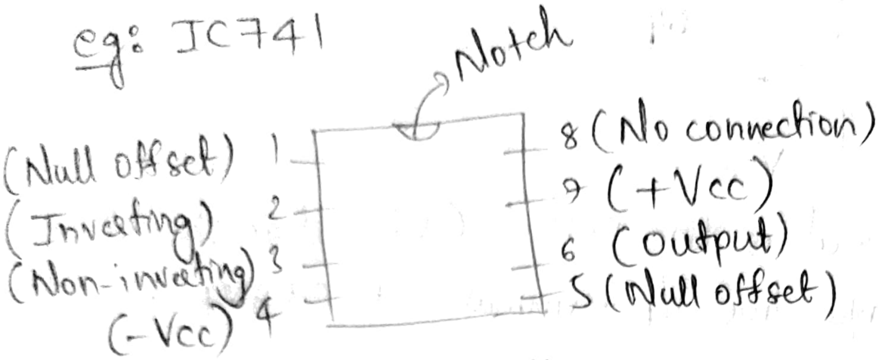
24/08/22

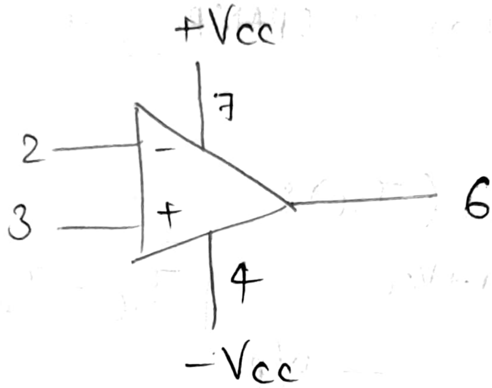
- **Operational Amplifier:**

  → performs mathematical operations

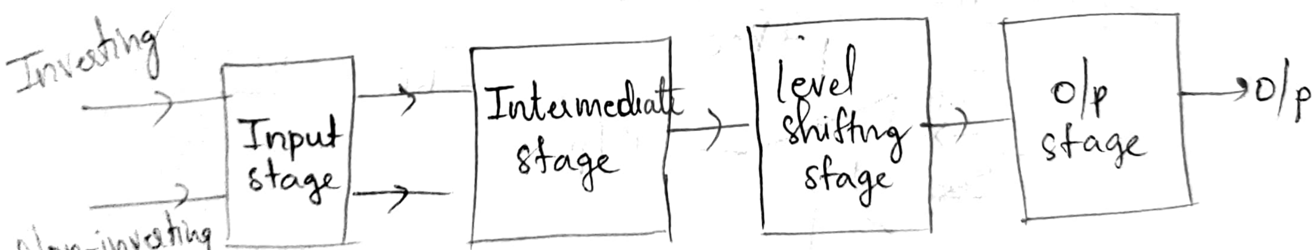
eg: IC741



(Pin Diagram)



- **Block diagram of OPAMP:**



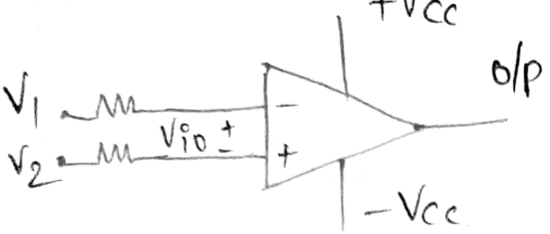
Dual i/p
Balanced o/p
Differential Amp

Dual i/p
Unbalanced o/p
Differential Amp

Emitter follower
with const
current source

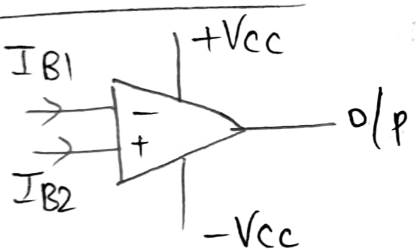
• Parameters of OPAMP:

(1) Input offset Voltage (V_{io}):



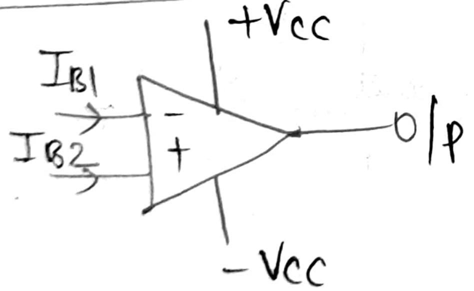
→ It is defined as the voltage (V_{io}) that should be supplied b/w two i/p terminals of OPAMP to make the o/p voltage of OPAMP zero.

(2) Input offset current (I_{io}):



$$\boxed{I_{io} = I_{B2} - I_{B1}}$$

(3) ~~Out~~Input Bias Current: (I_{BO})



$$I_{BO} = \frac{I_{B1} + I_{B2}}{2}$$

(4) Input Resistance (R_{in}):

→ It is defined as input resistance seen by one of the i/p terminal with other i/p terminal grounded.

⑤ Input capacitance (C_{in}):

→ It is defined as capacitance as seen by the one of i/p terminal with other i/p terminal grounded

⑥ Common mode rejection ration (CMRR):

$$CMRR = \frac{A_d}{A_c} = \frac{\text{Differential gain of OPAMP}}{\text{Common mode gain of OPAMP}}$$

$$V_d = V_1 - V_2$$

$$V_c = \frac{V_1 + V_2}{2}$$

$A_d >> A_c$

$$V_o = A_d V_d + A_c V_c$$

⑦ Power supply rejection ratio (PSRR):

→ It is defined as ratio of change in i/p offset voltage & change in ~~b~~ one of the biasing voltages with other biasing voltages remaining constant.

$$PSRR = \frac{dV_{io}}{dV_{cc}}$$

⑧ Slew rate: It is the maximum change in o/p voltage wrt time

$$\text{Slew rate} = \frac{dV_o}{dt}$$

⑨ Input offset Voltage drift:

$$\frac{dV_{io}}{dT}$$

Temperature

(10) Input offset Current drift:

$$\frac{dI_{io}}{dT} \rightarrow \text{Temperature.}$$

• Ideal OPAMP characteristics :

(1) $A_d = \infty$

(2) $R_{in} = \infty$

(3) zero o/p voltage when difference input $(V_1 - V_2) = 0$

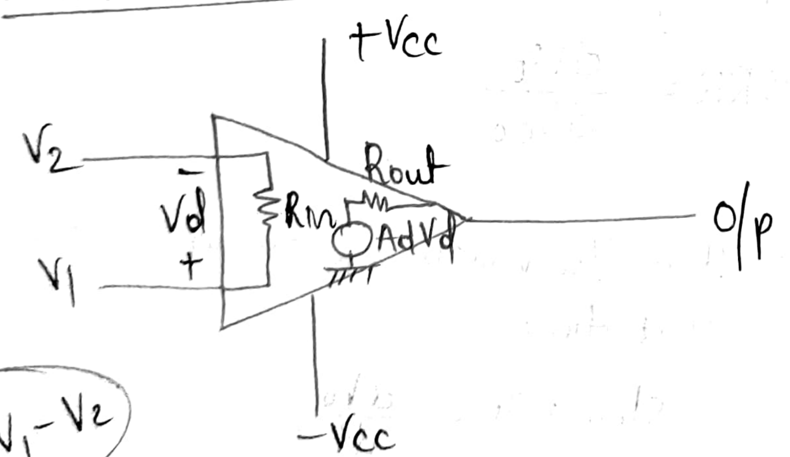
(4) Infinite Bandwidth

(5) CMRR $= \infty$

(6) Slew rate $= \infty$

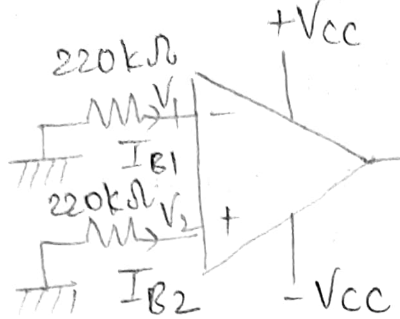
(7) $R_{out} = 0$

• Equivalent circuit of OPAMP:



$V_d = V_1 - V_2$

- Measurement of Input bias current and Input offset current :



Observation table:

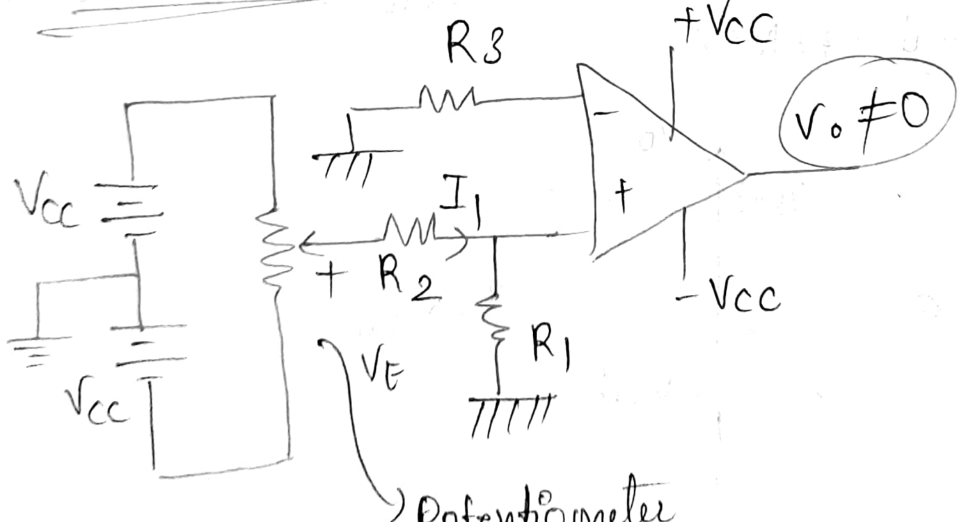
$V_1, V_2 \rightarrow$ given

$I_{B1} = \frac{V_1}{220k}$; $I_{B2} = \frac{V_2}{220k}$

Input bias current $= \frac{I_{B1} + I_{B2}}{2}$

Input offset current $= |I_{B1} - I_{B2}|$

- Measurement of input offset voltage:



$$\boxed{I_1 = \frac{V_E}{R_1 + R_2}}$$

$V_1 = R_1 \times I_1 = \frac{V_E R_1}{R_1 + R_2}$

$V_1 \rightarrow$ Input offset Voltage

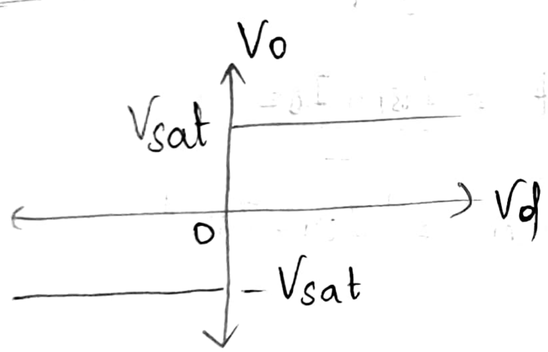
- Measurement of PSRR or SVRR:

① Set $V_{cc} = 13V$ & measure the corresponding input offset voltage (V_{io1}).

② set $V_{cc} = 17V$ & measure the corresponding input offset voltage (V_{io2})

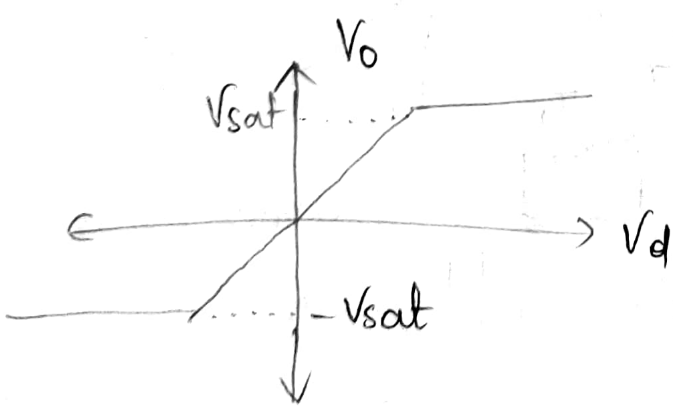
$$\boxed{PSRR = \frac{dV_{io}}{dV_{cc}}}$$

✱ Ideal transfer characteristics of OPAMP:

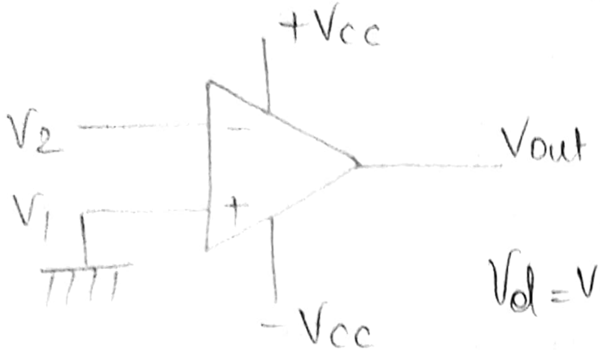


V_{sat} - Saturation voltage.

✱ Practical OPAMP:



- Virtual Ground Concept:

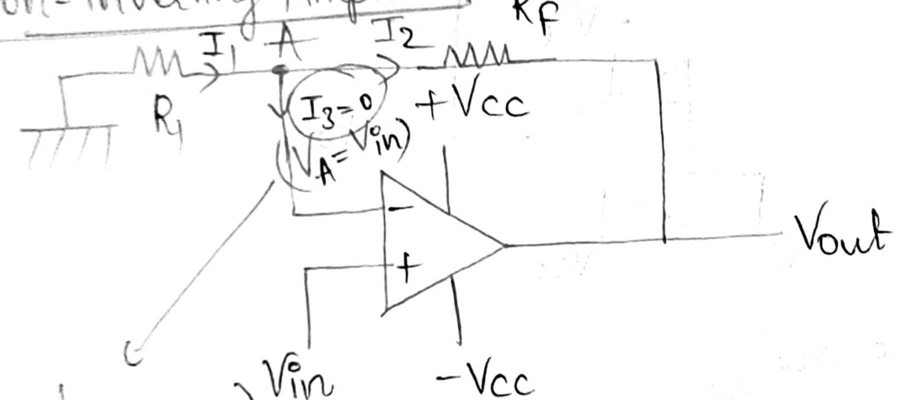


$V_d = V_1 - V_2 \Rightarrow \boxed{V_1 \simeq V_2}$

$V_{out} = (V_1 - V_2) A_d$ — differential gain.

$$V_1 - V_2 = \frac{V_o}{A_d} \simeq 0 \text{ (Since } A_d \text{ is very large)}$$

- Non-inverting Amplifier:



(Due to virtual ground)

$$I_1 = I_2 + \cancel{I_3}^{\,0}$$

$$\boxed{I_1 = I_2}$$

$$\frac{0 - V_{in}}{R_1} = \frac{V_{in} - V_o}{R_F}$$

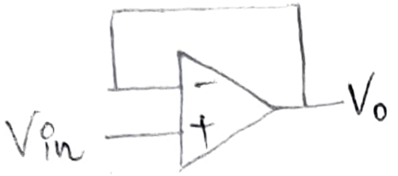
$$V_{in} - V_o = \frac{R_F}{R_1}(-V_{in})$$

$$V_{in} + \frac{R_F}{R_1} V_{in} = V_o$$

$$\boxed{\frac{V_o}{V_{in}} = 1 + \frac{R_F}{R_1}}$$

Gain of Non-inverting amplifier.

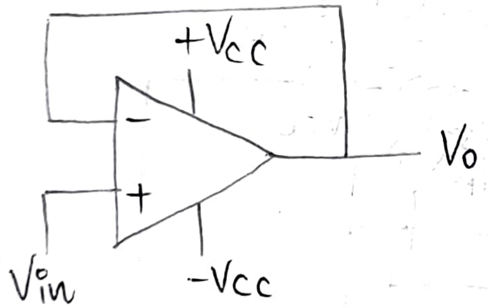
→ If $R_1 = \infty$ and $R_F = 0$ then the circuit is called voltage follower circuit.

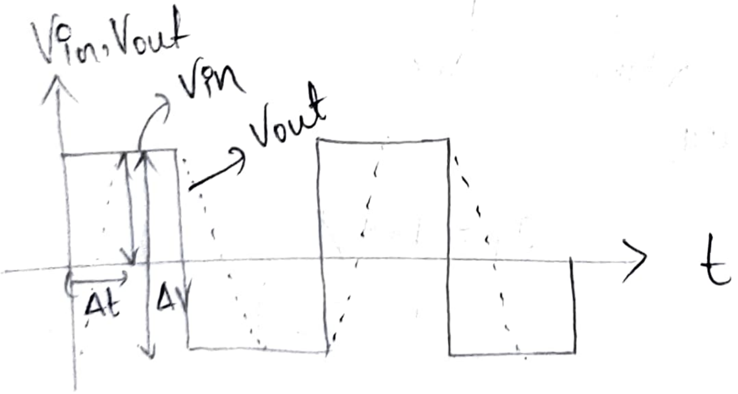


Voltage follower circuit

$$\frac{V_o}{V_{in}} = 1 \Rightarrow \boxed{V_o = V_{in}}$$

• Measurement of Slew rate:

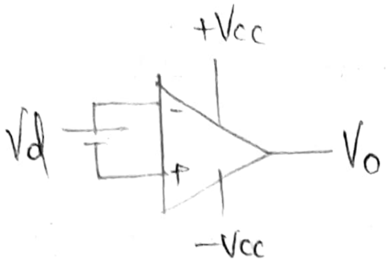




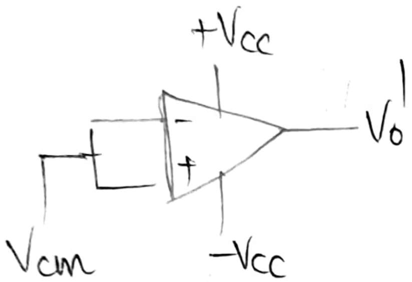
$$\text{Slew rate} = \frac{dV_o}{dt} = \frac{\Delta V}{\Delta t}$$

Measurement of CMRR:

Case-I:

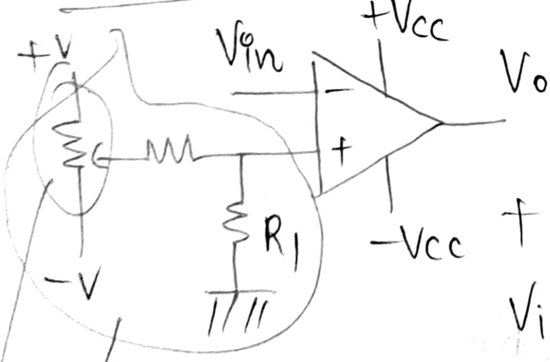


Case-II:



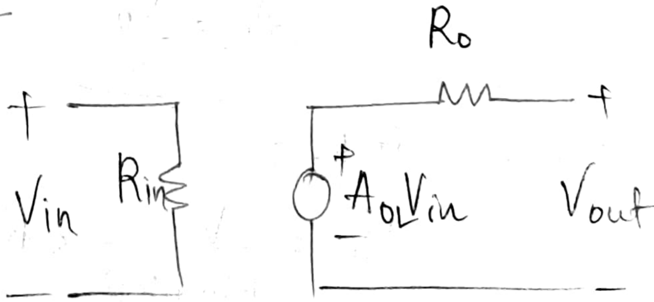
$$CMRR = \frac{A_d}{A_c} = \frac{\frac{V_o'}{V_d}}{\frac{V_o'}{V_{cm}}} = \frac{V_{cm}}{V_d}$$

Measurement of Open loop gain of Amplifier:



To compensate input offset Voltage.

Potentiometer

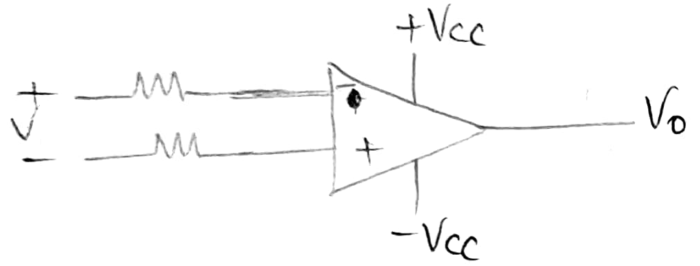


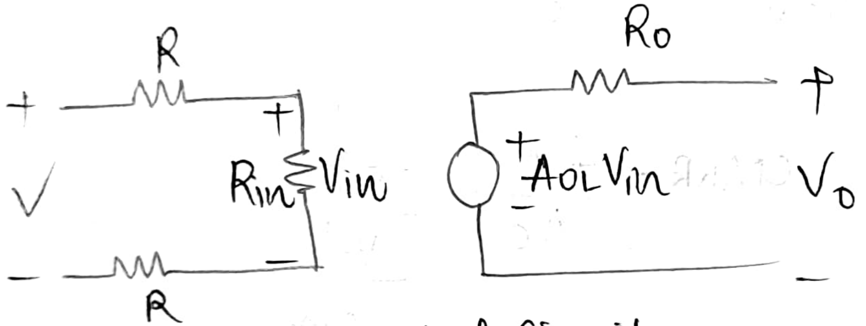
Equivalent Circuit

$V_o = A_{OL} V_{in}$

A_{OL} = open loop gain = $\dfrac{V_o}{V_{in}}$

- Measurement of Input Resistance of OPAMP:





Equivalent Circuit

Case-1: if $R = 0$

$V_o = A_{OL} V$

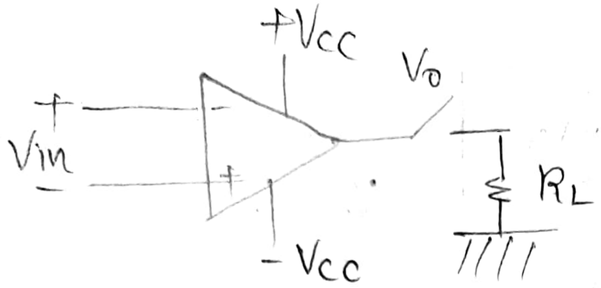
$$\boxed{A_{OL} = \frac{V_o}{V}}$$

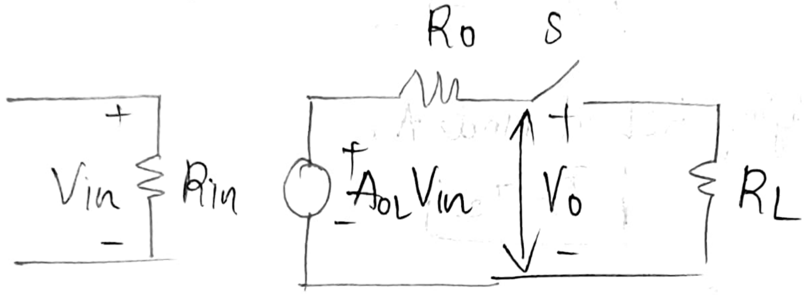
Case-2: if $R \neq 0$

$$V_{in} = \frac{V}{R_{in} + 2R} \times R_{in}$$

$$V_o' = \frac{A_{OL} \times V R_{in}}{R_{in} + 2R}$$

- Measurement of Output resistance of OPAMP:





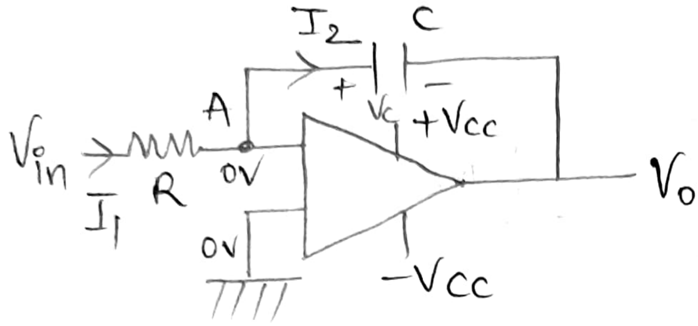
Case-1: (switch is open)

$$V_o = A_{OL} V_{in} \quad , \quad A_{OL} = \frac{V_o}{V_{in}}$$

Case-2: (switch is closed)

$$\frac{A_{OL} V_{in}}{R_o + R_L} \times R_L = V_o'$$

• Integrator using OPAMP :



Applying KCL at Node A,

$$\boxed{I_1 = I_2}$$

$$\frac{V_{in} - 0}{R} = I_2 = \frac{C\,dV_c}{dt} = C\frac{d}{dt}(0 - V_o)$$

$$\frac{V_{in}}{R} = \frac{-C\,dV_o}{dt}$$

$$\frac{dV_o}{dt} = \frac{-1}{RC} V_{in}$$

$$\boxed{V_o = \frac{-1}{RC}\int_0^t V_{in}\,dt}$$

$$\frac{V_o(s)}{V_{in}(s)} = -\frac{Z_f}{Z_1} = \frac{\frac{+1}{SC}}{R} = \frac{1}{SCR}$$

$$A_v \rightarrow \boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{1}{SCR}}$$

Voltage gain in laplace domain.

$S = j\omega$

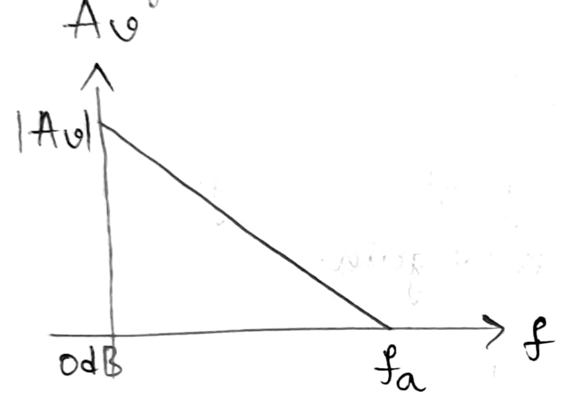
$$\boxed{A_v = \frac{V_o(j\omega)}{V_{in}(j\omega)} = \frac{1}{j\omega CR}}$$

$$|A_v| = \frac{1}{\omega CR} = \frac{1}{2\pi f RC}$$

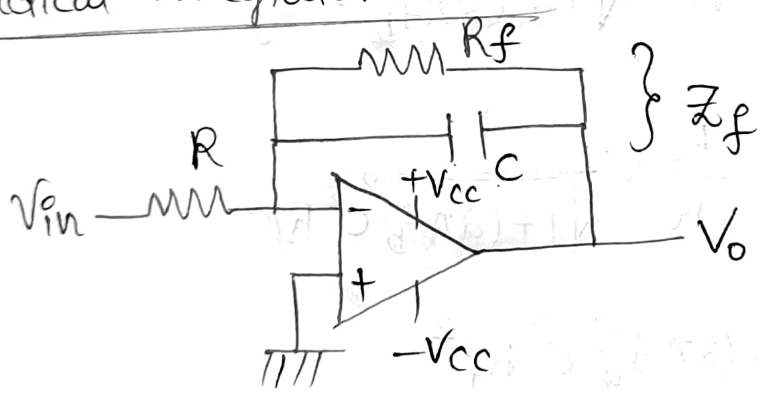
$f_a \rightarrow$ the frequency at which voltage gain is '1'.

$$1 = \frac{1}{2\pi f_a RC} \rightarrow \boxed{f_a = \frac{1}{2\pi RC}}$$

$$\boxed{|A_v| = \frac{f_a}{f}}$$



- Practical Integrator circuit :



$$Z_f = R_f \parallel \frac{1}{SC}$$

$$Z_1 = R$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{-Z_f}{Z_1} = \frac{-R_f \parallel \frac{1}{SC}}{R} = \frac{-\left[\frac{\frac{R_f}{SC}}{R_f + \frac{1}{SC}}\right]}{R}$$

$$\boxed{A_v(s) = \frac{-R_f}{(1+SCR_f)R}}$$

$s = j\omega$

$$\boxed{A_v(j\omega) = -\frac{R_f}{R}\left(\frac{1}{1+j\omega C R_f}\right)}$$

$$\boxed{|A_v| = \frac{R_f}{R} \times \frac{1}{\sqrt{1+\omega^2 C^2 R_f^2}}}$$

$$\boxed{|A_v|_{max} = \frac{R_f}{R}}$$

$f_b \Rightarrow$ It is the frequency at which gain is $\frac{1}{\sqrt{2}}$ times the max. gain.

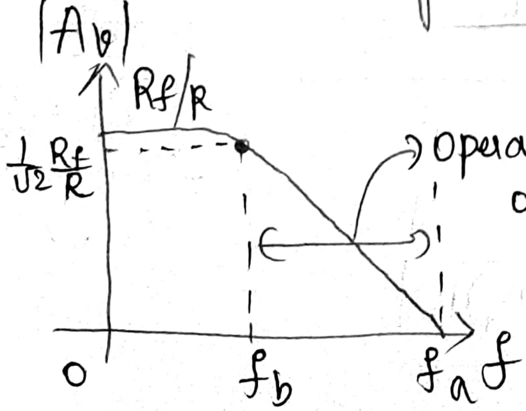
$$|A_v| = \frac{R_f}{R} \times \frac{1}{\sqrt{1+(2\pi f_b)^2 C^2 R_f^2}}$$

$$\frac{1}{\sqrt{2}} \times \cancel{\frac{R_f}{R}} = \cancel{\frac{R_f}{R}} \times \frac{1}{\sqrt{1+(2\pi f_b)^2 C^2 R_f^2}}$$

$$2 = 1 + (2\pi f_b)^2 C^2 R_f^2$$

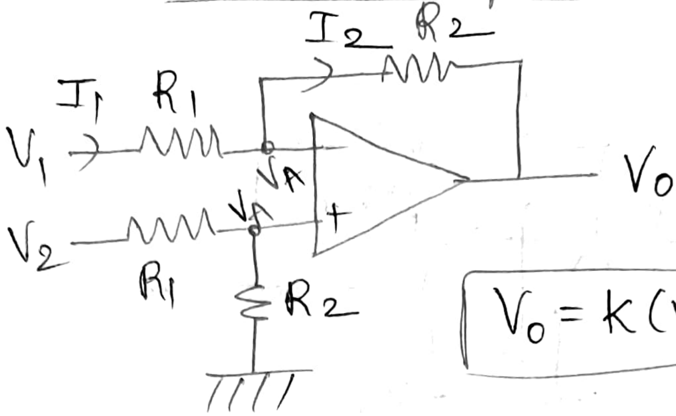
$$(2\pi f_b)^2 C^2 R_f^2 = 1$$

$$\boxed{f_b = \frac{1}{2\pi R_f C}}$$



operating region of integrator.

- Subtractor or Difference Amplifier:



$$\boxed{V_o = K(V_1 - V_2)}$$

$$V_A = \frac{V_2}{R_1 + R_2} \times R_2 \quad \text{— (1)}$$

Applying KCL,

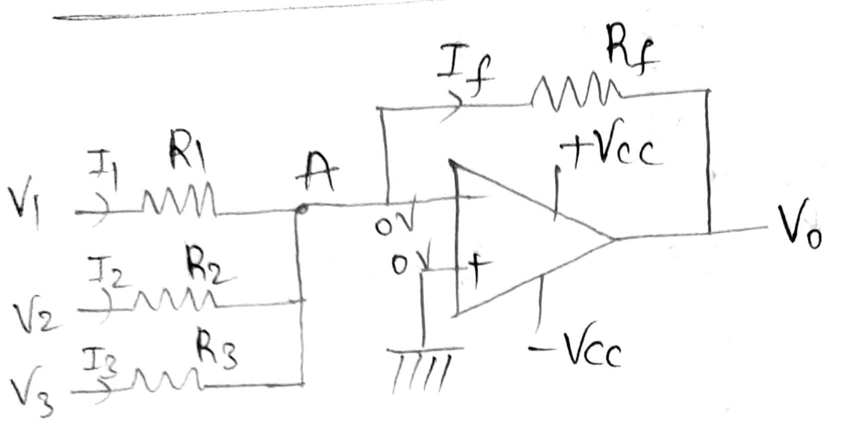
$$\boxed{I_1 = I_2}$$

$$\frac{V_1 - V_A}{R_1} = \frac{V_A - V_o}{R_2}$$

$$\frac{V_1}{R_1} - \frac{1}{R_1}\left(\frac{V_2 R_2}{R_1 + R_2}\right) = \frac{V_2 \cancel{R_2}}{(R_1 + R_2)\cancel{R_2}} - \frac{V_o}{R_2}$$

$$\boxed{V_o = -\frac{R_2}{R_1}(V_1 - V_2)}$$

• Summer circuit using Inverting OPAMP amplifier:



At Node A,

$$I_1 + I_2 + I_3 = I_f$$

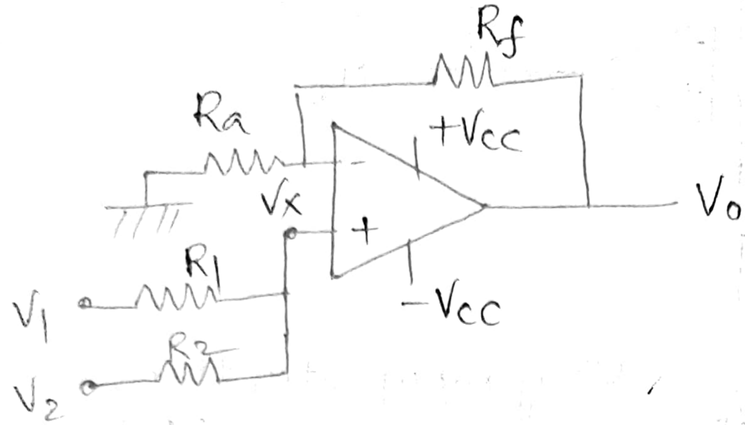
$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{0 - V_o}{R_f}$$

$$\boxed{V_o = -R_f\left[\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right]}$$

If $R_f = R_1 = R_2 = R_3$, then

$$\boxed{V_o = -(V_1 + V_2 + V_3)}$$

- Summing circuit using Non-inverting OP-AMP amplifier



Using superposition theorem,

Case-①: Apply only V_1, deactivating V_2.

$$V_{x1} = \frac{V_1}{R_1+R_2} \times R_2$$

Case-②: Apply only V_2, deactivating V_1.

$$V_{x2} = \frac{V_2}{R_1+R_2} \times R_1$$

$$V_x = V_{x1} + V_{x2}$$

$$= \frac{V_1 R_2}{R_1+R_2} + \frac{V_2 R_1}{R_1+R_2}$$

$$\boxed{V_x = \frac{V_1 R_2 + V_2 R_1}{R_1+R_2}}$$ → if $R_1 = R_2 = R$

then,

$$\boxed{V_o = \left(1+\frac{R_f}{R_a}\right)\left(\frac{V_1+V_2}{2}\right)}$$

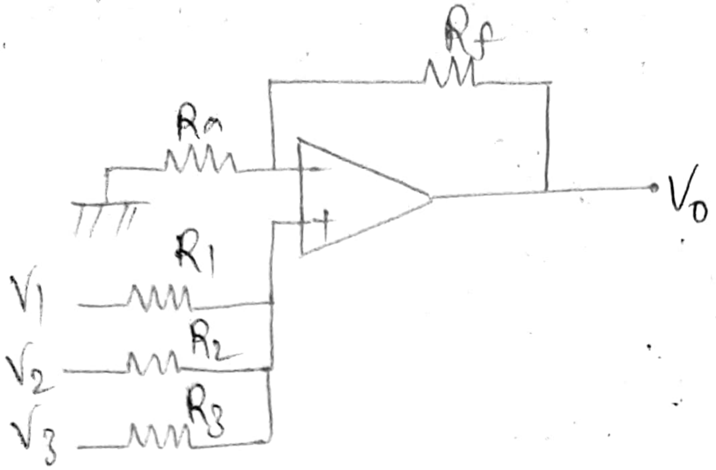
$$\boxed{V_o = \underbrace{\left(1+\frac{R_f}{R_a}\right)}_{\text{gain of non-inverting amplifier}}\underbrace{\left(\frac{V_1 R_2 + V_2 R_1}{R_1+R_2}\right)}_{\text{input voltage}}}$$

→ if $R_f = R_a$ & $R_1 = R_2 = R$

then,

$$\boxed{V_o = V_1 + V_2}$$

(*)



Using superposition theorem,

$$V_{x1} = \frac{V_1 \times \left(\frac{R_2 R_3}{R_2 + R_3}\right)}{R_1 + \left(\frac{R_2 R_3}{R_2 + R_3}\right)}$$

$$V_{x2} = \frac{V_2 \times \left(\frac{R_1 R_3}{R_1 + R_3}\right)}{R_2 + \left(\frac{R_1 R_3}{R_1 + R_3}\right)}$$

$$V_{x3} = \frac{V_3 \times \left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_3 + \left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

$$V_x = V_{x1} + V_{x2} + V_{x3}$$

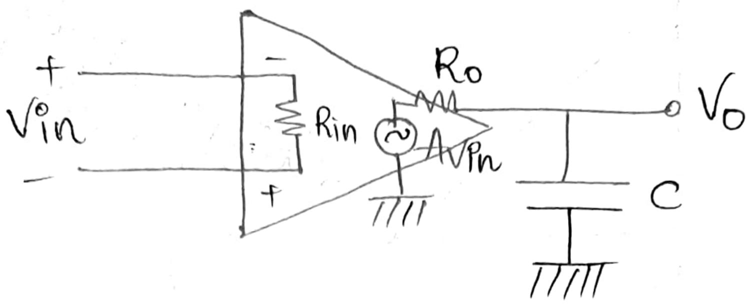
$$\boxed{V_x = \frac{V_1 R_2 R_3 + V_2 R_1 R_3 + V_3 R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}}$$

if $R_1 = R_2 = R_3 = R$

$$\boxed{V_x = \frac{V_1 + V_2 + V_3}{3}}$$

$$V_o = \left(1 + \frac{R_f}{R_a}\right) V_x \Rightarrow \boxed{V_o = \left(1 + \frac{R_f}{R_a}\right)\left(\frac{V_1 + V_2 + V_3}{3}\right)}$$

- High frequency OPAMP equivalent Circuit:



A – voltage gain at zero frequency.

$$V_o(s) = \frac{AV_{in}(s) \times \frac{1}{sC}}{R_o + \frac{1}{sC}}$$

$$\boxed{A_v(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{A}{1+sCR_o}}$$

(Gain in dB $= 20\log|A_{OL}(f)|$)

$s = j\omega$

$$\boxed{A_{OL}(f) = \frac{A}{1+j\omega CR_o} = \frac{A}{1+j(2\pi f)CR_o}}$$

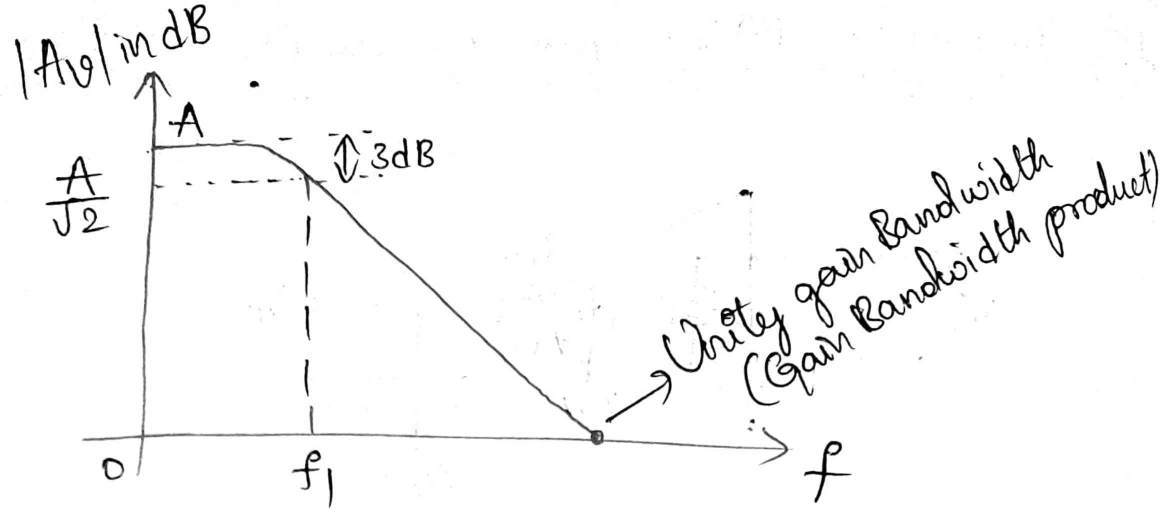
Open loop voltage gain.

$$\boxed{|A_{OL}(f)| = \frac{A}{\sqrt{1+(2\pi fCR_o)^2}}}$$

→ if $f=0$ then $|A_{OL}(f)|_{max} = A$

→ at $f = f_1$ (frequency at which gain is $\frac{1}{\sqrt{2}}$ times the max gain)

$$\boxed{f_1 = \frac{1}{2\pi CR_o}}$$ → Cut-off frequency | Break frequency.



for IC741 OPAMP

→ Break frequency for IC741 OPAMP is 5Hz.

→ At unity gain Bandwidth (UGB)

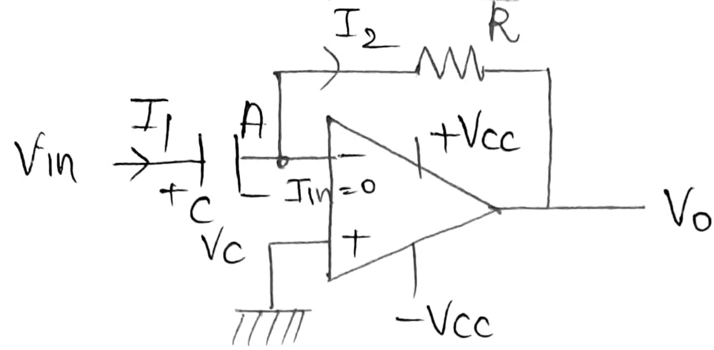
$$|A_{OL}(f)| = 1$$

$$1 = \frac{A}{\sqrt{1+\left(\frac{UGB}{f_1}\right)^2}}$$

$$1 = \frac{A}{\left(\frac{UGB}{f_1}\right)} \quad (\text{since } UGB >> 1)$$

$$\boxed{f_1 = \frac{UGB}{A}} \rightarrow \text{Break frequency}$$

- Differentiator using OPAMP:



at Node A,

$$\boxed{I_1 = I_2}$$

$$\frac{C\,dV_c}{dt} = \frac{0 - V_o}{R}$$

$$C\frac{d}{dt}(V_{in} - 0) = -\frac{V_o}{R}$$

$$\boxed{V_o = -RC\frac{dV_{in}}{dt}}$$

Apply laplace transform,

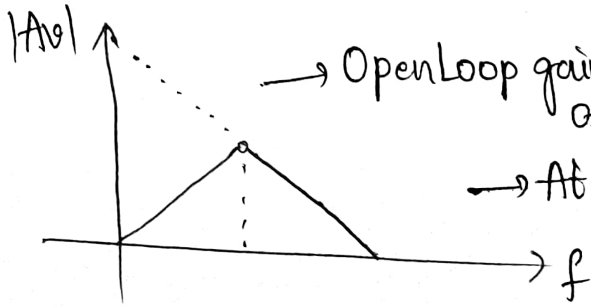
$$V_o(s) = -RCs\,V_{in}(s)$$

$$A_v(s) = \frac{V_o(s)}{V_{in}(s)} = -RCs = -j\omega RC$$

$$A_v(s) = -j(2\pi f)RC$$

$$\boxed{|A_v(s)| = 2\pi fRC}$$

→ This circuit cannot be used at low frequencies.

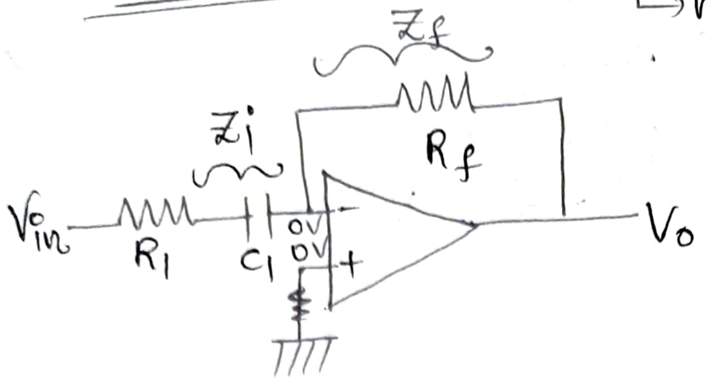


→ OpenLoop gain characteristics of OP-AMP.

→ At high frequencies, noise signal gets amplified.

- **Practical Differentiator using OPAMP :**

↳ noise will be still amplified.



$$\frac{V_o(s)}{V_{in}(s)} = -\frac{Z_f}{Z_i} = \frac{-R_f}{R_1 + \frac{1}{sC_1}}$$

$$V_o(s) = \frac{-V_{in}(s) \times R_f}{R_1 + \frac{1}{sC_1}}$$

$$V_o(s) = \frac{-V_{in}(s)}{\frac{R_1}{R_f} + \frac{1}{sC_1R_f}}$$

if $R_1 << R_f$, $\frac{R_1}{R_f} << 1$, so, $\longrightarrow V_o(s) = \frac{-V_{in}(s)}{\frac{1}{sC_1R_f}}$

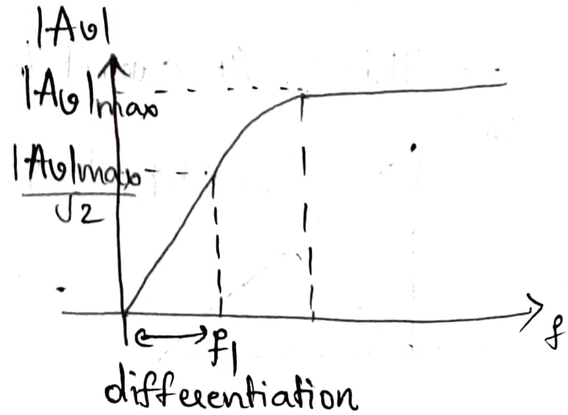
$$\boxed{V_o(s) = -sC_1R_fV_{in}(s)}$$

$$\boxed{V_o(t) \cong -R_fC_1\frac{dV_{in}}{dt}}$$

Frequency response,

$$A_v(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{-R_f}{R_1 + \frac{1}{sC_1}} = \frac{-R_f sC_1}{1+sR_1C_1} = \frac{-R_f j\omega C_1}{1+j\omega R_1C_1}$$

$$\boxed{|A_v| = \frac{R_f 2\pi f C_1}{\sqrt{1+\omega^2R_1^2C_1^2}}}$$



$f_1 \rightarrow$ cut-off frequency.

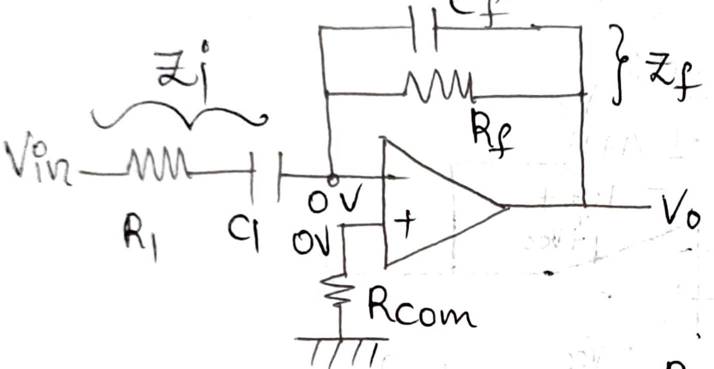
$$\frac{\cancel{R_f}}{R_1} \times \frac{1}{\sqrt{2}} = \frac{\cancel{R_f}(2\pi f_1)C_1}{\sqrt{1+(2\pi f_1)^2 R_1^2 C_1^2}}$$

$$1 + 4\pi^2 f_1^2 R_1^2 C_1^2 = 4\pi^2 f_1^2 R_1^2 C_1^2 \times 2$$

$$1 = 4\pi^2 f_1^2 R_1^2 C_1^2$$

$$\boxed{f_1 = \frac{1}{2\pi f_1 C_1 R_1}}$$ → frequency at which gain is $\frac{1}{\sqrt{2}}$ times max gain.

- Improved practical Differentiator using OPAMP:

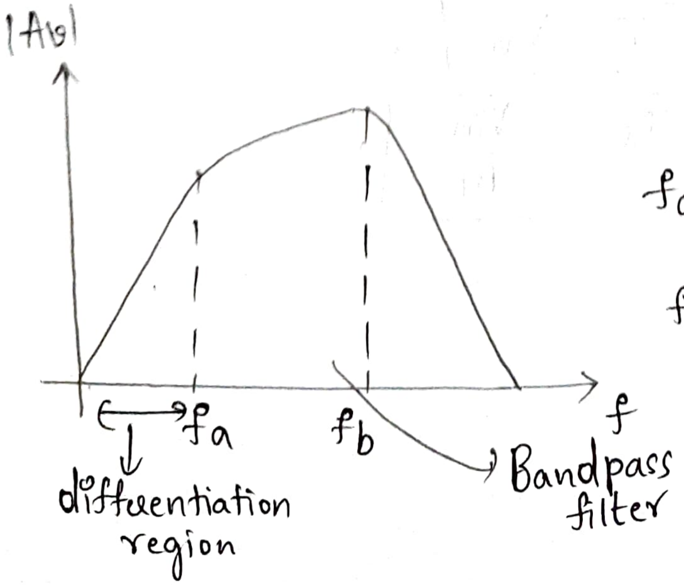


$$\frac{V_o(s)}{V_{in}(s)} = -\frac{Z_f}{Z_i} = \frac{\frac{1}{sC_f} \,\|\, R_f}{R_1 + \frac{1}{sC_1}} = \frac{\frac{R_f \times 1}{sC_f} \Big/ R_f + \frac{1}{sC_f}}{R_1 + \frac{1}{sC_1}}$$

$$A_v(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{R_f}{\cancel{sC_f}} \times \frac{\cancel{sC_f}}{1+R_fC_fs}}{\frac{1+R_1C_1s}{sC_1}}$$

$$\boxed{A_v(s) = \frac{sR_fC_1}{(1+sC_fR_f)(1+sC_1R_1)}}$$ → having two poles,

$$\left\{ \begin{aligned} s_1 &= \frac{-1}{C_fR_f} \\ s_2 &= \frac{-1}{C_1R_1} \end{aligned} \right\}$$



$$f_a = \frac{1}{2\pi R_1 C_1}$$

$$f_b = \frac{1}{2\pi R_f C_f}$$

when $f_a = f_b$,

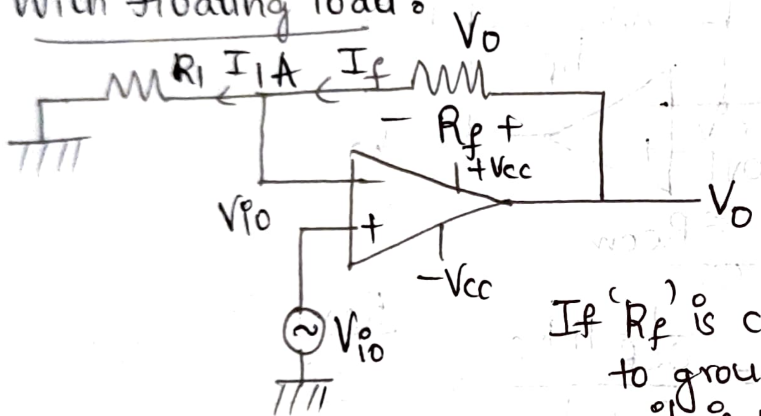
$$\frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_f C_f} \Rightarrow \boxed{R_1 C_1 = R_f C_f}$$

$$\boxed{|A_v(j\omega)| = \frac{\omega R_1 C_1}{\sqrt{(1+(\omega C_f R_f)^2) \times (1+(\omega C_1 R_1)^2)}}}$$

- Voltage to current convertor:

↳ produces o/p current which is proportional to i/p voltage.

① With floating load:



If 'R_f' is connected to ground then it is non-floating load.

At node A, (KCL):

$$I_1 = I_f$$

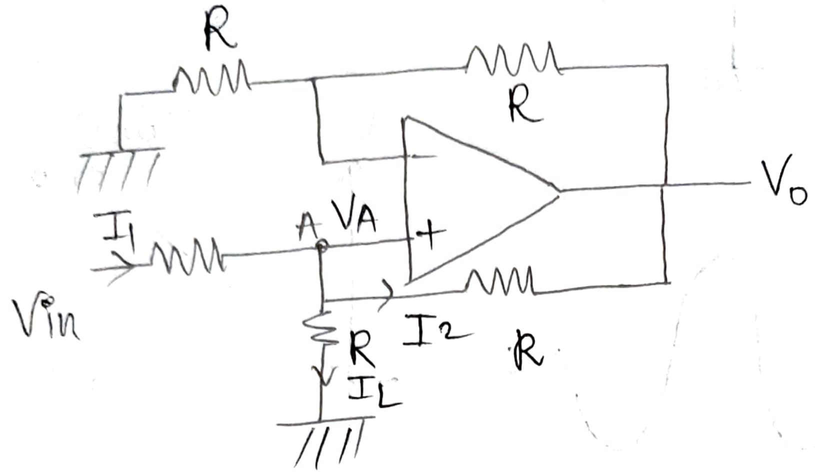
$$\frac{V_{in} - 0}{R_1} = \frac{V_o - V_{in}}{R_f} = I_f$$

$$\boxed{I_f \propto V_{in}}$$

$$\boxed{I_f = \frac{V_{in}}{R_1}}$$

- Voltage to current convertor:

② With grounded load



Apply KCL at node A,

$$I_1 = I_2 + I_L$$

$$\frac{V_{in} - V_A}{R} = \frac{V_A - V_o}{R} + I_L$$

$$\boxed{I_L = \frac{V_{in} - 2V_A + V_o}{R}} \quad -①$$

Since, it is non-inverting amplifier, Gain is $1 + \frac{R_f}{R_a}$

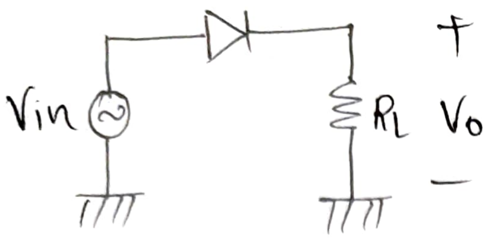
$$\frac{V_o}{V_A} = 1 + \frac{R}{R} = 2$$

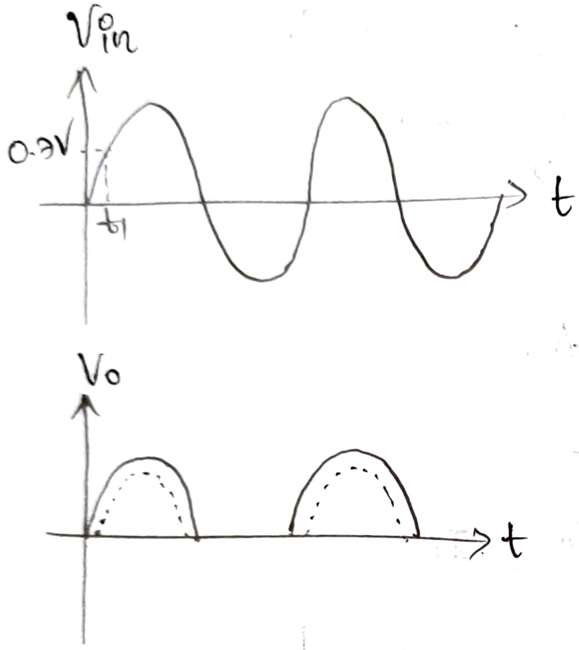
$$\boxed{V_o = 2V_A} \quad -②$$

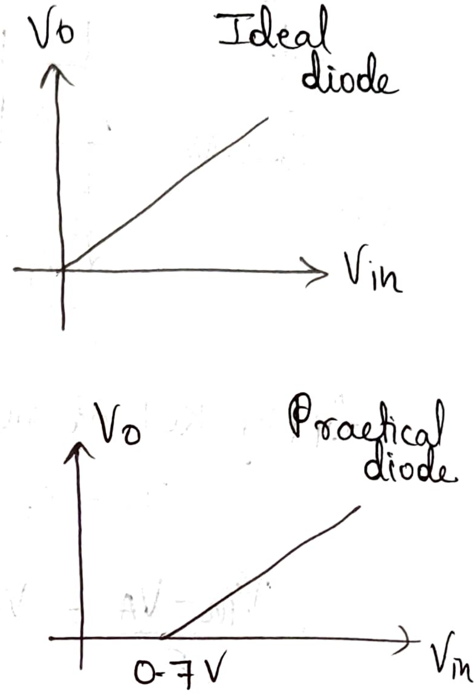
from ① & ②,

$$\boxed{I_L = \frac{V_{in}}{R}}$$

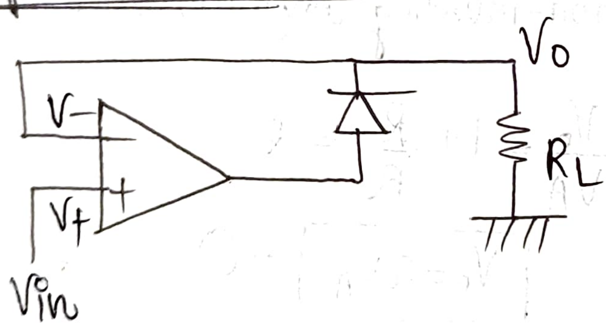
- Half-wave rectifier:

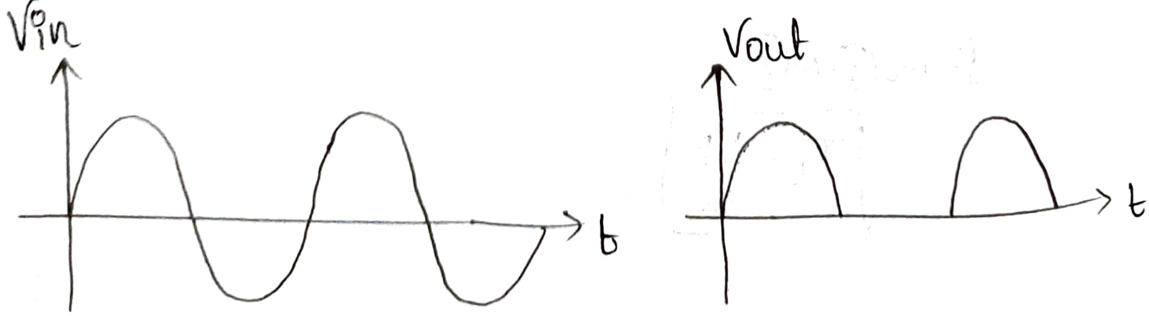






- Half-wave precision rectifier:

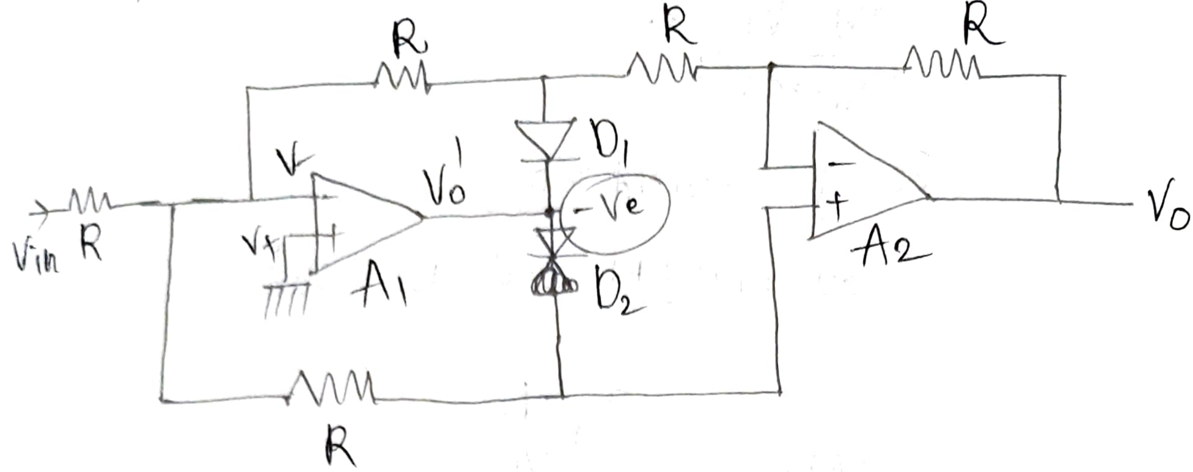




Whenever V_{in} is '+ve' and $V_o > 0.7V$ then diode conducts and is forward Biased.

Whenever V_{in} is '−ve' then diode is Reverse Biased.

- Full wave precision Rectifier:



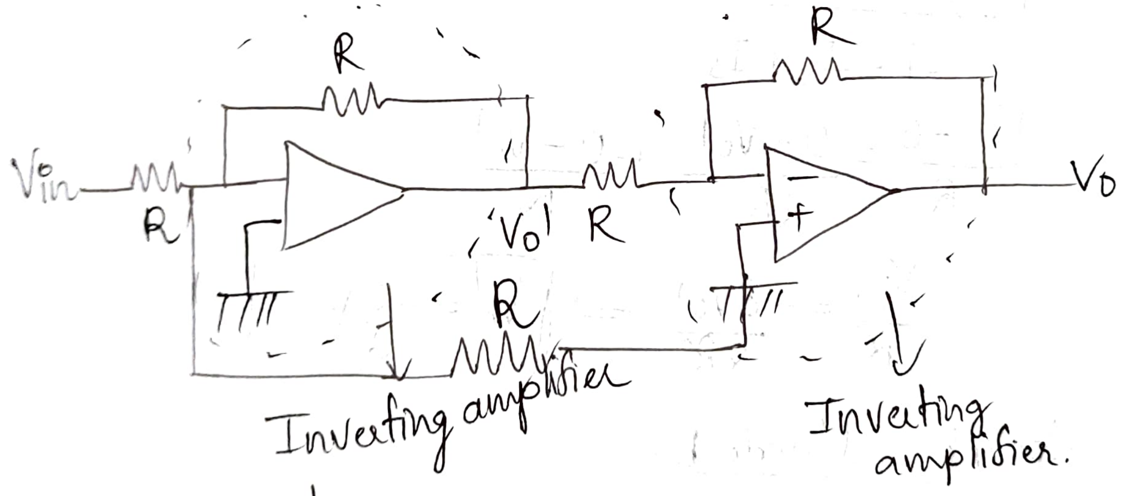
Whenever V_{in} is '+Ve'

$$V_d = V_+ - V_-$$

$$\boxed{V_d < 0}$$

$$V_o' = A_d V_d < 0$$

So, D_1 is ON and D_2 is OFF



$$\frac{V_o'}{V_{in}} = -\frac{R_f}{R_1} = -\frac{R}{R} = -1$$

$$\boxed{V_o' = -V_{in}}$$

$$\frac{V_o}{V_o'} = -\frac{R}{R} = -1$$

$$\boxed{V_o = -V_o' = V_{in}}$$

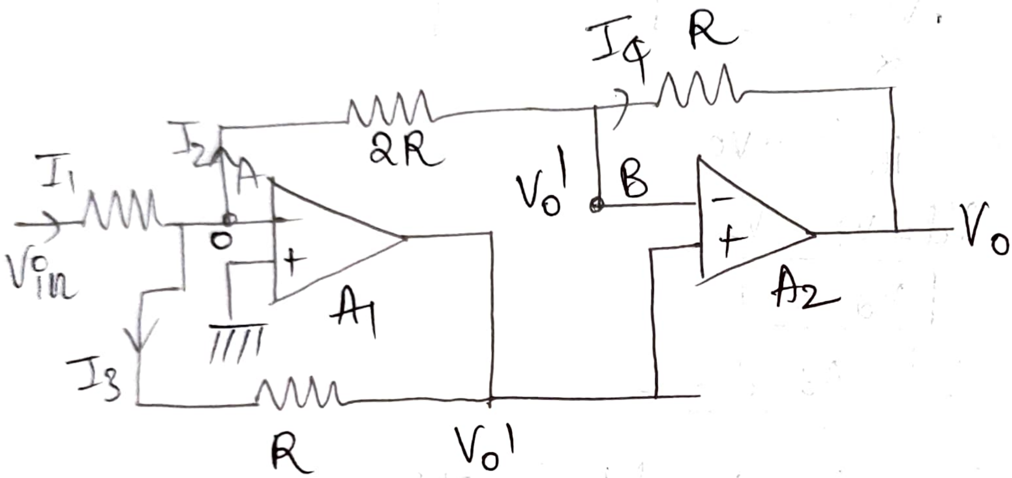
Whenever, V_{in} is '$-ve$',

$$V_d = V_+ - V_-$$

$$\boxed{V_d \geq 0V}$$

So, $V_o' = A_d V_d > 0$

So, D_1 is OFF and D_2 is ON



Apply KCL at Node A,

$$I_1 = I_2 + I_3$$

$$\frac{V_{in} - 0}{R} = \frac{0 - V_o'}{2R} + \frac{0 - V_o'}{R}$$

$$\frac{V_{in}}{\cancel{R}} = -\frac{3V_o'}{2\cancel{R}} \Rightarrow \boxed{V_o' = -\frac{2}{3}V_{in}}$$

Apply KCL at node B,

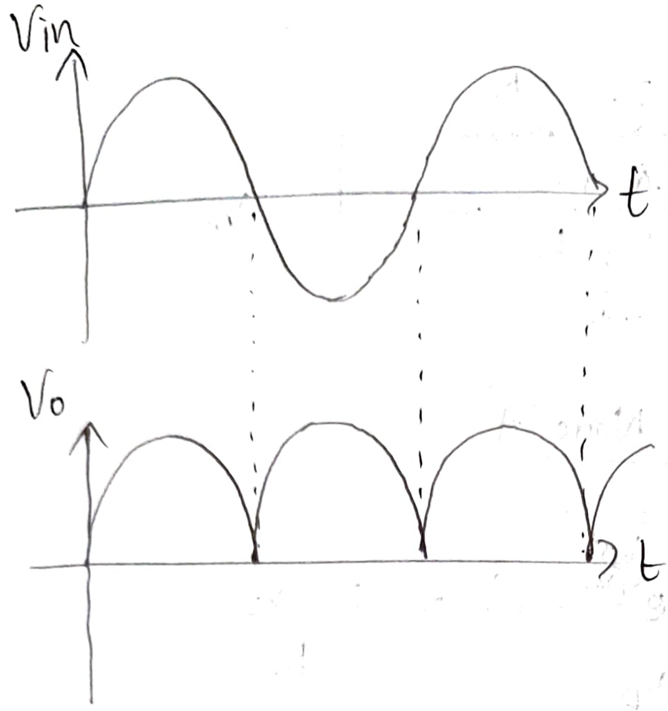
$$I_2 = I_4$$

$$\frac{0 - V_o'}{2\cancel{R}} = \frac{V_o' - V_o}{\cancel{R}}$$

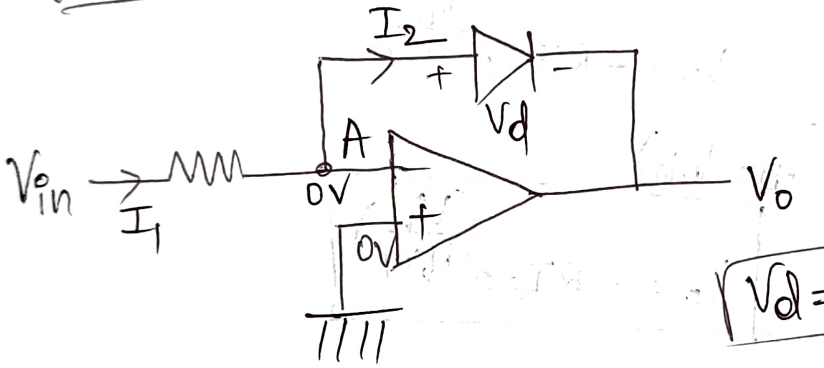
$$3V_o' = 2V_o$$

$$V_o = \frac{3}{2}V_o' \rightarrow V_o = \frac{\cancel{3}}{\cancel{2}}\left(-\frac{\cancel{2}}{\cancel{3}}V_{in}\right)$$

$$\boxed{V_o = -V_{in}}$$



- **Logarithmic Amplifier using OPAMP:**



$$\boxed{V_d = -V_o}$$

Apply KCL at node A,

saturation current,

$$I_1 = I_2$$

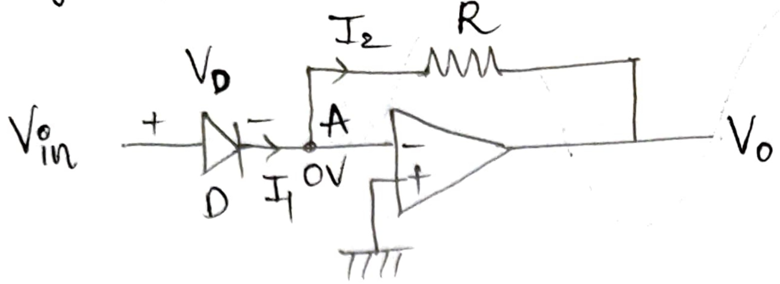
$$\frac{V_{in} - 0}{R} = I_2 = I_S\left(e^{\frac{V_d}{\eta V_T}} - 1\right)$$

$$\frac{V_{in}}{R} \simeq I_S \cdot e^{\frac{V_d}{\eta V_T}}$$

$$\ln\left(\frac{V_{in}}{I_S R}\right) = \frac{-V_o}{\eta V_T}$$

$$\boxed{V_o = -\eta V_T \ln\left(\frac{V_{in}}{I_S R}\right)}$$

Antilog Amplifier:



Apply KCL at Node 'A'.

$$I_1 = I_2$$

$$I_s\left(e^{\frac{V_D}{\eta V_T}} - 1\right) = \frac{0 - V_o}{R}$$

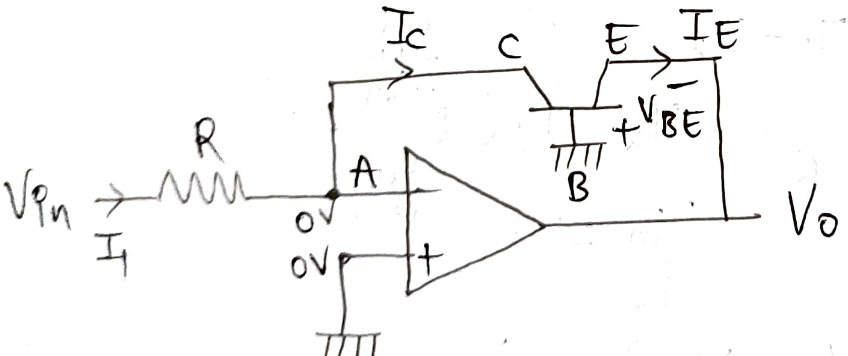
$$I_s e^{\frac{V_D}{\eta V_T}} \cong \frac{-V_o}{R}$$

$$V_o = -R I_s e^{\frac{V_D}{\eta V_T}}$$

$(V_D = V_{in})$

$$\boxed{V_o = -R I_s e^{\left(\frac{V_{in}}{\eta V_T}\right)}}$$

Logarithmic Amplifier using BJT:



$$\boxed{-V_{BE} \approx V_o}$$

$$\boxed{I_C \cong I_E}$$

$$\boxed{I_E \cong I_s\left(e^{\frac{V_{BE}}{V_T}}\right)}$$

Apply KCL at node A

$$I_1 = I_E$$

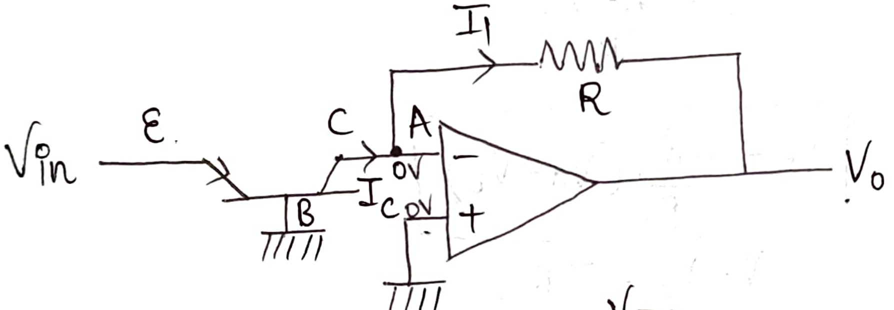
$$\frac{V_{in} - 0}{R} = I_S e^{\frac{V_{BE}}{V_T}}$$

$$\frac{V_{in}}{R} = I_S e^{\frac{-V_o}{V_T}}$$

$$\frac{V_{in}}{I_S R} = e^{\frac{-V_o}{V_T}}$$

$$\boxed{V_o = -V_T \ln\left(\frac{V_{in}}{I_S R}\right)}$$

- Anti-log Amplifier using BJT:



$$I_C \cong I_E \cong I_S e^{\frac{V_{EB}}{V_T}}$$

$$\boxed{I_C \cong I_S e^{\frac{V_{in}}{V_T}}}$$

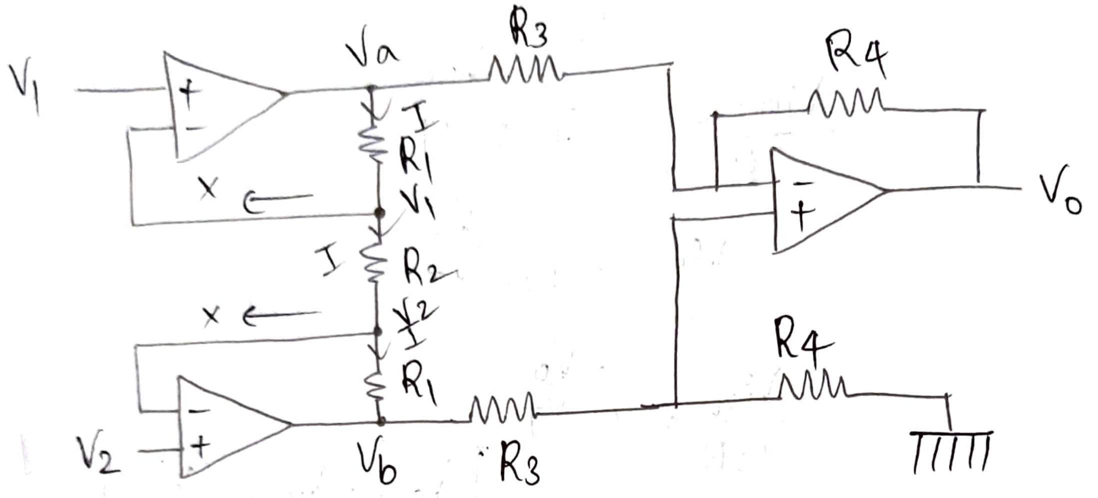
Apply KCL at Node A,

$$I_C = I_1$$

$$I_S e^{\frac{V_{in}}{V_T}} = \frac{0 - V_o}{R}$$

$$\boxed{V_o = -R I_S e^{\frac{V_{in}}{V_T}}}$$

Instrumentation Amplifier :



$$I = \frac{V_a - V_b}{2R_1 + R_2}$$

$$I = \frac{V_1 - V_2}{R_2}$$

$$\boxed{V_o = -\frac{R_4}{R_3}(V_a - V_b)}$$

$$V_1 - V_2 = IR_2$$

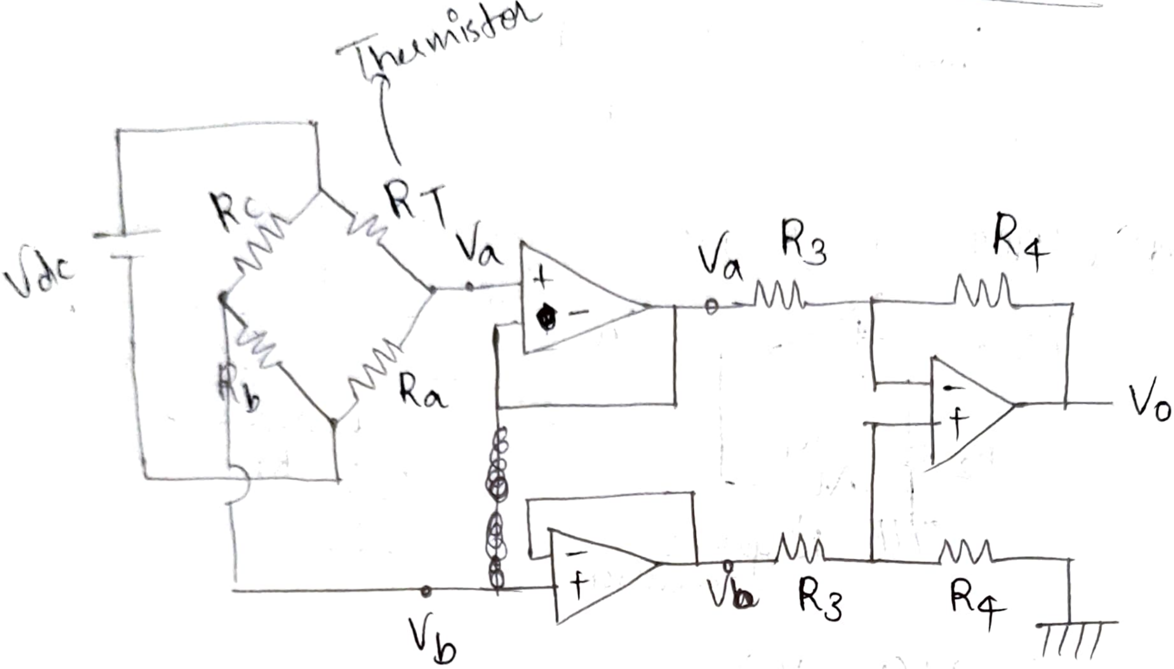
$$V_1 - V_2 = \frac{(V_a - V_b)R_2}{2R_1 + R_2}$$

$$V_a - V_b = \frac{(V_1 - V_2)(2R_1 + R_2)}{R_2}$$

$$\boxed{V_o = \frac{(2R_1 + R_2)R_4}{R_2 R_3}(V_2 - V_1)}$$

$$\boxed{V_o = \frac{R_4}{R_3}\left(1 + \frac{2R_1}{R_2}\right)(V_2 - V_1)}$$

- Instrumentation Amplifier with Resistive Transducer bridge :



① Initially bridge is balanced for some value of R_T

$$V_a = V_b$$

$$V_a = \frac{V_{dc}}{R_T + R_a} \times R_a$$

$$V_b = \frac{V_{dc}}{R_b + R_c} \times R_b$$

Since $V_a = V_b$ $\left(V_o = V_a - V_b = 0\,V \right)$

② If bridge is not balanced,

$$V_a = \frac{V_{dc} \times R_a}{R_T \pm \Delta R + R_a}$$

$$V_b = \frac{V_{dc} \times R_b}{R_c + R_b}$$

Assuming $R_a = R_b = R_c = R_T = R$

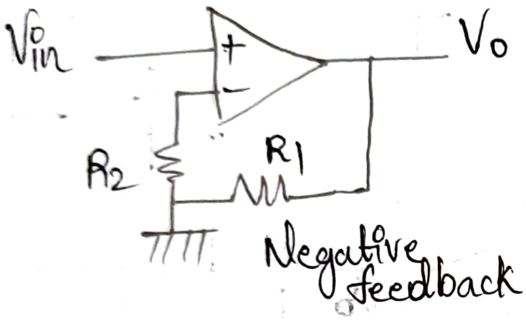
$$V_o = \cancel{V_a - V_b}\ V_b - V_a = \frac{V_{dc} \times \cancel{R}}{2\cancel{R}} - \frac{V_{dc} \times R}{2R \pm \Delta R} = \frac{\pm V_{dc}\,\Delta R}{4R \pm 2\Delta R}$$

$(4R >>> 2\Delta R)$ $\boxed{V_o = \pm\frac{V_{dc}\,\Delta R}{4R}}$

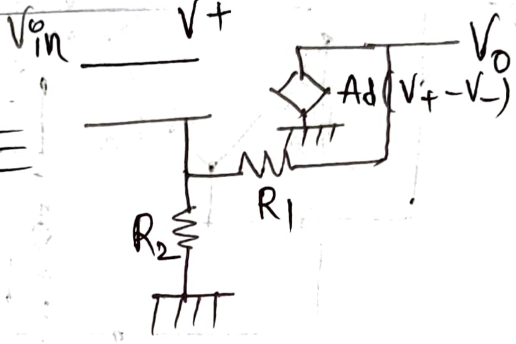
$$V_{out} = \frac{R_4}{R_3}(V_b - V_a)$$

$$\boxed{V_{out} = \frac{R_4}{R_3}\left[\frac{\pm V_{dc}\,\Delta R}{4R}\right]}$$

(*) OPAMP with Negative feedback: (Stable circuit)



$\equiv$



$$V_o = A_d(V_+ - V_-)$$

$$V_+ = V_{in}$$

$$V_- = \frac{V_o \times R_2}{R_1 + R_2}$$

$$V_o = A_d\left(V_{in} - \frac{V_o R_2}{R_1 + R_2}\right)$$

$$V_o\left[1 + \frac{R_2 A_d}{R_1 + R_2}\right] = A_d \times V_{in}$$

$$\frac{V_o}{V_{in}} = \frac{A_d}{1 + \frac{R_2 A_d}{R_1 + R_2}} \simeq \frac{\cancel{A_d}}{\frac{R_2 \cancel{A_d}}{R_1 + R_2}}$$

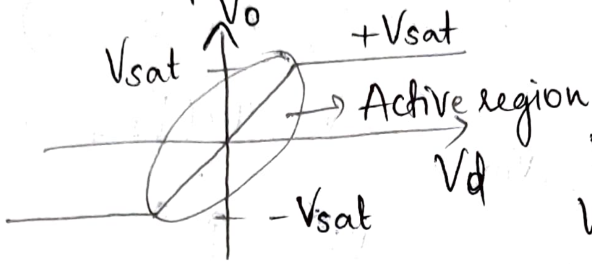
$$\boxed{\frac{V_o}{V_{in}} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}}$$

→ $V_+ = V_-$ – Virtual short circuit

This concept is applicable only when,

(i) OPAMP is connected in negative feedback.

(ii) OPAMP is operated in Active or linear region.



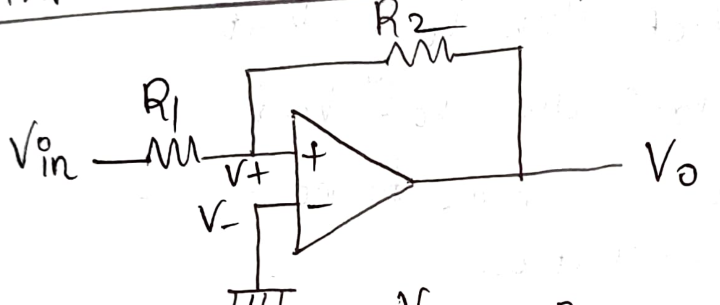
$V_d = V_+ - V_-$

$V_0 = A_d(V_+ - V_-)$

$V_+ = V_-$

$V_+ - V_- = \frac{V_0}{A_d} \simeq 0$

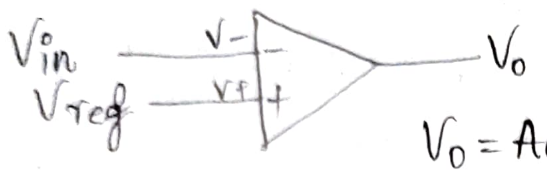
* OPAMP with Positive feedback: (Unstable circuit)



$$\frac{V_0}{V_{in}} = -\frac{R_2}{R_1}$$

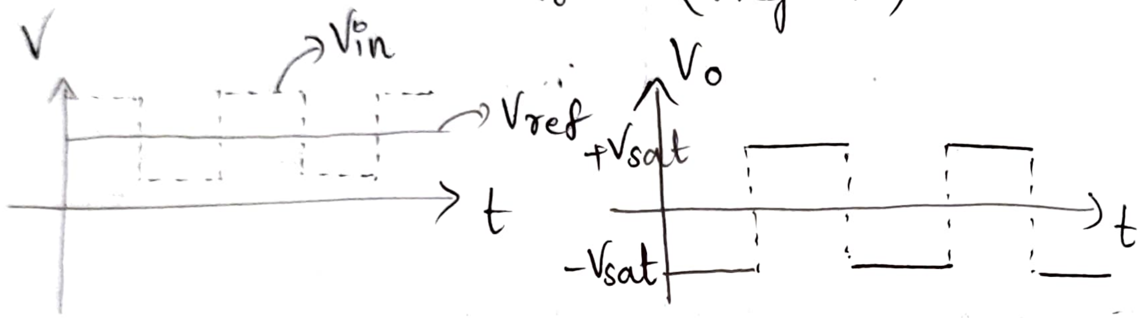
→ Output (V_0) is not stable, it gradually attains a large negative value.

(*) Inverting Comparator circuit :



$$V_0 = A_d(V_+ - V_-)$$

$$V_0 = A_d(V_{ref} - V_{in})$$

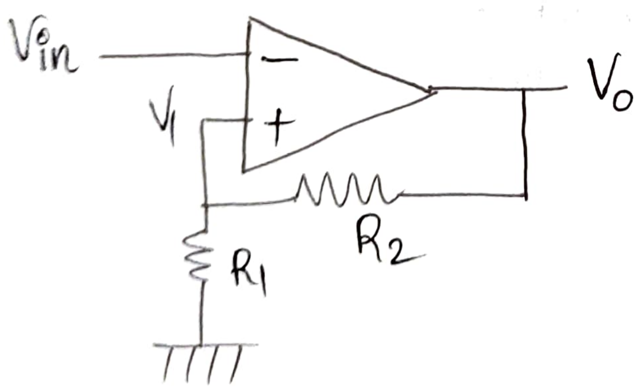


→ if $V_{in} > V_{ref}$ then $V_0 \cong -V_{sat}$.

→ if $V_{in} < V_{ref}$ then $V_0 \cong V_{sat}$.

→ This circuit is immune to noise because output is not stable.

(*) Inverting Schmitt Trigger circuit :



Assume that $V_0 = +V_{sat}$

$$V_d = V_+ - V_- = V_1 - V_{in}$$

$$\boxed{V_1 = \frac{+V_{sat} \times R_1}{R_1 + R_2} = V_{UT} \rightarrow \text{Upper threshold Voltage.}}$$

① If $V_{in} >$ Upper threshold Voltage (V_{UT}) then $V_0 = -V_{sat}$

② If $V_{in} < V_{UT}$ then $V_0 = +V_{sat}$.

Initial o
Voltage
$+V_{sat}$

Assume $V_0 = -V_{sat}$,

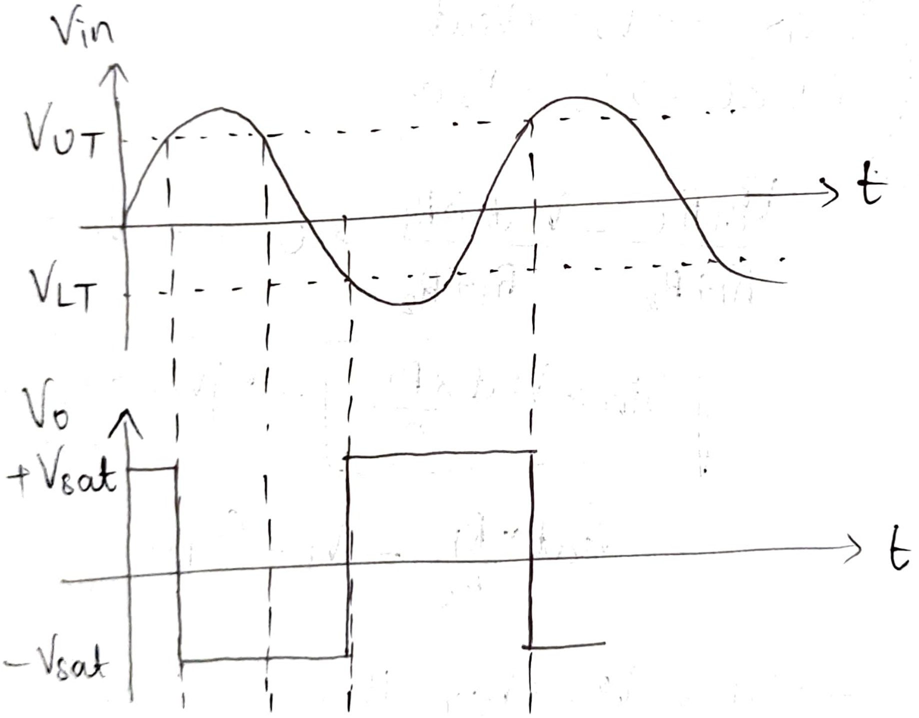
$$\boxed{V_1 = -\frac{V_{sat} \times R_1}{R_1 + R_2} = V_{LT}}$$ - lower threshold Voltage.

$$V_d = V_+ - V_- = V_{LT} - V_{in}$$

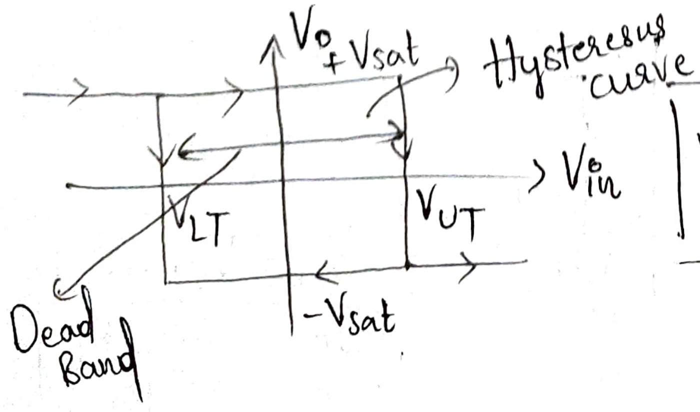
① If $V_{in} > V_{LT}$, $V_0 = -V_{sat}$

② If $V_{in} < V_{LT}$, $V_0 = +V_{sat}$.

Initial o/p voltage $-V_{sat}$

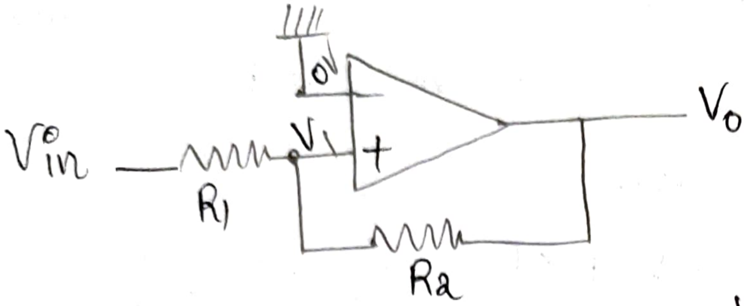


$$V_{UT} - V_{LT} = V_H \text{ (Hysteresus Voltage)}$$



$$\boxed{V_H = \frac{2V_{sat} \times R_1}{R_1 + R_2}}$$

⊛ Non-inverting Schmitt Trigger:



Positive feedback

① when, $V_o = -V_{sat}$ → initial o/p voltage

$$V_1 = \frac{V_{in} \times R_2}{R_1 + R_2} + \frac{V_o \times R_1}{R_1 + R_2}$$

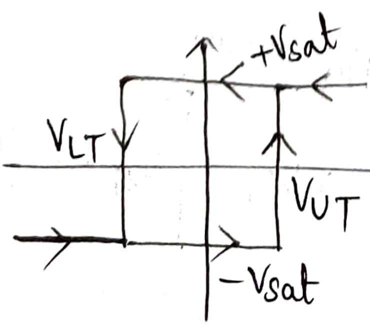
if $V_1 > 0 \rightarrow V_o = +V_{sat}$
if $V_1 < 0 \rightarrow V_o = -V_{sat}$

$$\frac{V_{in} R_2}{R_1 + R_2} - \frac{V_{sat} \times R_1}{R_1 + R_2} > 0 \rightarrow V_o = +V_{sat}$$

$$\boxed{V_{in} > V_{sat} \times \frac{R_1}{R_2}} \rightarrow \boxed{V_{in} > V_{UT}}$$

$$V_{sat} \times \frac{R_1}{R_2} = V_{UT}$$ (Upper threshold Voltage)

→ When $V_{in} > V_{UT}$ then $V_o = +V_{sat}$
→ When $V_{in} < V_{UT}$ then $V_o = -V_{sat}$



Hystereses curve (Transfer characteristics)

$$V_H = V_{UT} - V_{LT}$$

$$\boxed{V_H = \frac{2 V_{sat} R_1}{R_2}}$$

② when, $V_o = +V_{sat}$ → initial o/p voltage.

$$V_1 = \frac{V_{in} R_2}{R_1 + R_2} + \frac{V_{sat} R_1}{R_1 + R_2}$$

if $V_1 > 0 \rightarrow V_o = +V_{sat}$

if $V_1 < 0 \rightarrow V_o = -V_{sat}$

$$\frac{V_{in} R_2}{R_1 + R_2} + \frac{V_{sat} R_1}{R_1 + R_2} > 0 \rightarrow V_o = +V_{sat}$$

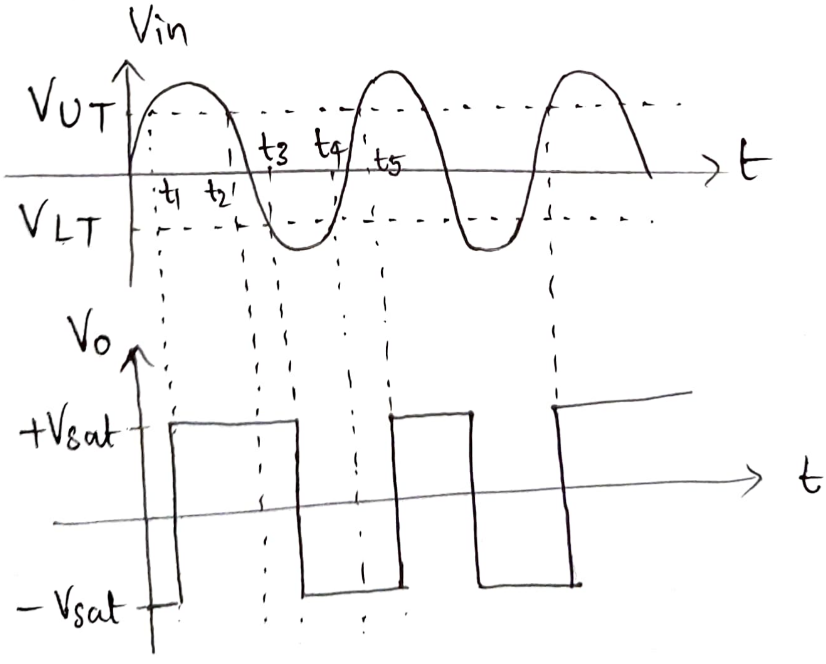
$$V_{in} > -V_{sat} \times \frac{R_1}{R_2} \rightarrow V_o = +V_{sat}$$

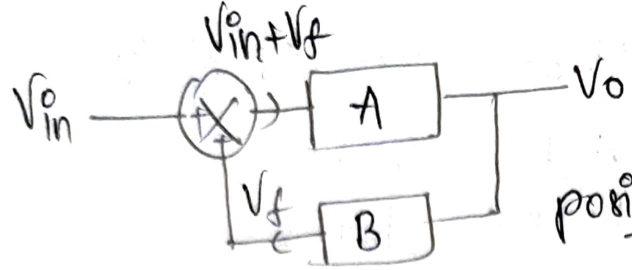
$$\boxed{V_{in} > V_{LT}}$$

$$V_{LT} = -V_{sat} \times \frac{R_1}{R_2}$$ (lower threshold Voltage)

→ When, $V_{in} > V_{LT} \rightarrow V_o = +V_{sat}$

→ When $V_{in} < V_{LT} \rightarrow V_o = -V_{sat}$





positive feedback,

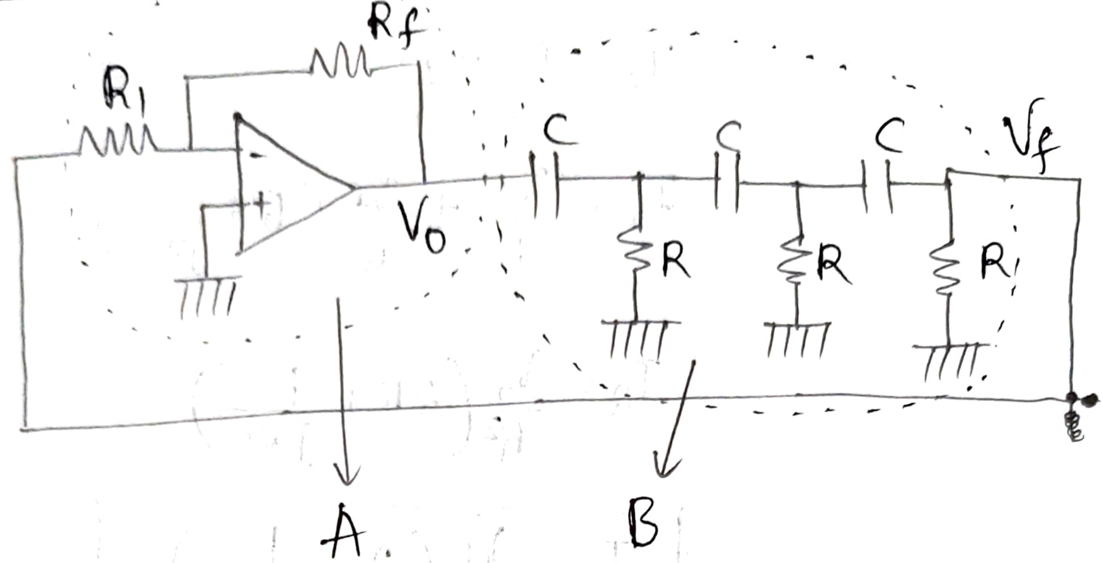
$$\frac{V_o}{V_{in}} = \frac{A}{1-AB}$$

AB – loop gain
AB → open loop gain

→ if $AB=1$ then $\frac{V_o}{V_{in}} = \frac{A}{0} = \infty$

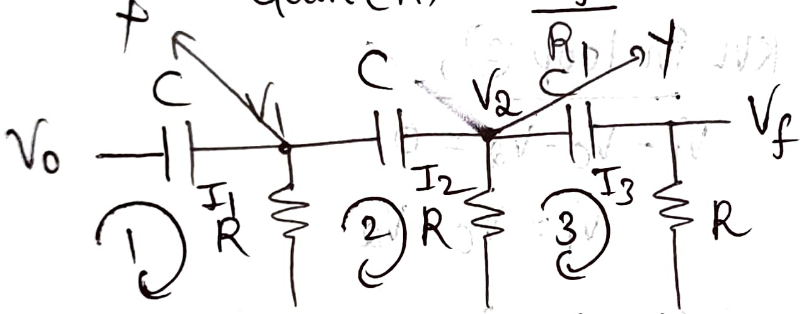
- Barkhausen criteria,
  ① Circuit will produce sustained oscillations when two conditions are satisfied,
  (i) loop gain $AB=1$
  (ii) Total phase shift around closed loop is 0° or integral multiples of 2π.

⊛ RC-phase shift oscillator:



Since it is inverting amplifier,

$$\text{Gain}(A) = -\frac{R_f}{R_1}$$



Apply KVL in loop 3,

$$V_2 - V_c - V_f = 0$$

$$V_2 = V_c + V_f$$

$$V_2 = \frac{1}{j\omega C} \times I_3 + V_f$$

But, $\boxed{I_3 = \frac{V_f}{R}}$

$$V_2 = \frac{1}{j\omega C} \times \frac{V_f}{R} + V_f$$

$$\boxed{V_2 = \frac{V_f}{R}\left(R + \frac{1}{j\omega C}\right)}$$

Apply KCL at node Y,

$$I_2 = I_3 + \frac{V_2}{R}$$

$$I_2 = \frac{V_f}{R} + \frac{V_f}{R^2}\left(R + \frac{1}{j\omega C}\right)$$

$$I_2 = \frac{V_f}{R^2}\left(2R + \frac{1}{j\omega C}\right)$$

$$\boxed{I_2 = \frac{V_f}{R}\left(2 + \frac{1}{j\omega CR}\right)}$$

Apply KVL in loop ②,

$$V_1 - V_C - V_2 = 0$$

$$V_1 = V_C + V_2$$

$$V_1 = V_f\left(1 + \frac{1}{j\omega RC}\right) + I_2 \times \frac{1}{j\omega C}$$

$$V_1 = V_f\left(1 + \frac{1}{j\omega RC}\right) + \frac{V_f}{R}\left(2 + \frac{1}{j\omega RC}\right) \times \frac{1}{j\omega C}$$

$$V_1 = V_f + \frac{3V_f}{j\omega RC} + \frac{V_f}{(j\omega RC)^2}$$

$$V_1 = V_f + \frac{3V_f}{j\omega RC} - \frac{V_f}{\omega^2 R^2 C^2}$$

$$\boxed{V_1 = V_f\left(1 + \frac{3}{j\omega RC} - \frac{1}{\omega^2 R^2 C^2}\right)}$$

Apply KCL at node X,

$$I_1 = I_2 + \frac{V_1}{R}$$

$$I_1 = \frac{V_f}{R}\left(2+\frac{1}{j\omega RC}\right) + \frac{V_f}{R}\left(1+\frac{3}{j\omega RC} - \frac{1}{\omega^2R^2C^2}\right)$$

$$\boxed{I_1 = \frac{V_f}{R}\left(3+\frac{4}{j\omega RC} - \frac{1}{\omega^2R^2C^2}\right)}$$

Apply KVL in loop 1,

$$V_0 - V_c - V_1 = 0$$

$$V_0 = V_1 + V_c = V_1 + \frac{I_1}{j\omega C}$$

$$V_0 = V_f\left(1+\frac{3}{j\omega RC} - \frac{1}{\omega^2R^2C^2}\right) + \frac{V_f}{j\omega RC}\left(3+\frac{4}{j\omega RC} - \frac{1}{\omega^2R^2C^2}\right)$$

$$\boxed{V_0 = V_f\left(1+\frac{6}{j\omega RC} - \frac{5}{\omega^2R^2C^2} - \frac{1}{j\omega^3R^3C^3}\right)}$$

$$\frac{1}{B} = \frac{V_0}{V_f} = 1+\frac{6}{j\omega RC} - \frac{5}{\omega^2R^2C^2} - \frac{1}{j\omega^3R^3C^3}$$

for 'B' to be real,

$$\frac{6}{j\omega RC} - \frac{1}{j(\omega RC)^3} = 0$$

$$\frac{6}{\omega RC} = \frac{1}{(\omega RC)^3}$$

$$\boxed{f_0 = \frac{1}{2\pi RC\sqrt{6}}} \leftarrow \boxed{\omega_0 = \frac{1}{\sqrt{6}RC}}$$

$$\frac{1}{B} = 1 - \frac{5}{\omega_0^2 R^2 C^2}$$

$$\frac{1}{B} = 1 - \frac{5}{\frac{1}{6\cancel{R^2}\cancel{C^2}} \times \cancel{R^2}\cancel{C^2}}$$

$$\frac{1}{B} = 1 - 30 \rightarrow \boxed{B = -\frac{1}{29}}$$

$$AB = 1$$

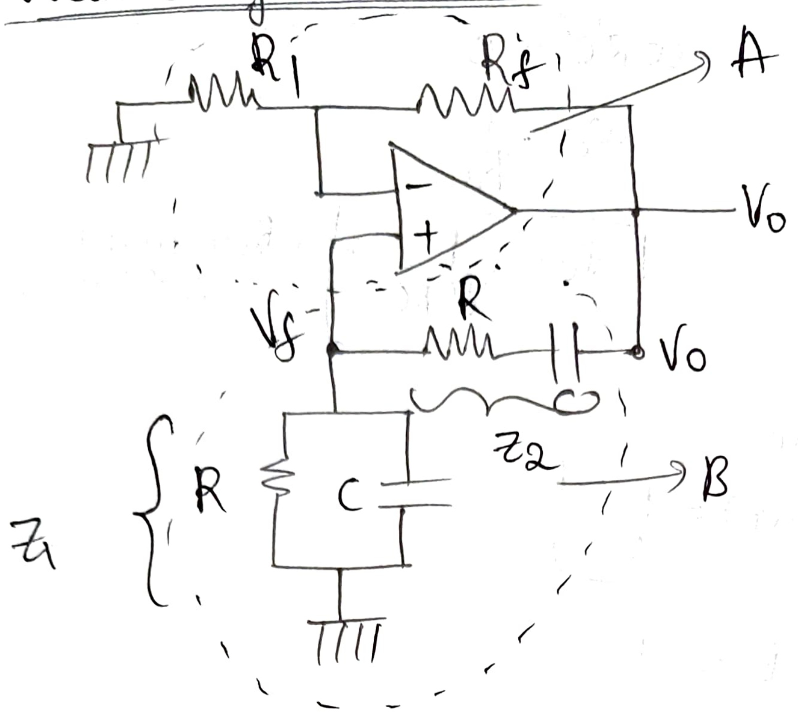
$$A \times \left(-\frac{1}{29}\right) = 1$$

$$\boxed{A = -29}$$

$$\cancel{-}\frac{R_f}{R_1} = \cancel{-}29$$

$$\boxed{R_f = 29 R_1}$$

* Wein-Bridge Oscillator:



$$\boxed{A = 1 + \frac{R_f}{R_1}}$$

$$\boxed{B = \frac{V_f}{V_o}}$$

$$V_f(s) = \frac{V_o(s) \times Z_1(s)}{Z_1(s) + Z_2(s)}$$

$$Z_1(s) = R \parallel \frac{1}{sC} = \frac{R}{1+sCR}$$

$$Z_2(s) = R + \frac{1}{sC} = \frac{1+sCR}{sC}$$

$$B(s) = \frac{V_f(s)}{V_o(s)} = \frac{Z_1(s)}{Z_1(s) + Z_2(s)} = \frac{R/1+sCR}{\frac{R}{1+sCR} + \frac{1+sCR}{sC}}$$

$$= \frac{sCR}{sCR + 1 + 2sCR + s^2C^2R^2}$$

$$B(s) = \frac{V_f(s)}{V_o(s)} = \frac{sCR}{1 + 3sCR + s^2C^2R^2}$$

Put $s = j\omega$,

$$B(j\omega) = \frac{j\omega RC}{1 + 3j\omega RC + (j\omega)^2 C^2R^2}$$

$$\boxed{B(j\omega) = \frac{j\omega RC}{1 + 3j\omega RC - \omega^2 R^2 C^2}}$$

For $B(j\omega)$ to be real,

$$1-\omega^2R^2C^2=0$$

$$\omega_0=\frac{1}{RC}$$

$$\boxed{f_0=\frac{1}{2\pi RC}}$$ → frequency of oscillation

At $\omega=\omega_0$

$$\boxed{B(j\omega_0)=\frac{1}{3}}$$

$$AB=1$$

$$A\times\frac{1}{3}=1$$

$$\boxed{A=3}$$

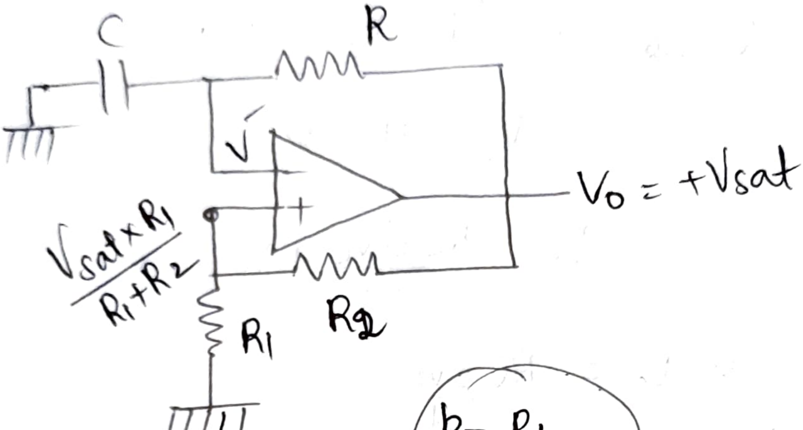
$$1+\frac{R_f}{R_1}=3$$

$$\boxed{R_f=2R_1}$$

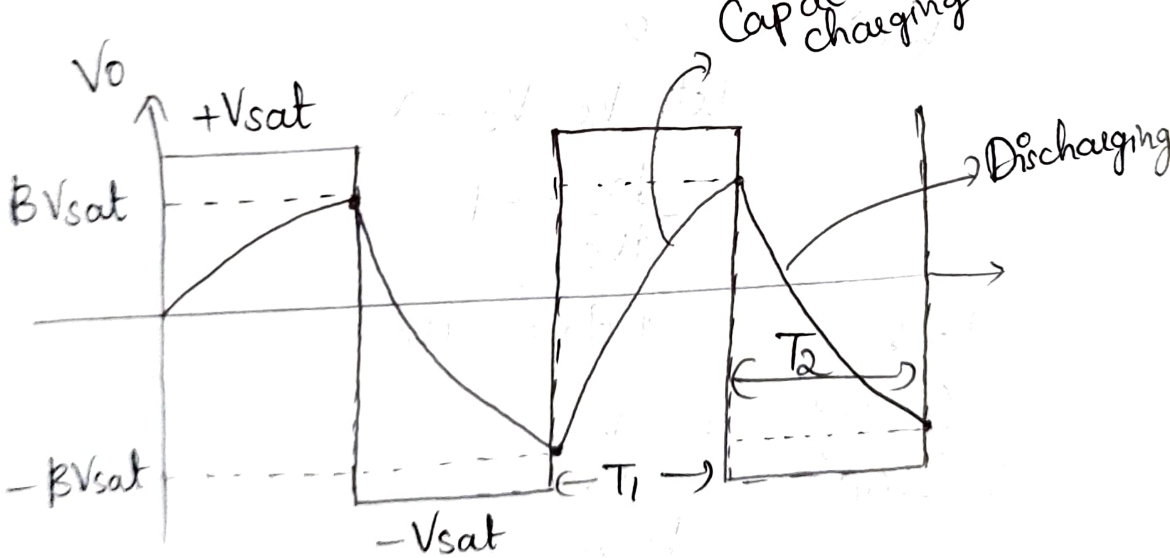
⊛ Astable Multivibrator (or) Square wave Generator :

→ It has zero stable states.

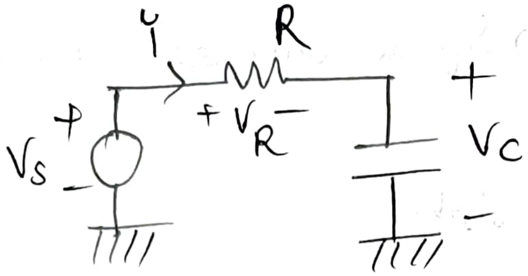
→ It has 2 Quasi stable states.
↓
temporary stable state.



$$\beta = \frac{R_1}{R_1 + R_2}$$



$$\left.\begin{aligned} V_0 &= +V_{sat} \\ &= -V_{sat} \end{aligned}\right\} \text{Two Quasi stable states.}$$



$i = C\dfrac{dV_c}{dt}$

RC-series circuit

$$V_S - iR - V_C = 0$$

$$V_S = iR + V_C$$

$$V_S = RC\frac{dV_C}{dt} + V_C$$

$$V_S - V_C = RC\frac{dV_C}{dt}$$

$$\int_{V_C(0)}^{V_C} \frac{dV_C}{V_S - V_C} = \int_0^t \frac{dt}{RC}$$

Let $V_S - V_C = x$

$$\frac{dx}{dV_C} = -1$$

$$\boxed{dx = -dV_C}$$

$$-\int_{V_S - V_C(0)}^{V_S - V_C} \frac{dx}{x} = \frac{t}{RC}$$

$$-\ln|x| \Big|_{V_S - V_C(0)}^{V_S - V_C} = \frac{t}{RC}$$

$$-\left[\ln|V_S - V_C| - \ln|V_S - V_C(0)|\right] = \frac{t}{RC}$$

$$\ln\left|\frac{V_S - V_C(0)}{V_S - V_C}\right| = \frac{t}{RC}$$

$$\boxed{t = RC\ln\left|\frac{V_S - V_C(0)}{V_S - V_C}\right|}$$

(i) $V_S = +V_{sat}$,

$V_C(0) = -\beta V_{sat}$

$V_C = +\beta V_{sat}$

$$T_1 = RC\ln\left|\frac{V_{sat}+\beta V_{sat}}{V_{sat}-\beta V_{sat}}\right|$$

$$T_1 = RC\ln\left|\frac{1+\beta}{1-\beta}\right| \quad \left(\beta = \frac{R_1}{R_1+R_2}\right)$$

$$T_1 = RC\ln\left|\frac{2R_1+R_2}{R_2}\right|$$

$$\boxed{T_1 = RC\ln\left|1+\frac{2R_1}{R_2}\right|} \rightarrow \text{Capacitor is charging}$$

(ii) $V_S = -V_{sat}$

$V_C(0) = +\beta V_{sat}$

$V_C = -\beta V_{sat}$

$$T_2 = RC\ln\left|\frac{-V_{sat}-\beta V_{sat}}{-V_{sat}+\beta V_{sat}}\right|$$

$$T_2 = RC\ln\left|\frac{1+\beta}{1-\beta}\right|$$

$$\boxed{T_2 = RC\ln\left|1+\frac{2R_1}{R_2}\right|} \rightarrow \text{Capacitor is discharging.}$$

T_0 = Time period = $T_1 + T_2$

$$\boxed{T_0 = 2RC\ln\left(1+\frac{2R_1}{R_2}\right)}$$

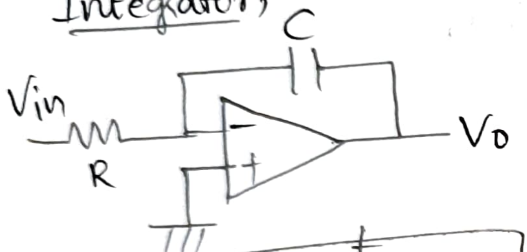
• Solving Differential Equations using OPAMP: (Analog Computer)

$$\frac{d^2V_0}{dt^2} + k_1\frac{dV_0}{dt} + k_2V_0 - V_1 = 0$$

$V_1 \rightarrow$ input voltage

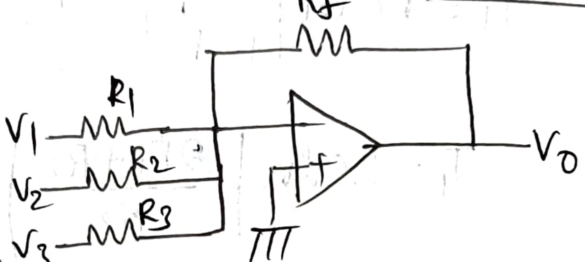
$k_1, k_2 \rightarrow$ real constants.

Integrator,

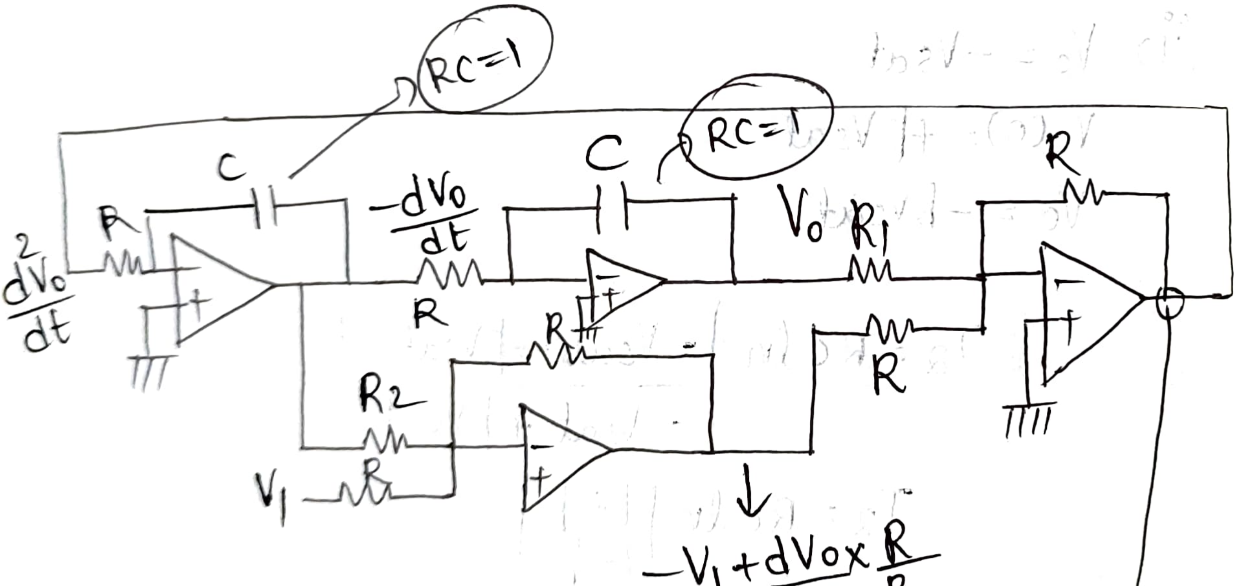


$$\boxed{V_0 = -\frac{1}{RC}\int_0^t V_{in}\,dt}$$

Summer circuit



$$\boxed{V_0 = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2}V_2 - \frac{R_f}{R_3}V_3}$$



$$-V_1 + \frac{dV_0}{dt} \times \frac{R}{R_2}$$

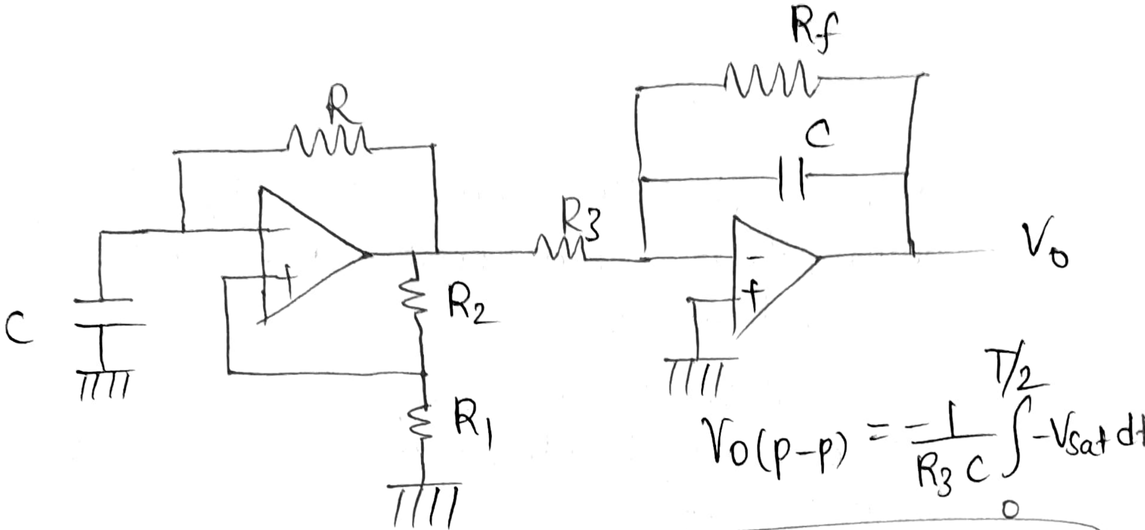
$$-V_0 \times \frac{R}{R_1} - \frac{R}{R_2}\frac{dV_0}{dt} + V_1$$

$$\frac{d^2V_0}{dt} = -V_0 \times \frac{R}{R_1} - \frac{R}{R_2}\frac{dV_0}{dt} + V_1$$

$$k_1 = \frac{R}{R_2} \qquad k_2 = \frac{R}{R_1}$$

Triangular wave generator using OPAMP:

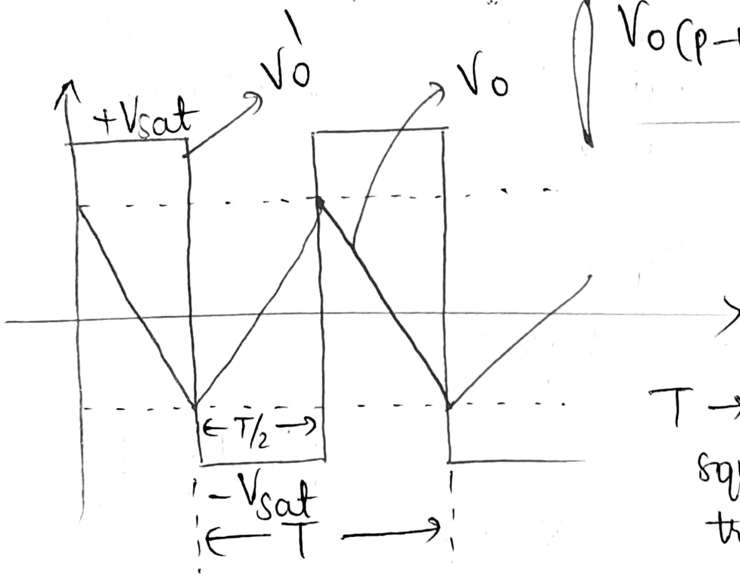
Astable multivibrator → Integrator → Triangular wave.



$$V_o(p-p) = \frac{-1}{R_3 C}\int_0^{T/2} -V_{sat}\,dt$$

$$\boxed{V_o(p-p) = \frac{+V_{sat}\,T}{2R_3 C}}$$

↓ Peak to peak output voltage.



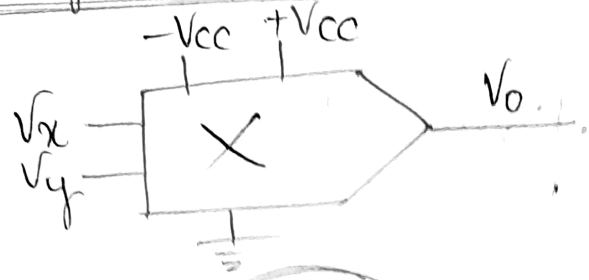
T → time period of square wave and triangular wave.

$$V_o = \frac{-1}{R_3 C}\int v_o'\,dt$$

$$\boxed{\frac{V_o(p-p)}{2} = \frac{V_{sat}\,T}{4R_3 C}}$$ → Amplitude of triangular wave.

$$\boxed{T = 2RC\ln\left(\frac{1+\beta}{1-\beta}\right)}$$ → $\beta = \frac{R_1}{R_1+R_2}$

Analog Multiplier:



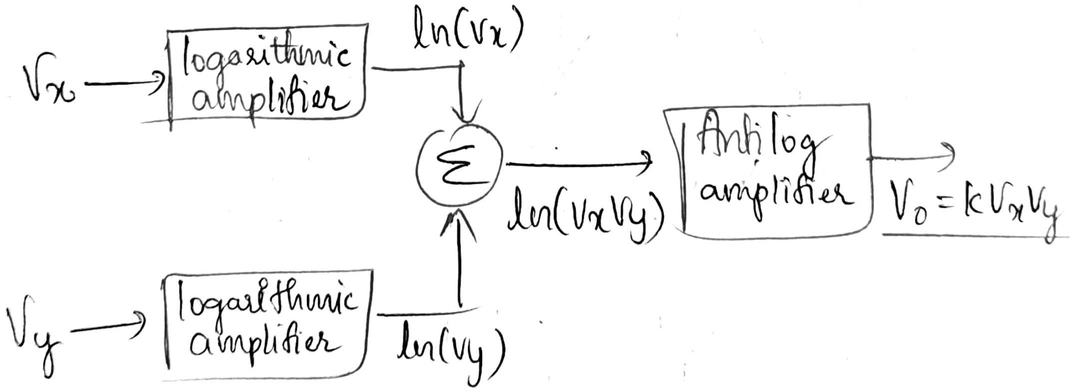
$$V_o = \frac{V_x V_y}{V_{ref}}$$

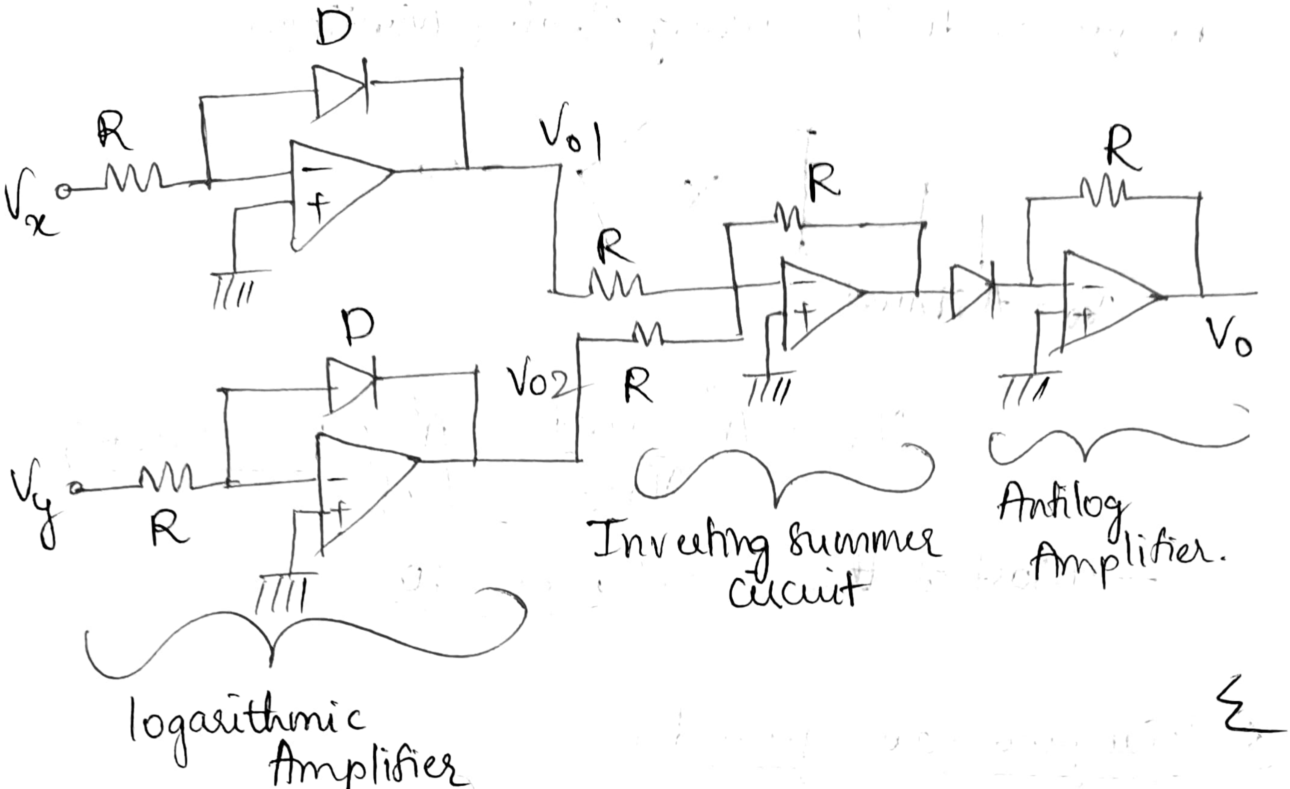
→ Analog multiplier IC which works only when both inputs V_x & V_y are positive and are called First Quadrant IC's.

→ In Second Quadrant Analog multiplier will produce correct output only when any of the input is '+ve'.

→ In Fourth Quadrant Analog multiplier will produce correct output only when V_x & V_y are either positive or negative.

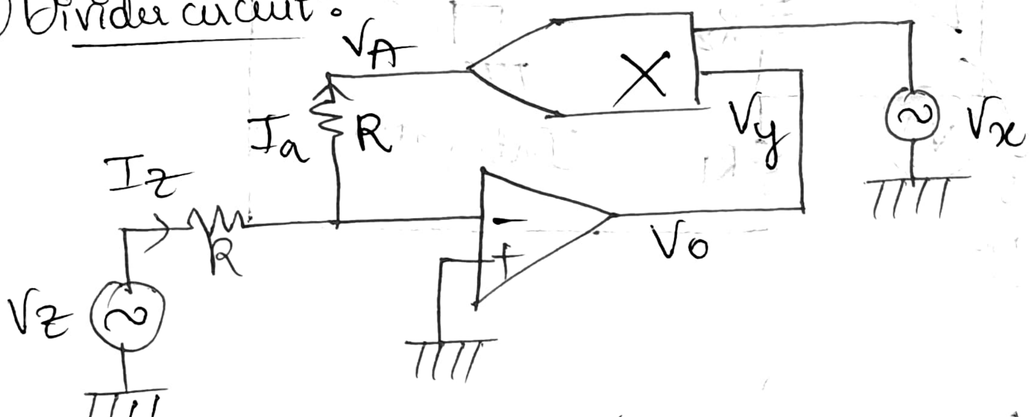
Simple Analog Multiplier:





(*) Applications of Analog Multiplier:

① Divider circuit:



$$I_z = \frac{V_z}{R}, \quad I_a = \frac{-V_A}{R} \longrightarrow I_z = I_a$$

$$V_z = -V_A$$

$$V_A = \frac{V_x V_y}{V_{ref}} = \frac{V_x V_o}{V_{ref}}$$

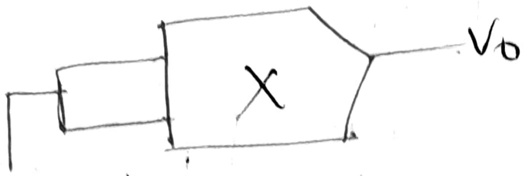
$$-V_z = \frac{V_x V_o}{V_{ref}}$$

$$\boxed{V_o = -V_{ref} \cdot \frac{V_z}{V_x}}$$

dividend

divisor.

② frequency doubler using Analog Multiplier:

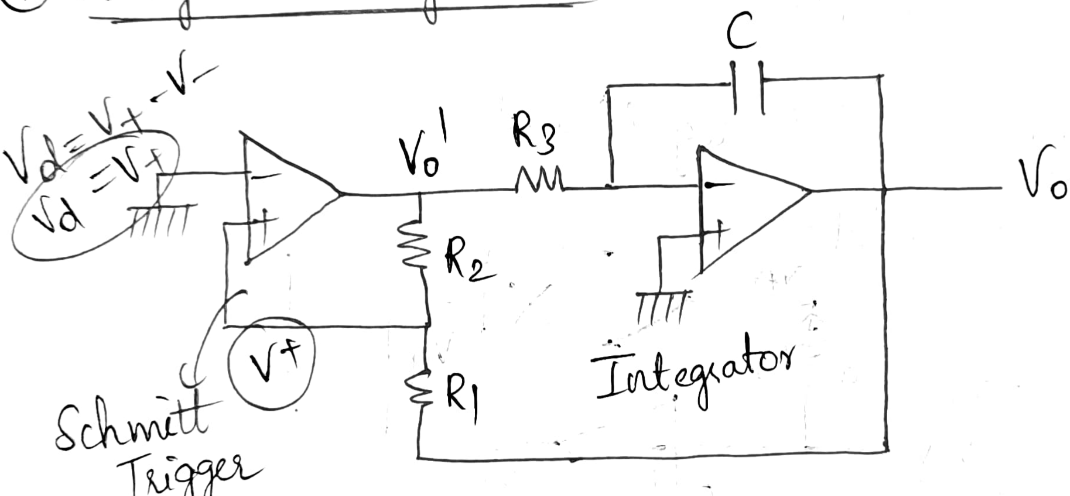


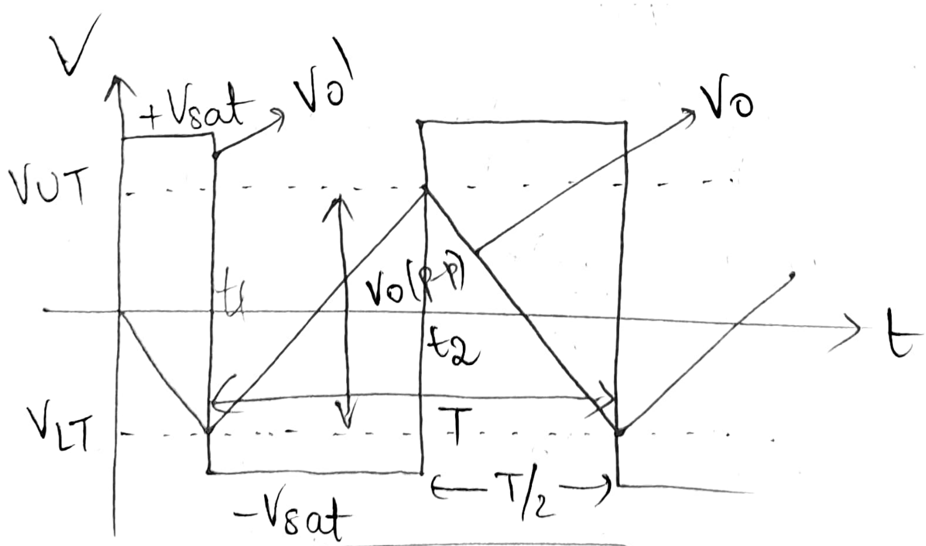
$V_0 = A \sin \omega t$

$$V_o = \frac{A^2 \sin^2 \omega t}{V_{ref}} = \frac{A^2}{V_{ref}}\left(\frac{1 - \cos 2\omega t}{2}\right)$$

→ Most of the cases, $V_{ref} = 10$

⊛ Triangular wave generator:





$$\boxed{V_P = \frac{V_0' R_1}{R_1 + R_2} + \frac{V_0 R_2}{R_1 + R_2}}$$

$$V_0 = \frac{-1}{R_3 C}\int_0^t V_{sat}\, dt$$

$$\boxed{V_0 = \frac{-V_{sat} \times t}{R_3 C}}$$

→ When $V_0' = +V_{sat}$,

$$V^+ = \frac{V_{sat} \times R_1}{R_1 + R_2} + \frac{V_0 \times R_2}{R_1 + R_2}$$

→ At instant t_1, $\boxed{V_+ = 0}$

$$0 = \frac{V_{sat} \times R_1}{R_1 + R_2} + \frac{V_0 \times R_2}{R_1 + R_2}$$

$$\boxed{V_0 = -V_{sat} \times \frac{R_1}{R_2} = V_{LT}}$$

→ When $V_0' = -V_{sat}$

$$V_+ = \frac{-V_{sat} \times R_1}{R_1 + R_2} + \frac{V_0 \times R_2}{R_1 + R_2}$$

At time instant, $t = t_2$;

$\boxed{V_+ = 0}$

$$-\frac{V_{sat} \times R_1}{R_1 + R_2} + \frac{V_0 \times R_2}{R_1 + R_2} \rightarrow \boxed{V_0 = \frac{V_{sat} \times R_1}{R_2} = V_{UT}}$$

$$V_{o(p-p)} = V_{UT} - V_{LT}$$

$$\boxed{V_{o(p-p)} = \frac{2V_{sat} \times R_1}{R_2}}$$

$$V_{o(p-p)} = \frac{-1}{R_3 C}\int_0^{T/2} -V_{sat}\, dt$$

$$\frac{2V_{sat} \times R_1}{R_2} = \frac{1}{R_3 C} V_{sat} \times \frac{T}{2} \rightarrow \boxed{T = \frac{4R_3 C R_1}{R_2}}$$

Q: Design a triangular wave generator using OPAMP, with $f = 1kHz$ & $V_{O(P-P)} = 2V$.

Sol:-

$$V_{O(P-P)} = \frac{2 V_{sat} \times R_1}{R_2}$$

If $V_{cc} = \pm 15V$ then assume $V_{sat} \cong 14V$

$$\cancel{2} = \frac{\cancel{2} V_{sat} \times R_1}{R_2}$$

$$\boxed{R_2 = 14 R_1}$$

If $R_1 = 10k\Omega$ then $R_2 = 140k\Omega$

$$f = \frac{R_2}{4 R_3 C R_1}$$

$$1 \times 10^3 = \frac{14\cancel{0} \times \cancel{10^3}}{4 \times R_3 \times C \times 1\cancel{0} \times \cancel{10^3}}$$

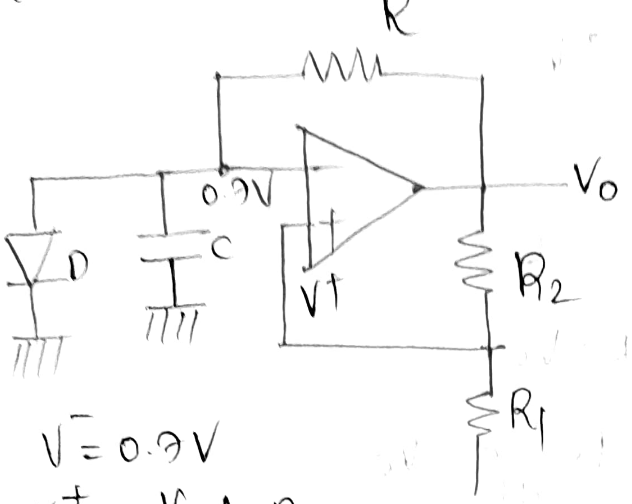
$$R_3 C = 3.5 \times 10^{-3}$$

$\rightarrow$ Assume $C = 1\mu F$

$$R_3 \times 10^{-6} = 3.5 \times 10^{-3}$$

$$\boxed{R_3 = 3.5 \times 10^3 \Omega}$$

Monostable Vibrator using OPAMP:



if Vo = +Vsat
then it is stable.

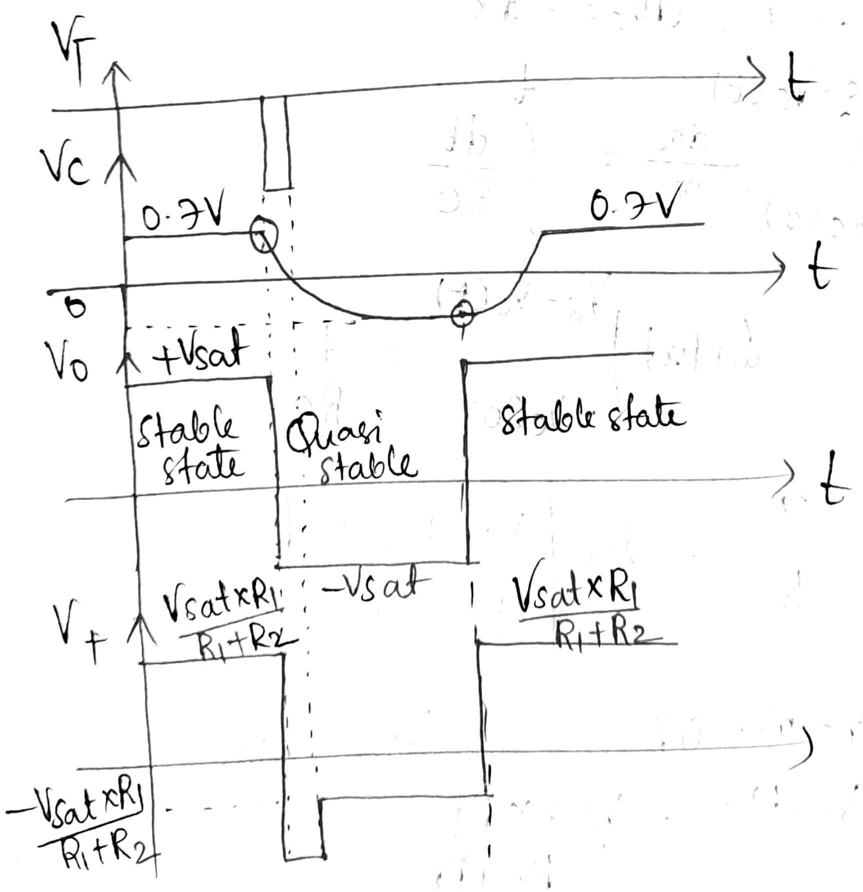
$V^- = 0.7V$

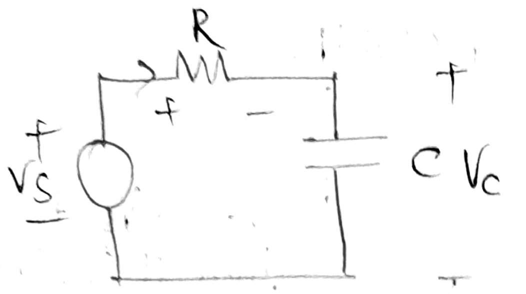
$$V^+ = \frac{V_{sat} \times R_1}{R_2}$$

(if Vo = +Vsat)

$V_d = V^+ - V^-$

$V_d = V^+ - 0.7$





$$V_S - iR - V_C = 0$$

$$V_S = iR + V_C$$

$$V_S = RC\frac{dV_C}{dt} + V_C$$

$$\int_{V_C(0)}^{V_C(t)} \frac{dV_C}{V_S - V_C} = \int_0^t \frac{dt}{RC}$$

$$V_S - V_C = x, \quad dV_C = -dx$$

$$\int_{-V_C(0)}^{V_S - V_C(t)} \frac{dx}{x} = \int_0^t \frac{dt}{RC}$$

$$\ln|x| \Big|_{-V_C(0)}^{V_S - V_C(t)} = \frac{t}{RC}$$

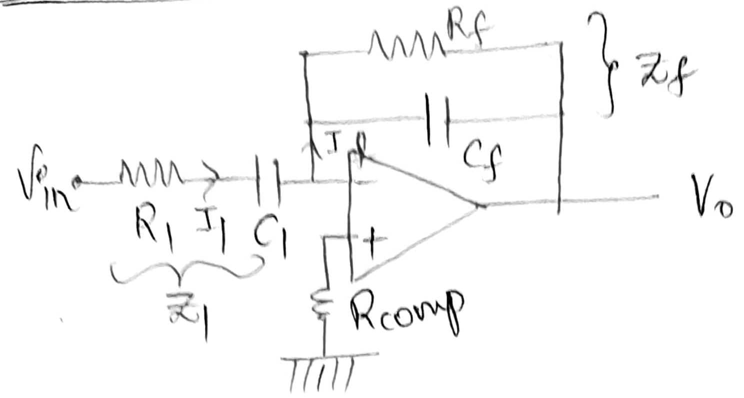
$$\frac{t}{RC} = \ln\left|\frac{V_S - V_C(0)}{V_S - V_C(t)}\right|$$

$$V_C(0) = 0V \qquad V_S = -V_{sat}$$

$$V_C(t) = -\frac{V_{sat} \times R_1}{R_2 + R_1}$$

$$\boxed{T = RC\ln\left(1 + \frac{R_1}{R_2}\right)}$$

Practical Differentiator using OPAMP:



$$Z_1(s) = R_1 + \frac{1}{sC_1} = \frac{sC_1R_1 + 1}{sC_1}$$

$$Z_f(s) = R_f \parallel C_f = \frac{R_f \times \frac{1}{sC_f}}{R_f + \frac{1}{sC_f}} = \frac{R_f}{1 + sC_fR_f}$$

$$I_1(s) = \frac{V_{in}(s) - 0}{Z_1(s)} \qquad I_2(s) = \frac{0 - V_o(s)}{Z_2(s)}$$

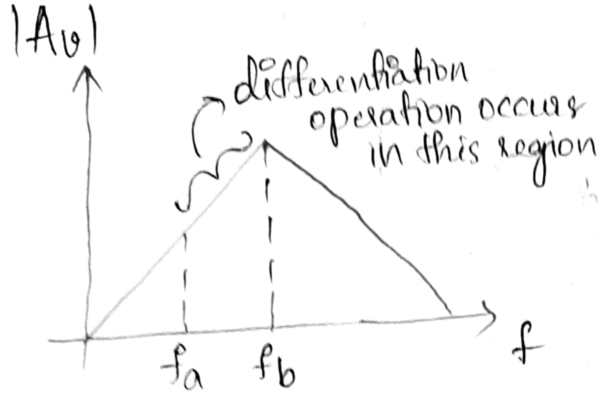
$$I_1(s) = I_2(s) \quad \text{(Virtual ground)}$$

$$\frac{V_{in}(s)}{Z_1(s)} = \frac{-V_o(s)}{Z_2(s)}$$

$$\frac{V_{in}(s)}{\frac{1 + sC_1R_1}{sC_1}} = \frac{-V_o(s)}{\frac{R_f}{1 + sC_fR_f}}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{-R_f}{(1 + sC_fR_f)} \times \frac{sC_1}{(1 + sC_1R_1)}$$

$$\boxed{|A_v(s)| = \frac{sR_fC_1}{(1 + sC_fR_f)(1 + sR_1C_1)}}$$



$f_a \rightarrow$ frequency at which gain $= 1$.

$$f_a = \frac{1}{2\pi R_f C_1}$$

$$f_b = \frac{1}{2\pi R_1 C_1}$$

For proper differentiation,

If $R_f C_1 >> R_f C_f$

$R_f C_1 >> R_1 C_1$ then

$$A_v(s) = -s R_f C_1$$

$$\frac{V_o(s)}{V_{in}(s)} = -s R_f C_1$$

$$V_o(s) = -s V_{in}(s) R_f C_1$$

$$\boxed{V_o(t) = -R_f C_1 \frac{d}{dt} V_{in}(t)}$$

If $R_1 C_1 = R_f C_f$

$$|A_v(s)| = \frac{s R_f C_1}{(1 + s C_1 R_1)^2}$$

$$|A_v(s)| = \frac{j\omega R_f C_1}{(1 + j\omega C_1 R_1)^2}$$

$$|A_v(s)| = \frac{f/f_a}{1 + (f/f_b)^2}$$

Q: Design a differentiator using op-amp to differentiate an input signal with $f_{min} = 200Hz$. Draw output waveform for sine wave with 1V peak at 200Hz.

Sol:- $f_{min} = f_a = 200Hz$

$$\frac{1}{2\pi R_f C_1} = 200$$

(let $C_1 = 0.1\mu F$)

$$R_f = \frac{1}{2\pi \times 0.1\times10^{-6} \times 200} = \underline{7.962k\Omega}$$

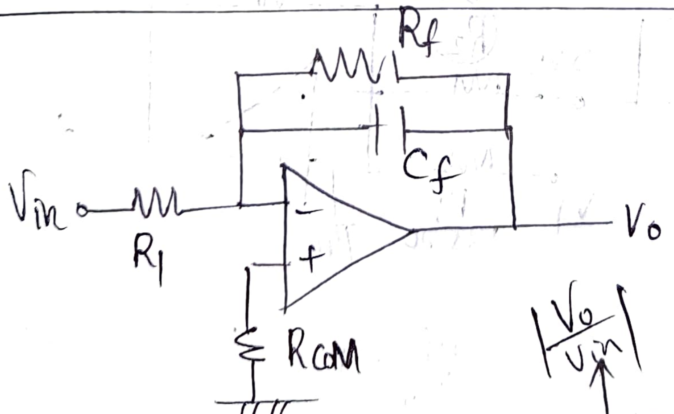
$$f_b = 10 f_a$$

$$\frac{1}{2\pi R_1 C_1} = 10\times 200$$

$$R_1 = \frac{1}{2\pi \times 2000 \times 0.1\times10^{-6}} = \underline{0.796k\Omega}$$

$$R_1 C_1 = R_f C_f$$

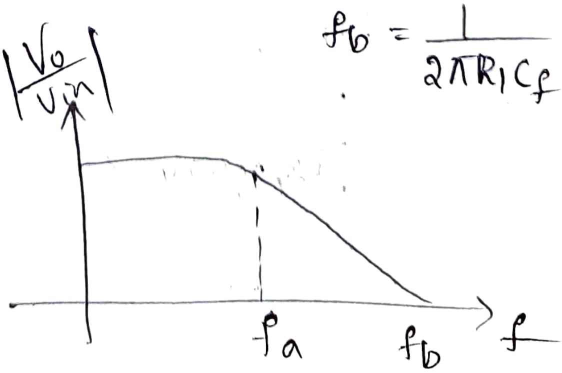
$$C_f = \frac{R_1 C_1}{R_f} = \underline{0.01\mu F}$$



$$f_b = 10 f_a$$

$$f_a = \frac{1}{2\pi R_f C_f}$$

$$f_b = \frac{1}{2\pi R_1 C_f}$$



Q: Design a practical integrator with lower limit of frequency is 160 Hz.

Sol: $f_a = 160\,Hz$

$$\frac{1}{2\pi R_f C_f} = 160 \qquad \text{Assume, } C_f = 0.01\,nF$$

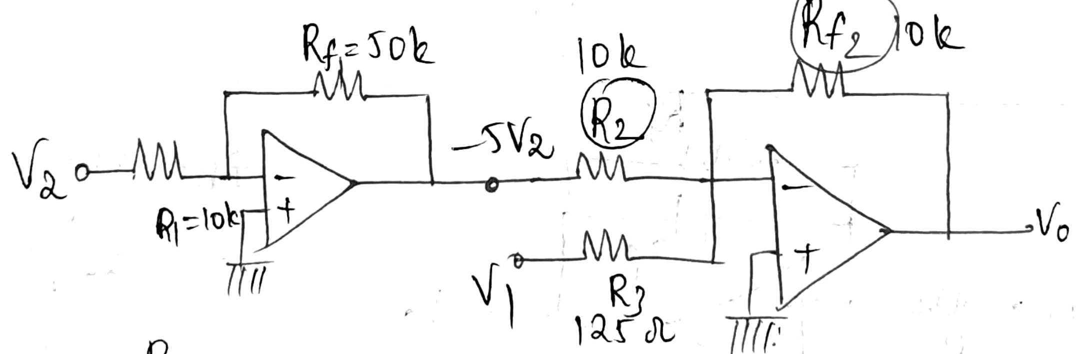
$$R_f = \frac{1}{2\pi \times 160 \times 0.01 \times 10^{-9}}$$

$f_b = 10 f_a = 1600\,Hz$

$$\frac{1}{2\pi R_1 C_f} = 1600 \rightarrow R_1 = \frac{1}{2\pi \times 1600 \times 0.01 \times 10^{-9}}$$

Q: Design a circuit using OPAMP, which gives output $V_o = 5V_2 - 8V_1$ where V_2 & V_1 are inputs.

Sol:



$$\frac{R_{f1}}{R_1} = 5$$

$R_{f1} = 50k$

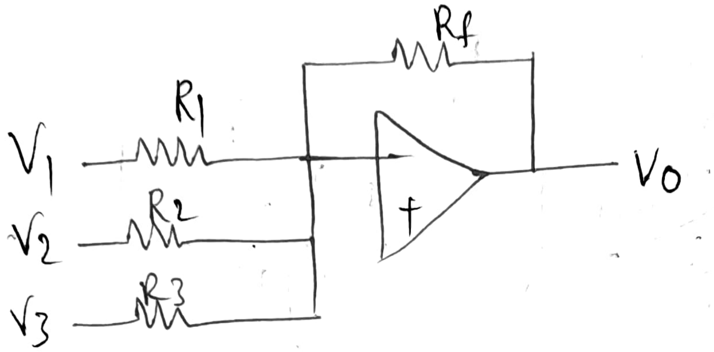
$$\frac{R_{f2}}{R_2} = 1 \rightarrow R_{f_2} = R_2 = 10k$$

$$\frac{R_{f2}}{R_3} = 8 \rightarrow R_3 = \frac{10k}{8} = 125\Omega$$

Q: Design an OPAMP circuit which performs following operation

$$V_0 = -V_1 - 2V_2 - 3V_3$$

Sol:



$$-\frac{R_f}{R_1} \times V_1 - \frac{R_f}{R_2} \times V_2 - \frac{R_f}{R_3} \times V_3 = V_0$$

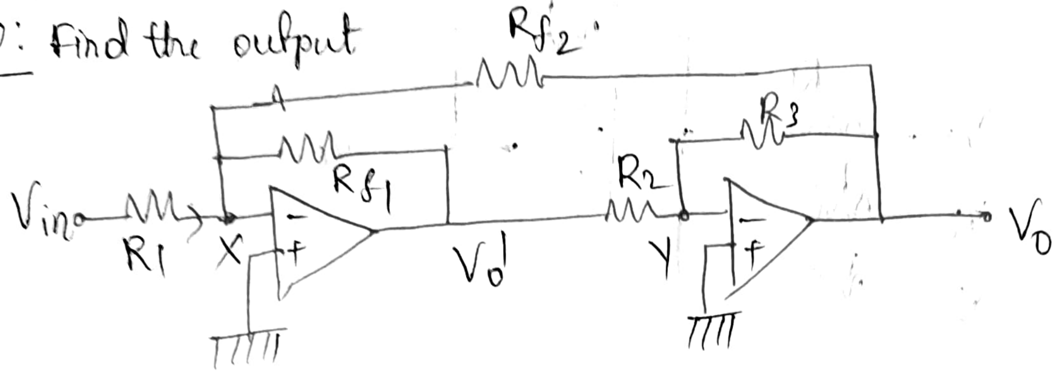
$$\frac{R_f}{R_1} = 1 \qquad \frac{R_f}{R_2} = 2 \qquad \frac{R_f}{R_3} = 3$$

Let $R_f = 100k$,

$$\boxed{\begin{aligned} R_1 &= 100k \\ R_2 &= 50k \\ R_3 &= \frac{100k}{3} \end{aligned}}$$

Q: Design a practical integrator which differentiates a signal having max freq 200 Hz.

Q: Find the output



Sol:

Apply KCL at Node X,

$$\frac{V_{in}}{R_1} = -\frac{V_0'}{R_{f1}} - \frac{V_0}{R_{f2}}$$

Apply KCL at Node Y,

$$\frac{V_0'}{R_2} = \frac{-V_0}{R_3}$$

$$\boxed{V_0' = -\frac{R_2}{R_3} \times V_0}$$

$$\frac{V_{in}}{R_1} = +\frac{R_2}{R_3 R_{f1}} V_0 - \frac{V_0}{R_{f2}}$$

$$= -V_0 \left(\frac{-R_2 R_{f2} + R_3 R_{f1}}{R_3 R_{f1} R_{f2}} \right)$$

$$\boxed{\frac{V_0}{V_{in}} = \frac{-R_3 R_{f1} R_{f2}}{R_1 (-R_2 R_{f2} + R_3 R_{f1})}}$$

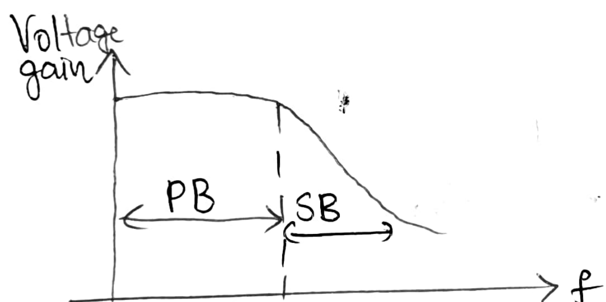
ACTIVE FILTERS

→ A filter is a frequency selective circuit, which allows signals of certain range of frequencies to pass through it and rejecting other frequencies.

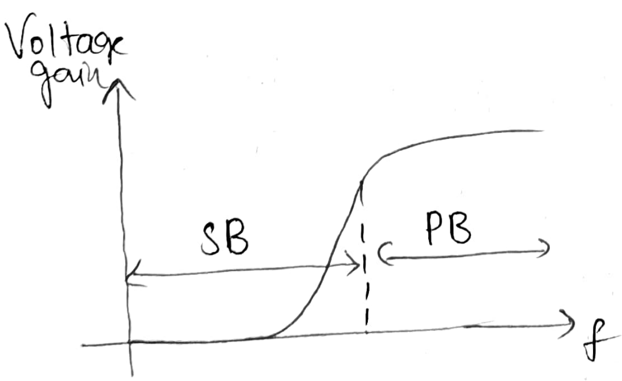
→ There are two kinds of filters
① Active filters – {made of passive + active elements}
② Passive filters – {made of only passive elements}

→ Types of Active filters
① Low pass filters
② High pass filters
③ Band pass filters
④ Band reject filter (or) Band stop filter.

→ frequency responses of active filters:

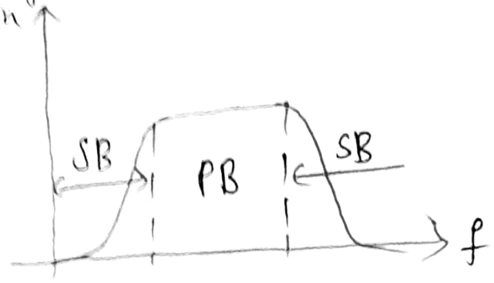


Low pass filter

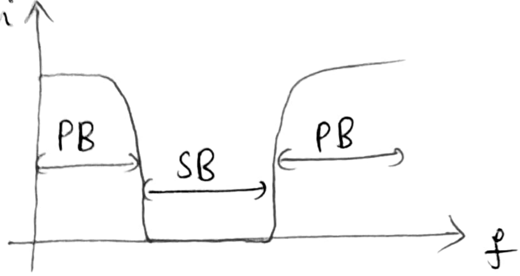


High pass filter

SB – stop Band
PB – pass Band



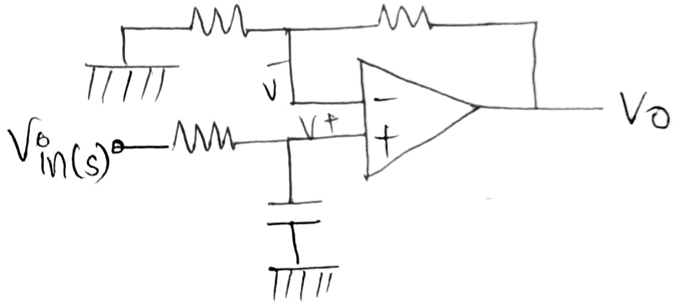
Band Pass filter



Band Reject filter

I) Low pass filter:

(i) First order Low pass filter!



→ It is first order because $A_v(s) = \dfrac{1+\frac{R_f}{R_1}}{1+sRC}$

↳ s has only 1 degree.

$$V^+(s) = \frac{V_{in}(s)}{R+\frac{1}{sC}} \times \frac{1}{sC}$$

$$V^+(s) = \frac{V_{in}(s)}{1+sRC}$$

$$V_o(s) = \left(1+\frac{R_f}{R_1}\right)V^+(s)$$

$$V_o(s) = \left(1+\frac{R_f}{R_1}\right)\frac{V_{in}(s)}{1+sRC}$$

$$A_{vf}(s) = \frac{V_o(s)}{V_{in}(s)} = \left(1+\frac{R_f}{R_1}\right)\times\frac{1}{1+sRC}$$

$s = j\omega$

$$A_{vf}(j\omega) = \left(1+\frac{R_f}{R_1}\right)\times\frac{1}{1+j\omega RC}$$

$$|A_{vf}(j\omega)| = \frac{\left(1+\frac{R_f}{R_1}\right)}{\sqrt{1+\omega^2R^2C^2}}$$

$|A_{vf}(j\omega)|_{max}$ when $\omega = 0$

$$|A_{vf}(j\omega)|_{max} = 1+\frac{R_f}{R_1}$$

$$|A_{vf}(j\omega)| = \frac{|A_{vf}(j\omega)|_{max}}{\sqrt{1+\omega^2R^2C^2}}$$

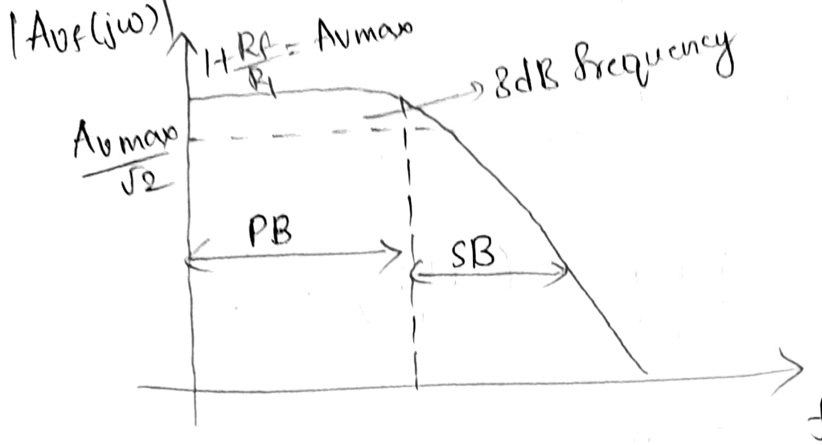
ω_0 $f_0 \rightarrow$ frequency at which gain is $\frac{1}{\sqrt{2}}$ times maximum gain.

$$\frac{|A_{vf}|\cancel{_{max}}}{\sqrt{2}} = \frac{|A_{vf}|\cancel{_{max}}}{\sqrt{1+\omega^2R^2C^2}}$$

Cutoff frequency.

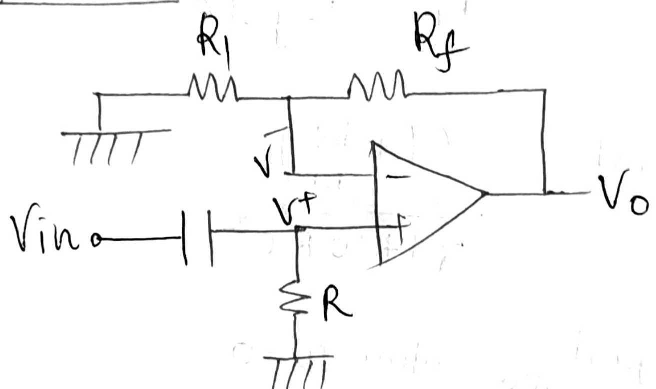
$$\omega_0 = \frac{1}{RC} \rightarrow \boxed{f_0 = \frac{1}{2\pi RC}}$$

→ When ω_0 is very large, then $|A_{vf}|_{max} = 0$



→ This is also called as Butterworth 1st order LPF.

② <u>High pass filter:</u>



→ This is first order HPF, also called as Butterworth 1st order HPF.

$$V^+(s) = \frac{V_{in}(s) \times R}{\frac{1}{SC} + R}$$

$$V^+(s) = \frac{V_{in}(s) \times SCR}{1 + SCR}$$

For non-inverting OP-AMP,

$$V_o(s) = V^+(s)\left(1 + \frac{R_f}{R_1}\right)$$

$$\boxed{V_o(s) = \frac{V_{in}(s) \times SCR}{1 + SCR}\left(1 + \frac{R_f}{R_1}\right)}$$

$$A_{vf}(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{sRC\left(1+\frac{R_f}{R_1}\right)}{1+sRC}$$

$s = j\omega$

$$A_{vf}(j\omega) = \frac{j\omega RC\left(1+\frac{R_f}{R_1}\right)}{1+j\omega RC}$$

$$|A_{vf}(j\omega)| = \frac{\omega RC\left(1+\frac{R_f}{R_1}\right)}{\sqrt{1+\omega^2R^2C^2}}$$

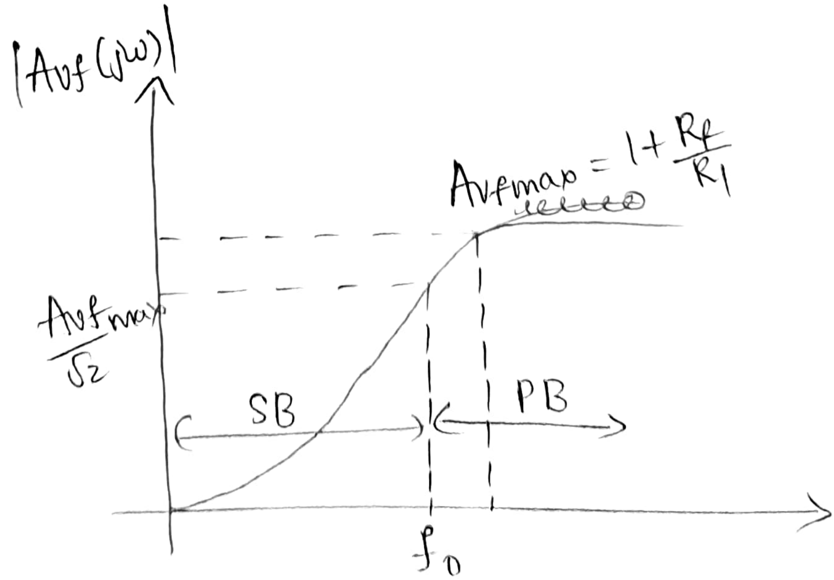
$$|A_{vf}(j\omega)| = \frac{\left(1+\frac{R_f}{R_1}\right)}{\sqrt{1+\frac{1}{\omega^2R^2C^2}}}$$

→ Maximum voltage gain occurs at $\omega = \infty$

$$|A_{vf}|_{max} = \left(1+\frac{R_f}{R_1}\right)$$

→ Minimum voltage gain occurs at $\omega = 0$

$$|A_{vf}|_{min} = 0$$



At $f = f_0$ (cut off frequency),
gain is $\frac{1}{\sqrt{2}}$ times the maximum gain.

$$|A_{vf}(2\pi f_0)| = \frac{A_{vf\,max}}{\sqrt{2}}$$

$$\frac{\cancel{A_{vf\,max}}}{\sqrt{2}} = \frac{\cancel{A_{vf\,max}}}{\sqrt{1+\frac{1}{\omega_0^2 R^2 C^2}}}$$

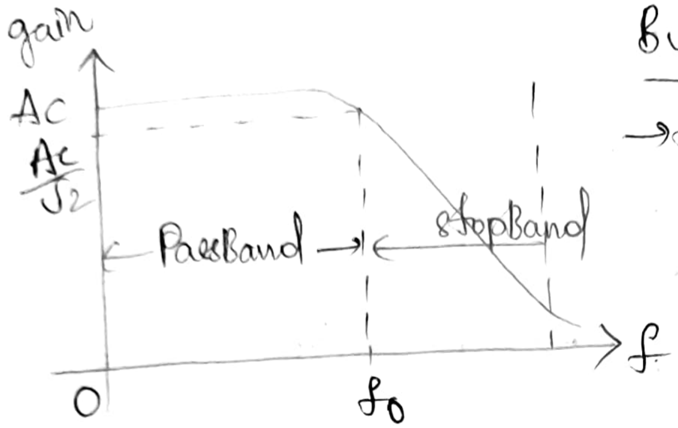
$$\frac{1}{\omega^2 R^2 C^2} = 2-1$$

$$\omega^2 R^2 C^2 = 1$$

$$\boxed{\omega_0 = \frac{1}{RC}}$$

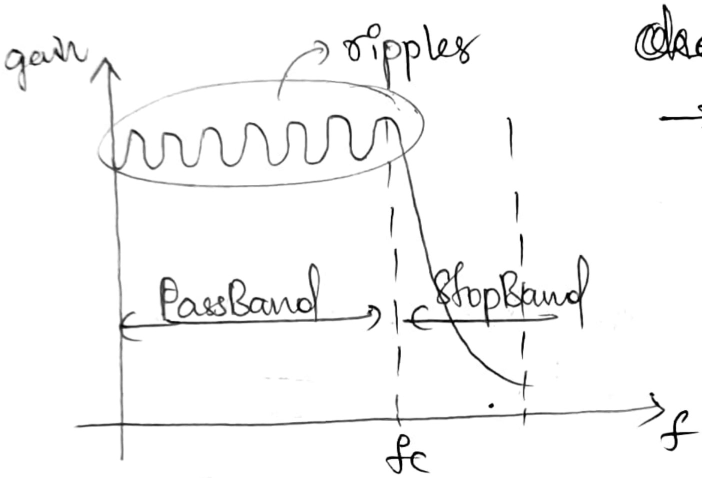
$$\boxed{f_0 = \frac{1}{2\pi RC}}$$

Butterworth LPF



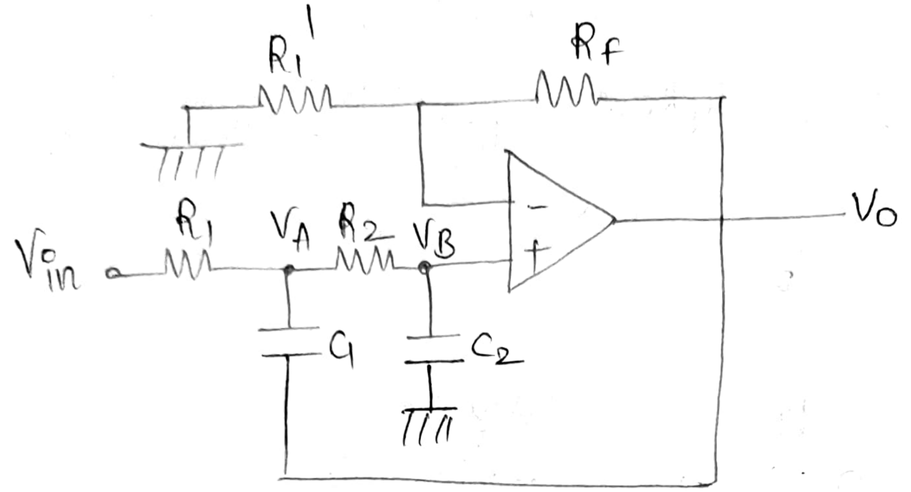
→ for faster transition from Passband to stopband Higher order filter is used

~~Chey~~ Chebyshev filter



→ for faster transition from passband to stopband Low order filter is used.

- Butterworth Second order Low pass filter:



Apply KCL at node A,

$$\frac{V_{in}(s) - V_A(s)}{R_1} = \frac{V_A(s) - V_B(s)}{R_2} + \frac{V_A(s) - V_o(s)}{\frac{1}{sC_1}} \quad \text{--- (1)}$$

$$V_B(s) = \frac{V_A(s) \times \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{V_A(s)}{1 + sC_2R_2} \quad \text{--- (2)}$$

$$V_o(s) = \left(1 + \frac{R_F}{R_1'}\right) V_B(s) \quad \text{--- (3)}$$

Substituting eq (2) & (3) in eq (1),

(4)

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{A_o / R_1R_2C_1C_2}{s^2 + s\frac{(C_2(R_1+R_2) + (1-A_o)R_1C_1)}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}}$$

Generally,

$$\boxed{H(s) = \frac{A_o\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}} \quad \text{--- (5)}$$

where, $Q = \frac{1}{2\xi}$

Comparing eq (4) & eq(5),

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \; ; \; 2\xi = \frac{C_2(R_1+R_2)+(1-A_0)R_1C}{\sqrt{R_1 R_2 C_1 C_2}}$$

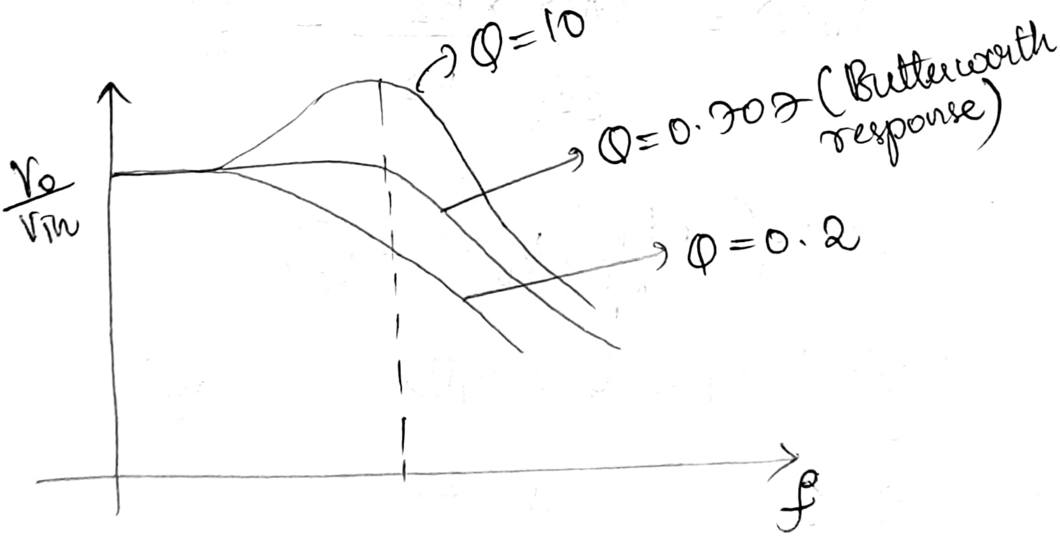
$$\frac{1}{2\xi} = Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1C_1 + R_2C_2 + R_1C_1(1-A_0)}$$

$$A_0 = 1 + \frac{R_F}{R_1}$$

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$ then,

$$f_c = f_n = \frac{1}{2\pi RC}$$

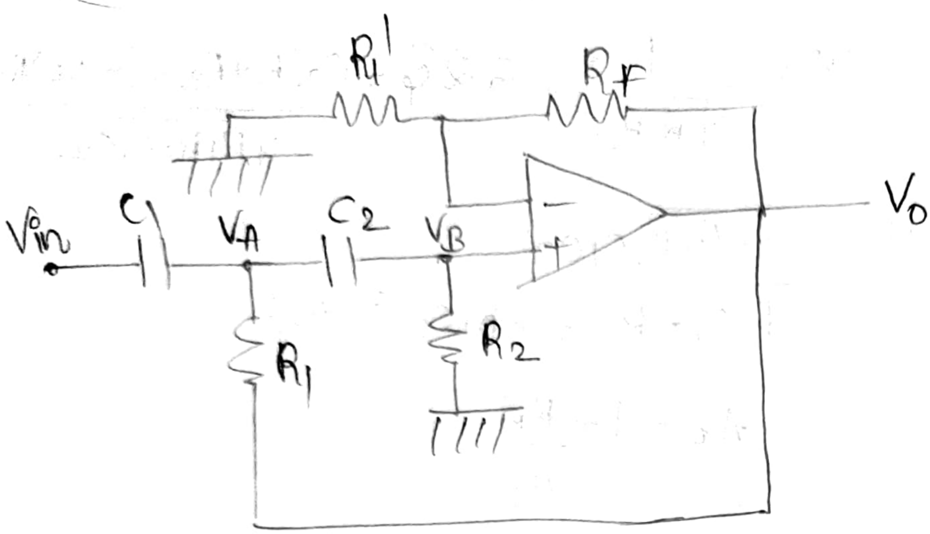
$$Q = \frac{1}{3 - A_0}$$



$$0.707 = \frac{1}{3 - A_0}$$ (for Butterworth response)

$A_0 = 1.586$ (when $Q = 0.707$)

- Butterworth Highpass filter (Second order):



Apply KCL at node A,

$$\frac{V_{in}(s) - V_A(s)}{\frac{1}{sC_1}} = \frac{V_A(s) - V_B(s)}{\frac{1}{sC_2}} + \frac{V_A(s) - V_o(s)}{R_1} \quad \text{— (1)}$$

$$V_B(s) = \frac{V_A(s) \times R_2}{\frac{1}{sC_2} + R_2} \quad \text{— (2)}$$

$$V_o(s) = \left(1 + \frac{R_F}{R_1'}\right) V_B(s) \quad \text{— (3)}$$

Substitute eq (2) & (3) in eq (1),

(4)

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{A_o s^2}{s^2 + s\frac{\left(R_1C_1 + R_1C_2 + R_2C_2(1-A_o)\right)}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}}$$

$$A_o = 1 + \frac{R_F}{R_1'}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_o s^2}{s^2 + \alpha \omega_o s + \omega_o^2} \quad \text{—} (5)$$

Comparing (4) & (5),

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\frac{\alpha}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{R_1 C_1 + R_1 C_2 + R_2 C_2 (1 - A_o)}{R_1 R_2 C_1 C_2}$$

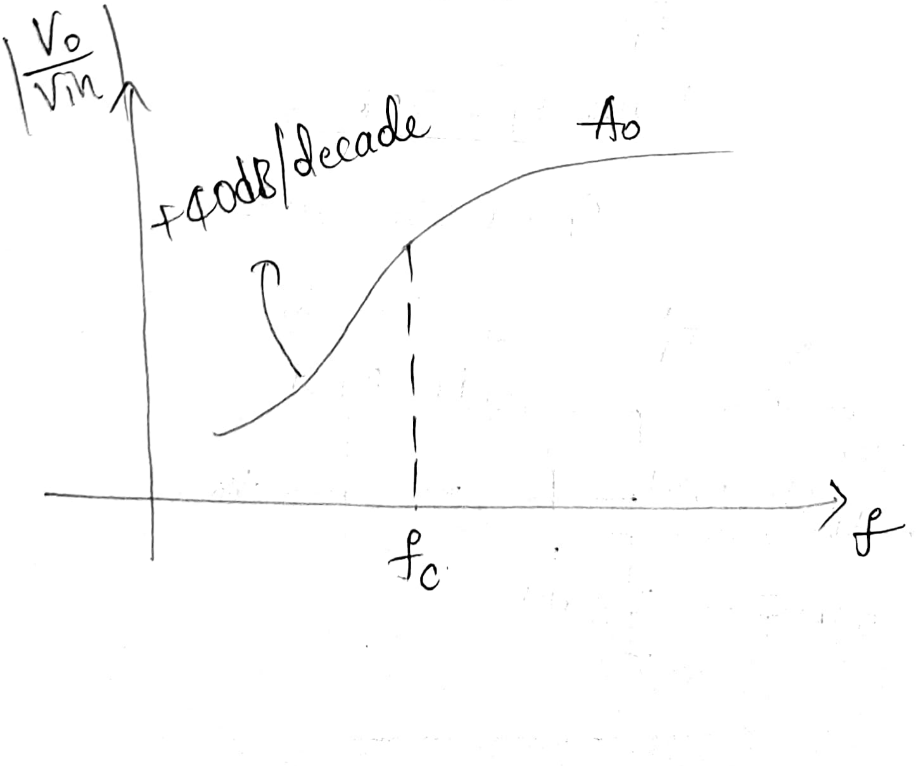
if $R_1 = R_2 = R$ & $C_1 = C_2 = C$,

then, $f_c = f_o = \frac{1}{2 \pi R C}$

$$\underline{\alpha = 3 - A_o}$$

for butterworth response, $\frac{1}{\alpha} = 0.707$

$$A_o = 1.586$$



Q: Design a butterworth Second order LPF with cutoff frequency of 1KHz?

Sol: ① Select a value of capacitor (C) $\leq 1\mu F$.

② Assume $R_1 = R_2 = R$ & $C_1 = C_2 = C$.

using eq $f_c = \frac{1}{2\pi RC}$, find 'R'.

③ passband gain = 1.586

$$1 + \frac{R_F}{R_1'} = 1.586$$

General procedure.

$C = 0.1\mu F$

$$f = \frac{1}{2\pi RC} \rightarrow 10^3 = \frac{1}{2\pi \times R \times 0.1 \times 10^{-6}}$$

$$R = \frac{10^4}{2\pi}$$

$\underline{R \simeq 1.6 k\Omega}$

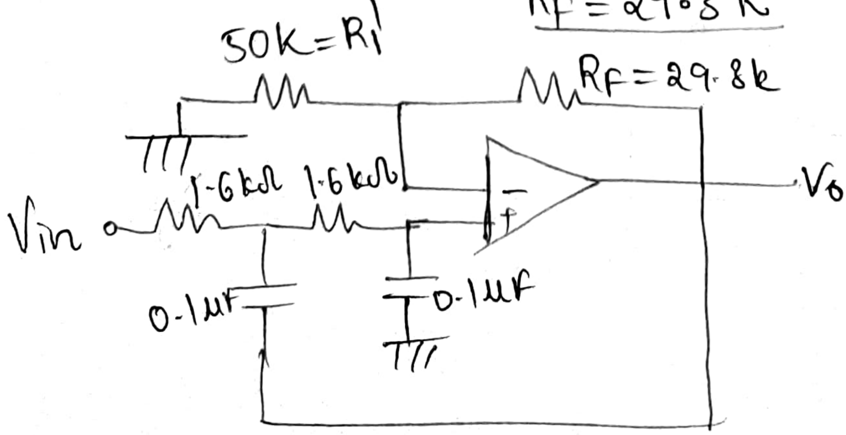
$$1 + \frac{R_F}{R_1'} = 1.586$$

$$\frac{R_F}{R_1'} = 0.586$$

Assume $\underline{R_1' = 50k}$,

$R_F = 0.586 \times 50\,k$

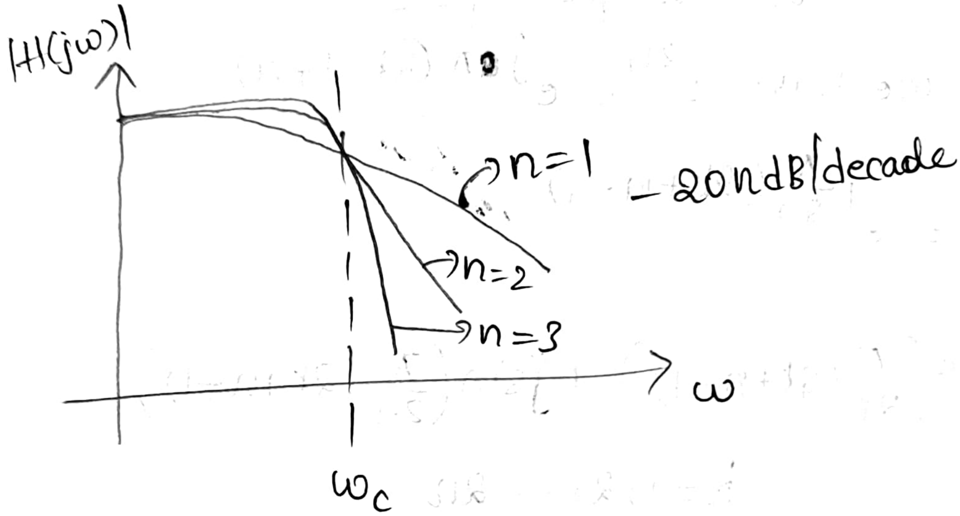
$\underline{R_F = 29.3\,K}$



* Butterworth filter (LPF):

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}}$$

$\omega_c \rightarrow$ cut-off frequency of the filter.
$n \rightarrow$ order of the filter.



* Normalised Butterworth LPF:

$$|H_n(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$ where $\omega_c = 1\,rad/s$.

$$|H_n(j\omega)|^2 = H_n(j\omega) \cdot H_n(-j\omega) = \frac{1}{1+\omega^{2n}}$$

Let $\omega = \frac{s}{j}$

$$|H_n(j\omega)|^2 = H_n(j\omega) \cdot H_n(-j\omega)$$
$$= H_n(s)\, H_n(-s)$$

$$|H_n(j\omega)|^2 = \frac{1}{1+\left(\frac{s}{j}\right)^{2n}}$$

→ The $2n$ poles of $H_n(s)H_n(-s)$ occur,

when $\left(\frac{s}{j}\right)^{2n} = -1$

$$s^{2n} = -(j)^{2n}$$

since, $-1 = e^{j\pi(2k-1)}$

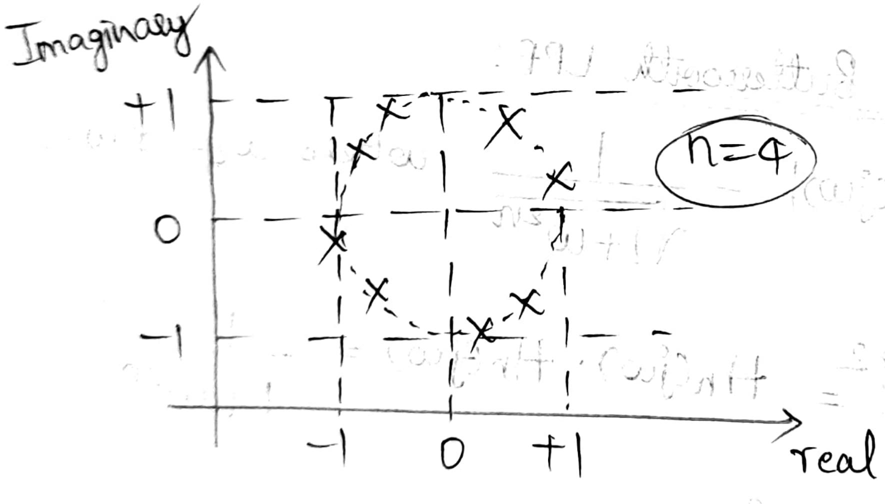
for k integer & $j = e^{j\pi/2}$

we have $s^{2n} = e^{j\pi(2k-1+n)}$

$$s_k = e^{j\frac{\pi}{2n}(2k+n-1)}$$

$$s_k = \cos\left(\frac{\pi}{2n}(2k+n-1)\right) + j\sin\left(\frac{\pi}{2n}(2k+n-1)\right)$$

$$k = 1, 2, \ldots 2n$$



n poles in LHP correspond to $H_n(s)$

n poles in RHP correspond to $H_n(-s)$

For obtaining stable & causal filter

we set,

$$H_n(s) = \frac{1}{(s-s_1)(s-s_2)\cdots(s-s_n)}$$

$$\boxed{H_n(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1} = \frac{1}{B_n(s)}}$$

Order (n)	Factors of polynomial $B_n(s)$
1	$s+1$
2	$s^2+1.414s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)$ $(s^2+1.9319s+1)$

* Scaling of Normalized filter:

① Replace 'ω' with $\frac{\omega}{\omega_c}$ in $H_n(\omega)$

② Replace 's' with $\frac{s}{\omega_c}$ in $H_n(s)$

$\omega_c \rightarrow$ cut-off frequency.

Q: Design a 3rd order Butterworth LPF with cut-off frequency 1 kHz.

Sol:

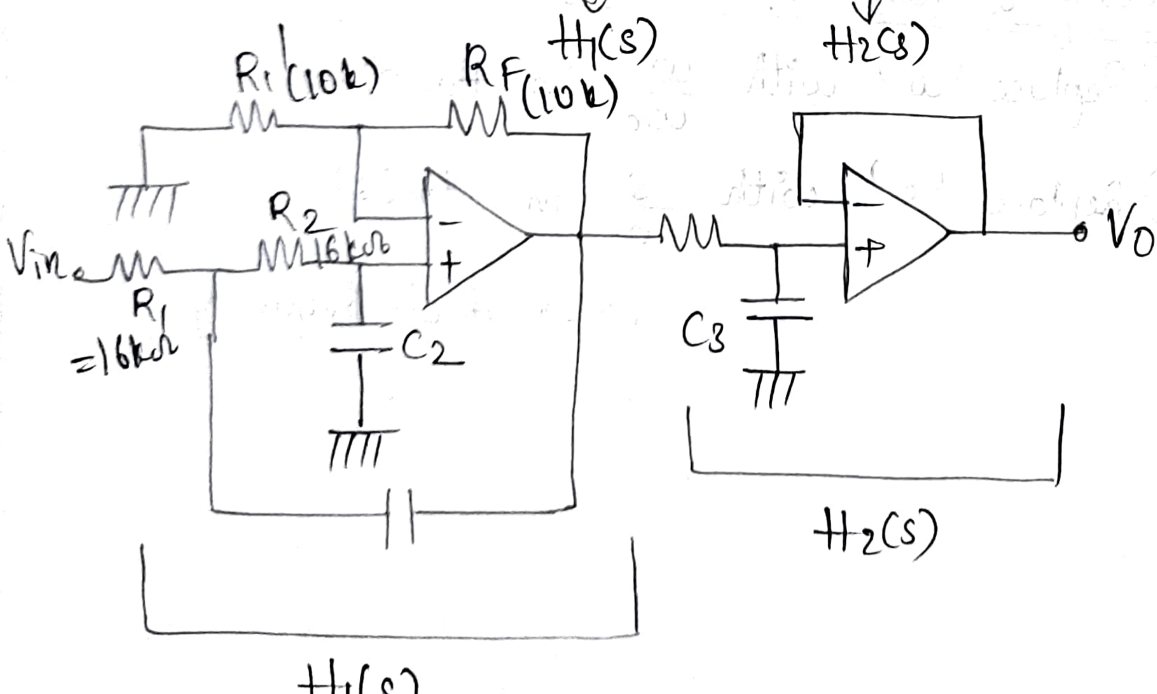
$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$H_n(s) = \left(\frac{1}{s^2+s+1}\right)\left(\frac{1}{s+1}\right)$$

Let cut off angular frequency $= \omega_0$

$$H_n(s) = \left[\frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right) + 1}\right] \times \left[\frac{1}{\left(\frac{s}{\omega_0}\right) + 1}\right]$$

$$\boxed{H_n(s) = \underbrace{\left(\frac{\omega_0^2}{s^2+\omega_0 s+\omega_0^2}\right)}_{H_1(s)}\underbrace{\left(\frac{\omega_0}{s+\omega_0}\right)}_{H_2(s)}}$$



when $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$f_0 = \frac{1}{2\pi RC} = 1\,kHz$$

Assume, $C = 0.01\,\mu F$

$$R = \frac{1}{2\pi f_o C} = \cancel{\text{}}$$

$$\boxed{R \simeq 16\,k\Omega}$$

$$H_1(s) = \frac{A_o / R_1 R_2 C_1 C_2}{s^2 + s\left(\frac{R_1C_2 + R_2C_2 + R_1C_1(1-A_o)}{R_1R_2C_1C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$\Downarrow$$

$$\boxed{H_1(s) = \frac{A_o / R^2C^2}{s^2 + s\frac{(3-A_o)}{RC} + \frac{1}{R^2C^2}}}$$

Comparing $H_1(s)$; $\omega_o = \frac{1}{RC}$

$$3 - A_o = 1$$

$$\boxed{A_o = 2}$$

$$1 + \frac{R_F}{R_1'} = 2$$

$$\frac{R_F}{R_1'} = 1 \longrightarrow \underline{R_F = R_1' = 10k}$$

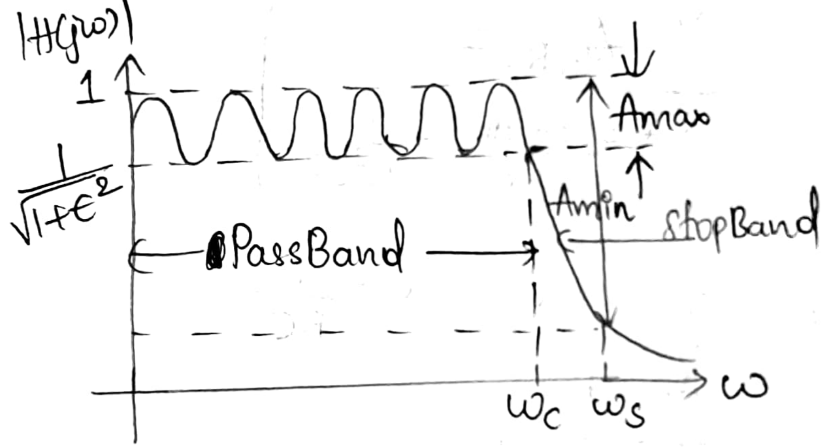
Chebyshev filter (LPF):

$$H(j\omega) = \frac{1}{\left(1+\varepsilon^2 C_n^2\left(\frac{\omega}{\omega_p}\right)\right)^{1/2}}$$

$\omega_p \to \omega_c$

where $\boxed{\epsilon = \left(10^{\frac{A_{max}}{10}} - 1\right)^{1/2}}$

$$H_n(j\omega) = \frac{1}{\left(1+\epsilon^2 C_n^2(\omega)\right)^{1/2}}$$



A_{max} & A_{min} are in dB.

$\omega_c = \omega_p$ = pass band edge

ω_s = stop band edge

A_{max} – maximum passband attenuation

A_{min} — minimum stopband attenuation

From the graph,

$$20\log 1 - 20\log\frac{1}{\sqrt{1+\varepsilon^2}} = A_{max}$$

$$10\log(1+\varepsilon^2) = A_{max}$$

$$1+\varepsilon^2 = 10^{A_{max}/10}$$

$$\boxed{\varepsilon = \left(10^{A_{max}/10} - 1\right)^{1/2}}$$

$f(\omega^2) = \epsilon^2 C_n^2(\omega) \rightarrow$ chebyshev polynomial.

$$\begin{cases} C_n(\omega) = \cos(n\cos^{-1}\omega) \text{ for } \omega \leq 1 \\ \quad\quad\;\; = \cosh(n\cosh^{-1}\omega) \text{ for } \omega > 1 \end{cases}$$

if $n=0 \rightarrow C_0(\omega) = 1$

if $n=1 \rightarrow C_1(\omega) = \omega$

$$\boxed{C_{n+1}(\omega) = 2\omega C_n(\omega) - C_{n-1}(\omega)}$$

n	$C_n(\omega)$
0	1
1	ω
2	$2\omega^2 - 1$
3	$4\omega^3 - 3\omega$
4	$8\omega^4 - 8\omega^2 + 1$

$$\boxed{\text{ripple width} = 1 - \frac{1}{(1+\epsilon^2)^{1/2}}}$$

- Order of chebyshev filter (LPF):

Given A_{min}, A_{max}, ω_s, ω_p, find 'n'.

$A_{min} = 10\log(1+\epsilon^2 C_n^2(\omega))$

for $\omega = \frac{\omega_s}{\omega_c} \rightarrow$ normalized

$$\epsilon^2 C_n^2(\omega) = \epsilon^2(\cosh(n\cosh^{-1}\omega))^2$$

$$= 10^{A_{min}/10} - 1$$

$$|A_{min}| = 10\log_{10}(1+\epsilon^2(\cosh(n\cosh^{-1}\omega))^2)$$

substitute $\epsilon^2 = 10^{A_{max}/10} - 1$ in above eq,

$$\cosh(n\cosh^{-1}\omega_0) = \left[\frac{10^{\frac{A_{min}}{10}}-1}{10^{\frac{A_{max}}{10}}-1}\right]^{1/2}$$

$$\boxed{n \geq \frac{\cosh^{-1}\left[\left(10^{\frac{A_{min}}{10}}-1\right)/\left(10^{\frac{A_{max}}{10}}-1\right)\right]^{1/2}}{\cosh^{-1}\left(\frac{\omega_s}{\omega_c}\right)}}$$

→ Poles,

$$1+\epsilon^2 C_n^2(\omega) = 0$$

$$s_K = (\sigma_K) \pm j\omega_K$$

where $K = 0, 1, 2, \ldots, 2n-1$

$$\sigma_k = \sinh a \sin(2k+1)\frac{\pi}{2n}$$

$$\omega_K = \cosh a \cos(2k+1)\frac{\pi}{2n}$$

$$\theta_K = (2k+1)\frac{\pi}{2n}$$

for $K = 0, 1, 2, \ldots, 2n-1$ & $\boxed{a = \frac{1}{n}\sinh^{-1}\left(\frac{1}{\epsilon}\right)}$

$$\frac{\sigma_k}{\sinh a} = \sin(2k+1)\frac{\pi}{2n}$$

$$\frac{\omega_K}{\cosh a} = \cos(2k+1)\frac{\pi}{2n}$$

$$\left(\frac{\sigma_k}{\sinh a}\right)^2 + \left(\frac{\omega_k}{\cosh a}\right)^2 = 1$$

Q: The specification for chebyshev filter is given by,

$A_{min} = 20dB$,

$A_{max} = 3dB$,

$\frac{\omega_s}{\omega_c} = 1.5$, determine order of the filter & confirm stopband requirement is fulfilled?

Sol:

$$n \geq \frac{\cosh^{-1}\left[(10^{2}-1)/(10^{0.3}-1)\right]^{1/2}}{\cosh^{-1}(1.5)}$$

$$n \geq \frac{\cosh^{-1}(9.97)}{\cosh^{-1}(1.5)}$$

$$\underline{n \geq 3.11}$$

choose $n=4$, $\epsilon = (10^{0.3}-1)^{1/2} = 0.99$

$$A_{min} = 10\log_{10}\left[1+1^2\{\cosh(4\cosh^{-1}(1.5))\}^2\right]$$

$= \underline{27.4dB}$ Requirement is satisfied.

Q: A chebyshev filter is required to provide the following specifications,

→ Passband max ripple width 0.5 dB upto 3kHz

→ Stopband minimum to be 60dB at 30kHz

Sol: Given $A_{max} = 0.5dB$ $\quad \omega_c = 3kHz \times 2\pi$

$A_{min} = 60 dB$ $\quad \omega_s = 2\pi \times 30kHz$

$$n \geq \frac{\cosh^{-1}\left[(10^{6}-1)/(10^{0.05}-1)\right]^{1/2}}{\cosh^{-1}(10)}$$

$\underline{n \geq 2.89}$

$\boxed{n=3}$

$$\epsilon = (10^{\frac{A_{max}}{10}} - 1)^{1/2}$$

$$\boxed{\epsilon = 0.35}$$

$$a = \frac{1}{3} \sinh^{-1}\left(\frac{1}{0.35}\right) = 0.59$$

$$\theta_K = (2k+1)\frac{\pi}{2n}$$

$$\theta_0 = \frac{\pi}{6} = 30^\circ$$

$$\theta_1 = 90^\circ$$

$$s_1 = -\sinh a = -\sinh(0.59)$$

$$\boxed{s_1 = -0.625}$$

$$s_2 = -0.625 \sin 30^\circ \pm j\, 1.18 \cos 30^\circ$$

$$\downarrow$$

$$\boxed{s_2 = -0.313 \pm j(1.022)}$$

$$s_3 = -0.625 \sin 90^\circ \pm j(1.18)\cos 90^\circ$$

$\boxed{s_3 = -0.625}$ → repeated root

$$H(s) = \frac{K}{(s-s_1)(s-s_2)(s-s_3)}$$

$$H(s) = \frac{K}{(s+0.625)(s+0.313+j(1.022))(s+0.313-j(1.022))}$$

$$\boxed{H(s) = \frac{K}{(s+0.625)(s^2+0.625s+1.142)}}$$

where K value occurs when $H(0) = 1$

$$\boxed{H(s) = \frac{0.625 \times 1.142}{(s+0.625)(s^2+0.625s+1.142)}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_N^2(\omega)}}$$

when $\omega = 0$

$$C_N(\omega=0) = \cos(N\cos^{-1}(0))$$

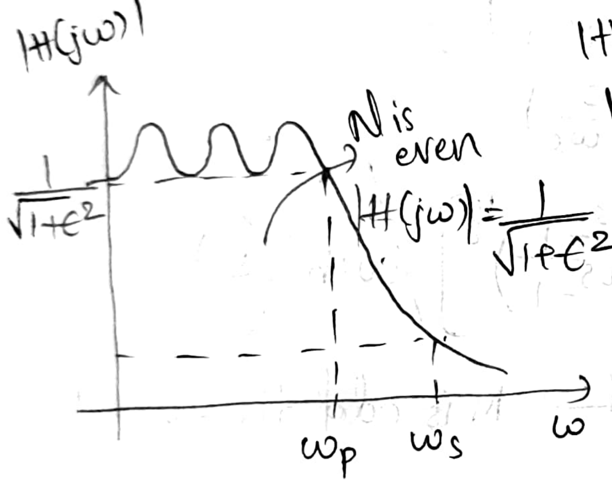
$$\boxed{C_N(\omega=0) = \cos(N \times \pi/2)}$$

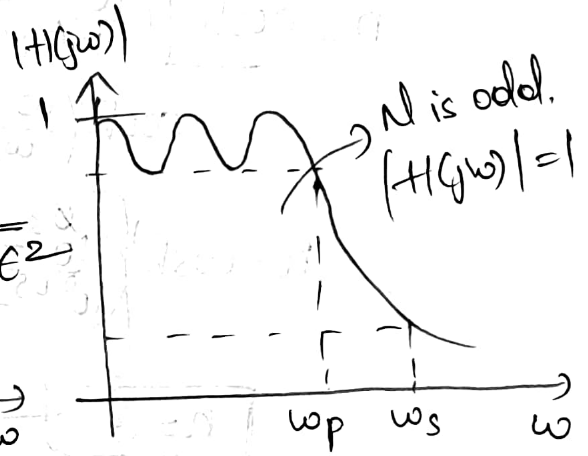
when $N \to$ even

$$C_N(\omega=0) = \pm 1 \to |H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2}}$$

when $N \to$ odd

$$C_N(\omega=0) = 0 \to |H(j\omega)| = 1$$





Q: Design a low pass filter with max gain of 5dB, with passband ripple of 0.5 dB & cut-off frequency of 2500 rad/s, stopband frequency of filter is 12,500 rad/s with stopband attenuation of 80dB or more.

Sol:- $A_{max} = 0.5\,dB$ $\quad A_{min} = 80\,dB$

$\omega_p = 2500\ rad/s$

$\omega_s = 12{,}500\ rad/s$

$$\epsilon = \left(10^{\frac{A_{max}}{10}} - 1\right)^{1/2} = \left(10^{0.05} - 1\right)^{1/2} = \underline{0.349}$$

$$n = \frac{\cosh^{-1}\left[\left(\dfrac{10^{\frac{A_{min}}{10}} - 1}{10^{\frac{A_{max}}{10}} - 1}\right)^{1/2}\right]}{\cosh^{-1}\left(\dfrac{\omega_s}{\omega_c}\right)}$$

$$n = \cosh^{-1}\left[\left(\frac{10^{8} - 1}{10^{0.05} - 1}\right)^{1/2}\right] \Big/ \cosh^{-1}(5)$$

$\boxed{n = 3} \rightarrow$ N is odd $H(s=0) = 1$

$$\boxed{H(s) = \frac{0.715}{(s+0.6264)(s^2+0.6264s+1.1424)}}$$

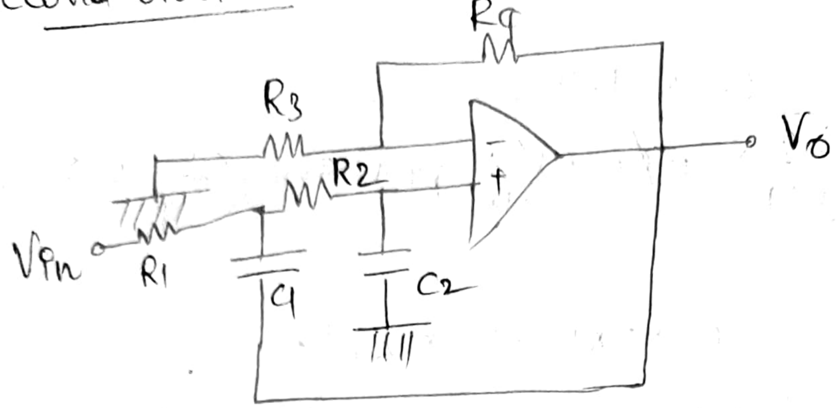
$$20\log_{10} x = 5$$

$$x = 10^{5/20} = \underline{1.7783}$$

Final transfer function is,

$$H(s) = \frac{0.715 \times 1.7783}{\underbrace{(s+0.6264)}_{1st\ order}\underbrace{(s^2+0.6264s+1.1424)}_{2nd\ order}}$$

Second order filter,



$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_o / R_1R_2C_1C_2}{s^2 + s\left[\frac{R_1C_2 + R_2C_2 + R_1C_1(1-A_o)}{R_1R_2C_1C_2}\right] + \frac{1}{R_1R_2C_1C_2}}$$

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_o / R^2C^2}{s^2 + \frac{1}{RC}s(3-A_o) + \left(\frac{1}{RC}\right)^2}$$

On comparing, $\left(\frac{1}{RC}\right)^2 = 1.1429$

$$RC = \sqrt{\frac{1}{1.1429}}$$

$$\frac{3-A_o}{RC} = 0.6264$$

$$(3-A_o) \times \sqrt{1.1429} = 0.6264$$

$$\underline{A_o = 2.41}$$

Set $A_o = 1$,

$$\boxed{\text{Then, } \frac{V_o(s)}{V_{in}(s)} = \frac{1/R_1R_2C_1C_2}{s^2 + \left[\frac{R_1C_1 + R_2C_2}{R_1R_2C_1C_2}\right] + \frac{1}{R_1R_2C_1C_2}}}$$

Since $\omega_p = 2500\ rad/s$

$$s \to \frac{s}{\omega_p}, \quad H(s) = \frac{0.215 \times 1.7783}{\left(\left(\frac{s}{\omega_p}\right)^2 + 0.6264\left(\frac{s}{\omega_p}\right) + 1.1424\right)\left(\frac{s}{\omega_p} + 0.6264\right)}$$

$$\boxed{H(s) = \frac{0.215 \times 1.7783 \times \omega_p^3}{(s^2 + 0.6264\, s\omega_p + 1.1424\,\omega_p^2)(s + 0.6264\,\omega_p)}}$$

On comparing,

$$\frac{1}{R_1 R_2 C_1 C_2} = 1.1424 \times \omega_p^2 = 1.1424 \times (2500)^2$$

$$\frac{R_1C_2 + R_2C_2}{R_1R_2C_1C_2} = 0.6264 \times 2500$$

~~Let $R_1 = R_2 = R = 10k\Omega$~~

$$\cancel{\frac{1}{C_1C_2} = 1.1424 \times (2500)^2 \times (10k)^2}$$

Assume $C_1 = C_2 = 1\mu F$

$$\frac{1}{R_1R_2} = 1.1424 \times (2500)^2 \times (1 \times 10^{-6})^2$$

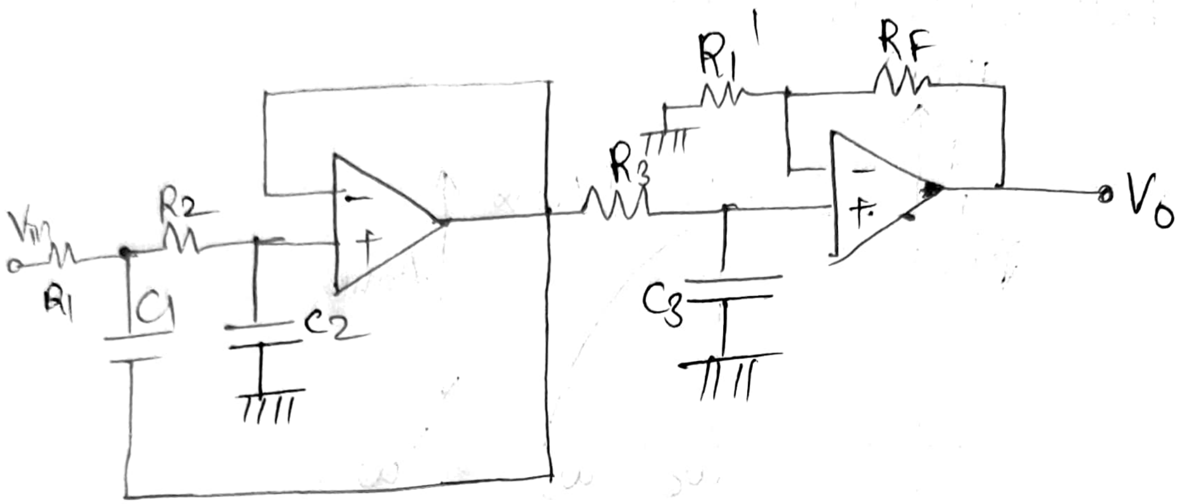
$$\frac{1}{R_1R_2} = 7.14 \times 10^{-6}$$

$$R_1R_2 = \frac{10^6}{7.14}$$

$$\frac{C_2(R_1 + R_2)}{R_1R_2} = 0.6264 \times 2500$$

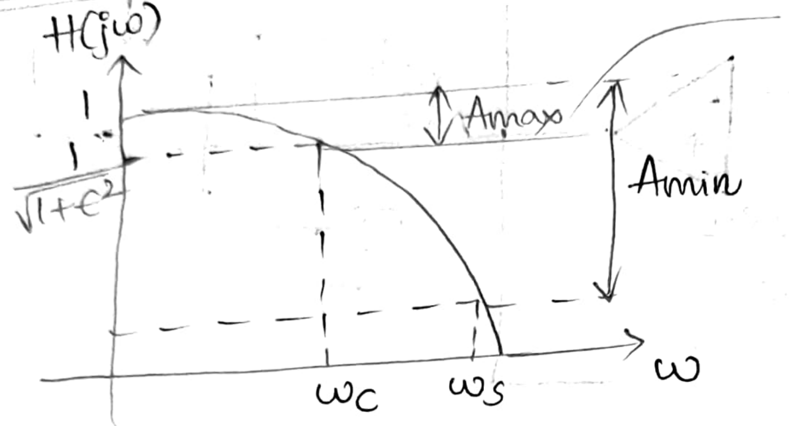
$$10^{-6}(R_1 + R_2) \times 7.14 \times 10^{-6} = 0.6264 \times 2500$$

Let $R_1 = R_2 = R$



V_{in}
R_1
R_2
C_1
C_2
R_1
R_F
R_3
C_3
V_0

Butterworth Approximation:



$\omega_c \rightarrow$ passband edge

$\omega_s \rightarrow$ stopband edge

$A_{max} \rightarrow$ Maximum passband attenuation. } (in dB)

$A_{min} \rightarrow$ Minimum stopband attenuation. }

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\omega^{2n}}}$$

$$|A_{max}| = 10\log_{10}(1+\epsilon^2)$$

$$\epsilon = \left(10^{A_{max}/10} - 1\right)^{1/2} \quad \text{---} \quad (1)$$

At $\omega = \dfrac{\omega_s}{\omega_c}$

$$|A_{min}| = 10\log_{10}(1+\epsilon^2\omega^{2n}) \quad \text{---} \quad (2)$$

Substituting eq (1) in (2)

$$A_{min} = 10\log_{10}\left(1 + \{10^{A_{max}/10} - 1\}(\omega^{2n})\right)$$

$$\omega^{2n} = \left\{ \frac{10^{A_{min}} - 1}{10^{A_{max}} - 1} \right\}$$

$$\omega = \frac{\omega_s}{\omega_c}$$

$$n = \frac{1}{2\log_{10}\left(\frac{\omega_s}{\omega_c}\right)} \times \log_{10}\left\{\frac{10^{A_{min}/10} - 1}{10^{A_{max}/10} - 1}\right\}$$

Q: A butterworth LPF is required to provide the following specifications passband upto 10 kHz with 3dB of loss, stopband to be at least 100dB down at 100kHz. Calculate the order of filter & roots of butterworth polynomials.

Sol:-

$A_{max} = 8\,dB$

$A_{min} = 100\,dB$

$f_s = 100\,kHz$

$f_c = 10\,kHz$

$n = 5,$

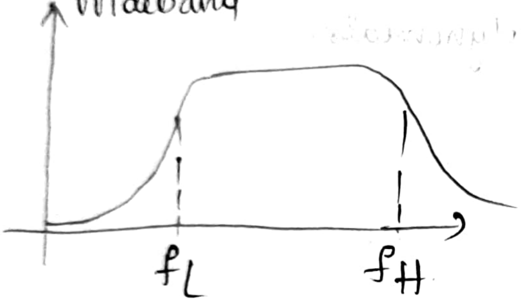
$$H(s) = \frac{1}{(s+1)(s^2+1.618s+1)\cdot(s^2+0.618s+1)}$$

Bandpass filters

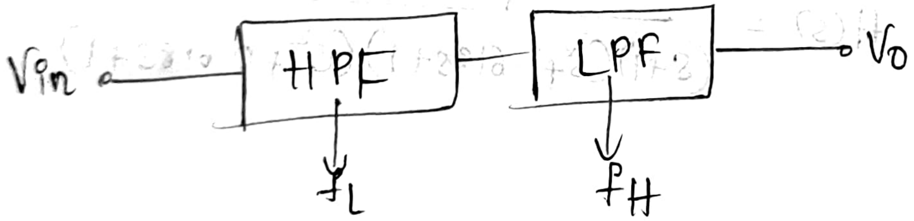
- Wideband pass ($Q < 10$)
- Narrow band pass ($Q > 10$)

$$Q = \frac{f_c}{B.W}$$

Wideband



→ **Wideband pass filters:**

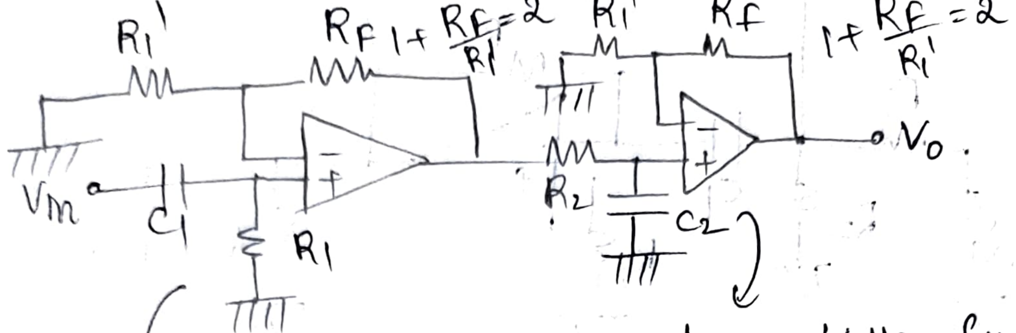


$f_H > f_L$

Q: Design a wide bandpass filter so that

$f_L = 200Hz$, $f_H = 1KHz$ & passband gain = 4

Sol:-



$$\frac{1}{2\pi R_2 C_2} = 1KHz = f_H$$

$$f_L = \frac{1}{2\pi R_1 C_1} = 200Hz$$

$C_2 = 0.1\mu F$

$C_1 = 0.1\mu F$

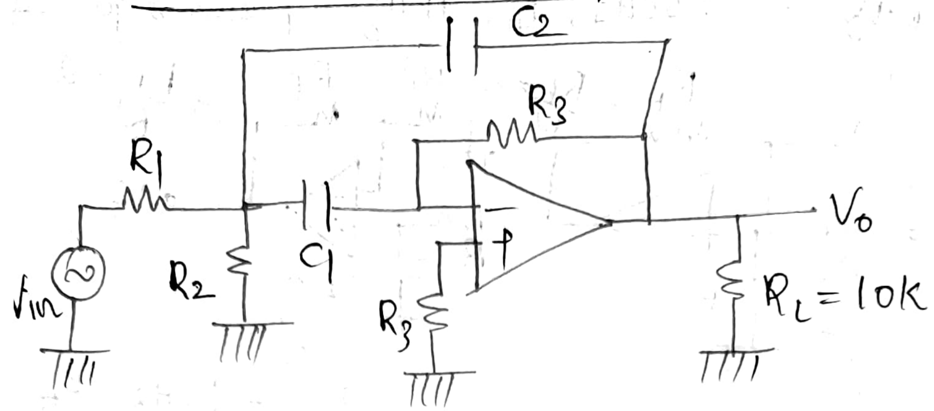
$$R_2 = \frac{1}{2\pi \times 10^{-7} \times 10^3}$$

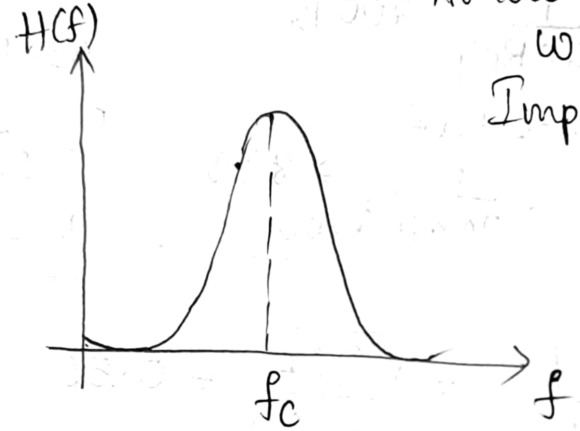
$$R_1 = \frac{1}{2\pi \times 10^{-7} \times 200} \cong 8k\Omega$$

$R_2 \simeq 1.6k\Omega$

$$Q = \frac{f_c}{BW} = \frac{\sqrt{f_L f_H}}{(f_H - f_L)} = 0.56$$

→ Narrow Bandpass filter:





At low frequencies,
$\omega = 0$,
Impedance is ∞

(*) Given f_c, Q, B.W. Design equations are
Assume $C_1 = C_2 = C$

$$R_1 = \frac{Q}{2\pi f_c C A_f}$$

$$R_2 = \frac{Q}{2\pi f_c C (2Q^2 - A_f)}$$

$$R_3 = \frac{Q}{\pi f_c C}$$

Passband gain $= A_f = \dfrac{R_3}{2R_1}$ $\quad A_f < 2Q^2$ → Condition must be satisfied.

→ Center frequency f_c can be changed to new center frequency f_c', keep gain & B.W constant by changing R_2 to R_2'. $\boxed{R_2' = R_2\left(\frac{f_c}{f_1}\right)^2}$

Q: (a) Design a narrow bandpass filter with $f_c = 1kHz$
$Q = 3$, $A_F = 10$

(b) Change the center freq to 1.5 kHz keeping
A_F & B.W constant.

Sol: $C_1 = C_2 = C = 0.01\,\mu F$

$R_1 = 4.72\,k\Omega$

$R_2 = 5.97\,k\Omega$

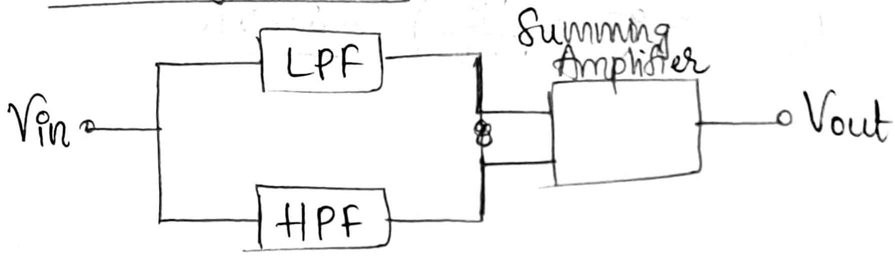
$R_3 = 95.5\,k\Omega$

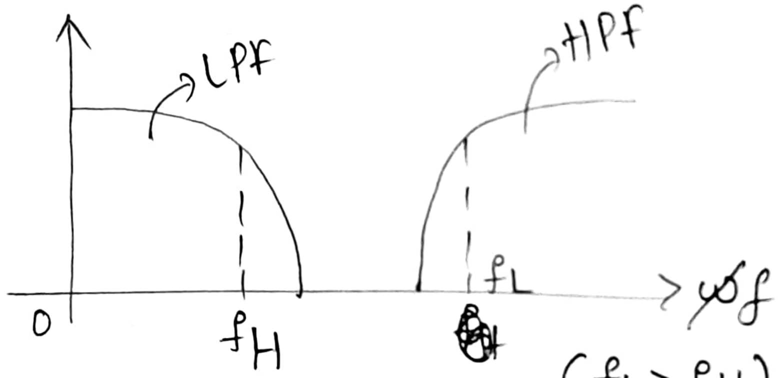
$R_2' = 5.97 \times \left(\frac{1k}{1.5k}\right)^2 = 2.65\,k\Omega$

Band Reject filters (Band stop)

- Wideband ($Q < 10$): $Q = \frac{f_c}{B.W}$
- Narrowband ($Q > 10$): $Q = \frac{f_c}{B.W}$

→ **Wideband reject filters**

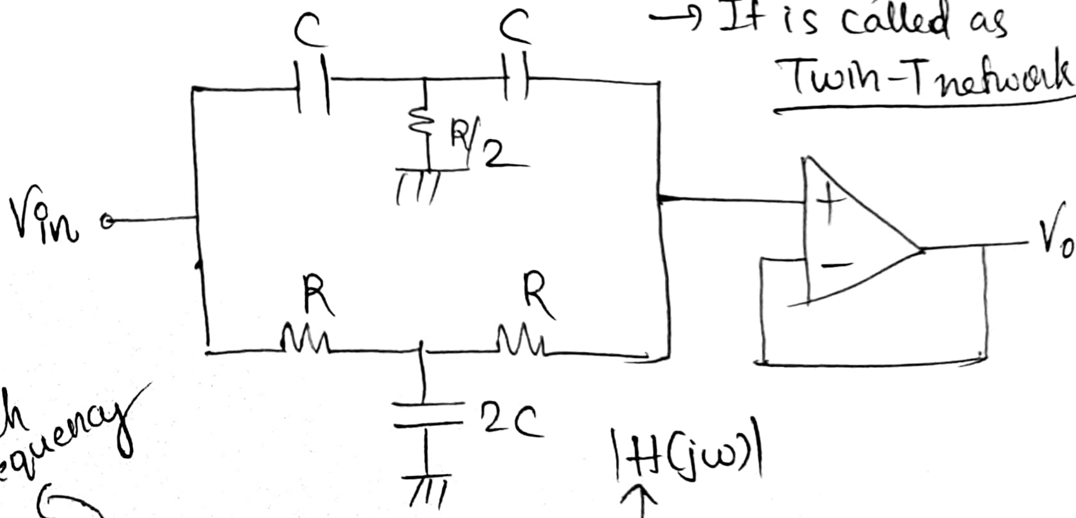




($f_L > f_H$)

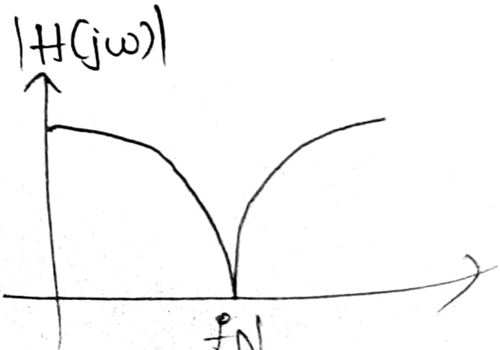
→ **Narrow Band Reject filter: (Notch filter)**

→ It is called as Twin-T network.



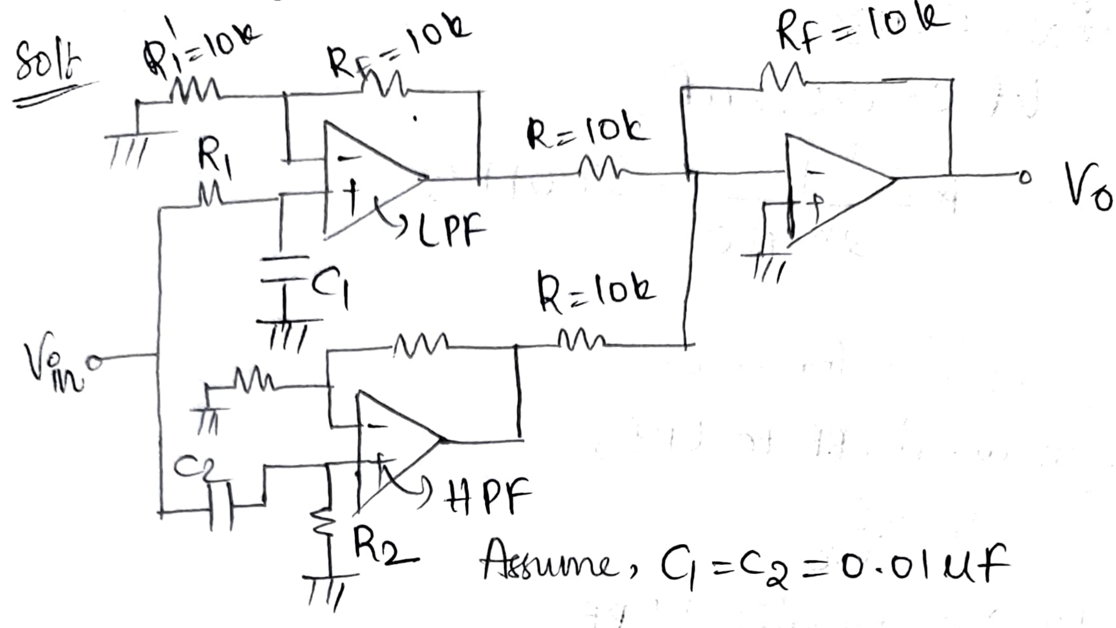
Notch frequency

$$\boxed{f_N = \frac{1}{2\pi RC}}$$



Q: Design a wide band reject filter with $f_H = 200\,Hz$ and $f_L = 1\,kHz$

Sol:



Assume, $C_1 = C_2 = 0.01\,\mu F$

LPF

$$\frac{1}{2\pi R_1 C_1} = 200$$

$$R_1 = \frac{1}{2\pi \times 200 \times 10^{-8}}$$

$$R_1 = 79.5\,k\Omega$$

HPF

$$\frac{1}{2\pi R_2 C_2} = 1000$$

$$R_2 = \frac{1}{2\pi \times 10^{3} \times 10^{-8}}$$

$$R_2 = 15.9\,k\Omega$$

Q: Design a 60Hz active notch filter

Sol: $f_N = 60Hz$

let $C = 0.068\ \mu f$

$$R = \frac{1}{2\pi f_N C} = \underline{39.01\ k\Omega}$$

(*) Frequency Transformations:

① Normalised LPF to LPF,

$$s \rightarrow \frac{s}{\omega_c}$$

$H(s) \rightarrow$ Normalized LPF

$$LPF \leftarrow H'(s) = H\left(\frac{s}{\omega_c}\right)$$

② Normalised LPF to HPF,

$$s \rightarrow \frac{\omega_c}{s} \qquad H(s) \rightarrow \text{Normalized LPF.}$$

$$H'(s) = H\left(\frac{\omega_c}{s}\right) \rightarrow HPF$$

③ Normalised LPF to BPF,

$$s \rightarrow \frac{s^2 + \omega_l \omega_h}{s(\omega_h - \omega_l)}$$

$$H'(s) = H\left(\frac{s^2 + \omega_l \omega_h}{s(\omega_h - \omega_l)}\right) \rightarrow \text{Band pass filter.}$$

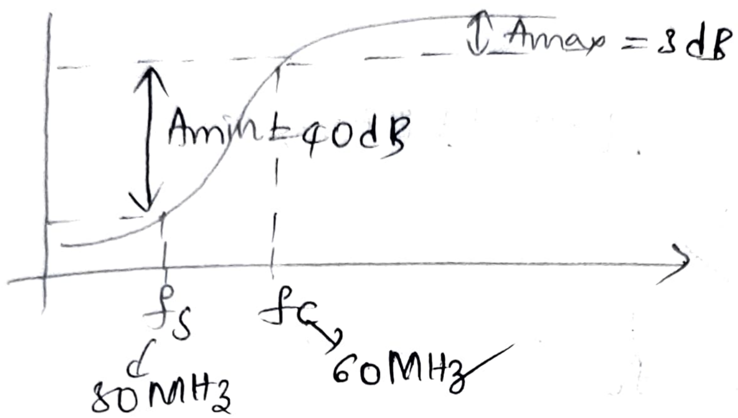
④ Normalised LPF to BSF,

$$s \rightarrow \frac{s(\omega_h - \omega_l)}{s^2 + \omega_l \omega_h}$$

$$H'(s) = H\left(\frac{s(\omega_h - \omega_l)}{s^2 + \omega_l \omega_h}\right) \rightarrow \text{Band stop filter.}$$

Q: A butterworth HPF has $f_c = 60 MHz$, $f_s = 80 MHz$
$A_{min} = 40 dB$, $A_{max} = 3 dB$
find order of filter.

Sol:-



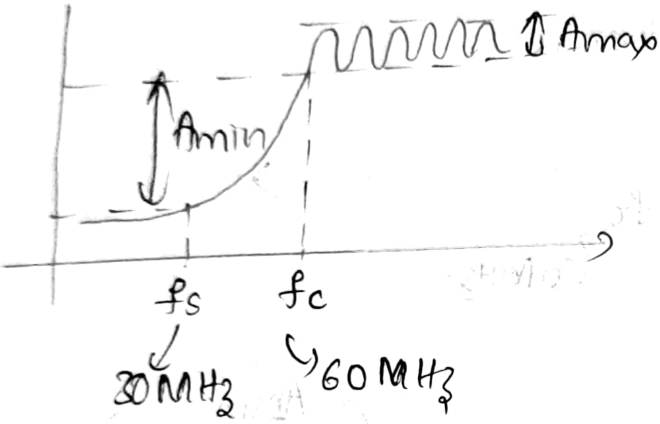
$$n = \frac{1}{2\log_{10}\left(\frac{\omega_c}{\omega_s}\right)} \times \log_{10}\left(\frac{10^{\frac{A_{min}}{10}} - 1}{10^{\frac{A_{max}}{10}} - 1}\right)$$

$$\underline{n = 6.64}$$

Q: A chebyshev HPF has $f_c = 60\,MHz$, $A_{min} = 40\,dB$
$f_s = 30\,MHz$, $A_{max} = 0.5\,dB$

find order of filter.

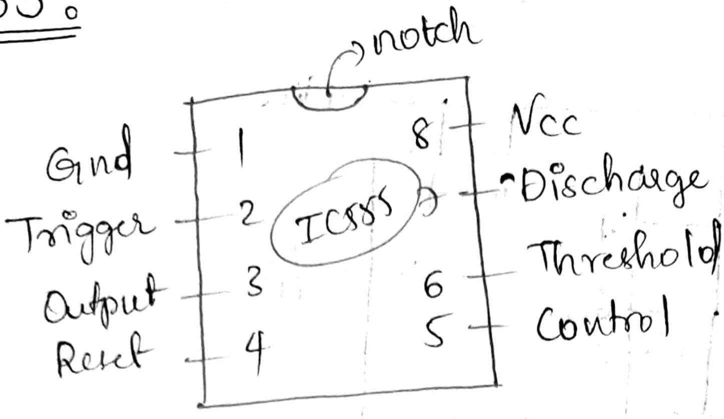
Sol:



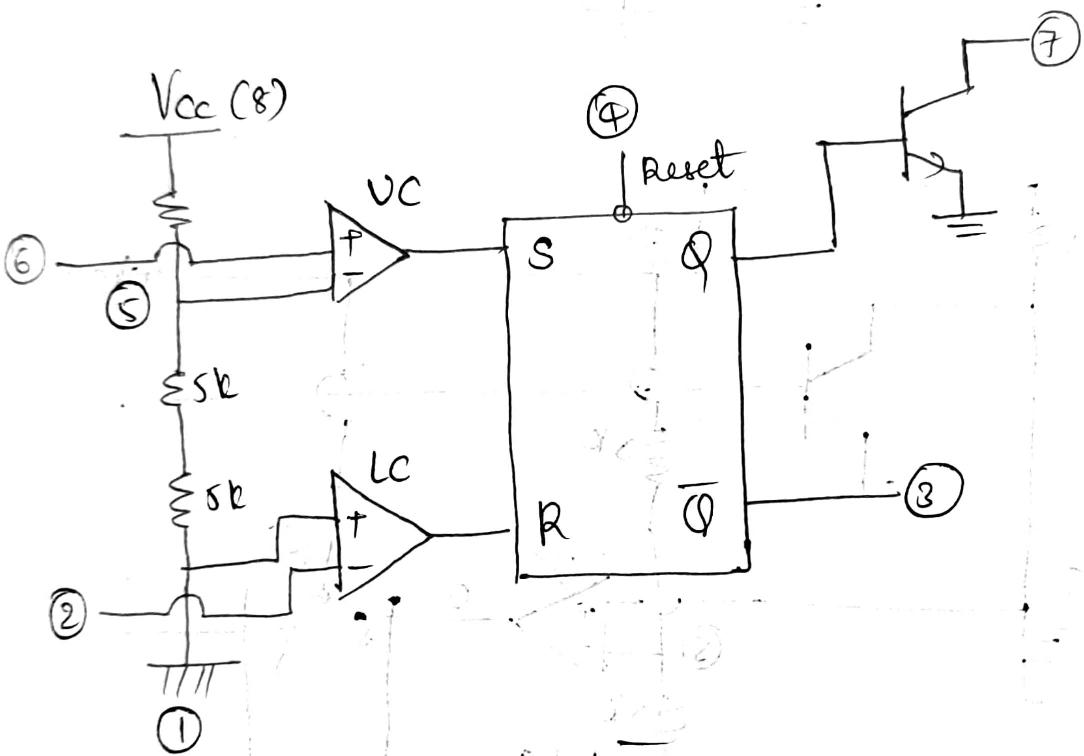
$$n = \frac{\cosh^{-1}\left(10^{\frac{A_{min}}{10}} - 1 \Big/ 10^{\frac{A_{max}}{10}} - 1\right)^{1/2}}{\cosh^{-1}\left(\frac{\omega_s}{\omega_c}\right)}$$

Normalized HPF.

IC555:

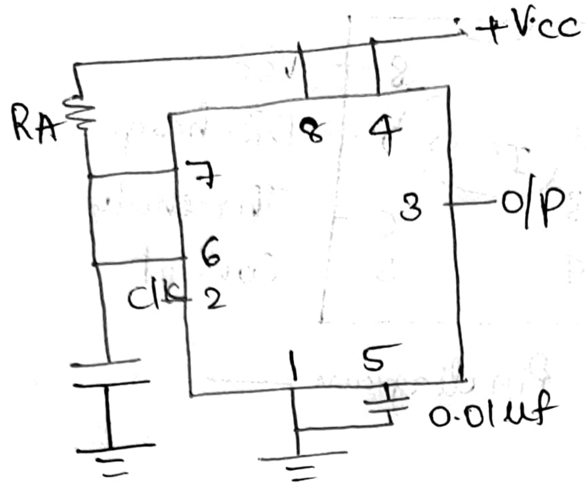


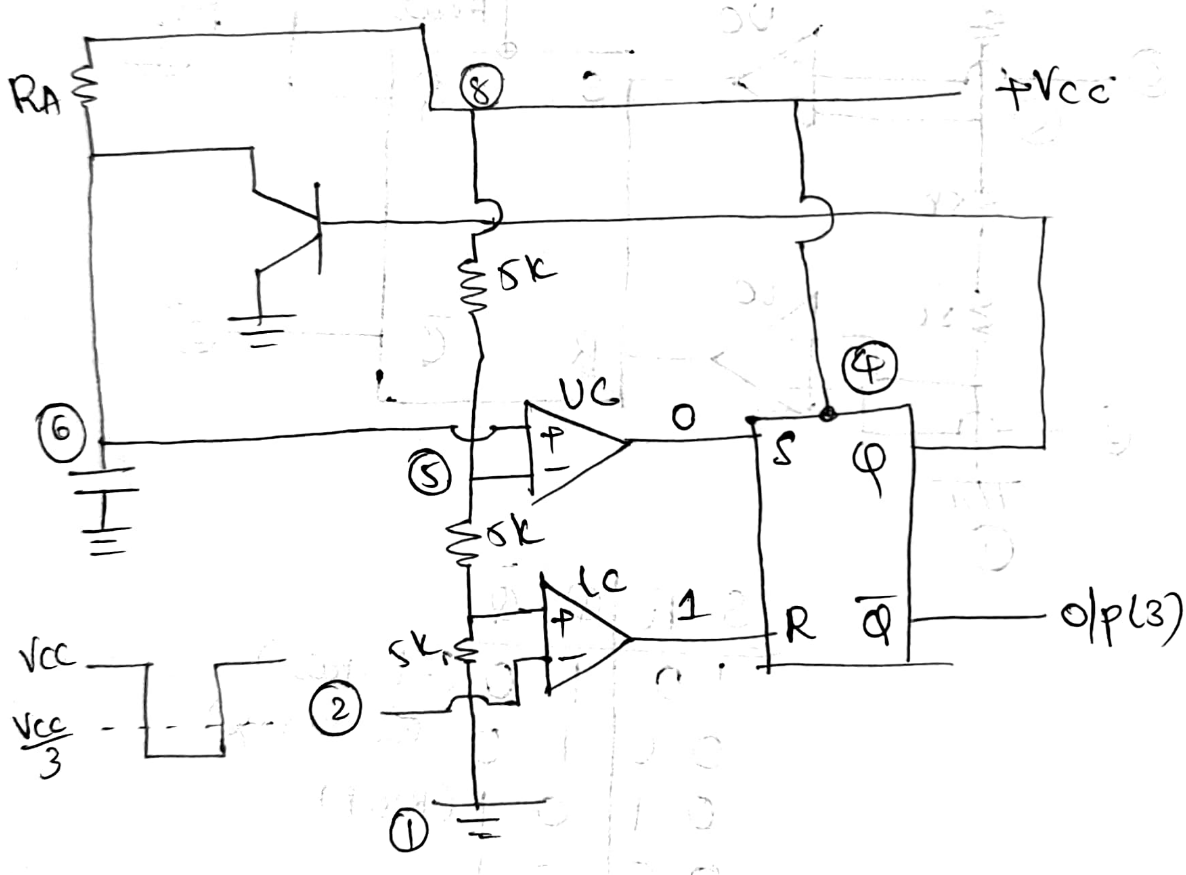
— Pin diagram —

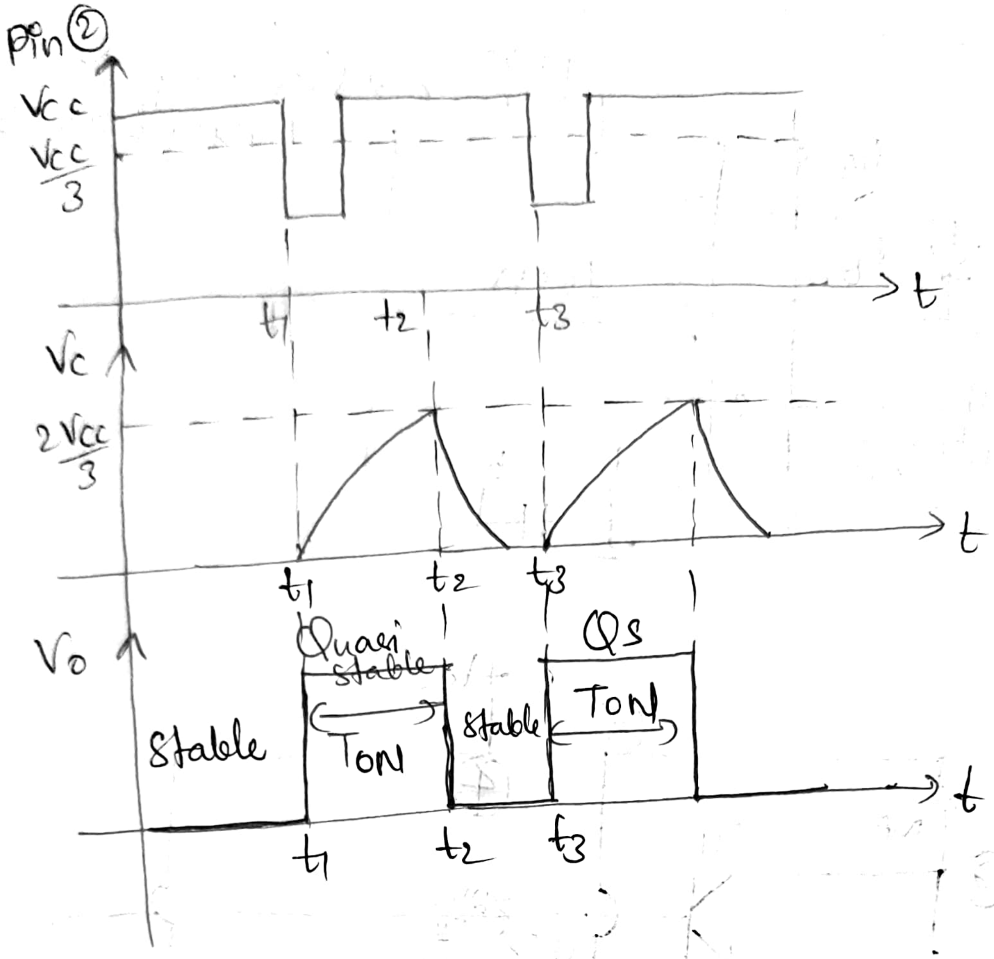


S	R	Q	$\overline{Q}$	
1	0	1	0	(Set state)
0	0	1	0	(No change)
0	1	0	1	(Reset)
0	0	0	1	
1	1	Invalid		

(*) IC555 as monostable multivibrator:







Time duration (T_{ON})

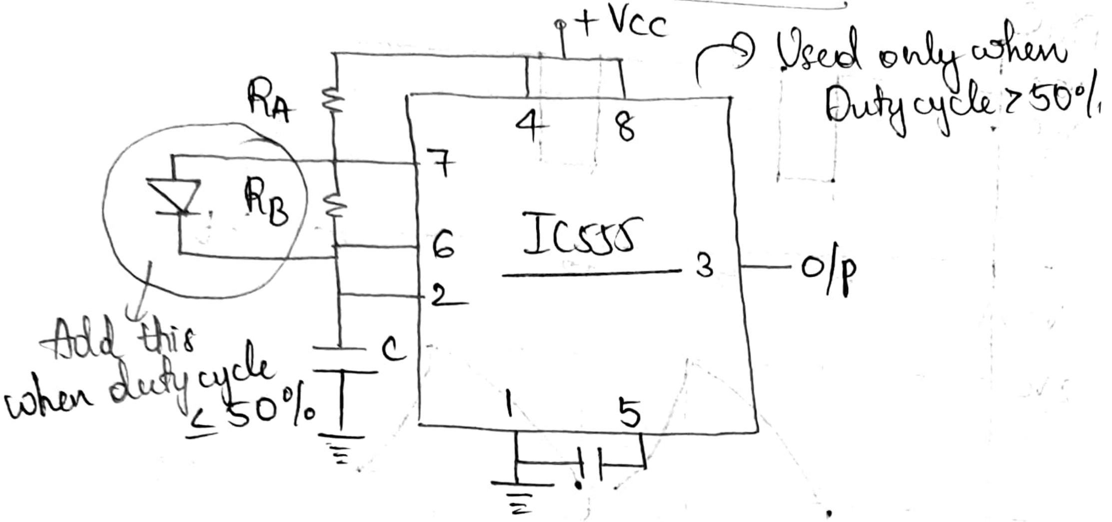
$$V_C = V_{CC}\left(1 - e^{-t/RC}\right)$$

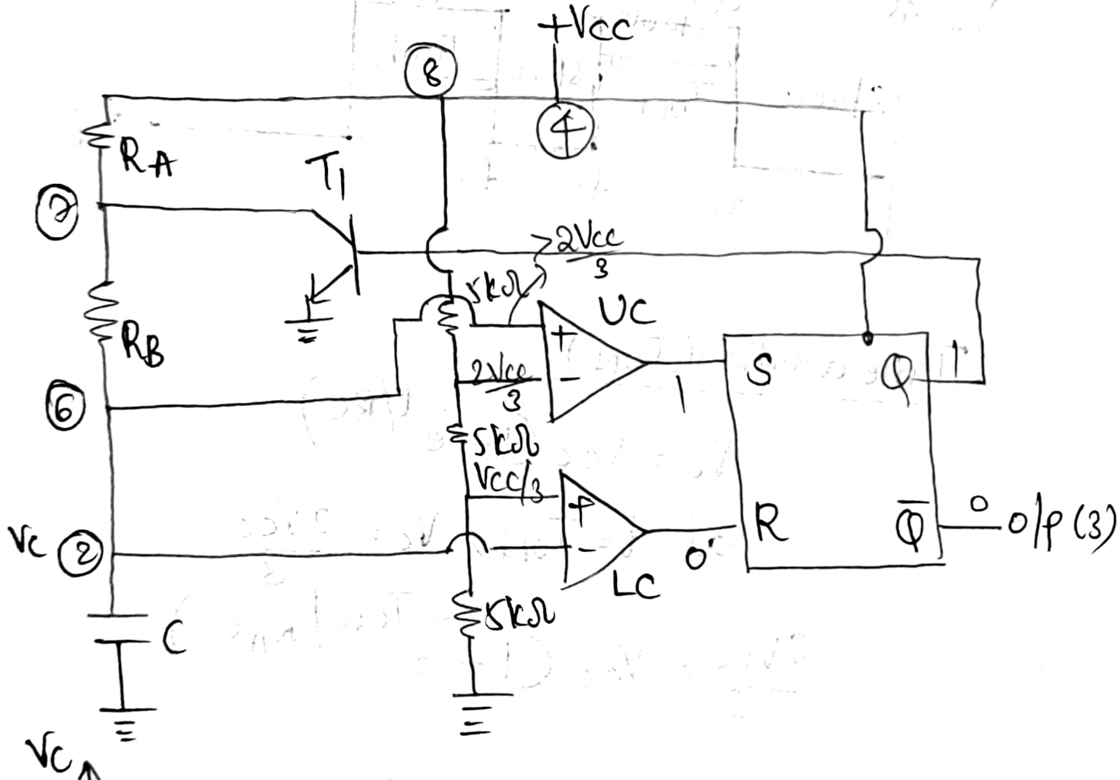
at $t = T_{ON}$, $V_C = \frac{2V_{CC}}{3}$

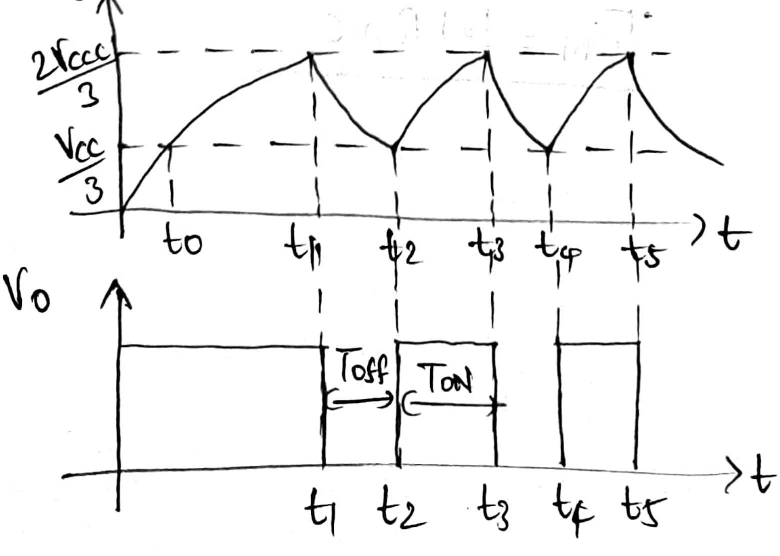
$$\frac{2V_{CC}}{3} = V_{CC}\left(1 - e^{-T_{ON}/R_A C}\right)$$

$$T_{ON} = 1.1\, R_A C$$

* Astable operation using IC555 timer:

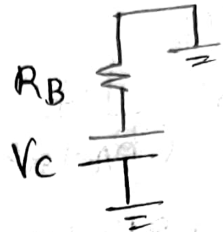






T_{ON} $(t_2 - t_3)$

T_{OFF} $(t_1 - t_2)$



$$T_{OFF} = 0.693 R_B C$$

$$T_{ON} = 0.693(R_A + R_B)C$$

$$\text{Timeperiod}(T) = T_{ON} + T_{OFF}$$

$$T = 0.693(R_A + 2R_B)C$$

$$\boxed{f = \frac{1}{T} = \frac{1.45}{(R_A + 2R_B)C}}$$ → frequency of oscillation.

$$\text{Duty cycle}(D) = \frac{T_{ON}}{T}$$

$$= \frac{0.693(R_A + R_B)C}{0.693(R_A + 2R_B)C}$$

$$\boxed{(D) = \frac{R_A + R_B}{R_A + 2R_B}}$$

Q: Design a astable multivibrator using IC555 timer which generates a signal with frequency 1kHz & duty cycle 75%.

Sol:

$$f = 1\,kHz = \frac{1.45}{(R_A + 2R_B)C}$$

$$(R_A + 2R_B)C = 1.45 \times 10^{-3} \quad \text{---} \quad (1)$$

$$D = \frac{R_A + R_B}{R_A + 2R_B} = 0.75$$

$$(R_A + R_B) = 0.75(R_A + 2R_B)$$

$$R_A(0.25) = R_B(1.5 - 1)$$

$$\boxed{R_A = 2R_B}$$

from (1),
$$4R_B C = 1.45 \times 10^{-3}$$

Assume $C = 0.1\,\mu F$

$$4R_B(10^{-7}) = 1.45 \times 10^{-3}$$

$$4R_B = 1.45 \times 10^{4}$$

$$R_B = 1.45 \times 25 \times 10^{2}$$

$$\boxed{R_B = 3625\,\Omega}$$

$$\begin{array}{r} 145 \\ \times\ 25 \\ \hline 3625 \end{array}$$

$$R_A = 7.25\,k\Omega$$

$$R_B = 3.625\,k\Omega$$

Q: Design a astable multivibrator using IC555 timer which generates a signal with $f = 1\,kHz$ & $D = 25\%$

Sol:

$$\left.\begin{aligned} T_{ON} &= 0.693\,R_A C \\ T_{OFF} &= 0.693\,R_B C \end{aligned}\right\} \text{ when } D \leq 50\%$$

$$T = 0.693\,(R_A + R_B)\,C$$

$$D = \frac{T_{ON}}{T} = \frac{R_A}{R_A + R_B}$$

$$f = 1\,kHz = \frac{1.45}{(R_A + R_B)\,C}$$

$$(R_A + R_B)\,C = 1.45 \times 10^{-3} \quad \text{— (1)}$$

$$0.25 = \frac{R_A}{R_A + R_B}$$

$$0.25\,R_B = 0.75\,R_A$$

$$\boxed{R_B = 3R_A}$$

from (1),

$$4R_A C = 1.45 \times 10^{-3}$$

Assume $C = 0.1\,\mu F$

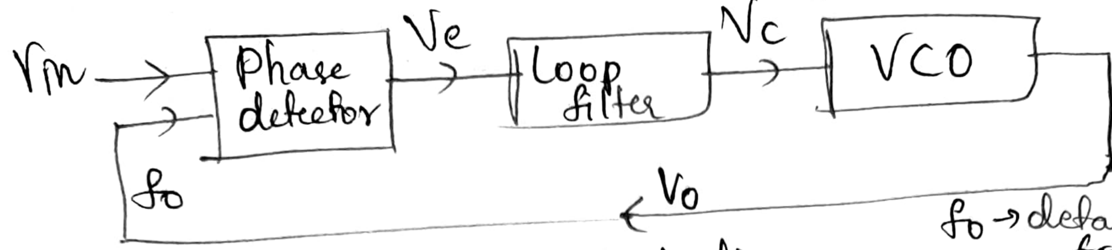
$$4R_A (10^{-7}) = 1.45 \times 10^{-3}$$

$$\underline{R_A = 3.625\,k\Omega}$$

$$\underline{R_B = 10.875\,k\Omega}$$

◦ PHASE LOCKED LOOP:

$V_e \rightarrow$ error voltage
$V_c \rightarrow$ low dc voltage



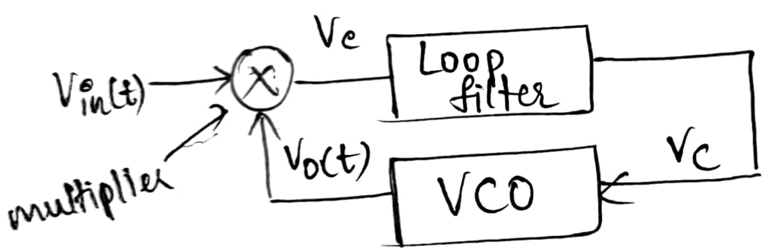
$f_o \rightarrow$ default freq.

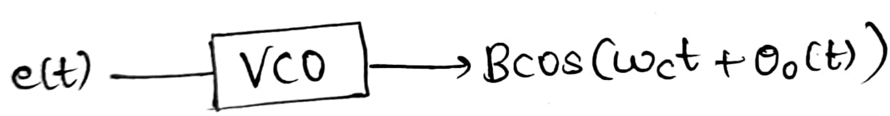
fig. Basic block diagram

VCO → Voltage Controlled Oscillator.

→ Modes of operation:

1. free running state. (f_o)
2. Capture state (when Vin is non-zero)
3. Lock state. (VCO frequency has changed to f_1 from f_o)





VCO o/p → $V_o(t) = B\cos(\omega_c t + \theta_o(t))$

Instantaneous angular freq of VCO o/p
$$= \omega_c + \dot{\theta}_o(t)$$

where, $\dot{\theta}_o(t) = c\,e(t)$

when $e(t) = 0 \longrightarrow \dot{\theta}_o(t) = 0$

So, instantaneous angular freq of VCO o/p is $\omega_c = 2\pi f_c$

where $f_c \rightarrow$ free running frequency of VCO.

Let $V_{in}(t) = A\sin(\omega_i t + \varphi_0) = A\sin(\theta_i(t) + \omega_c t)$

$$V_{in}(t) = A\sin(\omega_c t + (\omega_i - \omega_c)t + \varphi_0)$$

$$\boxed{\theta_i(t) = (\omega_i - \omega_c)t + \varphi_0}$$

$$\theta_i(s) = \frac{(\omega_0 - \omega_c)}{s^2} + \frac{\varphi_0}{s}$$

$$V_0(t) = B\cos(\omega_c t + \theta_0(t))$$

$$V_e(t) = AB\sin(\omega_c t + \theta_i(t))\cos(\omega_c t + \theta_0(t))$$

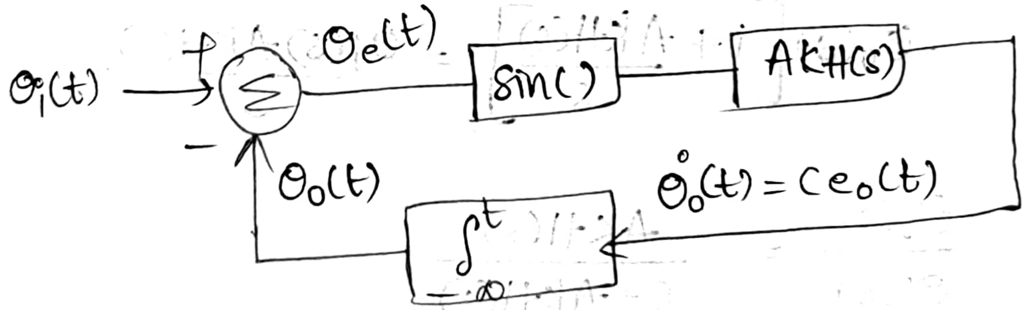
$$V_e(t) = \frac{1}{2}AB\left[\sin(\theta_i(t) - \theta_0(t)) + \sin(2\omega_c t + \theta_i(t) + \theta_0(t))\right]$$

Since loop filter is LPF,

$$V_c = \frac{1}{2}AB\left[\sin(\theta_i(t) - \theta_0(t))\right]$$

Let $H(s)$ be Transfer function of loop filter,

Interms of phase,



$$\dot{\theta}_0(t) = ce_0(t)$$

$$e(t) = h(t) * \frac{1}{2}AB\sin(\theta_i(t) - \theta_0(t))$$

$$ce(t) = ch(t) * \frac{1}{2}AB\sin(\theta_i(t) - \theta_0(t))$$

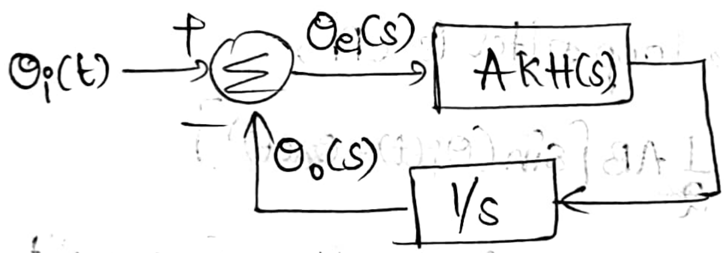
$$\theta_o(t) = c e_o(t) \qquad \left(K = \frac{1}{2} CB\right)$$

$$\dot{\theta}_o(t) = \frac{CAB}{2} \int_{-\infty}^{t} h(t-x) \sin(\theta_i(x) - \theta_o(x))\, dx$$

$$\dot{\theta}_o(t) = AK \int_{-\infty}^{t} h(t-x) \sin(\theta_e(t))\, dx$$

If $\sin(\theta_e(t)) \approx \theta_e(t)$, then entire circuit becomes linear & we can apply convolution. Assume $\theta_e(t) << \pi/2$, then

$$\sin(\theta_e) \approx \theta_e$$



$$\theta_o(s) = [\theta_i(s) - \theta_o(s)] \frac{AKH(s)}{s}$$

$$\theta_o(s)\left[1 + \frac{AKH(s)}{s}\right] = \frac{\theta_i(s) AKH(s)}{s}$$

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{AKH(s)}{s + AKH(s)}$$

$$\theta_e(s) = \theta_i(s) - \theta_o(s)$$

$$\theta_e(s) = \frac{s\,\theta_i(s)}{s + AKH(s)}$$

$$\theta_i(t) = (\omega_o - \omega_c)t + \psi_o$$

$$\theta_i(s) = \frac{\omega_i - \omega_c}{s^2} + \frac{\varphi_0}{s}$$

$$\theta_e(s) = \frac{s}{s + AKH(s)}\left[\frac{\omega_i - \omega_c}{s^2} + \frac{\varphi_0}{s}\right]$$

Let $H(s) = 1$, (1st order LPF)

$$\theta_e(s) = \frac{s}{s + Ak}\left[\frac{\omega_i - \omega_c}{s^2} + \frac{\varphi_0}{s}\right]$$

$$\theta_e(s) = \frac{\omega_i - \omega_c}{s(s + Ak)} + \frac{\varphi_0}{(s + Ak)}$$

$$\theta_e(s) = \frac{\omega_i - \omega_c / Ak}{s} - \frac{(\omega_i - \omega_c)/Ak}{(s + Ak)} + \frac{\varphi_0}{(s + Ak)}$$

$$\theta_e(t) = \frac{\omega_i - \omega_c}{Ak} - \frac{(\omega_i - \omega_c)}{Ak} e^{-Akt} + \varphi_0 e^{-Akt}$$

$$\boxed{\lim_{t \to \infty} \theta_e(t) = \left(\frac{\omega_i - \omega_c}{Ak}\right)} \rightarrow \text{Steady state response.}$$

Let $H(s) = \frac{s + a}{s}$

$$\theta_e(s) = \frac{s}{s + Ak\left(\frac{s + a}{s}\right)}\left[\frac{\omega_i - \omega_c}{s^2} + \frac{\varphi_0}{s}\right]$$

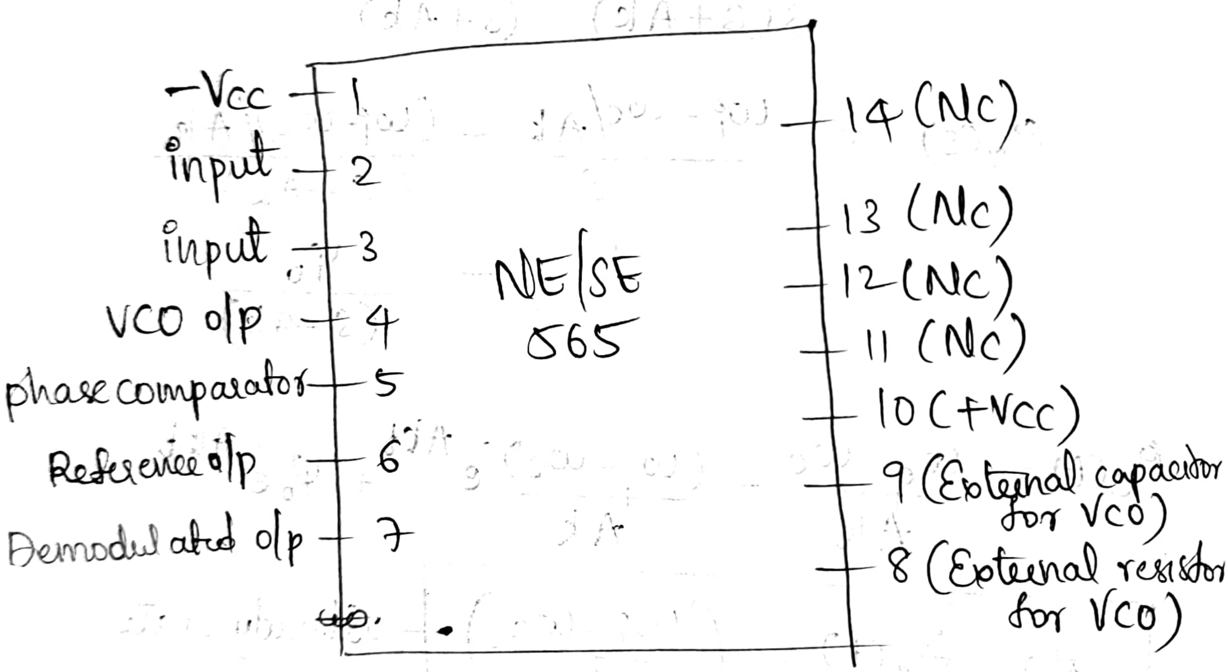
From final value theorem,

$$\theta_e(t=\infty) = \lim_{s \to 0} s\,\theta_e(s)$$

$$\underline{\theta_e(t=\infty) = 0}$$

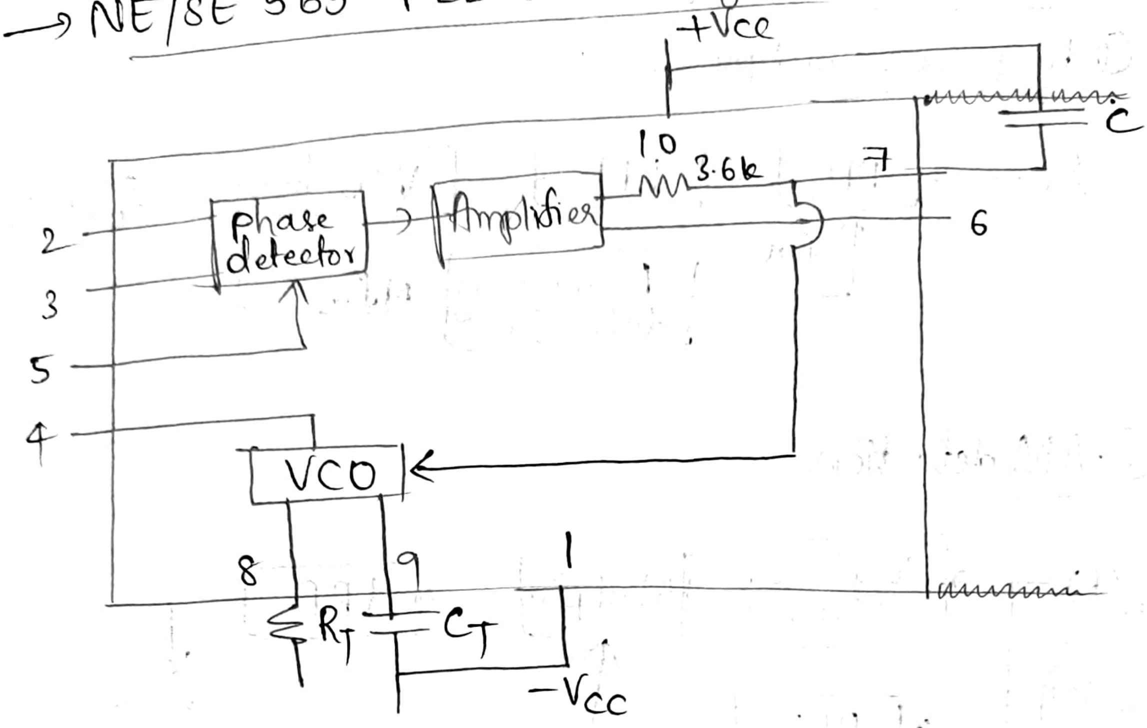
There is a exact phase difference of $\pi/2$ between i/p & o/p.

<u>IC PLL 565</u>



— Pin Diagram —

→ NE/SE 565 PLL Block diagram:



→ Important electrical parameters of 565 PLL,

- operating freq range = 0.001 Hz to 500kHz
- operating voltage range = ±6V to ±12V
- Input level : 10 mV rms (min) to 3V (p−p) max.
- o/p sink current = 1mA (typical)
- i/p impedance = 10 kΩ.

lock in range

$$\Delta f_L = \pm \frac{7.8 f_0}{V}$$

$$V = +V_{cc} - (-V_{cc})$$

Capture range

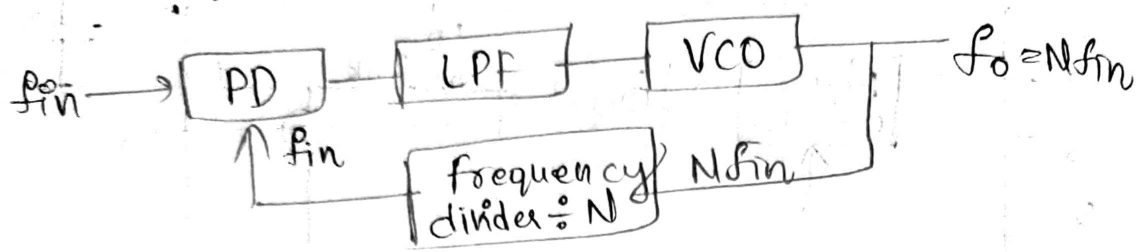
$$\Delta f_C = \pm \left[\frac{\Delta f_L}{(2\pi)(3.6)10^3 C} \right]^{1/2}$$

free running frequency

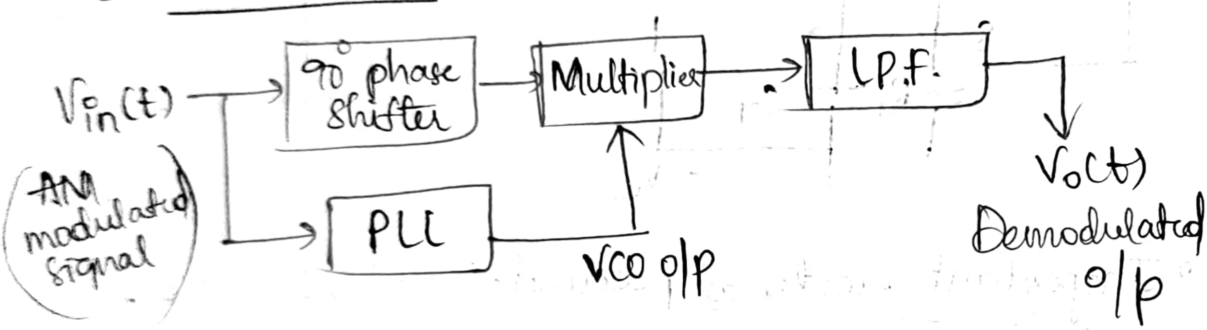
$$f_0 = \frac{0.25}{R_T C_T} \text{ Hz}$$

- Applications of PLL,

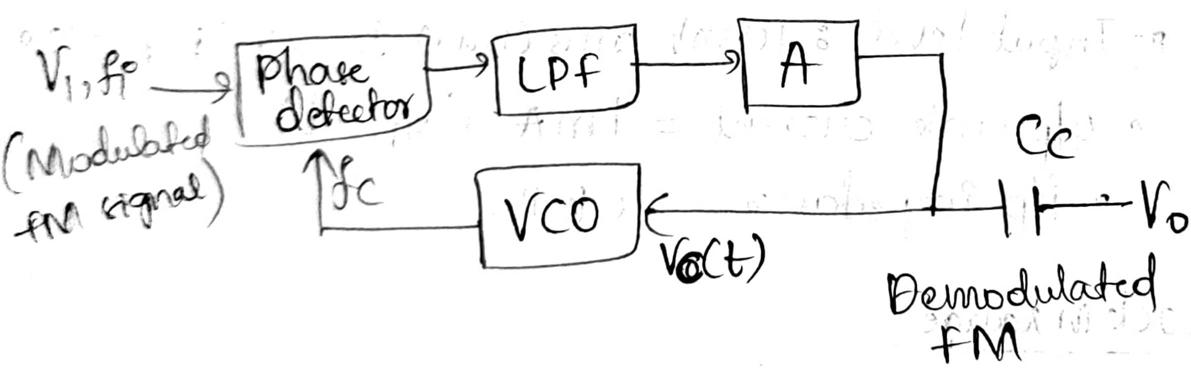
① frequency multiplier,



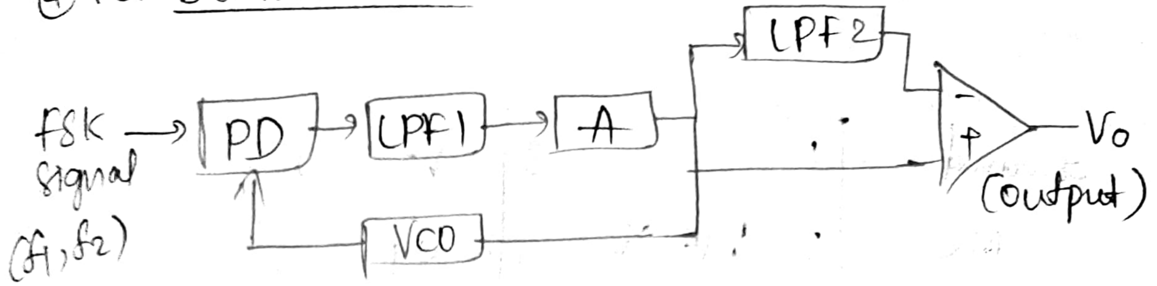
② AM detection



③ FM detection

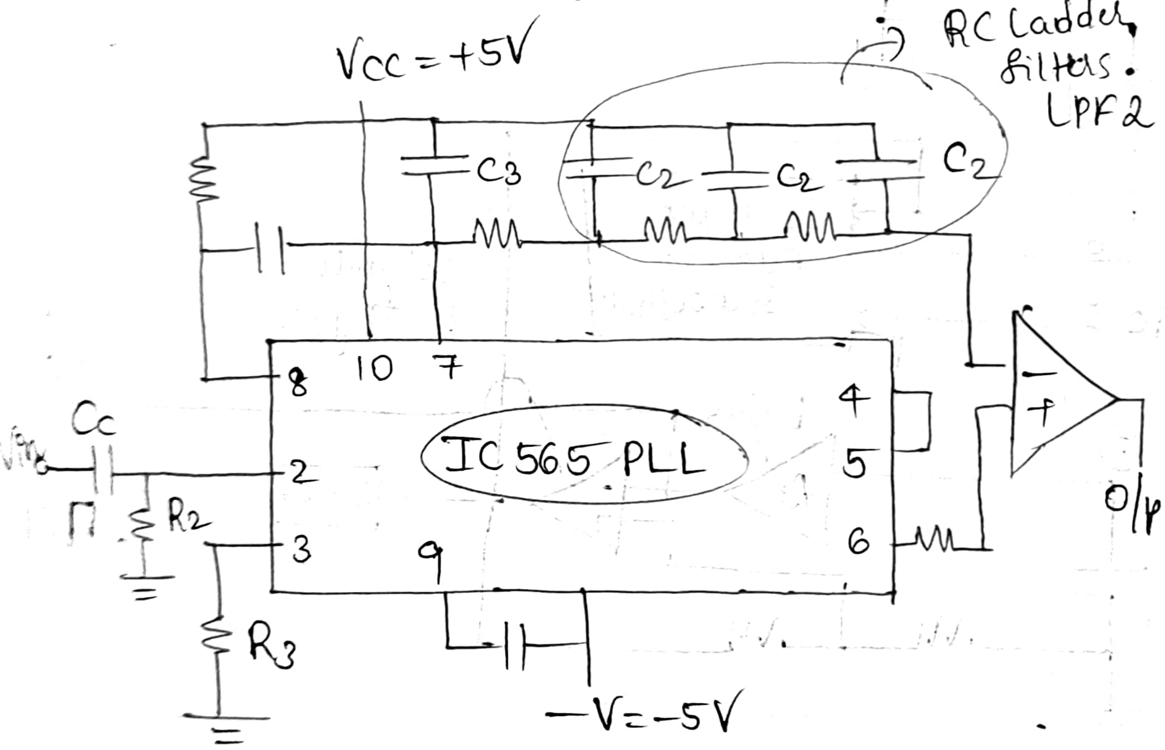


④ FSK Demodulation

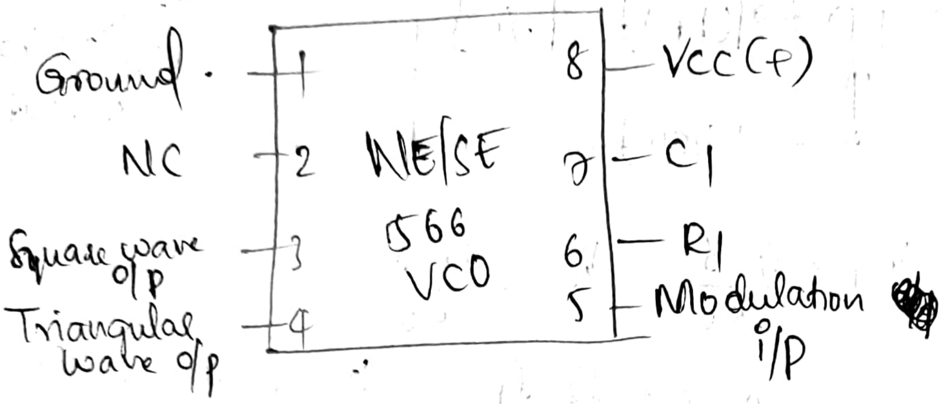


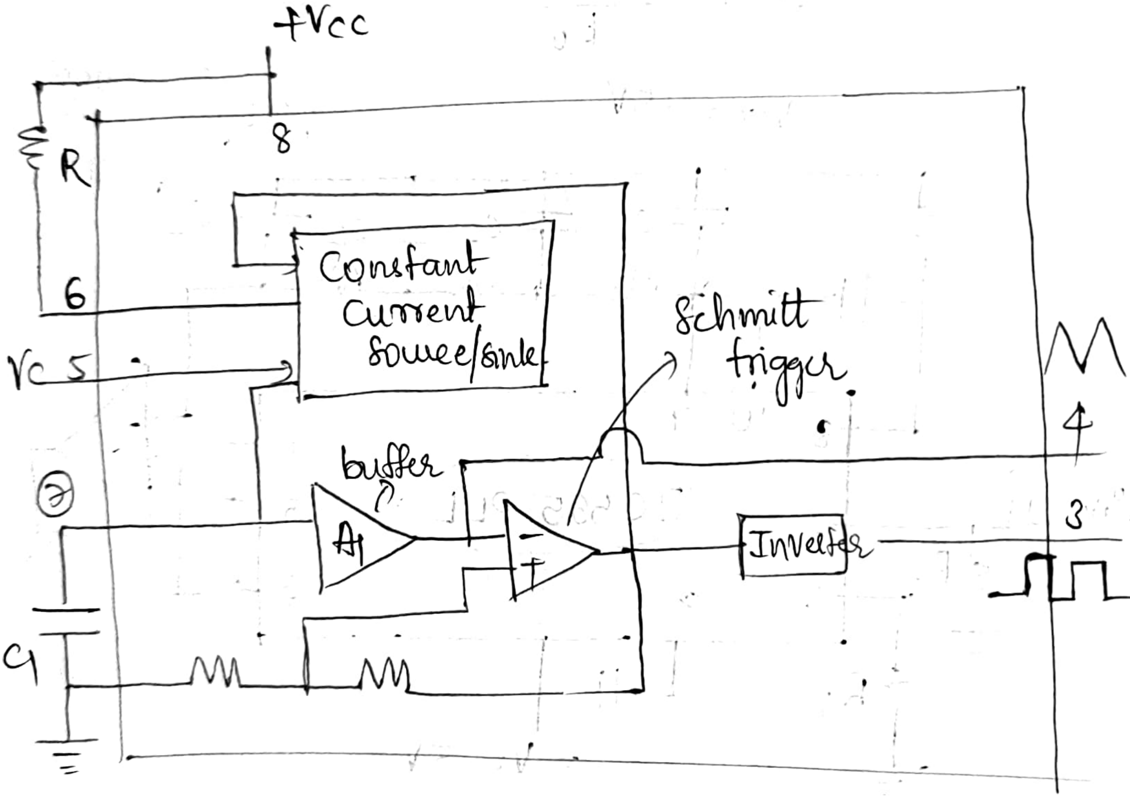
$$V_{f_1} = \frac{f_1 - f_0}{k_0} \qquad V_{f_2} = \frac{f_2 - f_0}{k_0}$$

$$\Delta V_f = \frac{f_2 - f_1}{k_0} \qquad k_0 - \text{parameter of VCO.}$$



- IC Voltage controlled oscillator NE/SE 566:





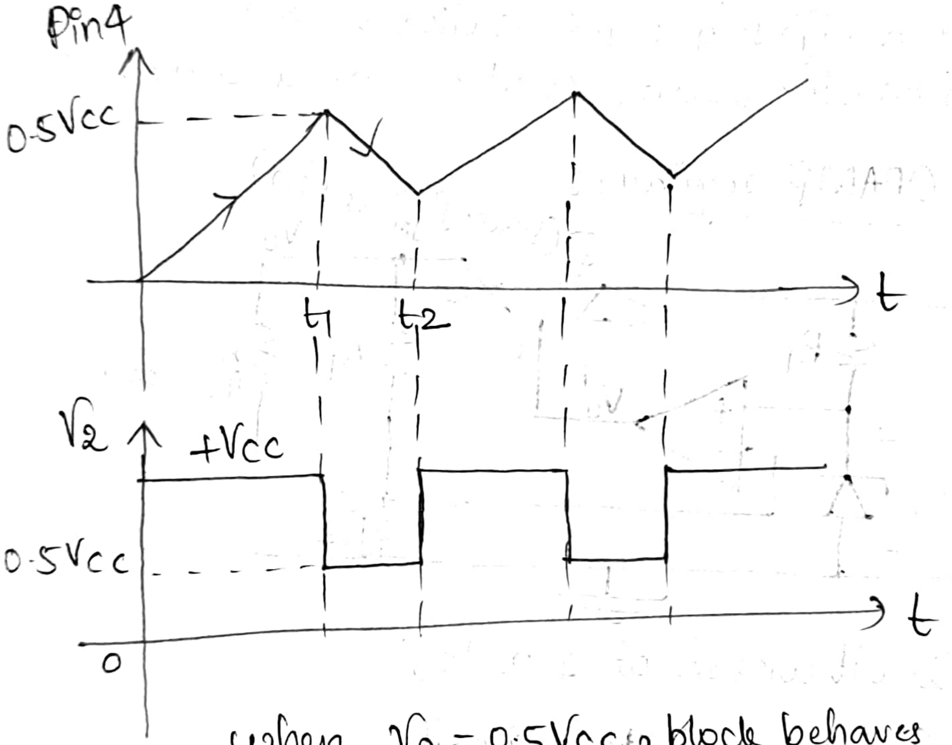
V_c - modulating input voltage

V_2 = o/p of OPAMP A_2 = $+V_{sat} = +V_{cc}$

$-V_{sat} = 0.5\,V_{cc}$

when $V_2 = +V_{cc}$, Block behaves as constant current source.

$$i = \frac{V_{cc} - V_c}{R_1}$$



when $V_2 = 0.5V_{cc}$, block behaves as constant current sink.

$Q = CV$

$$i = C\frac{dV}{dt}$$

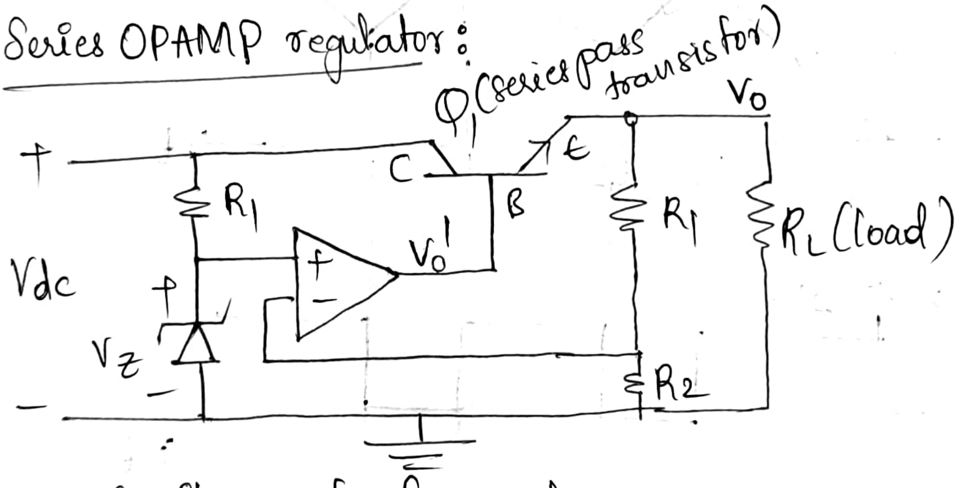
$$i = C_1\frac{\Delta V}{\Delta t} = \frac{C_1(0.25\,V_{cc})}{\Delta t}$$

$$\Delta t = \frac{C_1(0.25\,V_{cc})}{i}$$

$$i = \frac{V_{cc} - V_c}{R_1}$$

- **Voltage regulator:**
  - → It is a electronic circuit that provides a stable dc voltage independent of load current, temp etc..
  - → They can be classified as,
    1) Series regulator.
    2) Switching regulator.
  - → Series regulators use a power transistor connected in series b/w the unregulated dc input & the load.
  - → Switching regulators operate the power transistor as a high freq on/off switch so that power transistor doesn't conduct current continuously.

- **Series OPAMP regulator:**



→ Circuit consists of 4 parts,
① Reference Voltage circuit.
② Error amplifier.
③ Series pass transistor
④ Feedback n/w

$$V_o' = A(V_Z - \beta V_o)$$

$$\beta = \frac{R_2}{R_1 + R_2}$$

$$V_o' = V_o = A(V_Z - \beta V_o)$$

$$V_o = \frac{AV_Z}{1 + A\beta}$$

• IC Voltage regulators:

• Fixed Voltage Series Regulator:

→ 78XX series are 3 terminal, positive voltage fixed regulators.

→ In 78XX, the last 2 numbers (XX) indicate the o/p Voltage.

→ There are seven o/p voltage options available such as 5, 6, 8, 12, 15, 18 & 24V.

→ 79XX series generates -ve voltage (-ve DC Voltage)

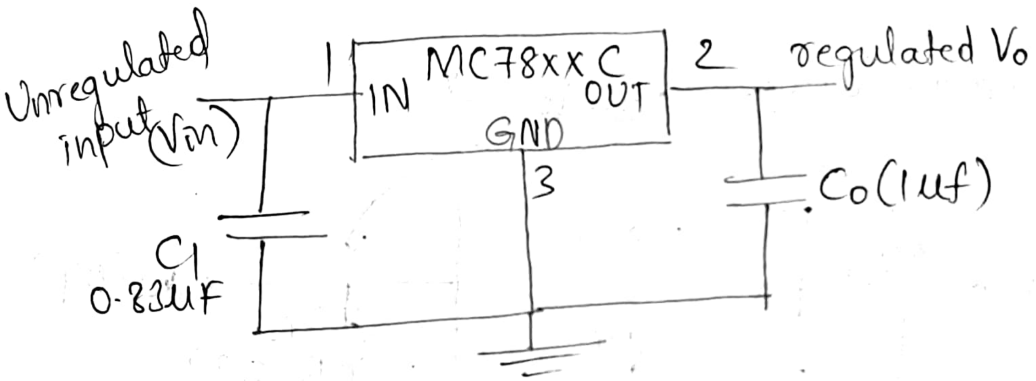


fig. Standard representation of a 3-terminal positive regulator.

Characteristics:

① V_o: The regulated o/p voltage is fixed value specified by manufacturer.

② $|V_{in}| \geq |V_o| + 2$ volts

The unregulated input voltage must be atleast 2V more than regulated o/p voltage.

③ $I_{o(max)}$: The load current may vary from '0' to rated maximum output current.

④ Thermal shutdown: The IC has temp sensor, which turns off the IC when it becomes hot (125°C to 150°C)

- **Line/Input regulation:**
  → It is defined as percentage change in o/p voltage for a change in input voltage.

- **Load regulation:**
  → It is defined as the change in o/p voltage for a change in load current.

- **Ripple rejection:**
  → The IC regulator not only keeps the o/p voltage constant but also reduces the amount of ripple.

- **Current Source:** $V_{in} - V_o = 2V$

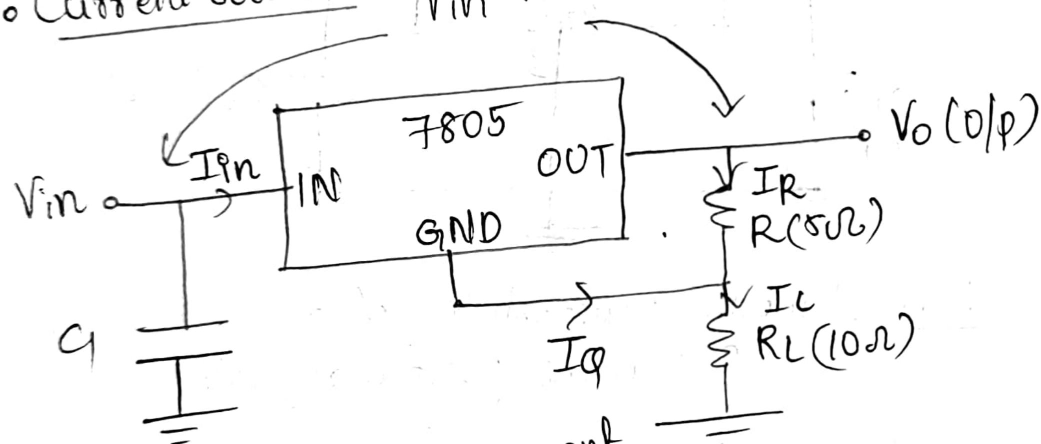


Fig. IC7805 as a current source.

→ The circuit is wired to supply a current of 1 Ampere to a 10Ω, 10 Watt load.

$$I_L = I_R + I_Q$$

where I_Q is Quiscent current & is ~~atleast~~ about 4.2mA for 7805

$$I_L = \frac{V_R}{R} + I_Q$$

Since $I_L = 1A$, $\frac{V_R}{R} \simeq 1A$
(neglecting I_Q)

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  → It is defined as percentage change in o/p voltage for a change in input voltage.

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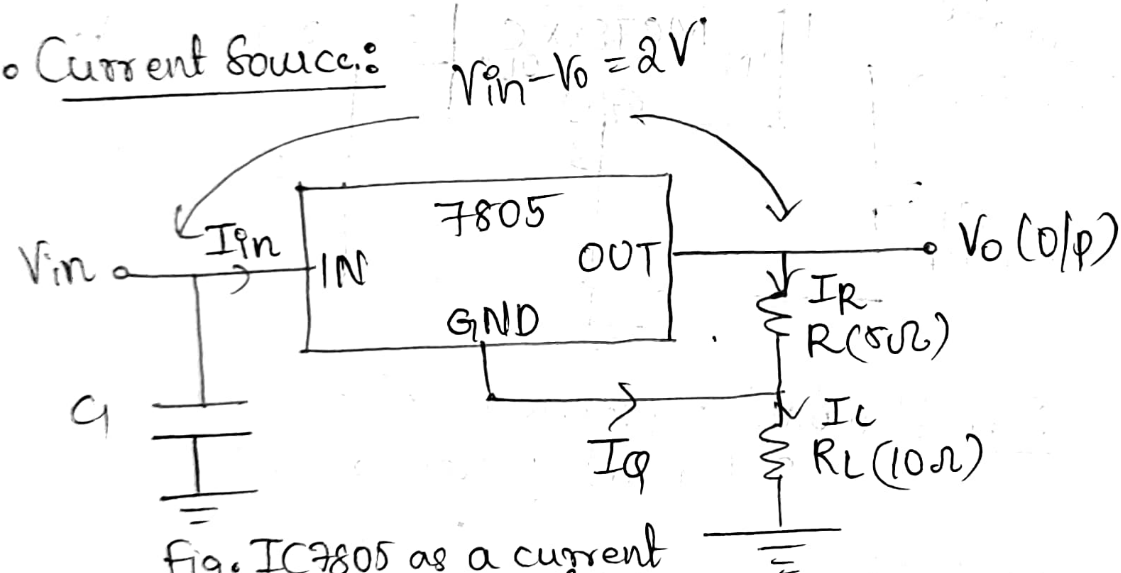

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Since $I_L = 1A$, $\frac{V_R}{R} \simeq 1A$
(neglecting I_Q)

- Boosting IC regulator o/p current :

→ It is possible to boost the o/p current of a 3 terminal regulator simply by connecting an external pass transistor in parallel with regulator.

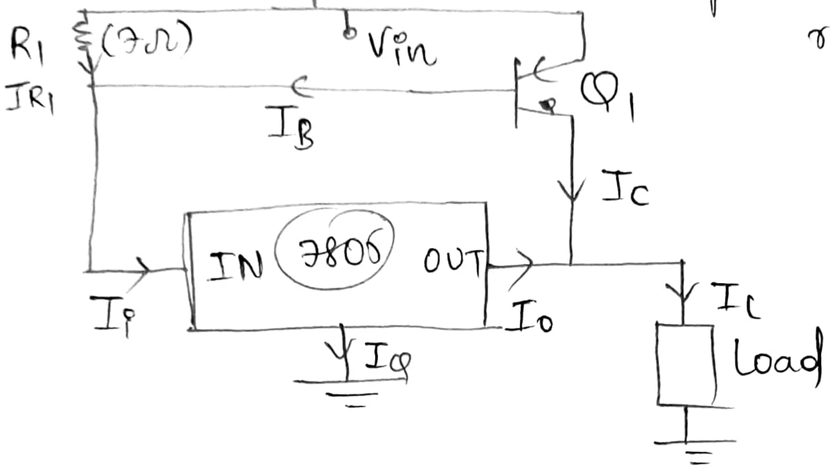


fig (b). Boosting a 3 terminal regulator

① For low load currents, the voltage drop across 'R' is insufficient (<0.7 V) to turn on transistor Q_1 & regulator itself is able to supply the load current

If I_L increases, V_{R_1} increases when $V_{R_1} = 0.7V$, transistor is turned 'on'

If $I_L = 100\,mA$, $V_{EB} = V_{R1} = 0.7V$

If $I_L > 100\,mA$, the transistor Q is turned 'ON' & it supplies extra current.

→ When Q1 is ON, $V_{EB}(ON)$ remains fairly constant, & excess current comes from Q_1

$$I_L = I_C + I_o$$

$$I_C = \beta I_B$$

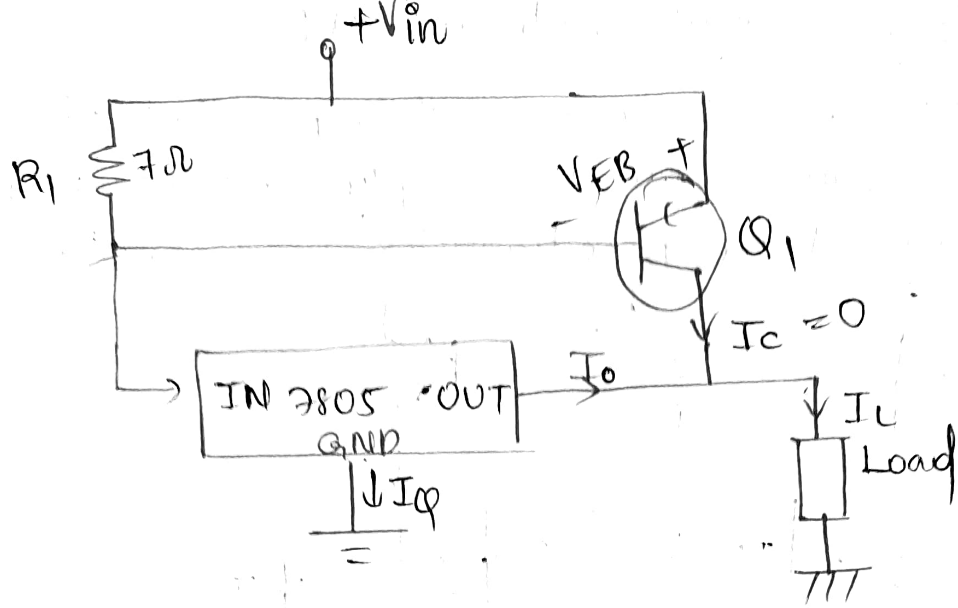
for regulator → $I_o = I_i - I_Q$, $I_o \simeq I_i$

$$I_B = I_i - I_{R_1} = I_o - I_{R_1}$$

$$I_B \cong I_o - \frac{V_{EB(ON)}}{R_1}$$

$$\boxed{I_L = (\beta+1) I_o - \frac{\beta V_{EB(ON)}}{R_1}}$$

Q:



Let $V_{EB(ON)} = 1V$ & $\beta = 15$.
Calculate the o/p current coming from 7805 & I_c coming from transistor Q_1 for loads (100Ω, 5Ω, 1Ω)

Sol: • For load 100Ω,

For 7805, the o/p voltage across load will be 5V.

$$I_L = \frac{5V}{100} = 50mA$$

Let Q_1 be off, then $V_{EB} = 50mA \times 7\Omega$

$$V_{EB} = 350mV$$

So, Q_1 is OFF,

$$I_c = 0 \,;\, I_L = I_0 = 50mA.$$

• For load 5Ω, Assume Q_1 is OFF! ($I_c = 0$)

$$I_L = \frac{5}{5} = 1A$$

$$V_{EB} = 1A \times 7\Omega$$
$$= 7V$$

Here Q_1 is 'ON'.

$$I_L = (\beta+1) I_o - \frac{\beta V_{EB(ON)}}{R_1}$$

$$1 = (16) I_o - \frac{15 \times 1}{7}$$

$$\underline{I_o = 196 mA}$$

$$I_o + I_c = I_L$$

$$196 mA + I_c = 1 A$$

$$\underline{I_c = 804 mA}$$

∘ <u>For load 1Ω,</u> Assume Q_1 is OFF ($I_c = 0$)

$$I_L = \cancel{0.2A} \; 5 = 5 A$$

$$V_{EB} = \cancel{0.2} \; 5 \times 7 = \cancel{1.4V} \; 35 V$$

So, Q_1 is (ON).

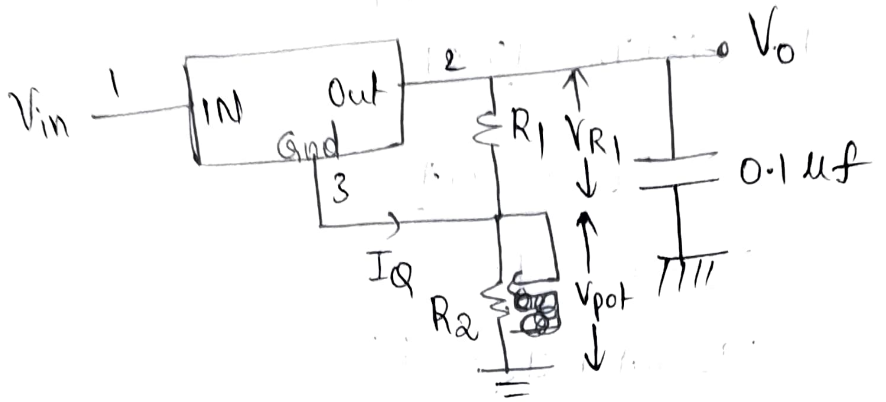
$$I_L = (\beta+1) I_o - \frac{\beta V_{EB(ON)}}{R_1}$$

$$\cancel{0.2} \; 35 = 16 I_o - \frac{15 \times 1}{7}$$

$$\underline{I_o = 446 mA}$$

$$\underline{I_c = 454 mA}$$

Fixed Regulator used as Adjustable regulator:



$$V_o = V_R + V_{pot}$$

$$V_o = V_R + (I_Q + I_{R1}) R_2$$

$$V_o = V_R + I_Q R_2 + \frac{V_R}{R_1} R_2$$

$$V_o \cong V_R\left(1 + \frac{R_2}{R_1}\right) \text{ ; } I_Q \text{ is neglected.}$$

eg: Specify suitable component value to get $V_o = 7.5V$ in the circuit of fig.1 using a 7805 regulator.

Sol: From datasheet, of 7805,

$$I_Q = 4.2 mA$$

we choose $I_{R_1} = 25 mA$

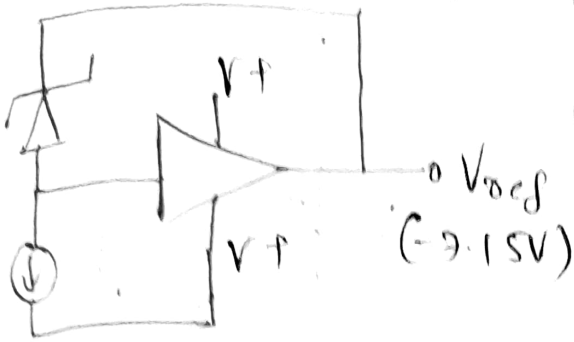
As $V_R = 5V$ for 7805

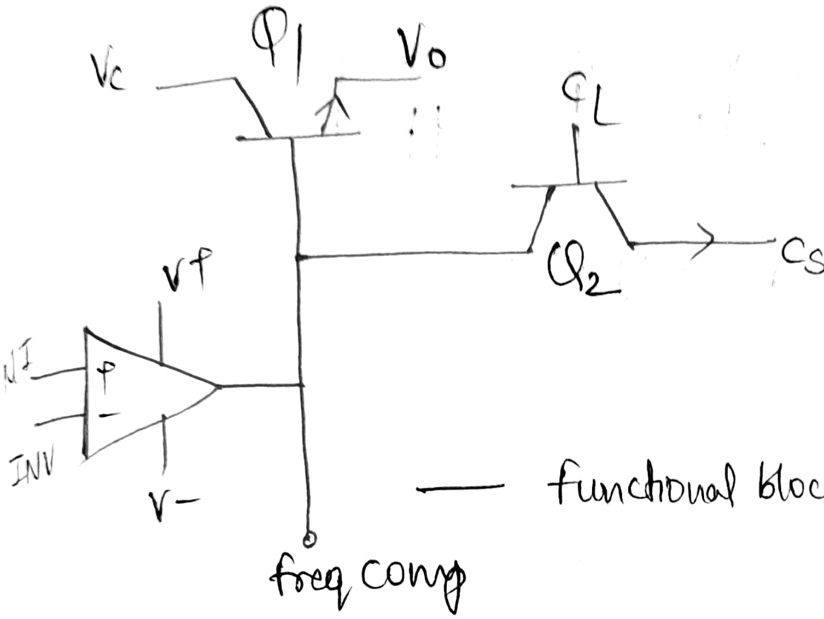
$$R_1 = \frac{5V}{25 mA} = 200\Omega$$

$$V_{R_2} = 2.5 V$$

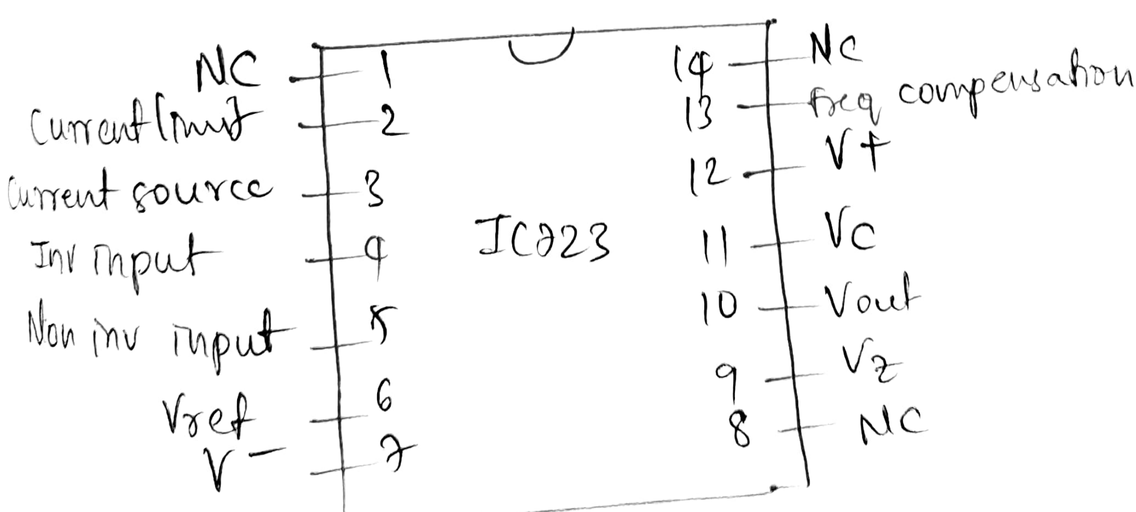
$$R_2 = \frac{V_{R2}}{I_{R_1} + I_Q} \rightarrow R_2 = 85.6\Omega$$

IC723:



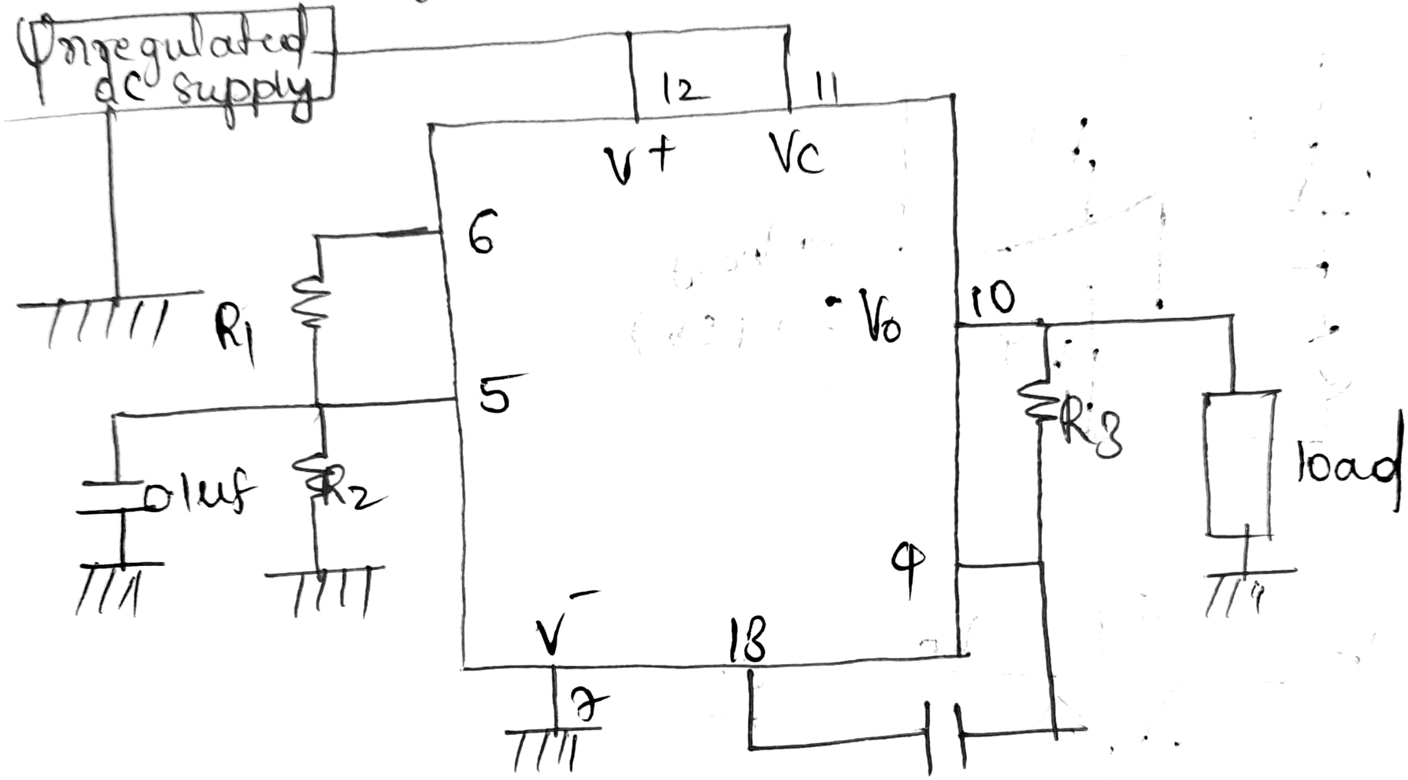


— functional block diagram —



Pin diagram

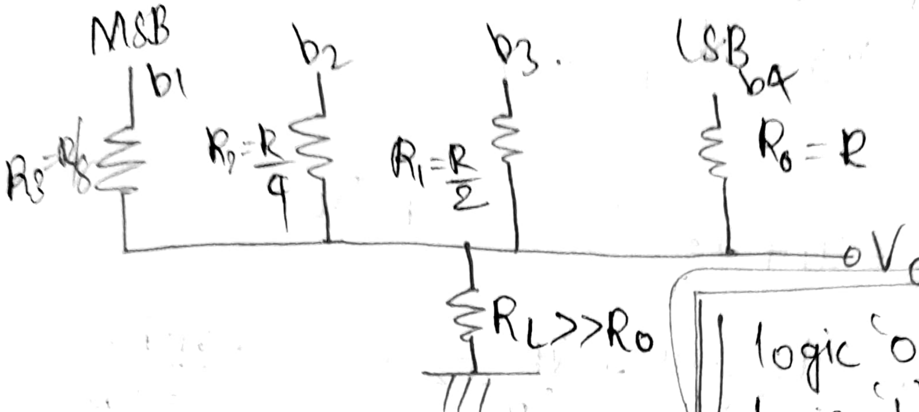
(*) Low voltage regulator :



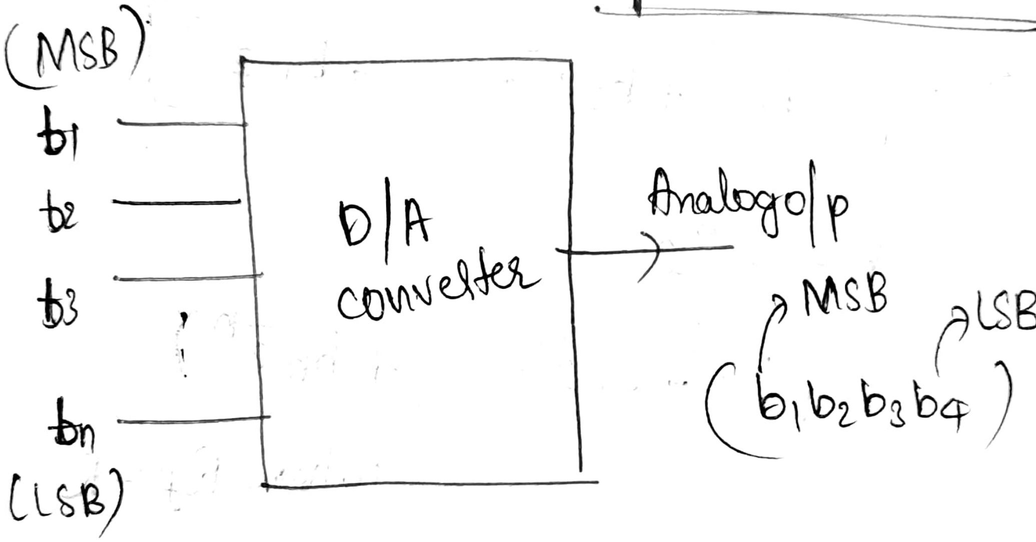
Pin diagram

* D/A converter:

① Weighted resistor type D/A converter,



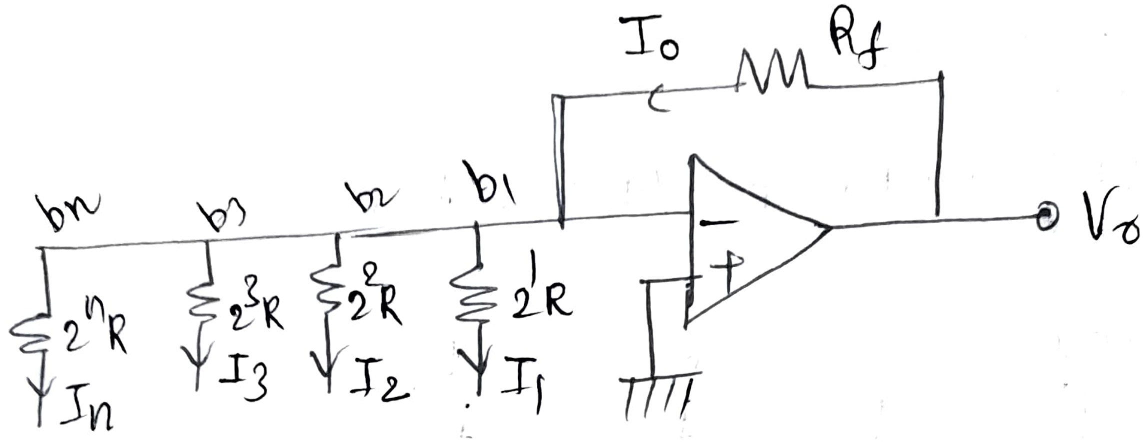
logic '0' = 0V
logic '1' = VR



From miller's theorem,

Voltage at any node in a resistive n/w is equal to sum of currents entering the node divided by sum of conductors connected at the node.

$$V_o = \frac{\frac{V_R}{R} + \frac{V_R}{R/2} + \frac{V_R}{R/4} + \frac{V_R}{R/8}}{\frac{1}{R} + \frac{1}{R/2} + \frac{1}{R/4} + \frac{1}{R/8}}$$



$$I_0 = I_1 + I_2 + I_3 + \cdots + I_n$$

$$I_0 = \frac{V_{R1} \times b_1}{2R} + \frac{V_R}{2^2R} b_2 + \cdots + \frac{V_R b_n}{2^n R}$$

$$I_0 = \frac{V_R}{R}\left[b_1 2^{-1} + b_2 2^{-2} + \cdots + b_n 2^{-n}\right]$$

$$V_0 = I_0 R_f$$

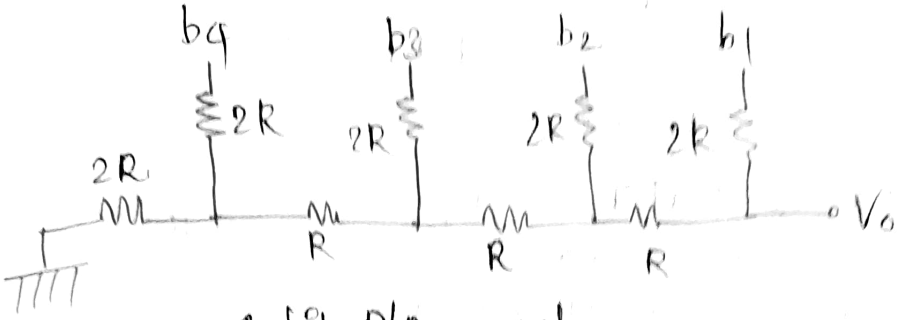
$$V_0 = V_R\left(b_1 2^{-1} + b_2 2^{-2} + \cdots + b_n 2^{-n}\right)$$

where $R_f = R$

- **Disadvantage of binary weighted D/A convertor:**

1) Requirement of wide range of resistors.
2) Fabrication of resistors beyond 'MΩ' is not practical.

(*) R-2R ladder D/A convertor:



— 4-bit D/A convertor —

logic '1' → V_R
logic '0' → 0

$b_1 b_2 b_3 b_4$

$1000 \rightarrow V_o = V_R/2$

$b_1 b_2 b_3 b_4$

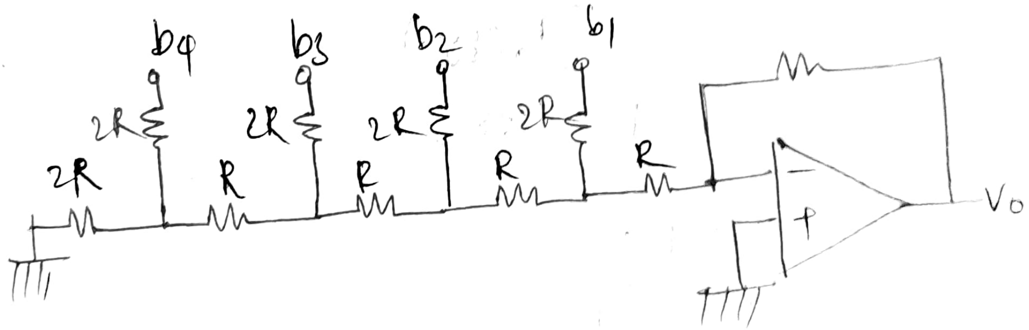
$0100 \rightarrow V_o = V_R/4$

$0010 \rightarrow V_o = V_R/8$

$0001 \rightarrow V_o = V_R/16$

(+)

$1111 \rightarrow V_o = \frac{V_R}{2} + \frac{V_R}{4} + \frac{V_R}{8} + \frac{V_R}{16}$



$$V_o = -\frac{V_R \times R_f}{R}\left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4}\right)$$

Assume $R_f = R_1$

$$V_o = -\frac{V_R}{2^n}\left(b_1 2^{n-1} + b_2 2^{n-2} + \dots + b_n 2^0\right)$$

resolution → $\frac{V_R \times R_f}{2^n \times R}$

Q: Consider R-2R 4-bit convertor R_f is variable

$R = 10k\Omega$, $V_R = 10V$

Determine the value of R_f that should be connected to achieve

(a) Value of 1LSB at o/p is 0.5V

(b) Analog o/p of 6V for binary input 1000.

(c) full scale o/p voltage of 10V.

Sol:- (a) $V_o = \frac{V_R \times R_f}{2^n \times R}$ $\underset{b_1}{0}\,\underset{b_2}{0}\,\underset{b_3}{0}\,\underset{b_4}{1}$

$$0.5 = \frac{10 \times R_f}{2^4 \times 10k\Omega}$$

$$\underline{R_f = 8k\Omega}$$

(b) $\underset{b_1}{1}\,\underset{b_2}{0}\,\underset{b_3}{0}\,\underset{b_4}{0}$

$$V_o = \frac{R_f \times V_R}{2^n \times R}$$

$$6 = \frac{R_f \times 10}{2^1 \times (10^4)}$$

$$\underline{R_f = 12k\Omega}$$

(c) $\underset{b_1}{1}\,\underset{b_2}{1}\,\underset{b_3}{1}\,\underset{b_4}{1}$

$$V_o = \frac{R_f \times V_R}{2^n \times R}.$$

$$10 = \frac{R_f \times 10}{10^4}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right)$$

$$R_f = \frac{10^4}{\frac{15}{16}} \rightarrow R_f = \frac{16}{15} \times 10^4$$

$$\underline{R_f = 10.667k\Omega}$$