Cont

$\vec{E} + \dot{\vec{A}} = -\nabla V$

$\nabla E + \nabla \cdot \dot{\vec{A}} = -\nabla \cdot \nabla V$

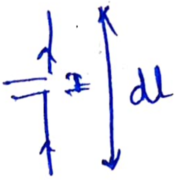
$\frac{\rho_v}{\varepsilon} + -\mu\varepsilon\ddot{V} = -\nabla^2 V$

$\nabla^2 V - \mu\varepsilon\ddot{V} = \frac{-\rho_v}{\varepsilon}$

$\boxed{\nabla^2 \vec{A} - \mu\varepsilon\ddot{\vec{A}} = -\mu\vec{J}}$ J, ρ

$\nabla \cdot \vec{A} = -\mu\varepsilon\dot{V}$

$\boxed{\nabla^2 V - \mu\varepsilon\ddot{V} = -\frac{\rho_v}{\varepsilon}}$



Converting into phasor domain

$\nabla^2 \bar{A}_p + \omega^2\mu\varepsilon\bar{A}_p = -\mu\bar{J}_p$ $\vec{A} = fn(\vec{J})$

$\nabla^2 V_p + \omega^2\mu\varepsilon V_p = -\frac{\rho_{vp}}{\varepsilon}$

$\nabla^2\bar{A}_p + \beta^2\bar{A}_p = -\mu\bar{J}_p$ $\beta = \omega\sqrt{\mu\varepsilon}$

(r, θ, ϕ)

$\boxed{\nabla^2 G + \beta^2 G = \delta(r)}$ Green's equation

$G \rightarrow$ Spatial Impulse Response $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$\lim_{r \to 0} \int_V \delta(r)\, dv = 1$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial G}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial G}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 G}{\partial\phi^2}$

$+ \beta^2 G = \delta$

G is only $f(r)$ because $\delta(r)$

$\Rightarrow \frac{\partial G}{\partial\theta} = \frac{\partial^2 G}{\partial\phi^2} = 0$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dG}{dr}\right) + \beta^2 G = \delta \qquad \psi = rG$$

$$\frac{d^2\psi}{dr^2} + \beta^2\psi = \underset{\lim\, r\to 0}{\delta} \qquad \frac{d^2y}{dt^2} + ay = 0 \qquad \begin{array}{l} m^2 + a = 0 \\ m = \pm j\sqrt{a} \end{array}$$

$$\psi = c_1 e^{j\beta r} + c_2 e^{-j\beta r} \qquad y(t) = c_1 e^{m_1 t} + c_2 e^{m_2 t}$$

The only solution exist is (∵ $-r$ dir is not possible, r is increasing in spher cor)

$$\psi = ce^{-j\beta r}$$

$$\psi = G\cdot r$$

$$G = \frac{c}{r}\cdot e^{-j\beta r}$$

$$\underset{\lim\, r\to 0}{\int}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\left(\frac{c}{r}e^{-j\beta r}\right)\right) + \beta^2\cdot\frac{ce^{-j\beta r}}{r} = \underset{\lim\, r\to 0}{\int_v}\delta(v)\,dv$$

$$\boxed{c = \frac{-1}{4\pi}}$$

$$\boxed{G = \frac{-1}{4\pi r}e^{-j\beta r}} \rightarrow \text{Green's function}$$

G is response due to impulse function.

$$\vec{A_p} \leftarrow -\mu\vec{J_p}$$

$$\vec{A_p} = G * -\mu\vec{J_p}$$

$$= \int_{r'} G(\bar{r}-\bar{r}')\cdot\mu\bar{J}(r')\,dv'$$

$$\bar{A_p} = \int \frac{1}{4\pi|\bar{r}-\bar{r}'|}e^{-j\beta(\bar{r}-\bar{r}')}\cdot\mu\bar{J}(r')\,dv'$$

Converting to time domain $\quad I \quad (r', \theta', \phi') \qquad \bar{A}_p \; (r, \theta, \phi)$

$$A_p e^{j\omega t} = \int_V \frac{1}{4\pi |\bar{r} - \bar{r}'|} \underbrace{e^{j\omega t} e^{-j\beta |\bar{r} - \bar{r}'|}} \mu J(r') \, dv$$

$$e^{j\omega\left(t - \frac{\beta}{\omega} |\bar{r} - \bar{r}'|\right)}$$

$$e^{j\omega\left(t - \frac{1}{c} |(\bar{r} - \bar{r}')|\right)}$$

delay

$$A_p e^{j\omega t} = \int_V \frac{\mu \bar{J}(r')\, dv'}{4\pi (\bar{r} - \bar{r}')} e^{j\omega\left(t - \frac{1}{c}(\bar{r} - \bar{r}')\right)} \quad \text{(retarded potential}$$

The A_p at point r depends on the current distribution at the prev time instant at r'.

$$\nabla \times \vec{A} = \vec{B}$$

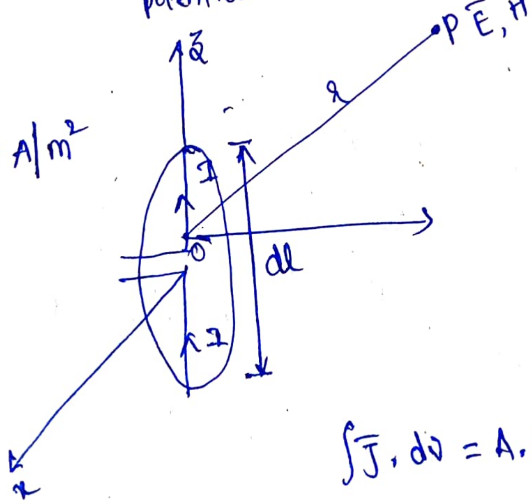
$$\nabla \times \vec{H} = J_c + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla^2 V - \mu \varepsilon V = \frac{-\rho_v}{\varepsilon}$$

$$V(r) = \int_V \frac{\rho_v(r')}{4\pi\varepsilon |\bar{r} - \bar{r}'|} dv' \, e^{-j\beta |\bar{r} - \bar{r}'|}$$

Scalar electric potential

$\vec{J} = A/m^2$



$\vec{A}, V$

$$\vec{A}(r) = \int \frac{\mu \bar{J}(r')}{4\pi |\bar{r} - \bar{r}'|} e^{-j\beta |\bar{r} - \bar{r}'|} dv'$$

$$= \int \frac{\mu J(0)}{4\pi r} e^{-j\beta r} dV$$

$$\int \bar{J} \cdot dv = A \cdot l$$

$$\int_V \vec{J} \cdot dv' = I \cdot \vec{dl}$$

$\underbrace{I \cdot \vec{dl}}_{\text{Current Element}}$

ρ_v & J_c are the sources of EM

P- independent of source, source free region

$$\bar{A} = \frac{\mu I \vec{dl}}{4\pi r} e^{-j\beta r} = \frac{\mu I dl}{4\pi r} e^{-j\beta r} \hat{a}_z$$

$$\vec{B} = \nabla \times \bar{A}$$

$$\vec{A}_r = A_z \cos\theta \cdot \hat{a}_r$$

$$\vec{A}_\theta = A_z \cos(90 - \theta) \hat{a}_\theta$$

$$\vec{A}_\phi = 0$$

$$\vec{B} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r.\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\ (0) \\ A_r & rA_\theta & r\sin\theta A_\phi \\ & & (0) \end{vmatrix}$$

$$\vec{B} = \hat{r}\underbrace{(0)}_{\text{zero}} + \hat{\theta}(0) + \frac{r\sin\theta\,\hat{\phi}}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]$$

$$\vec{B} = \frac{\mu I_o dl\, e^{-j\beta r}}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2}\right]\hat{\phi}$$

$$\vec{H} = \frac{I_o dl}{4\pi} e^{-j\beta r} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2}\right]\hat{\phi}$$

$$\nabla \times \vec{H} = \frac{\partial \bar{D}}{\partial t} \Rightarrow \nabla \times \bar{H}_p = j\omega\varepsilon \bar{E}_p$$

$$\boxed{\bar{E} = \frac{\nabla \times \vec{H}}{j\omega\varepsilon}}$$

$$\bar{E} = \frac{1}{j\omega\varepsilon}\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\,(=0) \\ 0 & 0 & rH_\phi \end{vmatrix}$$

$$\bar{E} = \frac{I_o\,dl\cos\theta}{4\pi\omega\varepsilon}e^{-j\beta r}\left\{\frac{\beta}{r^2} - \frac{j}{r^3}\right\}\hat{r}$$

$$+ \frac{I_o d\sin\theta}{4\pi\varepsilon}e^{-j\beta r}\left\{\frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right\}\hat{\theta}$$

$\beta = \omega\sqrt{\mu\varepsilon}$

$\frac{\beta^2}{\omega r} \Rightarrow \frac{\omega\mu\varepsilon}{r}$; $\propto$ freq

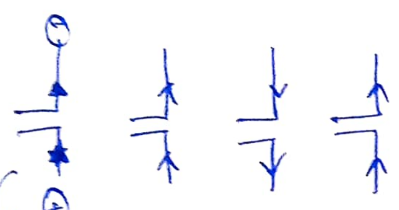
radiative field (far field)

$\frac{\beta}{\omega r} \Rightarrow$ independent of freq

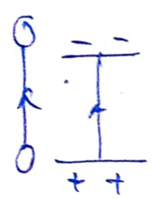
Due to EM Induction

$\frac{j}{\omega r^2} \Rightarrow$ Inversely $\propto$ to freq

Due to Electrostatic dipole nature



Dipole I changes having some induced current



In far field region;

$$\bar{E} \approx \frac{I_o\,dl\sin\theta}{4\pi\varepsilon r\omega}\,j\beta^2 e^{-j\beta r}\,\hat{a}_\theta$$

$$\bar{H} \approx \frac{I_o dl\sin\theta}{4\pi r}\,j\beta\,e^{-j\beta r}\,\hat{a}_\phi$$

In Near field region:

$$\frac{j\beta^2}{\omega r} = \frac{\beta}{\omega r^2}$$

$$r = \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

storing field (Inductive field)

$$r = \lambda/6$$

$\beta = \omega\sqrt{\mu\epsilon}$

$\frac{\beta^2}{\omega r} \Rightarrow \frac{\omega\mu\epsilon}{r}$ $\propto$ freq

$\frac{\beta}{\omega r} \Rightarrow$ independent of freq

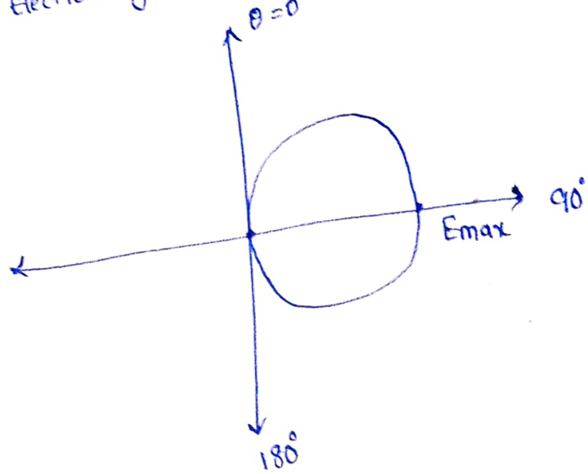
$\frac{j}{\omega r^2} \Rightarrow$ Inversely $\propto$ to freq

Induction

Far field reign

$\vec{E}_\theta$ $\vec{H}_\phi$ $\hat{r}$

TEM [Transverse Electromagnetic] wave



$$\frac{|E|}{|H|} = \sqrt{\frac{\mu}{\epsilon}} = \eta_0$$

$$\vec{H} = \frac{|E|}{\eta}$$

$$E_{max} = \frac{I_0\, dl\, \beta^2}{4\pi\epsilon\omega r}$$

$\vec{P}_{av} = ??$ $\qquad \bar{E} = \frac{E_{max}}{r} \cdot \sin\theta\, \hat{a}_\theta$

$$\vec{P}_{avg} = \frac{1}{2}\frac{|E|^2}{\eta_0} = \frac{1}{2\eta_0}\frac{\sin^2\theta}{r^2} E_{max}^2\, \hat{a}_r \quad W/m^2$$

$$W = \int \vec{P}_{avg} \cdot d\vec{S}$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{1}{2\eta_0}\frac{\sin^2\theta}{r^2} E_{max}^2 \cdot r^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{E_{max}^2}{2\eta_0} \cdot 2\pi \int_0^{\pi} \sin^3\theta\, d\theta$$

$$= \frac{E_{max}^2}{2\eta_0} \cdot 2\pi \cdot \frac{4}{3} \qquad (\eta_0 = 120\pi)$$

$$= \frac{E_{max}^2}{90} = \frac{(I_0\, dl)^2 \beta^4}{(4\pi\epsilon\omega)^2\, 90} \qquad \left\{ E_{max} = \frac{I_0\, dl\, \beta^2}{4\pi\epsilon\omega} \right\}$$

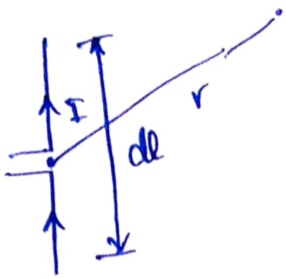
$$\boxed{W = 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2}$$

$$W = 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2$$

$$= \frac{1}{2} I_0^2 (R_r)$$

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

SUMMARY



$\bar{E}$,

$\bar{H}$,

$\vec{A} \propto \vec{J}$

$\vec{A} = \vec{J}$

$\hookrightarrow \vec{B}$

$\hookrightarrow \vec{E}$

far field

$$\bar{E} \cong \frac{I_0 dl \sin\theta}{4\pi\omega\varepsilon r} j\beta^2 e^{-j\beta r} \hat{a}_\theta$$

$$\bar{H} \cong \frac{I_0 dl \sin\theta}{4\pi r} j\beta e^{-j\beta r} \hat{a}_\phi$$

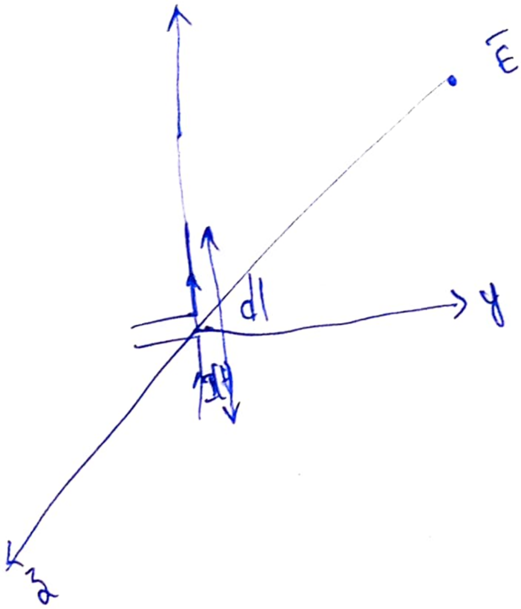
$$\bar{A} = \frac{\int \bar{J}\, dv\, e^{-j\beta r}}{4\pi r}$$

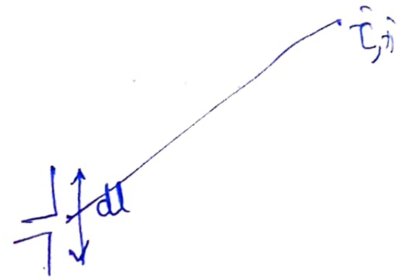
$$\frac{|\bar{E}|}{|\bar{H}|} = \eta_0$$

$$W = 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2 = \frac{1}{2} I_0^2 (R_r)$$

$$R_r = 80\pi^2 (dl/\lambda)^2$$

Hertzian dipole Antenna





Infinitesimally small dipole Antenna

$$\boxed{\begin{aligned} \nabla \cdot \vec{D} &= \rho_v \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \bar{B}}{\partial t} \\ \nabla \times \vec{H} &= \bar{J}_c + \bar{J}_D \\ \bar{J}_D &= \frac{\partial \bar{D}}{\partial t} \end{aligned}}$$

we assumed that in free space 0.

$\bar{B} = \mu \vec{H}$

$\bar{D} = \varepsilon \vec{E}$

$\nabla \cdot (\nabla \times \vec{A}) = 0$

$\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ = vector Magnetic potential

$$\nabla \times \vec{E} = -\dot{B}$$

$$= -(\nabla \times \dot{A}) = -(\nabla \times \dot{\vec{A}})$$

$$\nabla \times (\vec{E} + \dot{\vec{A}}) = 0.$$

$\vec{E} = \nabla C$

$\nabla \times (\nabla C) = 0$

$$\vec{E} + \dot{\vec{A}} = -\nabla V$$

Scalar

$$\nabla \times (-\nabla V) = 0$$

V = Scalar Electric potential

$$\vec{E} = -\nabla V - \dot{\vec{A}}$$

$$\nabla \times \vec{H} = J_c + \frac{\partial \vec{D}}{\partial t}$$

$$= \bar{J}_c + \varepsilon \dot{\bar{E}}$$

$$\nabla \times \frac{1}{\mu} \vec{B} = \frac{1}{\mu} \nabla \times (\nabla \times \bar{A})$$

$$= \bar{J}_c + \varepsilon \dot{\bar{E}}$$

$$\nabla \times \nabla \times \bar{A} = \mu \bar{J}_c + \mu\varepsilon (-\nabla \dot{V} - \ddot{\bar{A}})$$

$$\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 A$$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 A = \mu \bar{J}_c + \mu\varepsilon (-\nabla \dot{V} - \ddot{\bar{A}})$$

$$\nabla^2 \bar{A} - \mu\varepsilon \ddot{\bar{A}} = -\mu \bar{J}_c + \mu\varepsilon \nabla \dot{V} + \nabla(\nabla \cdot \bar{A})$$

$$\boxed{\nabla^2 \bar{A} - \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}} = -\mu \bar{J}_c + \mu\varepsilon \nabla \dot{V} + \nabla(\nabla \cdot \bar{A})$$

non-homoge wave equation

$$\bar{B} = \nabla \times \vec{A}$$

$$\nabla \cdot \vec{A} = ??$$

$$\mu\varepsilon \nabla \dot{V} + \nabla(\nabla \cdot \vec{A}) = 0 \rightarrow$$ Lorentz Gauge condition

$$\nabla(\underbrace{\mu\varepsilon \dot{V} + \nabla \cdot \vec{A}}) = 0$$

$$\mu\varepsilon \dot{V} + \nabla \cdot \vec{A} = 0$$

$$\nabla \cdot \vec{A} = -\mu\varepsilon \dot{V}$$

Cont