(1) $e(n) = x(n) - y(n) \rightarrow (1)$

$y(n) = e(n-1) \rightarrow (2)$

Replace n by $n-1$ in (1)

$e(n-1) = x(n-1) - y(n-1) \rightarrow (3)$

From (2), (3)

$y[n] = x[n-1] - y[n-1]$

$y[n] + y[n-1] = x[n-1]$

(1)(a) If $n=0$ $\quad y[0] + \cancel{y[-1]}^{0} = \cancel{\delta(-1)}^{0}$

$y[0] = 0$

If $n=1$ $\quad y[1] + y[0] = \delta(0)$

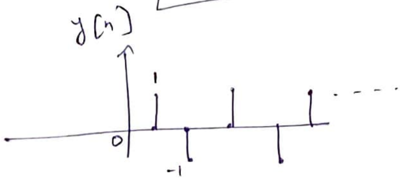
$\boxed{y(1) = 1}$

If $n=2$ $\quad y[2] + y[1] = \cancel{\delta(1)}^{0}$

$\boxed{y[-2] = -1}$

If $n=3$ $\quad y[3] + y[2] = \cancel{\delta(2)}^{0}$

$\boxed{y[3] = 1}$



1 (b)

$$y[n] + y[n-1] = u[n-1]$$

$n = 0 \rightarrow$ $y[0] + \cancel{y[-1]}^{\,0} = \cancel{u[-1]}^{\,0}$

$$\boxed{y[0] = 0}$$

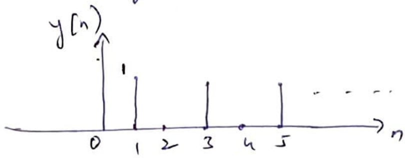
$n = 1 \rightarrow$ $y[1] + y[0] = u[0]$

$$y[1] + 0 = 1$$

$$\boxed{y[1] = 1}$$

$n = 2 \rightarrow$ $y[2] + y[1] = u[1]$

$$y[2] = 0$$



(2) (a)

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1]\, g[n-2k]$$

$$= g[n-2]$$

$$= u[n-2] - u[n-6]$$

2 (b)

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-2]\, g[n-2k]$$

$$= g[n-4]$$

$$= u[n-4] - u[n-8]$$

2 (c) Using 2(a) & 2(b) we conclude that the system is not LTI.

Because the I/P in 2(b) shifted right by 1 unit for the I/P ~~in~~ in 2(a).

However, the O/P ~~in~~ 2(b) is not shifted right by 1 unit for the O/P obtained in 2(a).

2 (d)

$$y[n] = \sum_{k=0}^{\infty} g[n-2k]$$

$$y[n] = g[n] + g[n-2] + g[n-4] + \cdots$$

$$= \left[u[n] - u[n-4]\right] + \left[u(n-2) - u(n-6)\right] + \left[u(n-4) - u(n-8)\right] + \cdots$$

$$y[n] = \begin{cases} 1, & n = 0, 1 \\ 2, & n \geq 2 \\ 0, & \text{otherwise} \end{cases} = 2u(n) - \delta(n) - \delta(n-1)$$

(3)

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$$

Let $\tau' = \tau - 2$

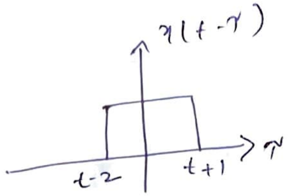
$d\tau' = d\tau$

$$y(t) = \int_{-\infty}^{t-2} e^{-(t-2-\tau')} x(\tau')\, d\tau'$$

Therefore $h(t) = e^{-(t-2)} u(t-2)$

(3)

(b)



$$\cancel{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau}$$

$$\cancel{= \int_{2}^{\infty}}$$

$$y(t) = \begin{cases} 0, & t < 1 \\ \int_{2}^{t+1} e^{-(\tau-2)}\, d\tau = 1 - e^{-(t-1)}, & 1 < t < 4 \\ \int_{t-2}^{t+1} e^{-(\tau-2)}\, d\tau = e^{-(t-4)}\left[1 - e^{-3}\right], & t > 4 \end{cases}$$

(4)

(a) Anti-Causal, because $h[n] = 0$ for $n > 0$

Stable, because $\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{5}{4} < \infty$

(b) Not causal, because $h[n] \neq 0$ for $n < 0$

Stable, because $\sum_{n=-\infty}^{3} 5^n = \frac{625}{4} < \infty$

(c) Not causal because $h[n] \neq 0$, $n < 0$

Unstable because second term become infinite as $n \to \infty$.

4 (g) Causal because $h[n]=0$ for $n<0$.

Stable because $\sum_{n=-\infty}^{\infty} |h(n)| = 1 < \infty$

5

(a) Not causal because $h(t) \neq 0$, $t<0$

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = \frac{e^{-2}}{2} < \infty$

(b) Not caused

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = \frac{1}{3} < \infty$

(c) Causal

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = 1 < \infty$

(d) causal

unstable, $\int_{-\infty}^{\infty} |h(t)|\, dt = \infty$

6

(a)

$$y(t) = \begin{cases} \dfrac{e^{-\beta t}\left(e^{-(\alpha-\beta)t} - 1\right)}{\beta - \alpha} ; & \alpha \neq \beta \\ t\, e^{-\beta t} u(t) ; & \alpha = \beta \end{cases}$$

6 (b)

$$y(t) = \begin{cases} \int_0^2 e^{2(t-\tau)}\,d\tau - \int_2^5 e^{2(t-\tau)}\,d\tau & ; t \le 1 \\ \int_{t-1}^2 e^{2(t-\tau)}\,d\tau - \int_2^5 e^{2(t-\tau)}\,d\tau ; & 1 \le t \le 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}\,d\tau, & 3 \le t \le 6 \\ 0, & t > 6 \end{cases}$$

$$y(t) = \begin{cases} \frac{1}{2}\left[e^{2t} - 2e^{2(t-2)} + 2e^{2(t-5)}\right. & ; t \le 1 \\ \frac{1}{2}\left[e^{2} + e^{2(t-5)} - 2e^{2(t-5)}\right. ; & 1 \le t < 3 \\ \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right], & 3 \le t \le 6 \\ 0 & ; t > 6 \end{cases}$$

6 (c)

$$y(t) = \begin{cases} 0 \; ; & t < 1 \\ \frac{2}{\pi}\left[1 - \cos\pi(t-1)\right] ; & 1 < t < 3 \\ \frac{2}{\pi}\left[\cos\{\pi(t-3)\} - 1\right] ; & 3 < t < 5 \\ 0 \; ; & t > 5 \end{cases}$$

6 (d)

$$y(t) = at + b$$

6 (e)

$$y(t) = \frac{1}{4} + t - t^2 ; \quad -\frac{1}{2} < t < \frac{1}{2}$$

$$= t^2 - 3t + \frac{7}{4} ; \quad \frac{1}{2} < t < \frac{3}{2}$$

The period of $y(t) = 2$