ELECTRICAL RESONANCE

→ Resonance is the freq. response of a ckt / n/w when the ckt operates at its natural freq called resonance freq.

→ Under resonance total supply vltg & supply current are in phase. So, $\phi = 0^\circ$ & $PF = \cos\phi = 1$ (UPF)

→ Under resonance the net impedance of the ckt becomes purely resistive & max power will be transfered to the ckt from source

---

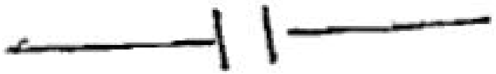
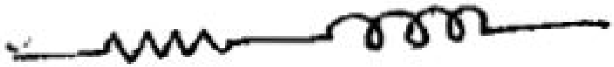
- Resonance can occur in any electrical n/w provided we have 2 similar but opposite natured energy storage components i.e. L & C.
- To undergo a good observable resonance

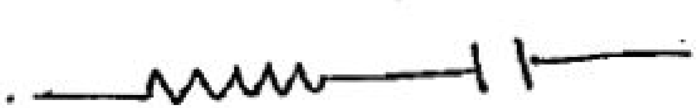
→ To undergo a good observable resonance for practical applications we need a good quality in these energy storage components which is measured as Quality factor or figure of merit given by:

$$Q\text{-factor} = 2\pi * \left[\frac{\text{Max. energy stored per cycle of supply}}{\text{Energy dissipated per cycle of supply}}\right]$$

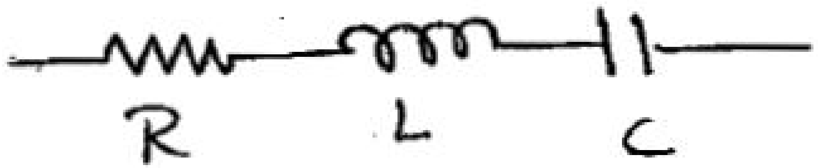
In practical applications $\boxed{Q \geq 10}$

→ Resonance phenomenon is useful in designing of passive filters, & antennas, receivers, SONARS, etc

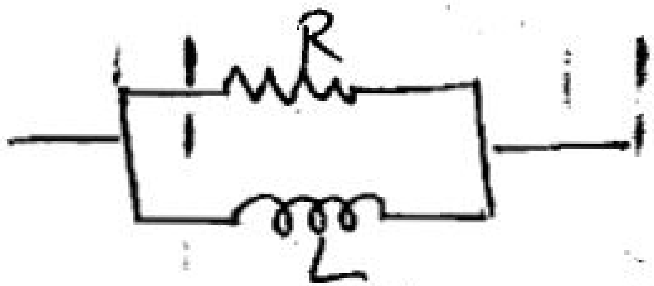
Element	Q-Factor
	0
	∞
	∞
	$\frac{\omega L}{R}$



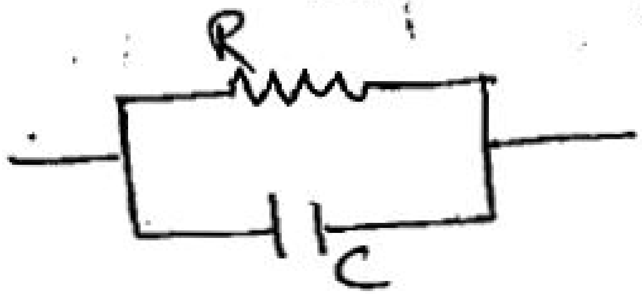
$$\frac{1}{\omega RC}$$



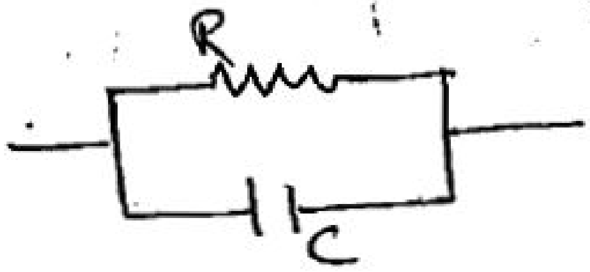
$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$



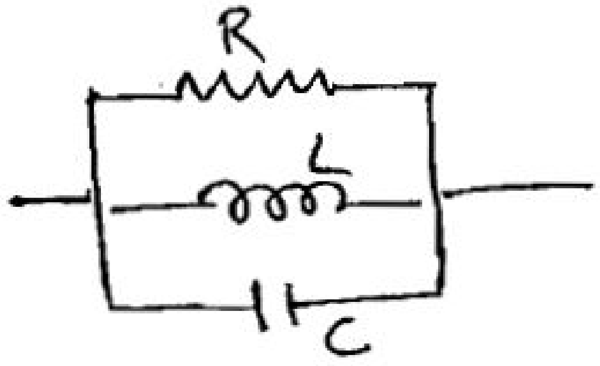
$$\frac{R}{\omega L}$$



$$\omega RC$$

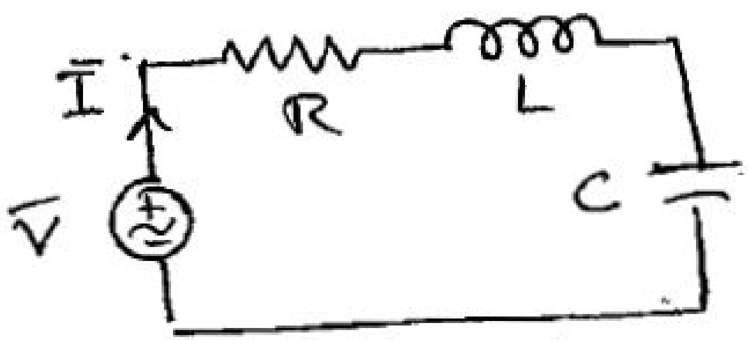


ωRC



$$Q_0 = R\sqrt{\frac{C}{L}}$$

⟨1⟩ Series Resonance :—



At resonance ; $\omega = \omega_0$

$\bar{V}$ & $\bar{I}$ in phase.

$\therefore$ $\phi = 0^\circ$; $Z = R$

But, $Z = R + j[X_L - X_C]$

but at resonance, net reactance = 0

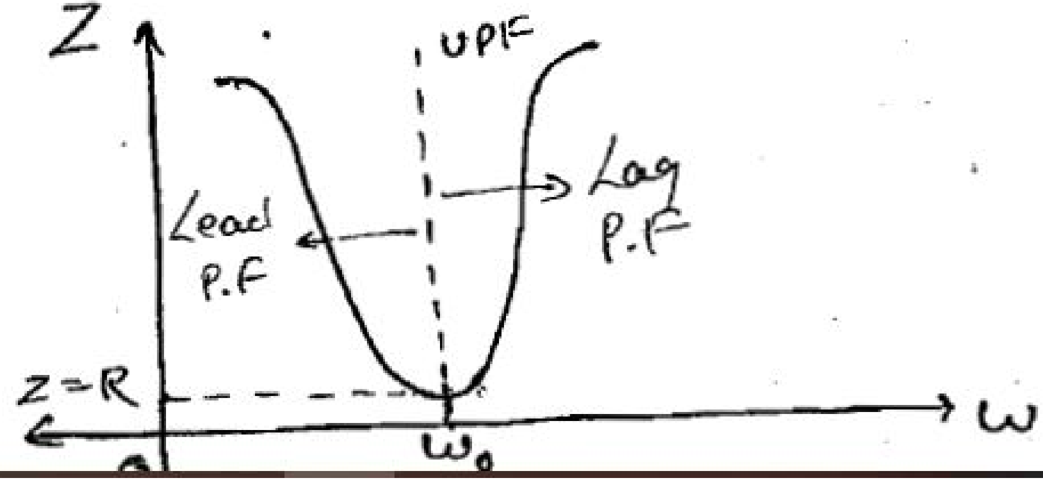
$\therefore X_L - X_C = 0 \Rightarrow X_L = X_C$

$\therefore \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad ; \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

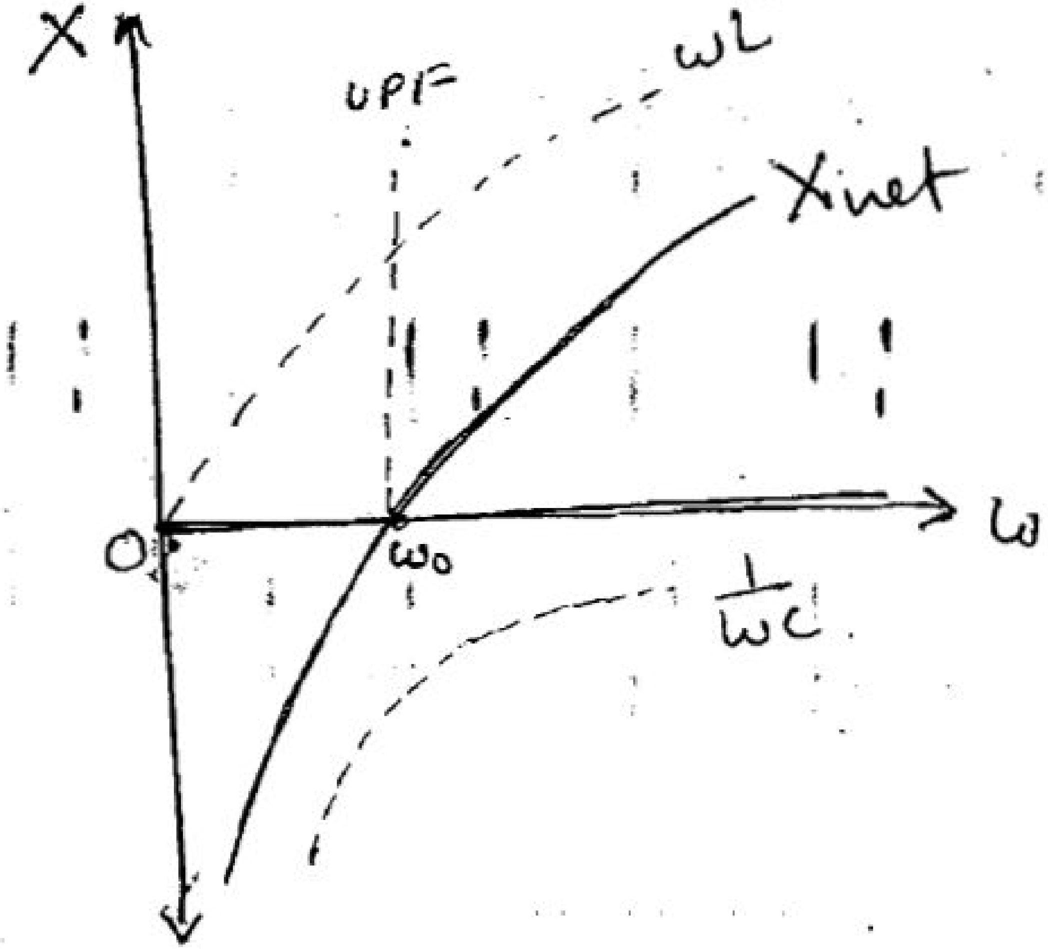
Graph (1)

Z vs ω



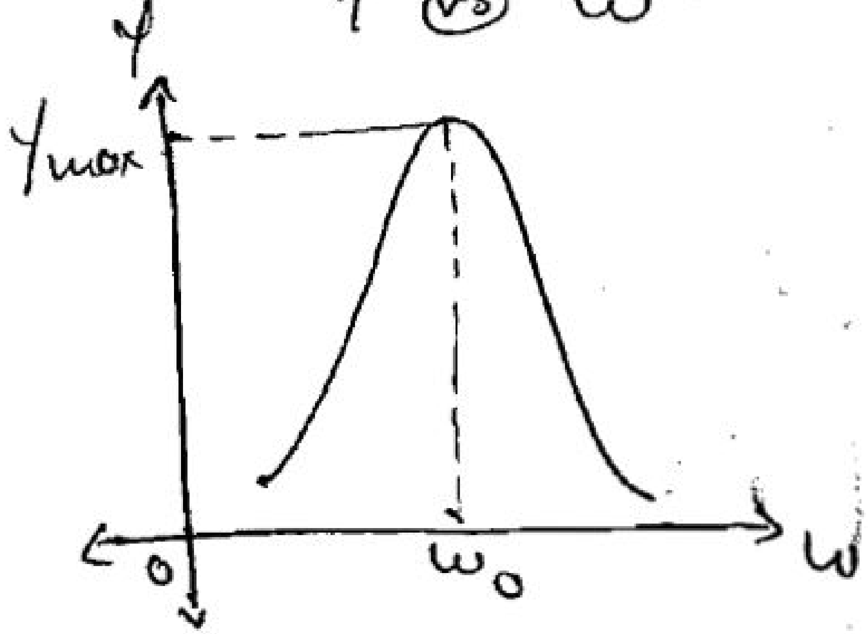
Graph ② X vs ω

$$Z = R + j\,[\underbrace{\omega L - \frac{1}{\omega C}}_{X_{net}}]$$



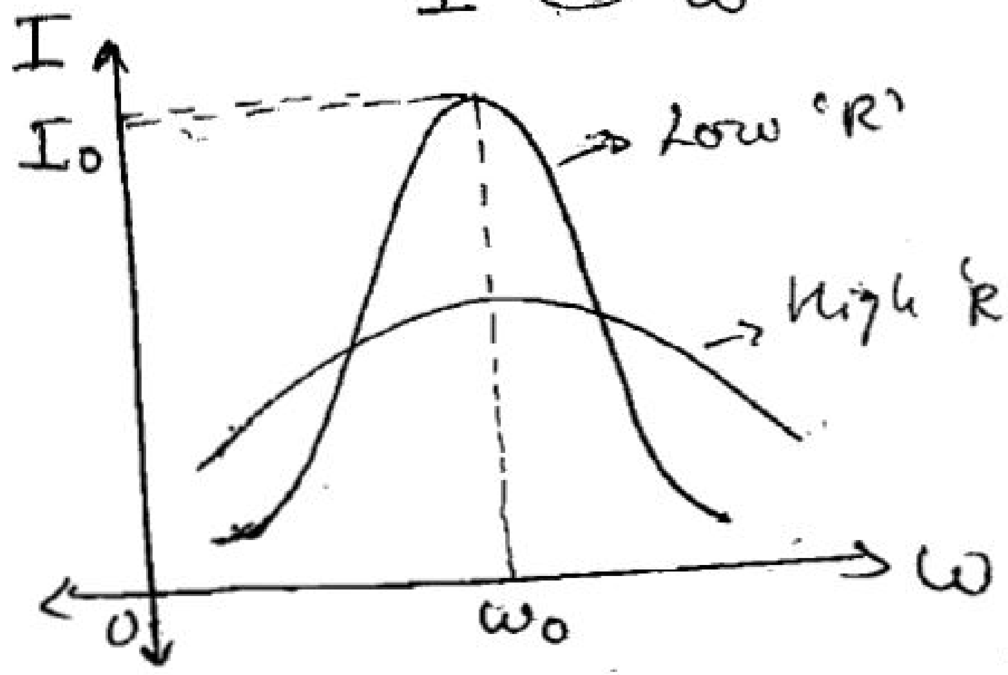
GRAPH ③

Y vs ω



Graph ④

I vs ω



Phasor diagram :-

(a) $\omega = \omega_0$

$Z = R$

(b) $\omega < \omega_0$

$Z = R - jX_{net}$

(c) $\omega > \omega_0$

$Z = R + jX_{net}$

(a) $\omega = \omega_0$

$Z = R$

↳ purely resis.

'I' in phase with 'V'

$\phi = 0^\circ$ [UPF)

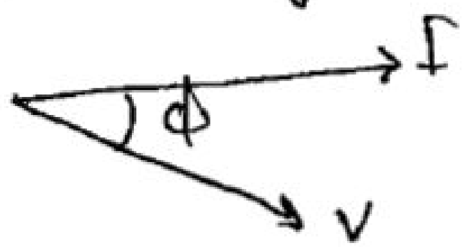
(b) $\omega < \omega_0$

$Z = R - jX_{net}$

↳ R-C ckt

'I' leads 'V' by $\phi < 90^\circ$

(leading PF)



ϕ is -ve with ~~wrt~~ respect to I (ref.)

(c) $\omega > \omega_0$

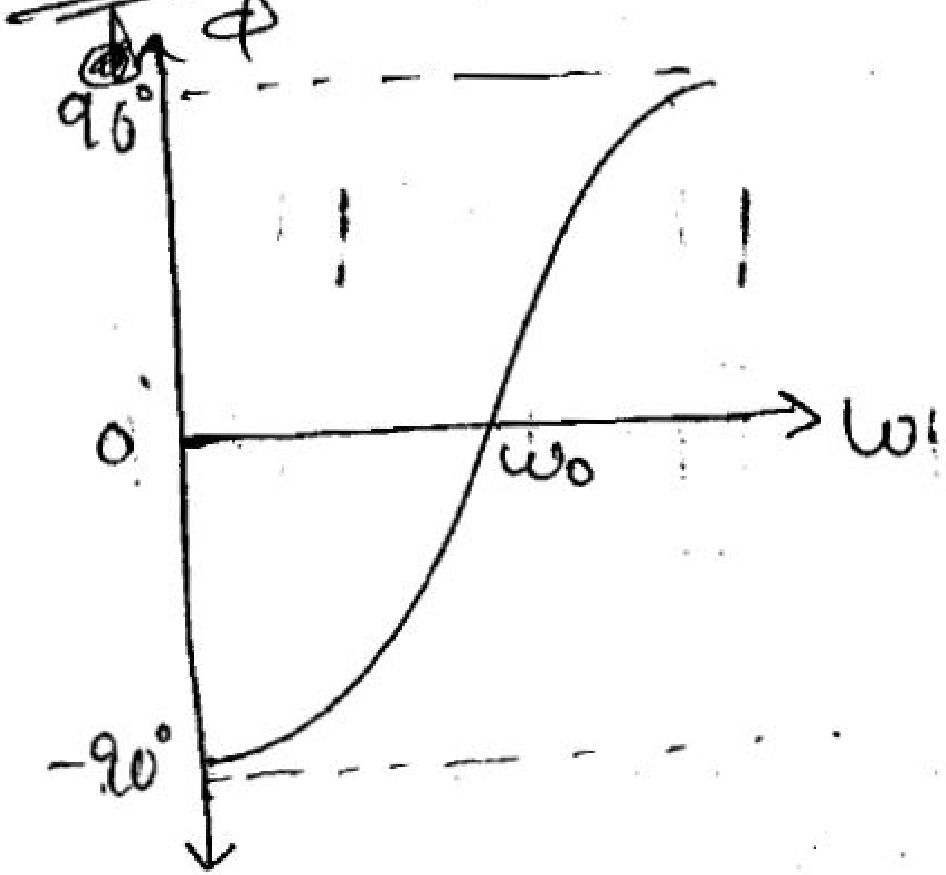
$Z = R + jX_{net}$

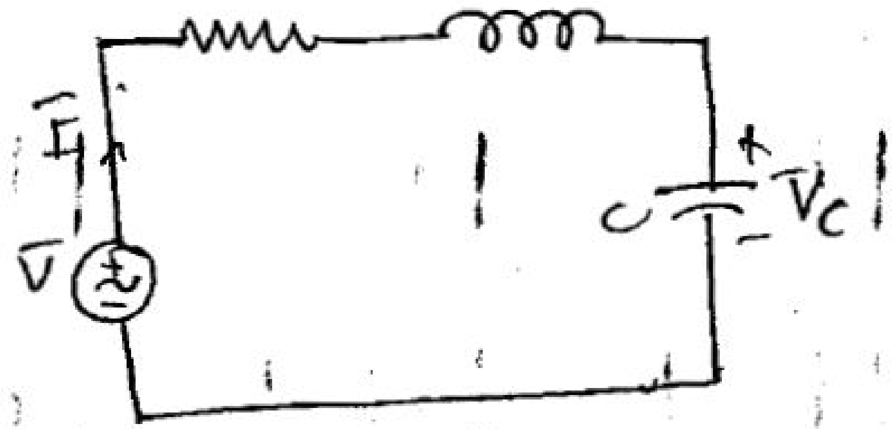
↳ R-L ckt

'I' lags 'V' by $\phi < 90^\circ$

(lagging PF)

GRAPH - 5

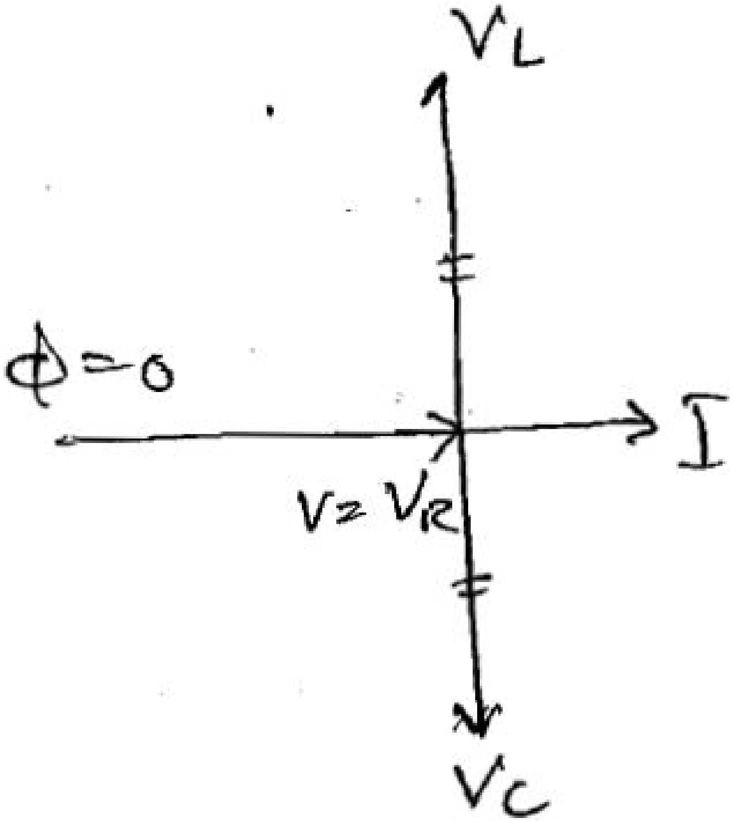




at Resonance $\omega = \omega_0$

$$|X_L| = |X_C|$$

$$|\bar{V}_L| = |\bar{V}_C|$$



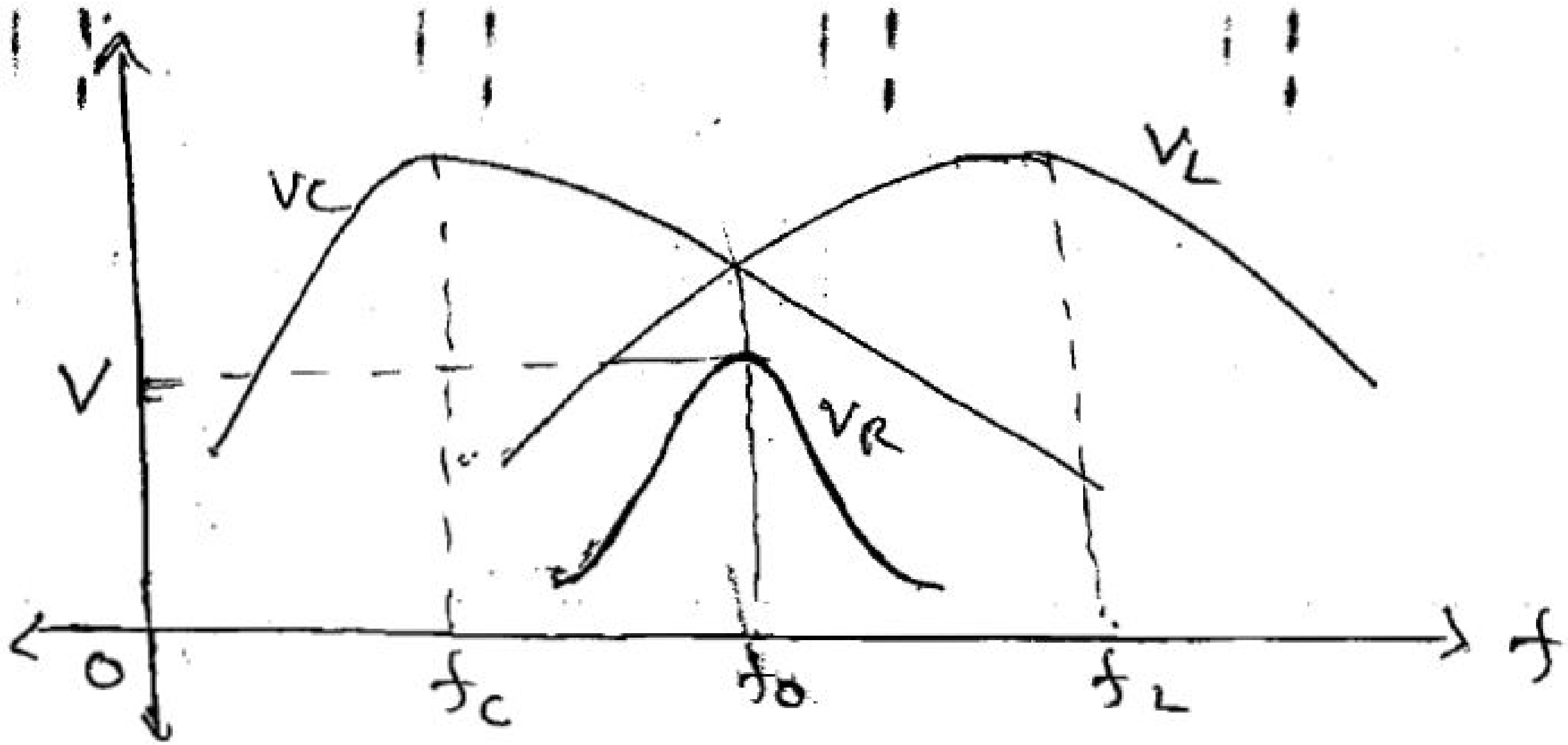
Q - Factor at Resonance.

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$

→ Under series resonance net impedance is min, so current is max. Hence it is called as acceptor ckt.

Variation of voltages across passive elements with change in freq.



The freq. at which max. voltg appears across capacitor

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{2L}}$$

$$f_C = f_0\sqrt{1 - \frac{R^2C}{2L}} \text{ Hz}$$

The freq. at which max. vltg appears across inductor:

$$f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}} = \frac{1}{2\pi\sqrt{LC\left[1 - \frac{R^2C}{2L}\right]}}$$

$$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} \text{ Hz}$$

But at $\omega = \omega_0$

$$|I|_0 = \frac{|V|}{\sqrt{R^2 + 0^2}} \Rightarrow \text{maximum}$$

So,

$$|I_0| = \frac{|V|}{R}$$

So, Power transfer also '<u>maximum</u>'

$$P_0 = I_0^2 R = \frac{|V|^2}{R} \quad W$$

Bandwidth :

$$\left.\begin{aligned} \omega_2 - \omega_1 &= \frac{R}{L} \quad \text{rad/sec} \\ f_2 - f_1 &= \frac{R}{2\pi L} \quad \text{Hz} \end{aligned}\right\}$$

B.W α R

BW is independent to 'f_0'

→ Resonance freq. is geometric mean of Bandwidth freq..

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$f_0 = \sqrt{f_1 f_2}$$

Selectivity :- (S)

→ Selectivity is the ability of a ckt/n/w to distinguish or discriminate desired & undesired freq.

→ $S \propto \frac{1}{|BW|}$

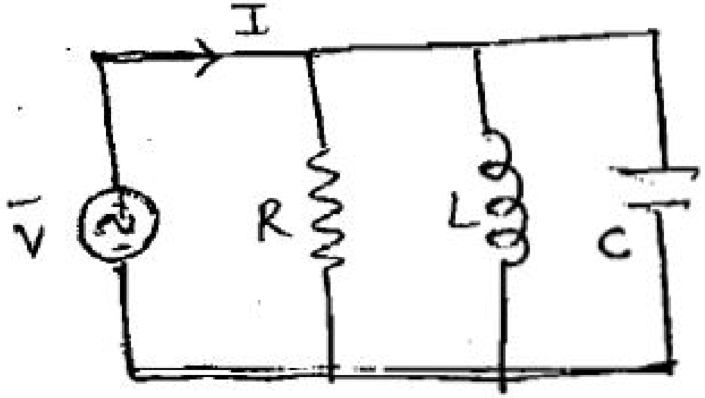
$$S = \frac{f_0}{|f_2 - f_1|} = \frac{\omega_0}{|\omega_2 - \omega_1|}$$

$$S = \left| \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi L}} \right| = \frac{1}{R}\sqrt{\frac{L}{C}} = Q! \quad !!$$

→ The value of selectivity is Q-factor under Resonance.

⟨2⟩ Parallel Resonance :—

① General ckt



At resonance
V & I (in phase)
$\phi = 0°$; PF = 1 (UPF)

$$Y_T = Y_R + Y_L + Y_C$$

$$Y_T = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} + \frac{j}{X_C} - \frac{j}{X_L} = \frac{1}{R} + j\left[\frac{1}{X_C} - \frac{1}{X_L}\right]$$

At resonance ($\omega = \omega_0$)

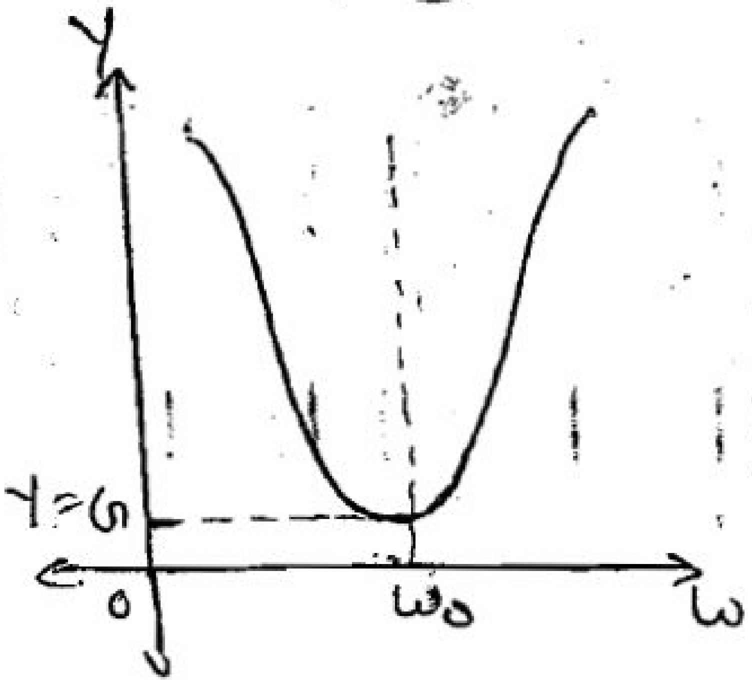
net susceptance $= 0$

$$\frac{1}{X_C} - \frac{1}{X_L} = 0 \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

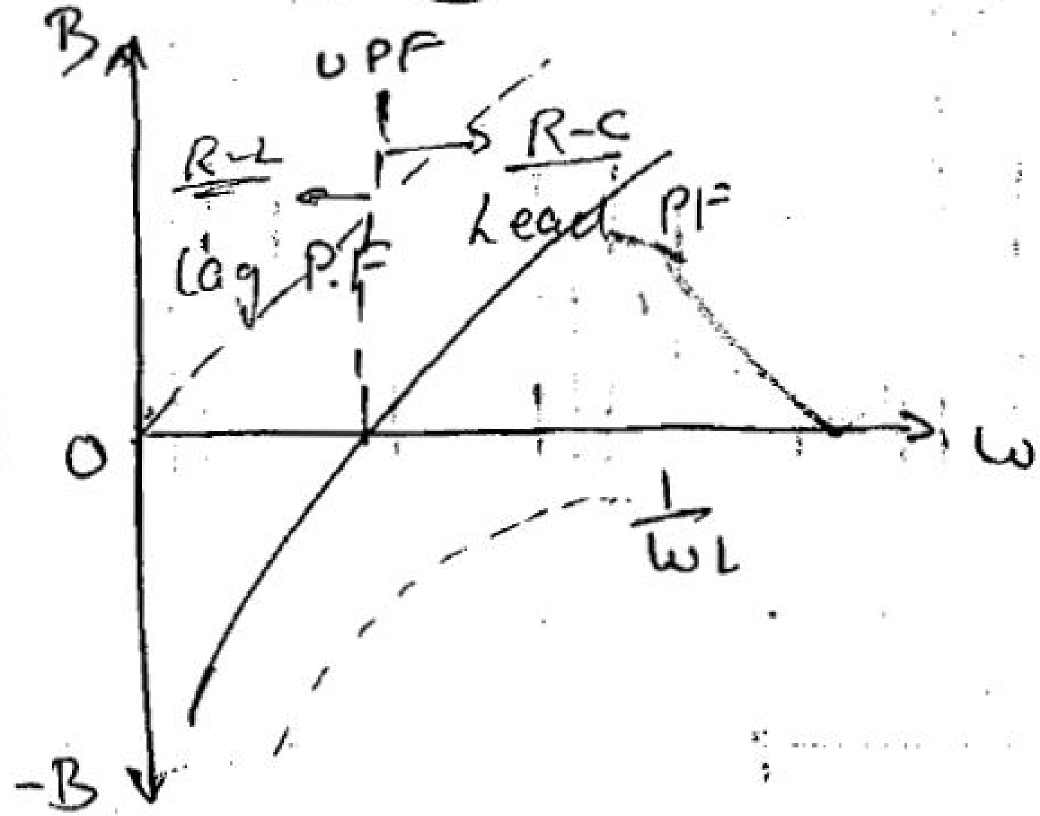
Graph ①

Y vs ω

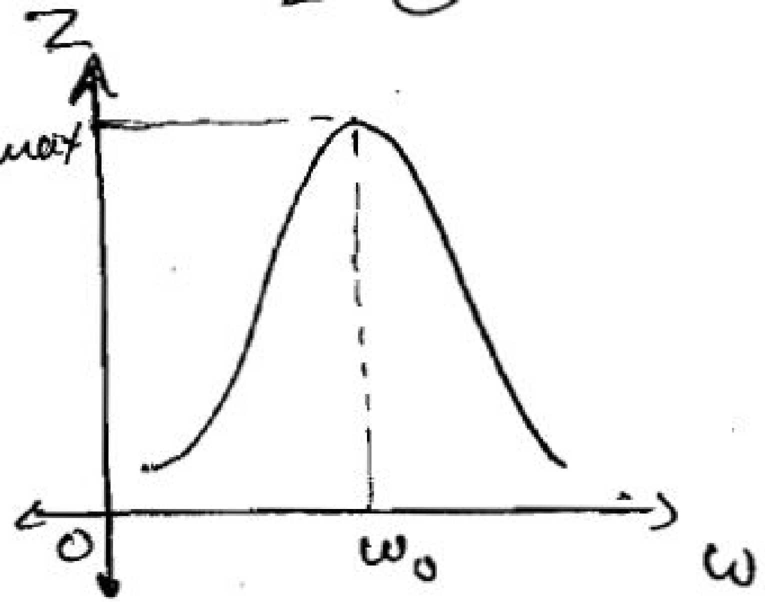


Graph ②

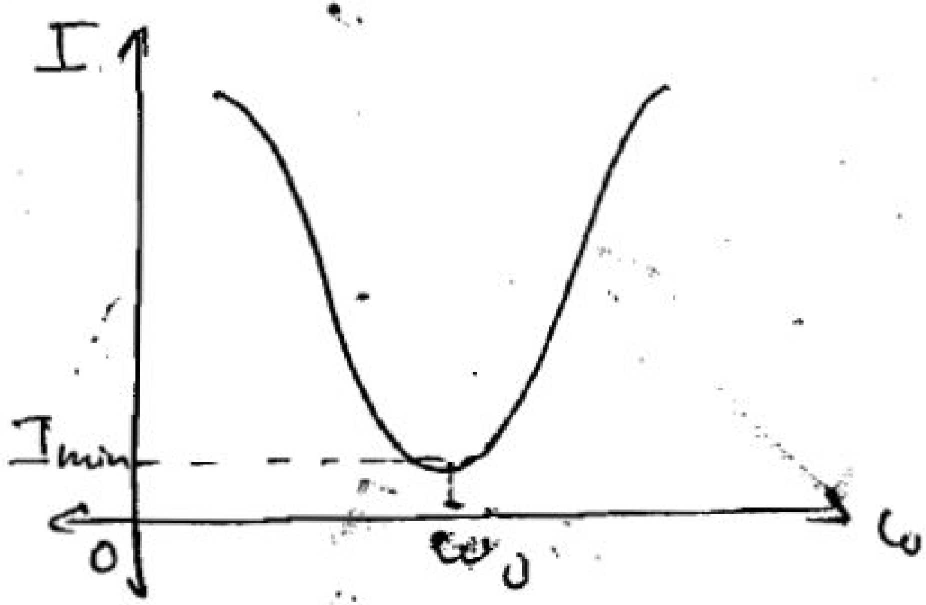
B vs ω



Graph ③ Z vs ω



Graph ④ I vs ω



Phasor diagrams :

(a) $\omega = \omega_0$

$Y = G$

↳ pure resis.

(b) $\omega < \omega_0$

$Y = G - jB_{net}$

↳ RL parallel

(c) $\omega > \omega_0$

$Y = G + jB_{net}$

↳ RC $\parallel^{el}$ ckt

Phasor diagrams :

(a) $\omega = \omega_0$

$Y = G$

↳ pure resis.

'I' in phase with 'V'.

$\phi = 0°$

PF = 1 (UPF

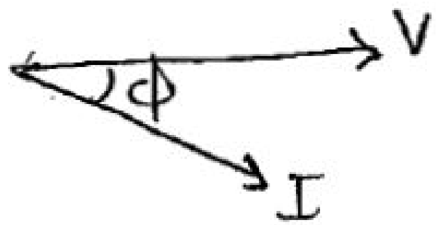
(b) $\omega < \omega_0$

$Y = G - jB_{net}$

↳ RL parallel ckt

'I' lags 'V' by $\phi < 90°$

(lagging PF)



ϕ is -ve w.r.t

(c) $\omega > \omega_0$

$Y = G + jB_{net}$

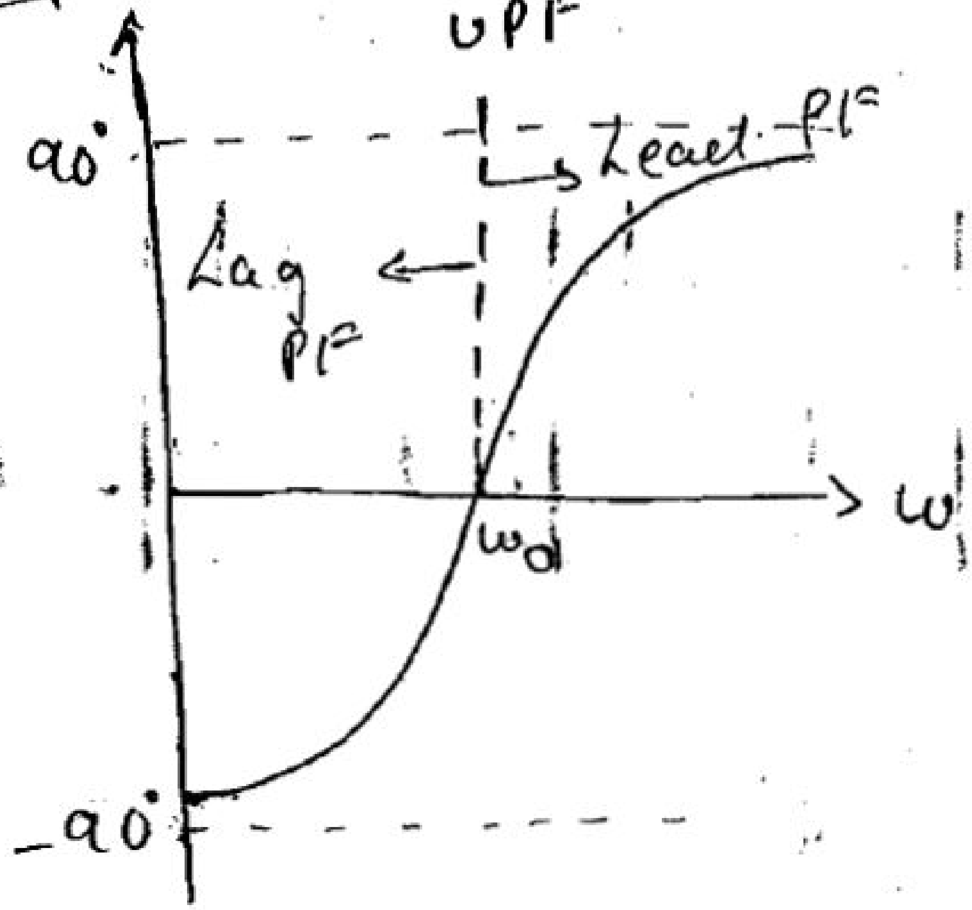
↳ RC $\parallel^{el}$ ckt

'I' leads 'V' by $\phi < 90°$

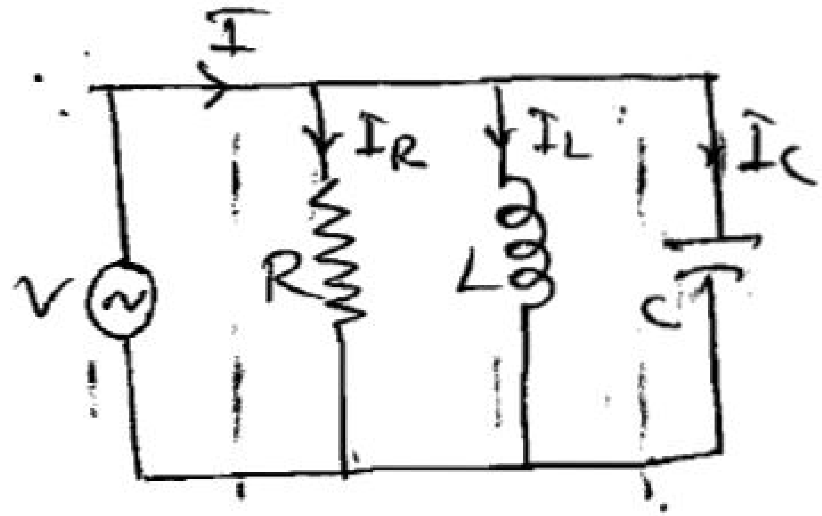
(leading PF)

ϕ is ~~+ve~~ w.r.t

Graph ⑤



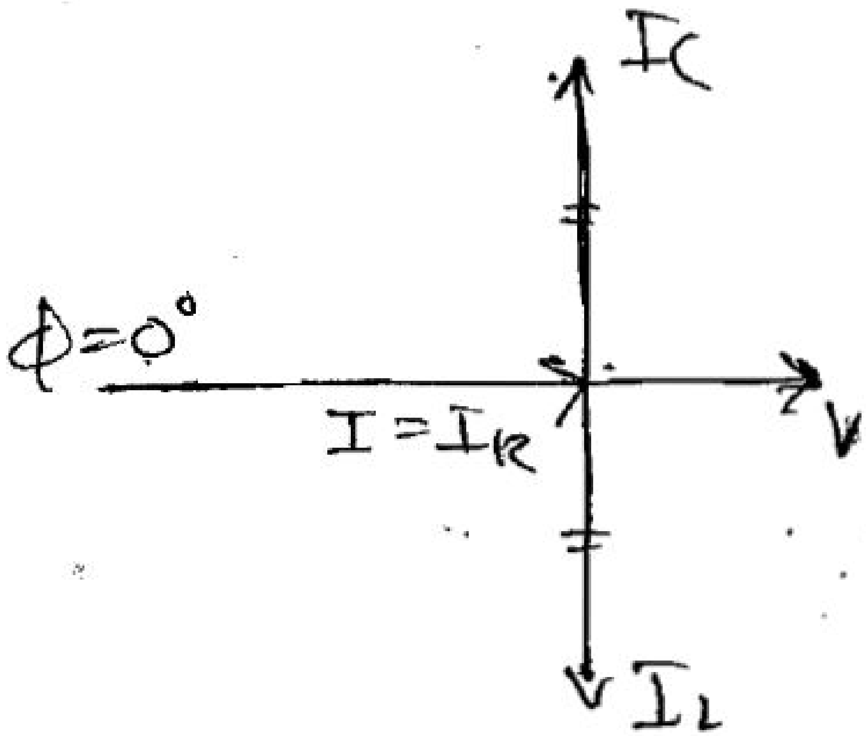
Complete phasor diag



At resonance

$$|B_L| = |B_C|$$

$$|I_L| = |I_C|$$



$I = I_R$

$\phi = 0^\circ$

$PF = 1$ (UPF)

Q-factor at Resonance

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q_0 = \frac{R}{\frac{1}{\sqrt{LC}} \times L} = \frac{1}{\sqrt{LC}} \times RC$$

$$Q_0 = R\sqrt{\frac{C}{L}}$$

At parallel resonance condition, net impedance is max. so current is minimum. Hence it is called rejector ckt.

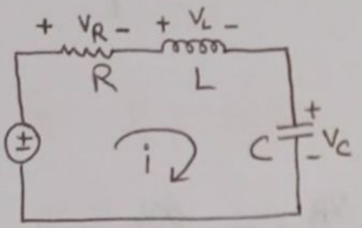
At ||el resonance freq. it is as if the total current flows only through resis. Hence it is called as current amplification circuit.

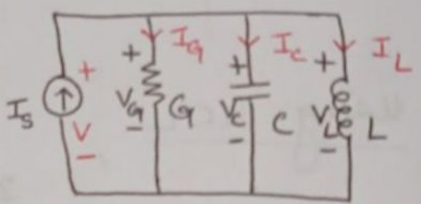
CHAPTER 2 : DUALITY & NETWORK TOPOLOGY

→ DUAL & DUALITY

Two circuits are duals of each other if the mesh equation that characterize one of them has the same mathematical form as the nodal equation that characterize the other.

Ex→





Mesh KVL

$-V_S + V_R + V_L + V_C = 0$

$V_S = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$

Nodal KCL

$-I_S + I_G + I_C + I_L = 0$

$I_S = VG + C\frac{dV}{dt} + \frac{1}{L}\int V\,dt$

Some dual elements

$V \longleftrightarrow I$

$V(t) \longleftrightarrow i(t)$

$V_m \sin \omega t \longleftrightarrow I_m \sin \omega t$

$R \longleftrightarrow G$

$L \longleftrightarrow C$

KVL ⟷ KCL

Series ⟷ Parallel

Mesh ⟷ Node

$\pi \longleftrightarrow T$

$\frac{di}{dt} \longleftrightarrow \frac{dv}{dt}$

$\int v\,dt \longleftrightarrow \int i\,dt$

OC ⟷ SC

Thevenin ⟷ Nortons

$Y \longleftrightarrow \Delta$

$\phi \longleftrightarrow q$

Tree $\longleftrightarrow$ Co tree

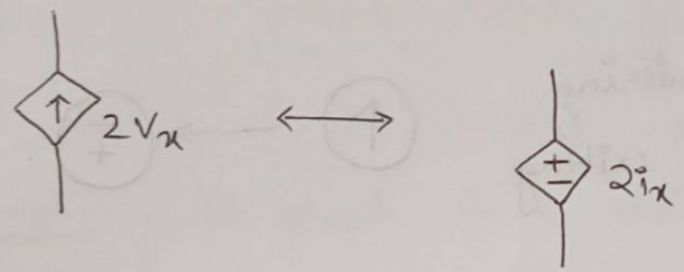
Twig $\longleftrightarrow$ Link / chord

cut set $\longleftrightarrow$ Tie set

$Z \longleftrightarrow Y$

$X \longleftrightarrow B$

Swith in series $\longleftrightarrow$ switch in parallel
(getting closed) (getting opened)



polarity voltage $\longleftrightarrow$ direction current

Ex $\rightarrow$ Dual of 2Ω resistance $\Rightarrow$ 2 seimens (conductance)

To solve problems

① Place a dot within each loop. These dots will become nodes of dual n/w.

② Place a dot outside of n/w, this dot will be ground node of dual n/w.

③ Carefully draw lines b/w nodes such that each line cuts only one element.

④ If any element exclusively present in a loop then connect the dual element in between node and ground.

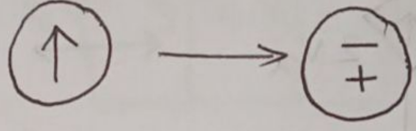
⑤ If an element is common b/w 2 loops, then dual element is placed in b/w two nodes.

⑥ For a voltage source producing clockwise current in a loop, its dual current source will have a direction from ground to non reference node.

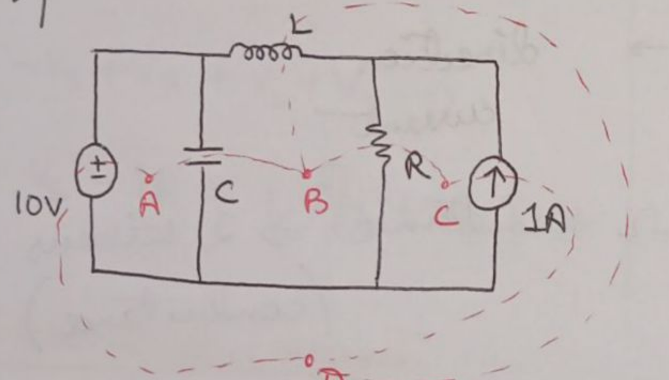
Anticlockwise

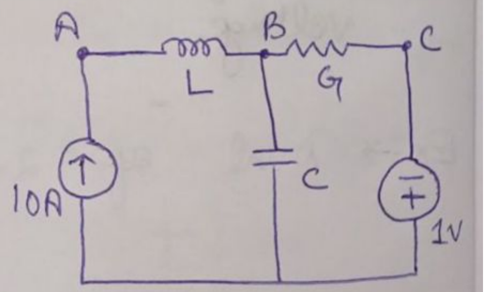
positive → entering

negative → exiting

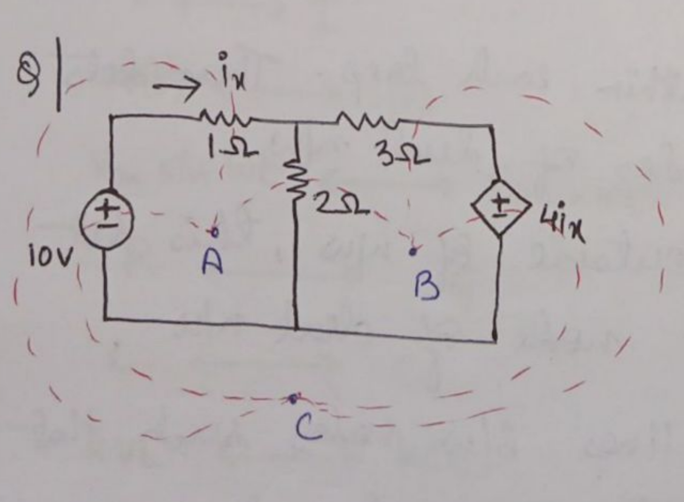


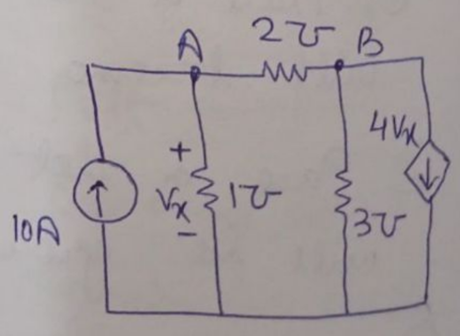
Q/





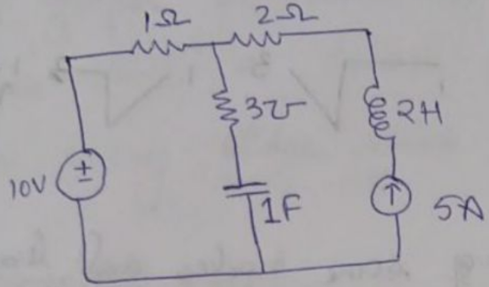
Q/





Topology – It is a branch of geometry applicable to electrical circuits where even by bending, stretching, swaping, tiling in knots, making the circuit upside down etc will not disturb the circuit property.

Ex



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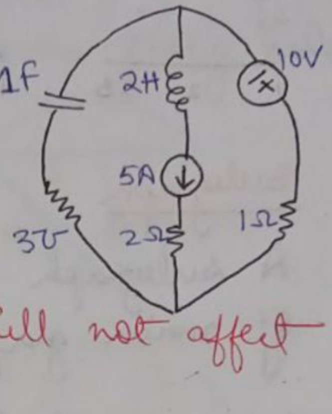


→ shape of a network ideally will not affect circuit analysis.

Graph – A graph is a skeleton representation of a n/w where every element is suppressed by its nature and represented as a simple line segment.

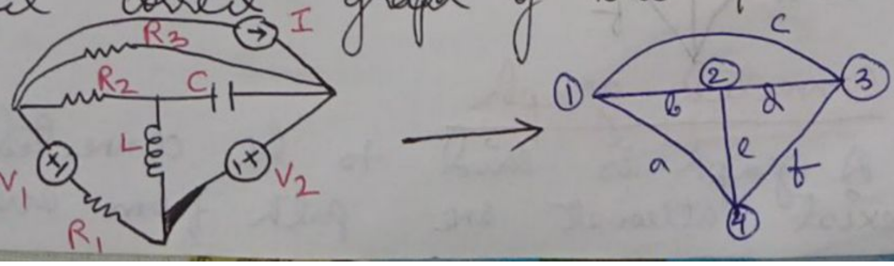
NOTE Ideal voltage source → SC
Ideal current source → OC

To simplify the order of matrix, we generally consider principle node only.

Nodes → vertices Branches → edges.

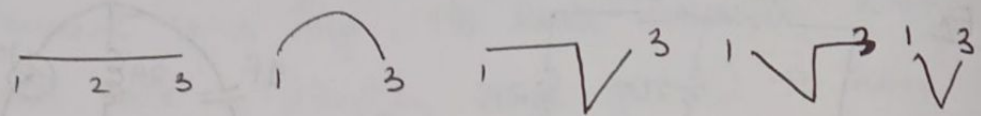
→ Nodes are numbers — ①, ②, ③, ...
Branches are named — a, b, c .....

Q| construct correct graph of the n/w shown below



Path — Path is a transversal from one node to another without crossing the same node twice.

Q/ No. of possible path for the above graph from node 1 to 3 is __5__.

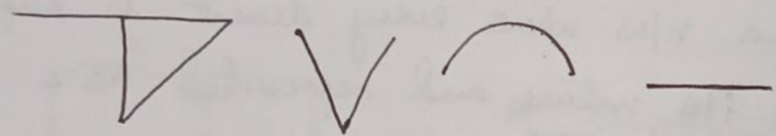


Subgraph

A subgraph consist of some nodes and branches of main graph.

Even a single edge can be a subgraph of the main graph.

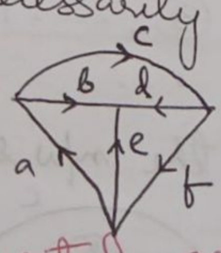
Ex



Directed graph

A graph is said to be directed if every edge is given a reference direction which is indicated by placing an arrow on every branch.

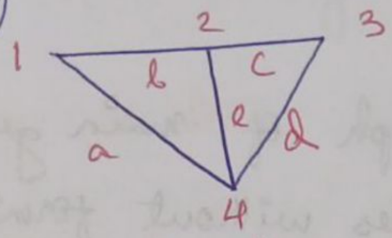
NOTE : This reference orientation need not necessarily indicate current dirⁿ.



Connected graph

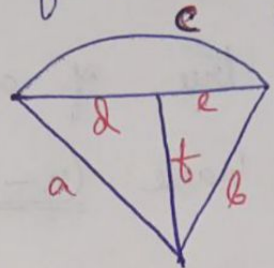
A graph is said to be connected if there exist atleast one path from every node to every other node.

Ex



complete graph | completely connected graph

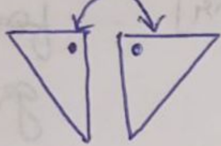
A graph is said to be complete graph if there exist a direct path from every node to every other node.

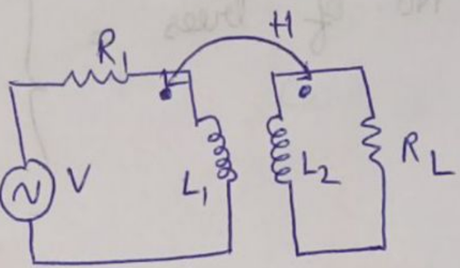


unconnected graph

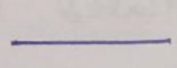
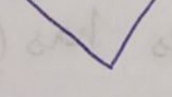
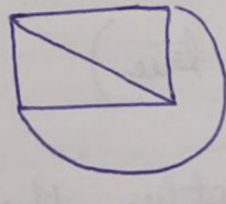
① Wireless comm. n/w.

② Magnetic circuits





Q) Minimum no. of edges to make a graph complete with n nodes is $\boxed{{}^nC_2}$.

nodes	Graph	Edges
2		$1 = \frac{2(2-1)}{2}$
3		$3 = \frac{3(3-1)}{2}$
4		$6 = \frac{4(4-1)}{2}$

TREE

A tree is a subgraph of main graph which connects all the nodes without forming closed loops.

* Rank of a tree with 'n' nodes = $(n-1)$

Any tree of a given graph with n nodes will have $(n-1)$ edges.

$$\text{No. of trees} = \begin{cases} n^{(n-2)} & ; \text{ for } n>2 \rightarrow \text{complete graph only} \\ \det \left| [A_r][A_r]^T \right| & ; \text{ for any graph} \end{cases}$$

where $[A_r]$ = reduced incidence matrix

Twig

Branch of a tree is specifically called as twig which is indicated by thick line segment.

Any tree with n nodes has $(n-1)$ twigs.

Co tree (compliment of tree)

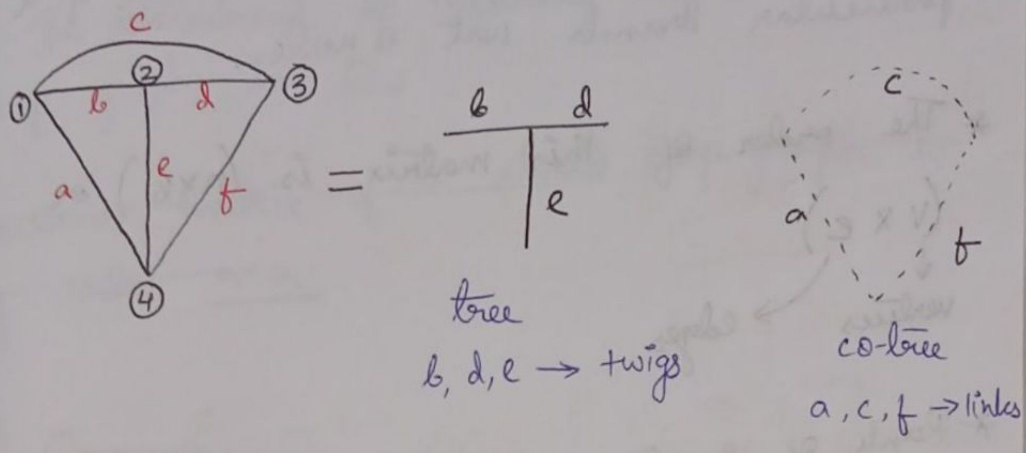
The set of branches other than tree branches in a graph forms co tree

Link / Chord

The branch of co tree is called as link

which is indicated by dotted line.

For any co tree, we have $b-(n-1)$ links



graph = Tree + co-tree

↓ ↓ ↓

edges twigs links

↓

$b = (n-1) + b-(n-1)$

Q) The no. of possible trees for the n/w graph shown below is ___ and draw them separately.

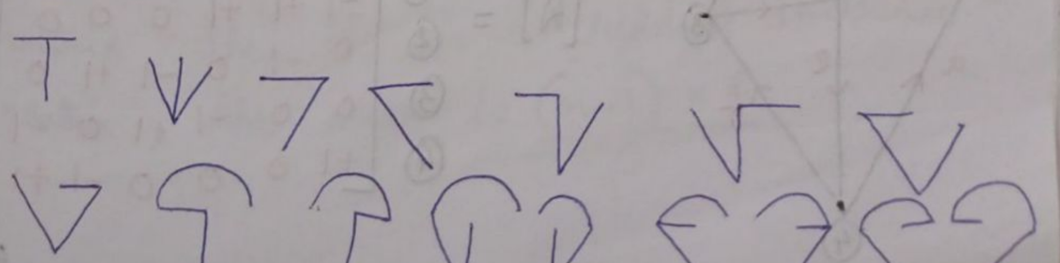
→ This is a complete graph

$\therefore$ No. of trees $= n^{(n-2)}$

$= 4^{(4-2)}$

$= 16$

Trees are



Incidence matrix [A]

Matrix that gives relation b/w no. of nodes and no. of branches and orientation of a particular branch wrt a node.

* The order of this matrix is $(n \times b)$ or $(v \times e)$

$(v \times e)$: $v \rightarrow$ vertices, $e \rightarrow$ edges

* Rank of incidence matrix with n nodes is $\underline{(n-1)}$

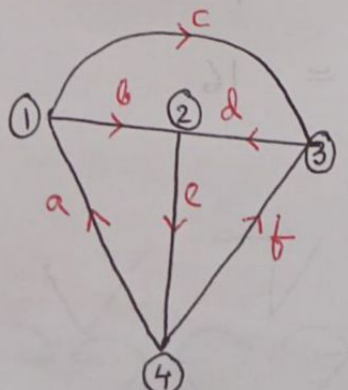
* The elements of this matrix $[A] = [a_{ij}]_{n \times b}$

where $a_{ij} = +1$; if j^{th} branch is incident with i^{th} node and oriented away from it.

$a_{ij} = -1$, if incident towards

$a_{ij} = 0$, if not incident

Q/ Construct the complete incidence matrix for the oriented graph shown below.



$$[A] = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \\ +1 & 0 & 0 & 0 & -1 & +1 \end{bmatrix} \end{array}$$

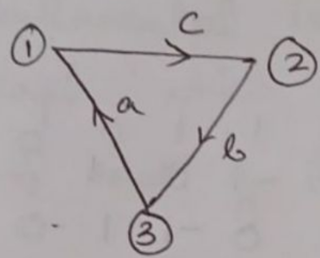
4×6

NOTE
The algebraic sum of the elements of every column vertically is zero.

Q| The determinant of incidence matrix of a closed loop graph is 0.

~~Incidence matrix of a closed loop graph is of order $n \times n$~~

Ex



$$[A] = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\det |A| = -1\left[0-(-1)\right] + 1\left(0-(-1)\right)$$

$$= -1 + 1 = 0$$

NOTE

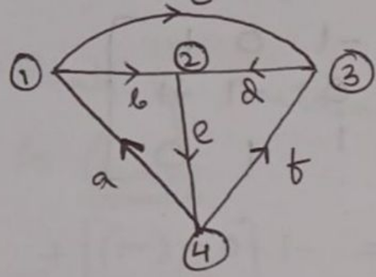
If the incidence matrices of 2 independent n/w graphs are identical then they are said to obey the principle of Isomorphism.

Reduced Incidence Matrix $[A_r]$

If one of the node in a given graph is considered as reference and that particular row is neglected while writing the incidence matrix, then it is a reduced incidence matrix, the order is $(n-1) \times b$

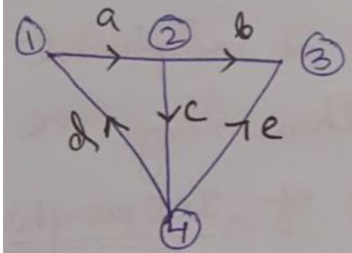
→ In computer methods of electrical circuits analysis by considering A_r, the memory space requirement and iteration time for solutions will be decreased.

Ex From the graph, if node (4) is considered as reference and that particular row (4) is neglected, then reduced incidence matrix is



$$[A_r] = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{bmatrix}$$

Q| No. of possible trees from n/w graph shown below is ____ and draw them separately.



It is just connected graph

No. of trees $= \det \left| [A_r] [A_r]^T \right|$

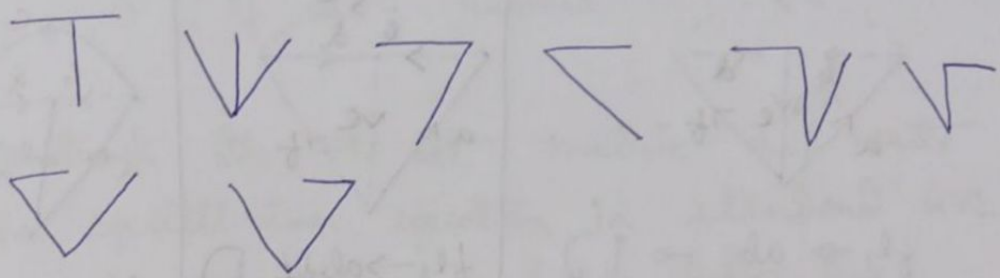
$$= \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\det \left| [A_r] [A_r]^T \right| = 2(6-1) - 1(0-(-2))$$

$$= 10 - 2 = 8 \text{ possible trees}$$

The trees are



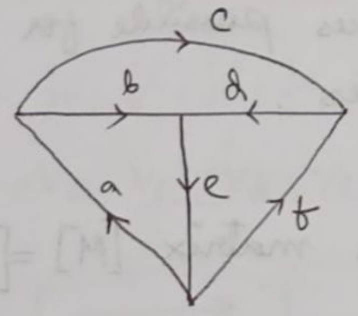
Concept of Fundamental Loops and Tie set current

Fundamental loops are closed path of the graph which are formed by only one link and rest of them are twigs.

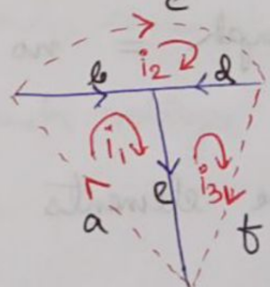
→ No. of fundamental loops for any given graph = No. of links i.e. $b-(n-1)$

→ These fundamental loop currents are called Tieset current and their orientation is governed by the link in it.

Ex →



⟹



No. of f-loops $= 6-(4-1)$

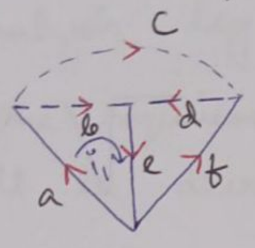
$= 3$

$fl_1 = abe \rightarrow i_1$

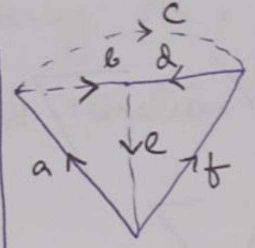
$fl_2 = bcd \rightarrow i_2$

$fl_3 = def \rightarrow i_3$

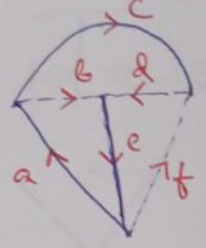
Q] Find fundamental loop.



$fl_1 \rightarrow abe \rightarrow i_1$
$fl_2 \rightarrow def \rightarrow i_2$
$fl_3 \rightarrow acf \rightarrow i_3$



$fl_1 \rightarrow abdf \; i_1$
$fl_2 \rightarrow acf \; i_2$
$fl_3 \rightarrow def \; i_3$



$fl_1 \rightarrow abe \rightarrow i_1$
$fl_2 \rightarrow acde \rightarrow i_2$
$fl_3 \rightarrow acf \rightarrow i_3$

Tie Set Matrix [M]

It is a matrix that gives the relation b/w the branch currents and tie set currents where every branch current can be expressed in terms of Tieset currents.

Order of this matrix is links x branches i.e. $\underline{[b-(n-1)] \times b}$

No. of tieset matrices possible for any graph = no. of trees.

The elements of this matrix $[M] = [a_{ij}]_{\text{links} \times \text{branches}}$

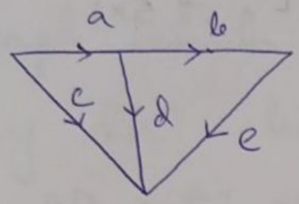
where

$a_{ij} = +1$, if j^{th} branch current is incident with i^{th} Tieset current and oriented in same dir^n.

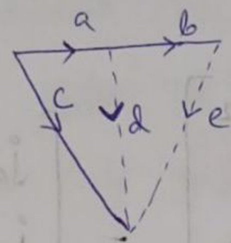
$a_{ij} = -1$, if incident and opposite

$a_{ij} = 0$, if not incident

Q) Construct the tie set matrix and write the equilibrium equation in standard KVL form for n/w graph shown below by considering a, b, c as tree branches.



$\Rightarrow$



$fl_1 \rightarrow acd \rightarrow i_1$

$fl_2 \rightarrow abec \rightarrow i_2$

No. of fundamental loops $= 5-(4-1) = 2$

$$[M] = \begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} i_1 \\ i_2 \end{array} & \begin{bmatrix} +1 & 0 & -1 & +1 & 0 \\ +1 & +1 & -1 & 0 & +1 \end{bmatrix}_{2\times5} \end{array}$$

Equilibrium equations

Let $j_a, j_b, j_c, j_d, j_e \longrightarrow$ branch currents

$V_a, V_b, V_c, V_d, V_e \longrightarrow$ branch voltages

KVL $[\longrightarrow]$

$$[M][V_b] = [0] \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}_{2\times5} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}_{5\times1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times1}$$

* Relation b/w j and i [↓]

$$[M]^T [I_l] = [J_b]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{5\times 2} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_{2\times 1} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5\times 1}$$

$$\Rightarrow \begin{bmatrix} i_1 + i_2 \\ i_2 \\ -i_1 - i_2 \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}$$

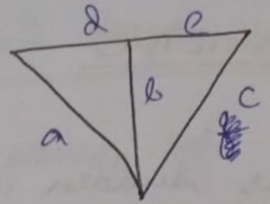
Concept of cut set

A cut set represents set of branches which when removed in a graph can be divided into 2 parts.

NOTE

The no. of cutsets simply represent the no. of possible ways that a graph can be divided into 2 parts.

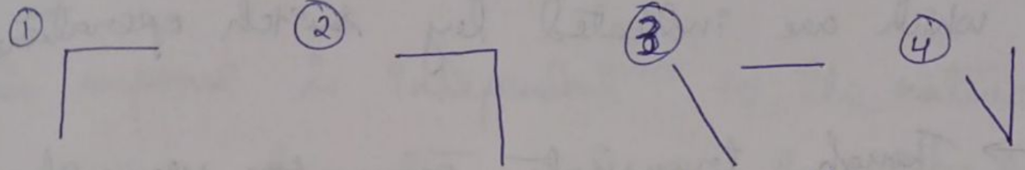
Q/ which of the following set of branches represent a proper cutset for n/w graph shown.



① acd ③ bdc

② ace ④ cde

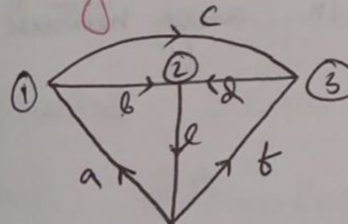
$\boxed{\text{Ans} \rightarrow bdc}$



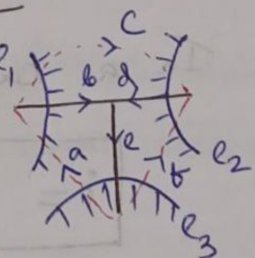
→ Fundamental cutset are cut through of a graph which can divide in 2 parts in any dirⁿ. but in path of cutting it should cut only one twig and rest of them as links.

No. of f cutset = No. of twigs (i.e. $n-1$)

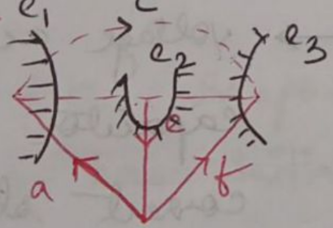
Orientation of f-cutset are governed by the twig in it.



Tree 1



Tree 2



No. of f cutsets = 4−1=3

$fc_1 \rightarrow a, b, c \rightarrow \overrightarrow{e_1}$

$fc_2 \rightarrow c, d, f \rightarrow \overleftarrow{e_2}$

$fc_3 \rightarrow a, e, f \rightarrow e_3 \downarrow$

$fc_1 \rightarrow a, b, c$

$fc_2 \rightarrow b, d, e$

$fc_3 \rightarrow c, d, f.$

Tree 3 and 4 please try yourself

ELECTRICAL TRANSIENTS

→ Transients are considered as sudden change in the state of a circuit or network which are indicated by switch operations

→ Though transients occur for very short duration in time, their impact is huge in determining entire steady state response.

State variables

(i) These are the critical parameters that must be observed to determine the transient state solution in any network.

(a) In a capacitor,

$$\boxed{i = C\frac{dv}{dt}}$$

→ voltage across capacitor is correct state variable.

KCL → Nodal

(b) In an inductor

$$\boxed{V = L\frac{di}{dt}}$$

→ current through inductor is correct state variable.

KVL → mesh

Forced Response

Response of any ckt/n/w when a source is present is called forced response

This response is independent to the nature of passive elements and can be different for different types of i/ps.

Natural response

The response of any circuit / n/w without any source is called natural response.

This response is independent of input but purely depends upon the nature of passive elements. It is always unique and determined by the characteristic eqn. governing the n/w.

Source free response is possible provided the n/w has initial stored energy.

$$\text{Complete response of a system} = \underset{\substack{\downarrow \\ \text{zero state response}}}{\text{forced response}} + \underset{\substack{\downarrow \\ \text{zero i/p response}}}{\text{Natural response}}$$

Any n/w or system reaches steady state after overcoming transient state.

Initial conditions

These are critical values of voltage across capacitor and current through inductors from previous steady state of a n/w :

$t = 0^-$ → instant just before switch operation (steady state before s/w operation)

$t = 0^+$ → instant just after switch operation (TRANSIENT state operation)

$t \to \infty$ → steady state after switch operation.

~~#~~Order of a n/w

The no. of energy storage components available in undistributed form in any circuit.

Ex → R–L , R–C → 1^{st} order n/w

R–L–C , R–L–L, C–C–R → 2^{nd} order

NOTE

① A capacitor will never allow sudden change in voltage across it.

$$V_c(0^-) = V_c(0) = V_c(0^+)$$

② Inductor will never allow sudden change in current through it.

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

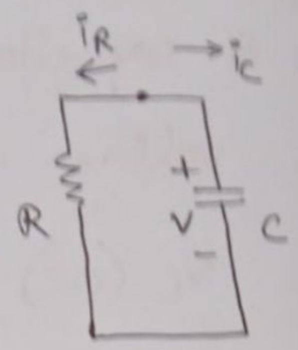
Behaviour of passive elements in transient state in comparison to steady state.

$\boxed{t \to 0^+}$ is considered as transient solution

Also, $\boxed{s \Rightarrow \infty}$ is also considered as transient solution.

Element	DC SS (steady state) $(s=0)$	AC SS $(s=j\omega)$	Transient state $\begin{pmatrix} t \to 0^+ \\ s \to \infty \end{pmatrix}$
R	R	R	R
L	S.C.	i lags V by $\phi = 90^\circ$	O.C
C	O.C	i leads V by $\phi = 90^\circ$	S.C.

(I) Source Free 1st Order



(a) RC circuit

Let $V(0) = V_0$

KCL

$$i_R + i_c = 0$$

$$\Rightarrow \frac{V}{R} + C\frac{dv}{dt} = 0$$

$$\Rightarrow C\frac{dv}{dt} = -\frac{V}{R}$$

$$\Rightarrow \int \frac{dv}{dt} = \int -\frac{dt}{RC}$$

$$\Rightarrow \ln V = -\frac{t}{RC} + \ln(A)$$

$$\therefore \ln\left(\frac{V}{A}\right) = -\frac{t}{RC}$$

$$\Rightarrow \boxed{V = A e^{-t/RC}}$$

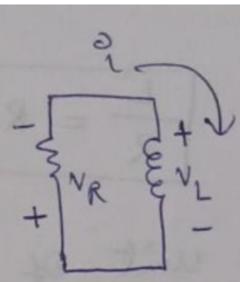
But at $t = 0$, $V = V_0$ then $A = V_0$

$$\therefore \boxed{V(t) = V_0 e^{-t/RC}}$$

$$\boxed{V(t) = V_0 e^{-t/\tau}}$$

$\tau = RC \rightarrow$ time const. of RC n/w.

(b) RL circuit



$i(0) = I_0$

KVL

$$v_R + v_L = 0$$

$$\Rightarrow iR + L\frac{di}{dt} = 0$$

$$\Rightarrow L\frac{di}{dt} = -iR$$

$$\Rightarrow \int \frac{di}{i} = -\int \frac{R}{L}\,dt$$

$$\Rightarrow \ln[i] = -\frac{R}{L}t + \ln[A]$$

$$\Rightarrow \ln\left(\frac{i}{A}\right) = -\frac{R}{L}t$$

$$\Rightarrow i = Ae^{-R/L\,t}$$

At $t=0$, $i = I_0$ then $A = I_0$

$$i(t) = I_0 e^{-R/L\,t} = I_0 e^{-t/(L/R)}$$

$$\boxed{i(t) = I_0 e^{-t/\tau}}$$

$\tau = \frac{L}{R}$ time const. of RL n/w

Time constant : It is the time taken by the response to reach 36.7% of its initial value (or) it is also defined as time taken by response to reach 63.4% to its final value.

→ unit of time const. is seconds.

$$\boxed{\frac{L}{R} = RC}$$

unit of $\frac{L}{RC} = \Omega$

unit of $R^2 C = H$

unit of $\frac{L}{R^2} = F$

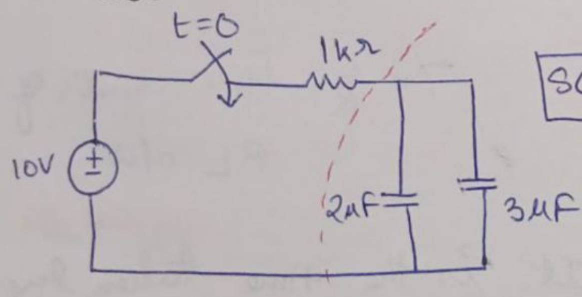
unit of $\frac{RC}{L} = \mho$

unit of $\frac{L}{C} = (\Omega)^2$

unit of $\sqrt{L/C} = \Omega$

unit of $\frac{R^2 C}{L} = 1$

Q/ Determine the time constant of the circuit



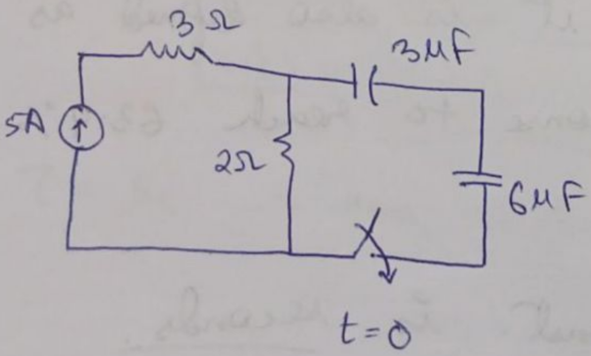
SC the Vsource

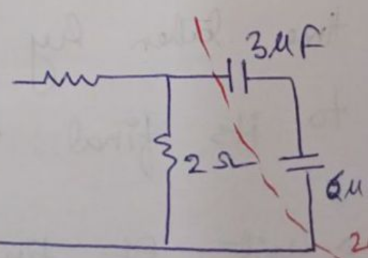
1kΩ   5μF

$\tau = R_{eq} C_{eq}$

$= 1k \times 5\mu$

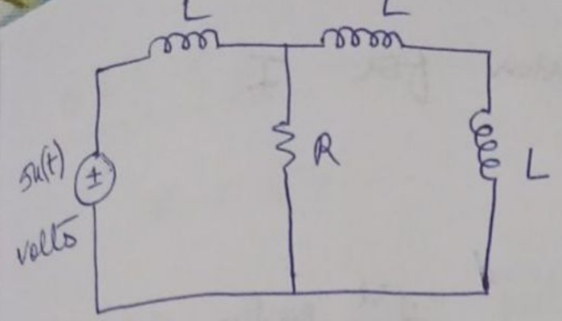
$= 5$ msec.





2Ω

$\tau = 2 \times 2 = 4$ μsec

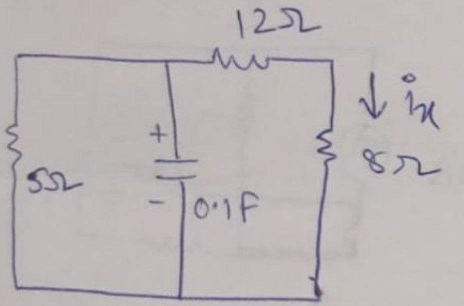


$$L_{eq} = L \parallel 2L$$

$$= \frac{2}{3} L$$

$$\tau = \frac{L}{R} = \frac{2}{3}\frac{L}{R}$$

Q) If $V(0) = 15V$, find complete expression for $i_x(t)$



This is a source free 1^{st} order RC n/w

$\boxed{\text{State variable 'V'}}$

$$V(t) = V_0 e^{-t/\tau}$$

$$\tau = R_{eq} \cdot C$$

$$V_0 = 15V \text{(given)}$$

$$= (5 \parallel 20) \cdot 0.1$$

~~$\tau = 5 \parallel 20$~~

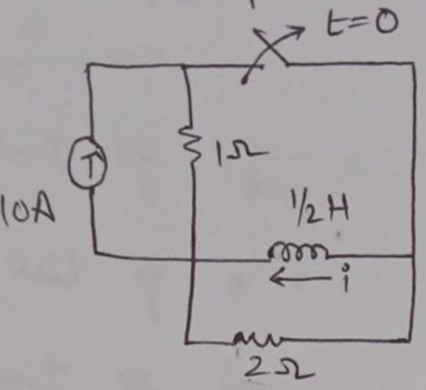
$$= 0.4 \text{ sec.}$$

$$V(t) = 15 e^{-t/0.4}$$

$$= 15 e^{-2.5t}$$

Now, $i_x(t) = \frac{V(t)}{12+8} = 0.75 e^{-2.5t}$ A

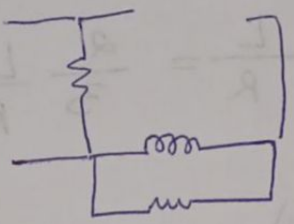
Q| Find complete expression for I.



1st order RL circuit.

state variable → i

$$i(t) = I_0 e^{-t/\tau}$$

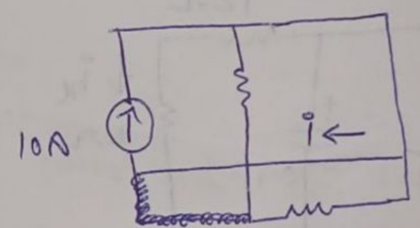


$$R_{eq} = 2\Omega$$

$$\tau = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4} \text{ sec}$$

$$i(0^-) = 10\,A = I_0$$

~~just~~ steady state just before switch operation



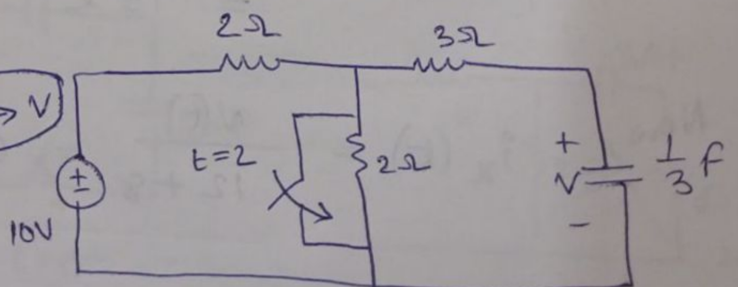
$$\therefore \quad i(t) = 10\, e^{-t/1/4}$$

$$= 10\, e^{-4t}\ A$$

Q| Determine the complete expression for V and determine energy stored in the capacitor upto 3rd second.

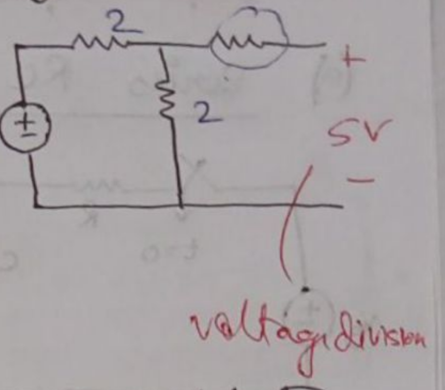
Soln

state variable → V



$\underline{0 < t \leq 2}$ → steady state only.

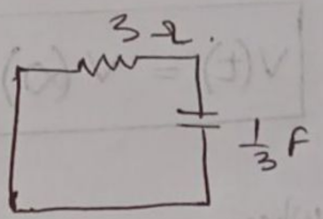
$V_0 = 5V$



$\underline{t > 2}$ transient solution

$$V = V_0 e^{-t/\tau}$$

Here V_0 means $V(2^-)$ from previous state.



$$\tau = R \cdot C$$

$$= \frac{1}{3} \times 3$$

$$= 1 \text{ sec}.$$

$$v(t) = 5 e^{-(t-2)/1}$$

Complete solution

$$V(t) = \begin{cases} 5V, & 0 < t \leq 2 \\ 5e^{-(t-2)}, & t \geq 2 \end{cases}$$

$$E = \frac{1}{2} C V^2$$

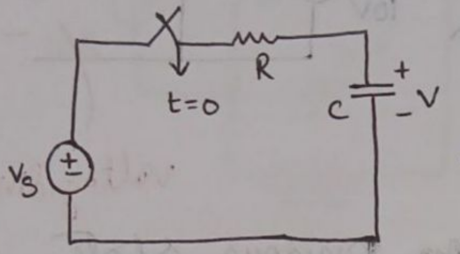
Now at $\boxed{t = 3}$

$$V(t=3) = 5e^{-(3-2)} = 1.84\,V$$

$$E_C = \frac{1}{2} \times \frac{1}{3} \times (1.84)^2 = 0.56\ J.$$

Step Response of 1st order circuit -

(a) Series RC circuit



$$\boxed{v(t) = v_{ss}(t) + v_{tr}(t)}$$

$v(t)$ → Total response

$v_{ss}(t)$ → steady state response

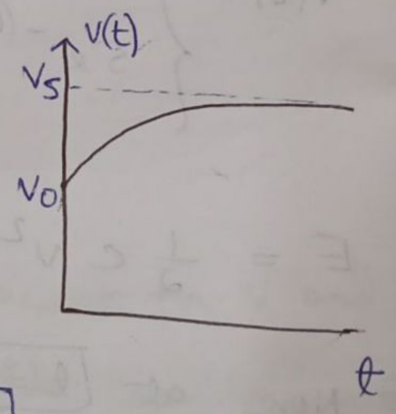
$v_{tr}(t)$ → transient response

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}}$$

where $v(0)$ → voltage across capacitor before switch operation and steady state

$v(\infty)$ → voltage across capacitor after switch operation and steady state.

$$\boxed{\tau = RC}$$



Case 1 : with initial condition given

Let $v(0) = V_o$

$$\boxed{v(t) = V_s + (V_o + V_s)\, e^{-t/\tau}}$$

Case 2 ; w/o initial condition,

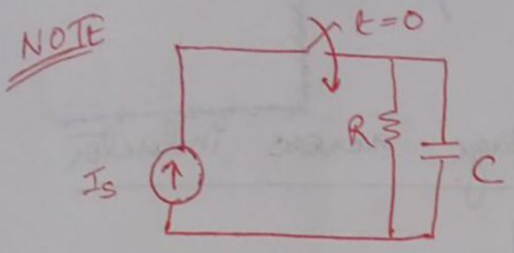
Let $v(0) = 0$

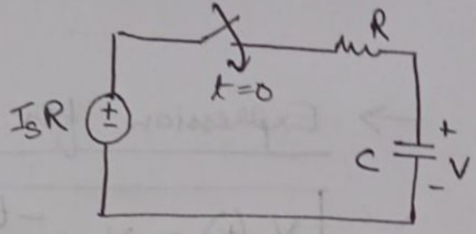
$$\boxed{v(t) = V_s\left(1 - e^{-t/\tau}\right)}$$

→ Expression for $i_c(t)$

$$\boxed{i_c(t) = \frac{V_s}{R} e^{-t/\tau}}$$

NOTE

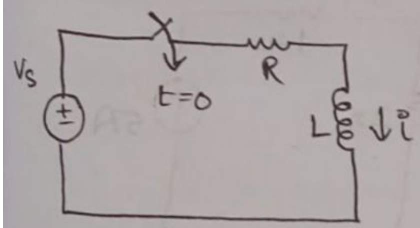




$$v(t) = I_s R + [V_0 - I_s R] e^{-t/\tau}$$

→ Here we cannot connect a voltage source across capacitor as it will cause sudden change of voltage across capacitor which will result in large current which may damage the ckt.

(b) RL ckt series



$$i(t) = i_{ss}(t) + i_{tr}(t)$$

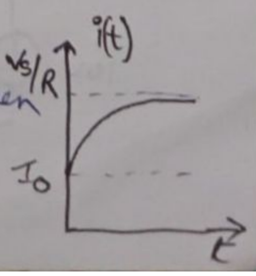
$$\boxed{i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}}$$

$I(0)$ → current through inductor before switch operation and steady state

$I(\infty)$ → current through inductor after switch operation and steady state

$$\boxed{\tau = \frac{L}{R}}$$

Case 1 : with initial condition given



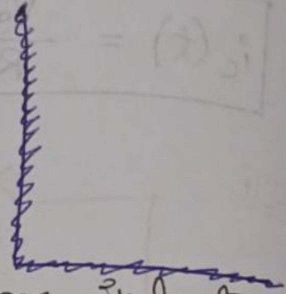
let $I(0) = I_0$

$$i_L(t) = \frac{V_s}{R} + [I_0 - V_s/R] e^{-t/\tau}$$

case 2 : without initial condition

$I(0) = 0$

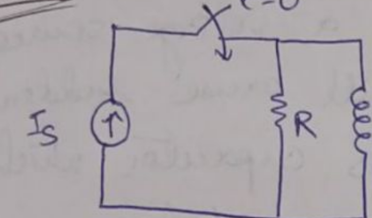
$$i_L(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

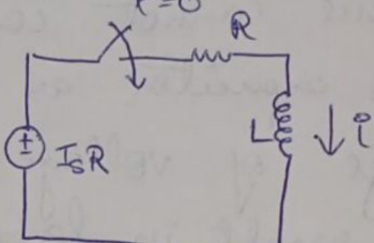


→ Expression for voltage across inductor

$$\boxed{V_L(t) = V_s e^{-t/\tau}}$$

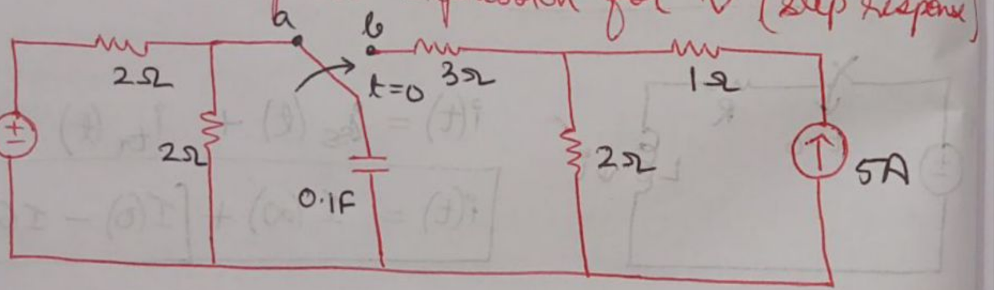
Note





$$\boxed{i(t) = I_s + \left[I_0 - I_s\right] e^{-t/\tau}} \qquad \boxed{\tau = L/R}$$

Q| Find the complete expression for V (step response)

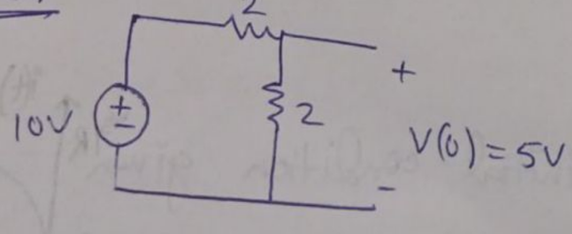


Sol^n This is step response of 1st order RC N/w

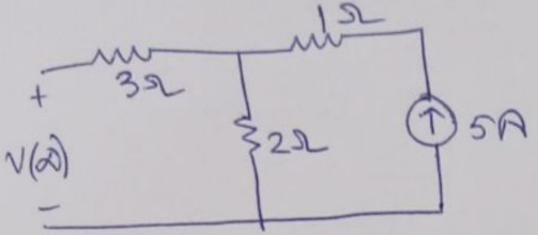
S.V. → V.

$$V(t) = V(\infty) + \left[V(0) - V(\infty)\right] e^{-t/\tau}$$

V(0)

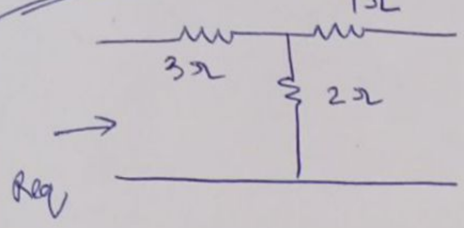


$V(\infty)$



$V(\infty) = 5 \times 2 = 10A$

$\tau,\ t>0$

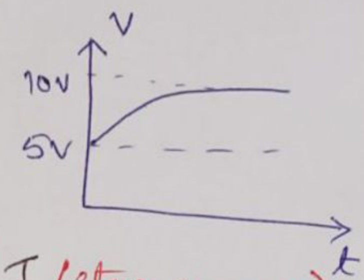


$R_{eq} = 5\Omega$

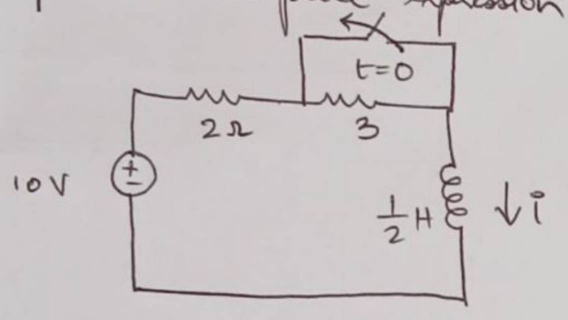
$\tau = R_{eq} \cdot C = 5 \times 0.1 = \frac{1}{2}$ sec.

$v(t) = 10 + (5 - 10)\, e^{-t/\frac{1}{2}}$

$v(t) = 10 - 5e^{-2t}$



Q Find complete expression for I (step response)

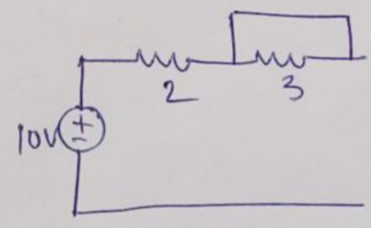


This is step response of 1st order RL n/w

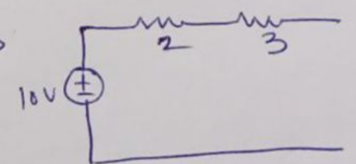
SV → i

$i(t) = I(\infty) + \left[I(0) - I(\infty)\right] e^{-t/\tau}$

$I(0) = \frac{10}{2} = 5A$



$I(\infty) = \frac{10}{5} = 2A$



$\tau = \frac{L}{R} = \frac{1/2}{5} = \frac{1}{10}$ sec

$\boxed{i(t) = 2 + 3e^{-10t}}$