatrix} 3 \\ 5 \end{bmatrix} + 0$$

$$= [1 \quad 1]\begin{bmatrix} s-2 & -3 \\ +4 & s+2 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= [1 \ 1] \frac{1}{(s^2+8)} \begin{bmatrix} (s+2) & -4 \\ 3 & s-2 \end{bmatrix}^T \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

2×2

$$= \frac{1}{s^2+8} [(s+2)+3 \quad 4+s-2] \begin{bmatrix} 3 \\ 5 \end{bmatrix} \qquad \frac{1}{s^2+4+12}$$

$$= \frac{1}{s^2+8} [s+5 \quad s+2] \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{3(s+5)+5(s+2)}{s^2+8}$$

$$8s+25$$

$$= [1 \ 1] \frac{1}{s^2+8} \begin{bmatrix} s+2 & 3 \\ -4 & s-2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{s^2+8} [s+2-4 \quad 3+s-2] \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{s^2+8} [s-2 \quad s+1] \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{s^2+8} [3s-6+5s+5] = \frac{8s-1}{s^2+8} \qquad \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

→ Nature of the system . $2\xi\omega_n = 0 \quad \xi = 0$

undamped system

Char. eqn

→ stability : $s^2+8 = s^2 = -8 \quad s = \pm 2\sqrt{2}j \qquad |sI-A| = 0$

Marginally st

$s^2+8=0$

$s = \pm 2\sqrt{2}$

Solution to the state equation.

$$x(t) = \underbrace{L^{-1}[(sI-A)^{-1}x(0)]}_{ZIR} + \underbrace{L^{-1}[(sI-A)^{-1}BU(s)]}_{ZSR}$$

$$= e^{At}x(0) + \int_0^t e^{A(t-\tau)} BU(\tau)\, d\tau$$

ZIR → zero i/p response

$\dot{x} = Ax+Bu$

$\frac{dx}{dt} = Ax + \cancel{Bu}^{0} \qquad D = +A \quad x = ke^{+At}$

ZSR → zero state respons

ZSR $\rightarrow$ Natural response, , free response, system response

ZSR $\rightarrow$ forced response

State transition matrix $\Big\}$ $\phi(t) = e^{At}$

$$= L^{-1}\left[(sI - A)^{-1}\right]$$

$$\phi(s) = (sI - A)^{-1}$$

$$ZSR = L^{-1}\left[\phi(s) B U(s)\right]$$

$$= \int_0^t \phi(t-\tau) \qquad A = d\phi/dt \Big|_{t=0}$$

properties STM:

1) $\phi(0) = I$

2) $\phi^k(t) = (e^{At})^k = e^{A(\underbrace{kt}_{t'})} = \phi(\underbrace{kt}_{t'})$

3) $\phi^{-1}(t) = \phi(-t)$

$$x = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

4) $\phi(t_1 + t_2) = \phi(t_1) . \phi(t_2)$

5) $\phi(t_2 - t_1)\, \phi(t_1 - t_2) = \phi(t_2 - t_0)$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}}_{A} x \quad \text{where} \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad y = \underbrace{\begin{bmatrix} 1 & -1 \end{bmatrix}}_{C} x$$

$$\phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$$

$$= \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin(\sqrt{2}t) & \cos\sqrt{2}t \end{bmatrix}$$

$\phi(t) = L^{-1}[(sI-A)^{-1}]$

$$= L^{-1}\left\{\begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}\right\} = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$x(t) = \phi(t)\, x(0)$

$$= \left[\cos\sqrt{2}t \cancel{t} \; \frac{1}{\sqrt{2}}\sin\sqrt{2}t \quad -\sqrt{2}\sin\sqrt{2}t \; \cos\sqrt{2}t\right] e^{-at}$$

$$= \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$Y = CX$

$$= [1 \quad -1]\begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$= \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t + \sqrt{2}\sin\sqrt{2}t - \cos\sqrt{2}t$$

$$= \frac{3}{\sqrt{2}}\sin\sqrt{2}t$$

→ Q:-

$$\dot{X} = \underbrace{\begin{bmatrix} 0 & -1 \\ +2 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 5 \end{bmatrix}}_{B} u \qquad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad y(t) = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} x(t)$$

$$sI - A = \begin{bmatrix} s & 0+1 \\ 0+2 & s+3 \end{bmatrix} \cdot \begin{bmatrix} s+3 & -(0+2) \\ -(0+1) & s \end{bmatrix}^T$$

$$(sI-A)^{-1} = \frac{1}{\cancel{s^2+3s} \; s^2+3s+2} \quad \frac{1}{s(s+3)+2} \quad \frac{1}{2}\begin{bmatrix} s+3 & -1 \\ +2 & s \end{bmatrix}$$

$$L^{-1}\{[(sI-A)^{-1}]\} = L^{-1}\begin{bmatrix} \frac{s+3}{s(s+3)+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s}{s^2+3s+2} \end{bmatrix} = L^{-1}\begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix}$$

$$L^{-1}[(sI-A)^{-1}] = \phi(t) = L^{-1}\begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$ verify $\phi(0) = 1$

$\{\cancel{L^{-1}((sI-A)^{-1}BU(s))}$

ZIR $\cancel{x(t)} = \phi(t)\,x(0) = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$

$2A \quad (s+2)(s+1) \quad (s)(s+2)$

$\frac{1}{2(s)} + \frac{1}{s+1} + \frac{1}{2(s+2)}$

$B = 1 \quad 2C =$

ZSR $= L^{-1}(\phi(s)\,B\,U(s))$

$$= \begin{bmatrix} 2.5 - 5e^{-t} + 2.5e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$$

$\frac{5}{s} + \frac{5}{s+1} \quad \frac{5}{s+2}$

$2s+4 - s - 1$

$s+3$

$$= L^{-1}\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$[\,8 \;\; 1/s\,]_{1\times 2}$

$2\times 2 \quad 2\times 1$

$2\times 1 \quad 1\times 1$

$$L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{5\cancel{s}}{\cancel{s}(s+1)(s+2)} \end{bmatrix}_{2\times 1}$$

$2\times 1 \quad 1\times 2$

2×1

$$L^{-1}\begin{bmatrix} \frac{5}{2s} + \frac{5}{s+1} + \frac{5}{2(s+2)} \\ \frac{5}{s+1} + \frac{-5}{s+2} \end{bmatrix}$$

2×2

$x(t) = Z_1 R + Z_2 S R$

$y(t) = Cx$

$= [0 \;\; 1]\, x(t)$

$= 3e^{-t} - 3e^{-2t}$

Controllability:-

→ The system is said to be controllable if it is possible to transfer the initial state to desired state in a finite time interval by the controllable i/p.

$Qc = [B \;\; AB \;\; A^2B \;\; \cdots \;\; A^{n-1}B]$ $n \times n$

→ Kalman's test

$|Qc| \neq 0$ rank = order = n → system is controllable   n → no. of state variables.

$|Qc| = 0$ rank < order → uncontrollable

Q:→ check the controllability of the transfer fn

$$T/F = \frac{1}{s^3 + 2s^2 + 3s + 4}.$$

Kalman's Test of Controllability

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$$

i/p u_1 i/p u_2

→ only one column of B must be used at a time to judge controllability using a particular i/p

$[B]_{n \times 1}$ $[AB]_{n \times 1}$ $A^2B = A \cdot AB$ $= n \times 1$

• if rank(Qc) = order = n → system is controllable

if rank (Qc) < order

no. of controllable state = rank

uncontrollable mode = order − rank

system is uncontrollable