National Institute of Technology Andhra Pradesh
Department of Electronics and Communication Engineering
EC 202: Signals and Systems
Assignment 2

1. Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]\,x[n-2]\,.$$

   (a) Is the system memoryless?

   (b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number. Is the system Invertible.

2. Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t))$$

.

   (a) Is this system causal?

   (b) Is this system linear?

3. Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by where n_0 is a finite positive integer.

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

   (a) Is this system linear?

   (b) Is this system time-invariant?

   (c) If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B and n_0 .

4. For each of the following input-output relationships, determine whether the corresponding system is linear, time invariant or both.

   (a) $y(t) = t^2 x(t-1)$

   (b) $y[n] = x^2[n-2]$

   (c) $y[n] = x[n+1] - x[n-1]$

   (d) $y(t) = \text{odd}\{x(t)\}$

5. A continuous-time signal $x(t)$ is shown in Figure 1. Sketch and label carefully each of the following signals:

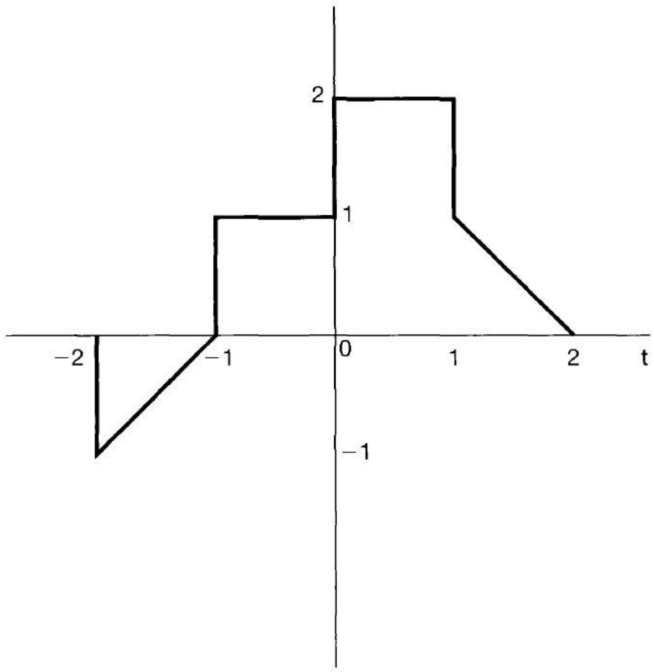

Figure 1

(a) $x(t-1)$
(b) $x(2-t)$
(c) $x(2t+1)$
(d) $x(4-\frac{t}{2})$
(e) $[x(t)+x(-t)]u(t)$
(f) $x(t)[\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})]$

6. A continuous-time signal $x(t)$ is shown in Figure 2. Sketch and label carefully each of the following signals:

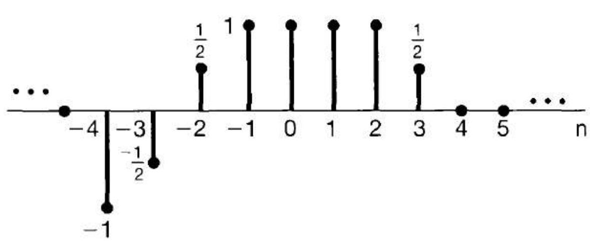

Figure 2

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(a) $x[n-4]$

(b) $x[3-n]$

(c) $x[3n]$

(d) $x[3n+1]$

(e) $x[n]u[3-n]$

(f) $x[n-2]\delta[n-2]$

(g) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$

(h) $x[(n-1)^2]$

7. Determine and sketch the even and odd parts of the signals depicted in Figure 3. Label your sketches carefully.

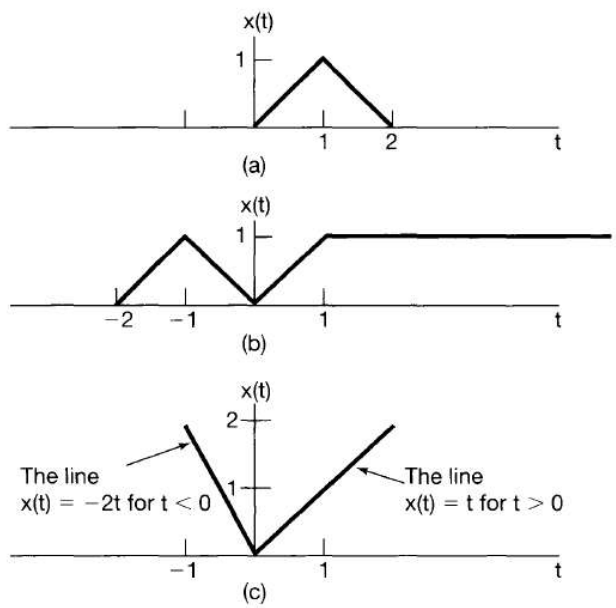

Figure 3

8. Determine and sketch the even and odd parts of the signals depicted in Figure 4. Label your sketches carefully.

9. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$

(b) $x(t) = e^{j(\pi t - 1)}$

(c) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$

(d) $x(t) = Even\,\{\cos(4\pi t)u(t)\}$

(e) $x(t) = Even\,\{\sin(4\pi t)u(t)\}$

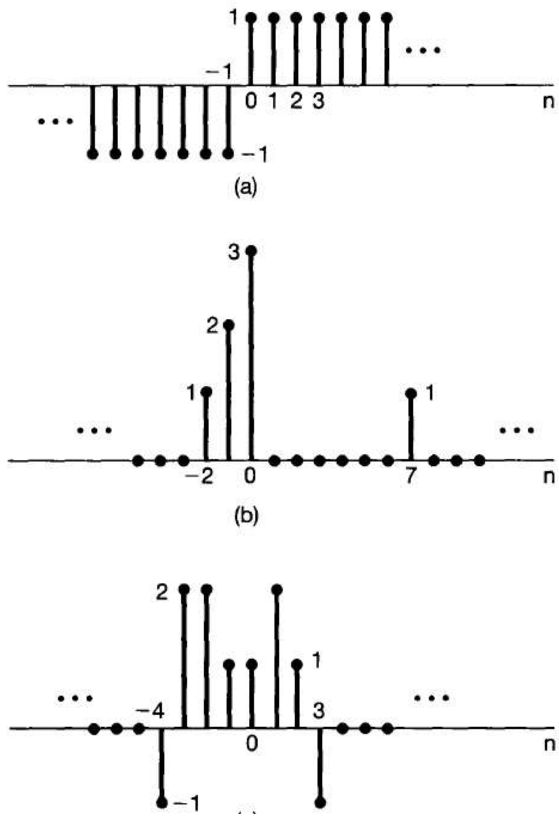

Figure 4

(f) $x(t) = \sum_{-\infty}^{\infty} e^{-(2t-1)} u(2t-1)$

10. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

    (a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$

    (b) $x[n] = \sin(\frac{n}{8} - \pi)$

    (c) $x[n] = \sin(\frac{\pi}{8}n^2)$

    (d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$

    (e) $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

11. In our previous lectures, we introduced a number of general properties of systems. In particular, a system may or may not be
    (1) Memoryless
    (2) Time invariant
    (3) Linear
    (4) Causal
    (S) Stable
    Determine which of these properties hold and which do not hold for each of the following

continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-2) + x(2-t)$

(b) $y(t) = [\cos(3t)]x(t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

(d) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

(e) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

(f) $y(t) = x(\frac{t}{3})$

(g) $y(t) = \frac{dx(t)}{dt}$

12. Determine which of the properties listed in Problem 11 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, y[n] denotes the system output and x[n] is the system input.

(a) $y[n] = x[-n]$

(b) $y[n] = x[n-2] - 2x[n-8]$

(c) $y[n] = nx[n]$

(d) $y[n] = Even\{x[n-1]\}$

(e) $y[n] = \begin{cases} x[n], & n > 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$

(f) $y[n] = \begin{cases} x[n], & n > 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

(g) $y[n] = x[4n+1]$

13. Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have the same output.

(a) $y(t) = x(t-4)$

(b) $y(t) = \cos[x(t)]$

(c) $y[n] = nx[n]$

(d) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(e) $y[n] = \begin{cases} x[n-1], & n > 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

(f) $y[n] = x[n]x[n-1]$

(g) $y[n] = x[1-n]$

(h) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau) d\tau$

(i) $y[n] = \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^{n-k} x[k]$

(j) $y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$

(k) $y(t) = x(2t)$

(l) $y[n] = x[2n]$

(m) $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$