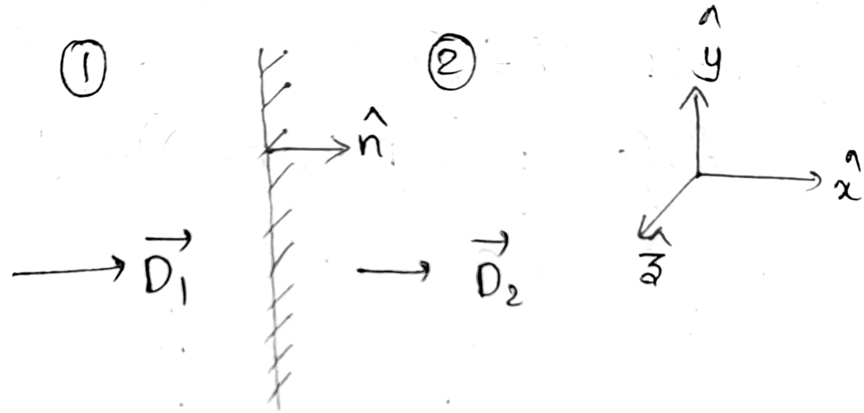
EC254:

16/02/2022

$$\hat{n}\cdot(D_2 - D_1) = \rho_s$$

$\hat{n} \rightarrow$ normal vector in direction moving from medium ① towards medium ②

Case-I:



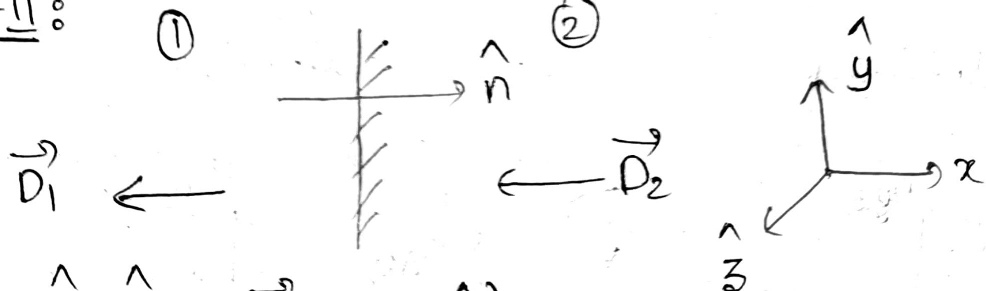
$\hat{n} = \hat{x}$ (+ve x direction)

$$\hat{n}\cdot(D_2 - D_1) = \rho_s$$

$$\hat{x}\cdot(D_2\hat{x} - D_1\hat{x}) = \rho_s$$

$$\boxed{D_2 - D_1 = \rho_s}$$

Case-II:



$\hat{n} = \hat{x}$

$\vec{D_1} = D_1(-\hat{x})$

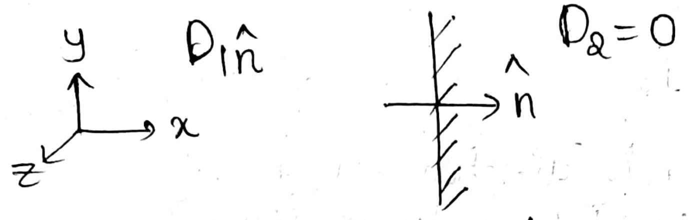
$\vec{D_2} = D_2(-\hat{x})$

$$\hat{n}\cdot(D_2 - D_1) = \rho_s$$

$$\hat{x}\,(-D_2\hat{x} + D_1\hat{x}) = \rho_s$$

$$\boxed{D_1 - D_2 = \rho_s}$$

Problem:



ρ_s b/w medium ① & ② $= 10\ c/m^2$

Sol:

$\rho_s = 10\ c/m^2$

$\hat{n} = \hat{x}$, $\vec{D_2} = 0$, $\vec{D_1} = D_{1x}\hat{x} + D_{1y}\hat{y} + D_{1z}\hat{z}$

$\hat{n}\cdot(D_2 - D_1) = \rho_s$

$\hat{x}\cdot\left(0 - (D_{1x}\hat{x} + D_{1y}\hat{y} + D_{1z}\hat{z})\right) = \rho_s$

$-D_{1x} = 10$

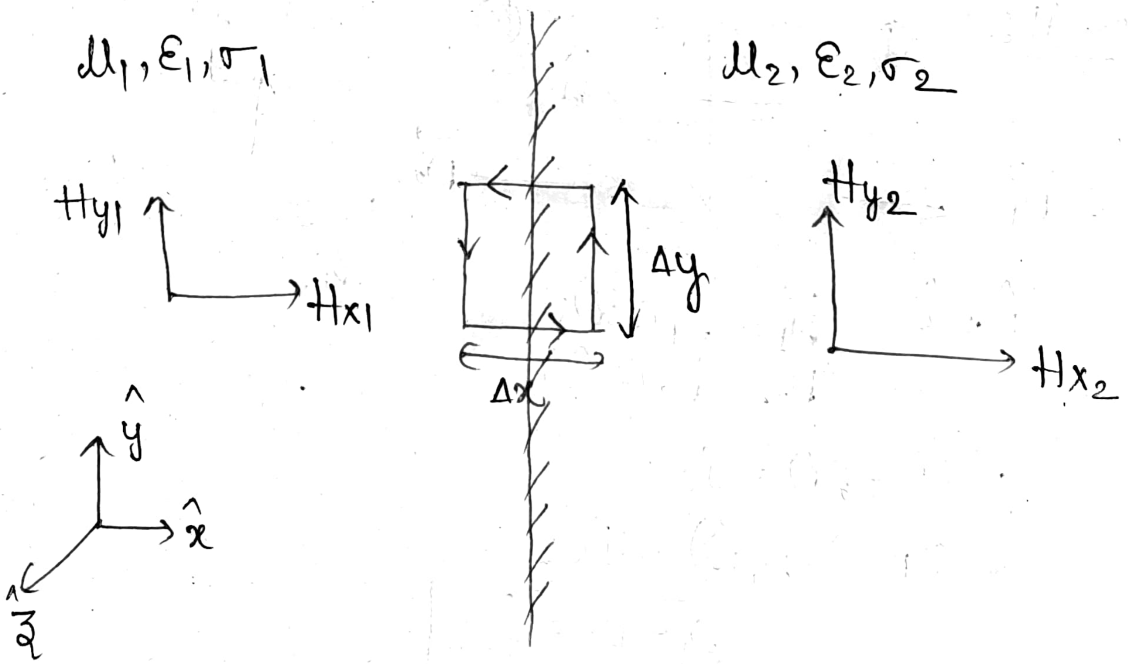
$\boxed{D_{1x} = -10\ c/m^2}$

Normal component of $D_1 = D_1\hat{x} = -10\hat{x}\ c/m^2$

II) Boundary condition - (Ampere's law)

$\oint \vec{H}\cdot\vec{dl} = I_{enclosed}$

$= \int_S \left(J_c + \frac{\partial \vec{D}}{\partial t}\right)\cdot\vec{dS}$



$$\oint_S \vec{H}\cdot\vec{dl} = \iint_S (\vec{J_c}+\vec{J_D})\cdot\vec{ds}$$

$$\oint \vec{H}\cdot\vec{dl} = H_{x1}\Delta x + H_{y2}\Delta y - H_{x2}\Delta x - H_{y1}\Delta y$$

$$\iint_S (\vec{J_c}+\vec{J_D})\cdot\vec{ds} = \left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)\cdot \Delta x\,\Delta y\,\hat{z}$$

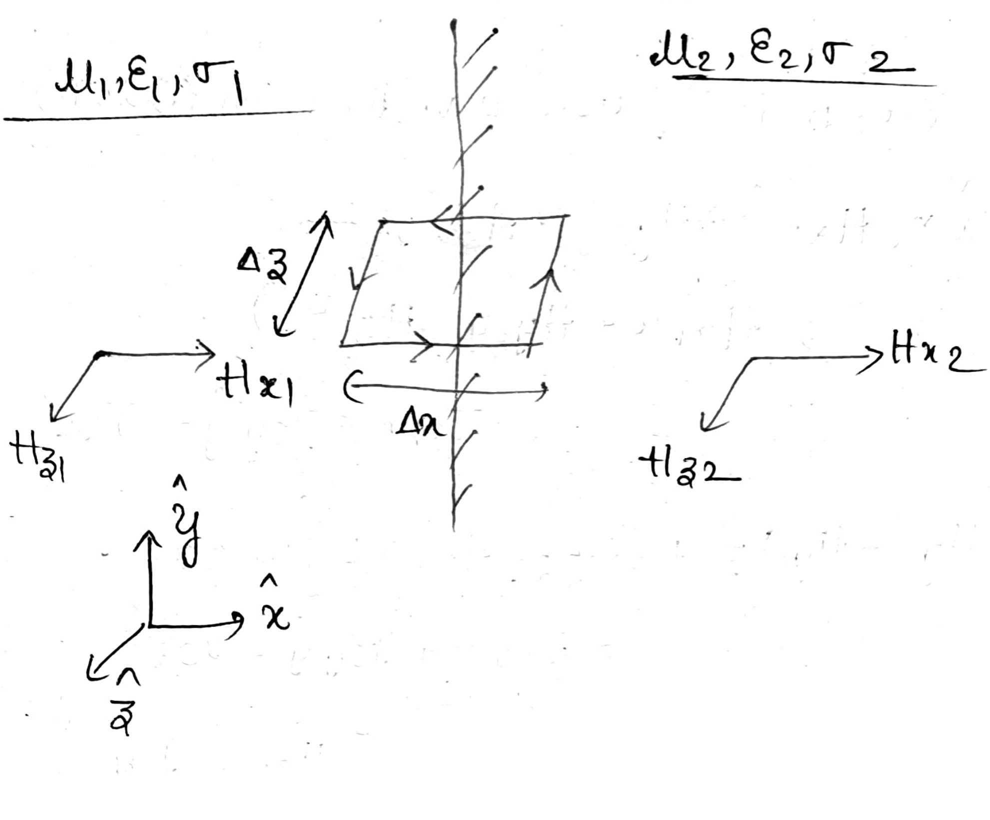
As, $\Delta x \to 0$, $\vec{J} \to \infty$

$$\underbrace{\left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)\cdot \Delta x\cdot\Delta y}\,\hat{z} = J_{sz}\cdot\Delta y$$

$J_s \to$ surface current density.

$$H_{y2}\cancel{\Delta y} - H_{y1}\cancel{\Delta y} = J_{sx}\cdot\cancel{\Delta y}$$

$$\boxed{H_{y2} - H_{y1} = J_{sz}} \longrightarrow (1)$$



$$\oint \vec{H}\cdot\vec{dl} = H_{x_1}\Delta x + H_{z_1}\Delta z - H_{x_2}\Delta x - H_{z_2}\Delta z$$

$$\iint_S (\vec{J}_c + \vec{J}_D)\cdot\vec{ds} = \underbrace{\left(\vec{J_c} + \frac{\partial \vec{D}}{\partial t}\right)}_{J_{sy}} \Delta x \cdot \Delta z\, \hat{y}$$

As $\Delta x \to 0$, $\vec{J} \to \infty$

$$(H_{z_1} - H_{z_2})\cancel{\Delta z} = J_{sy}\cdot\cancel{\Delta z}$$

$$\boxed{H_{z_1} - H_{z_2} = J_{sy}} \quad (2)$$

Across the Boundary, tangential component of $\vec{H}$ field differs by surface current density.

$$\hat{n}\times(\vec{H_2} - \vec{H_1}) = \vec{J_s}$$

$\hat{n}$ - Normal vector from medium (1) to medium (2)

In this case, $\hat{n}$ is $\hat{x}$ normal vector from (1) to (2)

$$\hat{x}\times(H_{x_2}\hat{x} + H_{y_2}\hat{y} + H_{z_2}\hat{z} - H_{x_1}\hat{x} + H_{y_1}\hat{y} + H_{z_1}\hat{z}) = J_{sx}\hat{x} + J_{sy}\hat{y} + J_{sz}\hat{z}$$

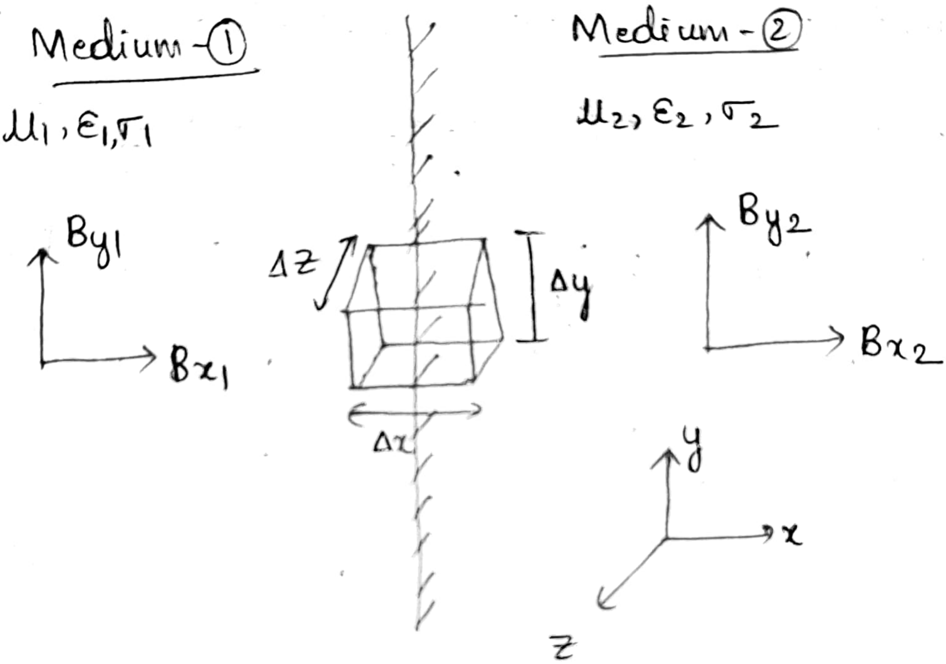
$$(H_{y_2} - H_{y_1})\hat{z} + (H_{z_2} - H_{z_1})(-\hat{y}) + 0\hat{x} = J_{sx}\hat{x} + J_{sy}\hat{y} + J_{sz}\hat{z}$$

$$\therefore \boxed{H_{y_2} - H_{y_1} = J_{sz}\ ;\ H_{z_1} - H_{z_2} = J_{sy}}$$

III) Boundary condition – (Gauss law for magnetism)

$$\oint_S \vec{B} \cdot \vec{ds} = 0$$

$$\iiint_V (\nabla \cdot \vec{B}) \cdot \vec{ds} = 0 \implies \boxed{\nabla \cdot \vec{B} = 0}$$



$$\oint \vec{B} \cdot \vec{ds} = 0$$

$$\underbrace{B_{x2} \Delta y \Delta z}_{\text{Left}} - \underbrace{B_{x1} \Delta y \Delta z}_{\text{Right}} + \underbrace{B_{y1} \Delta x \Delta z}_{\text{Top}} - \underbrace{B_{y2} \Delta x \Delta z}_{\text{Bottom}} = 0$$

As $\Delta x \to 0$

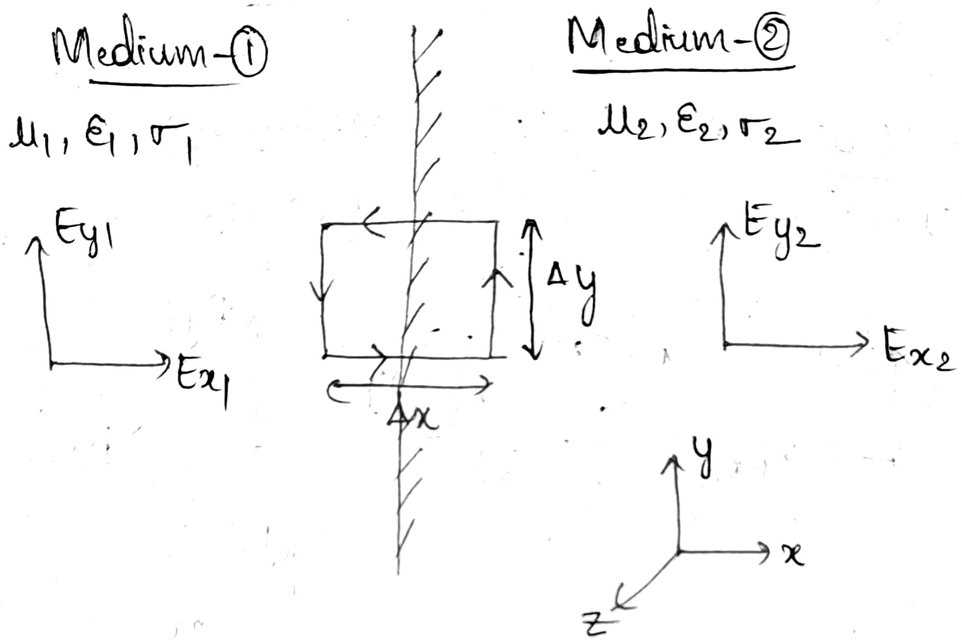
$$(B_{x2} - B_{x1}) \Delta y \Delta z = 0$$

$$\therefore \boxed{B_{x2} = B_{x1}}$$

→ Normal component of $\vec{B}$ remains continuous across the boundary.

IV) Boundary condition – (Faraday's law)

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t}\phi$$



$$\oint \vec{E} \cdot \vec{dl} = \iint_S -\frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$\oint \vec{E} \cdot \vec{dl} = E_{x1}\Delta x + E_{y2}\Delta y - E_{x2}\Delta x - E_{y1}\Delta y$$

$$\iint_S -\frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} = \frac{\partial B}{\partial t}\Delta x \cdot \Delta y \, \hat{z}$$

As, $\Delta x \to 0$

$$\oint \vec{E} \cdot \vec{dl} = (E_{y2} - E_{y1})\Delta y$$

$$\iint_S -\frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} = 0$$

$\therefore E_{y2} - E_{y1} = 0$

$\therefore \boxed{E_{y2} = E_{y1}}$ → Tangential component of $\vec{E}$ remains continuous across the Boundary

- Boundary conditions:

① $\hat{n} \cdot (\vec{D_2} - \vec{D_1}) = \rho_s$

② $\hat{n} \times (\vec{H_2} - \vec{H_1}) = J_s$

$$\left.\begin{aligned} &③\ \vec{B}_{n2} = \vec{B}_{n1} \\ &④\ \vec{E}_{t2} = \vec{E}_{t1} \end{aligned}\right\} \rightarrow \text{Universal Boundary conditions.}$$

- Perfect conductor:

→ Conductivity tends to infinity in a perfect conductor.

$(\sigma = \infty)$

$\vec{J_c} = \sigma \vec{E}$

$\vec{J_c}$ - finite value $\cancel{\sigma \neq \infty}$

$\sigma \rightarrow \infty ; \vec{E} \rightarrow 0$

E-field inside the conductor = 0

$\vec{H} = 0$ (inside the conductor)

(*)

Metal (Ideal conductor)

$\sigma = \infty$

$\overline{E_1}, \overline{H_1} = 0$

$\vec{D_1}, \vec{B_1} = 0$

Dielectric

$\mu_2, \epsilon_2, \sigma_2$

$\overline{E_2}, \overline{H_2}$

$\overline{D_2}, \overline{B_2}$

$$\vec{B_{n2}} = \vec{B_{n1}}$$

$$\therefore \boxed{\vec{B_{x1}} = 0 = \vec{B_{x2}}}$$

$$\therefore \boxed{\begin{matrix} \vec{E_{y1}} = 0 = \vec{E_{y2}} \\ \vec{E_{z1}} = 0 = \vec{E_{z2}} \end{matrix}}$$

Across metal surface,

① $\vec{E}$ field lies in normal direction

② $\vec{H}$ field lies in tangential direction.

$$\hat{n} \cdot (\bar{D_2} - \bar{D_1}) = \rho_s$$

$$\hat{x} \cdot (\bar{D_2} - 0) = \rho_s$$

$$\therefore \boxed{D_{x2} = \rho_s}$$

$$\vec{D_2} = \varepsilon_2 \vec{E_2}$$

$$\therefore \boxed{\vec{E_2} = \frac{\rho_s}{\varepsilon_2} \cdot \hat{x}}$$

(*) Surface currents are responsible for ohmic wall losses in the surface of metal.

$$\hat{n} \times (\vec{H_2} - \vec{H_1}) = \vec{J_s}$$

$$\hat{n} \times (\cancelto{0}{H_{x2}\hat{x}} + H_{y2}\hat{y} + H_{z2}\hat{z}) = J_{sx}\hat{x} + J_{sy}\hat{y} + J_{sz}\hat{z}$$

$$\hat{x} \times (H_{y2}\hat{y} + H_{z2}\hat{z})$$

$$H_{y2}\hat{z} - H_{z2}\hat{y} = \cancelto{0}{J_{sx}\hat{x}} + J_{sy}\hat{y} + J_{sz}\hat{z}$$

$$\therefore \boxed{H_{z2} = -J_{sy}} \qquad \therefore \boxed{H_{y2} = J_{sz}}$$

$$\therefore \boxed{\vec{H_2} = J_{sz}\hat{y} - J_{sy}\hat{z}}$$

$$\vec{B_2} = \mu_2 \vec{H_2}$$

$$\therefore \boxed{\vec{B_2} = \mu_2 (J_{sz}\hat{y} - J_{sy}\hat{z})}$$