Digital signal processing.

> CTS → DTS → Three steps.

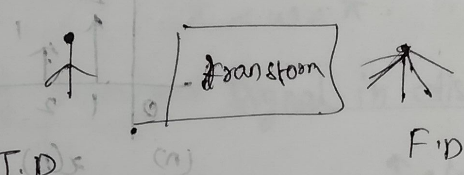
S + Q + E

S ↓ CT ↓ DT

Q ↓ CA ↓ DA

E ↓ amplitude to binary.

→ Impulse signal can trace any signal.



T.D   F.D

Z- transform for digital

-ve side means it has complex terms!

Speech $x(t)$

Image ← $f(p,q)$

Video ← $g(p,q,t)$

$x(t)$ → [Samp] $\xrightarrow{x[n]}$ [Quantize] $\xrightarrow{x_q[n]}$ [En] → $\{b_i\}$

$x(t)$ ← [Reconstruction] ← [look up Tool] ← [decode]

Analog	Digital
F	f
Ω	ω

$e^{j\theta}$ $= \omega_i$

$$\boxed{f = F/F_s}$$

$$\boxed{\omega = \Omega/F_s}$$

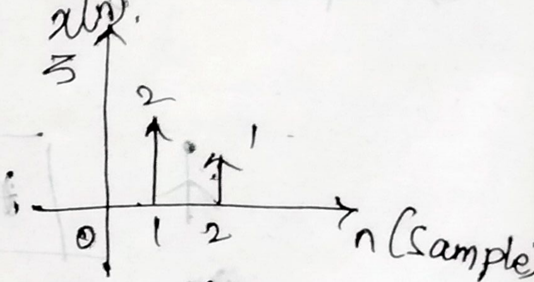
15/12/21

Distrete Time sequence

(sequence ↓ discrete (set of value))

$x(t) = A\cos 2\pi f t$

$t = nT_s$ $\quad x(nT_s)$



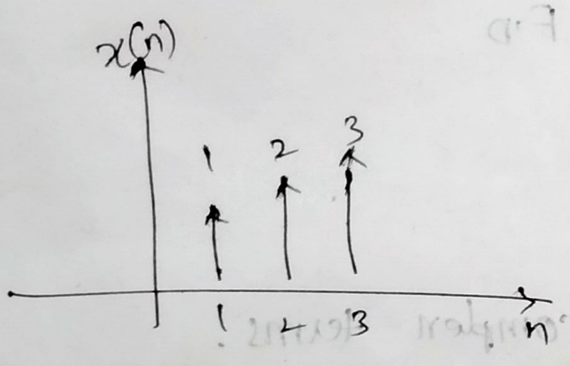
(n)	x(n)
0 →	3V
1 →	2V
2 →	1V.

$x[n] = \{\underset{\uparrow}{3}, 2, 1\}$

0 , 1 2

↑ indicate origin.

Ex.



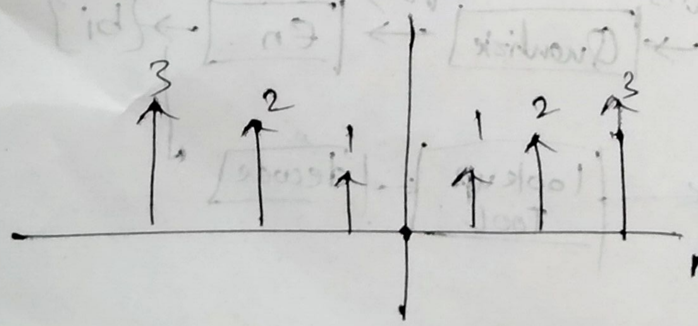
$x(n) = \{\underset{\uparrow}{0}, 1, 2, 3\}$

$x(n) = \{3, 2, 1, 0, 1, 2, 3\}$,

$x(n) = 3$ for $n=0$

2 for $n=1$

1 for $n=2$



$x(t) = A \cos 2\pi f t$.

$x(n) = A \cos 2\pi (nT_s) f$.

$= A \cos 2\pi (T_s F) n$.

digital freqen $= \boxed{\omega \Rightarrow T_s \times F}$

$\boxed{\omega = T_s . \Omega}$

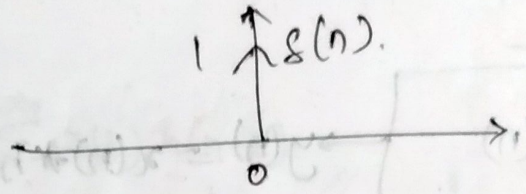
$-\infty < \Omega < \infty$ range of analog is $-\infty$ to ∞. $f = \infty$

$0 < \omega < 2\pi$ for digital limited.

$-\pi < \omega < \pi$.

Rep of standard signal in discrete form

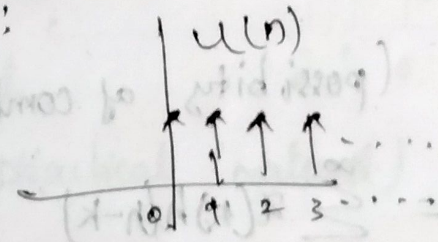
→ Impulse :



$\delta(n) = 1$ for $n = 0$

$= 0$ elsewhere

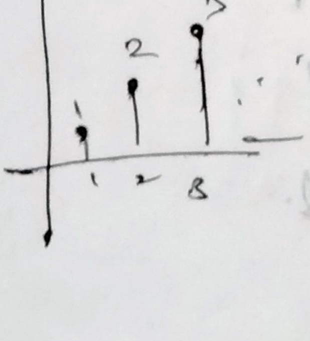
→ Step signal sequence :



$u(n) = 1 \quad n \geq 0$

$= 0$ other $n < 0$

ramp sequence



$r(n) = n \quad n \geq 0$

$= 0 \quad n < 0$

impulse is initial response

Classificatⁿ of discrete time signal.

→ symmetric,

discrete time Energy Sequence

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \qquad \text{for } u(n) = 0$$

(H/w signification for Energy power)

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \qquad \text{for } \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{1}{2}.$$

$$x(n) \to \boxed{h(n)} \to y(n) = x(n) * h(n).$$

linear convolution: (possibity of combining two signals).

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

↳ Folding, shifting, multiplication, addⁿ

$$y = T[x(n)]$$

$$x \to \boxed{T} \to y.$$

$x(n) = \{1, 2, 3\}$

$\uparrow$

0 1 2

$\delta(n)\ \delta(n-1)\ \delta(n-2)$

any signal can be rep. by impulse signal ($\delta(n)$)

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k).}$$

$y(n) = T[x(n)]$

$$= T\Big[\sum_{k=-\infty}^{\infty} \underbrace{x(k)}_{\text{amplitu}}\,\underbrace{\delta(n-k)}_{\text{position!}}\Big]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,\underbrace{T[\delta(n-k)]}_{\text{impulse resp.}}.$$

$$\boxed{\begin{array}{l} \text{if } x(n) = \delta(n) \\ y(n) = h(n) \end{array}}$$

$\Rightarrow$

$$\Rightarrow \sum_{k=-\infty}^{\infty} x(k).h(n-k).$$

$\downarrow$ finite $\downarrow$ finite then only O/p finite.

impulse — idle ideal

pulse — ~~pulse~~ practical (natural)

pulse Flat top. more practical.

1) Find the linear convolution of two given sequences

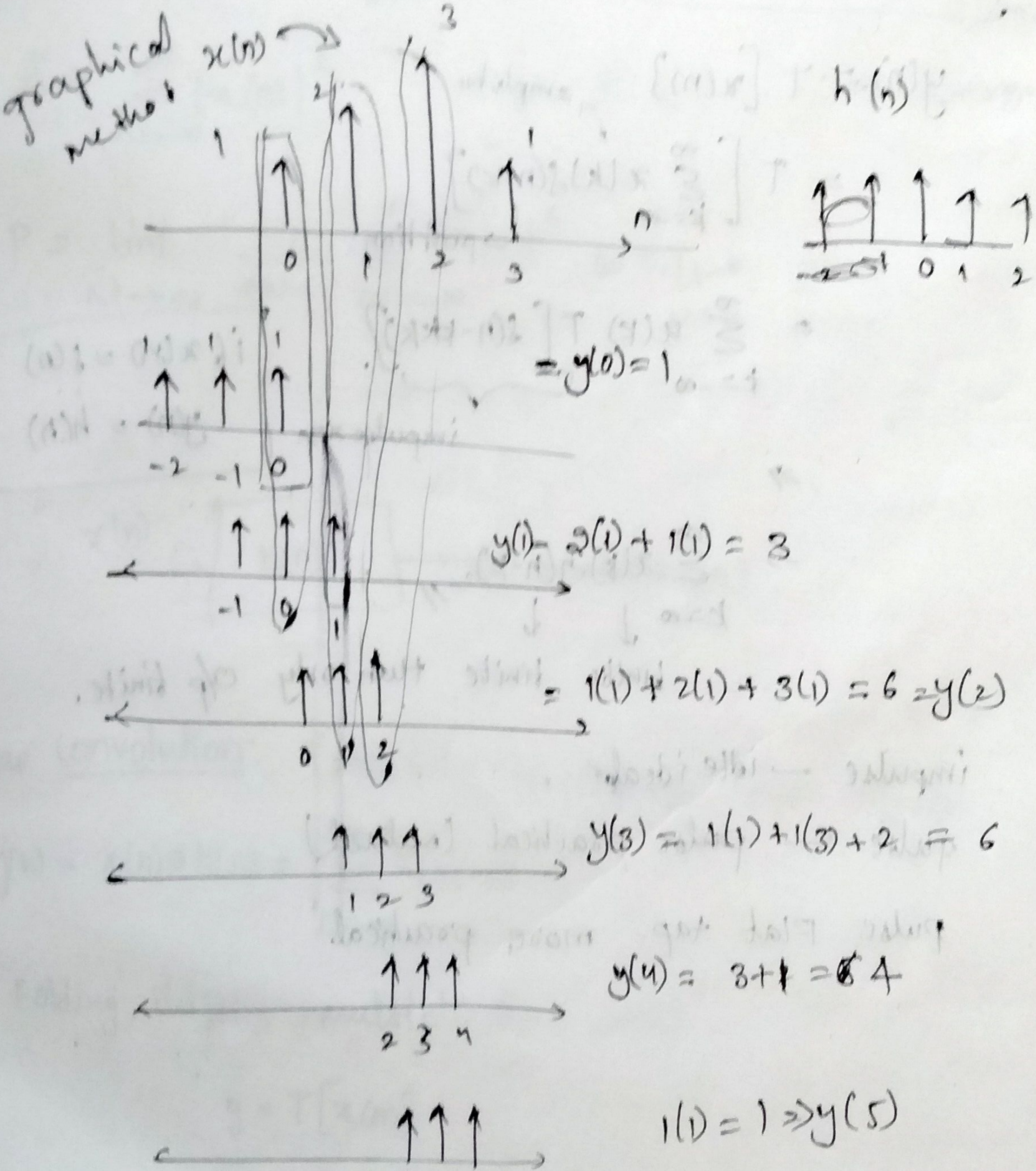
$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$. $\quad h(n) = \{\underset{\uparrow}{1}, 1, 1\}$.

length $= N_1$ $\qquad$ length N_2

output $= N_1 + N_2 - 1$

$4 + 3 - 1 = 6 \rightarrow$ '6 samples'

graphical method:



$= y(0) = 1$

$y(1) = 2(1) + 1(1) = 3$

$= 1(1) + 2(1) + 3(1) = 6 = y(2)$

$y(3) = 1(1) + 1(3) + 2 = 6$

$y(4) = 3 + 1 = \cancel{6}\ 4$

$1(1) = 1 \Rightarrow y(5)$

$y(6) = 0$

verification

Table method

$y(n) = \{$

$\& \ x(n) =$

Refen

$\therefore y(n) = \{\underset{\uparrow}{1}, 3, 6, 6, 4, 1\}$

verification

Table method

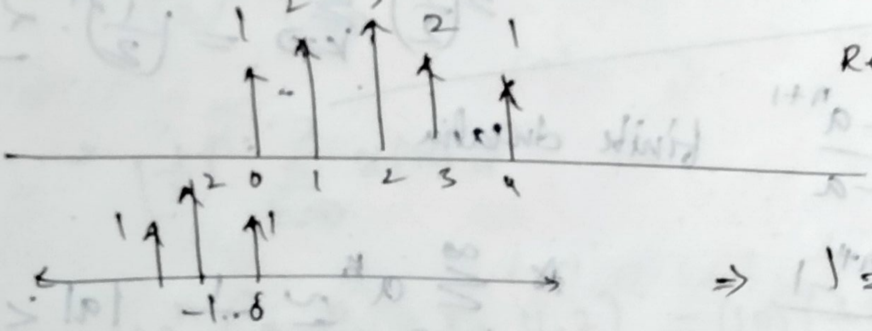
	1	2	3	1
1	1	2	3	1
1	1	2	3	1
1	1	2	3	1

→ Sum it.

Same start with same point.

$y(n) = \{1, 3, 6, 6, 4, 1\}$

& $x(n) = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$ $h(n) = \{\underset{\uparrow}{1}, 2, 1\}$.



Retendency in i/p
Retendency in o/p

$\Rightarrow 1 = y(0)$

$\Rightarrow 2 + 2 = 4 \Rightarrow y(1)$

$\Rightarrow 3 + 4 + 1 = 8 = y(2)$

$\Rightarrow 2 + 6 + 2 = 10 = y(3)$

$= 1 + 4 + 3 = 8 = y(4)$

$\Rightarrow$ ~~4 + 3 = 7 = y(5)~~

$\Rightarrow 2 + 2 = 4 \quad y(5)$

$1 = 1 \quad y(6)$.

$\Rightarrow$

	1	2	3	2	1
1	1	2	3	2	1
2	2	4	6	4	2
1	1	2	3	2	1

$= \{\underset{\underset{0}{\uparrow}}{1}, 4, 8, 10, 8, 4, 1\}$

Retendent data = repeating data.

onvolution of infinite duration Signals Sequences.

$x(n) = u(n)$. causal $\quad h(n) = \left(\frac{1}{2}\right)^n u(n)$ causal.

$y(n) = x(n) * h(n)$

$= \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$

$= \sum_{k=0}^{n} u(k) \left(\frac{1}{2}\right)^{n-k} u(n-k)$ $\quad \left(\frac{a(1-r^n)}{1-r^n}\right)$

$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k \Rightarrow \left(\frac{1}{2}\right)^n \cdot \left(\frac{1}{1-2}\right)$

$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k = \left(\frac{1}{2}\right)^n \cdot \frac{2^{n+1}-1}{2-1} u(n)$

* $\sum_{k=0}^{n} a^k \simeq \frac{1-a^{n+1}}{1-a}$ finite duration

$\simeq \frac{a^{n+1}-1}{a-1}$ $\quad$ * $\sum_{k=0}^{\infty} a^k \simeq \frac{1}{1-a} \quad |a| < 1$

xam $\quad x(n) = 2^n u(n) \quad h(n) = \left(\frac{1}{2}\right)^n u(n)$

$\Rightarrow \cancel{2^n} \cdot \frac{1}{\cancel{2^n}} \cdot \frac{2^{n+1}-1}{2-1}$ $\quad$ at last multiple by $u(n)$ bcau it sequence not scalar.

$= \frac{2^{n+1}-1}{1} u(n)$

hybrid

Ex. x

Ex: $h(n$

$x(n$

case (1)

when

case (2)

when

hybrid

Ex. $x(n) = \cancel{u(n)}$ (finite) . $h(n) = \left(\frac{1}{2}\right)^n u(n)$ (infinite)

$\{0, 1, 2)$
$\uparrow$

$$\Rightarrow \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left(\frac{1}{2}\right)^{n-k} u(n-k)$$

$n-k=0$
$n=k$
$k=n$
because it causal

$$= \sum_{k=0}^{n} x(k) \left(\frac{1}{2}\right)^{n-k} u(n-k)$$

$$= \left(\frac{1}{2}\right)^n \cdot \sum_{k=0}^{n} x(k) \cdot 2^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{2} x(k)\, 2^k$$

$$= \left(\frac{1}{2}\right)^n \cdot \left(0 \cdot 2^0 + 2 \cdot 1 + 4 \cdot 2\right) = (10)\left(\frac{1}{2}\right)^n \cdot u(n) \;//$$

Ex: $h(n) = \left(\frac{1}{2}\right)^n u(n)$.

$x(n) = u(n) - u(n-10)$ $\qquad y(n) = y_1(n) - y_2(n)$.

case (1) $x(n) * h(n) = y_1$
when $x(n) = u(n)$

$y_1(n) = \left(\frac{1}{2}\right)^n \cdot u(n) * u(n)$.

$y_1(n) = \left(\frac{1}{2}\right)^n \dfrac{\left(2^{n+1} - 1\right)}{2} u(n)$.

case (2) $x(n) * h(n) = y_2$
when $x(n) = u(n-10)$.

$y_2(n) = \left(\frac{1}{2}\right)^n u(n) * u(n-10)$.

$\cancel{\sum_{k=10}^{n} u(k-10) \cdot \frac{1}{2}}$

$$y_2 = \sum_{k=10}^{n} u(k-10)\cdot\left(\frac{1}{2}\right)^{n-k} u(n-k)$$

$$= \sum_{k=10}^{n} \left(\frac{1}{2}\right)^{n-k}$$

delay means n by $n-10$.

$=$

$$y_2(n) = \sum_{k=-\infty}^{\infty} u(k)\cdot\left(\frac{1}{2}\right)^{n-10-k} u(n-10-k).$$

$n-10-k=0$

$k = n-10$

$$= \sum_{k=0}^{n-10}$$

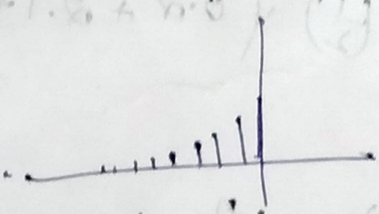
Exam $\left(\frac{1}{2}\right)^{n} u(-n)$. & $u(n)$

one way of scaling.

$$\& \sum_{\cancel{k=-\infty}\ 0}^{\cancel{\infty}\ n} \cancel{\left(\frac{1}{2}\right)^{k} u(-k)\cdot u(n-k)}$$

$n=k$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-k} u(-n-k)\cdot u(k).$$



linear convolution

(i) $x(n) = u(-n)$; $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$\sum_{k=-\infty}^{\infty} x(k).h(n-k) \Rightarrow \sum_{k=-\infty}^{0} u(-k)\left(\frac{1}{2}\right)^{n-k} u(n-k).$$

$$\Rightarrow \sum_{k=0}^{\infty} u(k)\left(\frac{1}{2}\right)^{n+k} \Rightarrow \left(\frac{1}{2}\right)^n \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \Rightarrow \left(\frac{1}{2}\right)^n \frac{1}{1-\frac{1}{2}}$$

$$\Rightarrow 2\left(\frac{1}{2}\right)^n u(n).$$

(ii) $x(n) = 2^n u(n)$. $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$\sum_{k=0}^{n} 2^k \left(\frac{1}{2}\right)^{n-k} \Rightarrow \left(\frac{1}{2}\right)^n . \sum_{k=0}^{n} 4^k \Rightarrow \left(\frac{1}{2}\right)^n \left\{\frac{4^{n+1}-1}{4-1}\right\} u(n).$$

(iii) $x(n) = \left(\frac{1}{4}\right)^n u(n)$; $h(n) = \left(\frac{1}{2}\right)^n u(n)$.

$$\sum_{k=0}^{n} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{n-k}.$$

$$\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{4} \times 2\right)^k \Rightarrow \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right) u(n).$$

$x(n) = a^n u(n)$ $\quad h(n) = b^n u(n)$ $\quad$ Check for $a > b$

$a < b$

$a = b$

$$\sum_{k=0}^{\infty} a^k u(k)\, b^{n-k} u(n-k) \qquad \cancel{a>b}$$

$$b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k \Rightarrow \text{if } a = b \Rightarrow \cancel{b^n}\ \cancel{(n+1)}\ (n+1)b^n$$

if $a < b$

$$b^n \cdot \sum_{k=0}^{\infty} \left(\frac{a}{b}\right)^k$$

$a < b$

$$\cancel{b^n \cdot \frac{1}{1-\frac{a}{b}} \Rightarrow \frac{b^{n+1}}{b-a}} \qquad b^n \cdot \left(\frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\left(\frac{a}{b}\right)}\right) u(n)$$

if $a > b$. $b^n \sum_{k=0}^{\infty} \left(\frac{a}{b}\right)^k \Rightarrow b^n \left(\frac{\left(\frac{a}{b}\right)^{n+1} - 1}{\frac{a}{b} - 1}\right) u(n)$

if i/p is more dominant than $h(n)$ than $y(n)$ will have more characteric of i/p.

DFT (discrete fourier transform). (syllabus starts).

$f \quad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$\hookrightarrow x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega.$

Time = continuous & periodic (original signal.

$\hookrightarrow$ DFT (means finite) (aperiodic

DTFT $\xrightarrow{}$ DFT $\omega = \frac{2\pi}{N} k$

DFS (infinite periodic)

$\boxed{\begin{array}{c} 0 \le \omega \le 2\pi \\ -\pi \le \omega \le \pi \end{array}}$ $\boxed{\omega = e^{-2j\pi}}$ $\rightarrow$ Twiddle factor

DFT

$(0 \to N-1)$ frequence components

$X(k) = X(e^{j\omega}) \Big|_{\omega = \frac{2\pi}{N} k} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} kn}$; $k = 0, 1 \ldots N-1$ $\quad \rightarrow W_N^{kn}$

frequency

$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}$; $n = 0, 1, \ldots \; N-1$ $\quad \rightarrow W_N^{-kn}$

$\boxed{W_N^{kn} = e^{-j\frac{2\pi kn}{N}}}$ $\rightarrow$ ~~twi~~ twiddle factor,

$2^n \rightarrow$ standard binary encode

$0 \longrightarrow N-1$

$2^n \rightarrow$ no: of binary bits require to level.

Ex determine the discrete frequency component of a Sequence $x(n) = \{1, 0, 1, 0\}$, $N = 4$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi \frac{kn}{N}}$$

$N-1$

$X(0) = x(0) + x(1) + x(2) + x(3)$ (First frequ. Component)

$= 2$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n}{4}}$$

$= x(0) + \cancel{x(1) e^{-j\pi/2}} + x(2) e^{-j\pi} + \cancel{x(3) e^{-j\frac{\pi}{2}\cdot 3}}$

$= 1 + (-1) = 0$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-\frac{2\pi \times 2}{4}\cdot n}$$

$X(2) = x(0) + \cancel{x(1) e^{-j\pi}} + x(2) e^{-j3\pi} + \cancel{x(3) e^{-j4\pi}}$

$= 1 + 1 = 2$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}\times 3n}$$

$= x(0) + \cancel{x(1) e^{-\frac{3\pi j}{2}}} + x(2) e^{-j\pi} + \cancel{x(3) e^{-j\frac{9\pi}{2}}}$

$= 1 + (-1) = 0.$

$X(K) = \{2, 0, 2, 0\}$.

$2 \rightarrow 1, 0$
$0 \rightarrow 0\ 0$
$2 \rightarrow 1\ 0$

H/w $x(n) = \{1,1,1,1,0,0,0,0\}$. H/w. give $\{1,2,1\}$

do 4 point

& 8 point

DFT: $X(k) = X(\omega)\big|_{\omega = \frac{2\pi}{N}k}$

$k \rightarrow$ discrete

frequency component

DTFT

$\infty \rightarrow$ finite

sampling of DTFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$k \rightarrow$ frequency component

$n \rightarrow$ time sample

$K = 0, 1, \ldots N-1$

'N' $\rightarrow$ N-point DFT

previously ∞ now 'N'

i/p length $\rightarrow \boxed{DFT} \rightarrow$ o/p length

$\underline{\underline{N}}$ time $\qquad$ N frequency.

$x(n) = \{0, 1, 0, 1\}$ (indices 0 1 2 3) calcul ~~k=~~ 4 point dFT.

$k=0$ $\quad X(0) = \sum_{n=0}^{3} x(n).e^{-j\frac{2\pi kn}{4}} = x(0) + x(1) + x(2) + x(3)$

$= \cancel{0} + 1 + 0 + 1 = 2.$

$k=1$ $\quad X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{4}} = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}n} = x(0) + x(1) e^{-j\frac{\pi}{2}} + x(2) e^{-j\frac{\pi}{2}\times 2} + x(3) e^{-j\pi/2 \times 3}$

$= 0 + 1(j) + 0 + 1.$

$-j + j = 0$

$K=2$ $\quad X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi 2n}{4}}$

$= x(0) + x(1) e^{-j\pi} + x(2) e^{-2j\pi} + x(3) e^{-3\pi j}$

$= 0 \cancel{+0} -1 + 0 \cancel{+} \overline{1} = \cancel{0} -2$

$K=3$ $\quad x(3) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi 3}{2}n}$

$= x(0) + e^{-\frac{3\pi}{2}j} + 0 + \cancel{0}\, x(3) e^{-\frac{3\pi}{2}\times 3j}$

$x(n)$

$$\{0, 1, 0 1\} \xrightarrow{DFT} \{2, 0, -2, 0\}$$

$4 \qquad\qquad 4$

$x(n) \qquad\qquad X(k)$

$$\text{leng } x(n) = \text{leng } X(k)$$

Ex

$$x(n) = \{\overset{0}{0}, \overset{1}{1}, \overset{2}{2}, \overset{3}{3}\}$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{4} kn} \qquad k = 0, 1, 2, 3$$

$$\cancel{x} X(0) = 0 + 1 + 2 + 3 + = 6.$$

$$X(1) = \cancel{x}\, 0e^{-j\frac{\pi}{2}} + 1e^{-j\pi} + 2e^{-j3\frac{\pi}{2}} + 3e^{-j2\pi}$$

$$= 0 + (-1) + 2(j) + 3(1) = \cancel{2-2j}\ 2j - 2$$

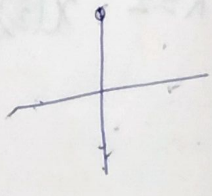
$$X(2) = 0e^{-j\pi} + 1e^{-j\pi} + 2e^{-j2\pi} + 3e^{-j3\pi}$$

$$0 \quad -1 \quad +2 \quad -3 \quad = -2$$

$$X(3) = 0e^{-j30} + 1e^{-j\frac{3\pi}{2}} + 2e^{-j\frac{3\pi}{2}2} + 3e^{-j\frac{3\pi}{2}3}$$

$$= 0 + j + 2(-1) + 3(-j) = -2j - 2$$

$\frac{DFT}{T \to F}$ $$\cancel{x} X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} kn}$$



$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N} kn}$$

Ex: $X(K) = \{\overset{0}{6}, \overset{1}{-2+2j}, \overset{2}{-2}, \overset{3}{-2-2j}\}$

$x(0),\ x(1),\ x(2),\ x(3)$

$$x(0) = \frac{1}{4}\sum_{n=0}^{3} X(K)e^{\ldots} = \frac{1}{4}\left[6\cancel{-2+2j}\cancel{-2}\cancel{-2}\cancel{-2j}\right) = 0$$

$$x(1) = \frac{1}{4}\sum_{k=0}^{3} X(K)e^{j\frac{2\pi k}{4}}$$

$$= \frac{1}{4}\left[6e^{0} + (-2+2j)e^{j\frac{\pi}{2}} + (-2)e^{j\pi} + (-2-2j)e^{j\frac{3\pi}{2}}\right)$$

$$= \frac{1}{4}\left(6 + \cancel{-2j}\cancel{-2} + \cancel{2} + \cancel{2j} - 2\right) = 1$$

$$x(2) = \frac{1}{4}\left[\frac{1}{4}\sum_{k=0}^{3} X(k)e^{j\frac{2\pi}{4}k\cdot 2}\right.$$

$$= \frac{1}{4}\left\{6e^{0} + (-2+2j)e^{j\pi} + (-2)e^{2j\pi} + (-2-2j)e^{j3\pi}\right]$$

$$= \frac{1}{4}\left(6 + 2\cancel{-2j} - \cancel{2} \cancel{+2} + \cancel{2j}\right) = 2$$

$(5\pi - \frac{\pi}{2})$

$$x(3) = \frac{1}{4}\sum_{k=0}^{3} x(k)e^{j\frac{2\pi 3}{2}k}$$

$3\times\frac{3\pi}{2}$ $\quad \frac{9\pi}{2}$ $\quad \frac{3\pi}{2}$ -sins

$$= \frac{1}{4}\left[6\cdot e^{0} + (-2+2j)e^{\frac{3\pi}{2}j} + (-2)e^{3\pi j} + (-2-2j)e^{\frac{9\pi}{2}j}\right\}$$

$$= \frac{1}{4}\left[6 + (-2+2j)(-j) + 2 + (-2-2j)(j)\right. = \frac{12}{4} = 3.$$

properties of DFT:-

1) linearity: $\alpha x_1(n) + \beta x_2(n) = \alpha X_1(k) + \beta X_2(k)$

$$DFT\{a_1 x_1(n) + a_2 x_2(n)\} = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

$$= \sum_{n=0}^{N-1} \{a_1 x_1(n) + a_2 x_2(n)\}\, e^{\frac{-j2\pi kn}{N}}$$

$$= a_1 \sum_{n=0}^{N-1} x_1(n) e^{\frac{-j2\pi kn}{N}} + a_2 \sum_{n=0}^{N-1} x_2(n) e^{\frac{-j2\pi kn}{N}}$$

$$= a_1 X_1(k) + a_2 X_2(k)$$

2) $x(n+N) = x(n)$ for all n, k value

$X(k+N) = X(k)$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi n/N}.$$

$$x(n+N) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi k}{N}(n+N)}$$

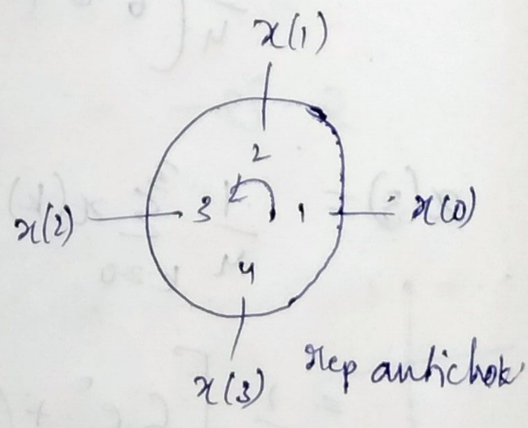
$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}} \cdot 1$$

$$= \cancel{x(X(k))}\, x(n)$$

3) circular shift of a finite sequence

$x(n) = \{1, 2, 3, 4\}$

$x(0)\ x(1)\ x(2)\ x(3)$



'−' delay ⇒ rotate in anticlockwise

'+' advance = rotate in clockwise

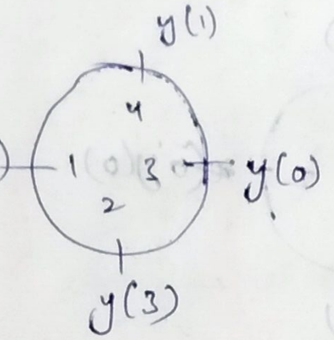
max is 4 only.

$y = x((n-2))_4 = \{3, 4, 1, 2\}$.

↳ max length.

Shifting shud not go beyond 4'

$y =$



Ex: $x(n) = \{1, 2, 3, 4\}$

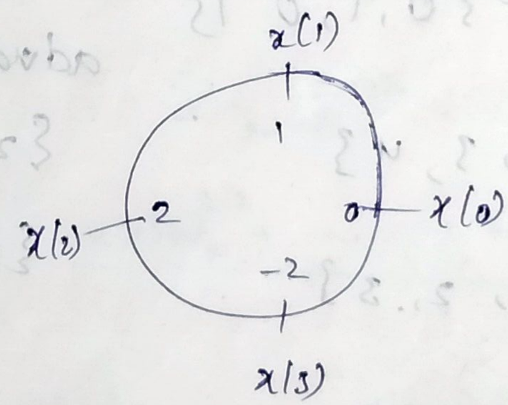
$x(n-1) = \{4, 1, 2, 3\}$

$x(n-2) = \{3, 4, 1, 2\}$.

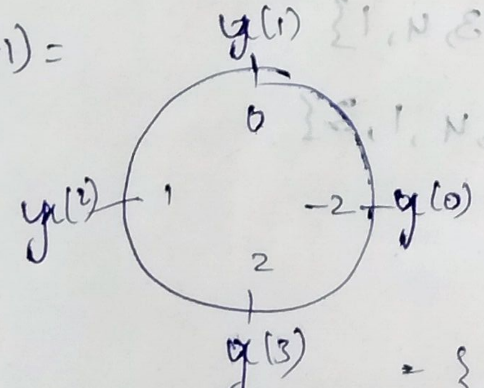
$x(n-3) = \{2, 3, 4, 1\}$

$x(n-4) = \{1, 2, 3, 4\}$

$x(n) = \{0, 1, 2, -2\}$

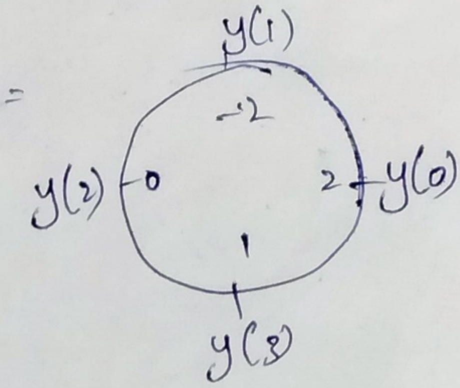


$x(n-1) =$



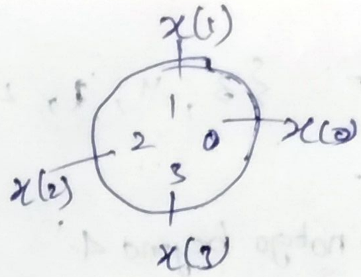
$= \{-2, 0, 1, 2\}$

$x(n+2) =$



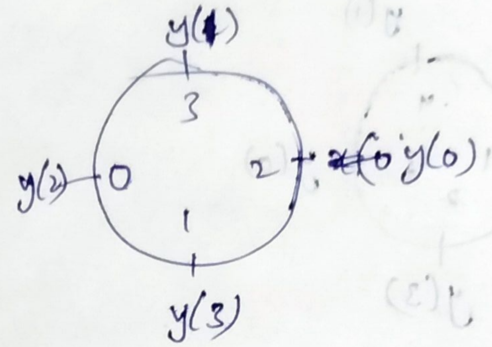
$= \{2, 1, 0, -2\}$

Ex $x(n) = \{0, 1, 2, 3\}$.



$y(n) = x((n-2))_4$

$\Rightarrow$



$\boxed{y(n) = x((n-n_0))_N = x(N + n - n_0)}$

$y(0) = x(4+0-2) = x(2)$

$y(1) = x(4+1-2) = x(3)$

$y(2) = x(4+2-2) = x(4)$

$y(3) = x(4+3-2) = x(5) = x(0)$ $= \{2, 3, 0, 1\}$

Ex $x(n) = \{1, 0, 10\}$

$y(n) = x(n+1) = \{0, 1, 0, 1\}$

delay.

$\{1, 2, 3, 4\}$

$x(n-1)$ $\{4, 1, 2, 3\}$

$x(n-2)$ $\{3, 4, 1, 2\}$

advance.

$\{2, 3, 4, 1\}$

$\{3, 4, 1, 2\}$

Circular Shift - property:

if $x(n) \xrightarrow{DFT} X(k)$.

$DFT\{x((n-n_0))_N\} \longrightarrow$?

IDFT $\Rightarrow x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}$

$$x(n-n_0) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi k(n-n_0)}{N}}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} \cdot e^{\frac{-j2\pi kn_0}{N}}$$

$$\therefore x(n-n_0) = x(n) e^{\frac{-j2\pi kn_0}{N}}$$

Apply DFT on b.s

$$DFT\{x(n-n_0)\} = e^{-j\frac{2\pi kn_0}{N}} \underbrace{\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}}}_{X(k)}$$

$$= X(k) e^{\frac{-j2\pi kn_0}{N}}$$

DFT as linear Transformation: (Matrix multiplication).

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}. \qquad W_N = e^{\frac{-j2\pi}{N}}$$

$$X(k) = x(0) + x(1) W_N^{k\cdot 1} + x(2) W_N^{k\cdot 2} + \dots + x(N-1) W_N^{k\cdot(N-1)}$$

$k = 0$ to $N-1$

$k=0$ $\quad X(0) = x(0)\cdot 1 + x(1)\cdot 1 + x(2)\cdot 1 \dots \dots \quad x(N-1)\cdot 1$

$k=1$ $\quad X(1) = x(0)\cdot 1 + x(1) W_N^{1} + x(2) W_N^{2} \dots \quad x(N-1) W_N^{1(N-1)}$

$k=2$ $\quad X(2) = x(0)\cdot 1 + x(1)\cdot W_N^{2} + x(2) W_N^{4} \dots \quad x(N-1) W_N^{2(N-1)}$

$\vdots$

$k=N-1$ $\quad X(N-1) = x(0)\cdot 1 + x(1) W_N^{(N-1)1} + x(2) W_N^{(N-1)2} \dots \quad x(N-1) W_N^{(N-1)(N-1)}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix} = W_N \times x_n$$

$$\underbrace{N\times N \quad N\times 1} = N \times 1$$

$$W = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & & & & \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$\boxed{X = W_N \times x_N}$$

freqn twiddle Sample (time) domain

Ex

$$X(k) = \sum_{n=0}^{3} x(n) W_4^{kn}$$

$$= x(0) + x(1) W_4^{k} + x(2) W_4^{2k} + x(3) W_4^{3k}$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}_{4\times 4} \quad W_4^{0 \Rightarrow 3}$$

① $W_N^{k+N} = W_N^{k}$

$W_N^{k+N/2} = -W_N^{k}$

0	1	2	3
4	5	6	7
8	9	10	11

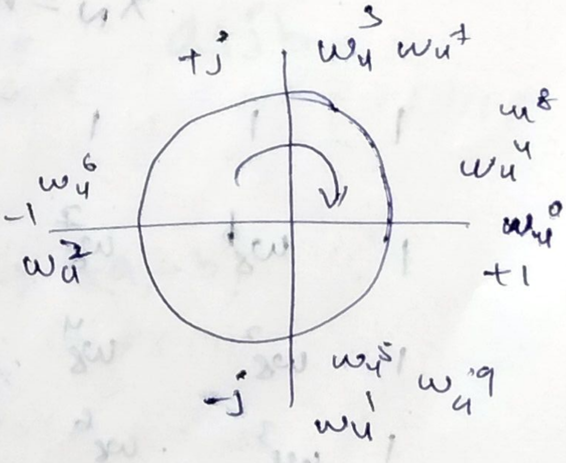
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4^1 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^0 & w_4^2 \\ 1 & w_4^3 & w_4^2 & w_4^1 \end{bmatrix}$$

$x(n) = \{0, 1, 2, 3\}$. 4 pt dft using Matrix

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4^1 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^0 & w_4^2 \\ 1 & w_4^3 & w_4^2 & w_4^1 \end{bmatrix}}_{4\times4} \cdot \underbrace{\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}}_{4\times1}$$

4-DFT : $X_{N}^{k} = W_N x_N$ $N=4$ $W_N^{k+N/2} = -W_N^{k}$

$$W_N = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4^1 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^0 & w_4^2 \\ 1 & w_4^3 & w_4^2 & w_4^1 \end{bmatrix}}_{4\times4}$$



$w_4^0 = 1$

$w_4^1 = e^{-j2\frac{\pi}{4}}$

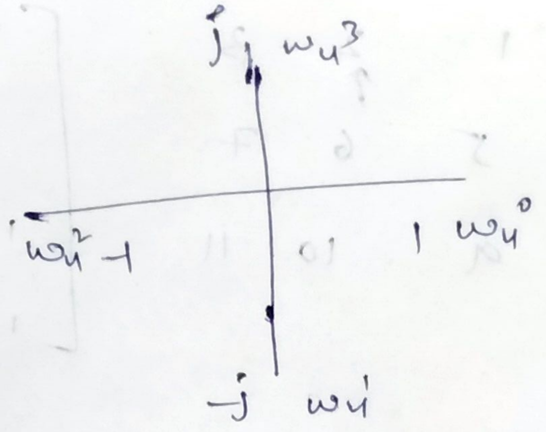
$= e^{-j\pi/2} = -j$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & +1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix}$$

$\{1, 0, 1, 0\}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 0 & -1 \\ 1 & j & -1 & -j \end{bmatrix} 4\times4$$

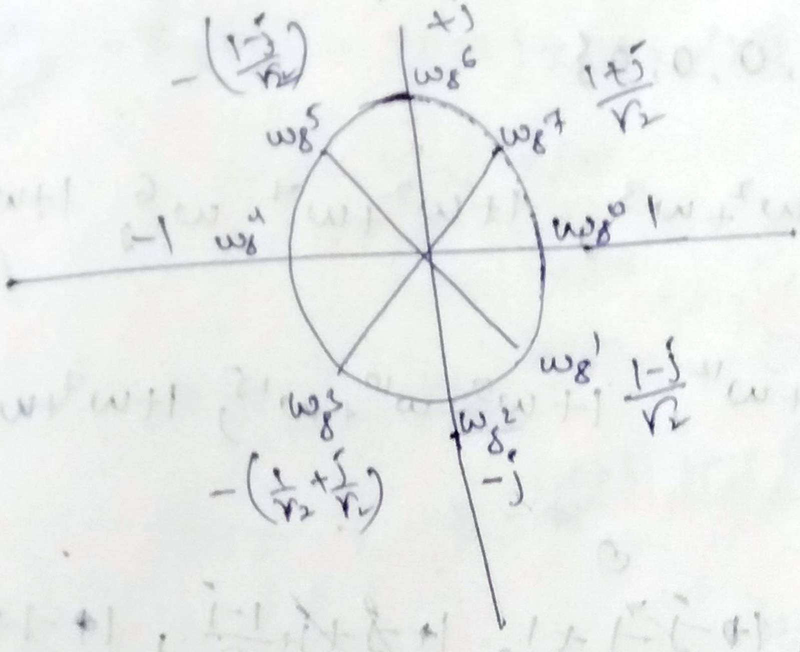


$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{pmatrix} 1+1 \\ 0 \\ 1-1 \\ 1+j \end{pmatrix}$$

8-pt dFT $N=8$.

$$X_N = W_N\, x_N.$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w_8^1 & w_8^2 & w_8^3 & w_8^4 & w_8^5 & w_8^6 & w_8^7 \\ 1 & w_8^2 & w_8^4 & w_8^6 & w_8^8 & w_8^{10} & w_8^{12} & w_8^{14} \\ 1 & w_8^3 & w_8^6 & w_8^9 & w_8^{12} & w_8^{15} & w_8^{18} & w_8^{21} \\ 1 & w_8^4 & 8 & 12 & 16 & 20 & 24 & 28 \\ 1 & w_8^5 & 10 & 15 & 20 & 25 & 30 & 35 \\ 1 & 6 & 12 & 18 & 24 & 30 & 36 & 42 \\ 1 & 7 & 14 & 21 & 28 & 35 & 42 & 49 \end{bmatrix}$$



$$w_8^0 = 1$$

$$w_8^1 = e^{\frac{-j2\pi}{8}} = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

$$w_8^2 = e^{\frac{-j2\pi \cdot 2}{8}} = -j$$

$$w_8^3 = e^{\frac{-j2\pi \cdot 3}{8}}$$

$$\Rightarrow \cos(-135) - j\sin(135)$$

$$\Rightarrow -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

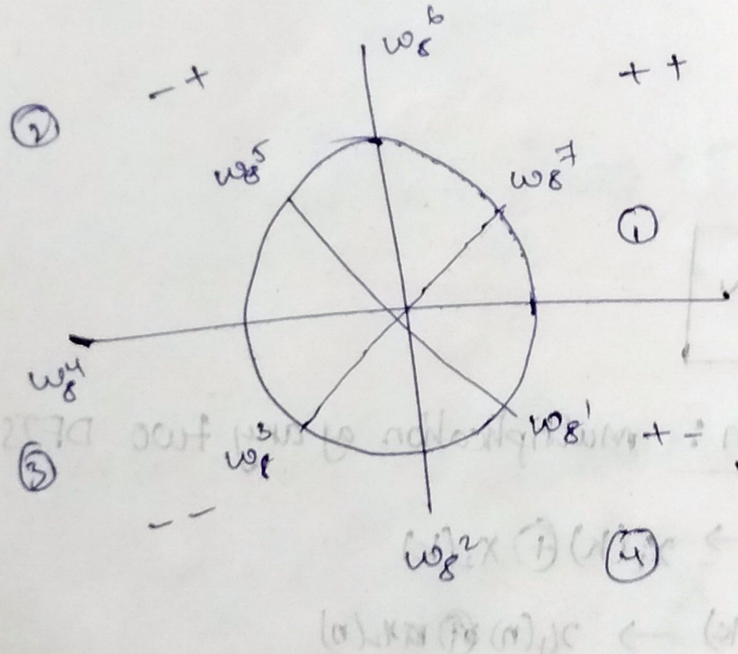
$$w_8^4 = -w_8^0 = -1$$

$$w_8^5 = \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$w_8^6 = +j$$

$$w_8^7 = \frac{1+j}{\sqrt{2}}$$

easier way



$a+jb$

$\rightarrow (a, -b)$

$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$X(k) = \{4,\ 1 + w^1 + w^2 + w^3,\ 1 + w^2 + w^4 + w^6,\ 1 + w^3 + w^6 + w^9,$

$1 + w^4 + w^8 + w^{12},\ 1 + w^5 + w^{10} + w^{15},\ 1 + w^7 + w^6 + w^5\}$

$= 4,\ 1 - j - \sqrt{2}j,\ 1 + \cancel{-j} - 1 + 1,\ 1 + \cancel{j} + \cancel{j} + \frac{1-j}{\sqrt{2}},\ 1 + \cancel{-1} + 1 - 1$

$1 + -\frac{1+j}{\sqrt{2}} - j\ \frac{1+j}{\sqrt{2}},\ 1+$

$X_N = W_N \cdot x_N$

$X_N W_N^{-1} = x_N$

$x_N = X_N W_N^{-1}$

IDFT $\boxed{x_N = \frac{1}{N} \cdot W_N^{*} \cdot X_N}$

<u>Circular convolution</u>: Multiplication of any two DFTs

$x_1(n) \times x_2(n) \longrightarrow X_1(k) \circledast X_2(k)$

$X_1(k) \times X_2(k) \rightarrow x_1(n) \circledast x_2(n)$

$$x(n) \rightarrow \boxed{\ h(n)\ } \rightarrow y(n) = x(n) * h(n)$$

$X(k) \times H(k) \xrightarrow{IDFT} y(n)$

Circular convolution = $\cancel{X}x_3(m) = IDFT\{X_1(k) \times X_2(k)\}$

$$X_3(k) = X_1(k) \cdot X_2(k)$$

$$\quad\downarrow \qquad \downarrow$$

$$x_1(n) \qquad x_2(n)$$

$$X_1(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \quad \text{--- } (1)$$

$$X_2(k) = \sum_{l=0}^{N-1} x(n) e^{-\frac{j2\pi kl}{N}} \quad (2)$$

$$IDFT \text{ of } \{X_3(k)\} = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k) e^{\frac{j2\pi}{N} km}$$

$$\cancel{x(m)}\, x_3(m) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k) e^{\frac{j2\pi km}{N}}$$

$$\Rightarrow x_3(m) = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \sum_{l=0}^{N-1} x_2(l) e^{-\frac{j2\pi kl}{N}} \cdot e^{j\frac{2\pi km}{N}}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \underbrace{\left\{ \sum_{k=0}^{N-1} e^{\frac{j2\pi k}{N}(m-n-l)} \right\}}_{\sum_{k=0}^{N-1} \left(e^{j\frac{2\pi}{N}(m-n-l)} \right)^k}$$

$$\sum_{k=0}^{N-1} \Big(\underbrace{e^{j\frac{2\pi}{N}(m-n-l)}}_{a} \Big)^k$$

$$m-n-(m-n+PN)$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & a=1 \\ \dfrac{1-a^{N-1+1}}{1-a} & a \neq 1 \end{cases}$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N & \boxed{l = m-n+PN} \end{cases}$$

$$((m-n))_N$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2(m-n)_N$$

$$x(n) * h(n)$$

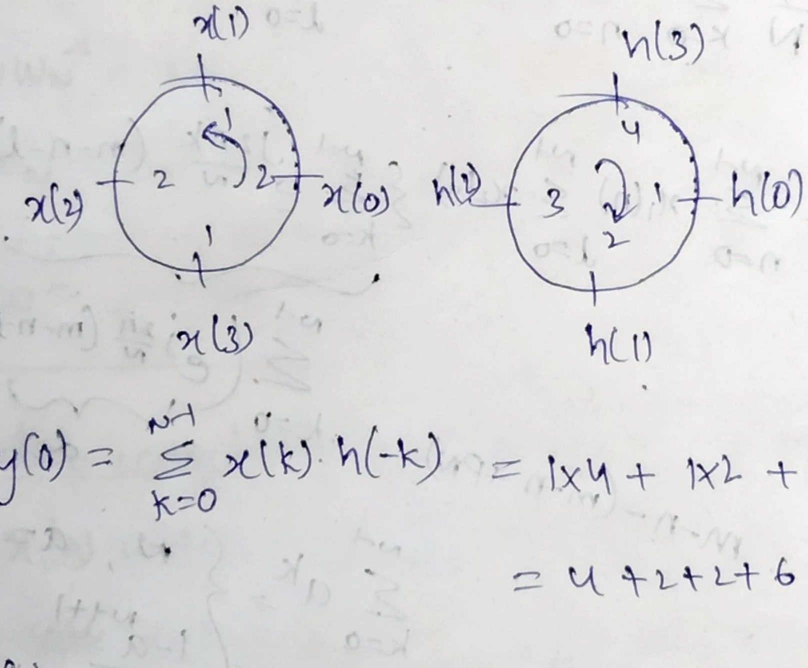
$$\text{linear} = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$x(n) \, (N) \, h(n) = \sum_{k=0}^{N-1} x(k)\, h(n-k)_N$$

Evaluation of circular convolution

(1) $x(n) = \{2, 1, 2, 1\}$, $h(n) = \{1, 2, 3, 4\}$.

$$y(n) = x(n) \, (N) \, h(n) = \sum_{k=0}^{N-1} x(k)\, h((n-k))_N$$

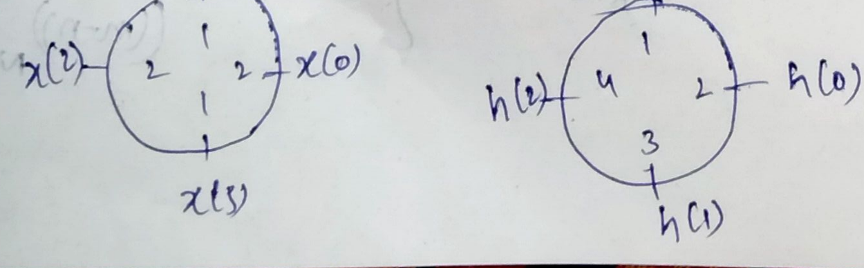


$$y(0) = \sum_{k=0}^{N-1} x(k) \cdot h(-k) = 1\times 4 + 1\times 2 + 1\times 2 + 2\times 3$$

$$= 4 + 2 + 2 + 6 = 14$$

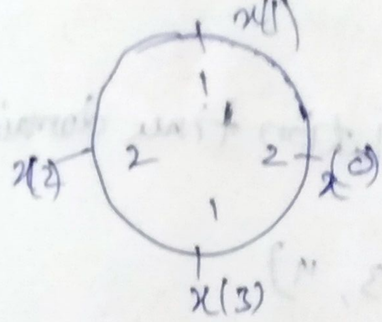
$$y(1) = x(k)\, h(1-k)$$

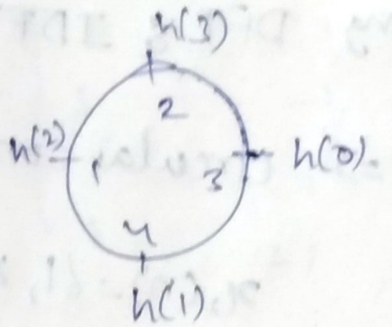
$h(-(k-1)$ Delay anticlock wise



$\Rightarrow 1\cdot 1 + 2\cdot 4 + 3\cdot 1 + 2\cdot 2$

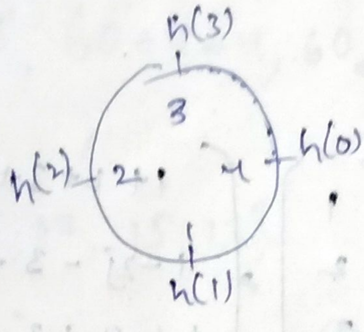
$\Rightarrow 16$





$\Rightarrow 2.1 + 3.2 + 4.1 + 2.4$

$= 14$



$= 3.1 + 2.4 + 1.1 + 2.2$

$= 16$

$\{14, 16, 14, 16\}$

Matrix method

$$\begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{matrix} 2.1 + 1.2 + 2.3 + 4.1 = 14 \\ 1.1 + 2.2 + 1.3 + 2.4 = 16 \\ 2.1 + 1.2 + 2.3 + 1.4 = 14 \\ 1.1 + 2.2 + 1.3 + 2.4 = 16 \end{matrix}$$

Ex $x(n) = \{1, 1, 2, 2\}$ $h(n) =$

$$\cancel{\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 2 & 1 \\ 2 & 2 & 1 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{matrix} \cancel{1.1 + 1.2 + 2.3 + 2.4 = 18} \\ \cancel{1.1 + 2.2 + 2.3 + 1.4 = 15} \\ 2.1 + 2.2 + 1.3 + 1.4 = \\ 2.1 + 1.2 + 1.3 + 2.4 \end{matrix}$$

$$\begin{pmatrix} 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{matrix} 1 + 2.2 + 2.3 + 4.1 = 15 \\ 1.1 + 1.2 + 2.3 + 4.2 = 17 \\ 2.1 + 1.2 + 1.3 + 2.4 = 15 \\ 2.1 + 2.2 + 3.1 + 1.4 = 13 \end{matrix}$$

Circular convolution using DFT & IDFT

IDFT $\{ X_1(k) \cdot X_2(k) \}$ => circular convolution time domain

$x_1(n) = (1, 1, 2, 2)$ $\qquad$ $x_2(n) = (1, 2, 3, 4)$

$$X_1(k) = \sum_{n=0}^{3} x_1(n) e^{-j}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -1+j \\ 0 \\ -1-j \end{bmatrix}$$

~~4x1 4x1~~

$= 60, \ (-1+j)(-2+2j), \ 0, \ (-1-j)(-2-2j)$

$= 60, \ \cancel{2} - 4j - \cancel{2}, \ 0, \ \cancel{2} + 2j + 2j - \cancel{2})$

$\therefore X_3(k) = (60, -4j, 0, 4j)$

IDFT $\Rightarrow x_3(n) = \frac{1}{N} \sum_{k=0}^{1} X_3(k) e^{-j\frac{2\pi kn}{N}}$

$x_3(n) = \frac{1}{N} \cdot W_N^{*} x_N$

$$\Rightarrow \frac{1}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}\begin{pmatrix}60\\ -4j\\ 0\\ 4j\end{pmatrix}$$

$$= \{15,\ 60+4+4,\ 60\cancel{+4j-4j}\ ,\ 60-4-4$$

$$= \{15, 17, 15, 13\}$$

(1) circl (2) matrix (3) DFT / IDFT.

Ex Evalut linea conv. of follow seqen.

$x(n) = \{1, 2, 2, 1\}$ $h(n) = \{1, 2, 3\}$.

$\Rightarrow 4 + 3 - 1 = 6$

	1	2	2	1
1	1	2	2	1
2	2	4	4	2
3	3	6	6	3

$= 1, 4, 9, 11, 8, 3$

$y(n) = x(n) * h(n) = \{1, 4, 9, 11, 8, 3\}$

Circl meth

$$\begin{bmatrix}1 & 0 & 0 & 1 & 2 & 2\\ 2 & 1 & 0 & 0 & 1 & 2\\ 2 & 2 & 1 & 0 & 0 & 1\\ 1 & 2 & 2 & 1 & 0 & 0\\ 0 & 1 & 2 & 2 & 1 & 0\\ 0 & 0 & 1 & 2 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\ 2\\ 3\\ 0\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 2+2+0+0+0\\ 2+4+3\\ 1+4+6\\ 0+2+6\\ 0+0+3\end{bmatrix} \Rightarrow \begin{bmatrix}1\\ 4\\ 9\\ 11\\ 8\\ 3\end{bmatrix}$$

Ex. Linear convolution using circular convolution

$x(n) = \{1\ 1\ 0\ 1\ 1\}$. $\quad h(n) = \{1\ -2\ -3\ 4\}$

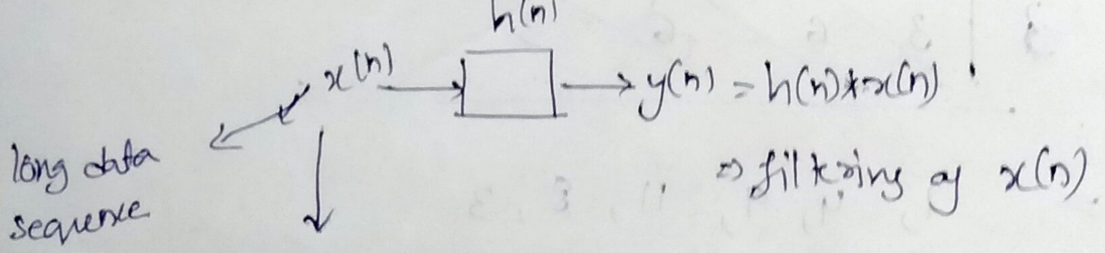
$5+4-1 = 8$.

$x(n) = \{1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\}$

$h(n) = \{1\ -2\ -3\ 4\ 0\ 0\ 0\ 0\}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -3 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -5 \\ 2 \\ 3 \\ -5 \\ 1 \\ 4 \end{bmatrix}$$

Filtering of long data sequences :-



length of the sequence ↑.

① overlap save method

② overlap add method.

* overlap save method:

divide and rule

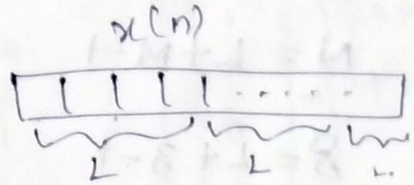
$x(n)$ = large $h(n)$ = small.

$h(n) = M$.

① Size of $h(n)$ is 'M'

$x(n) \rightarrow$ no. of blocks.

① determine M (size of $h(n)$)



② N = size of each block.

$N = L + M - 1$

$x_1(n) = \{ \underbrace{0, 0, \dots}_{M-1} \underbrace{x(0), x(1) \dots x(L-1)}_{L} \}$

(M-1) term

$x_2(n) = \{ \underbrace{x(L-M+1) \dots x(L-1)}_{M-1}, x(L), x(L+1) \dots x(2L-1) \}$

$x_3(n) = \{ \underbrace{x(2L-M+1), \dots x(2L-1)}_{M-1}, x(2L), \dots x(3L-1) \}$

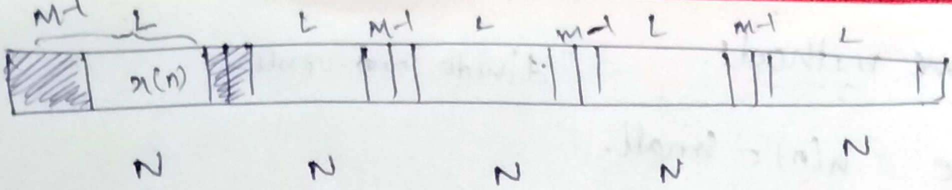
③ $y_1(n) = x_1(n) \, (N) \, h(n)$ = 1st (M-1) Remove

$h(n)$ shud be zero padded with 'L-1' no of zero

$y_2(n) = x_2(n) \, (N) \, h(n)$, 1st Remove (M-1) sample

$y_3(n) = x_3(n) \, (N) \, h(n)$. Remove 1st (M-1) sample

$y(n) = \{ y_1(n), y_2(n), y_3(n) \dots \}$



EX.

$$x(n) = \{3, 0, -2, 0, 2, 1, 0, -2, -1, 0\} \quad h(n) = \{1, 1, 1\}$$

$$M = 3$$

$$N = 2^m = 2^3 = 8$$

$$N = L + M - 1$$

$$8 = L + 3 - 1$$

$$L = 6$$

$$x_1(n) = \{\overbrace{0, 0}^{M-1}, \underbrace{3, 0, -2, 0, 2, 1}_{L}\}$$

$$x_2(n) = \{2, 1, 0, -2, -1, 0, 0, 0\}$$

EX

$$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\} \quad h(n) = \{1, 1, 1\}$$

$$N = 2^M = 2^3 = 8 \qquad M = 3$$

$$N = L + M - 1$$

$$8 = L + 3 - 1$$

$$\boxed{L = 6}$$

$$x_1(n) = \{0, 0, 3, -1, 0, 1, 3, 2\}$$

$$x_2(n) = \{3, 2, 0, 1, 2, 1, 0, 0\}$$

$y_1(n) = x_1(n) \circledast h(n)$ $\qquad$ $h(n) = \{1,1,1,0,0,0,0,0\}$

$y_2(n) = x_2(n) \circledast h(n)$

$$\begin{bmatrix} 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\ 3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+2+3 = 5 \\ 0+0+2 = 2 \\ 3+0+0 = 3 \\ -1+3+0 = 2 \\ 0+-1+3 = 2 \\ 1+0+(-1) = 0 \\ 3+1+0 = 4 \\ 2+3+1 = 6 \end{bmatrix}$$

discard M-1 sample from resul

$y_1 = \{\cancel{5}, \cancel{2}\ 3\ 2\ 2\ 0\ 4\ 6\}$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 2 & 1 & 0 & 2 \\ 2 & 3 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+0+0 = 3 \\ 2+3+0 = 5 \\ 0+2+3 = 5 \\ 1+0+2 = 3 \\ 2+1+0 = 3 \\ 1+2+1 = 4 \\ 0+1+2 = 3 \\ 0+0+1 = 1 \end{bmatrix}$$

$y_2 = [\cancel{3}\ \cancel{5}\ 5\ 3\ 3\ 4\ 3\ 1)$ $\qquad$ $y = (y_1, y_2)$

$y = [3\ 2\ 2\ 0\ 4\ 6\ 5\ 3\ 3\ 4\ 3\ 1)$

$x(n) = \{3, 0, -2, 0, 2, 1, 0, -2, -1, 0\}$

$h(n) = \{2, 2, 1\}.$

$N = 2^M = 2^3 = 8$

$L = 6$

$x_1(n) = \{0, 0, 3, 0, -2, 0, 2, 1\}$

$x_2(n) = \{2, 1, 0, -2, -1, 0, 0, 0\}$

$h(n) = \{2, 2, 1, 0, 0, 0, 0, 0\}$

$$y_1(n) = \begin{bmatrix} 0 & 1 & 2 & 0 & -2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 & -2 & 0 & 3 \\ 3 & 0 & 0 & 1 & 2 & 0 & -2 & 0 \\ 0 & 3 & 0 & 0 & 1 & 2 & 0 & -2 \\ -2 & 0 & 3 & 0 & 0 & 1 & 2 & 0 \\ 0 & -2 & 0 & 3 & 0 & 0 & 1 & 2 \\ 2 & 0 & -2 & 0 & 3 & 0 & 0 & 1 \\ 1 & 2 & 0 & -2 & 0 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 6 \\ 6 \\ -1 \\ -4 \\ 2 \\ 6 \end{bmatrix}$$

$y_1 = (\cancel{4}\ \cancel{1}\ 6\ 6\ 1\ -4\ 2\ 6)$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & -1 & -2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & -1 & -2 \\ -2 & 0 & 1 & 2 & 0 & 0 & 0 & -1 \\ -1 & -2 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -1 & -2 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 4 \\ -3 \\ -6 \\ -4 \\ -1 \\ 0 \end{bmatrix}$$

$y_1 = \{4\ -3\ -6\ -4\ -1\ 0\}$

$y = \{-6\ \ 6\ \ 1\ \ -4\ \ 2\ \ 6\ \ 4\ \ -3\ \ -6\ \ -4\ \ -1\ \ 0\}.$

Overlap - add method. ⇒ (M-1) no of zeroes to every sub sequnce.

$x(n) = \{x(0), x(1) \ldots\ldots \}$

$N = L+M-1$

$x_1(n) = \{x(0), x(1) \ldots x(L-1), \underbrace{0, 0, 0 \ldots M-1}_{(M-1)\ \#\ \text{of zero}}\}$

$x_2(n) = \{x(L), x(L+1) \ldots x(2L-1) \ldots \underbrace{0, 0, 0 \ldots M-1}_{(M-1)}\}$

$x_3(n) = \{x(2L), x(2L+1) \ldots x(3L-1) \ldots \underbrace{0\ 0\ 0 \ldots M-1}_{M-1}\}$

Ex $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ $h(n) = \{1, 1, 1\}$

$y(n) = 3 + 10 - 1 = 12$ $N = 2^m = 2^3 = 8$

$N = L+M-1 = 8 = L+2 \Rightarrow \boxed{L=6}$

$x_1(n) = \{3, -1, 0, 1, 3, 2, 0, 0\}$

$x_2(n) = \{0, 1, 2, 1, 0, 0, 0, 0\}$

$h(n) = (1, 1, 1, 0, 0, 0, 0, 0)$

$x_1(n) \circledN h(n)$

$$
\begin{bmatrix}
3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 \\
-1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 \\
0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\
1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\
3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\
2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 \\
0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \\
0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0
\end{bmatrix}
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{array}{c} y_1 \\ \begin{bmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix} \end{array}
$$

$y_1 = (3, 2, 2, 0, 4, 6, 5, 2)$

$x_2(n) \circledN h(n)$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 1 & 0
\end{bmatrix}
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}
$$

$y_2 = (0, 1, 3, 4, 3, 1, 0, 0)$

Now. $\{3, 2, 2, 0, 4, 6, \overline{5, 2}\}$

add

$\{\underline{0, 1}, 3, 4, 3, 1, \underline{0, 0}\}$ elimit

$= \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1, \cancel{0, 0}\}$

same problem by reducing 'N' value

$N = L+M-1$. let $N=5$

$5 = L+2$, $L=3$

$x_1(n) = \{3,-1,0,0,0\}$

$h(n) = \{1,1,1,0,0\}$

$x_2(n) = \{1,3,2,0,0\}$

$x_3(n) = \{0,1,2,0,0\}$

$x_4(n) = \{1,0,0,0,0\}$

$y_1(n) \; x_1(n) h(n)$

$$= \begin{bmatrix} 3 & 0 & 0 & 0 & -1 \\ -1 & 3 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 \\ 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 2 \\ 2 \\ -1 \\ 0 \end{bmatrix}$$

$y_2 \; x_2(n) h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix}$$

$y_3(n) \; x_3(n) h(n)$

$$\begin{bmatrix} 0 & 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$$

$y_4 \; x_4(n) h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

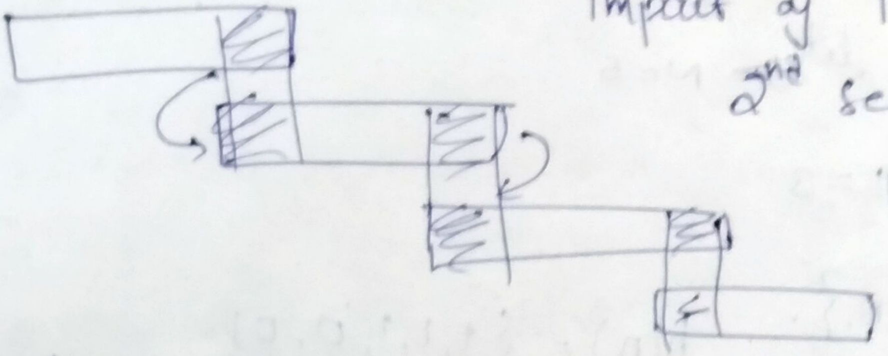
$\{3 \; 2 \; 2 \; -1 \; 0\}$

$\{1, 4, 6, 5, 2\}$

$\{0, 1, 3, 3, 2\}$ discard

$\{1, 1, 1, 0, 0\}$

$(3 \; 2 \; 2 \; 0 \; 4 \; 6 \; 5 \; 3 \; 3 \; 4 \; 3 \; 1)$



Impact of 1st will affect 2nd seque " save method (Aliasing)

in save, all subseanence are dependent.

in overlap add method aliasing problem with convolution

overlap save	overlap add
aliasing while making subseane sub seanence are depend $x_1(n)$, $x_2(n)$	aliasing while creatin $y(n)$ sub seane are independ.

Ex: $x(n) = \{1, 2, -1, 3, -2, -3, -1, 1, 2, -1\}$ $h(n) = \{1, 2\}$

$y(n) = 12$

$N = 2^m = 2^2 = 4$

$4 = L + 2 - 1$

$\boxed{L = 3}$

$x_1(n) = \{1, 2, -1, 0\}$

$x_2(n) = \{3, -2, -3, 0\}$ $h(n) = \{1, 2, 0, 0\}$

$x_3(n) = \{-1, 1, 1, 0\}$

$x_4(n) = \{2, -1, 0, 0\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \\ -2 \end{bmatrix} \qquad y_2(n) = \begin{bmatrix} 3 & 0 & -3 & -2 \\ -2 & 3 & 0 & -3 \\ -3 & -2 & 3 & 0 \\ 0 & -3 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -7 \\ -6 \end{bmatrix}$$

~~circular~~

$$y_3 \begin{bmatrix} -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 3 \\ 2 \end{bmatrix} \qquad y_4(n) = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$\Rightarrow$ 1, 4, 3, -2

3, 4, -7, -6

-1, -1, 3, 2

2, 3, -2, 0

$y(n)$ = 1 4 3 1 4 -7 -7 -1 3 4 3 -2 0

discard.

circular frequency shift property:

$$x(n) \xrightarrow{DFT} X(k)$$

$$x(n)\, e^{\frac{j2\pi l n}{N}} \xrightarrow{DFT} X((k-l))_N$$

$$x(n)\, e^{\frac{-j2\pi l n}{N}} \xrightarrow{DFT} X(k+l)_N$$

Proof $\quad X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}$

$$DFT\left(x(n)\, e^{\frac{j2\pi l n}{N}}\right) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{j2\pi l n}{N}}\, e^{-j2\pi \frac{kn}{N}}$$

$$\Rightarrow \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi}{N}(k-l).n} \Rightarrow \sum_{n=0}^{N-1} x(n)\, e^{-\frac{2\pi}{N}(N+k-l)n}$$

$= x(N+k-l) = x((k-l))_N$

Time reversal of the sequence

$\text{DFT}\{x(-n)_N\} = \text{DFT}\{x(N-n)\}$

$\longrightarrow X((-k))_N = X(N-k)$

Proof Assume $N-n=m$

$$\text{DFT}[x(N-n)] = \sum_{n=0}^{N-1} x(N-n)\, e^{-j\frac{2\pi kn}{N}}$$

at $n=0$ $\quad m=N$

$n=N-1$ $\quad m=1$

$$= \sum_{m=0}^{N-1} x(m).\, e^{-j\frac{2\pi}{N}k(N-m)}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{j\frac{2\pi km}{N}}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j\frac{2\pi k}{N}\cancel{(-1)}\cdot m\,(N-k)}$$

$$= X(N-k)_N = X(-k)_N$$

$\therefore \boxed{x(-n) \xrightarrow{\text{DFT}} X((-k))_N}$

complex conjugate property:

$$DFT\{x^*(n)\} \to ?$$

$$= \sum_{n=0}^{N-1} x^*(n) e^{\frac{-j2\pi kn}{N}}$$

$$= \left(\sum_{n=0}^{N-1} x(n) e^{\frac{j2\pi kn}{N}} \right)^*$$

$$= \left[\sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi n(N-k)}{N}} \right]^*$$

$$= X^*(N-k)_N \Rightarrow X^*((-k))_N$$

$$\therefore \boxed{x^*(n) \xrightarrow{DFT} X^*(-k)_N}$$

$$\boxed{x^*(-n) \xrightarrow{DF} X^*(k)_N}$$

if $x(n)$ is a real valued sequence

$$x(n) = x^*(n)$$

apply DFT on b.s

$$X(k) = X^*(-k)$$

$$X_R(k) + jX_I(k) = X_R(N-k) - jX_I(N-k)$$

$$\boxed{X_R(k) = X_R(N-k)} \quad \boxed{X_I(k) = -X_I(N-k)}$$

$$\therefore \boxed{X(k) = X^*(N-k)}$$

Let $X(k)$ is 14 pt DFT the ~~first~~ first 8 sample

$X(k) = \{12, -1+j3, 3+j4, 1-j5, -2+j2, 6+j3, -2-j3, 10\}$

Determine remain samples of $X(k)$ if $x(n)$ is real

$$X(k) = X^*(N-k)$$

$$X(8) = X^*(14-8) = X^*(6) = -2+j3$$

$$X(9) = X^*(14-9) = X^*(5) = 6-j3$$

$$X(10) = X^*(14-10) = X^*(4) = -2-j2$$

$$X(11) = X^*(14-11) = X^*(3) = 1+j5$$

$$X(12) = X^*(14-12) = X^*(2) = 3-j4$$

$$X(13) = X^*(14-13) = X^*(1) = -1-j3$$

$$X(14) = X^*(14-14) = X^*(0) = 12$$

Parseval's Energy theorem :: Energy Time = Energy Frequency

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$$

$$\sum_{n=0}^{N-1} x(n)\cdot x^*(n) = \sum_{n=0}^{N-1} x(n)\left[\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}}\right]^*$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\left[\sum_{k=0}^{N-1} X^*(k)\, e^{\frac{-j2\pi kn}{N}}\right]$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X^*(k)\cdot \underbrace{\sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}}_{X(k)}$$

$$\Rightarrow \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot X^{*}(k)$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^{2} \quad //$$

Fast Fourier Transform

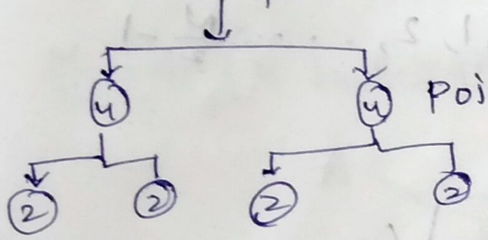
DFT limitation
↳ when 'N' is large frequency
more no of add^n & sub^n
→ high computional

→ N- point DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

→ computional complexity (to overcome this in DFT, we use FFT).

In FFT 8 point DFT



4 point DFT

2point DFT

Radix (or) base (or) minimum

→ FFT (or) Radix-2 or Butterfly diagram [Radix-2 → DIT, DIF]

eg $N=8$

$x(n) = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6), x(7)\}$ (8 point). DFT

$x_e(n) = \{x(0), x(2), x(4), x(6)\}$
$x_o(n) = \{x(1), x(3), x(5), x(7)\}$ } 4 point DFT

$x_{ee}(n) = \{x(0), x(4)\}$
$x_{eo}(n) = \{x(2), x(6)\}$
$x_{oe}(n) = \{x(1), x(5)\}$
$x_{oo}(n) = \{x(3), x(7)\}$ } 2 point DFT

→ $x(n)$

$x_e(n) = x(2n)$ $\quad n = 0, 1, \ldots, \frac{N}{2}-1$

$x_o(n) = x(2n+1)$ $\quad n = 0, 1, \ldots \ldots \frac{N}{2}-1$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$\Rightarrow \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2kn} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{k(2n+1)}$$

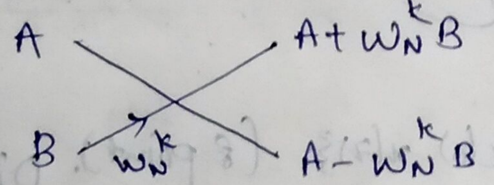
$$\Rightarrow \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{N/2}^{kn} + W_N^k \sum_{n=0}^{N/2-1} x(2n+1) W_{N/2}^{kn}$$

$$\boxed{X(k) = X_e(k) + W_N^k X_o(k)} \quad k = 0, 1, 2, \ldots \ldots \frac{N}{2}-1$$

$k \geq N/2$

$$\boxed{X(k) = X_e(k-N/2) - W_N^{(k-N/2)} X_o(k-N/2)} \left(k = \frac{N}{2} \ldots N-1\right)$$

→ Butterfly



→ DIT → decimation in Time domain

$\boxed{\text{DIT} \rightarrow \text{Radix} - 2}$

→ $N = 2^m$, m = no of stages, at every stage we have $N/2$ butterflies m=3,, 8 stage

0 - 0 0 0 $\xrightarrow{\text{reverse}}$ 0 0 0 → 0 ⇒ x(0)

1 - 0 0 1 ⟶ 1 0 0 → 4 ⇒ x(4)

2 - 0 1 0 ⟶ 0 1 0 → 2 ⇒ x(2)

3 - 0 1 1 ⟶ 1 1 1 → 6 ⇒ x(6)

4 - 1 0 0 ⟶ 0 1 1 → 1 ⇒ x(1)

5 - 1 0 1 ⟶ 1 0 1 → 5 ⇒ x(5)

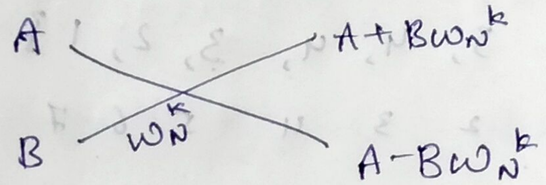
6 - 1 1 0 ⟶ 0 1 1 → 3 ⇒ x(3)

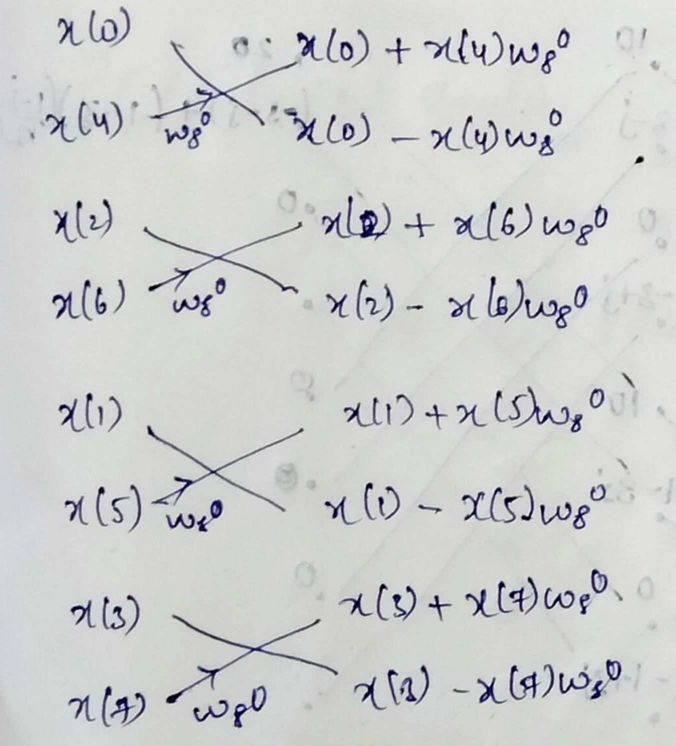
7 - 1 1 1 ⟶ 1 1 1 → 7 ⇒ x(7)

m=3. stages DFT.

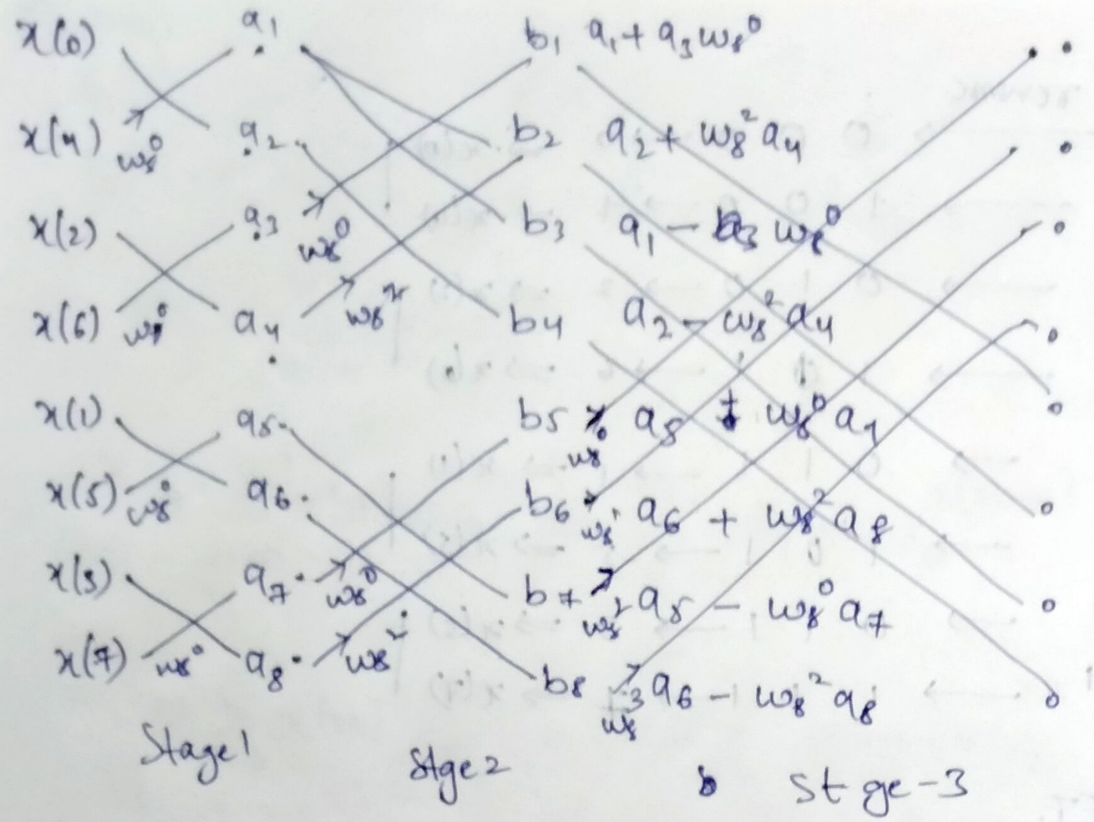
$N = 2^m = 8$

$\frac{N}{2} = 4$ (4 butterflies we have)





$k = \frac{Nt}{2^m}$ $\left(\begin{matrix} m=3 \\ t=0,1,2,3 \end{matrix}\right)$



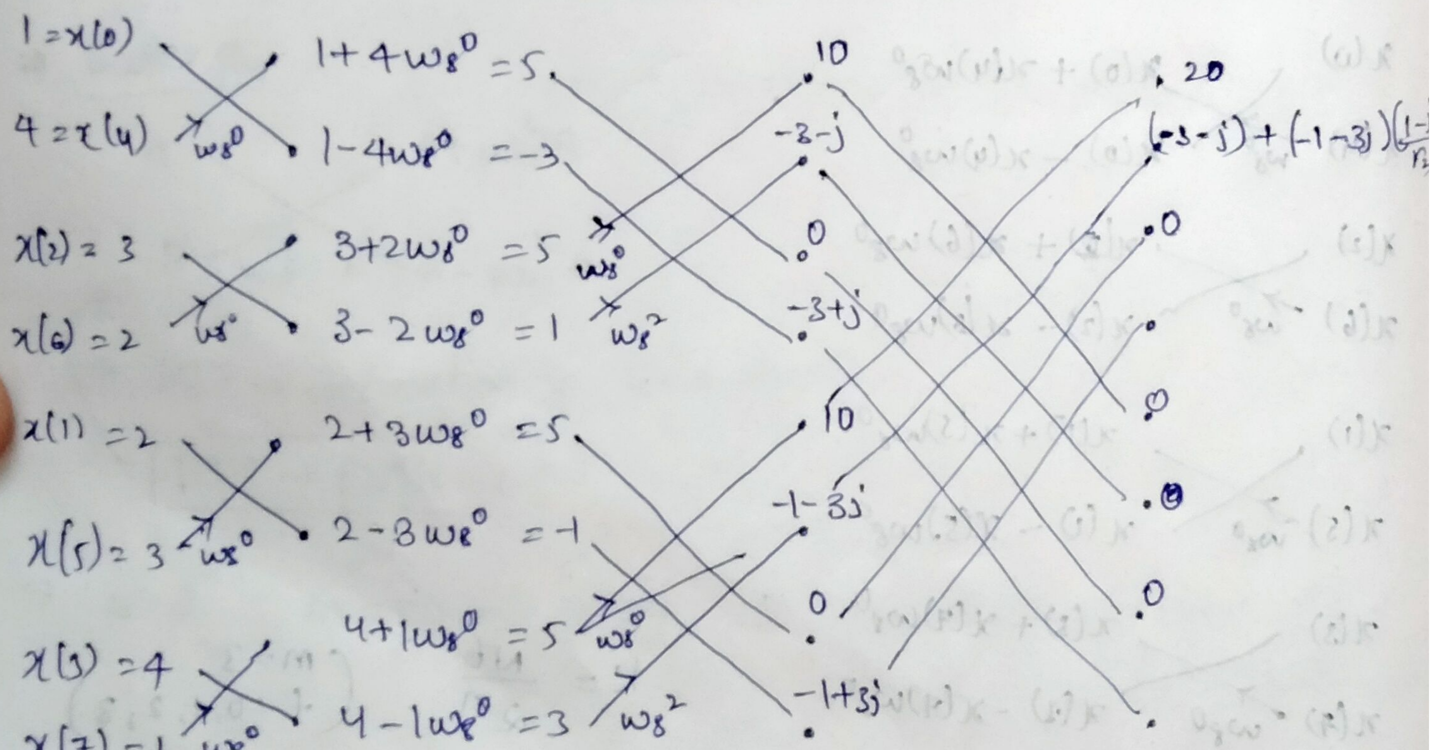
→ Evaluate 8 point FFT of a given sequence

($w_8^0 = 1$

$w_8^2 =$

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

0 1 2 3 4 5 6 7



$(-3+j)+(-1+2j)\left(\frac{1+j}{\sqrt{2}}\right)$

$\boxed{m=3}$

$= -3+j+\frac{1}{\sqrt{2}}(-1+3j-2)$

$N=2^m=8 \Rightarrow \frac{N}{2}=4$

$= -3+j+\sqrt{2}j-2\sqrt{2} \Rightarrow -3-2\sqrt{2}+j(1+\sqrt{2})$

$X(k)=$

Q) Given $x(n)=\{1,1,1,1,0,0,0,0\}$
$\quad\quad 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$

① $N=2^m=2^3=8 \Rightarrow M=3$ = no of stages

② Bit reversal

③ $N/2$ ④ 2^{l-1} sample $t=0,1,\ldots,2^{l-1}$

⑤ Twiddle factor $k=\frac{N\times t}{2^l}$; l = stage no

Stage 1 $\Rightarrow l=1$, $t=0 \Rightarrow k=0$

stage 2 $\Rightarrow l=2$ $t=0,1$

at $t=0$ $k=0$

$t=1$ $k=2$

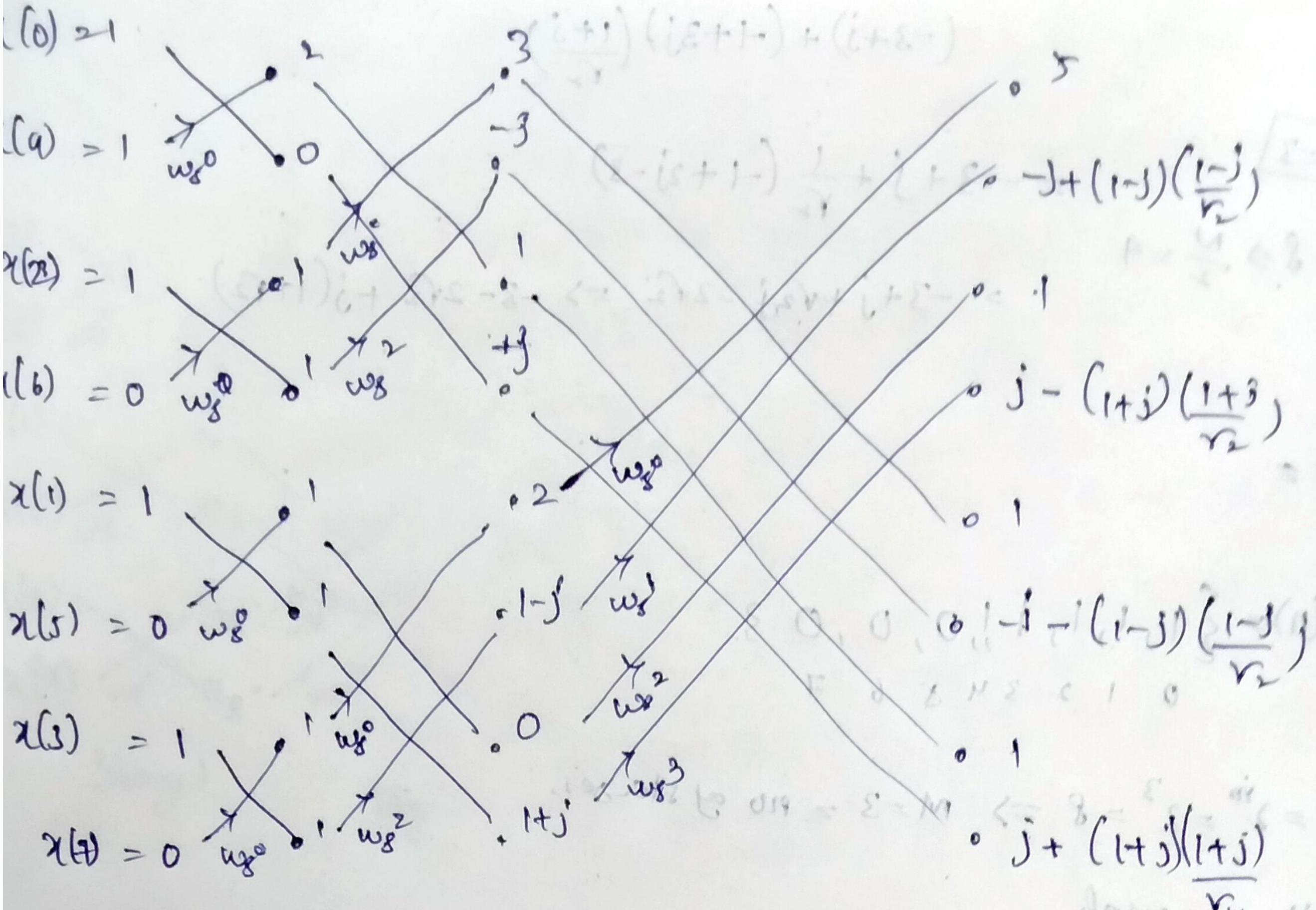
Stage 3 : $l=3$ $t=0,1,2,3$

$t=0$ $k=0$

$t=1$ $k=1$

$t=2$ $k=2$

$t=3$ $k=3$



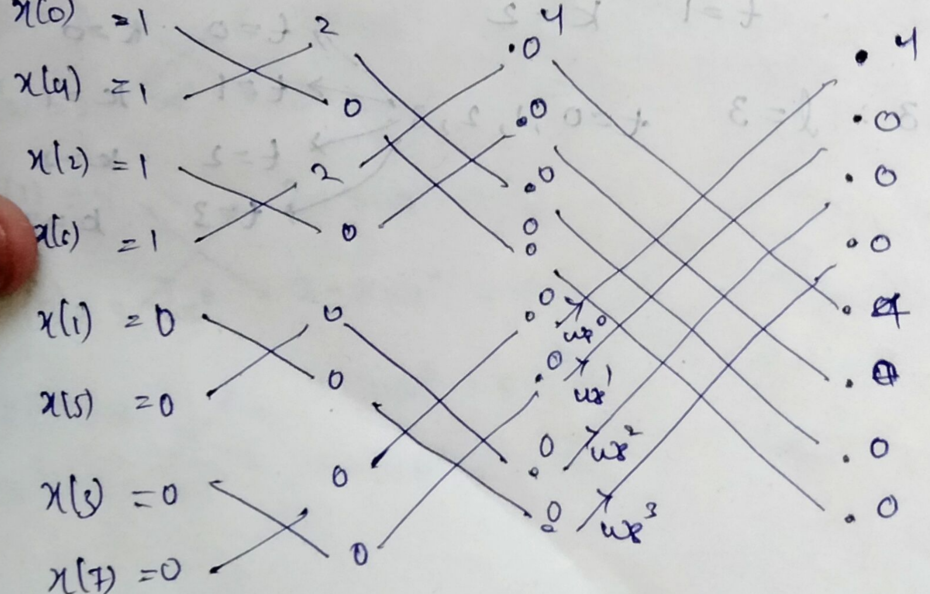
$-j-(1-j)w_8^1$

$= -j-(1-j)(0.707-j0.707)$

$= 0.414j$

$\therefore X(k) = \{5$

Q) $x(n) = \{1, 0, 1, 0, 1, 0, 1, 0\}$ → evaluate 8 point FFT using DIT
0 1 2 3 4 5 6 7



$X(k) = \{4, 0, 0, 0, 4, 0, 0, 0\}$.

no. of multiplication req : $\frac{N}{2} \times \log_2^N$

16-point FFT : DIT Radix-2

16 ⇒ $N = 16 = 2^4$ ⇒ $m = 4$

$N/2 = 8$ → butterflies req per row

Twiddle factor

S_1 : $k = \frac{N \times t}{2^l} = 0$ $\quad t = 0, 1, \ldots 2^{l-1} - 1$

S_2 : $t = 0, 1$ $\quad t = 0 \quad k = 0$

$k = 0, 4$ $\quad t = 1, \; k = \frac{16 \times 1}{2^2} = 4$

S_3 : $t = 0, 1, 2, 3$

$t = 0 \rightarrow k = 0$
$t = 1 \rightarrow k = 2$
$t = 2 \rightarrow k = 4$
$t = 3 \rightarrow k = 6$ $\quad [k = 0, 2, 4, 6]$

S_4 = $t = 0, 1, 2, 3, 4, 5, 6, 7.$

$t = 0 \quad k = 0$

$t = 1 \quad k = 1 \quad \ldots$ $\quad [k = 0, 1, 2, 3, 4, 5, 6, 7]$

Input order $\qquad$ bit reversal

0 – 0000 – x(0)
1 – 0001 – x(8)
2 – 0010 – x(4)
3 – 0011 – 12
4 – 0100 – 10
7 – 0111 – 14
8 – 1000 = 1
9 – 1001 = 9
10 – 1010 = 5

11 – 1011 – 13
12 – 1100 – 3
13 – 1101 – 11
14 – 1110 – 7
15 – 1111 – 5

Order

$x(0)$	$x(8)$
$x(4)$	$x(12)$
$x(2)$	$x(10)$
$x(6)$	$x(14)$
$x(1)$	$x(9)$
$x(5)$	$x(13)$
$x(3)$	$x(11)$
$x(7)$	$x(15)$