1. An elliptically polarized wave travelling in −ve z direction is received by circularly polarized antenna whose main lobe is along $\theta = 0$ direction. The unit vector describing the polarization of the incident wave is given by,

$$\hat{\rho}_w = \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}}$$

find polarization loss factor (PLF) when the wave that would transmitted by,

(a) RHCP

(b) LHCP

$\overrightarrow{+z}$ $\quad$ $\overleftarrow{-z}$

Sol:-

$$\left\{\begin{array}{l} \vec{A}_1 = \hat{a}_x + j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y) \Rightarrow \text{LHCP} \\ \vec{A}_2 = a_x - j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x - j\hat{a}_y) \Rightarrow \text{RHCP} \end{array}\right.$$

if wave is travelling $+z$ direction.

$$\left\{\begin{array}{l} \vec{A}_1 = \hat{a}_x - j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x - j\hat{a}_y) \Rightarrow \text{LHCP} \\ \vec{A}_2 = \hat{a}_x + j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y) \Rightarrow \text{RHCP} \end{array}\right.$$

if wave is travelling $-z$ direction

$$\hat{\rho}_w = \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}}$$

~~LHCP~~ $\cancel{\hat{\rho}_w \cdot \hat{\rho}_a = \sqrt{\frac{2}{5}} \ldots}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

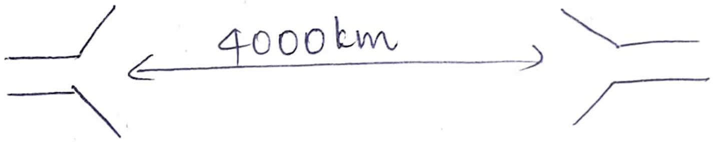
$$= \left|\frac{2}{\sqrt{10}} + \frac{1}{\sqrt{10}}\right|^2 = \frac{9}{10} \rightarrow \text{LHCP}$$

$$\text{PLF} = \left|\frac{1 \times 2}{\sqrt{10}} - \frac{1}{\sqrt{10}}\right|^2 = \frac{1}{10} \rightarrow \text{RHCP}$$

~~RHCP~~ $\cancel{\hat{\rho}_w \cdot \hat{\rho}_a = \sqrt{\frac{2}{5}} + \frac{j}{\sqrt{10}}}$

2. A 30dB, RHCP antenna in a radio link radiates 5W power at 2GHz. The receiving antenna has a impedance mismatch at terminals which leads to VSWR of 2. The receiving antenna is about 95% efficient and has a field pattern of near the beam maximum given by $\vec{E}_r = (2\hat{a}_x + j\hat{a}_y) F_r(\theta, \phi)$. The distance b/w two antennas is 4,000km and receiving antenna is required to deliver 10^{-14} W to the receiver. Determine max effective aperture of receiving antenna.

Sol:-



Given

$G = 30\,dB$

$f = 2\,GHz$

$P_t = 5W$

RHCP

$VSWR = 2$

$\eta = 95\%$

$P_r = 10^{-14}\,W$

$A_{er} = ?$

$|\Gamma_L| = 1/3$

$(1 - |\Gamma_L|^2) = 8/9$

for receiving antenna,

$$W = \frac{P_t}{4\pi r^2} \cdot D \;\; W/m^2$$

$$P_r = \frac{P_t}{4\pi r^2} \cdot D \cdot A_{er} \cdot (0.95) \cdot (8/9) \cdot 10^3$$

$$\hat{\rho}_r = \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}}$$

$$\hat{\rho}_t = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}$$

$$PLF = |\hat{\rho}_r \cdot \hat{\rho}_t|^2$$

$$= \left| \frac{2}{\sqrt{10}} + \frac{1}{\sqrt{10}} \right|^2$$

$$= 9/10$$

$$10^{-14} = \frac{5}{4\times\pi\times(4000\times10^3)^2}\times(0.95)\times(8/9)\times10^3\times\frac{9}{10}\times A_{er}$$

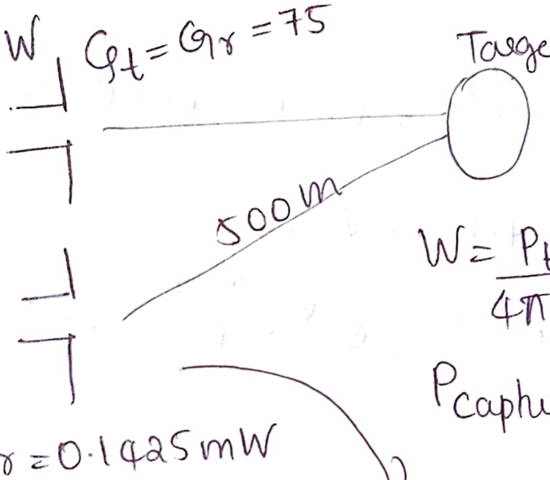
$$A_{er} = \frac{10^{-14}\times4\pi\times16\times10^{12}\times9\times10}{5\times(0.95)\times8\times10^8} = \frac{1808.64\times10^{-2}}{3800}$$

$$A_{er} = 0.475\times10^{-2}\ m^2$$

$$A_{er} = 0.00476\ m^2$$

3. Problem - 6 in Assignment - 1,

$P_t = 1000W$, $G_t = G_r = 75$



$P_r = 0.1425 mW$

$$W = \frac{P_t}{4\pi r^2}\cdot D\ W/m^2$$

$$P_{captured} = \sigma\cdot W$$

$$W_s = \frac{P_c}{4\pi R^2}$$

$$P_r = W_s\cdot A_{er}.$$

$$P_r = \frac{P_c}{4\pi R^2}\cdot\frac{\lambda^2}{4\pi}\cdot D_r$$

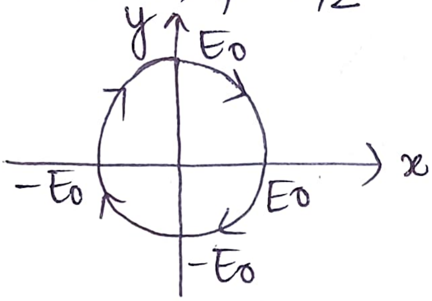
$$P_r = \frac{\sigma\cdot P_t}{(4\pi R^2)^2}\cdot D_t\cdot\frac{\lambda^2}{4\pi}\cdot D_r$$

$$\sigma = 0.01257\ m^2$$

$\Rightarrow \vec{E} = E_{01}\cos(\omega t - \beta z)\,\hat{a}_x + E_{02}\cos(\omega t + \phi - \beta z)\,\hat{a}_y$

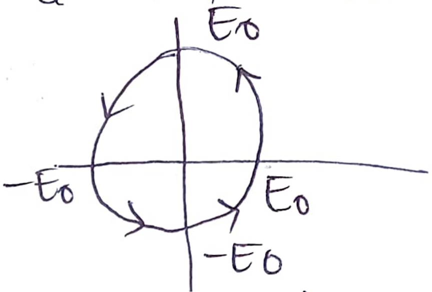
$E_{01} = E_{02} \quad \phi = \pm \pi/2$

if $z = 0$, $\phi = \pi/2$



LHCP(+z-direction)

if $z = 0$; $\phi = -\pi/2$



RHCP(+z-direction)

if $z = 0 \Rightarrow \vec{E} = E_0\cos(\omega t)\,\hat{a}_x + E_0\cos(\omega t + \phi)\,\hat{a}_y$

$\vec{E}_p = E_{01}\,\hat{a}_x + E_{02}\angle\phi\,\hat{a}_y$

$$\boxed{\vec{E}_p = E_{01}\,\hat{a}_x + E_{02}\,e^{j\phi}\,\hat{a}_y} \rightarrow (*)$$

$(e^{j\phi} = \cos\phi + j\sin\phi)$

for +z-direction,

$\phi = \pi/2 \rightarrow$ LHCP

$\phi = -\pi/2 \rightarrow$ RHCP

for −z-direction,

$\phi = \pi/2 \rightarrow$ RHCP

$\phi = -\pi/2 \rightarrow$ LHCP

→ for elliptical polarization;

for +z-direction,

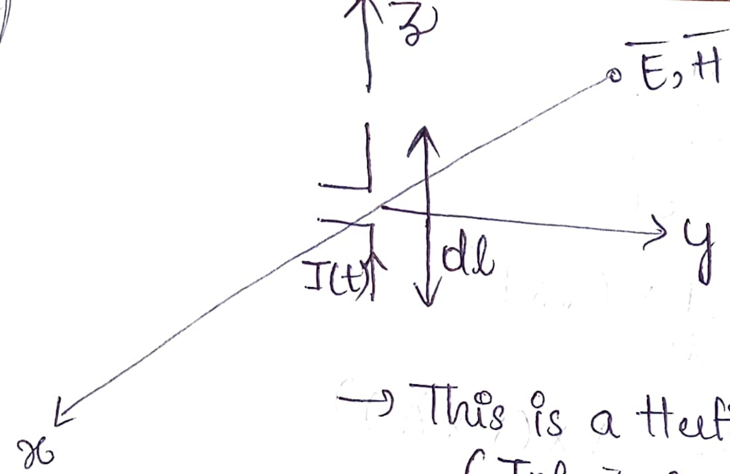
$\phi = +ve \rightarrow$ LHEP

$\phi = -ve \rightarrow$ RHEP

for −z-direction,

$\phi = +ve \rightarrow$ RHEP

$\phi = -ve \rightarrow$ LHEP.



→ This is a Hertzian dipole Antenna (Infinity Small Antenna).

→ Maxwell's equations,

(i) $\nabla \cdot \overline{D} = \rho_v$

(ii) $\nabla \cdot \overline{B} = 0$

(iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\nabla \times \vec{H} = \vec{J_c} + \frac{\partial \vec{D}}{\partial t}$

→ $\boxed{\vec{B} = \mu \vec{H}}$

→ $\boxed{\vec{D} = \varepsilon \vec{E}}$

→ $\nabla \cdot \vec{B} = 0$

But $\nabla \cdot (\nabla \times \vec{A}) = 0$

So, $\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ → Vector magnetic potential.

→ $\nabla \times \vec{E} = -\dot{\vec{B}}$

$= -(\nabla \times \dot{\vec{A}})$

$\boxed{\nabla \times \vec{E} = -(\nabla \times \dot{\vec{A}})}$

$\boxed{\nabla \times (\overline{E} + \dot{\overline{A}}) = 0}$

$\boxed{\overline{E} + \dot{\overline{A}} = -\nabla V}$ → $\boxed{V \rightarrow \text{scalar electric potential}}$

$\nabla \times (-\nabla V) = 0$

$$\rightarrow \nabla \times \vec{H} = J_c + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J_c} + \varepsilon \dot{\vec{E}}$$

$$\nabla \times \frac{1}{\mu}\vec{B} = \frac{1}{\mu} \times (\nabla \times \vec{B})$$

$$= \frac{1}{\mu}(\nabla \times (\nabla \times \vec{A}))$$

$$= \vec{J_c} + \varepsilon \dot{\vec{E}}$$

$$\frac{1}{\mu}(\nabla \times (\nabla \times \bar{A})) = \bar{J_c} + \varepsilon(-\nabla \dot{V} - \dot{\bar{A}})$$

$$\boxed{\nabla \times (\nabla \times \bar{A}) = \bar{J_c}\mu + \mu\varepsilon(-\nabla \dot{V} - \ddot{\bar{A}})}$$

But $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J_c} + \mu\varepsilon(-\nabla \dot{V} - \ddot{\bar{A}})$$

$$\nabla^2 \bar{A} - \mu\varepsilon \ddot{\bar{A}} = -\mu \vec{J_c} + \mu\varepsilon \nabla \dot{V} + \nabla(\nabla \cdot \bar{A})$$

$$\nabla^2 \bar{A} - \mu\varepsilon \frac{\partial^2}{\partial t}\bar{A} = -\mu \vec{J_c} + \underbrace{\mu\varepsilon \nabla \dot{V} + \nabla(\nabla \cdot \bar{A})}$$

$$\nabla(\mu\varepsilon \dot{V} + \nabla \cdot \bar{A}) = 0$$

$$\mu\varepsilon \dot{V} + \nabla \cdot \bar{A} = 0$$

$$\boxed{\nabla \cdot \bar{A} = -\mu\varepsilon \dot{V}}$$

~~loo~~ Lorentz Gauge

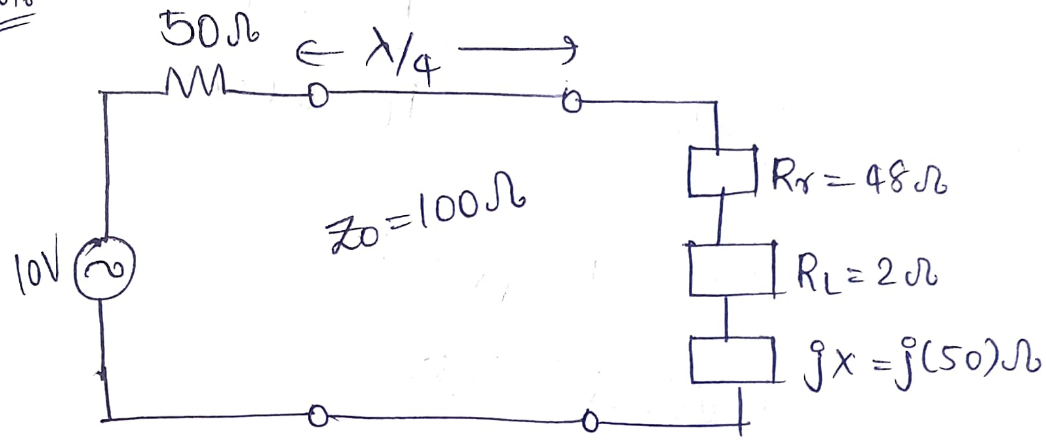
$$\boxed{\nabla^2 \bar{A} - \mu\varepsilon \ddot{\bar{A}} = -\mu \vec{J}} \quad \text{—— (1)}$$

Problem:

Radiation ~~resh~~ resistance $= 48\,\Omega$
loss resistance $= 2\,\Omega$
Reactance $= 50\,\Omega$
} of antenna.

An antenna is connected to generator with open circuit voltage of 10V & internal impedance of $50\,\Omega$ via $\lambda/4$ long transmission line with characteristic impedance of $100\,\Omega$.

Sol:-



$$Z_L = R_L + jX_L$$

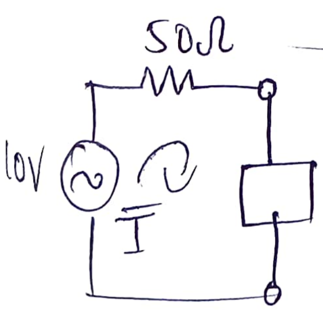
$$Z_L = 50 + j(50)$$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = 0.5 + j(0.5)$$

$$\bar{Z}_{in} = \frac{1}{\bar{Z}_L} = \frac{1}{0.5 + j(0.5)} = (1 - j)$$

$$Z_{in} = (1 - j)\, Z_0$$

$$\boxed{Z_{in} = 100 - j(100)}$$



$Z_{in} = 100 - j(100)$

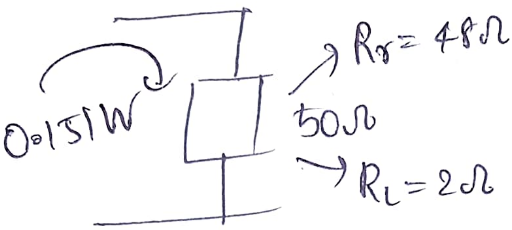
$$\bar{I} = \frac{V}{150 - j(100)} \Rightarrow \bar{I} = \frac{1}{15 - j(10)}$$

$$\bar{I} = \frac{15 + j(10)}{\sqrt{325}} \rightarrow \boxed{\bar{I} = 0.0055\angle 33.69^\circ}$$

Power delivered to load $= \frac{1}{2}|I|^2 \cdot Z_0$

$= \frac{1}{2}|I|^2 \times 100$

$$\boxed{P_{delivered} = 0.151\,W}$$



$P_r = P_D \cdot \frac{48}{50}$

$$\boxed{P_r = 0.145\,W}$$

$$\boxed{P_L = 0.006\,W}$$

$\frac{1}{2}|I|^2 \times 50 = 0.075\,W$

Power supplied $= \frac{Re}{2}(V \times I^*)$

$P_{supplied} = 0.231\,W$

Problem:

$$E = \begin{cases} 1 & 0^\circ \leq \theta \leq 45^\circ \\ 0 & 45^\circ \leq \theta \leq 90^\circ \\ 1/2 & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

(a) What is directivity of this antenna.

(b) What is radiation resistance of antenna at 200m from it if the field is equal to 10 V/m (rms) for $\theta = 0^\circ$ at that distance and terminal current (5A). rms.

Sol: Directivity $= \dfrac{U_D}{U_{avg}}$

But, $U_{avg} = \dfrac{P_{rad}}{4\pi}$

$$U = \frac{\text{Power}}{\text{Solid angle}}$$

$$r^2 \cdot W = \frac{\text{Power}}{\text{Area}} \cdot r^2$$

$$W = \frac{1}{2} \cdot \frac{|E_0|^2}{\eta_0} = \frac{|E_{rms}|^2}{\eta_0}$$

$$U = \frac{|E_{rms}|^2}{\eta_0} \cdot r^2$$

$$U = W \cdot r^2 = \begin{cases} \dfrac{r^2}{\eta_0} & 0 \leq \theta \leq 45^\circ \\ 0 & 45^\circ \leq \theta \leq 90^\circ \\ \dfrac{r^2}{4\eta_0} & 90^\circ \leq \theta \leq 180^\circ \end{cases}$$

$$U_0 = \frac{r^2}{\eta_0}$$

$$P_r = \int W \cdot dA$$

$$P_r = \int_{\phi=0}^{2\pi} \int_0^{2\pi} U \cdot \sin\theta \, d\theta \, d\phi$$

$$= 2\pi \left[\int_0^{\pi/4} \left(\frac{r^2}{\eta_0}\right) \sin\theta \, d\theta + \int_{\pi/2}^{\pi} \left(\frac{r^2}{4\eta_0}\right) \sin\theta \, d\theta \right]$$

$$= 2\pi \left[\frac{r^2}{\eta_0} (-\cos\theta)_0^{\pi/4} + \frac{r^2}{4\eta_0} (-\cos\theta)_{\pi/2}^{\pi} \right]$$

$$= \frac{2\pi r^2}{\eta_0} \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{4}[1-0] \right)$$

$$\begin{array}{r} 1.250 \\ -0.707 \\ \hline 543 \end{array}$$

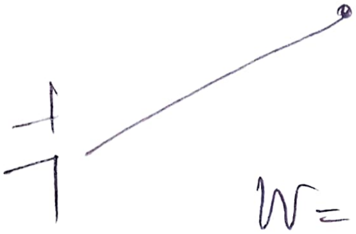
$$P_r = \frac{2\pi r^2}{\eta_0}(0.543)$$

$$U_{avg} = \frac{P_r}{4\pi} = \frac{\cancel{2\pi} r^2}{\eta_0} \times (0.543) \times \frac{1}{\cancel{4\pi}\,2}$$

$$\boxed{U_{avg} = \frac{0.271 r^2}{\eta_0}}$$

$$\text{Directivity} = \frac{U_0}{U_{avg}} = \frac{1}{0.271} = 3.69$$

$$= 5.67 \text{ dB}$$



$E_{rms} = 10$ V/m

$I_{rms} = 5$ A

$$W = \frac{|E_{rms}|^2}{\eta_0} = \frac{|10|^2}{\eta_0} = \frac{100}{\eta_0}$$

Power radiated $= \iint_S W \cdot ds$

$$P_r = \frac{2\pi}{\eta_0} \times (0.543) \times 100 \times (200)^2$$

$$= \frac{2\pi}{120\pi} \times (0.543) \times 100 \times (200)^2$$

$$P_r = 36,193 \text{ W}$$

$$P_r = I_{rms}^2 \cdot R_r$$

$$36,193 = (25) \cdot (R_r)$$

$$\underline{\underline{R_r = 1447.72\ \Omega}}$$

Problem:

Normalized radiation intensity of antenna is,

$U = \sin\theta \cdot \sin^3\phi$

The intensity exists only in $0 \le \theta \le \pi$, $0 \le \phi \le \pi$ region, and it is zero elsewhere. Find

(a) Exact directivity

(b) Azimuthal & elevation plane half-power Beamwidths.

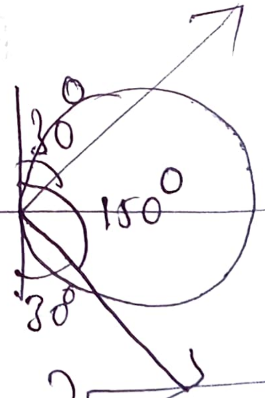
Sol:

$U_{avg} = \frac{P_r}{4\pi}$ $\quad D = \frac{U_0}{U_{avg}}$

$P_r = \int_\Omega U \, d\Omega$

$= \int_0^\pi \int_0^\pi \sin\theta \sin^3\phi \, d\theta \, d\phi$

$\boxed{P_r = 2\pi/3}$



$U = \sin\theta \sin^3\phi$

$\rightarrow \phi = \pi/2$

$U = \sin\theta$

$\frac{1}{2} = \sin\theta \rightarrow \boxed{\theta_{HP} = 30^\circ}$

$\boxed{\theta = 150^\circ}$ $\Big\}$ $\boxed{\theta_{HPBW} = 120^\circ}$

$\rightarrow \theta = \pi/2$

$U = \sin^3\phi$

$\frac{1}{2} = \sin^3\phi \rightarrow \sin\phi = \sqrt[3]{\frac{1}{2}} \rightarrow \phi = 52.53^\circ$

$\phi = 127.46^\circ$

$\boxed{\phi_{HPBW} = 74.94^\circ}$

$$\bar{E} + \mathring{\bar{A}} = -\nabla V$$

$$\nabla \cdot \bar{E} + \nabla \cdot \mathring{\bar{A}} = -\nabla \cdot \nabla V$$

$$\frac{\delta_v}{\varepsilon} + (-\mu \varepsilon \ddot{V}) = -\nabla^2 V$$

$$\boxed{\nabla^2 V - \mu \varepsilon \ddot{V} = -\frac{\delta_v}{\varepsilon}} \quad \text{— (2)}$$

from (1) & (2),

- $\nabla^2 \bar{A_p} + \omega^2 \mu \varepsilon \bar{A_p} = -\mu \vec{J_p}$
- $\nabla^2 V_p + \omega^2 \mu \varepsilon V_p = -\frac{\delta_{vp}}{\varepsilon}$

} Writing in terms of phasors.

$$\rightarrow \nabla^2 \bar{A_p} + \beta^2 \bar{A_p} = -\mu \vec{J_p} \quad (\beta = \omega\sqrt{\mu\varepsilon})$$

$$\nabla^2 G + \beta^2 G = \delta(r)$$

G → spatial Impulse response

$$\int_{\lim\limits_{r \to 0} r} \delta(r)\, dr = 1$$

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial G}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \cancel{\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial G}{\partial \theta}\right)}^{0} +$$

$$\frac{1}{r^2 \sin^2\phi} \cancel{\frac{\partial^2 G}{\partial \phi^2}}^{0} + \beta^2 G = \delta$$

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial G}{\partial r}\right) + \beta^2 G = \delta \qquad (\psi = rG)$$

$$\boxed{\frac{d^2 \psi}{dr^2} + \beta^2 \psi = \delta}$$

$$\rightarrow \boxed{\psi = C_1 e^{j\beta r} + C_2 e^{-j\beta r}}$$

$$\psi = \cancel{c_1 e^{j\beta r}} + c_2 e^{-j\beta r}$$

$$\psi = c e^{-j\beta r}$$

$$\frac{d^2 y}{dt^2} + ay = 0 \qquad m = \pm j\sqrt{a}$$

$$y(t) = c_1 e^{m_1 t} + c_2 e^{m_2(t)}$$

But, $(\psi = r G)$

$$c e^{-j\beta r} = r G$$

$$\boxed{G = \frac{c}{r} e^{-j\beta r}}$$

Substituting G in $\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dG}{dr}\right) + \beta^2 G = \delta$

$$\int_{\lim r \to 0} \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr}\left(\frac{c}{r} e^{-j\beta r}\right)\right) + \beta^2\left(\frac{c}{r} e^{-j\beta r}\right) = \int_{\lim r \to 0} \delta(v)\, d r$$

On solving, we get $\boxed{c = \frac{-1}{4\pi}}$

$$\boxed{\therefore G = \frac{-1}{4\pi r} e^{-j\beta r}} \rightarrow \text{Spatial Impulse Response.}$$

from, $\nabla^2 G + \beta^2 G = \delta(r)$

$$G \leftarrow \delta$$

$$\vec{A_p} \leftarrow -\mu \vec{J_p}$$

$$\underbrace{\overline{A_p}}_{y(t)} = \underbrace{G}_{r(t)} * \underbrace{-\mu \vec{J_p}}_{x(t)}$$

$$= \int G(\bar{r} - \bar{r}') \cdot \mu \bar{J}(r') \, dv'$$

$$y(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

$$\vec{A_p} = \int \frac{1}{4\pi|\bar{r}-\bar{r}'|} \cdot e^{-j\beta(\bar{r}-\bar{r}')} \cdot \mu \bar{J}(r')\, dv'$$

$$A_p e^{j\omega t} = \int_v \frac{1}{4\pi|\bar{r}-\bar{r}'|}\, e^{j\omega t} \cdot e^{-j\beta(\bar{r}-\bar{r}')} \cdot \mu \bar{J}(r')\, dv'$$

$$\boxed{A_p e^{j\omega t} = \int_v \frac{1}{4\pi|\bar{r}-\bar{r}'|} \cdot e^{j\omega\left(t-\frac{1}{c}(\bar{r}-\bar{r}')\right)} \cdot \mu \bar{J}(r')\, dv'}$$

$$\nabla \times \bar{A} = \bar{B}$$

$$\nabla \times \bar{H} = \bar{J_c} + \frac{\partial \vec{D}}{\partial t}$$

$$V(r) = \int_v \frac{\rho_v(r)}{4\pi|\bar{r}-\bar{r}'|\,\varepsilon}\, dv' \cdot e^{-j\beta|\bar{r}-\bar{r}'|}$$