#INDUCTORS

Properties

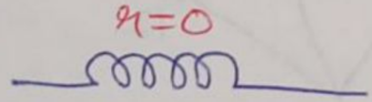
$$\boxed{V = L\frac{di}{dt}}$$

(1) For dc excitation, $\frac{di}{dt} = 0$

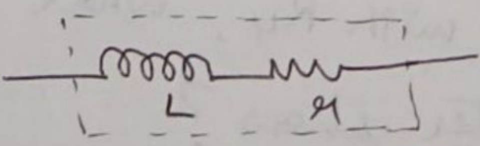
$\therefore \boxed{V = 0} \longrightarrow$ inductor is S.C. for ideal DC

(2) An inductor never allows sudden change in current through it.

(3) An ideal inductor is a coiled wire with zero internal resistance, so power dissipated is zero.



(4) Practical inductors will have small internal resistance and they are represented as coil shown below which allow some power losses.



(5) Inductors are available in different shapes and sizes and they are classified on the basis of type of core material on which winding is done.

(6) Inductors are used as filters, compensator etc in communication and power system.

Capacitors

Properties

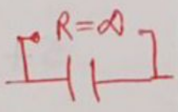
$$i = C \frac{dv}{dt}$$

(1) For dc excitation, $\frac{dv}{dt} = 0$, $i_c = 0$

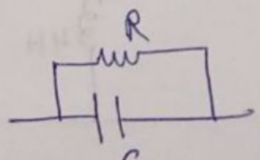
capacitor acts as open circuit for ideal D.C.

(2) capacitor never allows sudden change in voltage across it.

(3) Ideal capacitors are considered to have infinite dielectric resistance b/w electrodes so dielectric losses are zero but conduction is through polarization



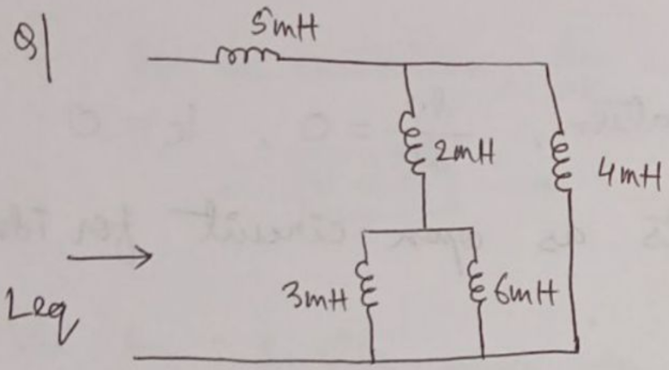
(4) Practical capacitors are considered to have very large dielectric resistance b/w electrodes so they undergo losses.



⑤ capacitors are available in different shapes and sizes and they are classified on the bases of dielectric material b/w the electrodes

⑥ They are used as filters, compensators, power factor correcting equipments, wave shaping etc in communication system

Q|



$3 \| 6 = 2\text{ mH}$

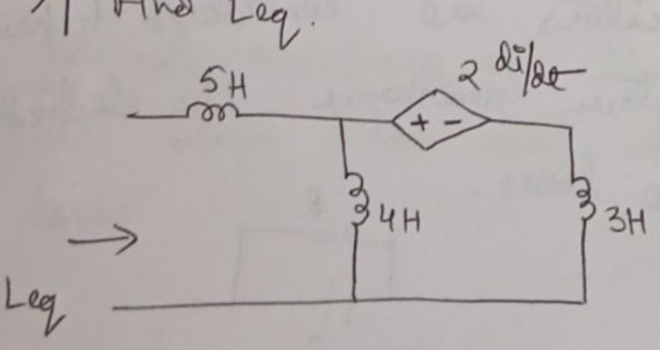
$2+2 = 4\text{mH}$

$4 \| 4 = 2\text{mH}$

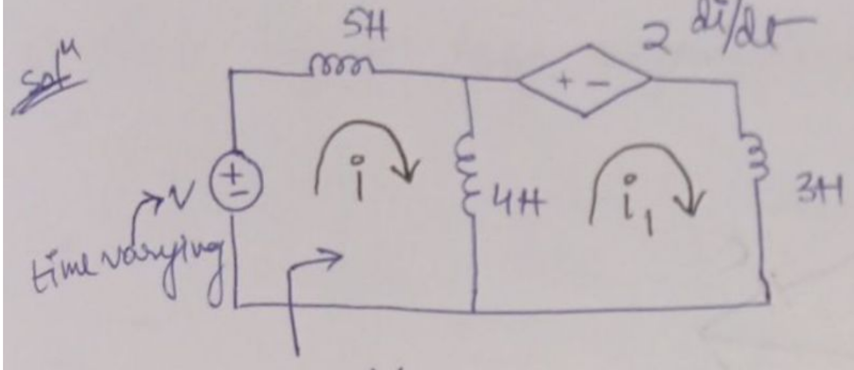
$5+2 = 7\text{ mH}$

$$L_{eq} = 5 + \left\{\left[2 + (3\|6)\right] \| 4\right\}$$
$$= 5 + \left[(2+2) \| 4\right]$$
$$= 5 + 2$$
$$= 7\text{ mH}$$

Q| Find Leq.



Sol^n



$$L_{eq} = \frac{V}{(di/dt)}$$

Mesh

$$-V + 5\frac{di}{dt} + 4\left(\frac{di}{dt} - \frac{di_1}{dt}\right) = 0$$

Let $\quad X = \dfrac{di}{dt}$

$\quad Y = \dfrac{di_1}{dt}$

$$\therefore \; 9X - 4Y = V \longrightarrow (1)$$

$$4\left(\frac{di_1}{dt} - \frac{di}{dt}\right) + 2\frac{di}{dt} + 3\frac{di_1}{dt} = 0$$

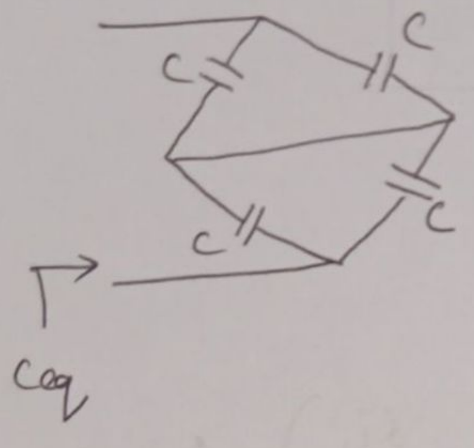
$$\Rightarrow \; -2X + 7Y = 0 \longrightarrow (2)$$

$\therefore$ solving (1) and (2),

$$V = \left(\frac{63-8}{7}\right)X = \frac{55}{7}\frac{di}{dt}$$

$$\therefore \; L_{eq} = \frac{55}{7}\text{ H}$$

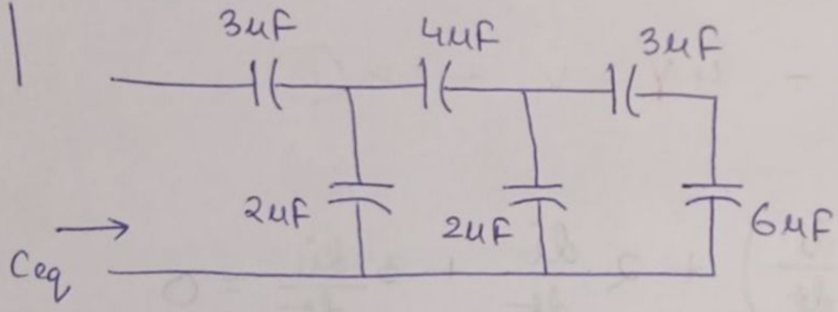
Q| Find Ceq



$$C_{eq} = 2C \parallel 2C$$

$$= \frac{4C^2}{4C} = C$$

Q|



3 series 6 = 2 uF

2 || 2 = 4 uF

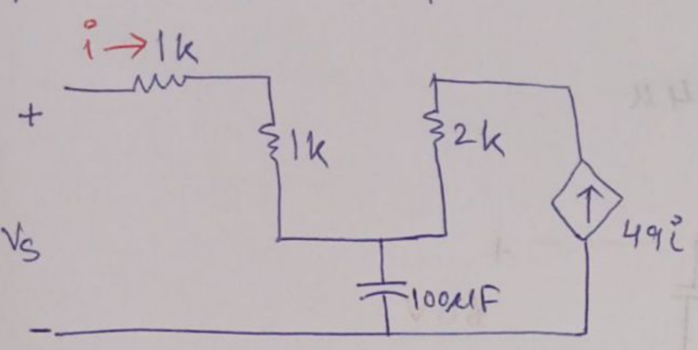
4 series 4 = 2 uF

2 || 2 = 4 uF

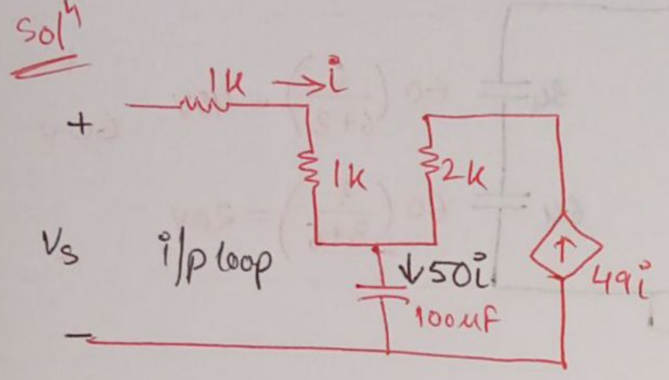
3 series 4 = $\frac{12}{7}$ uF

$\therefore C_{eq} = \frac{12}{7}$ uF

Q/ Find input loop capacitance



Sol^n



KVL

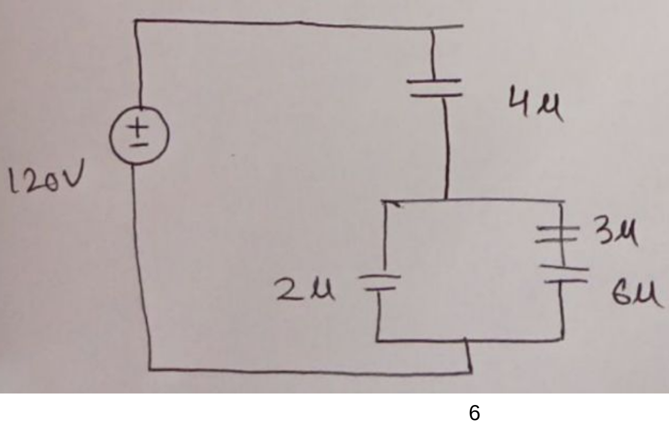
$V = \frac{1}{C}\int i\,dt$

$$-V_S + 2000\,i + \frac{1}{100\mu}\int 50i\,dt = 0$$

$$\Rightarrow V_S = 2000\,i + \frac{1}{\cancel{50\mu}\;2\mu}\int i\,dt$$

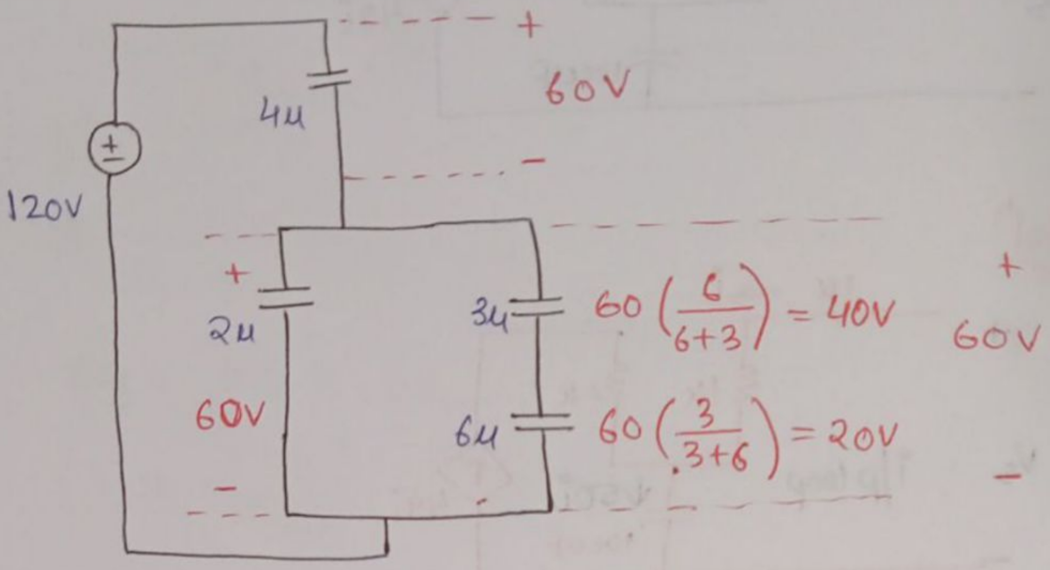
$\therefore C_{loop} = 2\mu F$, $R_{loop} = 2000\ \Omega$

Q/ Determine steady state voltage across each capacitor and energy stored in each. Also find charge.



$3 \text{ series } 6 = 2\mu$

$2 \parallel 2 = 4\mu$



<u>4uF</u>

$q_4 = CV = 4 \times 60 = 240\,\mu C$

$E_4 = \frac{1}{2} CV^2 = \frac{1}{2} \times 4 \times (60)^2 = 7200\,\mu J$

$q_2 = CV = 2 \times 60 = 120\,\mu C$

$E_2 = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times (60)^2 = 3600\,\mu J$

$q_3 = CV =$

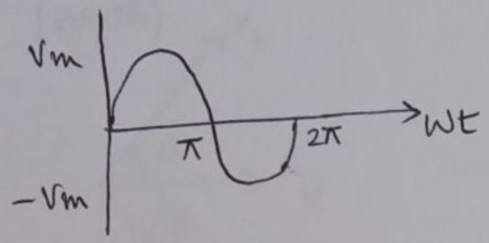
$E_3 = \frac{1}{2} CV^2 =$

$q_6 = CV =$

$E_6 = \frac{1}{2} CV^2 =$

Unit 5 — Steady State Analysis of Circuits for Sinusoidal Excitations

→ Radian



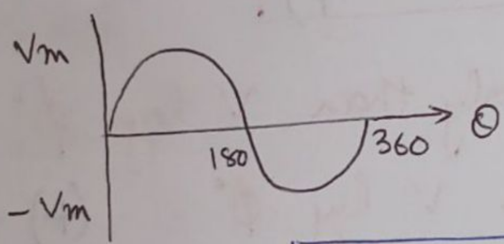
$V_m \rightarrow$ amplitude

$\omega \rightarrow$ angular frequency (radian / sec)

$$\boxed{V = V_m \sin \omega t}$$

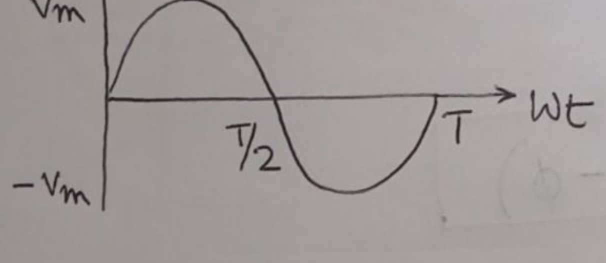
$$\omega = 2\pi f = \frac{2\pi}{T}$$

→ Degree



$$\boxed{V = V_m \sin \theta}$$

→ Time



$$V = V_m \sin\left(\frac{2\pi}{T} \cdot t\right)$$

India

Power frequency = 50 Hz $\Rightarrow T = \frac{1}{50} = 20ms$

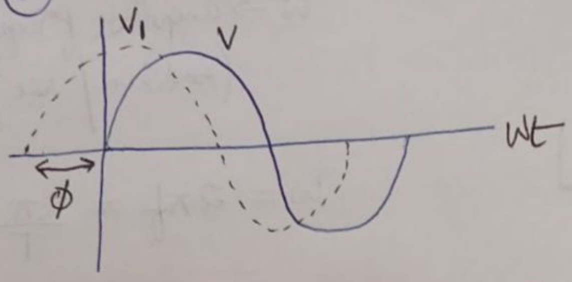
$$\boxed{1T = 2\pi = 360^\circ = 20ms}$$

Standard sine wave

$$\boxed{V_x = V_m \sin(\omega t + \phi)}$$

$\phi \rightarrow$ phase shift (degree)

(1)

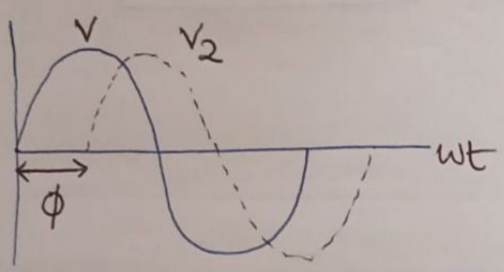


$$\boxed{V_1 = V_m \sin(\omega t + \phi)}$$

V_1 comes early than V by $\phi°$.

$\therefore V_1$ leads V by $\phi°$. $(+) \rightarrow$ leading

(2)



$$\boxed{V_2 = V_m \sin(\omega t - \phi)}$$

V_2 lags V by $\phi°$.

$(-) \rightarrow$ lagging

India

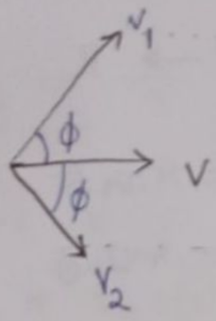
$$\boxed{\phi = 60°}$$

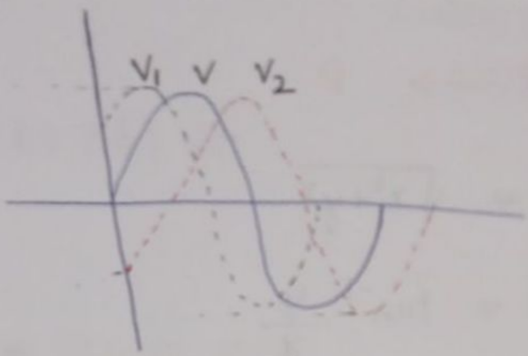
$360° = 20$ msec (Already we know)

$60° = t_{shift}$

$\therefore t_{shift} = \frac{60}{360} \times 20ms = 3.33$ ms

$$\boxed{\omega = 2\pi f = 2\pi(50) = 100\pi \text{ rad/sec}}$$



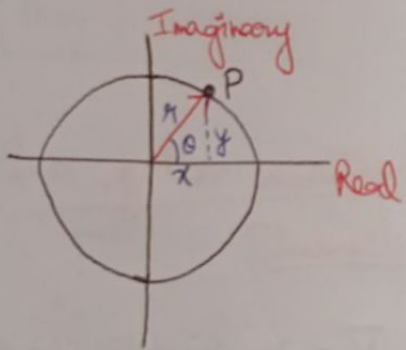


$V = |V_m| \angle 0°$

$V_1 = |V_m| \angle \phi°$

$V_2 = |V_m| \angle -\phi°$

S plane



$\boxed{P}$ rectangular form

$P = x + iy$

$\boxed{P}$ polar form

$P = r \angle \theta$

$\boxed{P}$ Euler's form

$P = re^{j\theta}$

Rectangular	→	Polar
$0 + j0$	→	$0 \angle 0°$
$1 + j0$	→	$1 \angle 0°$
$0 + j1$	→	$1 \angle 90°$
$1 + j1$	→	$\sqrt{2} \angle 45°$
$0 - j1$	→	$1 \angle -90°$

$j \rightarrow$ powerful operator

$+j \rightarrow$ lead

$-j \rightarrow$ lag

Rectangular coordinates $\rightarrow x, y$

Polar coordinates $\rightarrow r, \theta$

$R \longrightarrow P$

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$P \longrightarrow R$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Ex $\rightarrow$ $1+j0$ to polar

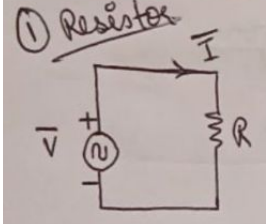
$$r = \sqrt{1^2+0^2} = 1$$

$$\theta = \tan^{-1} \frac{0}{1} = 0$$

$$\therefore \; 1+j0 = 1\angle 0^\circ$$

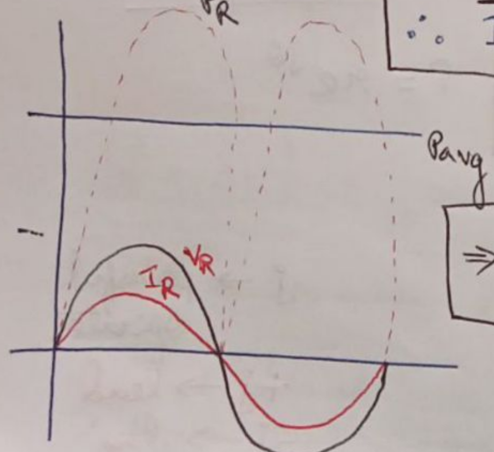
Phasor relationship between voltages and current in passive elements

① Resistor



let $\overline{V} = V_m \sin \omega t \longrightarrow ①$

$$V = IR$$



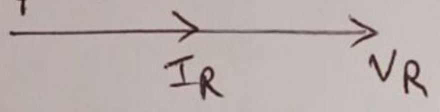
$$\therefore \; \overline{I} = \frac{\overline{V}}{R} = \frac{V_m \sin \omega t}{R}$$

$$= \frac{V_m}{R} \sin \omega t$$

$$\Rightarrow \overline{I} = I_m \sin \omega t \longrightarrow ②$$

Phasor diagram

$\phi = 0^\circ$



Power factor

$$\cos\phi = \cos 0^\circ = 1$$

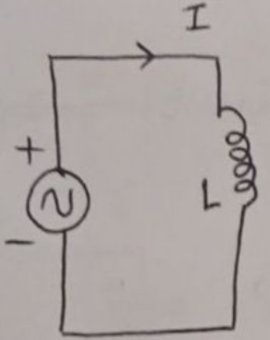
(unity power factor)

$$P_{avg} = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$$

$$\boxed{P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}}$$

$$\boxed{P_{avg} = V_{rms} \cdot I_{rms}}$$

② <u>Inductor</u>



Let $\overline{I} = I_m \sin \omega t \longrightarrow (1)$

Now $V = L \frac{dI}{dt}$

$\therefore V = L \frac{d}{dt} (I_m \sin \omega t)$

$= L \omega I_m \cos \omega t$

$= \omega L I_m \sin(\omega t + 90^\circ)$

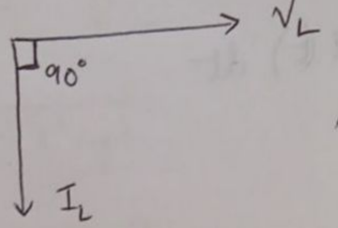
$= \omega L I_m \sin(\omega t) [j]$

$\therefore$ $\overline{V} = j\omega L \overline{I}$ $\longrightarrow$ voltage is j operator times the current

$\overline{V} = +j X_L \overline{I}$ $\longrightarrow$ V leads I by 90°

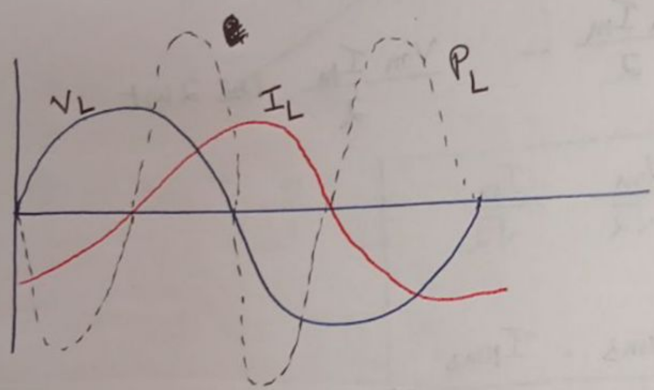
where $X_L = WL = 2\pi f L$
↳ inductive reactance

Phasor diagram

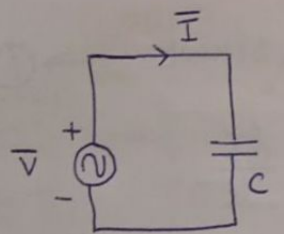


voltage leads I by 90°

power factor $= \cos\phi = \cos 90° = 0$



③ capacitor



Let $\bar{V} = V_m \sin Wt \longrightarrow ①$

$$I = C\frac{dv}{dt}$$

$$\Rightarrow \bar{I} = C\frac{dv}{dt}(V_m \sin Wt)$$

$$= C.W.V_m \cos Wt$$

$$= WCV_m \sin(Wt + 90°) \rightarrow ②$$

$$= WCV_m \sin Wt\,[j]$$

$$\bar{I} = jWC\bar{V}$$

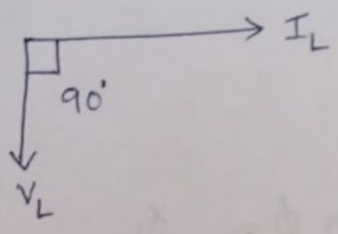
$$\bar{V} = \frac{\bar{I}}{jWC} = \frac{-j}{WC}\bar{I}$$

$$\boxed{\overline{V} = -jX_C\overline{I}} \longrightarrow \text{voltage lags } I \text{ by } 90^\circ$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

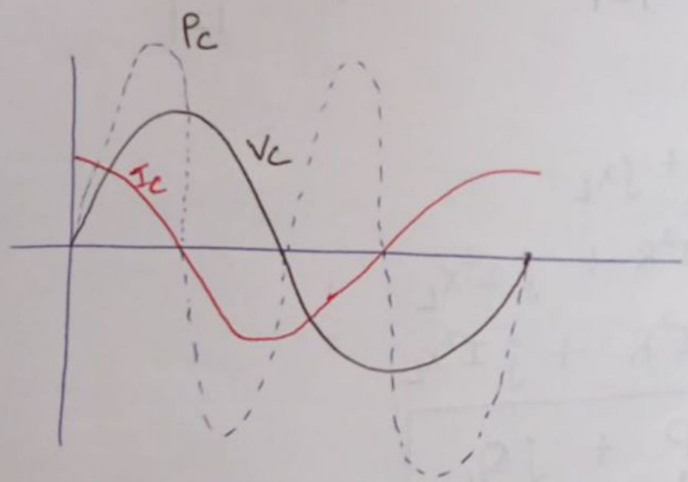
↳ capacitive reactance

Phasor diagram

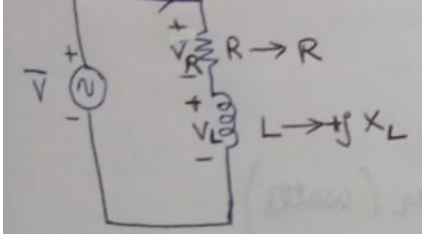


Power factor

$\cos\phi = \cos 90^\circ = 0$



series R–L



KVL

$$-\overline{V} + \overline{V}_R + \overline{V}_L = 0$$

$$\Rightarrow \overline{V} = IR + jX_L I$$

$$\Rightarrow \overline{V} = I(R + jX_L)$$

$$\Rightarrow \boxed{\overline{V} = IZ}$$

$$\boxed{\text{Impedance } Z\ (\Omega) = R + jX_L}$$

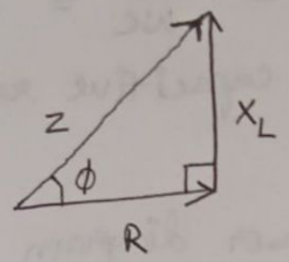
Impedance triangle

$$Z = R + jX_L$$

$$|z| = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \tan\left(\frac{\omega L}{R}\right)$$



Power factor

$$\cos\phi = \frac{R}{|z|} \qquad \sin\phi = \frac{X_L}{|z|}$$

→

$$Z = R + jX_L$$

$$I^2 Z = I^2 R + j I^2 X_L$$

$$VI = I^2 R + j I^2 X_L$$

$$\boxed{S = P + jQ_L}$$

S → Total | Apparent power (VA) (associated with source)

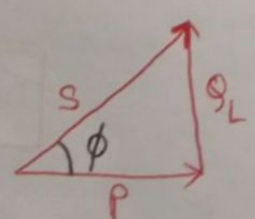
P → Active | Real | True power (watts) (consumable part of energy)

Q_L → Reactive power (VAR) (flux, charge)

Power triangle

$$|S| = \sqrt{P^2 + Q_L^2}$$

$$\phi = \tan^{-1}\left(\frac{Q_L}{P}\right)$$



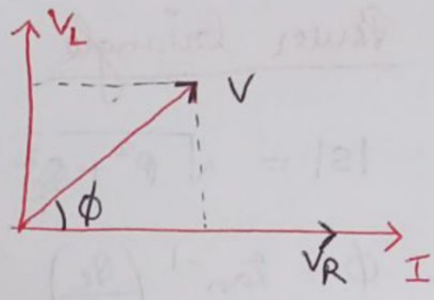
$\cos\phi = \frac{P}{S} \Rightarrow P = \cos\phi \times S = VI\cos\phi$ Watts

$\sin\phi = \frac{Q_L}{S} \Rightarrow Q_L = S\sin\phi = VI\sin\phi$ VAR's

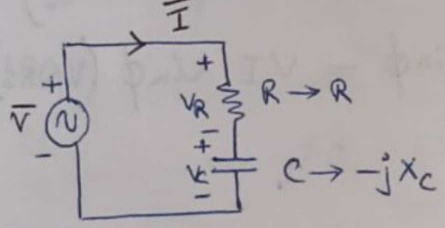
Here $V, I \longrightarrow$ rms value

Phasor diagram

I lags V by $\phi < 90^\circ$
$\downarrow$
less than



Series RC



KVL

$$-\bar{V} + \bar{V}_R + \bar{V}_C = 0$$

$$\Rightarrow \bar{V} = IR - jX_C I$$

$$\Rightarrow \bar{V} = \bar{I}(R - jX_C)$$

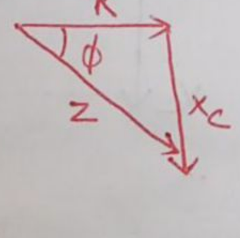
$$\Rightarrow \boxed{\bar{V} = IZ}$$

$$\boxed{Z = R - jX_C}$$

Impedance triangle

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{wRC}\right)$$



Power factor

$$\cos\phi = \frac{R}{|Z|} \qquad \sin\phi = \frac{X_C}{|Z|}$$

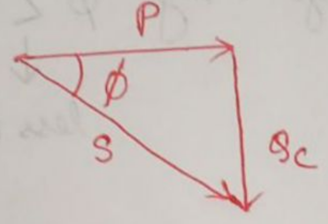
$$IZ^2 = I^2R - jX_C I^2$$

$$VI = I^2R - jI^2X_C$$

$$\boxed{S = P - jQ_C}$$

$\longrightarrow$ reactive power (VAR's)

Power triangle



$$|S| = \sqrt{P^2 + Q_C^2}$$

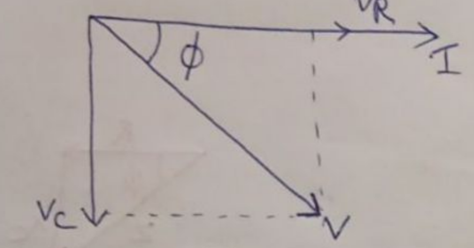
$$\phi = \tan^{-1}\left(\frac{Q_C}{P}\right)$$

$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = VI\cos\phi \text{ (watts)}$$

$$\sin\phi = \frac{Q_C}{S} \Rightarrow Q_C = S\sin\phi = VI\sin\phi \text{ (VAR's)}$$

$V, I \longrightarrow$ rms value

Phasor diagram

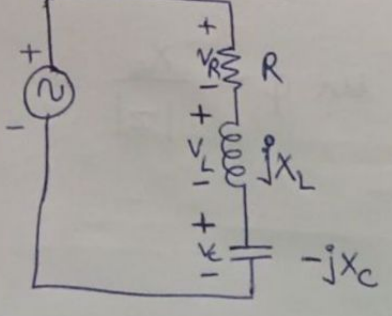


I leads V by $\phi < 90^\circ$

$\downarrow$

less than

RLC Circuit (Series)



KVL

$$-\bar{V} + \bar{V}_R + \bar{V}_L + \bar{V}_C = 0$$

$$\Rightarrow \bar{V} = IR + jX_L I - jX_C I$$

$$\Rightarrow \bar{V} = I[R + j(X_L - X_C)]$$

$V = IZ$

Here, $\boxed{Z = R + j\underbrace{(X_L - X_C)}_{}}$ → X_{net} → net reactance

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

net impedance angle

Power factor

$$\cos\phi = \frac{R}{|Z|} \qquad \sin\phi = \frac{|X_L - X_C|}{|Z|}$$

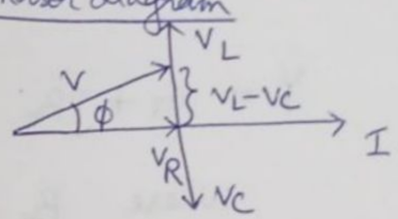
Case 1 : If $X_L > X_C$ → General nature of electrical system

$\boxed{Z = R + jX_{net}}$

↓

series RL circuit

Phasor diagram



I lags V by $\phi < 90°$ (lagging power factor)

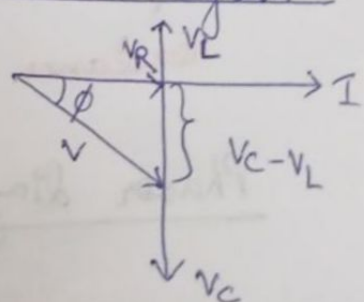
Case 2 : If $X_L < X_C$

$\boxed{Z = R - jX_{net}}$

↓

Series RC circuit

Phasor Diagram



I leads V by $\phi < 90°$ (leading power factor)

Case 3: If $X_L = X_C$

$$\boxed{Z = R} \longrightarrow \text{purely resistive}$$

I in phase with V $\Rightarrow \phi = 0^\circ$, PF = 1
unity power factor

$$\boxed{Z = R \pm jX}$$

$\rightarrow Z = R + jX_L$

$\rightarrow Z = R - jX_C$

$$\boxed{Y = G \pm jB}$$

Y = admittance $= \frac{1}{Z}$

G = conductance $= \frac{1}{R}$

B = susceptance $= \frac{1}{X}$

$Y = G - jB_L$

where $B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L}$

$\rightarrow$ inductive susceptance ($\mho$)

$Y = G + jB_C$

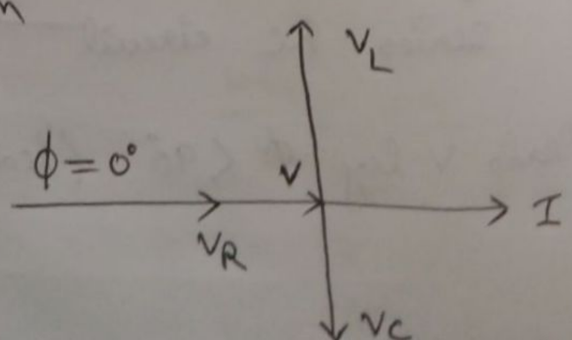
where $B_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C = 2\pi f C$

$\rightarrow$ capacitive susceptance ($\mho$)

impedance $\longrightarrow$ admittance

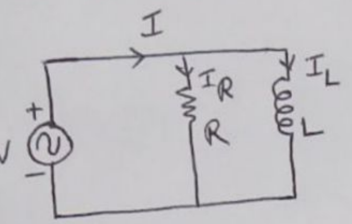
reactance $\longrightarrow$ susceptance

Phasor diagram



PARALLEL CIRCUITS → V — reference

→ Parallel RL



$$I_R = \frac{V}{R} \angle 0^\circ$$

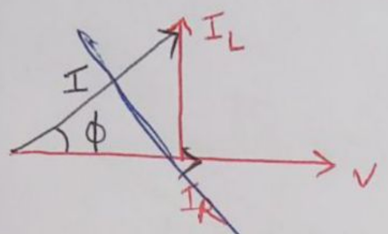
$$I_L = \frac{V}{jX_L} = \frac{V}{X_L} \angle -90^\circ$$

$$I = I_R + I_L$$

$$|I| = \sqrt{I_R^2 + I_L^2}$$

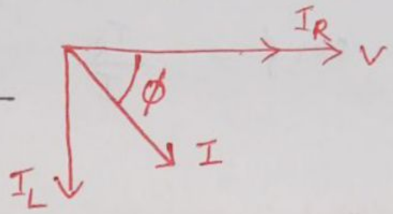
$$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$

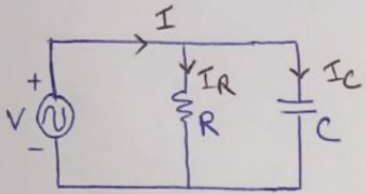


~~V lags I by $\phi < 90^\circ$~~

V leads current by $\phi < 90^\circ$



→ Parallel RC



$$I_R = \frac{V}{R} \angle 0^\circ$$

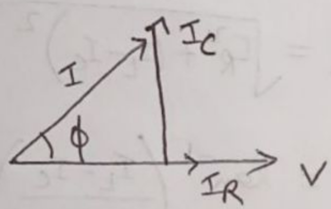
$$I_C = \frac{V}{-jX_C} = \frac{V}{X_C} \angle 90^\circ$$

$$I = I_R + I_C$$

$$|I| = \sqrt{I_R^2 + I_C^2}$$

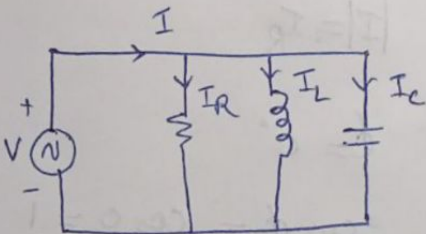
$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$



V lags I by $\phi < 90^\circ$

→ Parallel RLC

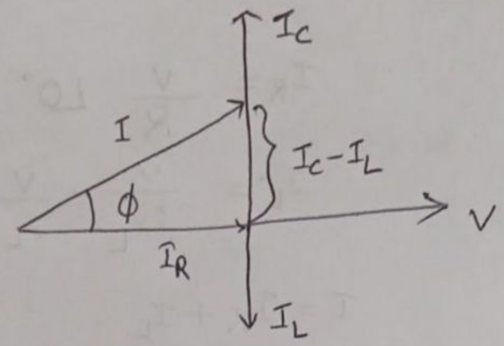


$$I_R = \frac{V}{R} \angle 0^\circ$$

$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

Case 1 : $X_L > X_C \rightarrow \cancel{I_L} \quad I_L < I_C$

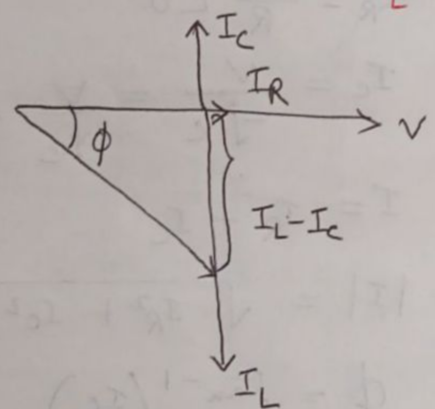


$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$

Case 2 : $X_L < X_C \Rightarrow I_L > I_C$

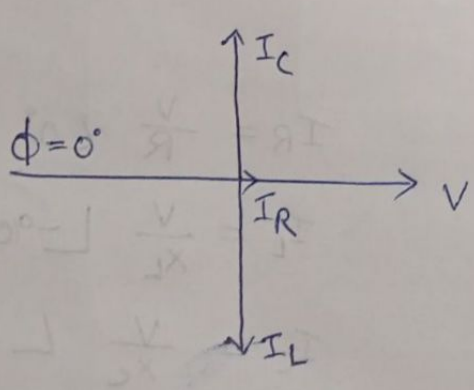


$$|I| = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$

Case 3 : $X_L = X_C \Rightarrow I_L = I_C$



$$|I| = I_R$$

$$\phi = 0^\circ$$

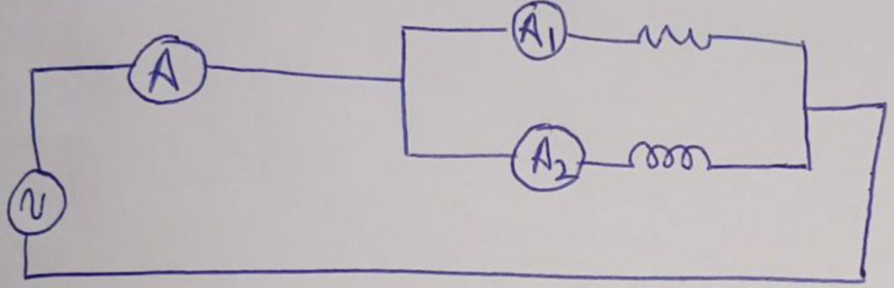
$$\cos\phi = \cos 0 = 1$$

Unity power factor.

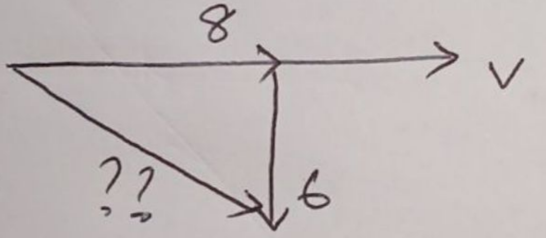
Q| If A_1 reads 8A

A_2 reads 6A

A reads ?? Also find pF.



Sol^n



$$I = \sqrt{I_R^2 + I_L^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

$$P.F. = \frac{I_R}{I} = \frac{8}{10} = 0.8 \text{ (lagging)}$$

RMS value | True | Effective value

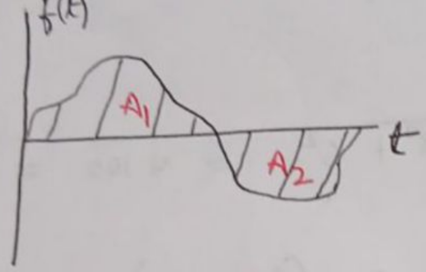
It is that steady value of a time varying voltage or current waveform which could produce the same value of heat as given by the original waveform for definite period of time.

$$\boxed{V_{rms} = \sqrt{\frac{1}{T}\int_0^T [V(t)]^2 \, dt}}$$

Average value | Mean value

It is that steady value of time varying voltage or current waveform which could develop same amount of charge as given by the original waveform for a definite period of time in a circuit.

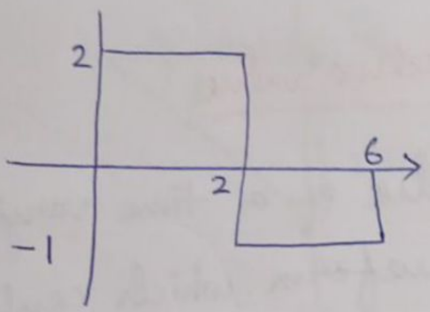
Concept of Symmetry



$|A_1| = |A_2| \Rightarrow$ Symmetrical

$|A_1| \neq |A_2| \Rightarrow$ Asymmetrical

Ex



+ve area $= |2 \times 2| = 4$

-ve area $= |4 \times (-1)| = 4$

$\therefore$ Symmetrical

NOTE

Average value of any symmetrical waveform for one full cycle is always zero.

1. For symmetrical waveform

$$V_{avg} = \begin{cases} 0 \longrightarrow \text{full cycle} \\ \frac{1}{(T/2)} \int_0^{T/2} v(t)\,dt \longrightarrow \text{half cycle} \end{cases}$$

2. For asymmetrical waveform

$$V_{avg} = \frac{1}{T}\int_0^T V(t)\,dt \longrightarrow \text{Full cycle}$$

⇒ Peak factor / Crest factor

$$\frac{V_{max}}{V_{rms}}$$

⇒ Form factor / Shape factor $= \dfrac{V_{rms}}{V_{avg}}$

⇒ Peak to peak value $V_{P-P} = |V_{max} - V_{min}|$

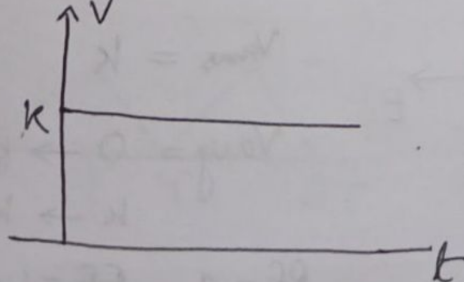
NOTE

Most of our electrical applications involves heat generation so we talk RMS values in general.

Ex → 1 ϕ, domestic supply in India
= 230V ⟶ RMS value

However applications like battery charging electroplating, electrorefining etc involves charge so we calculate average values.

STANDARD WAVEFORMS

1. 

$V_{rms} = K$

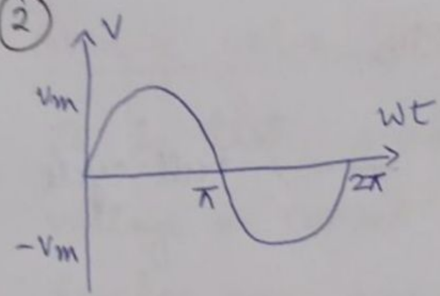
$V_{avg} = K$

P.F. = 1

F.F = 1

$V_{PP} = K$

(2)



$V_{rms} = \frac{V_m}{\sqrt{2}}$

$V_{avg} = 0 \longrightarrow$ full cycle

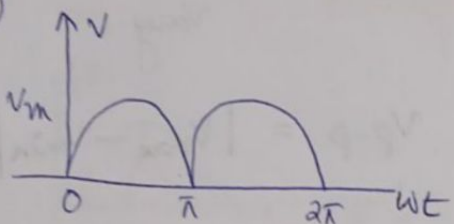
$= \frac{2V_m}{\pi} \longrightarrow$ half cycle

$PF = \sqrt{2}$

$FF = \frac{\pi}{2\sqrt{2}} = 1.11$

$V_{pp} = 2V_m$

(3)



$V_{rms} = \frac{V_m}{\sqrt{2}}$

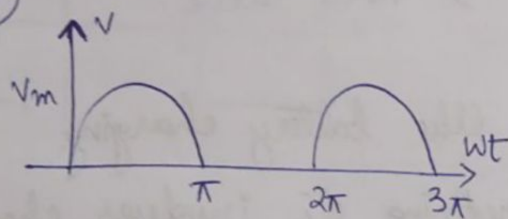
$V_{avg} = \frac{2V_m}{\pi}$

$PF = \sqrt{2}$

$FF = 1.11$

$V_{pp} = V_m$

(4)



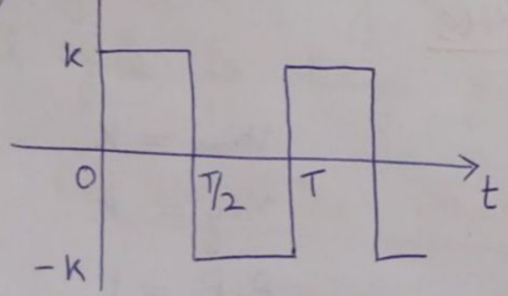
$V_{rms} = \frac{V_m}{2}$

$V_{avg} = \frac{V_m}{\pi}$

$PF = 2 \quad FF = 1.57$

$V_{pp} = V_m$

(5)



$V_{rms} = k$

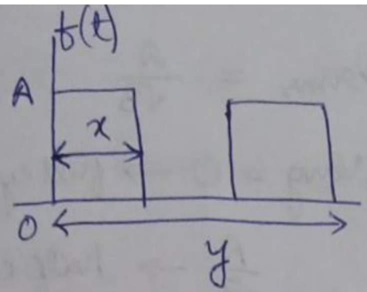
$V_{avg} = 0 \rightarrow$ full cycle

$k \rightarrow$ half cycle

$PF = 1 \quad FF = 1.57$

$V_{pp} = V_m$

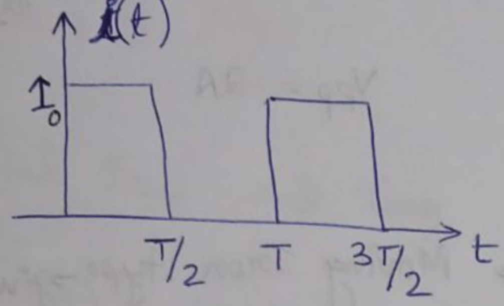
(6)



$$f_{rms} = A\sqrt{\frac{x}{y}}$$

$$f_{avg} = A\left(\frac{x}{y}\right)$$

(7)



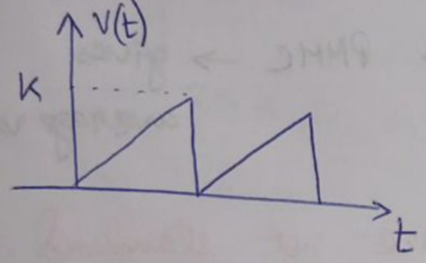
$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$I_{avg} = \frac{I_0}{2}$$

$PF = \sqrt{2}$ $\quad FF = \sqrt{2}$

$I_{PP} = I_0$

(8)

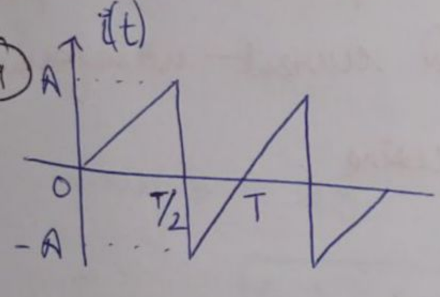


$$V_{rms} = \frac{k}{\sqrt{3}}$$

$$V_{avg} = \frac{k}{2}$$

$PF = \sqrt{3}$ $\quad V_{PP} = k$

(9)



$$I_{rms} = \frac{A}{\sqrt{3}}$$

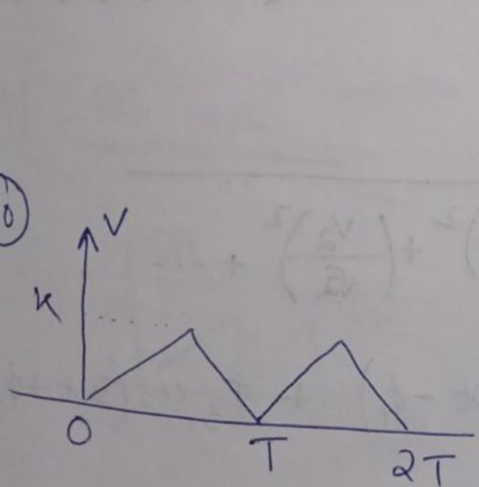
$I_{avg} = 0 \rightarrow$ full cycle

$= \frac{A}{2} \rightarrow$ half cycle

$PF = \sqrt{3}$ $\quad FF = \frac{2}{\sqrt{3}}$

$I_{PP} = 2A$

(10)



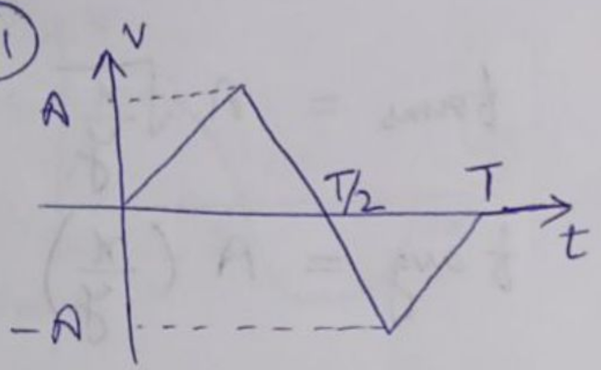
$$V_{rms} = \frac{k}{\sqrt{3}}$$

$$V_{avg} = \frac{k}{2}$$

$PF = \sqrt{3}$ $\quad FF = \frac{2}{\sqrt{3}}$

$V_{PP} = k$

(11)



$V_{rms} = \frac{A}{\sqrt{3}}$

$V_{avg} = 0 \rightarrow$ full cycle

$\frac{A}{2} \rightarrow$ half cycle

$PF = \sqrt{3}$ $FF = \frac{2}{\sqrt{3}}$

$V_{PP} = 2A$

NOTE

* AC Analog meters $\longrightarrow$ Moving Iron type $\rightarrow$ gives RMS values

* DC analog meters $\rightarrow$ PMMC $\rightarrow$ gives average value

* Practical waveforms are not standard so we use Fourier series expansion to express the practical voltages and current waveforms in terms of Sine or cosine

* Practical waveforms are not standard so we use Fourier series expansion to express the practical voltages and current waveforms in terms of sine or cosine.

Ex : $v(t) = V_0 + V_1 \sin \omega t + V_2 \sin 2\omega t + \dots$

$\therefore$ $V_{avg} = V_0$

$$V_{rms} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \dots}$$

$i(t) = I_0 + I_1 \cos(\omega t - \phi_1) + I_3 \cos(3\omega t + \phi_3)$

$\therefore$ $I_{avg} = I_0$

$$I_{rms} = \sqrt{I_0^2 + \left(I_1/\sqrt{2}\right)^2 + \left(I_3/\sqrt{2}\right)^2}$$

** NOTE : If frequency are same, dont use Fourier series concept.

$$v(t) = V_1 \sin(\omega t + \phi) + V_2 \sin(\omega t - \phi)$$
$$= V_1 \angle \phi + V_2 \angle -\phi$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

Q| Find I_{rms} and I_{avg}

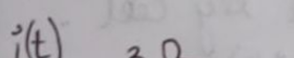
$$i(t) = 20 + 10 \sin(\underline{\omega t} - 30^\circ) + 7 \cos(\underline{\omega t} + 40^\circ)$$

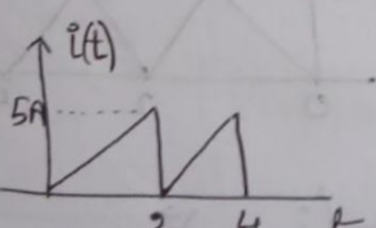
frequency same

$$= 20 + 10 \sin(\omega t - 30^\circ) + 7 \sin(\omega t + 130^\circ)$$
$$= 20 + (10 \angle -30^\circ) + (7 \angle +130^\circ)$$
$$= 20 + 4.16 + 0.36j$$
$$= 20 + [4.17 \angle 4.94^\circ]$$

$$I_{avg} = 20$$

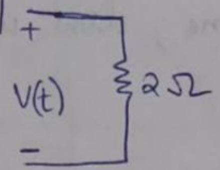
$$I_{rms} = \sqrt{(20)^2 + \left(\frac{4.17}{\sqrt{2}}\right)^2} = 20.21$$

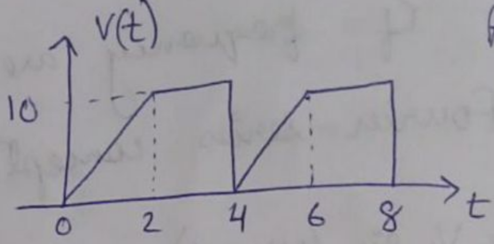
Q|  Find P_{lost}



$$P_{lost} = I_{rms}^2 \times R$$
$$= \left(\frac{5}{\sqrt{3}}\right)^2 \times 3$$
$$= 25\,W$$

Q|



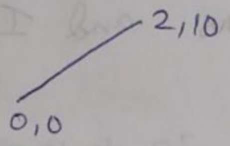


Find P_{lost} for $t = 4$

$$P_{lost} = \frac{V_{rms}^2}{R}$$

$0 < t < 2$

$$v(t) = 5t$$



$$\frac{10-0}{2-0} = 5$$

$2 < t < 4$

$$v(t) = 10$$

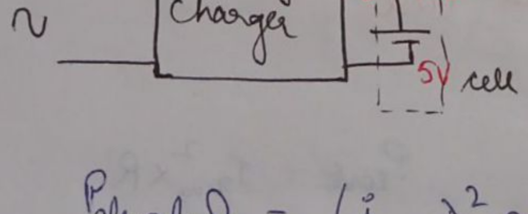
$$V_{rms} = \sqrt{\frac{1}{T}\left[\int_0^2 v(t)^2\,dt + \int_2^4 v(t)^2\,dt\right]}$$

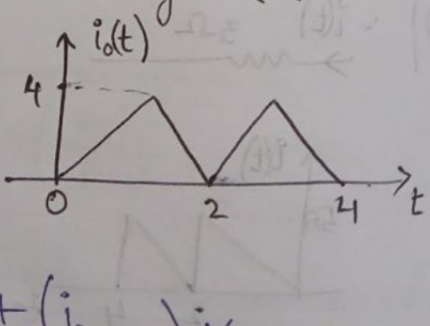
$$= \sqrt{\frac{1}{4}\int_0^2 (5t)^2\,dt + \int_2^4 (10)^2\,dt}$$

$$= \sqrt{66.66}$$

$$P_{lost} = \frac{(V_{rms})^2}{R} = \frac{66.66}{2} = 33.33\,W$$

Q| Find total power absorbed by cell.





$$P_{absorbed} = (i_{o_{rms}})^2 R + (i_{o_{avg}})\,V$$

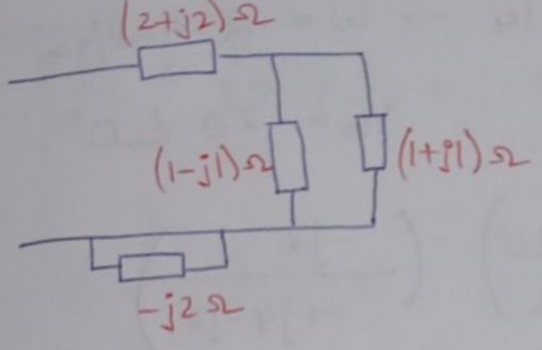
Now, from standard waveforms, $i_{o\,rms} = \frac{4}{\sqrt{3}}$

$i_{o\,avg} = \frac{4}{2} = 2$

$$P_{abs} = \left(\frac{4}{\sqrt{3}}\right)^2 \times 1 + 2\times 5$$

$$= \frac{16}{3} + 10 = 15.33W$$

Q) find Zeq.



$$Zeq = (2+j2) + \left[(1-j1) \parallel (1+j1)\right]$$

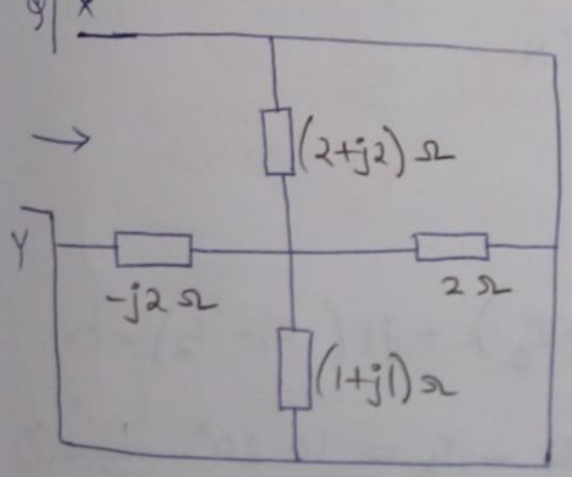
$$= (2+j2) + \frac{(1-j1)(1+j1)}{(1+j1)+(1-j1)}$$

$$= (2+2j) + \frac{1+\cancel{j1}-\cancel{j1}-j^2 1}{1+\cancel{j1}+1-\cancel{j1}}$$

$$= (2+2j) + \frac{2}{2}$$

$$Zeq = 3+j2$$

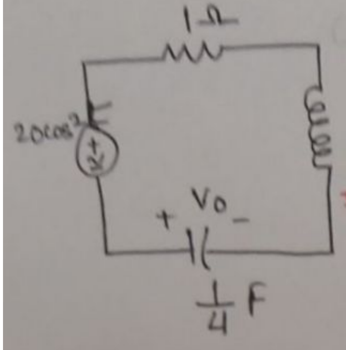
Q)



Zeq ?

$$Zeq = 0$$

Q| find V_0.



$2H = +j\omega L = +j\,2\times 2 = j4$

Here,
$V = 20\cos 2t$
$\therefore\ \omega = 2$ rad/sec

$V_m = 20\angle 0^\circ$

$\frac{1}{4}F = -j/\omega C = -j/2\times\frac{1}{4} = -j2$

$$\therefore\ V_0 = -\left(\frac{20\angle 0^\circ}{\sqrt{2}}\right)\left(\frac{-j2}{1+j4-j2}\right)$$

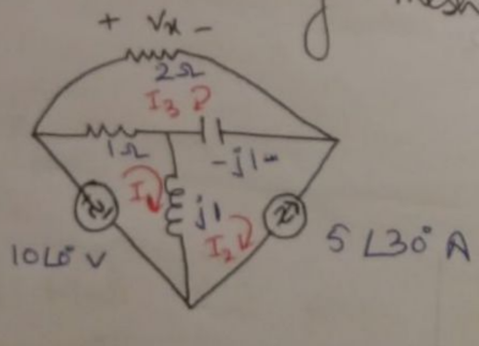
voltage division rule

$\uparrow$ V_{rms}

$$= \frac{-(10\sqrt{2}\angle 0^\circ)(-j2)}{1+j2}$$

$$= (11.3 + 5.6j)\ V$$

Q| find V_x using mesh and nodal analysis.



Mesh

$$-10\angle 0^\circ + 1(I_1 - I_3) + j1(I_1 - I_2) = 0$$
$$\Rightarrow (1+j1)I_1 - jI_2 - I_3 = 10\angle 0^\circ \longrightarrow (1)$$

$$2I_3 - j1(I_3 - I_2) + 1(I_3 - I_1) = 0$$
$$\therefore\ -I_1 + jI_2 + (3-j)I_3 = 0 \longrightarrow (2)$$

$$I_2 = -(5\angle 30^\circ) \longrightarrow (3)$$

From (1) and (3),

$$(1+j1)\, I_1 - I_3 = 10 - \underbrace{5\angle 120^\circ}_{} \quad 12.5 - j4.33 \longrightarrow (A)$$

From (2) and (3)

$$-I_1 + (3-j)\, I_3 = \underbrace{5\angle 120^\circ}_{-2.5 + j4.33} \longrightarrow (B)$$

$$\therefore (A) \Rightarrow (1+j1)\, I_1 - I_3 = 12.5 - j4.33$$

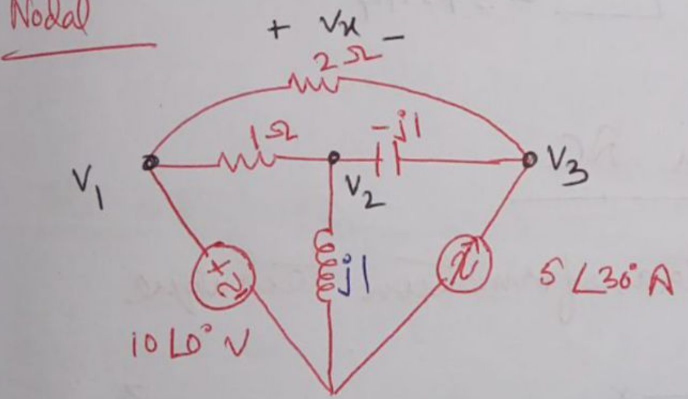
$$(B)\times(1+j) \Rightarrow -(1+j)I_1 + (4+2j)\, I_3 = -6.83 + j1.83$$

$$(3+j2)\, I_3 = 5.67 - j2.5$$

$$\therefore I_3 = 1.71 \angle -57.48^\circ \text{ A}$$

Now, $V_x = 2 I_3 = 3.42 \angle -57.48^\circ$ V

Nodal



$$V_1 = 10 \longrightarrow (1)$$

$$\frac{V_2 - 10}{1} + \frac{V_2}{j} + \frac{V_2 - V_3}{-j1} = 0$$

$$\Rightarrow V_2 - 10 - jV_2 + j(V_2 - V_3) = 0$$

$$\Rightarrow V_2 - jV_3 = 10 \longrightarrow (2)$$

$$-(5\angle 30^\circ) + \frac{V_3 - V_2}{-j1} + \frac{V_3 - 10}{2} = 0$$

$$\Rightarrow \frac{j(V_3 - V_2)}{1} + \frac{V_3 - 10}{2} = 5\angle 30^\circ$$

$$\Rightarrow -j2\,V_2 + (1+j2)\,V_3 = 10 + 10\angle 30^\circ$$

$$= 18.66 + j5 \longrightarrow (3)$$

(2) $\times j2 \Rightarrow j2\,V_2 + 2V_3 = j20$

(3) $\Rightarrow -j2V_2 + (1+j2)\,V_3 = 18.66 + j5$

$$V_3 = \frac{18.66 + j5}{3+j2}$$

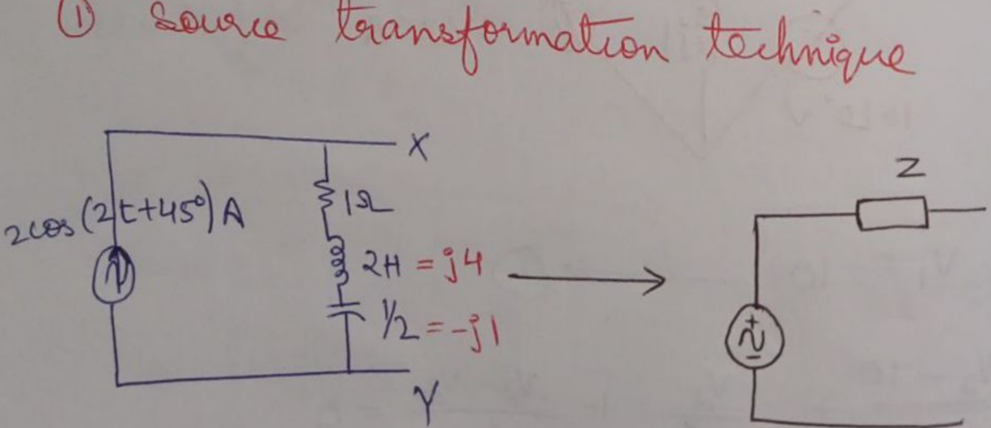
$V_3 = 8.65 \angle 19.57^\circ$

$V_x = V_1 - V_3 = 10 - (8.66 \angle 19.57)$

$V_x = 3.43 \angle -57.44$

Theorems in AC

(1) source transformation technique



Here, $I = 2\cos(2t + 45^\circ) = 2\angle 45^\circ$

$w = 2$

$z = 1 + j4 - j1 = (1+j3)\,\Omega$

$V = I \cdot z$

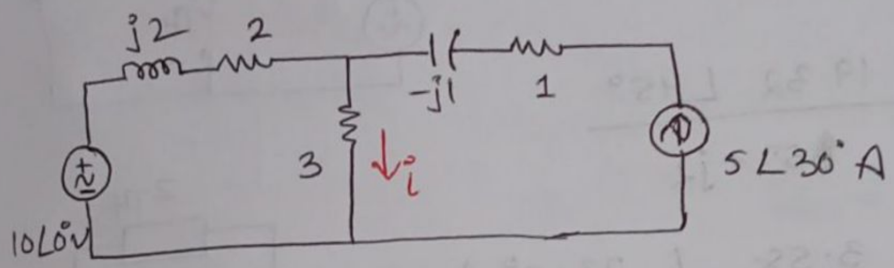
$= 2\angle 45^\circ\ (1+j3)$

$= 2\angle 45^\circ \times 3.16\angle 71.56^\circ$

$= 6.32\angle 116.56^\circ$

$\therefore\ v = 6.32\cos(2t + 116.56^\circ)\ V$

② Superposition, Thevenins, Nortons theorem



Find i

① Superposition Theorem

10V only :
$$i' = \frac{10\angle 0^\circ}{5+j2}$$

$$i' = 1.857\angle 21.8^\circ$$

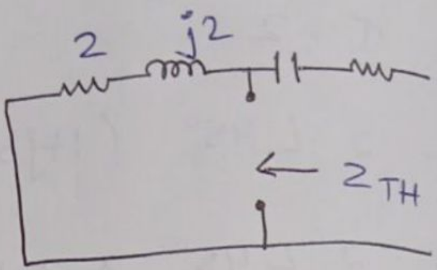
5A only :
$$i'' = 5\angle 30^\circ\left(\frac{2+j2}{5+j2}\right)$$

$$\cancel{= 5\angle 30^\circ (1}$$

$$= 2.61\angle 53.19^\circ$$

$$i = i' + i'' = 3.58\angle 23.2^\circ$$

Thevenin theorem

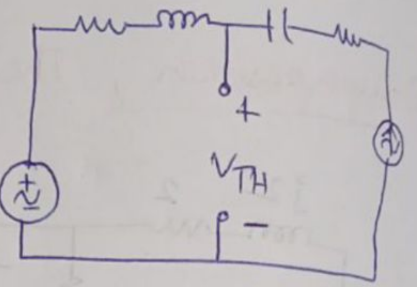


$Z_{TH} = (2+j2)\ \Omega$

KVL

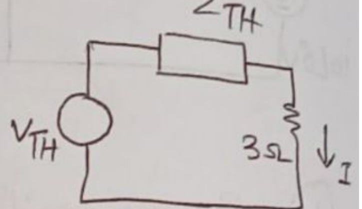
$$-10\angle 0^\circ + V_{TH} - (2+j2)\, 5\angle 30^\circ = 0$$

$$V_{TH} = 19.32\angle 45^\circ$$



$$I = \frac{19.32\angle 45^\circ}{5+j2}$$

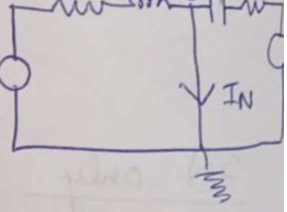
$$= 3.58\angle 23.2^\circ\ A$$



Nortons theorem

$$I_N = \frac{10\angle 0^\circ}{2+j2} + 5\angle 30^\circ$$

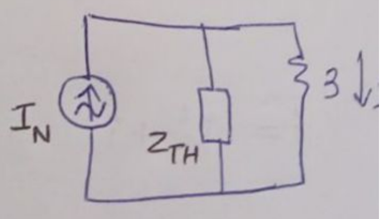
$$= \frac{10\angle 0^\circ}{2.82\angle 45^\circ} + 5\angle 30^\circ$$



$$= 6.83\angle 0^\circ$$

$$I = 6.83\angle 0^\circ \left(\frac{2+j2}{5+j2}\right)$$

$$= 3.58\angle 23.2^\circ\ A$$



Maximum Power Transfer Theorem in AC

Though there are 3 types of physical power existing in AC steady state n/ws, the power that is consumable / utilizable / convertible in any other form is Active power or real power in watts.

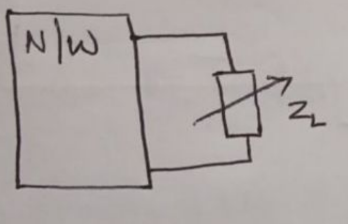
Max^m. power transfer theorem is confined to active power only.

General case

P_{max} occurs in the load when $\boxed{Z_L = Z_{TH}^*}$

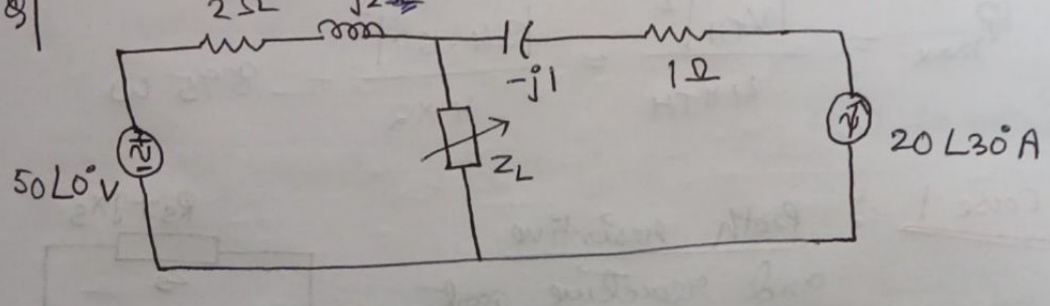
$$\boxed{P_{max} = \frac{|V_{TH}|^2}{4\,R_{TH}}}$$

$$\boxed{R_{TH} = \text{Real part}\,[Z_{TH}^*]}$$



Z_L is controllable

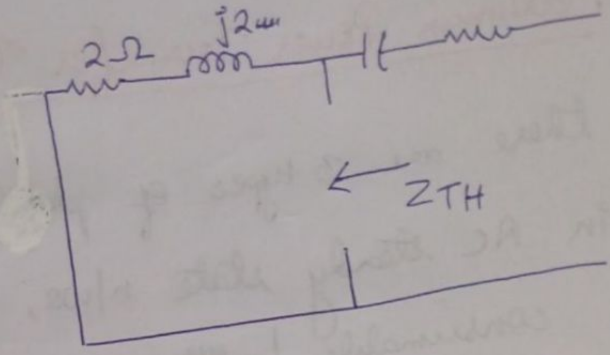
Q|



Find value of Z_L for which P_{max} occurs and find P_{max}.

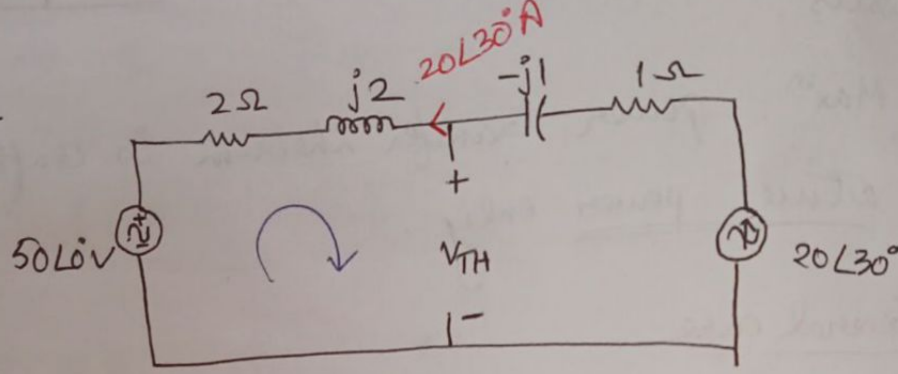
Determine Thevenin equivalent across Z_L.

Z_{TH}



$$Z_{TH} = (2 + j2)\ \Omega$$

V_{TH}



$$-(50\angle 0) - (2+j2)\, 20\angle 30^\circ + V_{TH} = 0$$

$$\Rightarrow \quad V_{TH} = 84.64 \angle 40.2^\circ$$

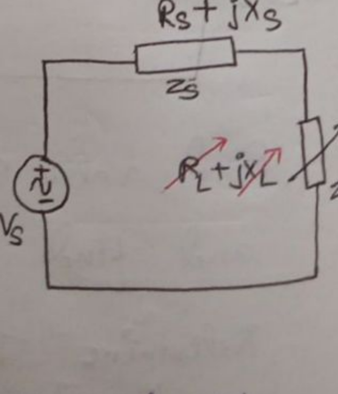
Now,

$$Z_L = Z_{TH}^* = (2 - j2)\ \Omega$$

$\uparrow$ R_{TH}

$$P_{max} = \frac{|V_{TH}|^2}{4R_{TH}} = \frac{|84.64|^2}{4 \times 2} = 895\ W$$

Case 1 : Both resistive and reactive part are controllable

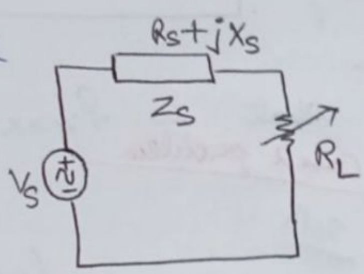


P_{max} occurs in load if load impedance is complex conjugate of equivalent source impedance seen by it.

$$\boxed{Z_L = Z_s^*} = R_s - jX_s$$

$$P_{max} = \frac{|V_s|^2}{4R_s}$$ $V_s \longrightarrow$ RMS value

Case 2 : Load is controllable resistor

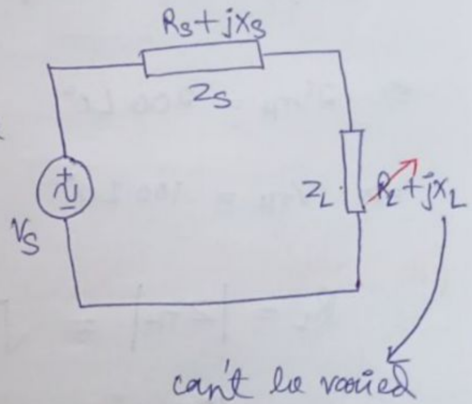


Load is purely resistive but source has some reactance, we cannot avoid reactive power in n/w.

P_{max} occurs in load if

$$\boxed{\begin{aligned} R_L &= |Z_s| = \sqrt{R_s^2 + X_s^2} \\ P_{max} &= I_{rms}^2 \times R_L \end{aligned}}$$

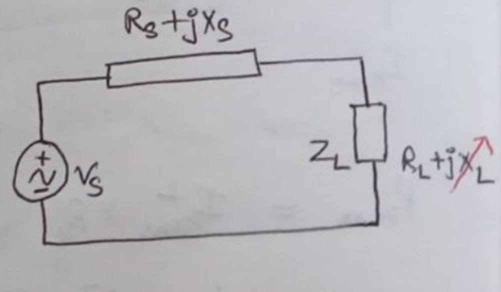
Case 3: R_L variable, X_L non variable



$$R_L = |R_s + jX_s + jX_L|$$

$$= \sqrt{R_s^2 + (X_L + X_s)^2}$$

Case 4: X_L variable, R_L constant



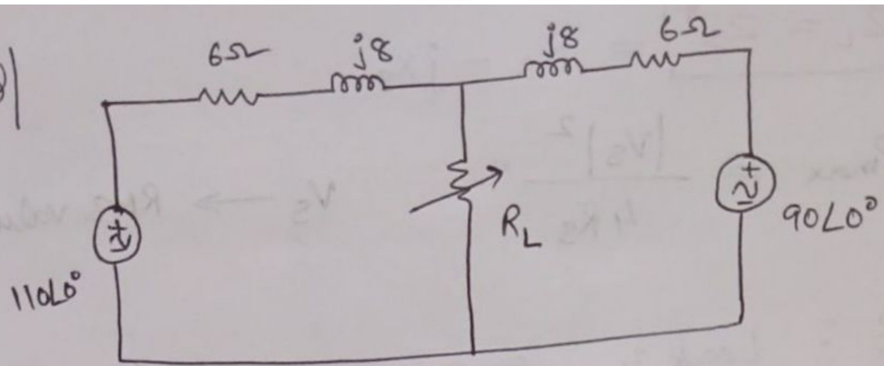
For P_{max},

$$X_L = -X_s$$

$$\Rightarrow X_L + X_s = 0$$

P_{max} occurs in load if $\boxed{X_L = -X_s}$

Q)

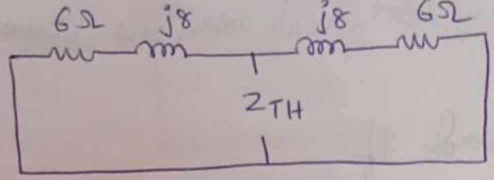


What is P_{max} transferred to load?

Case 2 problem

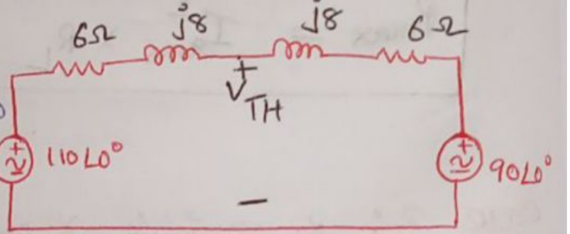
Sol^n

$$Z_{TH} = (6+j8) \parallel (6+j8) = (3+j4)\ \Omega$$



V_{TH} Nodal Analysis

$$\frac{V_{TH} - 110\angle 0^\circ}{6+j8} + \frac{V_{TH} - 90\angle 0^\circ}{6+j8} = 0$$



$$\Rightarrow 2V_{TH} = 200\angle 0^\circ$$

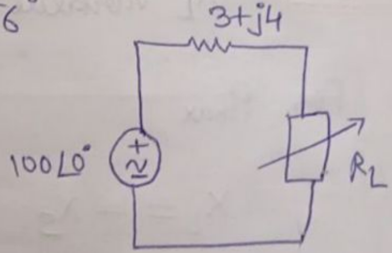
$$\Rightarrow V_{TH} = 100\angle 0^\circ$$

$$R_L = |Z_{TH}| = \sqrt{3^2+4^2} = 5\ \Omega$$

$$I_{5\Omega} = \frac{100\angle 0^\circ}{3+j4+5} = 11.18\angle -26.56^\circ$$

↑

I_{rms}

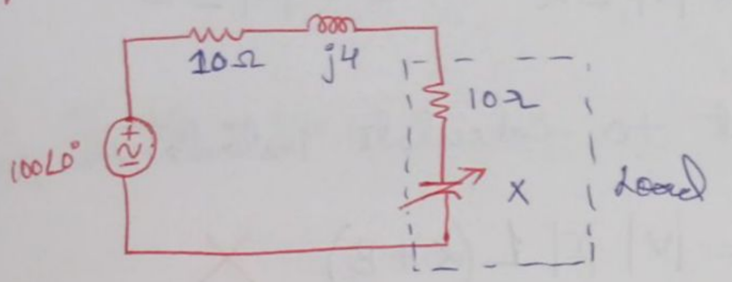


$$P_{max} = I_{rms}^2 \times R_L$$

$$= |11.18|^2 \times 5$$

$$= 625\ W$$

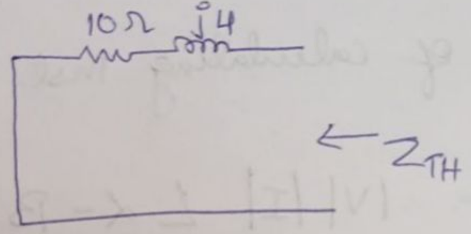
Q/ Find P_{max} transferred to load.



Case 4 problem

Sol^n

Z_{TH} =



$\therefore Z_{TH} = 10 + j4$

$\therefore X_L = -X_S$

$\Rightarrow X = -j4$

$$P_{max} = \left(\frac{100\angle 0^\circ}{10+j4+10-j4}\right)^2 \times 10 = I^2 R$$

$= 250\ W$

Complex Power (S^*) → volt ampere (VA)

$$\boxed{S^* = VI^*}$$

It is used to calculate instantaneous power in AC circuit.

I^* → complex conjugate of I

$I = 10\angle 20^\circ A \Rightarrow I^* = 10\angle -20^\circ A$

NOTE ① If $V = |V| \angle \alpha$, $I = |I| \angle B$

If we want to calculate instantaneous power

$V \cdot I = |V| \, |I| \, \angle(\alpha + B)$ ✗

Correct way of calculating inst. power

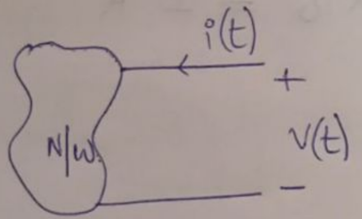
$V \cdot I^* = |V| \, |I| \, \angle \alpha - B$ ✓

② If $V = |V| \angle \alpha$, $I = |I| \angle -B$

$V I = |V| \, |I| \, \angle \alpha - B$ ✗

correct way : $V \cdot I = |V| \, |I| \, \angle(\alpha + B)$ ✓

Q/



Here, $v(t) = 20 + j12$

$i(t) = 5 + j4$

Calculate P and Q.

Sol^n

$v(t) = 20 + j12 = 23.32 \angle 30.96° \text{ V}$

$i(t) = 5 + j4 = \cancel{5+j4} \; 6.4 \angle 38.65° \text{ A}$

complex power, $S^* = V I^*$

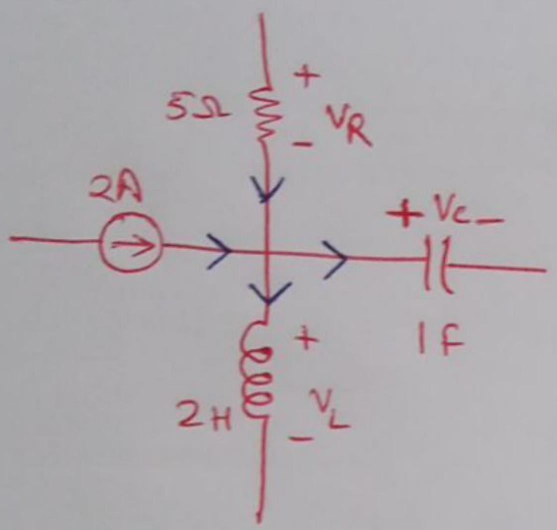
$= |23.32| \, |6.4| \, \angle (30.96 - 38.65)$

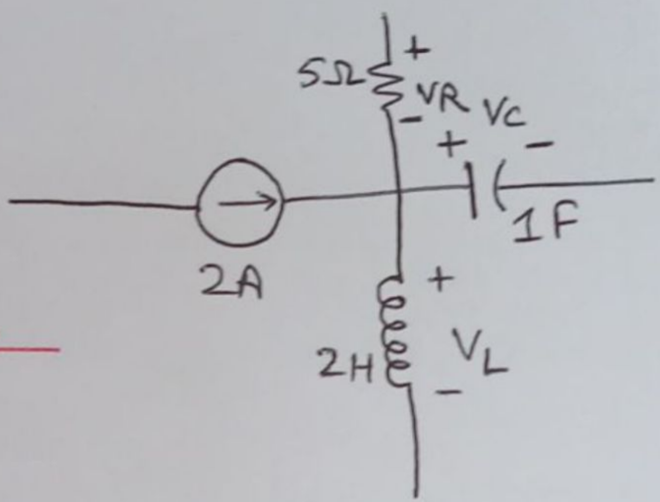
$= 149.248 \angle -7.69°$

P $= 147.9 - j19.97$ Q

$P = 147.9$ watts , $Q = 19.97$ VAR's

Q/ If $V_R = 5V$, $V_c = 4\sin 2t$ V , $V_L = ?$





<u>using KCL</u> ,

$$2 + \frac{V_R}{5} = \frac{dV_c}{dt} + \frac{1}{2}\int V_L\, dt$$

$$\Rightarrow \frac{1}{2}\int V_L\, dt = 2 + 1 - 8\cos 2t$$

$$\Rightarrow \int V_L\, dt = 6 - 16\cos 2t$$

$$\Rightarrow V_L = \frac{d}{dt}\left(6 - 16\cos 2t\right)$$

$$\Rightarrow V_L = 32\sin 2t \ \text{V}$$