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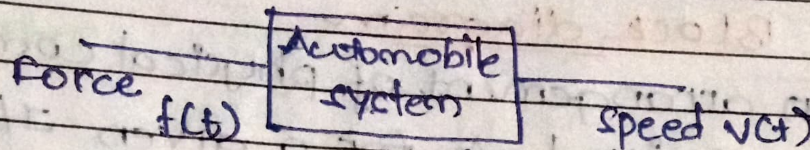
Expt.Name : **EC301: CONTROL SYSTEMS** Date : Page No :

Objective: i/p signal → control systems → o/p signal → o/p is controlled by i/p

results :

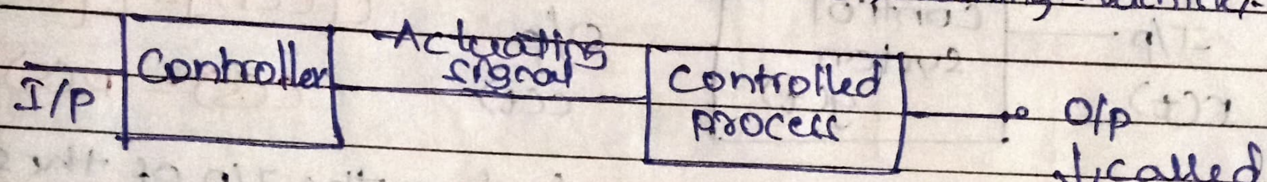
Eg:



1] Open Loop system :

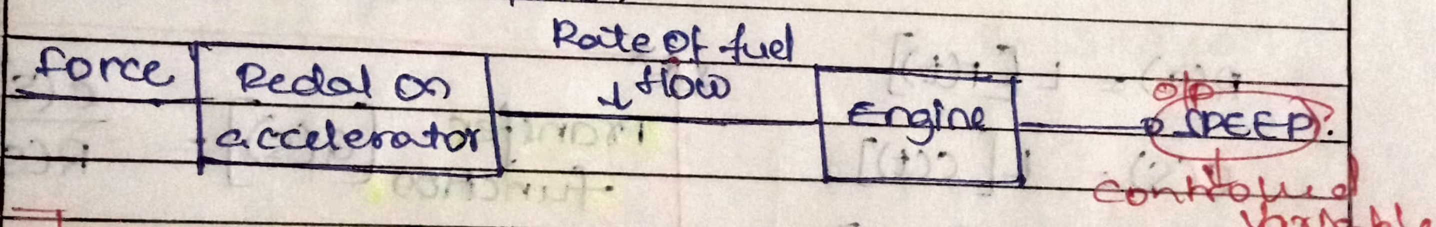
- O/P are not depends or have feedback on controller.

Eg: Handdryer, Manual Ac, Washing Machine, ...etc



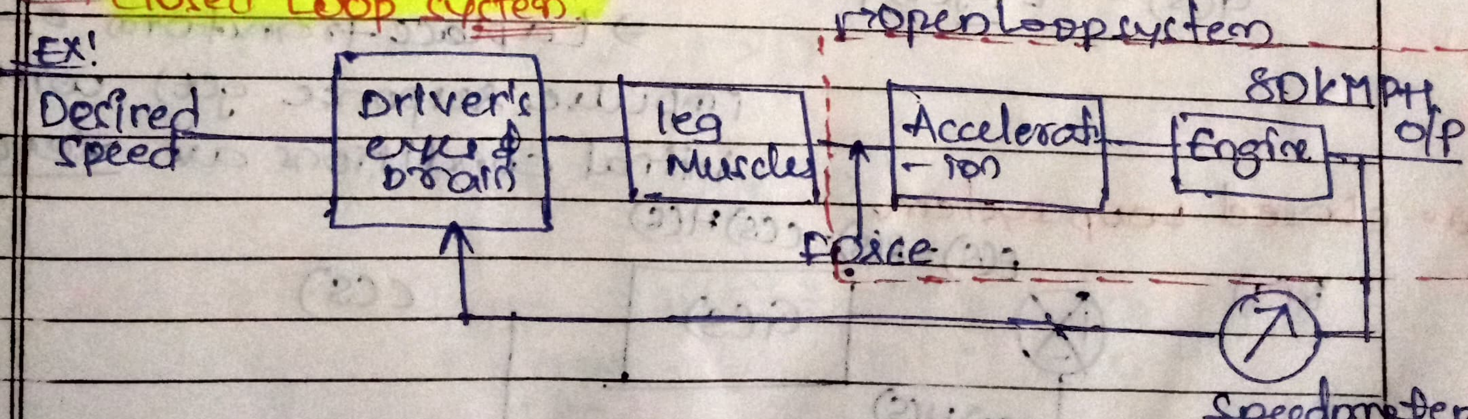
BLOCK DIAGRAM

Eg: Automobile system



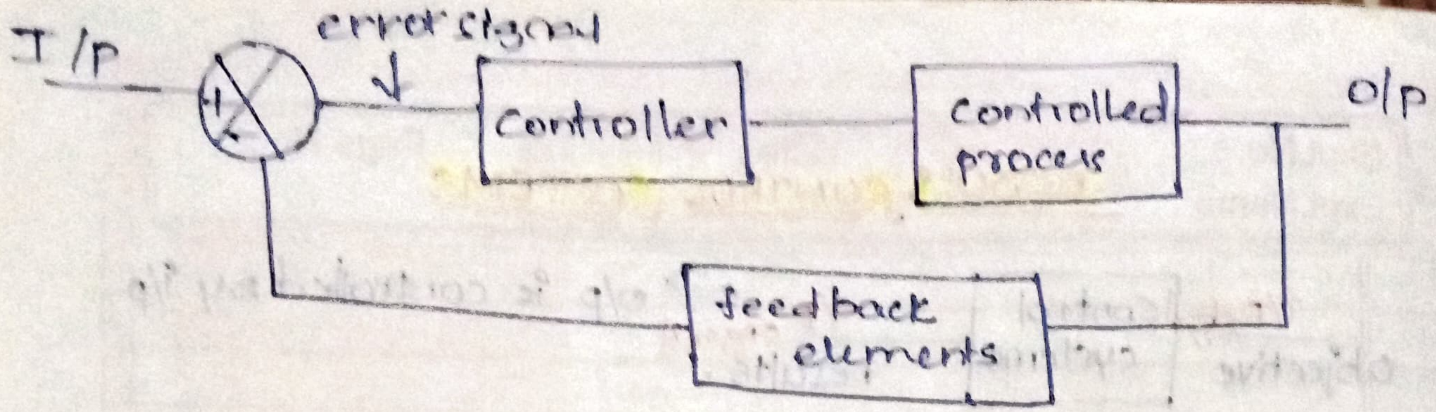
2] Closed Loop system:

EX:



Signature ....

S.K.B.W

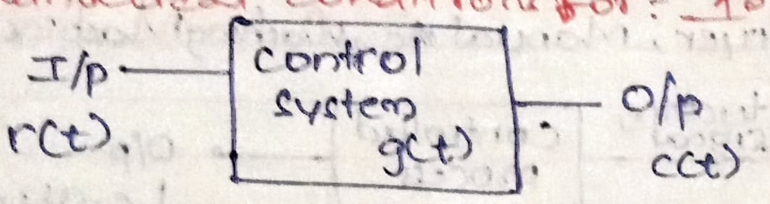


Block diagram

Definition: It is an arrangement of physical components such that it give desired o/p for a given I/p by means of regulator or controller.

→ This is known as Control system.

Mathematical conditions for: 1° open Loop system.



$L[c(t)] = L[r(t) * g(t)]$

$C(S) = R(S) \times G(S)$

g(t) → Impulse response (when the I/p of the system is impulse signal)

$R(S) = L[r(t)]$

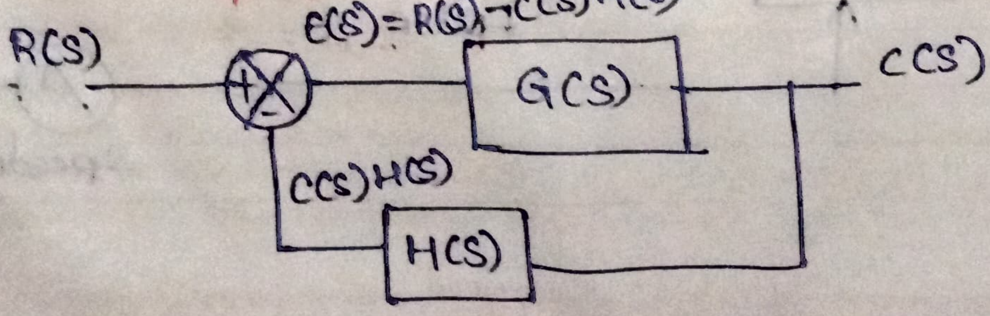
$C(S) = L[c(t)]$

$G(S) = L[g(t)]$

** $\boxed{\text{Transfer function } [G(S)] = \frac{C(S)}{R(S)}}$

⇒ Laplace transform of impulse response g(t) with initial conditions are Zero.

2° closed Loop system.



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$E(S) = R(S) - C(S)H(S)$ — ①

$C(S) = E(S) \times G(S)$ — ②

$C(S) = [R(S) - C(S)H(S)]\, G(S)$

Transfer ftn : $\boxed{\dfrac{C(S)}{R(S)} = \dfrac{G(S)}{1+G(S)H(S)}}$ → Closed Loop.

disadvantage;

feedback of the closed loop system reduces the gain by a factor of $1+G(S)H(S)$.

LAPLACE TRANSFORM :

If $f(t)$ satisfies $\int_0^{\infty} |f(t)e^{-at}|\, dt < \infty$

$$\boxed{F(S) = L[f(t)] = \int_0^{\infty} f(t)e^{-st}\, dt}$$

$\boxed{S = \sigma + j\omega}$ eg: $S = 3+2j\omega$.

Q) $f(t) = u(t) = 1, \; t>0$
$\qquad\qquad\quad 0, \; t<0$

$\int_0^{\infty} |e^{-\sigma t}|\, dt < 0 \Rightarrow \sigma > 0$

$$\boxed{L\{u(t)\} = \frac{1}{s}}, \boxed{\sigma > 0.}$$

Q) $f(t) = e^{-at}u(t); \; a>0$ $\quad f(t) = e^{-at} \; ; t>0$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad 0 \; ; \; t<0$

$\sigma + a > 0$

$\sigma > -a$

$$\boxed{F(S) = \frac{1}{S+a} \; ; \; \sigma > -a}$$

Signature ....

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⑩] $f(t) = e^{at} u(t)$; $a > 0$

$$\boxed{F(S) = \frac{1}{S-a} \quad ; \sigma > a}$$

Theorems of Laplace T/F

1] Multiplication by a constant

$$L[K f(t)] = K\, L[f(t)]$$

2] Sum and difference:

$$L[f_1(t) \pm f_2(t)] = L[f_1(t)] \pm L[f_2(t)]$$

3] Differentiation:

$$L\left[\frac{d^n f(t)}{dt}\right] = S^n F(S) - S^{n-1} f(0) - S^{n-2} f'(0)$$

$$\text{where } f^K(0) = \left.\frac{d^K f(t)}{dt^K}\right|_{t=0}$$

$$n=1 \Rightarrow L\left[\frac{df(t)}{dt}\right] = SF(S) - f(0)$$

4] Integration:

$$L\left[\int_0^{t_n}\int_0^{t_{n-1}} \cdots \int_0^{t_1} f(t)\, d\tau\, dt_1 \cdots dt_{n-1}\right] = \frac{F(S)}{S^n}$$

$$n=1 \Rightarrow L\left[\int_0^{t_1} f(t)\, dt_1\right] = \frac{F(S)}{S}$$

5] Shift in time domain:

$$L[f(t-\tau)] = e^{-\tau S} F(S)$$

6] Complex shift:

$$L[e^{\mp\alpha t} f(t)] = F(S \pm \alpha)$$

7] Real convolution:

$$L[f_1(t) * f_2(t)] = F_1(S) F_2(S)$$

8] complex convolution:

$$L[f_1(t)\, f_2(t)] = F_1(S) * F_2(S)$$

9] Initial Value Theorem:

$$\lim_{t\to 0} f(t) = \lim_{S\to\infty} SF(S)$$

10] Final Value Theorem:

$$\underset{t\to\infty}{Lt}\, f(t) = \underset{S\to 0}{Lt}\, SF(S)$$

If SF(S) doesn't contain poles on right side or on imaginary axis of the S-Plane.

⇒ $G(S) = P(S)/Q(S)$

If $Q(S) = 0 \Rightarrow$ POLEs

$P(S) = 0 \Rightarrow$ zeros

Ex: $F(S) = \dfrac{5}{S(S^2+S+2)}$, find final value.

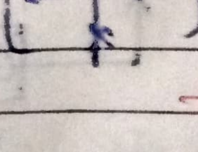
$$\underset{t\to\infty}{Lt}\, f(t) = \underset{S\to 0}{Lt}\, SF(S)$$

$$\underset{t\to\infty}{Lt}\, f(t) = \underset{S\to 0}{Lt}\, \frac{5}{S^2+S+2} = 5/2$$

Ex: $F(S) = \omega/(s^2+\omega^2)$, find final value

$$\underset{t\to\infty}{Lt}\, f(t) = \underset{S\to 0}{Lt}\, \frac{S\omega}{S^2+\omega^2} = \underset{S\to 0}{Lt}\, \frac{\omega}{2S} = \infty;\ \text{undefined}$$

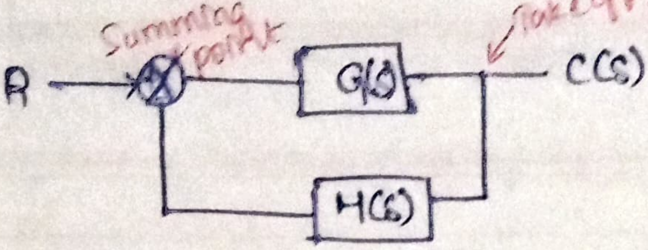
Poles: $S = \pm j\omega$



as poles are located on imaginary axis, final value theorem is not defined for it

Block Diagram Algebra: → block + summing point + take-off point

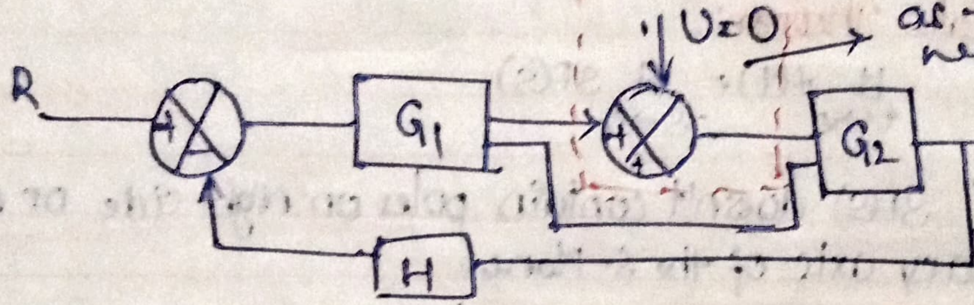
⇒



$$C = \frac{G}{1+GH} \cdot R$$

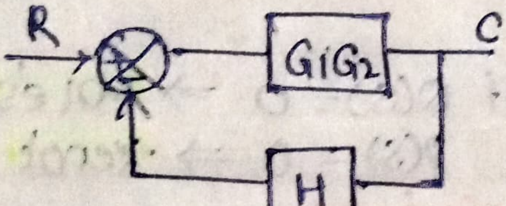
If $GH = -1$, it becomes undefined. So, to make it stable, we will add another feedback.

⇒



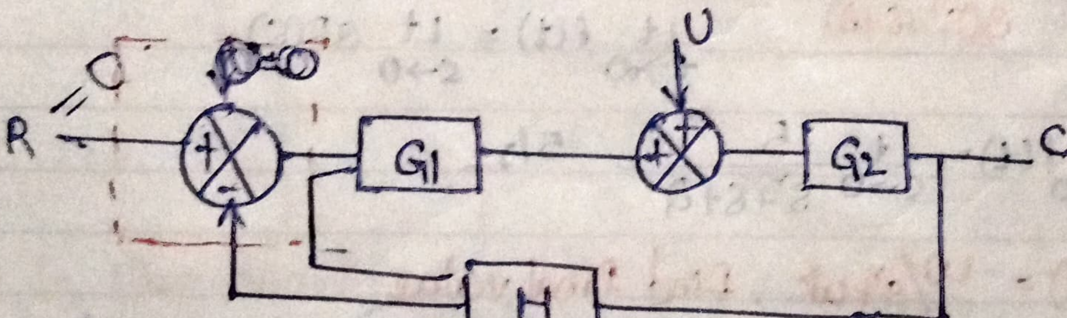
as feedback = 0 we can S.C

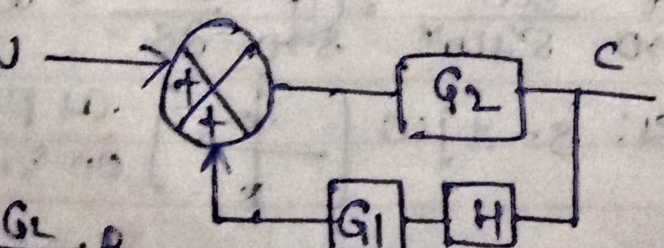
≡



$$\boxed{C_R = \frac{G_1G_2}{1+G_1G_2H} \cdot R}$$

⇒





$$C_R = \frac{G_1G_2}{1+G_1G_2} \cdot R$$

$$C_U = \frac{G_2}{1-G_1G_2H} \times U$$

Transfer function:

$$\boxed{C = C_R + C_U}$$

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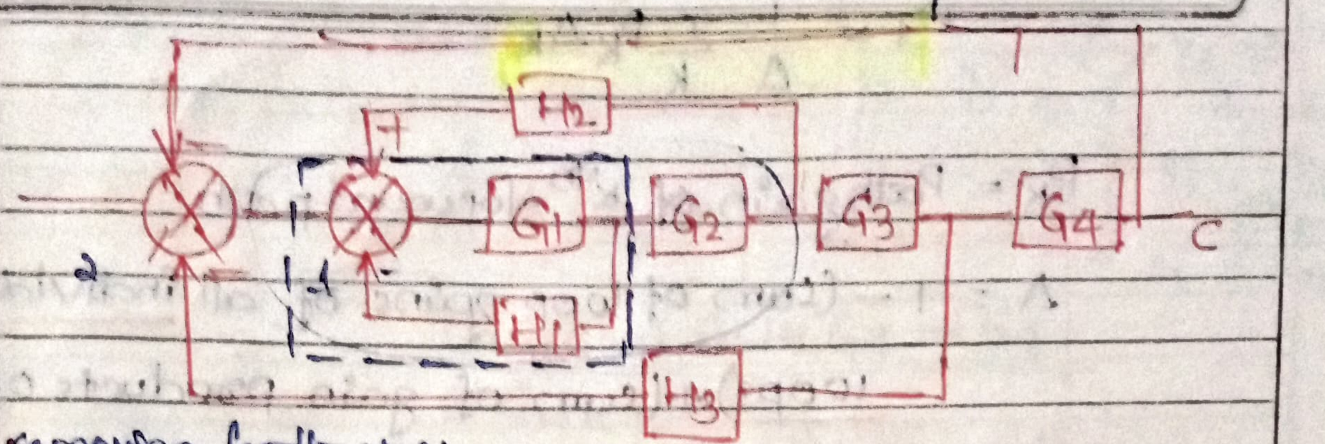
Q.



removing feedback :

$$C_1 = \frac{G_1}{1+G_1H_1} \rightarrow C_2 = \frac{G_1G_2}{1+\cancel{G_1G_2H_1}+G_1H_1}$$

$$C_3 = \frac{G_1G_2}{1+\cancel{G_1G_2H_1}-G_1G_2H_2+G_1H_1}$$

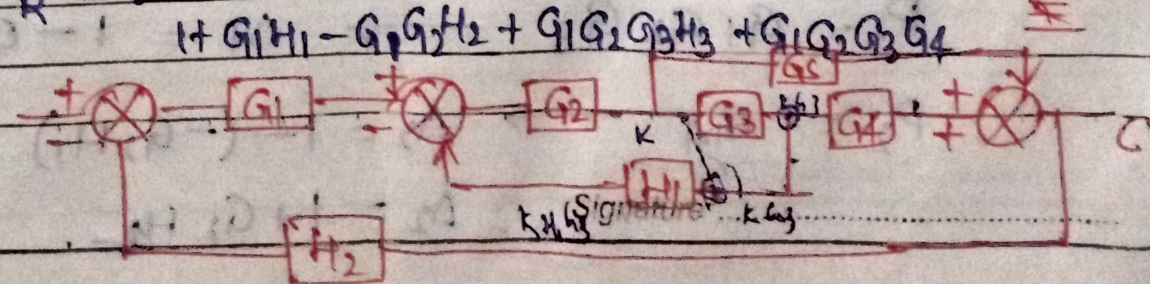
$$C_4 = \frac{G_1G_2G_3}{1+\cancel{G_1G_2H_1}-G_1G_2H_2+G_1H_1} ; \quad C_5 = \frac{G_1G_2G_3}{1+\cancel{G_1G_2H_1}-G_1G_2H_2+G_1H_1+G_1G_2G_3H_3}$$

$$C_5 = \frac{G_1G_2G_3G_4}{1+\cancel{G_1G_2H_1}-G_1G_2H_2+G_1G_2G_3H_3+G_1H_1}$$

$$C_6 = \frac{G_1G_2G_3G_4}{1+\cancel{G_1G_2H_1}-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4+G_1H_1}$$

Ans! $$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4}$$

Q



Signature ....

Signal flow graph – Manson's gain formula

The Transfer function (T) is,

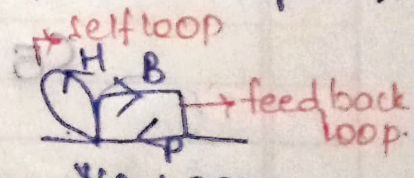
$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

P_K = Path gain of K^{th} forward path.

Δ = 1 – (sum of loop gains of all individual loops) + (sum of gain products of all possible combinations of two non-touching loops) – (sum of gain products of all possible comb. three non-touching loops) + - - -

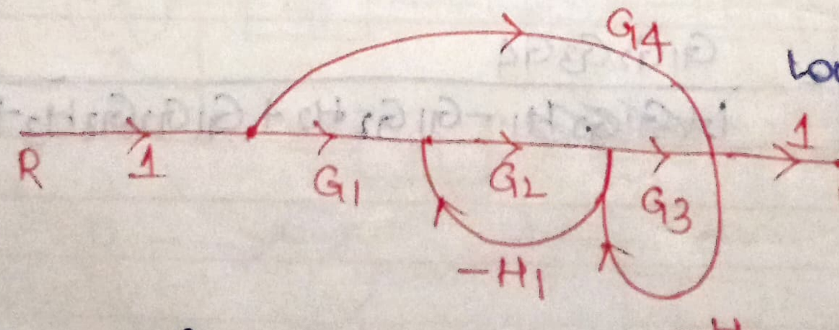
Δ_K = The value of Δ for the part of the graph not touching K^{th} forward path.

Loop: Staring and ending node should be same.



∴ 2 loops.
loop gain = BP.

eg:



Find Transfer ftn.

There are two forward paths:

$P_1 = G_1 G_2 G_3 \longrightarrow \Delta_1 = 1 - 0 = 1$

$P_2 = G_4 \longrightarrow \Delta_2 = 1 - (-G_2 H_1)$

$\therefore \Delta_2 = 1 + G_2 H_1$.

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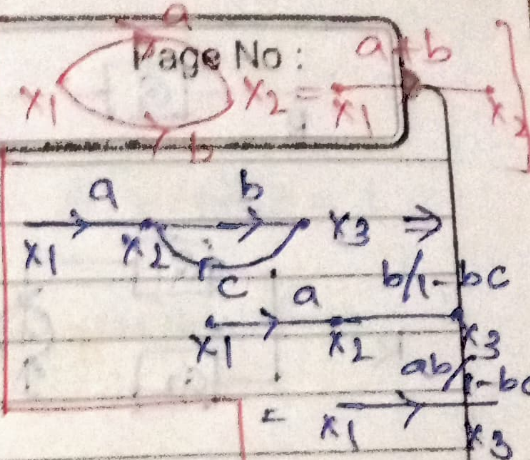
NOTE



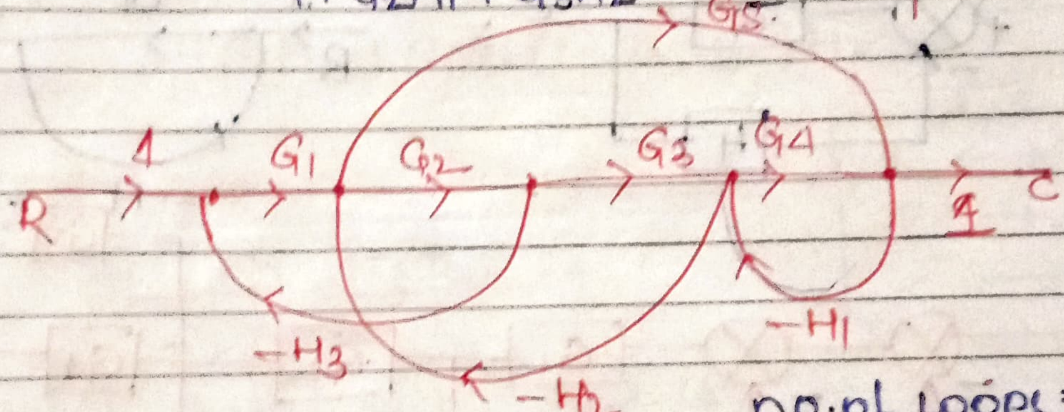
$$\Delta = 1 - (-G_2H_1 - G_3H_2)$$

$$\therefore \Delta = 1 + G_2H_1 + G_3H_2$$

$$\boxed{T = \frac{1}{\Delta}\sum_K P_K \Delta_K}$$

$$\therefore T = \frac{G_1G_2G_3 + G_2H_1G_4 + G_4}{1 + G_2H_1 + G_3H_2}$$

1)



no. of loops = 4.

$\Rightarrow$ $P_1 = G_1G_2G_3G_4 \rightarrow \Delta_1 = 1$

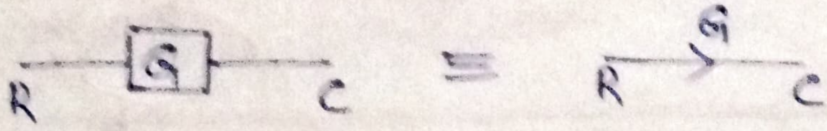
$P_2 = G_1G_5 \longrightarrow \Delta_2 = 1$

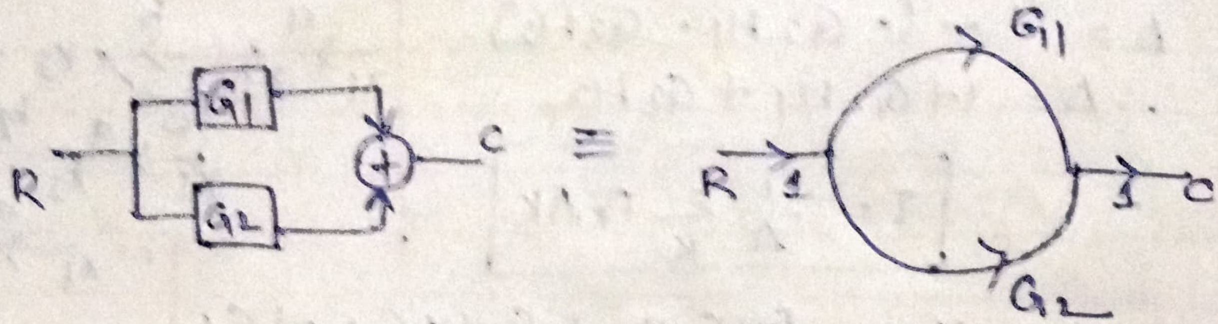
$$\Delta = 1 - (-G_1G_2H_3 - G_2G_3H_2 - G_4H_1 + G_5H_1H_2) + [(-G_1G_2H_3)(-G_4H_1)]$$

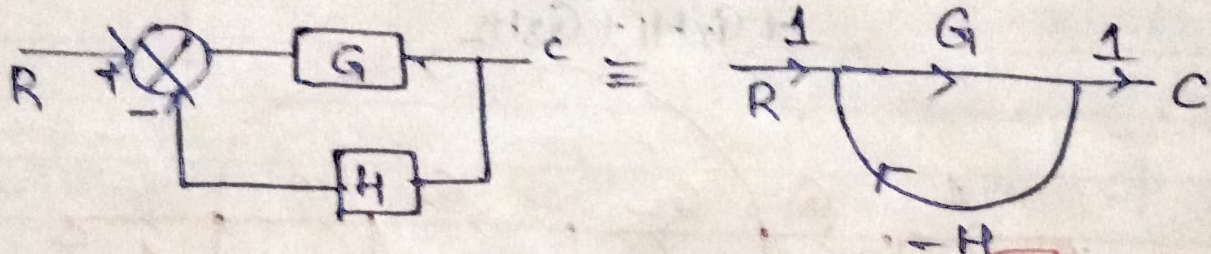
$$\boxed{T = \frac{1}{\Delta}\sum_K P_K \Delta_K}$$

Signature ....

Block to Signal-flow Conversion







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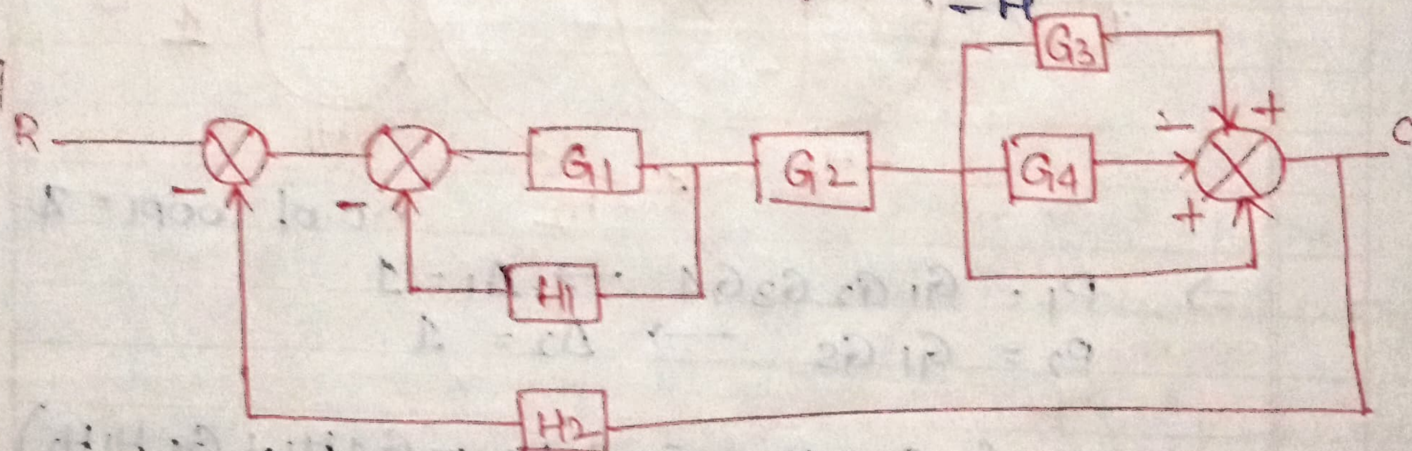
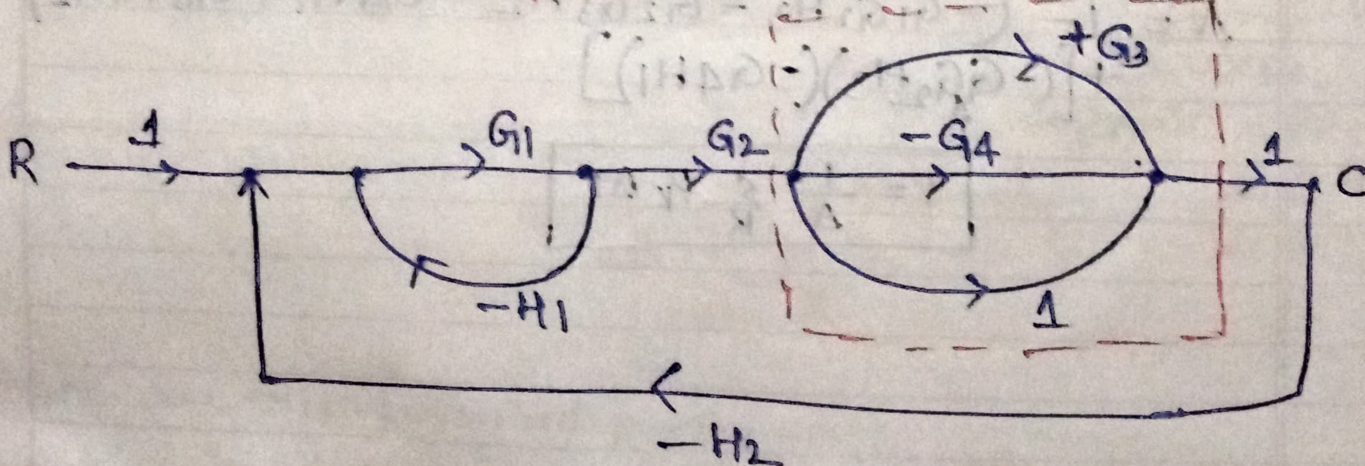
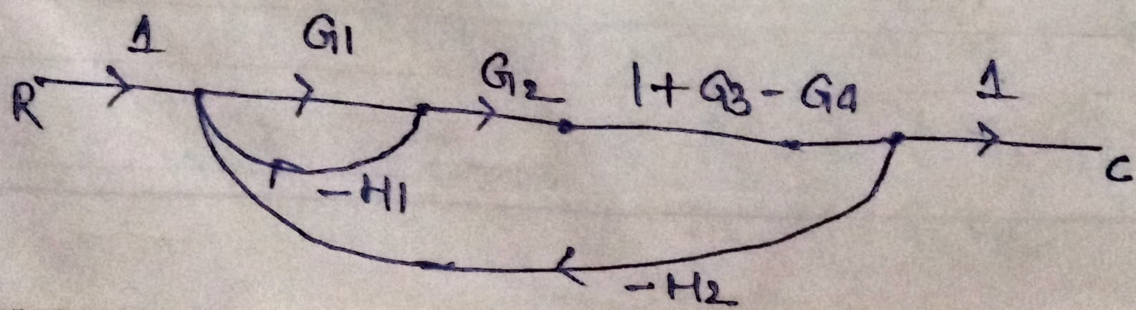






$$P_1 = G_1 \cdot G_2 (1 + G_3 - G_4) \Rightarrow \Delta_1 = 1 - 0 = 1$$

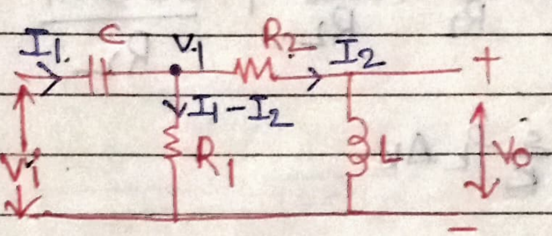
$$\Delta = 1 - (-G_1H_1 - G_1G_2(1 + G_3 - G_4)H_2)$$

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

$$\therefore T = \frac{G_1G_2(1+G_3-G_4)}{1+G_1H_1+G_1G_2(1+G_3-G_4)H_2}$$

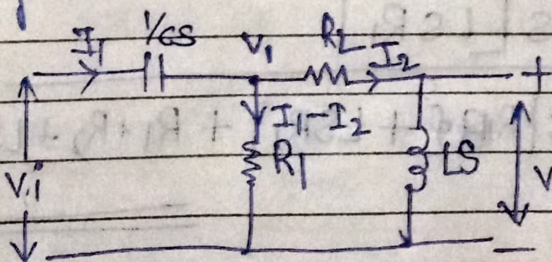
Signal flow graphs for electrical N/W

L.T
$L \longleftrightarrow L \cdot S$
$R \longrightarrow R$
$C \longrightarrow 1/CS$



1. Identify node voltages & Branch currents.
   1. node voltage - V_1
   2. Branch current - I_1, I_2

• converting the circuit to Laplace Transform.



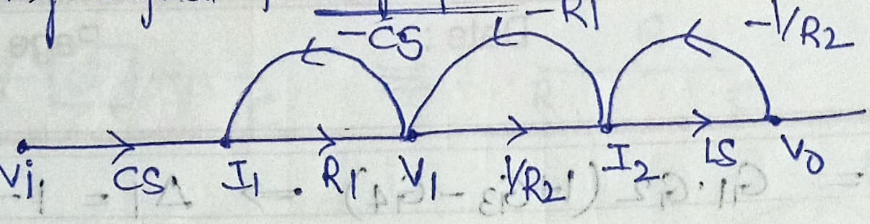
$$I_1 = \frac{V_i - V_1}{1/CS} = CS[V_i - V_1] \quad -(1)$$

$$I_2 = \frac{V_1 - V_0}{R_2} \quad -(2) \; ; \quad V_1 = [I_1 - I_2]R_1 \quad -(3)$$

$$V_0 = LS\, I_2 \quad -(4)$$

Signature' ....

• constructing signal flowgraph.



Transfer function.

$$P_1 = (CS)(R_1)(1/R_2)(LS)$$

$$= \frac{LCS^2R_1}{R_2} \longrightarrow \Delta_1 = 1$$

$$\Delta = 1 - \left(-CSR_1 - R_1/R_2 - \frac{LS}{R_2}\right) + \left((-CSR_1)(-LS/R_2)\right)$$

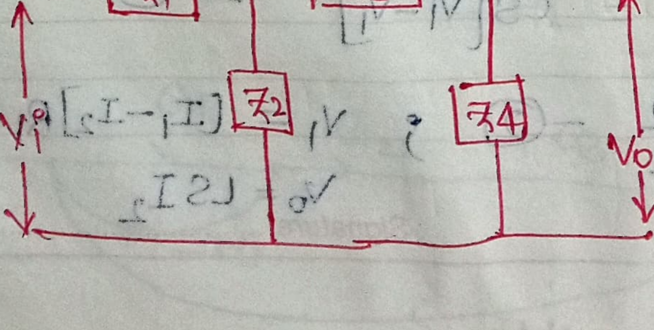
$$= 1 + CSR_1 + \frac{R_1}{R_2} + \frac{LS}{R_2} + \frac{LCS^2R_1}{R_2}$$

$$T = \frac{1}{\Delta} \sum_K P_k \Delta_k$$

$$= \frac{CLS^2R_1/\cancel{R_2}}{\dfrac{R_2 + CSR_1R_2 + R_1 + LS + LS^2CR_1}{\cancel{R_2}}}$$

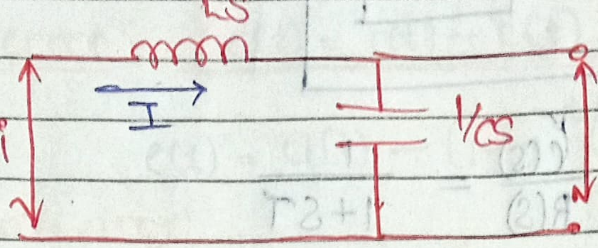
$$\frac{V_0}{V_1} = \frac{CS[LSR_1]}{CS[R_1R_2 + LSR_1] + R_1 + R_2 + LS}$$

D



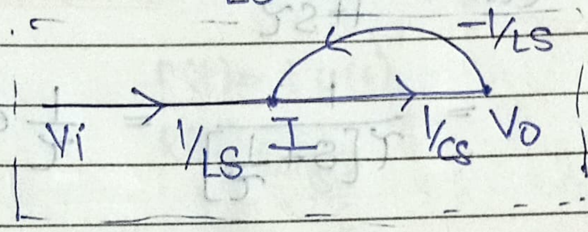
$$TS = \frac{Z_2Z_4}{Z_1[Z_2+Z_3+Z_4] + Z_2[Z_3+Z_4]}$$

Q)



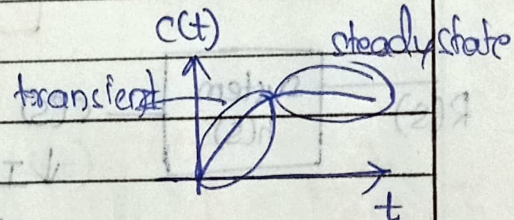
→ Zero node voltages , → 1 branch current

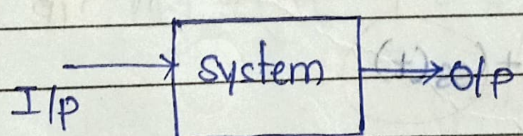
$I = \frac{V_i - V_o}{LS}$; $V_o = I/cs$



$\frac{V_o}{V_i} = \frac{1/cs}{1/cs + LS} = \frac{1}{1+s^2LC}$

Time domain Analysis:



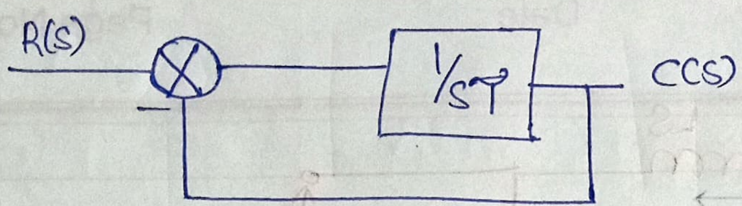


$c(t) = c_{tr}(t) + c_{ss}(t)$

Transient response: $\boxed{\lim_{t \to \infty} c_{tr}(t) = 0}$

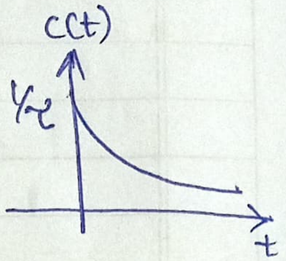
eg) $c(t) = \underbrace{5 + 10\sin 2t}_{c_{ss}(t)} + \underbrace{e^{-10t}\cos 5t}_{c_{tr}(t)}$

Time response of first order system:



$$\frac{C(S)}{R(S)} = \frac{1}{1+ST}$$

1] Impulse Responce:-



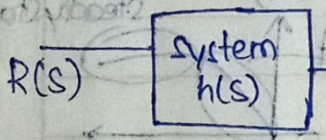
$$r(t) = \delta(t)$$

$$R(S) = 1$$

$$C(S) = \frac{1}{1+S\tau}$$

$$= \frac{1}{\tau\left[S+\frac{1}{\tau}\right]} = \frac{1}{\tau} e^{-t/\tau} u(t) \rightarrow C_{tr}(t)$$

$$\boxed{\therefore h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)}$$



$\downarrow$ ILT

$$c(t) = C_{tr}(t) + C_{ss}(t)$$

$C_{tr}(t)$ → caused due to system h(s)

$C_{ss}(t)$ → " " i/p R(S).

2] Step response:

$$r(t) = u(t)$$

$$R(S) = 1/S$$

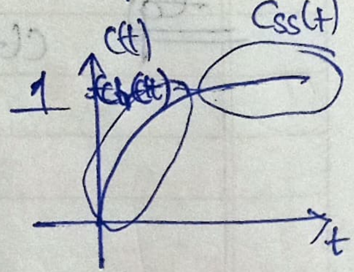
$$C(S) = \frac{1}{S} \cdot \frac{1}{1+S\tau} = \frac{1}{S} - \frac{\tau}{\tau S+1}$$

$\downarrow$ ILT

$$c(t) = u(t) - e^{-t/\tau} u(t)$$

$$c(t) = \left(1 - e^{-t/\tau}\right) u(t)$$

$1 \rightarrow C_{ss}(t)$, $e^{-t/\tau} \rightarrow C_{tr}(t)$



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• system error : $\boxed{e(t) = r(t) - c(t)}$

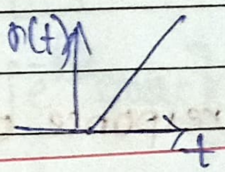
$$e(t) = u(t) - (1 - e^{-t/\tau})u(t) = e^{-t/\tau}u(t)$$

• steady state error :

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

$$= 0$$

3 Ramp Input :



$$r(t) = t\,u(t)$$

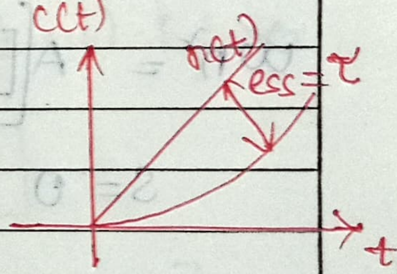
$$R(s) = \frac{1}{s^2}$$

$$\boxed{L\{t^n\} = n!/s^{n+1}}$$

$$C(s) = \frac{1}{s^2} \cdot \frac{1}{\tau s + 1}$$

$$C(s) = \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{\tau s + 1}$$

$$\therefore c(t) = (\underbrace{t - \tau}_{c_{ss}(t)} + \underbrace{\tau e^{-t/\tau}}_{c_{tr}(t)})u(t)$$



$$e(t) = r(t) - c(t) = (\tau - \tau e^{-t/\tau})u(t)$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \tau$$

• Parabolic I/p: $r(t) = \frac{t^2}{2} u(t)$

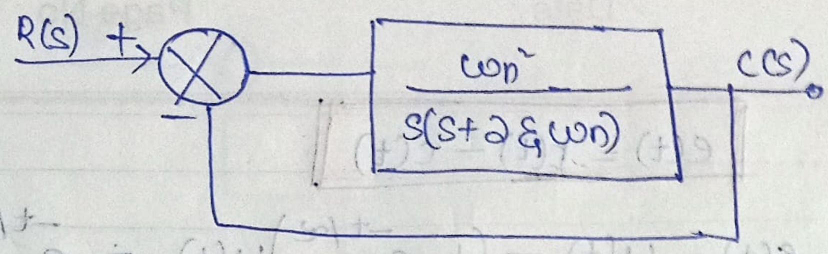
$$R(s) = 1/s^3$$

$$C(s) =$$

Response of second order system:- It have two poles.

ζ = damping ratio

ω_n = natural freq.



unity gain closed loop feedback system.

$$\frac{C(s)}{R(s)} = \frac{\frac{\omega_n^2}{s(s+2\zeta\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\zeta\omega_n)}} = \frac{\omega_n^2}{\omega_n^2+s(s+2\zeta\omega_n)}$$

step response of second order system:

$$R(s) = 1/s$$

$$C(s) = \frac{1}{s}\cdot\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{1}{s}\cdot\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\boxed{\omega_n^2(1-\zeta^2)}} = \frac{A}{s}+\frac{Bs+C}{(\ )^2+(\ )}$$

ω_d^2

$$\omega_n^2 = A\left[[s+\zeta\omega_n]^2+\omega_n^2(1-\zeta^2)\right] + Bs^2+Cs$$

$$s=0 \Rightarrow \omega_n^2 = A\left[\cancel{(\zeta\omega_n)^2}+\omega_n^2-\cancel{\omega_n^2\zeta^2}\right]$$

$$\boxed{A=1} \quad \text{— (1)}$$

$$s=-\zeta\omega_n \Rightarrow \cancel{\omega_n^2} = A[\cancel{\omega_n^2}-\omega_n^2\zeta^2]+B\omega_n^2\zeta^2-C\omega_n\zeta$$

$$B\omega_n^2\zeta^2 - C\omega_n\zeta = \omega_n^2\zeta^2 \quad \text{— (2)}$$

$s=1$

$$\cancel{\omega_n^2} = \left[s^2+\cancel{\zeta^2\omega_n^2}+2s\zeta\omega_n+\cancel{\omega_n^2}-\cancel{\omega_n^2\zeta^2}\right]+B+C$$

$$B + C = -s^2 - 2s\xi\omega_n \quad -③$$

solve ② and ③

$$B\omega_n^2\xi^2 - [-s^2 - 2s\xi\omega_n - B][\omega_n\xi] = \omega_n^2\xi^2$$

$$\boxed{B = -1}, \quad \boxed{C = -1}$$

$$\left[L\{\sin at\} = \frac{a}{s^2+a^2}\right]$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2} \qquad \left[L\{e^{-bt}\sin at\} = \frac{a}{(s+b)^2+a^2}\right]$$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} \times \frac{\omega_d}{\omega_d}$$

$\Big\downarrow$ ILT

$$\Rightarrow C(t) = u(t) - e^{-\xi\omega_n t}\cos(\omega_d t) + \frac{\xi\omega_n}{\omega_d}e^{-\xi\omega_n t}\sin(\omega_d t) \qquad \left[L\{e^{-bt}\cos at\} = \frac{s+b}{(s+b)^2+a^2}\right]$$

(or)

$$C(t) = 1 - e^{-\xi\omega_n t}\cos(\omega_d t) + \frac{\xi\omega_n}{\omega_d}e^{-\xi\omega_n t}\sin(\omega_d t)\ ; t > 0$$

$$C(t) = 1 - e^{-\xi\omega_n t}\left[\cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t)\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos(\omega_d t) + \xi\sin(\omega_d t)\right]$$

Let, $\xi = \cos\phi\ ; \sqrt{1-\xi^2} = \sin\phi$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\phi\cos(\omega_d t) + \cos\phi\sin(\omega_d t)\right]$$

$$\therefore C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi), \quad 0 \le \xi \le 1$$

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Transfer fn.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi) \quad , 0 \le \xi \le 1$$

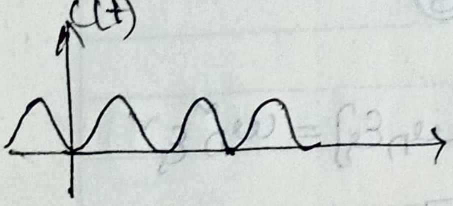
Special cases:

1] Undamped system: $\xi = 0 \Rightarrow \cos\phi = 0 \Rightarrow \phi = \pi/2$

$$\Rightarrow c(t) = 1 - \sin(\omega_d t + \phi) \quad ; \omega_d = \omega_n\sqrt{1-\xi^2}$$

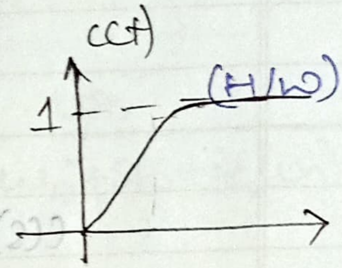
$$= 1 - \sin(\omega_n t + \pi/2)$$

$$= 1 - \cos(\omega_n t)$$



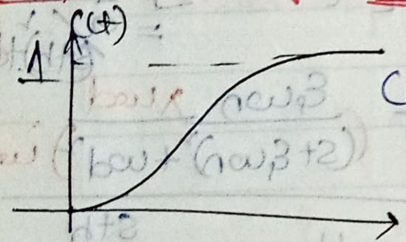
2] Critically damped system: $\xi = 1$

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$



3] over damped system: $\xi > 1$

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})} + \frac{e^{-(\xi + \sqrt{\xi^2+1})\omega_n t}}{2\sqrt{\xi^2-1}\,(\xi + \sqrt{\xi^2-1})}$$

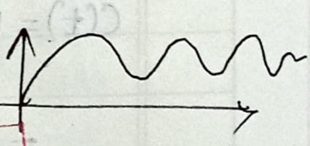


at partial fraction

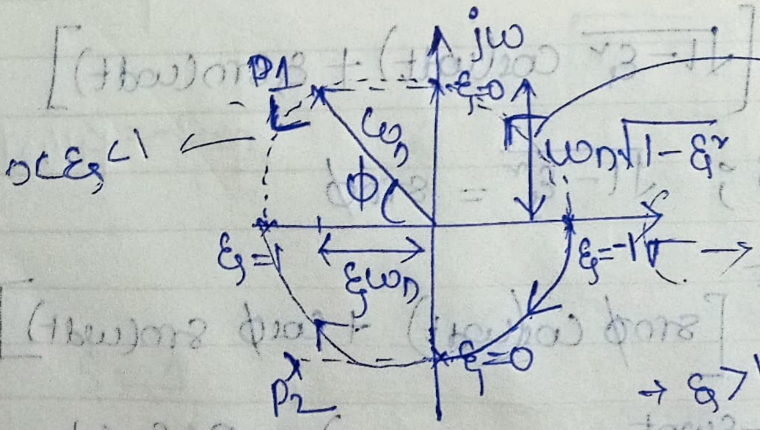
$$s^2 + 2\xi\omega_n s + \omega_n^2 = (s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})$$

4] underdamped system:

$$0 < \xi < 1, \quad c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$



poles: $s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$



$\boxed{-1 < \xi < 0}$

negatively damped system.

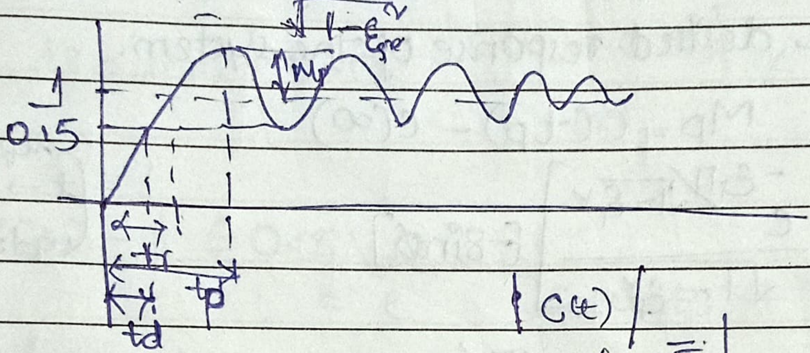
$\rightarrow \xi > 1 \Rightarrow$ poles will be only on real axis but far from real axis.

Transient response specifications:

1. Rise Time: Time required to reach the final value.

e undamped system.

$$c(t) = \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right] u(t)$$



$\omega_d = \omega_n\sqrt{1-\xi^2}$

$$c(t)\Big|_{t=t_r} = 1$$

$\sin(\omega_d t + \phi) = 0$

$(\omega_d t_r + \phi) = \pi$, $t = \dfrac{\pi - \phi}{\omega_d}$

2. Peak time (t_p): Time required to reach its first peak.

$$\frac{d\,c(t)}{dt}\Bigg|_{t=t_p} = 0$$

$$\cancel{\frac{-e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}}\left[\cos(\omega_d t + \phi)\right]\omega_d + \frac{\sin(\omega_d t + \phi)\cancel{e^{-\xi\omega_n t}}}{\cancel{\sqrt{1-\xi^2}}}(\xi\omega_n) \neq 0$$

$$+\left[\omega_d \cos(\omega_d t + \phi)\right] \pm \left[\xi\omega_n \sin(\omega_d t + \phi)\right]$$

$$\tan(\omega_d t + \phi) = \frac{\omega_d}{\xi\omega_n} = \frac{\sqrt{1-\xi^2}}{\xi} = \tan\phi$$

$$t = \frac{1}{\omega_d}\left[\tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] - \phi\right]$$

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$$\omega_d t_p \phi = n\pi + \phi$$

$$t_p = \frac{n\pi}{\omega_d}, \quad n = 1, 2, 3, \ldots$$

$$t_p = \frac{\pi}{\omega_d} \longrightarrow 1^{st} \text{ peak}$$

$$t_p = \frac{2\pi}{\omega_d} \longrightarrow 2^{nd} \text{ peak}$$

Peak overshoot:

It is the maximum peak value of the response curve measured from the desired response of the system.

$$M_p = c(t_p) - c(\infty)$$

$$c(t_p) = 1 - \left[\frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\right][-\sin\phi]$$

replace, $t \to t_p$; $t_p = \frac{\pi}{\omega_d}$; $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

and $$c(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$= e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

Delay time (t_d): Time required for the response to reach half of its final value.

$$c(t)\Big|_{t=t_d} = 1/2$$

$$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

Settling time (t_s): The last moment of the step response entres the 5% strip and never leaves from that point on.

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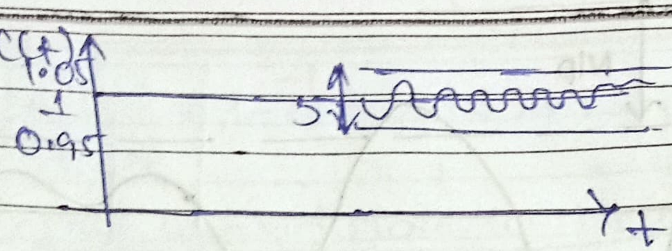

$$t_s = \min\left\{ t > 0, \frac{|c(t') - c(\infty)|}{c(\infty)} < 0.05, \forall t' \geq t \right\}$$

$$|c(t_s) - 1| \leq 0.05$$

$$= e^{-\xi\omega_n t_s}\left|\cos\omega_d t_s + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t_s\right| \leq 0.05$$

In general $e^{-\xi\omega_n t} \leq 0.05$

$$t_s = \frac{-\ln 0.05}{\xi\omega_n} = \frac{3}{\xi\omega_n}$$

$t_s = 3/\xi\omega_n$ [5% tolerance band]

$= 4/\xi\omega_n$ [2% tolerance band]

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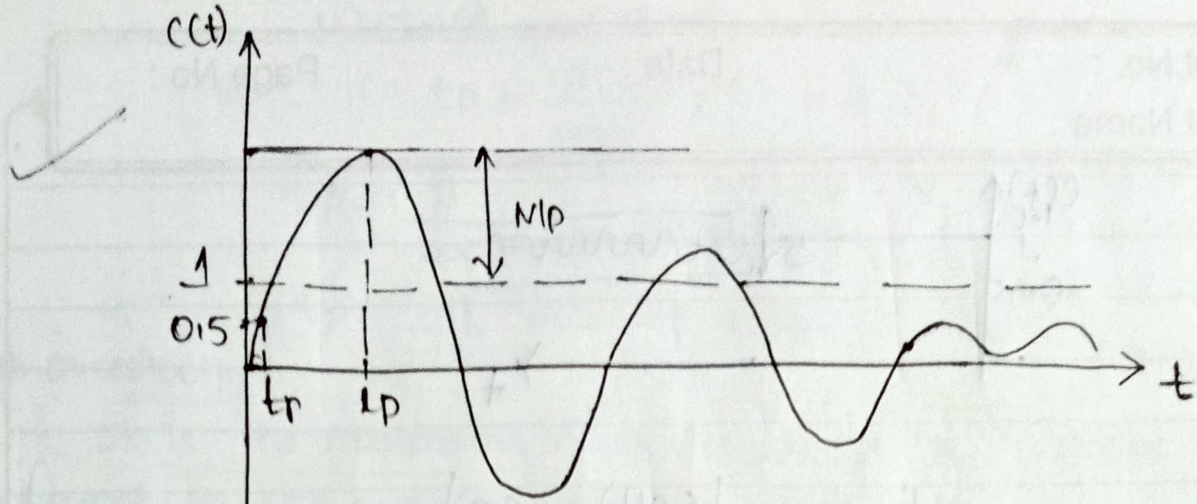


(rice time) $t_r = \dfrac{\pi - \cos^{-1}(\xi)}{\omega_d}$; (settling time) $t_s = \dfrac{3}{\xi\omega_n}$ [5% tollerance]

(Peak overshoot) $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$; $t_s = \dfrac{4}{\xi\omega_n}$ [2% tollerance]

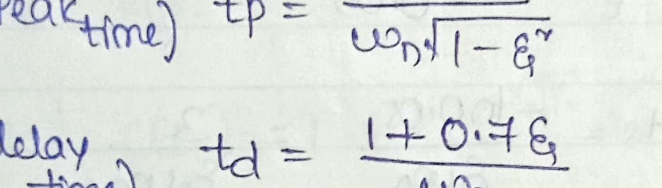
(Peak time) $t_p = \dfrac{n\pi}{\omega_n\sqrt{1-\xi^2}}$

(delay time) $t_d = \dfrac{1 + 0.7\xi}{\omega_n}$

As $\xi\uparrow = t_p\uparrow,\ t_d\uparrow,\ t_r\uparrow,\ M_p\downarrow$

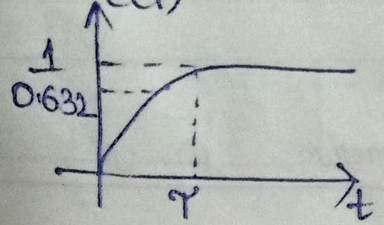
Time constant: defined as Time required for the signal to attain 63.2% of its final Value, when i/p of system is step signal. (response)

→ How fast the system reaches to final value.



τ = time constant

$$\frac{C(S)}{R(S)} = \frac{1}{1+S\tau}$$

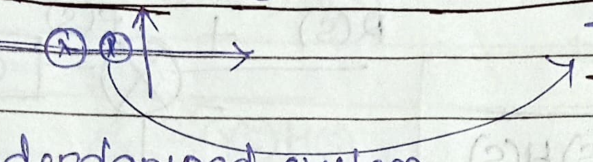


$R(S) = 1/s$

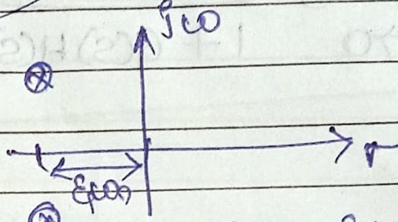
$c(t) = (1 - e^{-t/\tau})u(t)$

$$c(t)\Big|_{t=\tau} = 1 - e^{-1} = 0.632$$

→ pole located close to the origion, denotes the Time constant. $\left(\tau = -\frac{1}{s}\right)$



→ for underdamped system,

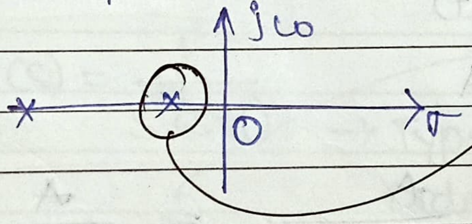


$$s_1 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

for calculating τ, only real part has to taken.

$$\tau = \frac{-1}{(-\xi\omega_n)} = \frac{1}{\xi\omega_n}$$

→ for over damped system.



Q) $G(S) = \dfrac{25}{S(S+4)}$, $H(S) = 1$. find all the time domain specifications

2nd order system

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} = \frac{25}{S^2 + 4S + 25}$$

$$\omega_n = 5, \quad \xi = 0.4$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4.5 \text{ rad/sec}$$

$t_d = 0.256$ sec

$t_r = 0.44$ sec

$t_p = 0.69$ sec

$t_s = 2$ sec ($\pm 2\%$ error)

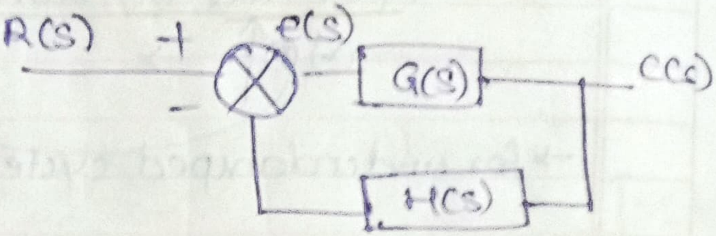
$= 1.5$ sec ($\pm 5\%$ error)

Steady state errors:

It is the deviation of o/p from the reference I/p at the steady state $t \to \infty$; $e(t) = r(t) - c(t) * h(t)$

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)}$$ [final value theorem]

$$\boxed{e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s)H(s)}}$$



$E(s) = R(s) - H(s)C(s)$

$= R(s) - E(s)G(s)C(s)$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s)H(s)}}$$

1. Step I/p: $r(t) = Au(t)$

$R(s) = \frac{A}{s}$

$e_{ss} = \lim_{s \to 0} \frac{A}{1 + G(s)H(s)} = \frac{A}{1 + \lim_{s \to 0} G(s)H(s)}$

Let, $K_p = \lim_{s \to 0} G(s)H(s)$

= State position error const

$\therefore e_{ss} = \frac{A}{1 + K_p}$

2. Ramp I/p: $r(t) = Atu(t)$

$R(s) = A/s^2$

$e_{ss} = \lim_{s \to 0} \frac{A}{s + sG(s)H(s)} = \frac{A}{\lim_{s \to 0} sG(s)H(s)} = \frac{A}{K_v}$

→ Velocity error const

$L\{t^n \cdot u(t)\} = \frac{n!}{s^{n+1}}$

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3) parabolic I/P: $r(t) = \frac{At^2}{2}u(t)$, $R(s) = A/s^3$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{A}{s^2 G(s)H(s)}$$ $K_a = \underset{s\to 0}{Lt}\, s^2 G(s)H(s)$

$$= \frac{A}{\underset{s\to 0}{Lt}\, s^2 G(s)H(s)}$$ $K_a \to$ acceleration error const.

$$\therefore e_{ss} = A/K_a$$

Type and order of the system:

Type – no. of poles located at origin

Order – no. of poles of the system

Closed loop system; $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$; O.L.S $\frac{C(s)}{R(s)} = G(s)$

eg. ① $G(s)H(s) = \frac{1}{s^2(s+1)}$ $\to$ Type = 2, Order = 3

order of numerator is known as degree.

② $G(s) = \frac{10(s+2)}{s^2(s+4)(s+10)}$, $H(s) = 1$, $r(t) = \left(1+4t+\frac{t^2}{2}\right)u(t)$

Find steady state error.

Type = 2

Order = 4

$R(s) = \frac{1}{s} + \frac{4}{s^2} + \frac{1}{s^3}$

$$e_{ss} = \underset{s\to 0}{Lt}\, s \cdot \frac{R(s)}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{Lt}\, s \frac{\left(\frac{1}{s} + 4\frac{1}{s^2} + \frac{1}{s^3}\right)}{1+GH} = \frac{1}{1+K_p} + \frac{4}{K_v} + \frac{1}{K_a}$$

$$= 0 + 0 + 2 = 2$$

$K_p = \infty \to \underset{s\to 0}{Lt}\, G(s)H(s)$

$K_v = \infty \to \underset{s\to 0}{Lt}\, sG(s)H(s)$

$K_a = 1/2 \to \underset{s\to 0}{Lt}\, s^2 G(s)H(s)$

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S.K.B.W

Type = Degree of I/p signal , ess = ?

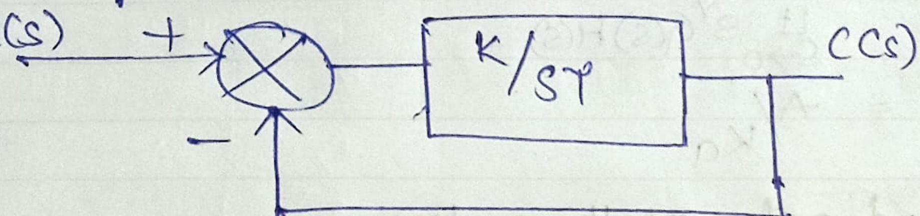
Type > " , ess = 0

Type < " , ess = ∞

degree → numerator
Type → denominator

→ Steady state gain (or) DC gain

Ratio of steady state o/p of a system to its constant i/p; i.e steady state of unit step response.



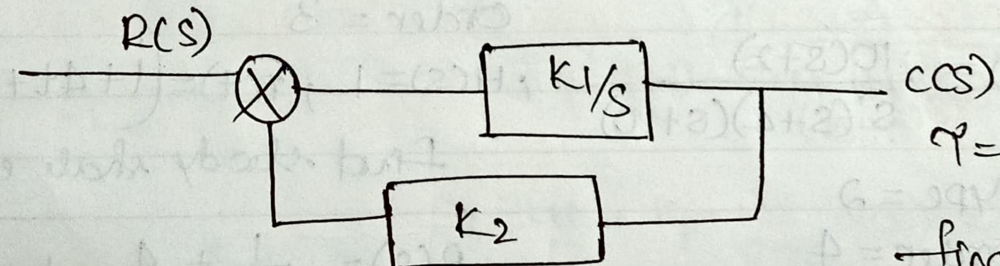
$$\frac{C(S)}{R(S)} = \frac{K}{K+ST}$$

$$C(S) = \frac{1}{S} \cdot \frac{1}{1+ST}$$

The steady state gain

$$\boxed{\lim_{t\to\infty} c(t) = \lim_{S\to 0} S\,C(S)}$$

eg.



$T = 0.4$, steady state gain = 2

find K_1, K_2?

$$\frac{C(S)}{R(S)} = \frac{K_1/S}{1+\frac{K_1K_2}{S}} = \frac{K_1}{S+K_1K_2}$$

$$T = \frac{1}{K_1K_2} = 0.4$$

$$\text{steady state gain} = \lim_{S\to 0} \cancel{S} \cdot \frac{1}{\cancel{S}} \cdot \frac{K_1}{S+K_1K_2} = \frac{1}{K_2} = 2$$

$\boxed{\therefore K_2 = 0.5}$

$\boxed{K_1 = 5}$

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Reeponce of the system for exponential I/P!

$r(t) = e^{st}$ → LTI system $h(t)$ → $c(t) = r(t) * h(t)$

$$= \int_{-\infty}^{\infty} h(\tau) r(t-\tau) d\tau$$

$$c(t) = |H(j\omega)| r(t) e^{j\angle H(j\omega)}$$

$$= \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$$

$$\boxed{H(s) = |H(s)| e^{j\phi} \quad \phi = \angle H(s)}$$

$$\therefore c(t) = e^{st} \cdot H(s)$$

$$= |H(s)| e^{st + j\phi}$$

$s = \sigma + j\omega$ $j\omega$

$\sigma = 0$

LT → FT

$$\Rightarrow \boxed{c(t) = |H(j\omega)| e^{j(\omega t + \phi)}}$$

→ Difference b/n LT & FT

Q) $r(t) = 10\cos(2t + 45^\circ)$

$H(s) = \frac{C(s)}{R(s)} = \frac{s+1}{s+2}$; Determine the o/p of the system:

$\omega = 2$.

$$c(t) = |H(j\omega)| e^{j(\omega t + \phi)}$$

$$c(t) = |H(2j)| r(t) e^{j\angle H(2j)}$$

$$H(2j) = \frac{2j+1}{2j+2} = \frac{(2j+1)(2j-2)}{-4-4} = \frac{+4+4j-2j+2}{+8} = \frac{2j+6}{8}$$

$$|H(2j)| = \frac{1}{8}\sqrt{4+36} = \sqrt{\frac{40}{64}} = \sqrt{\frac{5}{8}}$$

$$\therefore c(t) = \sqrt{\frac{5}{8}}\, r(t)\, e^{j\,18.43}$$ $\angle H(j\omega) = \tan^{-1}\left[\frac{1}{3}\right] = 18.43$.

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Q) $r(t) = \sin t$, $\frac{C(s)}{R(s)} = \frac{1}{s+1}$ → $\boxed{c(t) = |H(j\omega)|\, r(t)\, e^{j\angle H(j\omega)}}$

$H(s) = \frac{1}{s+1} = \frac{s-1}{s^2-1}$

$c(t) = \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$

$= \frac{j\omega - 1}{-\omega^2 - 1} = \frac{1 - j\omega}{1+\omega^2}$

$c(t) = 10 \times \sqrt{\frac{5}{8}} \cos(2t + 63.45)$

$\omega = 1$

$\Rightarrow H(j\omega) = \frac{1-j}{2}$

$\phi = \tan^{-1}[-1] = -\frac{\pi}{4}$

→ $\boxed{\text{stability:}}$

$r(t) \longrightarrow \boxed{\text{LTI system}} \longrightarrow c(t)$

→ If $r(t)$ is bounded, then the o/p $c(t)$ should be bounded

$\int_{-\infty}^{\infty} |r(t)| < \infty$ $\int_{-\infty}^{\infty} |c(t)|\, dt < \infty$

• Marginally | critically | limited stable:

$r(t) \longrightarrow \boxed{\text{system}} \longrightarrow c(t)$

$r(t)$ bounded

$c(t)$ → not bounded.

maintains constant Amplitude & frequency.

Absolutely stable:

$r(t) \longrightarrow \boxed{\text{system}} \longrightarrow c(t)$

$r(t)$ bounded, $c(t)$ bounded.

$c(t)$ is always bounded.

for all system properties.

Conditionally stable: A system is stable only for certain range of system parameters.

$H(s) = \frac{C(s)}{R(s)} = \frac{G}{1+GH}$; $\boxed{1+GH = 0}$ ** characteristic eqn.

Based on poles we comment on stability

→ Routh Hurwitz (RH) criterion:

Consider a n^{th} order characteristic equation

$$a_0 s^n + a_1 s^{n-1} + \cdots + a_n = 0$$

Highest power coeff

→ system is stable if all these coefficients are +ve. (1st column)

s^n	a_0	a_2	a_4	...
s^{n-1}	a_1	a_3	a_5	...
s^{n-2}	$\frac{a_1a_2 - a_0a_3}{a_1}$	$\frac{a_1a_4 - a_0a_5}{a_1}$		
⋮	⋮			
s^0	a_n			

eg: characteristic eqn: $as^3 + bs^2 + cs + d = 0$

s^3	a	c
s^2	b	d
s^1	$\frac{bc-ad}{b}$	0
s^0	d	0

→ system is said to be stable, if all the coefficients are greater than zero.

$a>0$; $b>0$; $d>0$

$bc > ad$;

~~$c>0$~~

* → If any of the term (of any degree) is missing the ~~the system is Unstable.~~

$\Rightarrow bc < ad \rightarrow$ unstable

$\Rightarrow bc = ad \rightarrow$ Marginally stable.

b] CE: $s^2+5s+10=0 \longrightarrow$ stable

s^2	1	10
s^1	5	0
s^0	10	

CE: $s^3+10s^2+3s+30=0 \rightarrow$ Marginally stable.

$bc = ad$

s^2	~~10~~	~~30~~
~~s^1~~	3	0
s^0	~~$\frac{90}{3}$~~	~~30~~

s^3	1	3
s^2	10	30
s^1	0	-
s^0		

CE: $s^4+2s^3+6s^2+8s+10=0 \longrightarrow$ Unstable

s^4	1	6	10
s^3	2	8	
s^2	2	10	
s^1	-2	0	
s^0	10		

1 change

1 change

If (1st column all +ve $\Rightarrow$ stable system)

elif: (one/more -ve no) unstable

$\Rightarrow$ 2 poles in right side

$\Rightarrow$ rest 2 poles in left side.

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Q) $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0} \to \epsilon$	-12	
s^2	$\frac{2\epsilon+12}{\epsilon}$	15	
s^1		0	
s^0	15		

→ whenever you get zero in first row ⇒ replace it by ϵ (Epsilon).

$\epsilon \to 0$

→ system is Unstable.

→ If −ve sign means some poles on RHS plane.

$$\frac{\left(\frac{2\epsilon+12}{\epsilon}\right)(-12) - 15\epsilon}{\frac{2\epsilon+12}{\epsilon}} = \frac{-15\epsilon^2 - 24\epsilon - 144}{2\epsilon+12}$$

Q) $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$

s^5	1	3	2
s^4	1	3	2
s^3	$\cancel{0} \to 4$	$\cancel{0} \to 6$	
s^2	$3/2$	2	
s^1	$2/3$		
s^0	2		

Auxilary equation:

$\phi = s^4 + 3s^2 + 2$

$\frac{d\phi}{ds} = 4s^3 + 6s$

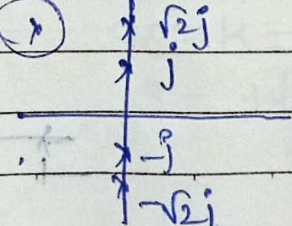
$\phi = 0$

$s^4 + 3s^2 + 2 = 0$

$(s^2+2)(s^2+1) = 0$

$s = \pm\sqrt{2}j, \pm j$

5th pole on left side



→ whenever, a row has zeros
⇒ There are two poles on imaginary axis (can be obtained from Auxilary eqn)

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Q] $s^6+3s^5+4s^4+6s^3+5s^2+3s+2=0$

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	$\cancel{0}$ 8	$\cancel{0}$ 8		
s^2	2	2		
s^1	$\cancel{0}$ 4			
s^0	2			

Auxilary eqn:

$\phi_1 = 2s^4+4s^2+2=0$

$\frac{d\phi_1}{ds} = 8s^3+8s$

$\phi_2 = 2s^2+2$

$\frac{d\phi_2}{ds} = 4s$

$\phi_1 = 0$

$2[s^2+1]^2=0$

$s=\pm j$ (1)

$\phi_2 = 0$

$s^2+1=0$

$s=\pm j$ (2)

→ If poles are repeated on on imagnary axis then system is unstable.

⇒ Unstable.

Q] $s^4+s^3+0s^2-s-1=0$ ⇒ find the no. of poles in the LH of s-plane

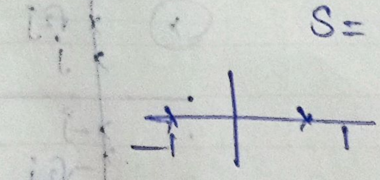
s^4	1	0	-1
s^3	1	-1	
s^2	1	-1	
s^1	$\cancel{0}$ 2		
s^0	-1		

$\phi_1 = s^2-1$

$\frac{d\phi_1}{ds} = 2s$

$s^2-1=0$

$s=\pm 1$



1 sign change → 1 pole on RH of s-plane change.

3 poles on LH of s-plane.

Q] find the ~~rank~~ Value of K, for which the system is stable. ①
2] K → Marginally stable.
3] If the system is Marginally stable, find the frequency of oscillation.

$$s^3+9s^2+4s+K=0$$

RH Table:

s^3	1	4
s^2	9	K
s^1	$\frac{36-K}{9}$	0
s^0	$\frac{K\frac{(36-K)}{9}}{\left(\frac{36-K}{9}\right)} = K$	

1) for system to be stable;

$K>0$ and $\frac{36-K}{9}>0 \Rightarrow 36>K \Rightarrow K<36$

$\boxed{0<K<36}$

2] if $\boxed{K=36}$ ⇒ Marginally stable.

Auxiliary eqn: $\phi_1 = 9s^2+K$

$$\frac{d\phi_1}{ds} = 18s$$

and $\phi_1 = 0$

$9s^2+K=0$

$s = \pm\frac{\sqrt{K}}{3}j = \pm 2j = \pm j\omega_n$

$\omega_n = 2$ rad/sec

Q) find k → for system is Marginally stable.

$$GH = \frac{K}{S(S+2)(S+4)(S+6)}$$

characteristic eq(n) $1+GH=0$

$$S(S+2)(S+4)(S+6)+K=0$$

$$(S^2+2S)(S^2+10S+24)+K=0$$

$$(S^4+12S^3+44S^2+48S+K)=0$$

RH Table:

S^4	1	44	K
S^3	12	48	0
S^2	40	K	
S^1	$\frac{1920-12K}{40}$	0	
S^0	K		

⇒ ~~Marginally~~ stable: $1920 = 12K$

$K = 160$

and $K>0$

$$\Rightarrow \boxed{0<K<160}$$

Marginally stable

Q) system transfer ftn:

$$G(S) = \frac{K(S+1)}{S^3+PS^2+3S+1}$$

oscillates at 2 rad/sec

find P & K.

Take

$H(S) = 1$ → unity feedback.

$$1+GH=0$$

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$$s^3 + Ps^2 + 3s + 1 + K(s+1) = 0$$

$$s^3 + Ps^2 + (3+K)s + (1+K) = 0 \rightarrow CE.$$

s^3	1	$3+K$
s^2	P	$1+K$
s^1	$\frac{P(3+K)-(1+K)}{P}$	0
s^0	$1+K$	

Marginally stable : $P(3+K) = (1+K)$ — ①

$$\phi_1 = Ps^2 + (1+K)$$

$$\frac{d\phi_1}{ds} = 2Ps$$

$$Ps^2 + (1+K) = 0$$

$$s^2 = -\frac{(1+K)}{P}$$

$$s = \pm j\sqrt{\frac{1+K}{P}} \Rightarrow \omega = 2 \Rightarrow 4 = \frac{1+K}{P}$$

$$4P = 1+K \quad \text{— ②}$$

from ① and ②, $\cancel{P}(3+K) = 4\cancel{P}$

$$\boxed{K=1} \Rightarrow \boxed{P=0.5}$$

(Q)

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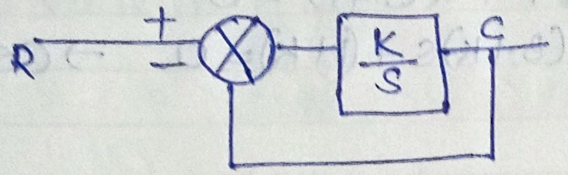
ROOT LOCUS :(RL)

→ RL is nothing but variation of closed Loop system poles as the system gain (K) varies.

Eg)



characteristic Eqn:

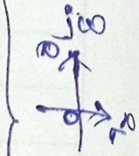
$1 + GH = 0$

$1 + K/s = 0$

$\Rightarrow s = -K$

⇒ no. of branches = order of C. Eqn.

K	s
0	0
1	−1
2	−2
⋮	⋮
∞	−∞



⇒ Loopgain / Loop transfer ft^{n}: GH which is enough to solve root locus.

Now;

$$GH = \frac{K\,N(s)}{D(s)}$$

Ch. Eqn: $1 + GH = 0$

$D(s) + K\,N(s) = 0$

case (i) $K = 0 \Rightarrow D(s) = 0$

Poles of GH = poles of closed Loop system

case (ii) $K = \infty \Rightarrow N(s) = 0$

Zeros of GH = poles of CL system.

⇒ The root locus diagram starts from K=0 (poles of GH) and ends at K=∞ (zeros of GH)

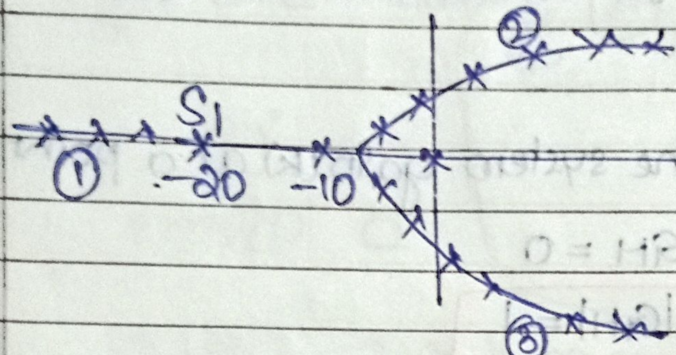
Q) Find start and end points of the RL diagram.

$$GH = \frac{K(s+5)}{s(s+10)(s+20)}$$

no. of Branches = order of ce = no. of poles of GH
= 3 branches.

start points ; $s = 0, -10, -20$

End points ; $s = -5, \infty, \infty$



Angle conditions

let $S = S_0$

$1 + G(s_0)H(s_0) = 0$; only then

we can say whether the point 's_0' is on RL or not

$\angle GH = \angle -1$

$= \pm(2q+1)\,180°$; $q = 0, 1, 2 \ldots$

Q] Check whether the following points lies on RL or not

① $s = -0.75$ ② $-1+j4$

$$GH = \frac{k}{s(s+2)(s+4)}$$

$$\angle GH \Big|_{s=-0.75} = \frac{\angle K}{\angle -0.75 \; \angle 1.25 \; \angle 3.25} = \frac{0°}{\pm 180° \; 0° \; 0°} = \pm 180°$$

$\therefore s = -0.75$ is on RL.

$$\angle GH\Big|_{s=-1+j4} = \frac{\angle K}{\angle -1+j4 \ \angle 1+j4 \ \angle 3+j4}$$

$$= \frac{0^\circ}{104^\circ . 76^\circ , 53^\circ} = -233^\circ \Rightarrow \text{which is not a odd multi of } 180^\circ$$

$\Rightarrow s = -1+j4$ is not on the RL.

Magnitude Condition:

Purpose: To find the system gain (K) at a point on RL.

$$1 + GH = 0$$

$$\boxed{|GH| = 1}$$

Ex! $GH = \frac{K}{S(S+4)}$ find the system gain at a point $S = -2+j5$.

(i) find whether 's' is located on RL or not.

$$\angle GH\Big|_{s=-2+j5} = \frac{\angle K}{\angle(-2+j5) \ \angle 2+j5}$$

$$= \frac{0^\circ}{\angle 68^\circ \angle 111.8^\circ} = -180^\circ$$

$\Downarrow$

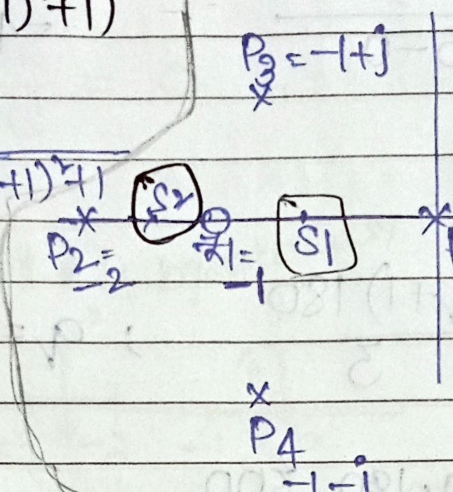
s is on RL

(ii) find system gain:

$$|GH|\Big|_{S=-2+j5} = 1$$

$$\frac{K}{(\sqrt{25+4})^2} = 1 \Rightarrow \boxed{K = 29}$$

Q] $GH = \dfrac{s+1}{s(s+2)((s+1)^2+1)}$



$$\angle GH\Big|_{S_1} = \frac{\angle S_1+1}{\angle S_1 \;\angle S_1+2 \;\angle (S_1+1)^2+1}$$

$$= \frac{\angle 0^\circ}{\pm 180^\circ \angle 0^\circ \;\; 0^\circ}$$

$= \pm 180 \Rightarrow S_1$ is on root locus.

$$\angle GH\Big|_{S_2} = \frac{\angle S_2+1}{\angle S_2 \;\angle S_2+2 \;\angle (S_2+1)^2+1} = \frac{\pm 180}{\pm 180^\circ \;\; 0^\circ \;\; 0^\circ} = 0^\circ$$

$\Downarrow$

S_2 is not on RL

NOTE!

If 's' is real, then it is on the RL, ~~if~~ only if $\Rightarrow$ there are odd no. of real open poles (+) ~~and~~ zeros to right side of S.

Asymptotes: ~~no. of branches~~ = no. of asymptotes = $n-m$

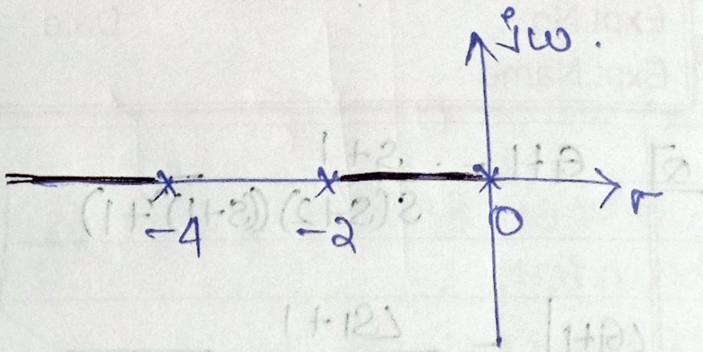
$$\boxed{\phi = \frac{(2q+1)180^\circ}{n-m}} \quad ; q = 0, 1, 2, \ldots, n-m+1$$

$n \to$ no. of poles of GH ; $m \to$ no. of ~~poles~~ zeros of GH

Centroid!

$$\sigma = \frac{(\text{sum of real part of poles of GH}) - (\text{sum of real part of zeros of GH})}{n-m}$$

Q4: $GH = \dfrac{K}{s(s+2)(s+4)}$

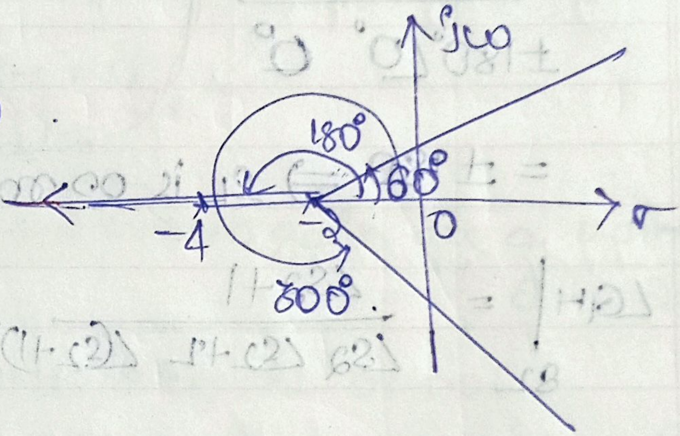


$$\text{Centroid} = \frac{(0-2-4)-(0)}{3-0}$$

$$= -2$$

$$\phi = \frac{(2q+1)\,180}{3} \quad ; \; q = 0, 1, 2$$

$$= 60, 180, 300$$

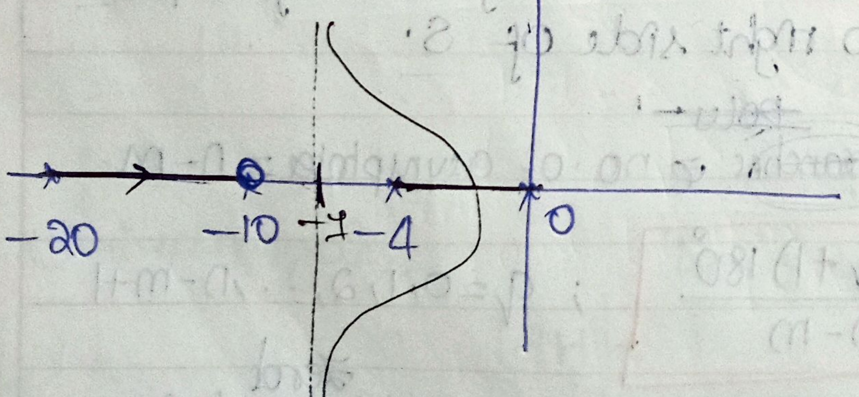


Q7 $GH = \dfrac{K(s+10)}{s(s+4)(s+20)}$

$$\text{Centroid} = \frac{(0-4-20)-(-10)}{2} = -7$$

$$\phi = \frac{(2q+1)\,180}{2}$$

$$= 90, 270$$



Break Points: for only real axis.

Break away points: Two or more poles located adjacent to each other.

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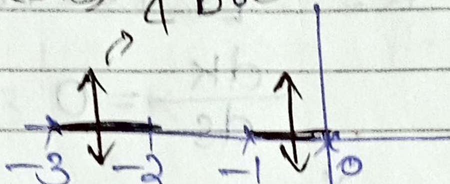
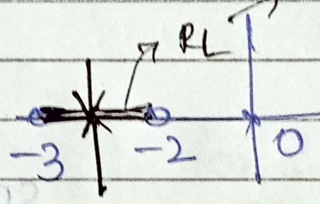
Break in points: Two or more zeros located adjacent to eachother.

$$\frac{dK}{ds} = 0$$

Q) $GH = \frac{K}{s(s+1)(s+2)(s+3)}$



Q) $GH = \frac{K(s+2)(s+3)}{s^2}$



Q) $GH = \frac{K}{s(s+2)}$

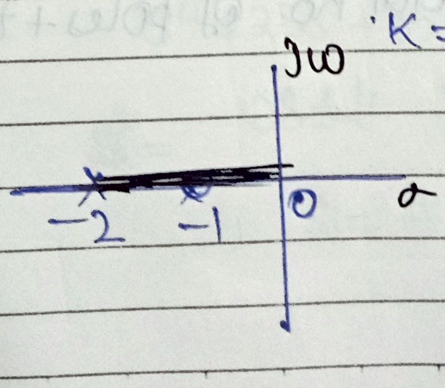
CE: $1+GH = 0$

$s(s+2) + K = 0$

$K = -s(s+2) \Rightarrow \frac{dK}{ds} = 0 \quad -\frac{d}{ds}(s^2+2s) =$

$\frac{dK}{ds} = \cancel{(s+2) \cdot s(2) \Rightarrow s+2+2s}$

$K = -s^2 - 2s \Rightarrow \frac{dK}{ds} = 0 \Rightarrow \boxed{s = -1}$



$s = -1$ is Breakaway point.

should check whether s lies on RL or not by angle condition

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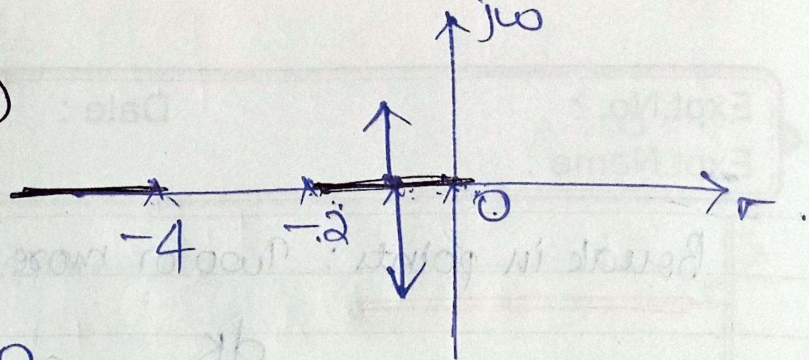
Q] $GH = \dfrac{K}{S(S+2)(S+4)}$



$1 + GH = 0$

$s(s+2)(s+4) + K = 0$

$s(s^2 + 6s + 8) + K = 0$

$K = -[s^3 + 6s^2 + 8s]$

$\dfrac{dK}{ds} = 0$

$24 \times 4 = 96$

$s = \dfrac{-12 \pm \sqrt{144 - 4(24)}}{2(3)}$

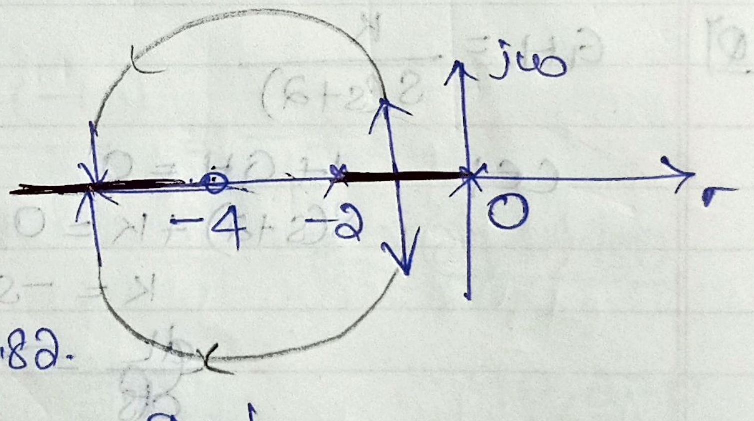
$3s^2 + 12s + 8 = 0 \Rightarrow s = -0.84, -3.15$

-0.84: It lies on RL

-3.15: It doesn't lie on RL

So, $s = -0.84$ is break away point.

Q]. $GH = \dfrac{K(s+4)}{s(s+2)}$



$1 + GH = 0$

$\dfrac{dK}{ds} = 0$.

$s = -1.17, -6.82$.

-1.17: Breakaway point

-6.82: Breakin poin

Total no. of poles + zeros = odds.

Q1 find the DC gain K and the intersection points with imaginary axis.

$$GH = \frac{K}{S(S+1)(S+3)(S+5)}$$

$$1 + GH = 0$$

$$s^4 + 9s^3 + 23s^2 + 15s + K = 0$$

s^4	1	23	K
s^3	9	15	
s^2	21.3	K	
s^1	$\frac{21.3 \times 15 - 9k}{21.3}$	0	
s^0	K		

for Marginally stable;

$$\frac{21.3 \times 15 - 9k}{21.3} = 0$$

$$\boxed{K = 35.5}$$

$$s^4 + 9s^3 + 23s^2 + 15s + 35.5 = 0.$$

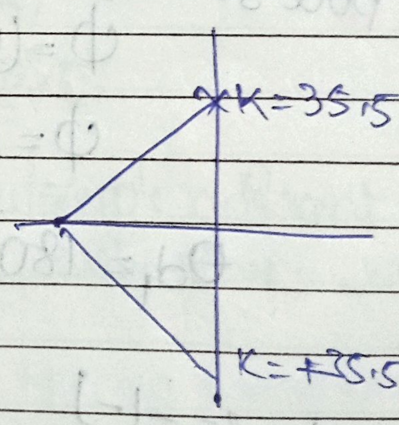
$S = j\omega$

Real + Img

0

$$s^4 + 23s^2 + 35.5 = 0$$

$s^2 =$ ~~128±1.1~~



$$S = \pm 1.28j, \pm 4.69j$$

- Angle of departure & - Angle of arrival :

$\boxed{\text{If pole/zero is complex no.}}$ → angles are only calculated when

- Angle of arrival $\theta_a = 180 + \phi$

- Angle of departure $\theta_d = 180 - \phi$

$$\phi = \sum_i \theta_{p_i} - \sum_j \theta_{z_j}$$

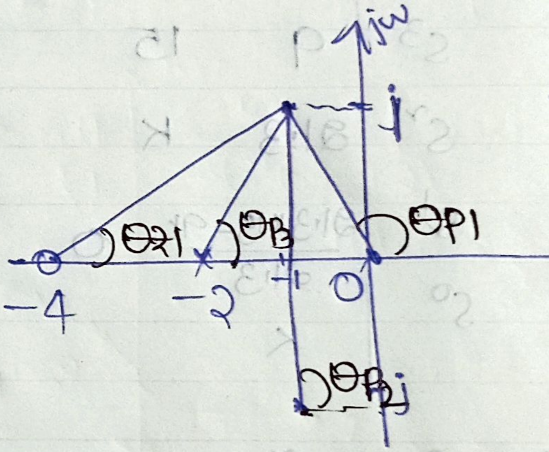
Q. find θ_d for the system $GH = \dfrac{K(S+4)}{S(S+2)(S^2+2S+2)}$

$S^2 + 2S + 2 = 0$

$S = \dfrac{-2 \pm \sqrt{4-4(2)}}{2}$

$= \dfrac{-2 \pm 2j}{2}$

$S = -1 \pm j$; $S = -2, 0.$



$\theta_{P1} = 135^\circ$

$\theta_{P2} = 90^\circ$

$\theta_{P3} = 45^\circ$

$\theta_{z1} = 18.4^\circ$

for pole $S = -1+j$

$\phi = (135 + 90 + 45) - (18.45)$

$\phi = 251.5^\circ$

$\theta_{d_1} = 180 - 251.5 = -72^\circ$

for pole $S = -1-j$

$\theta_{d_2} = 72^\circ$

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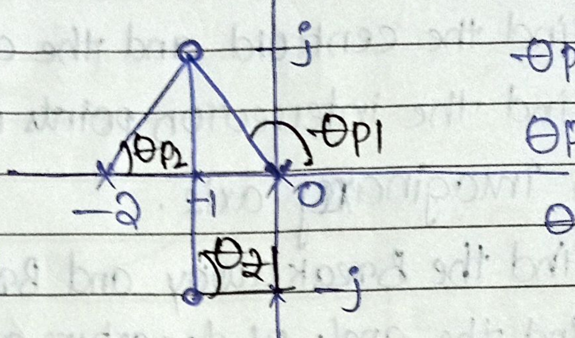
Q) Find the angle of arrival for the sys. $GH = \frac{K(s^2+2s+2)}{s(s+2)}$

s^2+2s+2

$\Rightarrow s = -1 \pm j$



$\theta_{P1} = 135^\circ$

$\theta_{P2} = 45^\circ$

$\theta_{Z1} = 90^\circ$

$s = -1+j$:

$\phi = (135^\circ + 45^\circ) - (90^\circ)$
$= 90^\circ$

$\theta_{a_1} = 180 + 90 = 270^\circ$

$s = -1-j \Rightarrow \theta_{a_2} = 270^\circ$

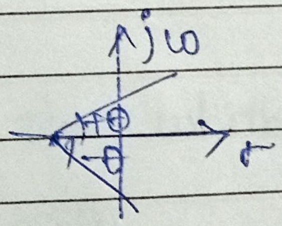
Note!

for poles and zeros are real / for real axis poles and zero's

Angle of arrival $\theta_a = \pm 180^\circ / a$

Angle of departure $\theta_d = \pm 180^\circ / a$

a = no. of poles/zero's adjacent to each other.



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Rules for construction of Root locus

Step 1: Locate the open loop poles and zeros in the s-plane.

2: Find the no. of Root locus branches.
= no. of O.L poles / order of CE eqn.

3: Identify and draw the real axis root locus branches.

4: Find the centroid and the angle of asymptodes.

5: Find the intersection points of Root locus branches with an imaginary axis.

6: Find the Breakaway and Break in points.

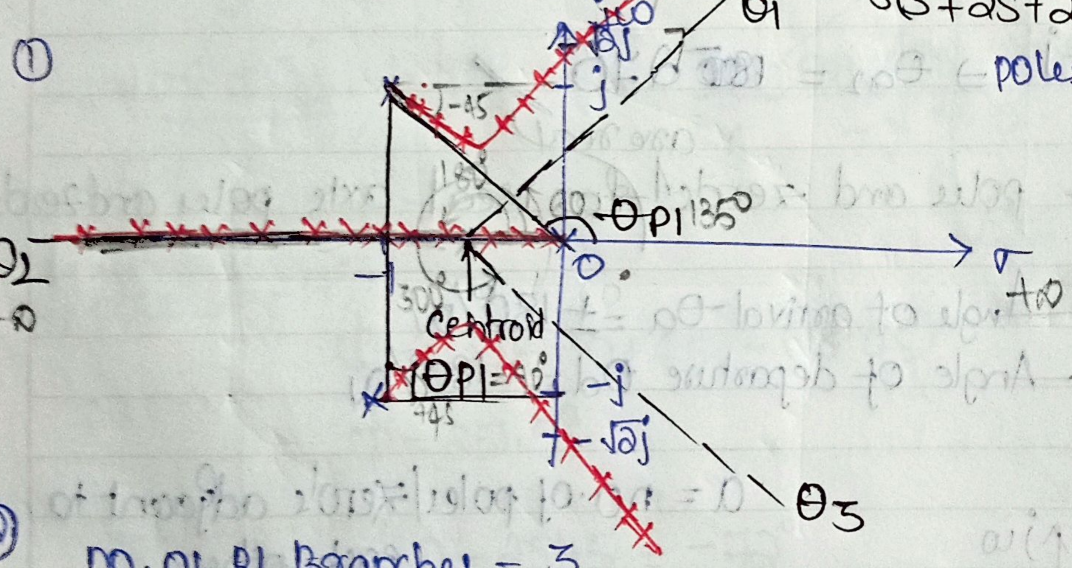
7: Find the angle of departure and angle of arrival.

Q]. Draw the RL for the system: $GH = \frac{K}{s(s^2+2s+2)}$

poles: $s = 0$

$s^2+2s+2=0$

$\Rightarrow s = -1 \pm j$

① 

② no. of RL Branches = 3

③ any point on left half plane.

④ $\text{centroid} = \frac{(0-1-1)-(0)}{3-0} = -0.67$

$$\phi = \frac{(2q+1)180}{n-m} \Rightarrow \phi = \frac{(2q+1)\,180}{3}$$

$\theta_1 = 60°, \theta_2 = 180°, \theta_3 = 300°.$

⑤ K=?

$$1+GH=0$$

$$s^3+2s^2+2s+K=0$$

$$K=-[s^3+2s^2+2s]$$; at K=... for system is Marginaly stable.

$$\frac{dK}{ds}=0$$

s^3	1	2
s^2	2	K
s^1	$\frac{4-K}{2}$	
s^0	K	

$$\frac{4-K}{2}=0$$

$$\boxed{K=4}$$

$$2s^2+K=0$$

$$2s^2+4=0$$

$$s=\pm\sqrt{2}j$$

⑥ $1+GH=0$

$$\frac{dK}{ds}=0$$

$$\Rightarrow 3s^2+4s+2=0$$

$$s=\frac{-4\pm\sqrt{16-4(6)}}{2(3)}$$

$$=\frac{-\cancel{4}^{2}\pm\cancel{2}\sqrt{-2}}{\cancel{2}(3)}=-0.67\pm j0.47$$

check whether the 's' points lie on RL or not.

$$\angle GH\Big|_{s=-0.67+j0.47}=-164.11^\circ \Rightarrow \text{not odd multiple of } 180^\circ$$

∴ These points are neither Break away or Break in points

⑦ Angle of departure: ϕ_d

$$\phi_d = 180 - \phi$$

$$\phi = \sum_{1} \theta_{p1} - \sum_{1} \theta_{z1}$$

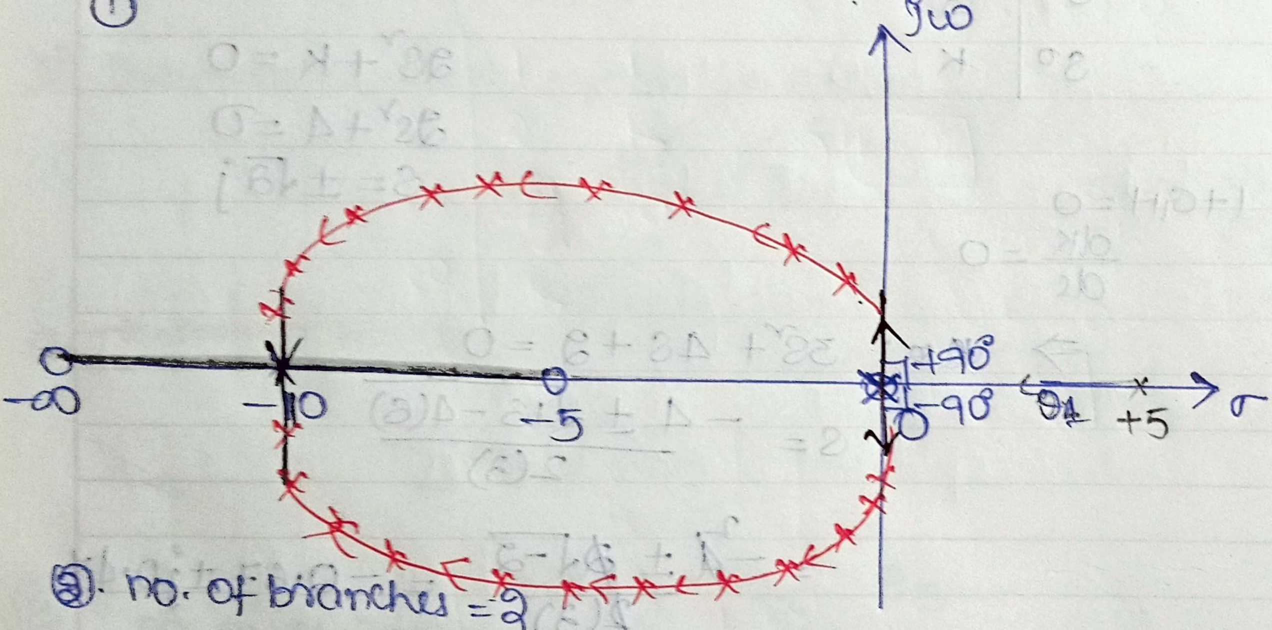
$$\phi = (135+90) - (0) = 225$$

$s = -1+j$. $\therefore \phi_{d1} = -45$ | $s = -1-j$ $\therefore \phi_{d2} = +45$

Q) Draw RL:

$$GH = \frac{K(s+5)}{s^2}$$

①



②. no. of branches = 2

③ after -5

④ centroid = $\frac{(0)-(-5)}{1} = 5 \Rightarrow \sigma = +5$

$$\phi = \frac{(2q+1)180°}{n-m} = \frac{(2q+1)180°}{1} \Rightarrow \theta_1 = 180$$

⑤ There will be no Intersection point (from graph) $(K=0)$

→ as asymptote is not cutting Img. axis.

→ ROOTLOCUS IS SYMETRIC ABOUT REAL AXIS

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⑥ $1+GH=0$

$$\frac{dK}{ds}=0$$

$$s^2+K(s+5)=0$$

$$K=-s^2/s+5$$

$$\frac{(s+5)(2s)-(s^2)(1)}{(s+5)^2}=0.$$

$$s\left[(s+5)2-s\right]=0$$

$$\Rightarrow s=0,\ \boxed{s=-10}$$ ✓

not considered

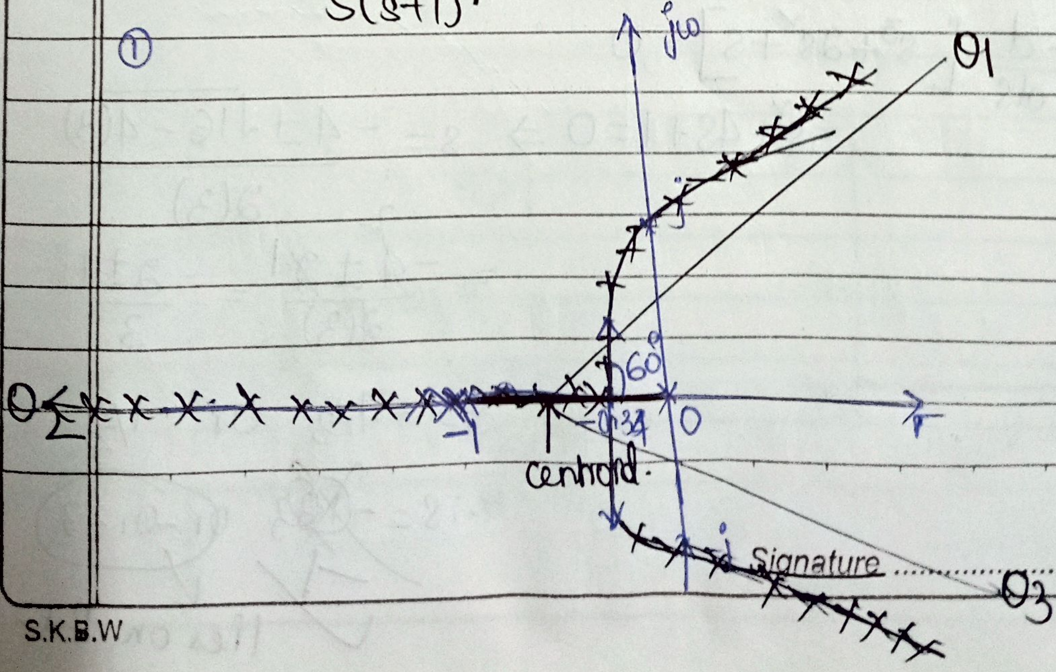
Break away points.

Break in points

⑦ $\theta_a = 180/a = 180/2 = 90^\circ$

Q] $GH = \frac{K}{s(s+1)^2}$ Draw the root locus.

①



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② no. of branches = 3

③

④ centroid(σ) = $\dfrac{(-1-1+0)-(0)}{3-0} = -2/3 = -0.67$.

$$\theta = \frac{(2q+1)180^\circ}{n-m} = \frac{(2q+1)180^\circ}{3} \Rightarrow \begin{array}{l}\theta_1 = 60\\ \theta_2 = 180^\circ\\ \theta_3 = 300^\circ\end{array}$$

⑤ Intersection point:

$$GH = \frac{K}{s(s+1)^2}$$

$$1 + GH = 0 \Rightarrow s[s^2+1+2s] + K = 0$$

$$s^3 + 2s^2 + s + K = 0$$

$$\begin{array}{c|cc}
s^3 & 1 & 1 \\
s^2 & 2 & K \\
s^1 & \frac{2-K}{2} & 0 \\
s^0 & K &
\end{array} \Rightarrow \boxed{K = 2}$$

Auxiliary eqn:

$$2s^2 + K = 0$$

$$\boxed{s = \pm j}$$

⑥ Break away points: $\dfrac{dK}{ds} = 0$

$$-\frac{d}{ds}\left[s^3 + 2s^2 + s\right] = 0$$

$$3s^2 + 4s + 1 = 0 \Rightarrow s = \frac{-4 \pm \sqrt{16 - 4(3)}}{2(3)}$$

$$= \frac{-\cancel{4}^{\,2} \pm \cancel{2}^{\,1}}{\cancel{2}(3)} = \frac{-2 \pm 1}{3}$$

$\therefore s = -4/3$ or $-1/3$

$\therefore s = \cancel{-4/3}$ -1 ✓ or -0.33

lies on RL

7. $\theta d = 180/a$
$= 180/3 = \underline{60^\circ}$

BODE PLOTS:

1. To analyze the freq. response of openloop Transfer fn (Loopgain - GH)
2. To analyze the closed loop system stability
3. To compute gain Margin and phase Margin

procedure:

1. Replace $s = j\omega$ in GH function
2. Find the magnitude (dB) and phase of GH

$$M_{dB} = 20\log_{10}|G(j\omega)H(j\omega)|$$

$$\phi = \tan^{-1}\left[\text{Img part}/\text{Real part}\right]$$

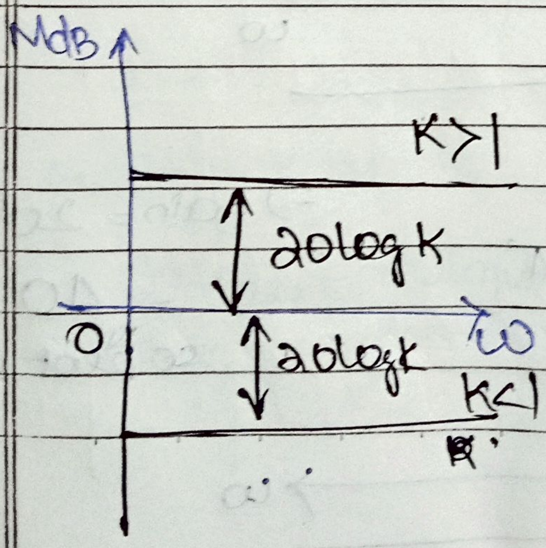
3. Draw M_{dB} & ϕ wrt ω.
   ↳ plots are logarithmic.

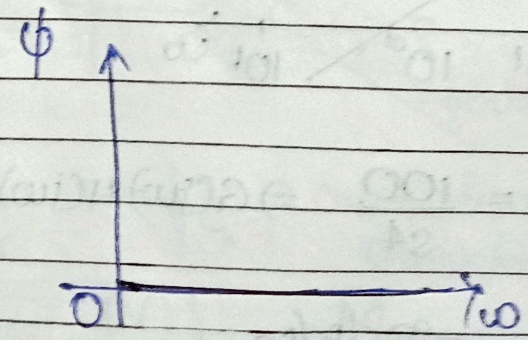
Ex: $GH = K$

$G(j\omega)H(j\omega) = K$

$M_{dB} = 20\log_{10}K$

$\phi = 0^\circ$.





Ex! $GH = 1/s$

$G(j\omega)H(j\omega) = \frac{1}{j\omega}$

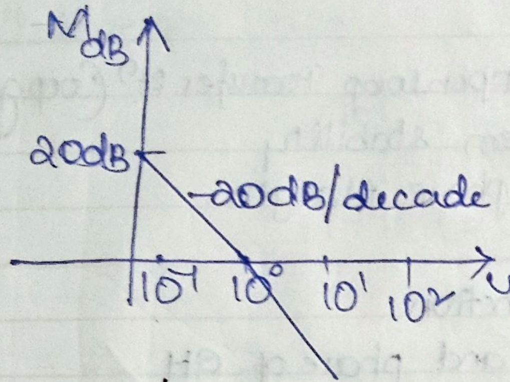
$M_{dB} = 20\log\left|\frac{1}{\omega}\right| = -20\log\omega$

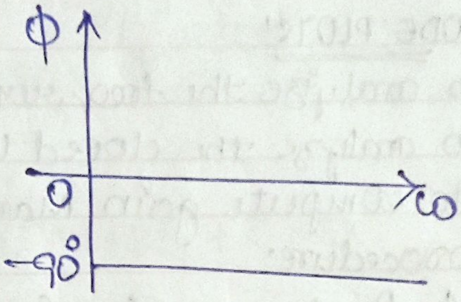
$\phi = -90°$.

slope $= \frac{d[M]}{d\log\omega}$

$= \frac{d}{d\log\omega}[-20\log\omega]$

$= -20$.



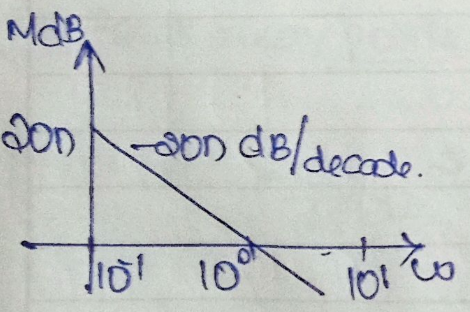


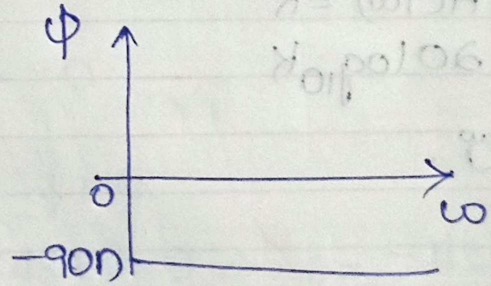
Ex! $GH = 1/s^n$, $n>0$

$G(j\omega)H(j\omega) = \frac{1}{(j\omega)^n}$

$M_{dB} = -20n\log\omega$

$\phi = -90°n$



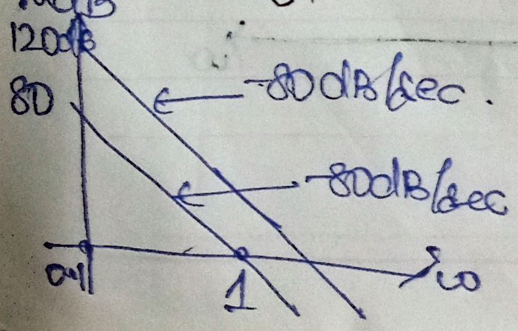


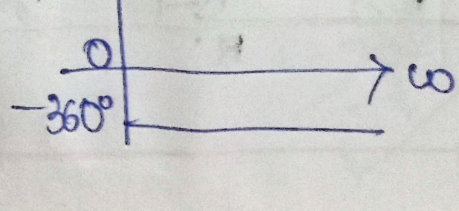
Ex! $GH = \frac{100}{s^4}$ $\Rightarrow G(j\omega)H(j\omega) = \frac{100}{(j\omega)^4}$

$\Rightarrow$ gain $= 20\log_{10}100$

$= 40$

so plot will shift by 40dB



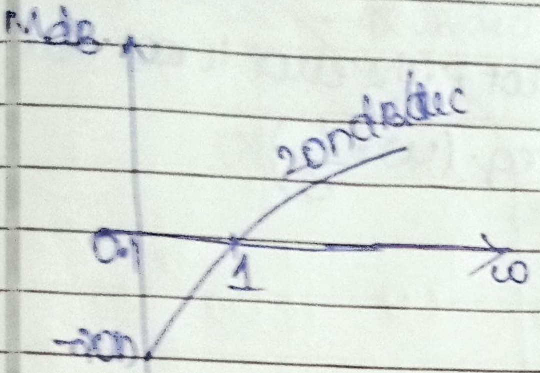


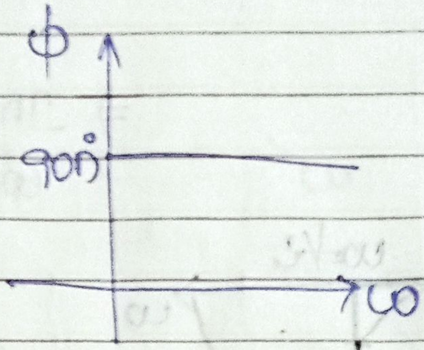
Ex: $GH = s^n$

$G(j\omega)H(j\omega) = (j\omega)^n$

$M_{dB} = 20n \log_{10}\omega$

$\phi = 90n$





Ex: $GH = \frac{1}{1+s\tau}$

$G(j\omega)H(j\omega) = \frac{1}{1+j\omega\tau}$

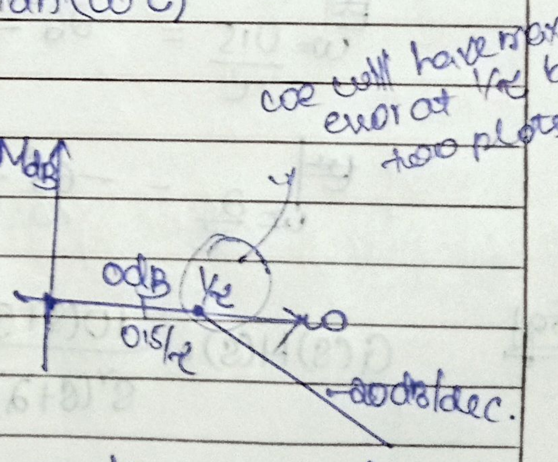
$M_{dB} = -20\log\sqrt{1+\omega^2\tau^2}$ $\phi = -\tan^{-1}(\omega\tau)$

Asymptotic / approximation plot

Case i: $\omega\tau << 1$

$M_{dB,asy} = 0dB$

$\phi_{asy} = 0$



Case ii: $\omega\tau >> 1$

$M_{dB_{asy}} = -20\log\omega\tau$

$\phi_{asy} = -90^\circ$

$\omega = 1/\tau \rightarrow$ corner freq. / Break frequency.

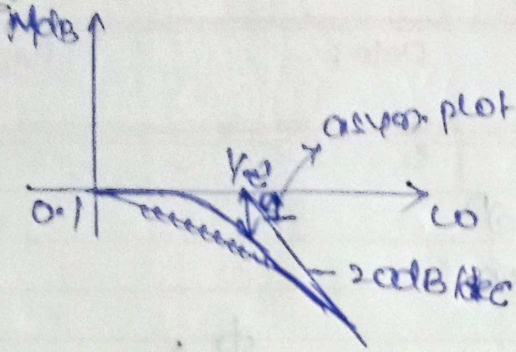
Error

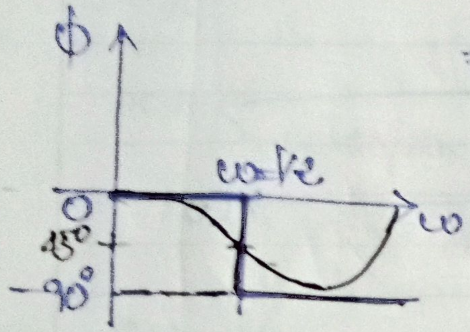
$E|_{\omega=1/\tau} = M_{actual} - M_{asym.}$

$= -3 - 0 = -3dB \rightarrow$ Max error

$E|_{\omega=0.5/\tau} = -0.96dB - 0dB = -0.96dB.$

- Actual plot:





⇒ In phase plot, the error is ~~not~~ max. at corner freq. ($\omega = \frac{1}{\tau}$)

$$E\Big|_{\omega=1/\tau} = -45^\circ - 0^\circ = -45^\circ$$

$$E\Big|_{\omega=\frac{0.5}{\tau}} = -26^\circ - 0^\circ = -26^\circ$$

$$E\Big|_{\omega=\frac{2}{\tau}} = -63^\circ + 90^\circ = 27^\circ$$

Considering Magnitudes

$$\left| E\big|_{\omega=1/\tau} \right| = 45^\circ$$

⇒ Max Error at $1/\tau$

Eg:

$$G(s)H(s) = \frac{10(s+5)^2}{s^2(s+2)(s+10)} = \frac{K(1+sZ_1)(1+sZ_2)}{(1+sP_1)(1+sP_2)}$$

$$= \frac{12.5\left(1+\frac{s}{5}\right)^2}{s^2\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)} = \frac{\overset{12.5}{\cancel{10}(25)}\left(1+\frac{s}{5}\right)^2}{s^2\times(2)\left[1+\frac{s}{2}\right]\times\cancel{10}\left[1+\frac{s}{10}\right]}$$

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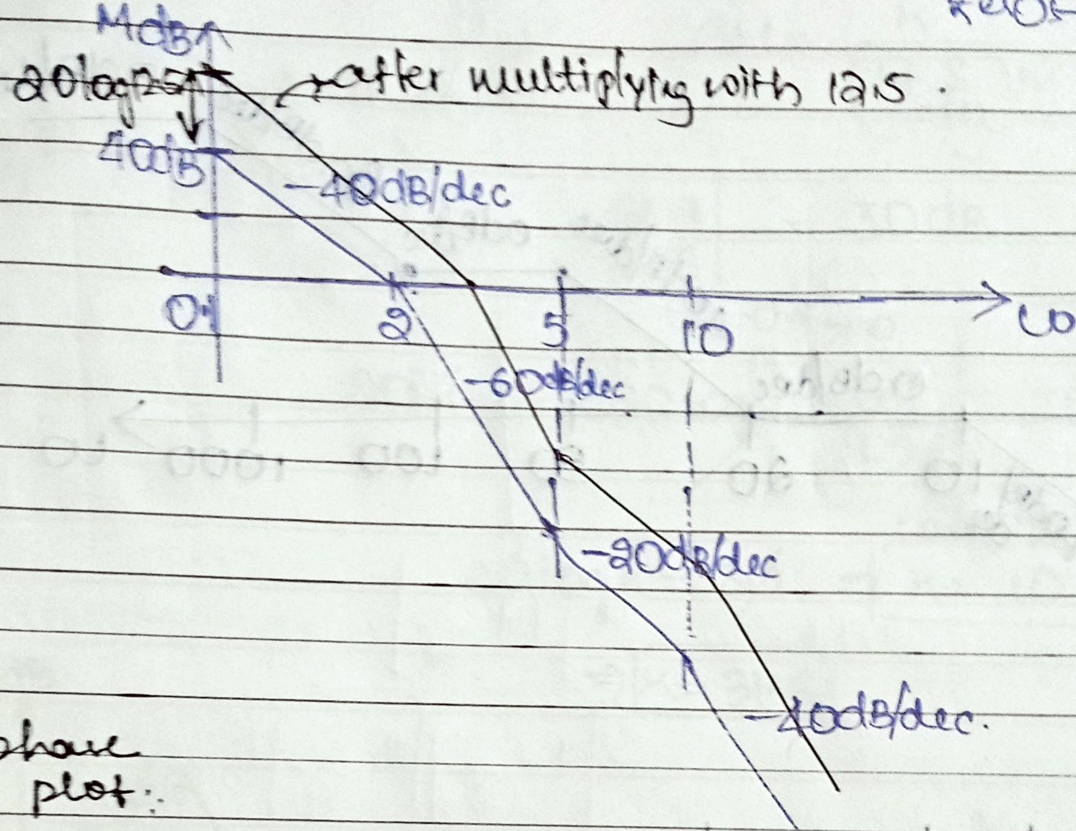
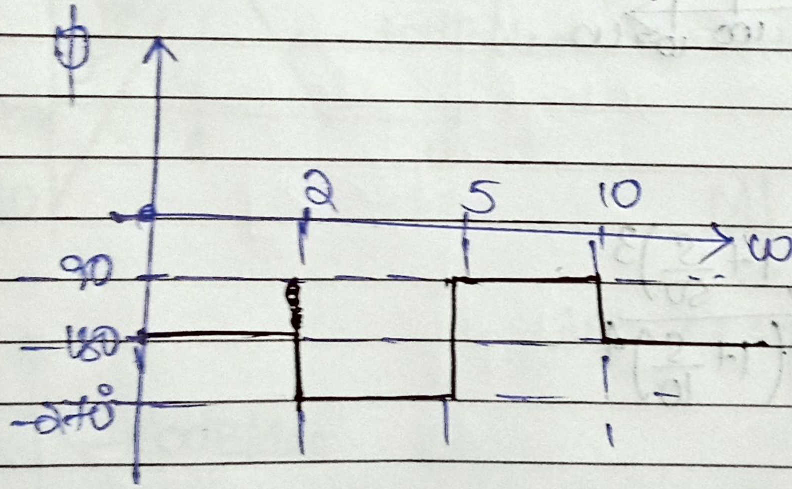
asymptotic plot.

pole → -20dB/dec
zero → +20 dB/dec.



phase plot:



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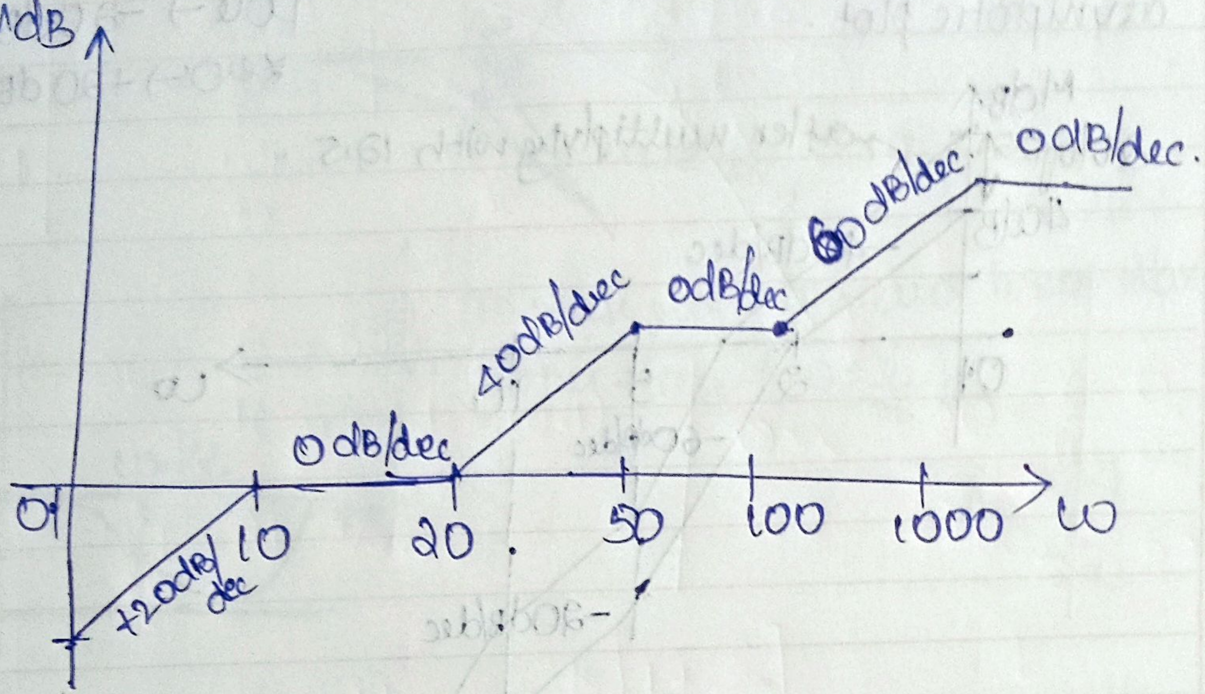
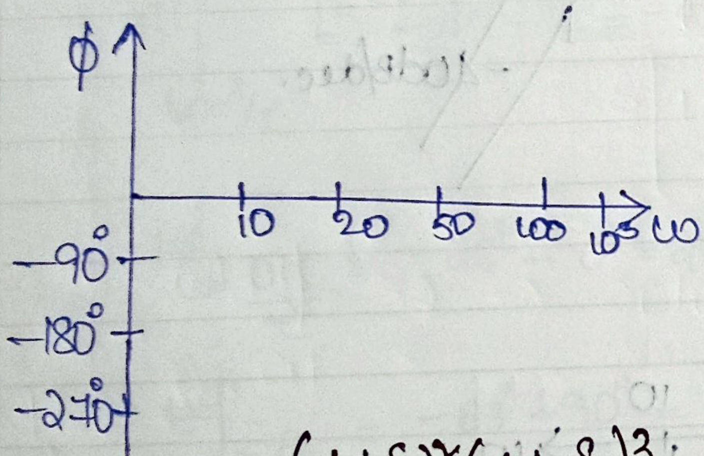
Q] $$G(S)H(S) = \frac{S\left(1+\frac{S}{20}\right)^2\left(1+\frac{S}{100}\right)^3}{\left(1+\frac{S}{10}\right)\left(1+\frac{S}{50}\right)^2\left(1+\frac{S}{1000}\right)^3}$$

Zero — +20dB/dec

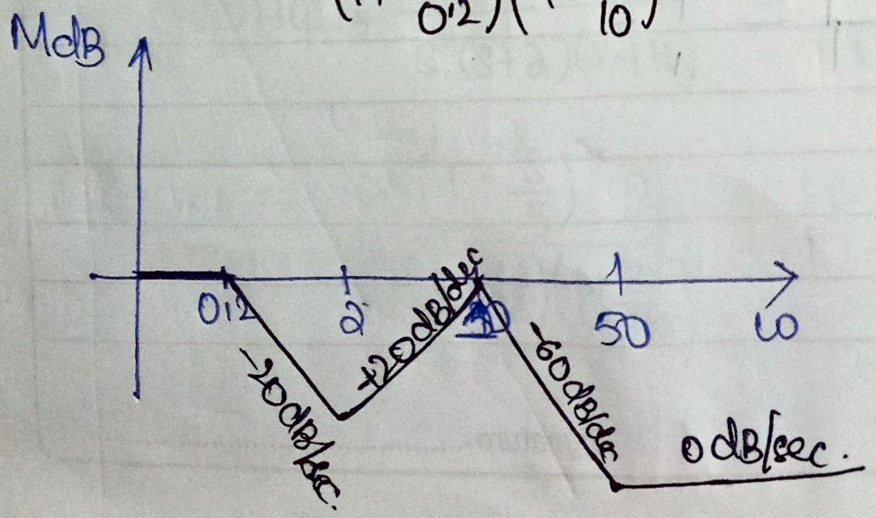
pole — -20 dB/dec.

(1st) Mark corner frequency

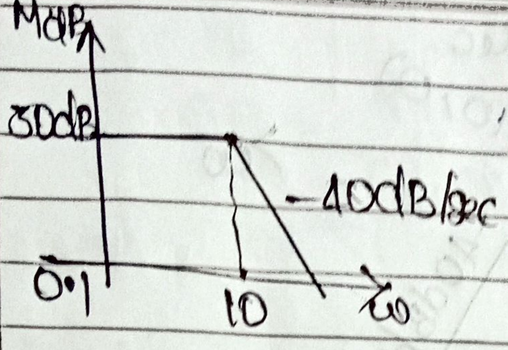




Q] $$G(S)H(S) = \frac{\left(1+\frac{S}{2}\right)^2\left(1+\frac{S}{50}\right)^3}{\left(1+\frac{S}{0.2}\right)\left(1+\frac{S}{10}\right)^4}$$



Q)



$$GH = \frac{K}{\left(1+\frac{s}{10}\right)^2}$$

$$M_{dB}\Big|_{\omega=0.1} = 30\,dB$$

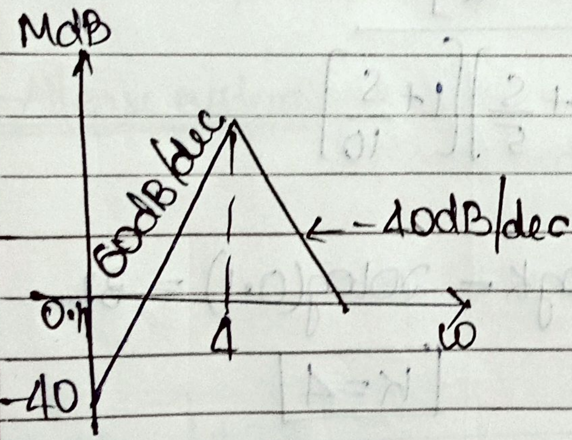
$$20\log K - \cancel{20\log\left(1+\frac{\omega^2}{100}\right)}\Big|_{\omega=0.1} = 30$$

$\to 0$

$$20\log K = 30 \Rightarrow \boxed{K = 10^{1.5}}$$

$$\Rightarrow \boxed{K = 31.6}$$

Q)

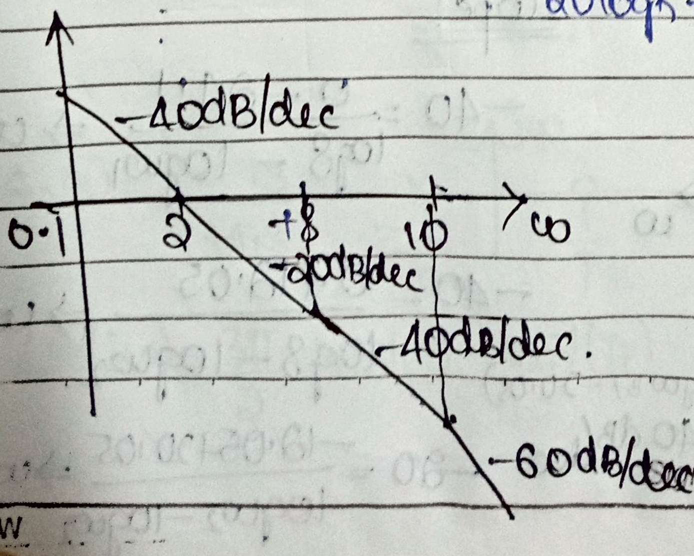


$$\Rightarrow GH = \frac{Ks^3}{\left(1+\frac{s}{4}\right)^5}$$

$$M_{dB}\Big|_{0.1} = -40$$

$$20\log K + 60\log 0.1 = -40 \Rightarrow \boxed{K = 10}$$

Q)



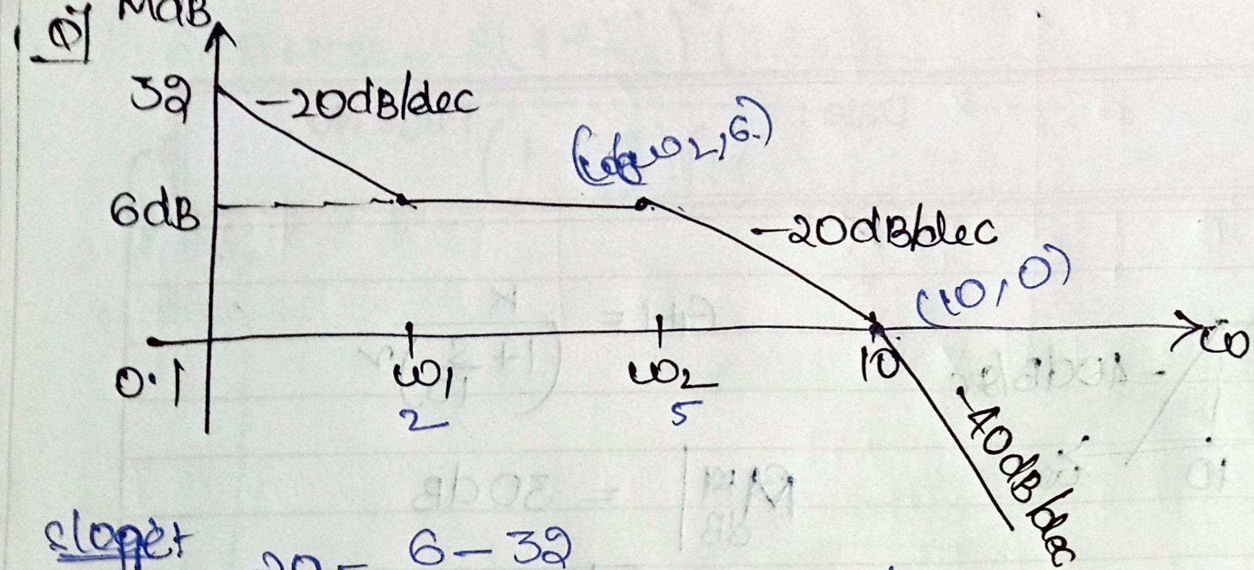
$$GH = \frac{K\cdot\left[1+\frac{s}{2}\right]}{s^2\cdot\left[1+\frac{s}{8}\right]\left[1+\frac{s}{10}\right]}$$

$$M_{dB}\Big]_{\omega=2} = 0$$

$$20\log K - 40\log 2 + 20\log\sqrt{2} = 0$$

$$K \simeq 2.8$$

Q]



Slope:

$$-20 = \frac{6-32}{\log\omega_1 - \log(0.1)}$$

$\omega_1 = 2$ rad/sec

$$-20 = \frac{6-0}{\log\omega_2 - \log 10}$$

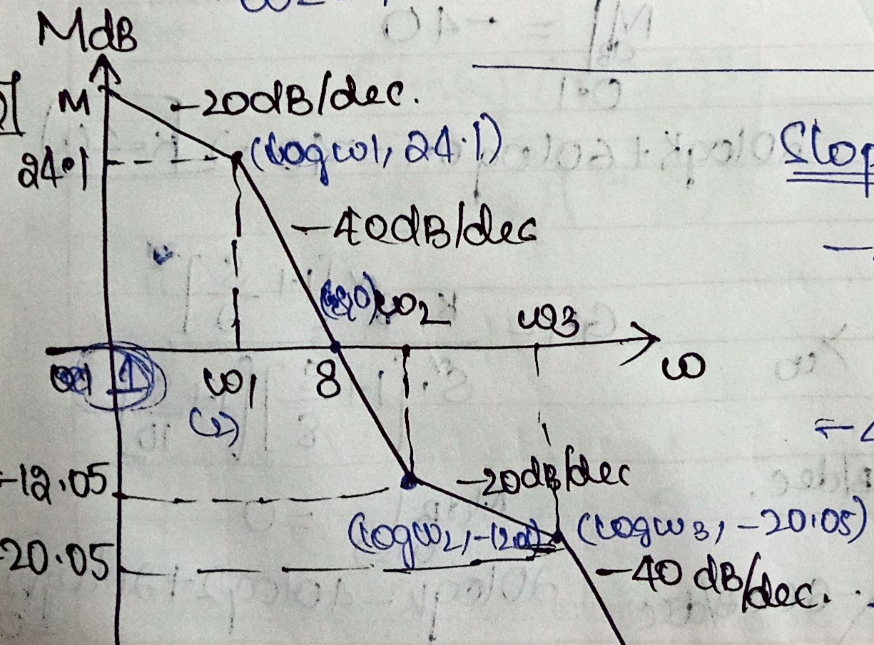
$\Rightarrow \omega_2 = 5$ rad/sec.

$$GH = \frac{K\left[1+\frac{s}{2}\right]}{s\left[1+\frac{s}{5}\right]\left[1+\frac{s}{10}\right]}$$

$M_{dB}\Big|_{\omega=0.1} = 32 \Rightarrow 20\log K - 20\log(0.1) = 32$

$\boxed{K=4}$

Q]



Slope:

$$-40 = \frac{0-24.1}{\log 8 - \log\omega_1} \Rightarrow \omega_1 = 2\text{ rad}$$

$$-40 = \frac{0+12.05}{\log 8 - \log\omega_2} \Rightarrow \omega_2 = 16\text{ rad}$$

$$-20 = \frac{-12.05+20.05}{\log\omega_2 - \log\omega_2} \Rightarrow \omega_3 = 40\text{ rad}$$

$$GH = \frac{K\left[1+\frac{S}{16}\right]}{S\left[1+\frac{S}{2}\right]\left[1+\frac{S}{40}\right]}$$

$$-40 = \frac{M - 24.1}{\log 0.1 - \log \omega_1}$$

$$\Rightarrow M = 50.22\,dB$$

Minimum phase and non-Minimum phase systems.

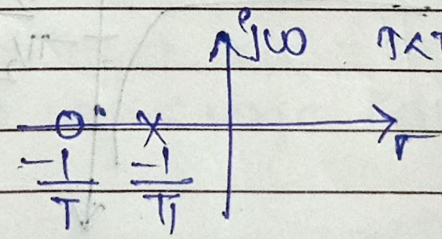
Transfer fthc having neither poles nor zero's on right half S-plane are called Minimum phase system.

The Transfer fthc having poles and/or zero's in the left half of s-plane are called non-Minimum phase system.

All pass system: A system in which zero's lies in right half S-plane and poles lies in left half of S-plane and locus of pole-zero pair is symetric about imaginary plane.

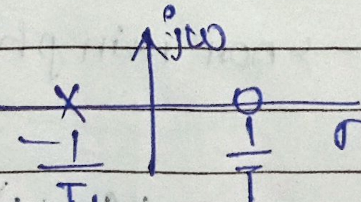
$\Rightarrow$

$$G_1(S) = \frac{1+TS}{1+T_1S}$$



→ Minimum phase system

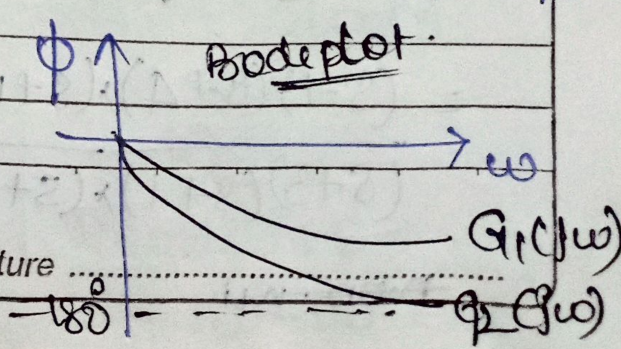
$$G_2(S) = \frac{1-TS}{1+T_1S}$$



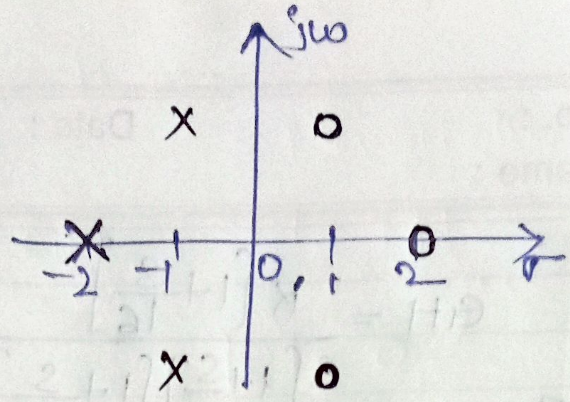
→ non-Minimum phase system

$$\angle G_2(j\omega) = \frac{3\pi}{2} + \tan^{-1}(T) - \tan^{-1}(T_1)$$

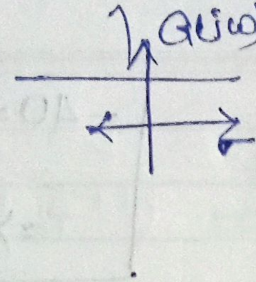
Bodeplot.



$$\Rightarrow G_3(s) = \frac{(s-2)(s^2-2s+2)}{(s+2)(s^2+2s+2)}$$



$|G_3(s)| = 1$



→ (4) All pass system

i) As every zero/pole has symmetric pole/zero about origin

→ It passes all the frequencies - All pass system poles & zeros are symmetric about Imaginary axis.

$$\Rightarrow G_3(s) = \frac{s-2}{s+2}$$

$$G_3(s) = \frac{j\omega - 2}{j\omega + 2}$$ (-1+) (+,+)

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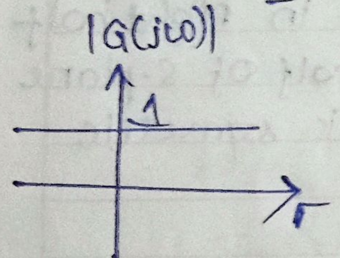
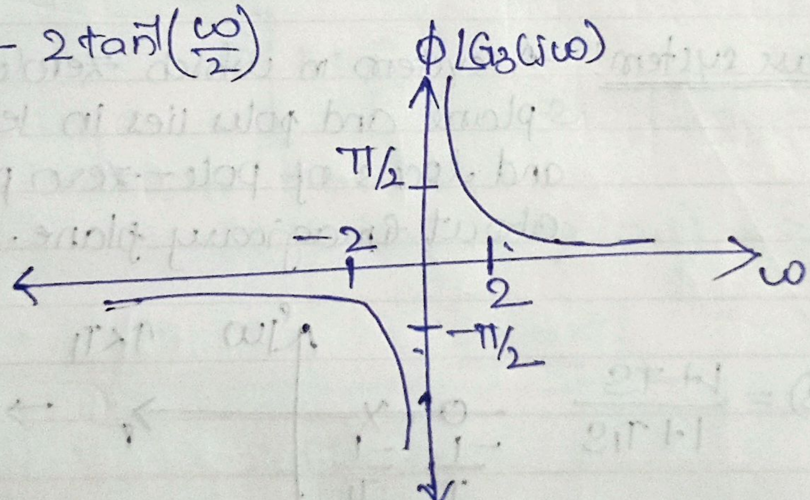
III | IV

$$\angle G_3(j\omega) = \left[\pi - \tan^{-1}\left(\frac{\omega}{2}\right)\right] - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= \pi - 2\tan^{-1}\left(\frac{\omega}{2}\right)$$





Eg: $$G_4(s) = \frac{(s-1)(s+4)}{(s+3)(s+1)}$$ → non min. phase system.

$$= \frac{(s-1)(s+4)\times(s+1)}{(s+3)(s+1)\times(s+1)} = \underbrace{\frac{(s+1)(s+4)}{(s+1)(s+3)}}_{\text{Min. phase system}} \times \underbrace{\frac{(s-1)}{(s+1)}}_{\text{All pass system}}$$

~~non mi~~

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⇒ non min. phase system can be expressed as min phase system and all pass system

Gain Margin & phase margin:

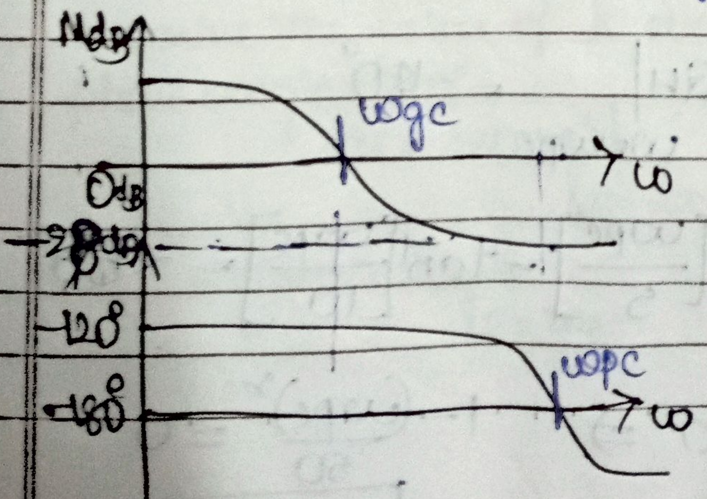
Gain margin (GM) = $-20\log M\Big|_{\omega=\omega_{pc}}$; $M = |GH|$

Phase margin (PM) = $180+\phi\Big|_{\omega=\omega_{gc}}$; ϕ = phase angle of GH

ω_{pc} - phase crossover frequency → $\phi = -180°$
⇒ The freq. at which phase = $-180°$
ω_{gc} - gain cross over frequency → $|M| = 0dB = 1$
⇒ freq. at which gain = 1/0dB.

- for a closed loop system to be stable, GM and PM should be positive.

→ for stability → $\omega_{pc} > \omega_{gc}$ [GM, PM are +ve]
for marginally stability → $\omega_{pc} = \omega_{gc}$ [GM = 0dB, PM = 0°]
for unstable → $\omega_{pc} < \omega_{gc}$ [GM, PM are −ve]



$\omega = \omega_{pc}$

$20\log M = -20\,dB$

$GM = -20\log M$
$= -(-20)$
$= 20\,dB$

$PM = 180° + \phi$
$= 180° - 120° = 60°$

Signature ....

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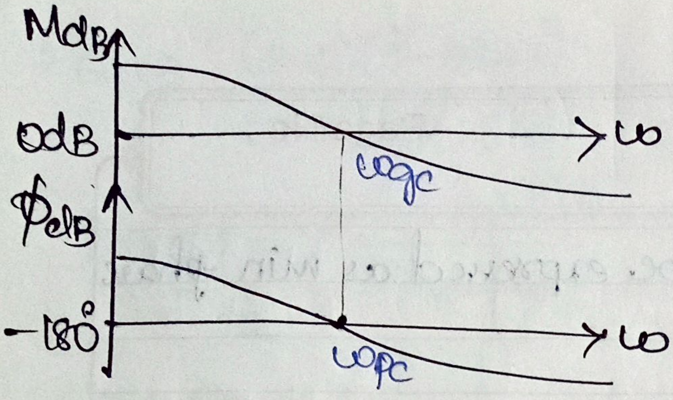


$\omega_{pc} = \omega_{gc}$

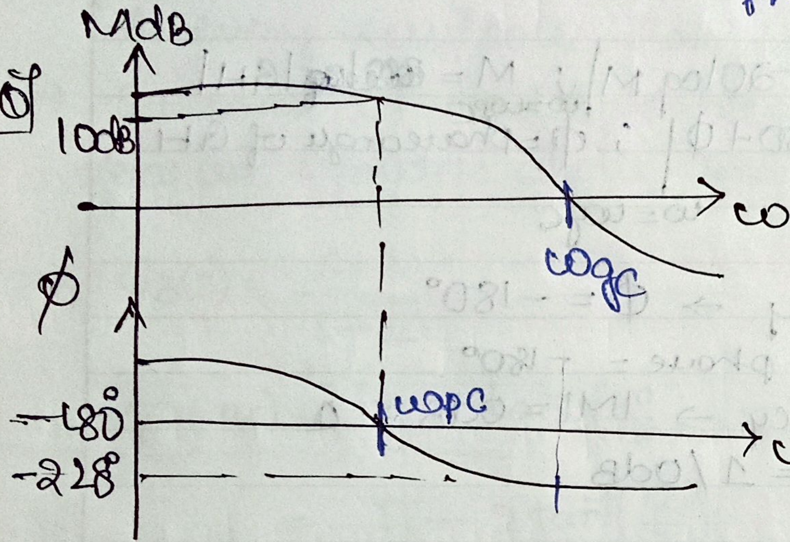
$\Rightarrow$ Marginally stable

$GM = -20\log M\Big|_{\omega=\omega_{pc}}$

$= -[0] = 0$

$PM = 180+\phi\Big|_{\omega=\omega_{gc}} = 180-180 = 0^\circ$

Q]



$GM = -20\log M$

$= -[10] = \underline{-10dB}$

$PM = 180+\phi\Big|_{\omega=\omega_{gc}}$

$= 180-228 = \underline{-48^\circ}$

$\omega_{pc} < \omega_{gc}$

$\Rightarrow$ unstable system.

Q] $GH = \dfrac{1}{s(s+5)(s+10)}$

phase crossover freq: $\angle GH\Big|_{\omega=\omega_{pc}} = -180^\circ$

$$-90 - \tan^{-1}\left[\frac{\omega_{pc}}{5}\right] - \tan^{-1}\left[\frac{\omega_{pc}}{10}\right] = -180^\circ$$

$$\tan^{-1}\left[\frac{\frac{\omega_{pc}}{5} + \frac{\omega_{pc}}{10}}{1 - \frac{(\omega_{pc})^2}{50}}\right] = 90 \Rightarrow 1 - \frac{(\omega_{pc})^2}{50} = 0$$

$$\Rightarrow \boxed{\omega_{pc} = \sqrt{50}\ rad/sec}$$

gain crossover freq:

$$|M|\Big|_{\omega=\omega_{gc}} = 1 \quad \text{or} \Rightarrow 20\log M = 0$$

$$\frac{1}{\omega_{gc}\sqrt{\omega_{gc}^2+25}\sqrt{\omega_{gc}^2+100}} = 1$$

$$1 = (\omega_{gc})^2(\omega_{gc}^2+25)(\omega_{gc}^2+100)$$

let $\omega_{gc}^2 = x$

$$1 = x(x+25)(x+100)$$

$$GM = -20\log|M|\Big|_{\omega=\omega_{pc}}$$

$$= -20\log\frac{1}{\sqrt{50\times 75\times 180}} = 20\log 750 \ (+ve)$$

$$PM = 180 + \phi\Big|_{\omega=\omega_{gc}}$$

Q. The open loop Transfer ftn of a sys. is $GH = \frac{K}{S(S+2)(S+4)}$ determine the value of K such that phase margin = 60°.

$$PM = 180 + \phi\Big|_{\omega=\omega_{gc}} = 60 \Rightarrow \phi\Big|_{\omega=\omega_{gc}} = -120°.$$

$$GM = -20\log M\Big|_{\omega=\omega_{gc}}$$

$$|GH|\Big|_{\omega=\omega_{gc}} = 1$$

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$$\left.\frac{K}{\omega(\sqrt{\omega^2+4})(\sqrt{\omega^2+16})}\right|_{\omega=\omega_{gc}} = 1$$

$$-\tan^{-1}(\omega_{gc}) - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{4}\right) = -120 \quad \text{--- (1)}$$

$$\boxed{\omega_{gc} = 0.72 \text{ rad/sec}}$$

$$\underline{K = 2}$$

Polar plot:

Rules for drawing polar plot: $\quad GH = \frac{KN(S)}{D(S)}$

1. Substitute $\boxed{S = j\omega}$ in GH.
2. find magnitude and phase at $\omega = 0$

   $|M|\big|_{\omega=0}$ → starting magnitude; $\angle\phi\big|_{\omega=0}$ → starting phase.

3. $|M|\big|_{\omega=\infty}$ → ending magnitude $\quad \angle\phi\big|_{\omega=\infty}$ → ending phase

   compute ending Mag. ending phase.

4. Identify starting direction (whether clockwise or anticlockwise)
   - If finite pole of GH is near imaginary axis then starting direction is clockwise.
   - If finite zero is near img. axis then starting direction is anticlockwise.
5. Identify ending direction.

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$\phi = \phi_1 - \phi_2$ ϕ_1 – starting phase angle
ϕ_2 – Ending " "

$\phi > 0 \rightarrow$ Ending direction is C.W
$\phi < 0 \rightarrow$ " " " Anti C.W

Ex) $GH = \frac{1}{1+s\tau}$

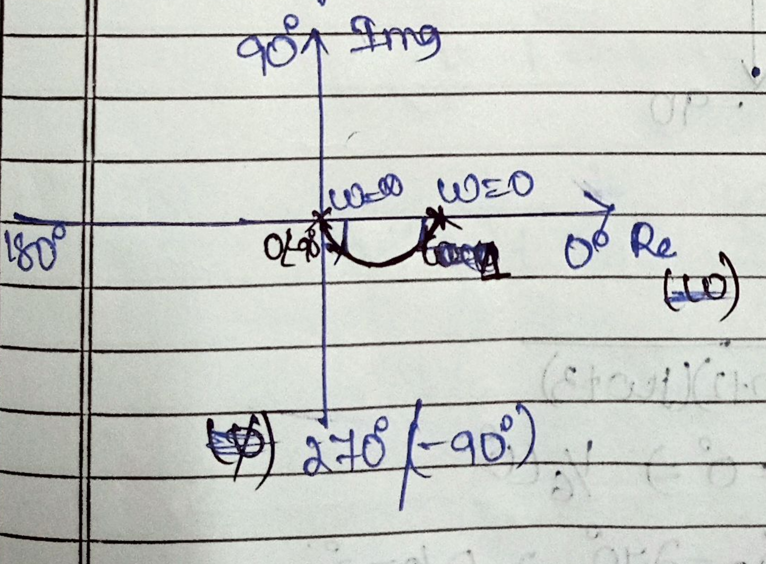
① $s = j\omega \Rightarrow G(j\omega)H(j\omega) = \frac{1}{1+j\omega\tau}$

② $\omega = 0$
$|GH| = 1$, $\phi = 0°$
$\Rightarrow 1\angle 0°$

③ $\omega = \infty$
$|GH| = 0$, $\phi = -90°$
$0\angle -90°$

④ Starting direction – C.W [clock wise] as finite pole near imag axis

⑤ Ending direction – C.W as $\phi = 0 - (-90°) = 90°$.



At $\omega = 1/\tau$
$\hookrightarrow 0.707\angle -45°$.

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Q] $GH = \frac{1}{(s+1)(s+2)}$

① $s = j\omega$; $G(j\omega)H(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$

② $\omega = 0$; $|M| = 1/2$ $\angle\phi = 0^\circ$

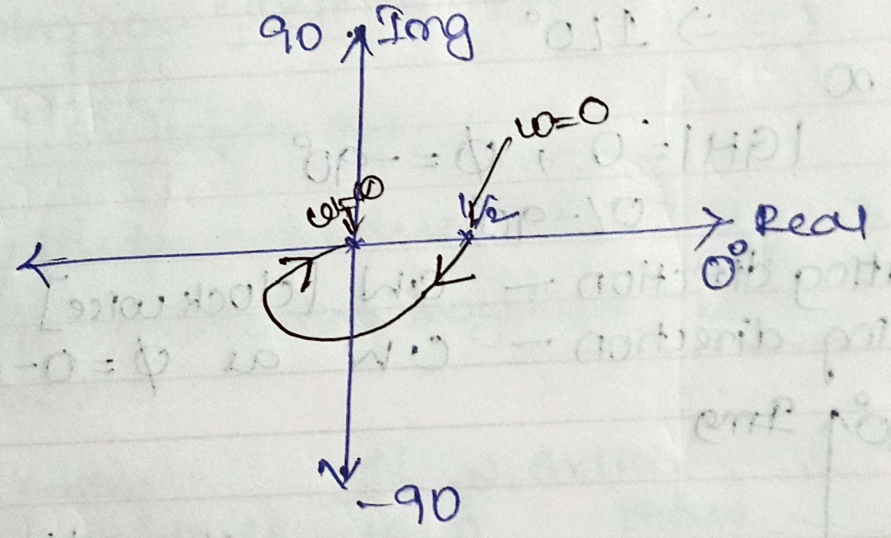
$1/2 \angle 0^\circ$

③ $\omega = \infty$; $|M| = 0$; $\angle\phi = -180^\circ \Rightarrow 0\angle -180^\circ$

④ Starting direction :- Clockwise

⑤ Ending " :- $\phi = 0 - (-180^\circ) = 180$

$\Rightarrow$ Clockwise.



Q] $GH = \frac{1}{(s+1)(s+2)(s+3)}$

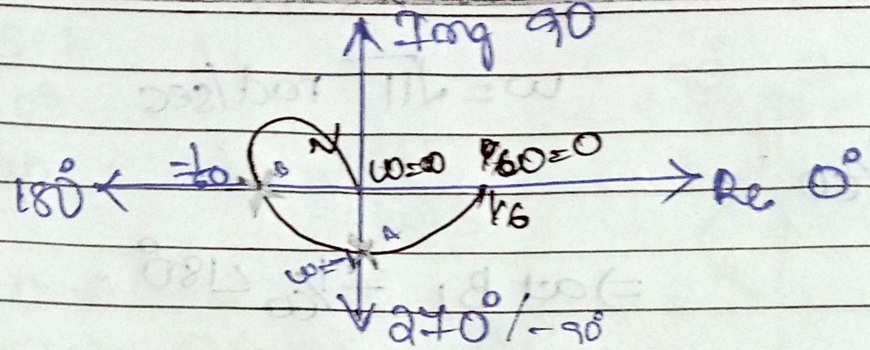
① $s = j\omega \Rightarrow GH = \frac{1}{(j\omega+1)(j\omega+2)(j\omega+3)}$

② $\omega = 0 \Rightarrow |M| = 1/6$, $\angle\phi = 0^\circ \Rightarrow 1/6 \angle 0^\circ$

③ $\omega = \infty \Rightarrow |M| = 0^\circ$, $\angle\phi = -270^\circ \Rightarrow 0\angle -270^\circ$

④ SD → C.W

⑤ ED → CW



$$-\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) = -90$$

$$\cancel{\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\frac{\omega^2}{6}}{1-\frac{\omega^2}{6}}\right) = 90^\circ}$$

$$\boxed{\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right)}$$

$$\cancel{\frac{\left(\frac{\frac{\omega^2}{6}}{1-\frac{\omega^2}{6}}\right)\omega}{1-\left(\frac{\frac{\omega^2}{6}}{1-\frac{\omega^2}{6}}\right)\times\omega} = \frac{1}{0}}$$

$$\cancel{1 = \omega\left[\frac{\omega^2}{6-\omega^2}\right]} \Rightarrow \cancel{6-\omega^2 = \omega^3}$$

$$\cancel{\omega^3+\omega^2-6=0}$$

$$\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\frac{\omega}{2}+\frac{\omega}{3}}{1-\frac{\omega^2}{6}}\right) = 90$$

$$1 - \omega\left[\frac{\frac{\omega}{2}+\frac{\omega}{3}}{1-\frac{\omega^2}{6}}\right] = 0$$

$$1 = \omega\left[\frac{3\omega+2\omega}{6-\omega^2}\right]$$

$$6-\omega^2 = 5\omega^2 \Rightarrow \omega^2 = 1$$

$$\boxed{\omega = \pm 1}$$

$$|M|\Big|_{\omega=1} \Rightarrow \frac{1}{\sqrt{2}\times\sqrt{5}\times\sqrt{10}} = 0.1$$

At B) $\phi = 180^\circ$

$$-\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) = 180^\circ$$

$\omega = \sqrt{11}$ rad/sec

$$|M|_{\omega=\sqrt{11}} = -1/60$$

$\Rightarrow$ at B) $-1/60 \angle 180^\circ$.

Q] $GH = \dfrac{1}{s^2(1+s)}$

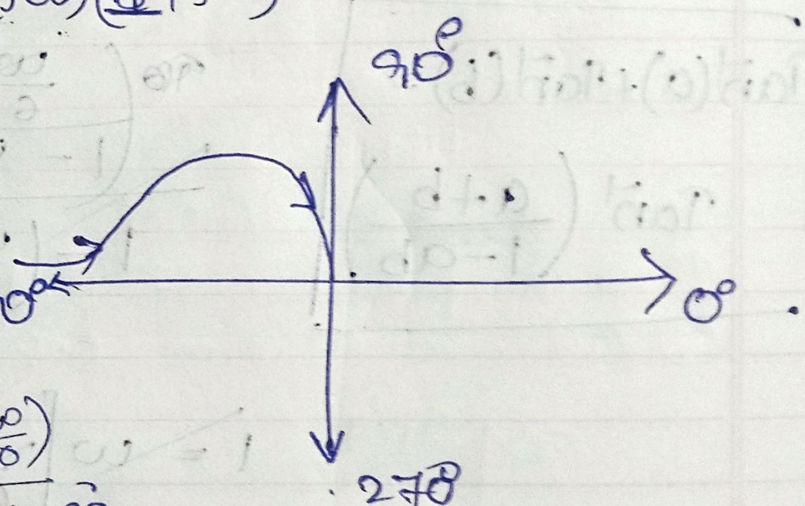
① $s = j\omega \Rightarrow GH = \dfrac{1}{(j\omega)(j\omega)(1+j\omega)}$

② $\omega = 0 \Rightarrow \infty \angle -180^\circ$

③ $\omega = \infty \Rightarrow 0 \angle -270^\circ$

④ SD → CW

⑤ ED → C.W



Q] $GH = \dfrac{1}{s(s+1)}$ $\quad \dfrac{\tan^{-1}\left(\frac{\omega}{0}\right)}{\angle 90}$

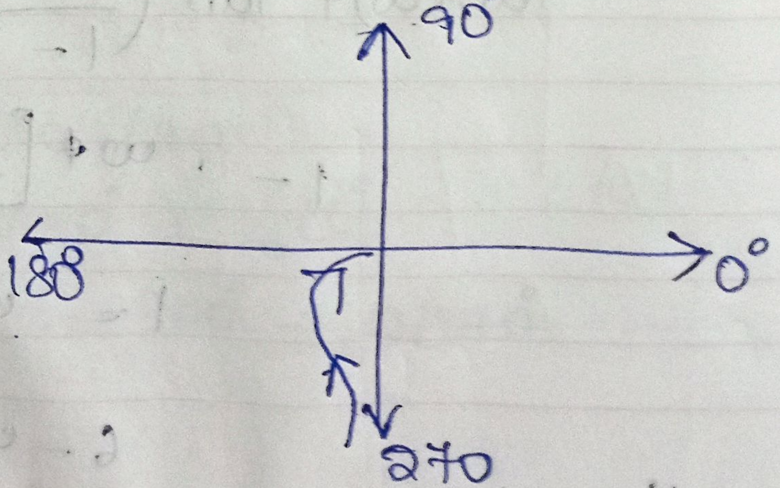
① $s = j\omega \Rightarrow GH = \dfrac{1}{j\omega(1+j\omega)}$

② $\omega = 0 \Rightarrow \infty \angle -90^\circ$

③ $\omega = \infty \Rightarrow 0 \angle -180^\circ$

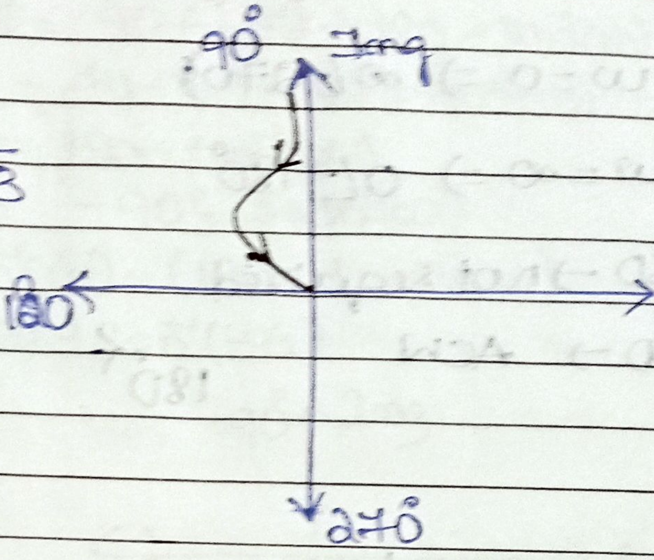
④ SD → CW

⑤ ED ⇒ CW



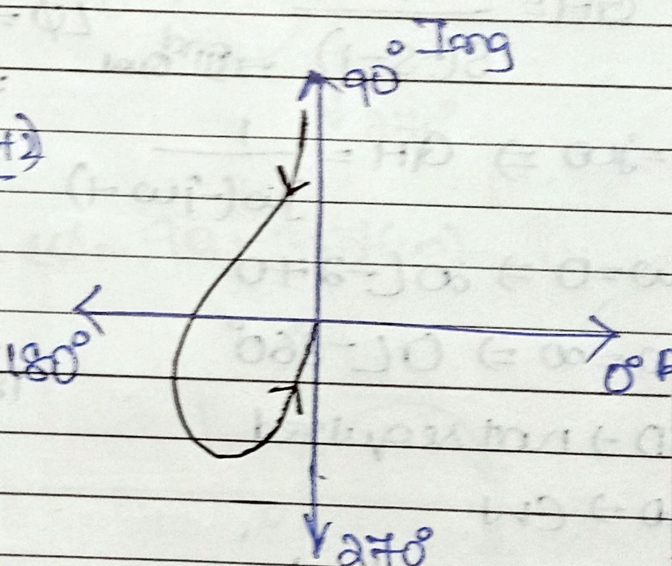
Q) $GH = \frac{s+1}{s^3}$

1. $s = j\omega$; $GH = \frac{j\omega+1}{(j\omega)^3}$
2. $\omega = 0 \Rightarrow \infty\angle-270°$
3. $\omega = \infty \Rightarrow 0\angle-180°$
4. SD → ACW
5. ED → ACW



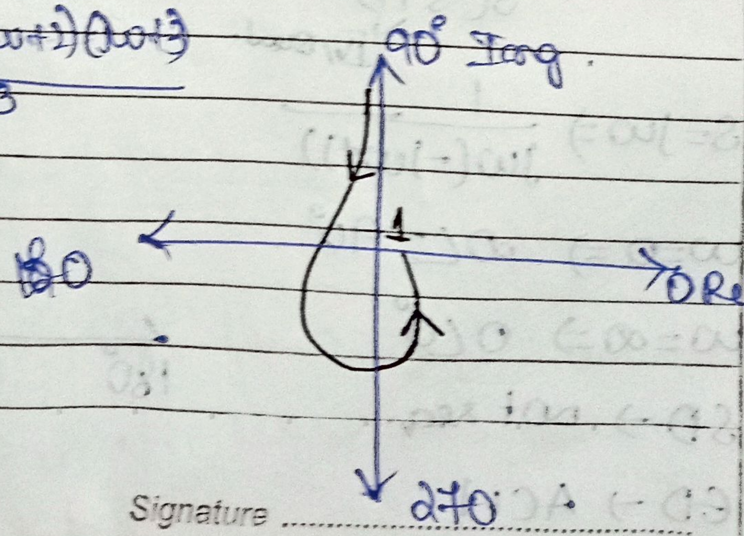
Q) $GH = \frac{(s+1)(s+2)}{s^3}$

1. $s = j\omega \Rightarrow GH = \frac{(j\omega+1)(j\omega+2)}{(j\omega)^3}$
2. $\omega = 0 \Rightarrow \infty\angle-270°$
3. $\omega = \infty \Rightarrow 0\angle-90°$
4. SD ⇒ ACW
5. ED ⇒ ACW.



Q) $GH = \frac{(s+1)(s+2)(s+3)}{s^3}$

1. $s = j\omega \Rightarrow GH = \frac{(j\omega+1)(j\omega+2)(j\omega+3)}{(j\omega)^3}$
2. $\omega = 0 \Rightarrow \infty\angle-270°$
3. $\omega = \infty \Rightarrow 1\angle0°$
4. SD ⇒ ACW
5. ED ⇒ ACW



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Q] $GH = \dfrac{1}{S(S-1)}$ → II quad

$\angle\phi = -90 - (180 - \tan^{-1}(\omega))$
$= -270 + \tan^{-1}(\omega)$

⇒ If T.F consists of -ve terms, we should not consider S.D

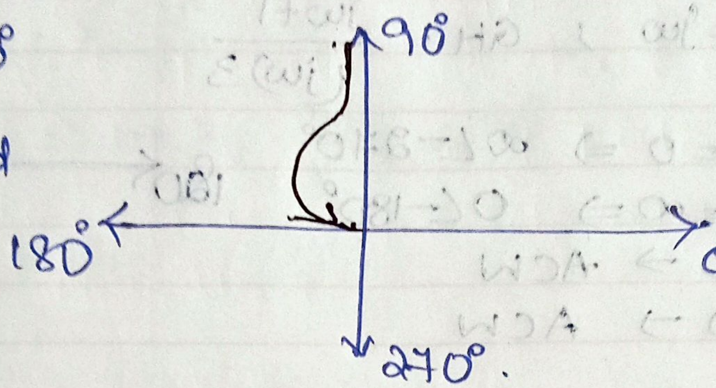
① $S = j\omega \Rightarrow GH = \dfrac{1}{j\omega(j\omega-1)}$

② $\omega = 0 \Rightarrow \infty\angle -270°$

③ $\omega = \infty \Rightarrow 0\angle -180°$

④ SD → not required

⑤ ED → ACW



Q]. $GH = \dfrac{1}{S(-S-1)}$ → IIIrd Quad

$\angle\phi = -90 - (180 + \tan^{-1}(\omega))$
$= -270° - \tan^{-1}(\omega)$

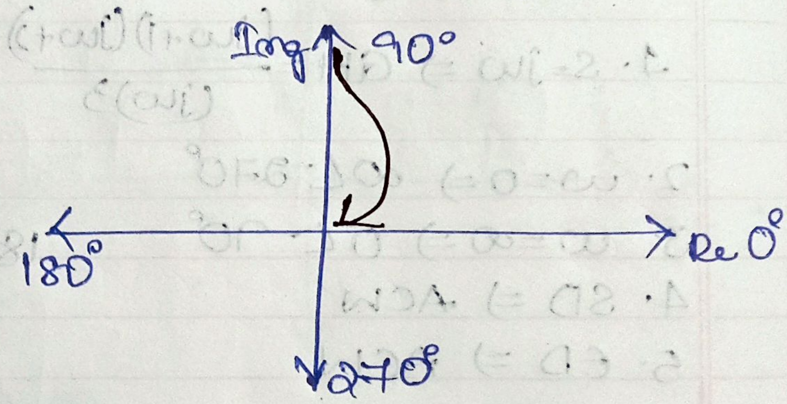
① $S = j\omega \Rightarrow GH = \dfrac{1}{j\omega(-j\omega-1)}$

② $\omega = 0 \Rightarrow \infty\angle -270$

③ $\omega = \infty \Rightarrow 0\angle -360°$

④ SD → not required

⑤ ED → CW



Q] $GH = \dfrac{1}{S(-S+1)}$ → IV, Quad.

$\angle\phi = -90 - (-\tan^{-1}(\omega))$
$= -90° + \tan^{-1}(\omega)$

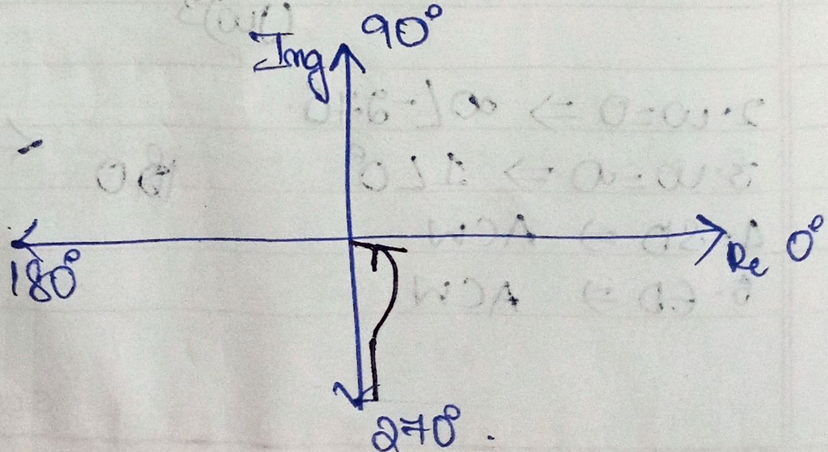
① $S = j\omega \Rightarrow \dfrac{1}{j\omega(-j\omega+1)}$

② $\omega = 0 \Rightarrow \infty\angle -90°$

③ $\omega = \infty \Rightarrow 0\angle 0°$

④ SD → not req.

⑤ ED → ACW



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Q] $GH = \frac{S+2}{(S+1)(S-1)}$

$$\angle\phi = 90 - [\cancel{90} + 180 - \tan^{-1}(\omega)] \quad (-\tan^{-1}(\omega))$$
$$= \cancel{90 - 90} - 180 + \tan^{-1}(\omega)$$
$$= \cancel{180 + \tan^{-1}(\omega)}$$
$$= -90 - 2\tan^{-1}(\omega)$$

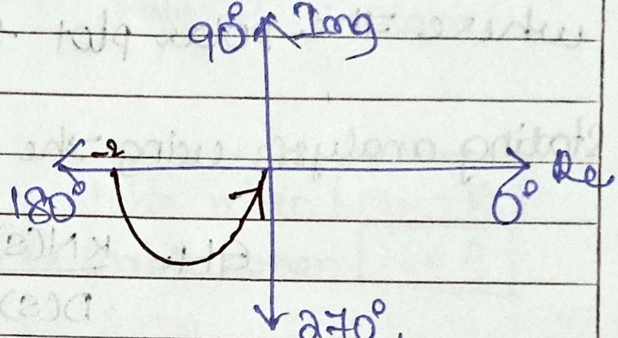
$$\angle\phi = \tan^{-1}(\omega/2) - \tan^{-1}(\omega) - (180 - \tan^{-1}(\omega))$$
$$= -180 + \tan^{-1}(\omega/2)$$

② $\omega = 0 \Rightarrow -2\angle -180^\circ$

③ $\omega = \infty \Rightarrow 0\angle -90^\circ$

④ SD ⇒ not req.

⑤ ED ⇒ ACW



Q] $GH = \frac{S}{S+1}$

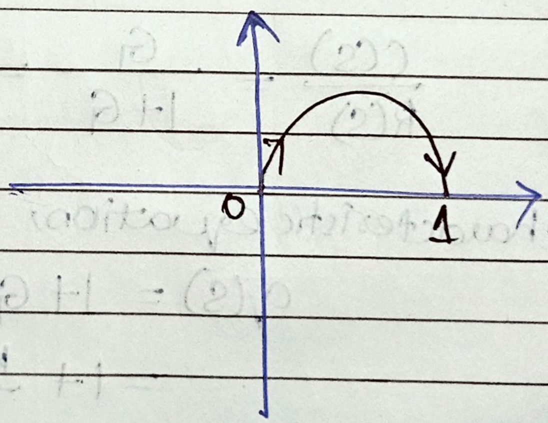
① $S = j\omega \Rightarrow GH = \frac{j\omega}{j\omega + 1}$ $\quad \angle\phi = 90 - \tan^{-1}(\omega)$

② $\omega = 0 \Rightarrow 0\angle 90^\circ$

③ $\omega = \infty \Rightarrow 1\angle 0^\circ$

④ SD ⇒ CW

⑤ ~~ACW~~ ED ⇒ CW



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Rules for Drawing Nyquist plot!

1. Draw the polar plot.
2. Draw the mirror image of polar plot
3. The no. of infinite radius half circles will be equal to no. of poles/zeros at origin.
4. The infinite radius half circle will start at the point where the mirror image of polar plot ends and the infinite radius half circle at the point
5. where the polar plot starts.

Q) Stating analysis using the Nyquist plot.

$$GH = \frac{KN(S)}{D(S)} \quad -① \quad \text{(openloop T/F)}$$

Assume it is a unity gain system.

The closed loop Transfer function of the system.

$$\frac{C(S)}{R(S)} = \frac{G}{1+G} = \frac{G(S)\,D(S)}{D(S)+K\,N(S)} \quad -②$$

characteristic equation,

$$q(S) = 1+G$$

$$= 1+\frac{KN(S)}{D(S)}$$

$$= \frac{D(S)+K(N(S)}{D(S)}$$

$\Rightarrow$ poles of CE = zero's of CLS

lly with zeros.

Poles Of CE = poles of OL T/F

~~poles~~ zeros of CE = poles of C.L. T/F

Nyquist stability criteria:

$$N = P - Z$$

N = no. of encirclements of the point $-1+j0$

P = poles of CE (poles of O.L T/F) in the right half of S-plane.

Z = Zeros of CE (pole of CL T/F) in the right half of S-plane.

Note:

1. The OL system is said to be stable when $\boxed{N = -Z}$
2. The CL system is said to be stable when $\boxed{N = P}$

Q] $GH = \frac{10}{S+5}$. Draw Nyquist plot

1. Draw polar plot.

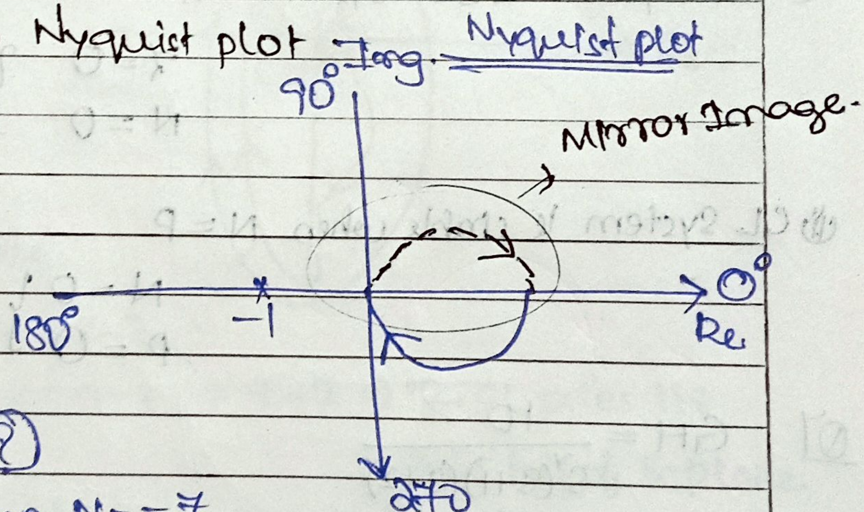
$\omega = 0 \Rightarrow 2\angle 0^\circ$

$\omega = \infty \Rightarrow 0\angle -90^\circ$

SD ⇒ CW

ED ⇒ CW

→ stability - ?



(i) OL system is stable when $N = -Z$

$CE = \frac{G}{1+GH}$

$\left.\begin{matrix} Z = 0 \\ N = 0 \end{matrix}\right\}$ OL system is stable

(as plot is not touching $-1+j0$)

(ii) CL system $N = P$

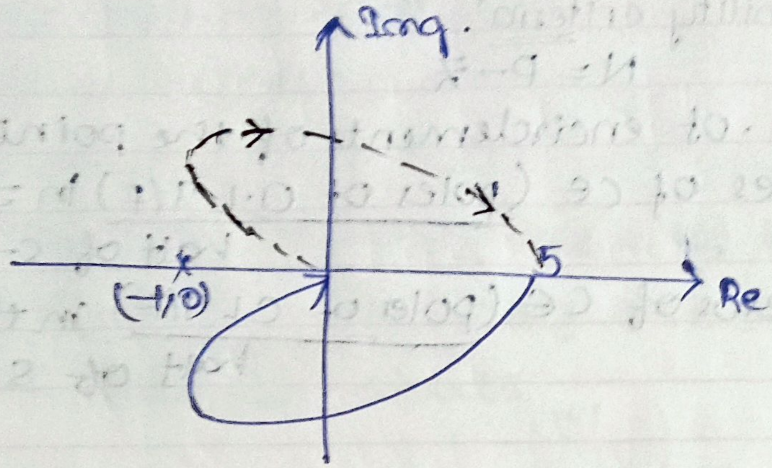
$\left.\begin{matrix} P = 0 \\ N = 0 \end{matrix}\right\}$ CL system is stable.

Signature ....

Q] $GH = \frac{10}{(s+1)(s+2)}$

① $\omega = 0 \Rightarrow 5\angle 0^\circ$; $\omega = \infty \Rightarrow 0\angle -180^\circ$.

②
Take mirroring.



Nyquist plot.

(i) OL system is stable when $N = -Z$

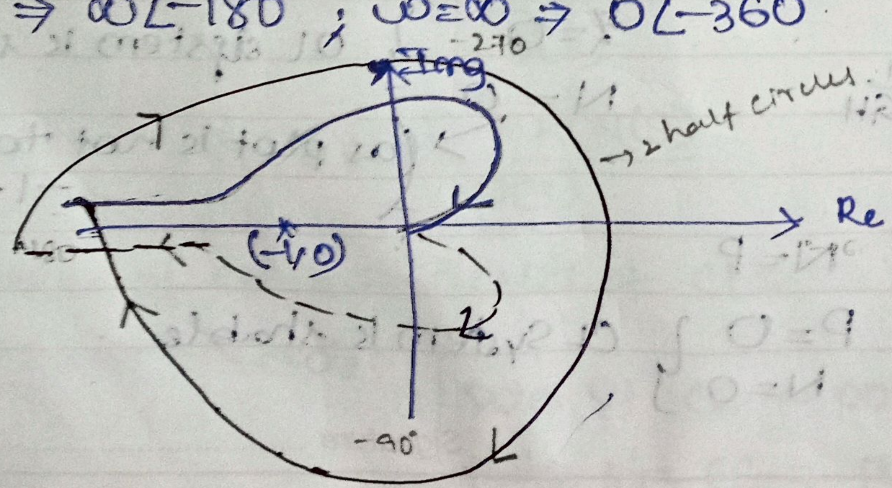
$Z = 0$
$N = 0$ } OL is stable.

(ii) CL system is stable when $N = P$

$N = 0$
$P = 0$ } CL system is stable.

Q] $GH = \frac{10}{s^2(s+1)(s+2)}$

① $\omega = 0 \Rightarrow \infty\angle -180^\circ$; $\omega = \infty \Rightarrow 0\angle -360^\circ$



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(i) OL loop system stable $N = -Z$

$N = -2$

$Z =$

(ii) CL system → $N = P$

$\left.\begin{array}{l} P = 0 \\ N = -2 \end{array}\right\}$ Unstable.

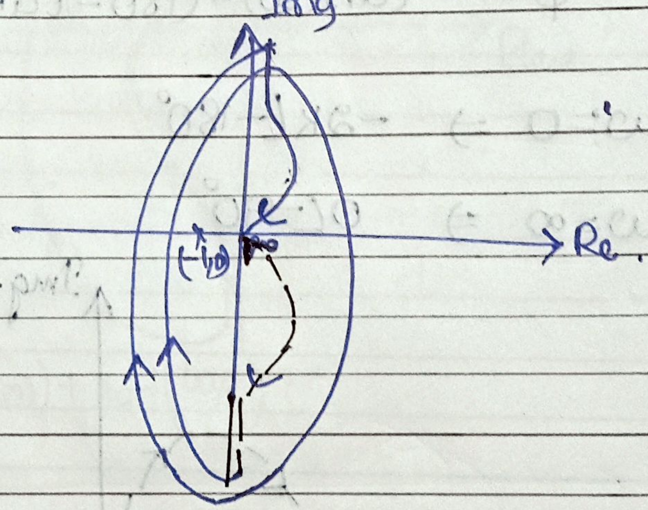
→ -ve because the circles are CW

Q] $GH = \dfrac{1}{S^3(S+1)}$

1. $\omega = 0 \Rightarrow \infty\angle -270$

$\omega = \infty \Rightarrow 0\angle -360$

→ 3 half circles.



CL system stable when $N = P$

$\left.\begin{array}{l} N = -2 \\ P = 0 \end{array}\right\}$ unstable.

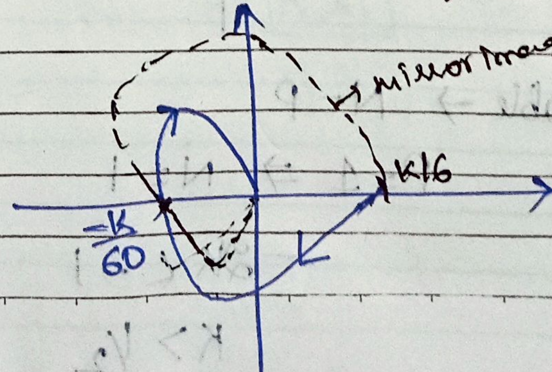
$N = P - Z$

$-2 = 0 - Z \Rightarrow Z = 2 \Rightarrow$ 2 CL poles lis In Right half of S-plane

Q. $GH = \dfrac{K}{(S+1)(S+2)(S+3)}$, find 'K' value ⇒ OL system is stable. for which CL system is stable.

$\omega = 0 \rightarrow \dfrac{K}{6}\angle 0°$

$\omega = \infty \quad 0\angle -270°$



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CL system stability, $N = P$

$P = 0 \Rightarrow N = 0$

$-\frac{K}{60} > -1 \Rightarrow \boxed{K < 60}$

and $\quad -\frac{K}{6} > -1 \Rightarrow \boxed{K > -6}$

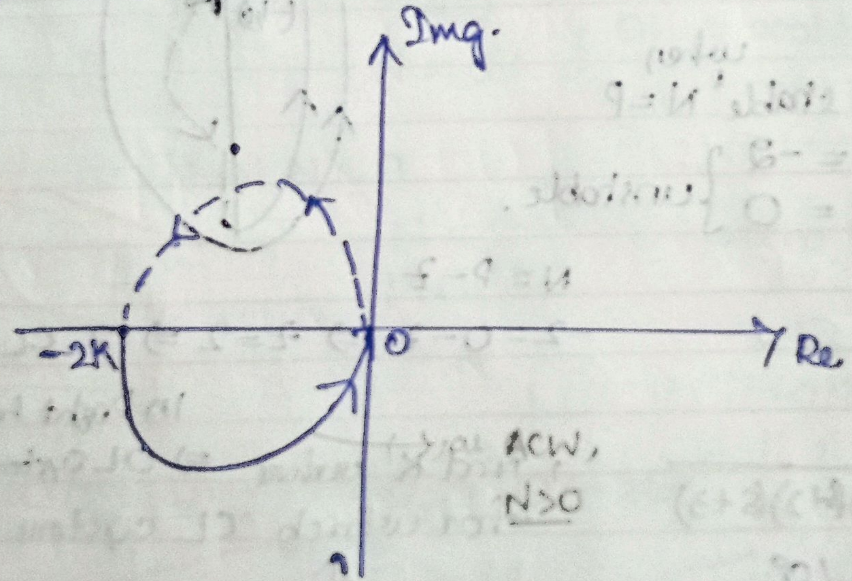
$\therefore \underline{-6 < K < 60}$

Q] $GH = \frac{K(S+2)}{(S+1)(S-1)} \rightarrow K = ?$, CL sys. is stable.

$\phi = -\tan^{-1}(\omega) - (180 - \tan^{-1}(\omega)) + \tan^{-1}\left(\frac{\omega}{2}\right)$

$\omega = 0 \Rightarrow -2K\angle -180^\circ$

$\omega = \infty \Rightarrow 0\angle -90^\circ$



CL system stable $\rightarrow N = P$

$P = 1 \Rightarrow N = 1$

$-2K < -1$

$K > 1/2$

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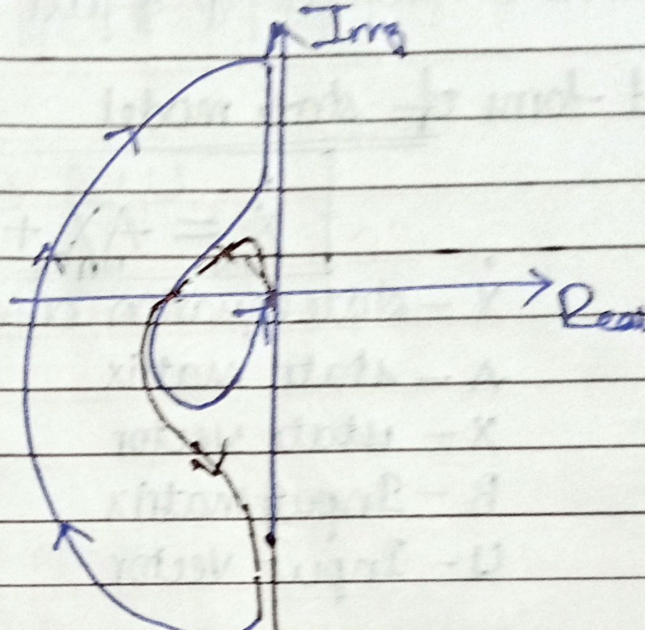
Q] $GH = \frac{K(s+3)}{s(s-1)}$

$\phi = -90 - (180 - \tan^{-1}(\omega)) + \tan^{-1}(\omega/2)$

$= -270 + \tan^{-1}(\omega) + \tan^{-1}(\omega/3)$

$\omega = 0 \Rightarrow \infty\angle -270^\circ$

$\omega = \infty \Rightarrow 0\angle -90^\circ$



point at which it cuts Re-axis can be found by taking $\phi = 180^\circ$, finding 'ω' and substituting magnitude at that ω.

$180 = -90 - (180 - \tan^{-1}(\omega)) + \tan^{-1}(\omega/3)$

$\Rightarrow \boxed{\omega = \sqrt{3}}$

$M\Big|_{\omega=\sqrt{3}} = K$ and CL stability : $N = P$

as $P = 1 \Rightarrow \boxed{N = 1}$

$-K < -1 \rightarrow$ System is stable.

$\Rightarrow \boxed{K > 1}$

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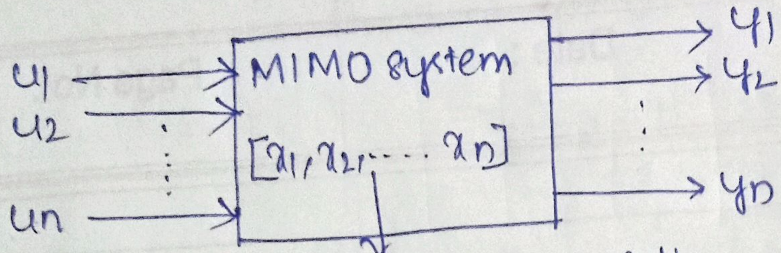
State Space - Analysis: MIMO - Multiple Input Multiple Output



MIMO system is described by state variable.

O/p depends on present i/p & past O/p's

standard form of state model

$$\boxed{\dot{X}_{n\times 1} = A_{n\times n} X_{n\times 1} + B_{n\times m} U_{m\times 1}}$$

$\dot{X}$ - state equation or differential state vector

A - state matrix

X - state vector

B - Input matrix

U - Input vector

$$\boxed{Y_{p\times 1} = C_{p\times n} X_{n\times 1} + D_{p\times m} U_{m\times 1}}$$

Y - O/p vector

C - O/p matrix

D - Transition matrix

Q) $\dfrac{d^2y}{dt^2} + 10\dfrac{dy}{dt} + 5y = 10u(t)$

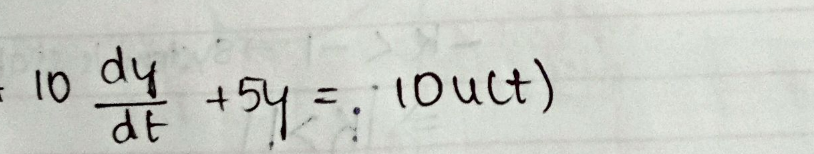
u(t) - i/p

y(t) - o/p

1 I/P 1 O/P

One I/P → m = 1

one O/P → P = 1



no. of statevariable $\xrightarrow{\text{wrt}}$ = order of (system) Dea^n.
⇒ 2 order = 2 → no. of S.V = 2

$$U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}$$

$$\dot{X}_{2\times1} = A_{2\times2} X_{2\times1} + B_{2\times1} U_{1\times1}$$

$$Y_{1\times1} = C_{1\times2} X_{2\times1} + D_{1\times1} U_{1\times1}$$

Eg: $y''' + 2y'' + 3y' + y = u$

1 i/p → m = 1

1 o/p → P = 1

order = 3 ⇒ no. of state variables = 3.
$[x_1, x_2, x_3]$

we have to express that DE in terms of State Variables.

$$\left.\begin{aligned} x_1 &= y \\ \dot{x}_1 &= \dot{y} = x_2 \\ \dot{x}_2 &= \ddot{y} = x_3 \\ \dot{x}_3 &= \dddot{y} \end{aligned}\right\} \longrightarrow DE \Rightarrow$$

$\dot{x}_3 + 2\dot{x}_2 + 3\dot{x}_1 + x_1 = u$

⇒ $\dot{x}_3 = -2x_3 - 3x_2 - x_1 + u$

$$\dot{X} = AX + BU$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1} u$$

$$Y = CX + DU$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}_{1\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} \cancel{0} \end{bmatrix}_{1\times1} u$$

$$\underline{y = x_1}$$

Q] $y'''' + 10y''' - 6y'' + 7y' + 5y = 10\,u(t)$ $\quad m=1,\ n=4,\ p=1$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & -7 & 6 & -10 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Q] $\dfrac{y(s)}{U(s)} = \dfrac{10s+5}{s^3+6s^2+7s+8}$ $\quad \boxed{\dot{x}_n = s^n}$

S power = no. of variables

$$x_1 = s^0$$
$$x_2 = \dot{x}_1 = s$$
$$x_3 = \dot{x}_2 = s^2$$
$$\dot{x}_3 = s^3$$

$$U(s) = s^3 + 6s^2 + 7s + 8$$
$$= \dot{x}_3 + 6\dot{x}_2 + 7\dot{x}_1 + 8x_1$$
$$= \dot{x}_3 + 6x_3 + 7x_2 + 8x_1$$
$$\dot{x}_3 = -8x_1 - 7x_2 - 6x_3 + U(s)$$

$$Y(s) = 10s + 5$$
$$= 10\dot{x}_1 + 5x_1$$
$$= 10x_2 + 5x_1$$

$\to y(s)$ - o/p
$U(s)$ - i/p

n - no. of state variables.
n=3
m - no. of i/p ⇒ m=1
p - no. of o/p → P = 1

$$\dot{X}_{3\times1} = A_{3\times3} X_{3\times1} + B_{3\times1} U_{1\times1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -7 & -6 \end{bmatrix}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1} [U]$$

$$Y_{1\times1} = C_{1\times3} X_{3\times1} + D_{1\times1} U_{1\times1}$$

$$Y = \begin{bmatrix} 5 & 10 & 0 \end{bmatrix}_{1\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0][u]$$

$\dot{x}_2$ $\dot{x}_1$

x_3 x_2 x_1

Q] $$\frac{Y(s)}{U(s)} = \frac{s^2+5s+10}{s^4+3s^3+6s+5}$$ $n=4,\ m=1,\ p=1$

$$\dot{X}_{4\times1} = A_{4\times4} X_{4\times1} + B_{4\times1} U_{1\times1}$$

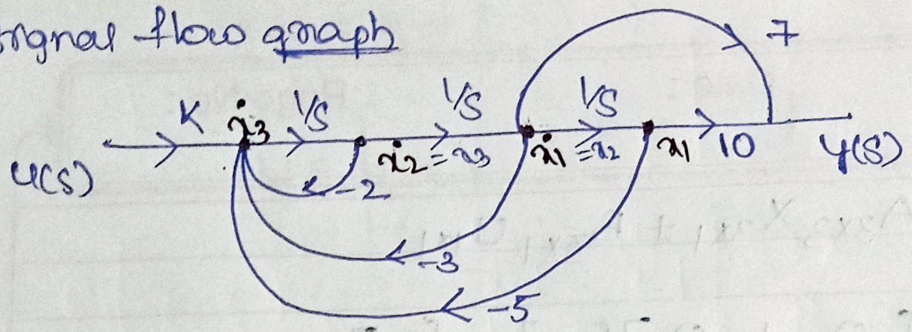
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & -6 & -3 & \cancel{-0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [u]$$

$$Y_{1\times1} = C_{1\times4} X_{4\times1} + D_{1\times1} U_{1\times1}$$

$$Y = \begin{bmatrix} 10 & 5 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0][u]$$

3 - S.V
as 3(1/s)

Signal flow graph



$\dot{x}_1 = x_2$

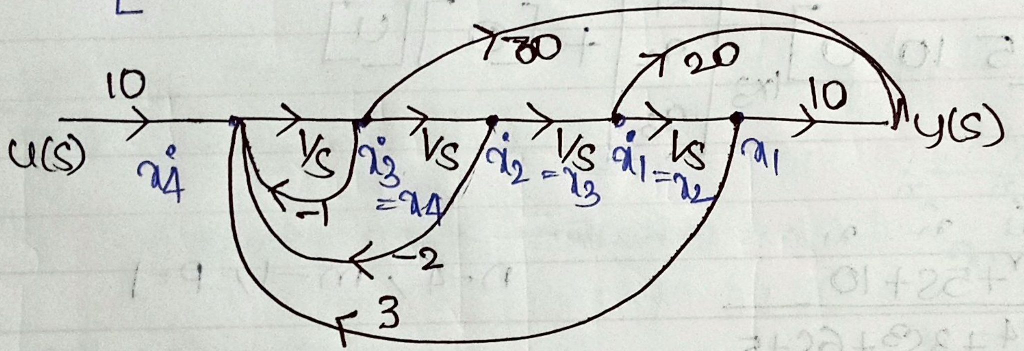
$\dot{x}_2 = x_3$

$$\dot{x}_3 = -5x_1 - 3x_2 - 2x_3 + K\,U(s)$$

$$y(s) = 10x_1 + 7x_3$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} \quad C = [10 \;\; 7 \;\; 0] \quad D = [0]$$

Q)



$$\dot{x}_4 = -x_4 - 2x_3 + 3x_1 + 10\,U(s)$$

$$Y(s) = 10x_1 + 20x_2 + x_4(30)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & -2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \quad C = [10 \;\; 20 \;\; 0 \;\; 30]$$

$$D = [0]$$

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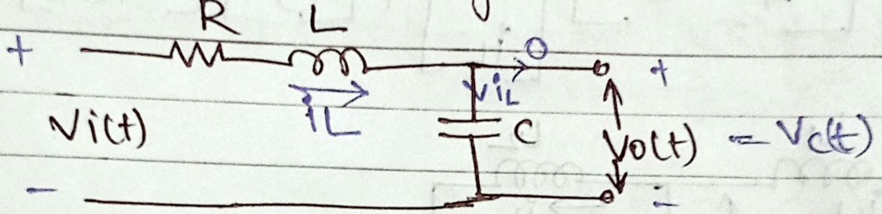
Procedure for obtaining the state eqn for electrical N/W

1. Select the state variables such that voltage across capacitor and current through inductor.
2. Write the independent KCL and KVL equations for the Electrical Network
3. The resultant eqn must consist of state variables, differential state variable, i/p and o/p Variables

Q] ① Obtain state model for given Electrical N/W



no. of SV = no. of (Inductors + Capacitors)
= 2

⇒ SV are, $V_C(t)$ and $i_L(t)$

$$i_L = C\frac{dV_C}{dt} \quad -①$$

KVL in loop →

$$V_i(t) - i_L(R) - L\frac{di_L}{dt} - V_C(t) = 0$$

$$\therefore V_i(t) = R\,i_L(t) + L\frac{di_L}{dt} + V_C(t)$$

let, $\frac{dV_C}{dt} = \dot{V}_C$

$\frac{di_L}{dt} = \dot{i}_L$

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$$\dot{V}_C = \frac{i_L}{C} \quad -③$$

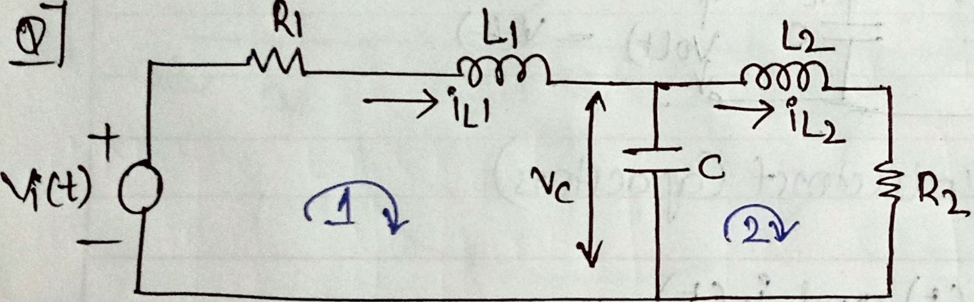
$$\dot{i}_L = \frac{V_i(t)}{L} - \frac{i_L(t) \times R}{L} - \frac{V_C(t)}{L} \quad -④$$

$$\begin{bmatrix} \dot{V}_C(t) \\ \dot{i}_L(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1/C \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}}_{A} \begin{bmatrix} V_C \\ I_L \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_{B} V_i$$

Output, $V_o(t) = V_C(t)$

$$V_o(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} V_i(t) \end{bmatrix}$$

Q]



KCL: $i_{L1} = i_{L2} + C \cdot \frac{dV_C}{dt}$

$$\dot{V}_C = \frac{1}{C}\left[i_{L1} - i_{L2}\right] \quad -①$$

KVL: Loop 1: $V_i(t) = i_{L1}R_1 + L_1\frac{di_{L1}}{dt} + V_C$

$$\dot{i}_{L1} = \frac{1}{L_1}\left[V_i(t) - V_C - i_{L1}R_1\right] \quad -②$$

Loop 2: $V_C = L_2\frac{di_{L2}}{dt} + R_2 i_{L2}$

$$\dot{i}_{L2} = \frac{1}{L_2}\left[V_C - i_{L2}R_2\right] \quad -③$$

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$$\begin{bmatrix} \dot{V}_c \\ \dot{i}_{L1} \\ \dot{i}_{L2} \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L_1 & -R/L_1 & 0 \\ 1/L_2 & 0 & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} V_C \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_1 \\ 0 \end{bmatrix} V_i[S]$$

$\Rightarrow \dot{X} = Ax + Bu$

Laplace Transform $\Rightarrow$ $SX(S) = AX(S) + BU(S)$

$X(S) = (SI-A)^{-1} BU(S)$ — (1)

$\Rightarrow Y = CX + DU$

Laplace Transform $\Rightarrow$ $Y(S) = CX(S) + DU(S)$

$= C(SI-A)^{-1}BU(S) + DU(S)$

$$\frac{Y(S)}{U(S)} = C(SI-A)^{-1}B + D$$

Q) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [u]$$

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\dot{X} = AX + BU$, $Y = CX + DU$

$\Rightarrow D = 0.$

$$\frac{Y(S)}{U(S)} = C[SI-A]^{-1}B + D$$

$$SI-A = [S]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} S-2 & -3 \\ 4 & S+2 \end{bmatrix}$$

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$$[SI - A]^{-1} = \frac{1}{(s^2-4)+12}\begin{bmatrix} s+2 & 3 \\ -4 & s-2 \end{bmatrix}$$

$$C[SI-A]^{-1}B = \underset{1\times 2}{[1 \quad 1]}\underset{2\times 2}{\begin{bmatrix} s+2 & 3 \\ -4 & s-2 \end{bmatrix}}\underset{2\times 1}{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} \times \frac{1}{(s^2-4)+12}$$

$$[s-2 \quad s+1] = \begin{bmatrix} s+2+s-2 \\ -4+s-2 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} \times \frac{1}{(s^2-4)+12}$$

$$= [3s-6+5s+5] \times \frac{1}{s^2+8}$$

$$= \frac{8s-1}{s^2+8}$$

$$s^2 = -8 \; ; \; s = \pm j8.$$

(i) Nature: Undamped system

(ii) Stability: Marginally stable.

Natural response | Free response | System impulse response

ZIR - Zero input response

ZSR - Zero state response / forced respon

Solution to the state equation:

$$x(t) = \underbrace{L^{-1}\left[(SI-A)^{-1}x(0)\right]}_{ZIR} + \underbrace{L^{-1}\left[(SI-A)^{-1}BU(s)\right]}_{ZSR}$$

$$= e^{At}x(0) + \int_0^t e^{A(t-\tau)}BU(\tau)\,d\tau$$

$e^{At} \rightarrow$ state transition Matrix

$$\phi(t) = e^{At}$$

$$= L^{-1}\left[(SI-A)^{-1}\right]$$

$$\phi(s) = (SI-A)^{-1} \Longleftarrow$$

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$ZSR = L^{-1}[\phi(S) B U(S)]$

$= \int_0^t \phi(t-\tau) B U(\tau) d\tau$

Properties of state-transition Matrix (STM)

1. $\phi(0) = I$

2. $\phi^K(t) = (e^{At})^K = e^{A(Kt)} = \phi(Kt)$

3. $\phi^{-1}(t) = \phi(-t)$

4. $\phi(t_1 + t_2) = \phi(t_1) \cdot \phi(t_2)$

5. $\phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0)$

Q] $\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} X$ where $X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $Y = \underbrace{[1, -1]}_{c} X$

$\phi(t) = L^{-1}[(SI - A)^{-1}]$

$= \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$

TPO

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Q $\phi(t) = L^{-1}[(SI-A)^{-1}]$

$$= L^{-1}\left\{\begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}\right\} = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}_{2\times 1}$$

$$x(t) = \phi(t)x(0)$$

$$= \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$y = Cx$$

$$= [+1\ -1]\,x = \frac{3}{\sqrt{2}}\sin\sqrt{2}t$$

Q $\dot{x} = \underbrace{\begin{bmatrix} 0 & -1 \\ +2 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 5 \end{bmatrix}}_{B} u; \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad y(t) = \underbrace{[0\ \ 1]}_{C}\, x(t)$

$$\phi(t) = L^{-1}[(SI-A)^{-1}]$$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ +2 & -3 \end{bmatrix} = \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$s^2 + 3s + 2$$
$$s^2 + 2s + s + 2$$
$$s(s+2) + (s+2)$$
$$(s+2)(s+1)$$

$$(SI-A)^{-1} = \frac{1}{s(s+3)+2}\begin{bmatrix} s+3 & -1 \\ +2 & s \end{bmatrix}$$ Laplace $\Rightarrow$ Transform^{-1} $$\left\{\begin{bmatrix} \frac{s+3}{(s+2)(s+1)} & \frac{-1}{(s+2)(s+1)} \\ \frac{2}{(s+2)(s+1)} & \frac{s}{(s+2)(s+1)} \end{bmatrix}\right\}$$

$$\phi(t) = L^{-1}\begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$\rightarrow$ verify $\phi(0) = 1$

ZIR $x(t) = \phi(t)\cdot x(0) = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$

ZSR $= L^{-1}[\phi(s) B U(s)] = \begin{bmatrix} 2.5 - 5e^{-t} + 2.5e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$

$x(t) = ZIR + ZSR$

$y(t) = Cx$

$= [0 \ 1]\, x(t) = 3e^{-t} - 3e^{-2t}$

Controllability:

A system is said to be controllable if it is possible to transfer the initial state to desired state in a finite time interval by the controllable i/p.

n - no. of state variables.

$$Q_c = \begin{bmatrix} B & AB & A^2B \ldots & A^{n-1}B \end{bmatrix}$$ → Kalman's test.

$|Q_c| \neq 0 \Rightarrow$ system is controllable.

Q) Check the controllability of the Transfer fn. $T/F = \frac{1}{S^3 + 2S^2 + 3S + 4}$

$n = 3$

$$A_{3\times3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}_{3\times3} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1}$$

$$Q_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$|Q_c| \neq 0$

$\Rightarrow$ controllable.

Observability:

A system is said to be observable, if it is possible to determine the initial states of the system by observing the O/p in finite time interval.

$$Q_O = [C^T \quad A^T C^T \quad (A^T)^2 C^T \dots (A^T)^{n-1} C^T]$$

$|Q_O| \neq 0 \rightarrow$ system is observable. C^T – C Transpose.

Q) $\dot{X} = \underbrace{\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{B} u \; ; \; Y = \underbrace{[1 \quad 1]}_{C} X$

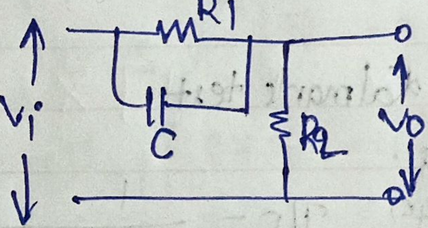
$|Q_O| = 0 \rightarrow$ System is not observable.

Compensators:

Redesigning the system by use of external device is called as compensation of system.

The external physical device which is used for redesigning the system is known as compensator.

① **Lead Compensator:** → called High pass filter.



$$\frac{V_o(s)}{V_{in}(s)} = \frac{R_2}{R_2 + (R_1 \parallel 1/Cs)}$$

$$= \frac{\left[s + \frac{1}{CR_1}\right]}{\left[s + \frac{1}{\left(\frac{R_2}{R_1+R_2}\right) CR_1}\right]}$$

$0 < \alpha \leq 1$

let $\alpha = \frac{R_2}{R_1 + R_2}$

$\tau = R_1 C$

$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(s\tau + 1)}{\alpha s\tau + 1}$$

pole: $s = \frac{-1}{\alpha\tau}$ and zero: $s = -1/\tau$