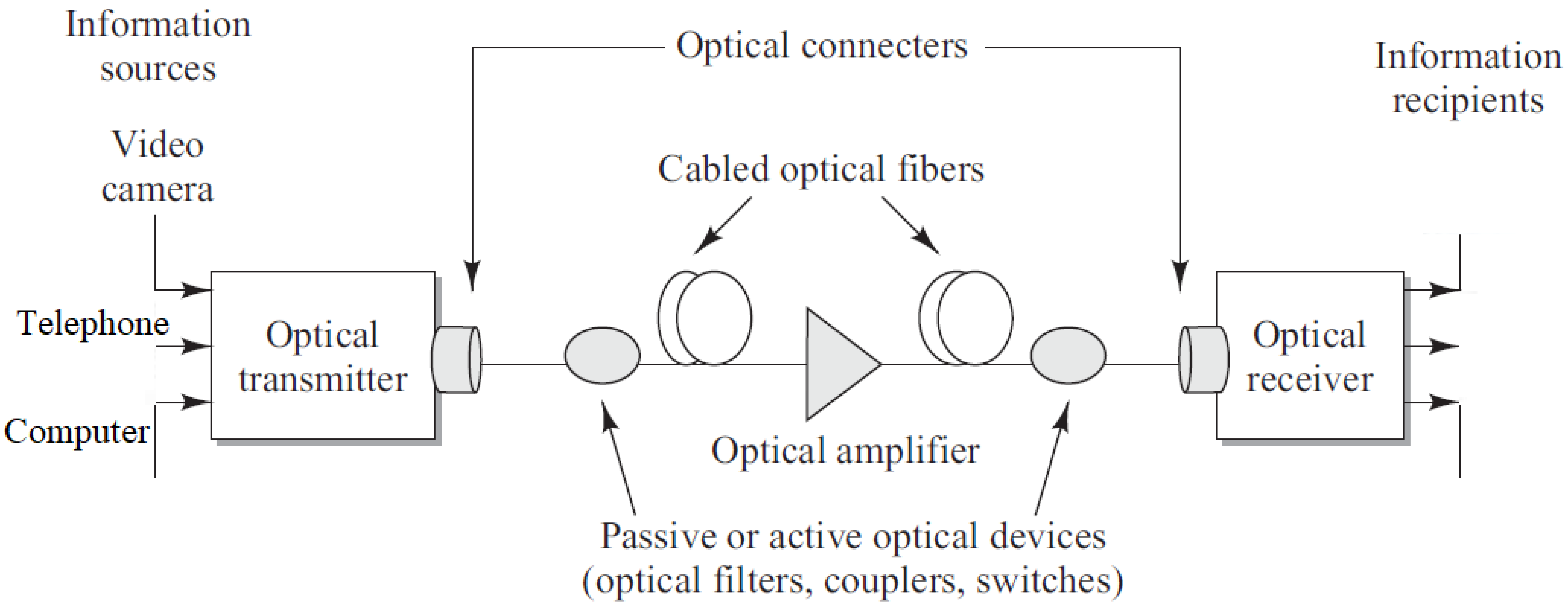
COURSE: *OPTICAL COMMUNICATION* (EC317)

UNIT-IV OPTICAL RECEIVER

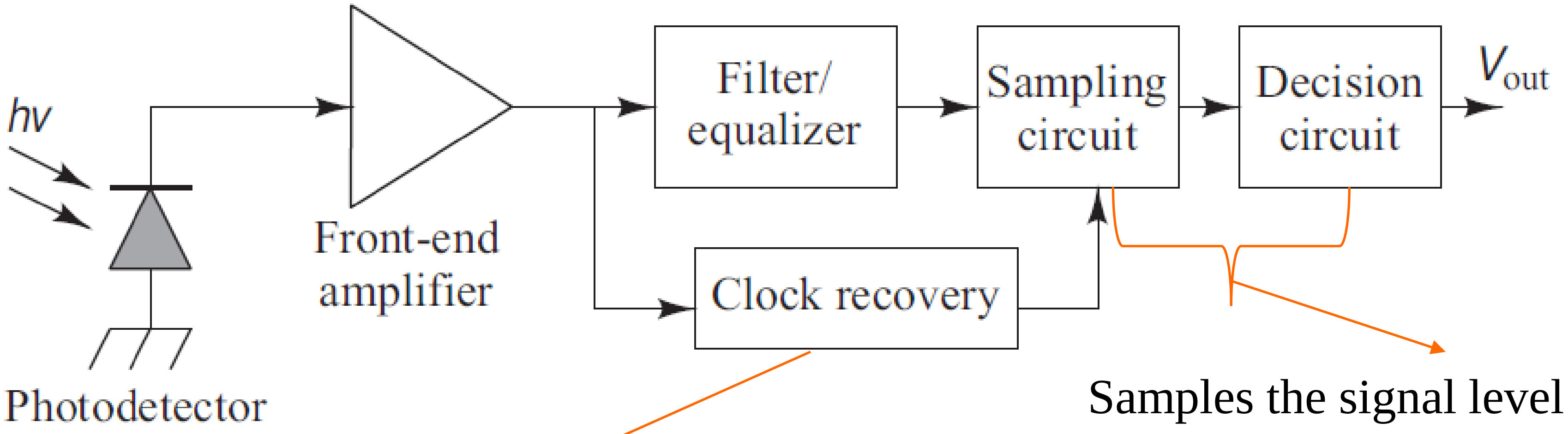
M. RAJARAO (Ad-hoc faculty)
Department of Electronics and Communication Engineering,
NIT Andhra Pradesh, Tadepalligudem,
Andhra Pradesh, INDIA.

OPTICAL FIBER COMMUNICATION SYSTEM



OPTICAL RECEIVER

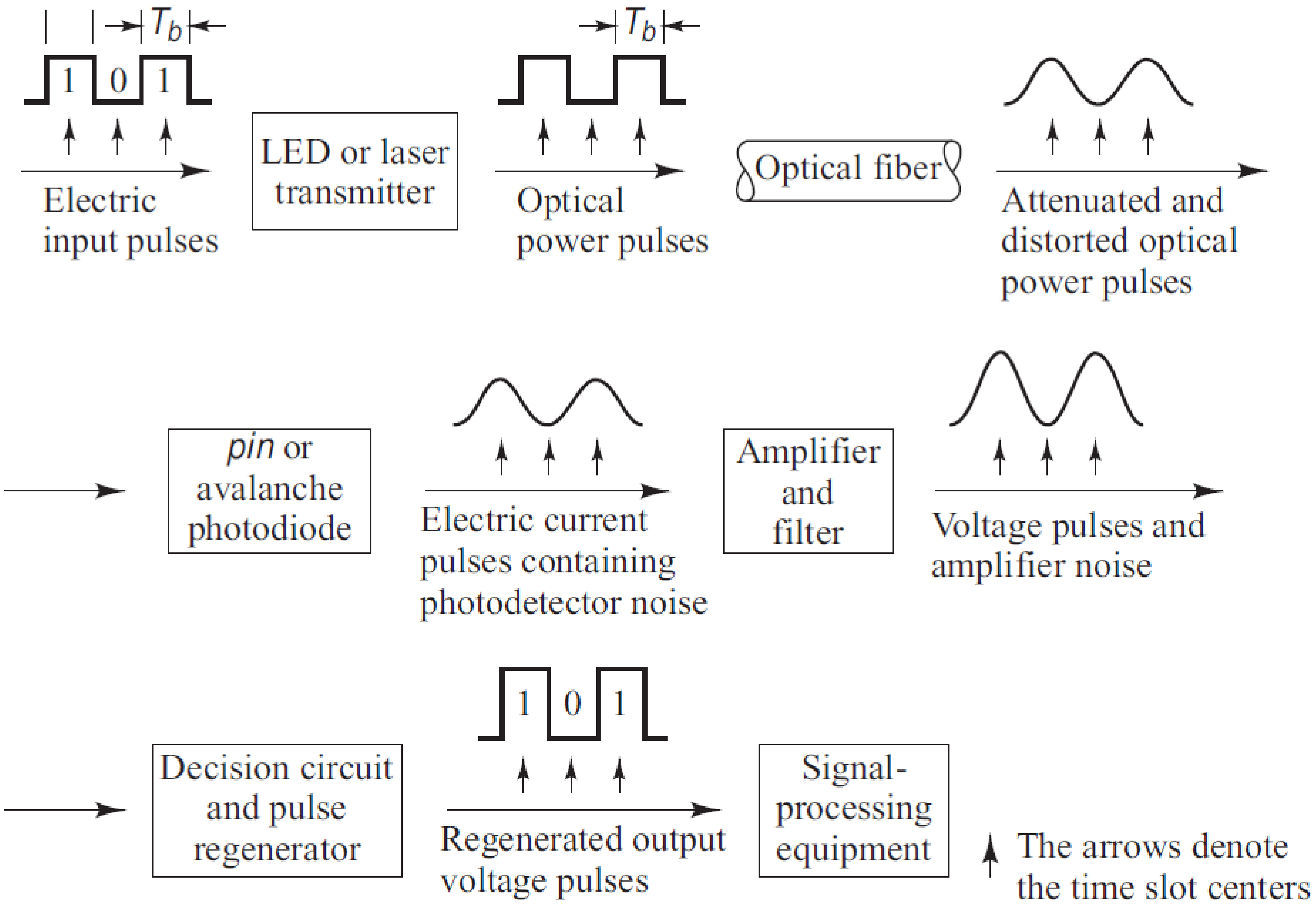
- **Basic function of the receiver**: able to detect weak, distorted signals and make decisions on what type of data was sent based on an amplified and reshaped version of this distorted signal.



provides the bit boundaries (start /end). The clock has a periodicity equal to the bit interval.

Samples the signal level at the midpoint of each time slot and compares it with a the threshold level (V_{Th}).

$$received\ bit = \begin{cases} 1;\ V_r > V_{th} \\ 0; V_r < V_{th} \end{cases}$$

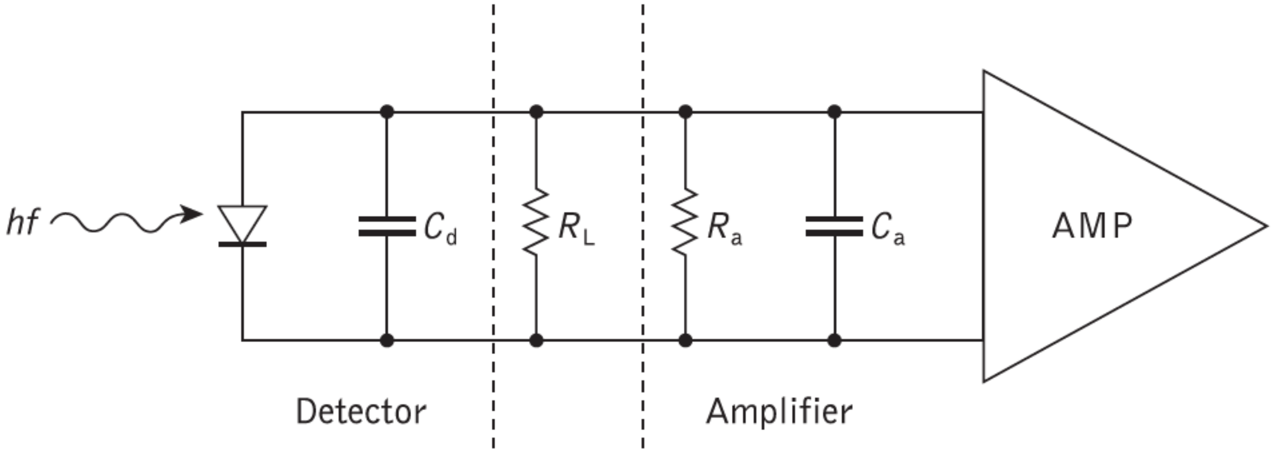


T_b
1 0 1
Electric input pulses
LED or laser transmitter
T_b
Optical power pulses
Optical fiber
Attenuated and distorted optical power pulses
pin or avalanche photodiode
Electric current pulses containing photodetector noise
Amplifier and filter
Voltage pulses and amplifier noise
Decision circuit and pulse regenerator
1 0 1
Regenerated output voltage pulses
Signal-processing equipment
The arrows denote the time slot centers

Error Sources

- Photon detection quantum noise (Poisson fluctuation)
- Dark current
- Statistical gain fluctuation (for an APD)
- Thermal noise
- Amplifier noise

$$\langle i_s{}^2 \rangle = 2qI_p\, B_e \qquad \langle i_d{}^2 \rangle = 2qI_d\, B_e \qquad \langle i_T{}^2 \rangle = \frac{4k_B T}{R_L}\, B_e$$



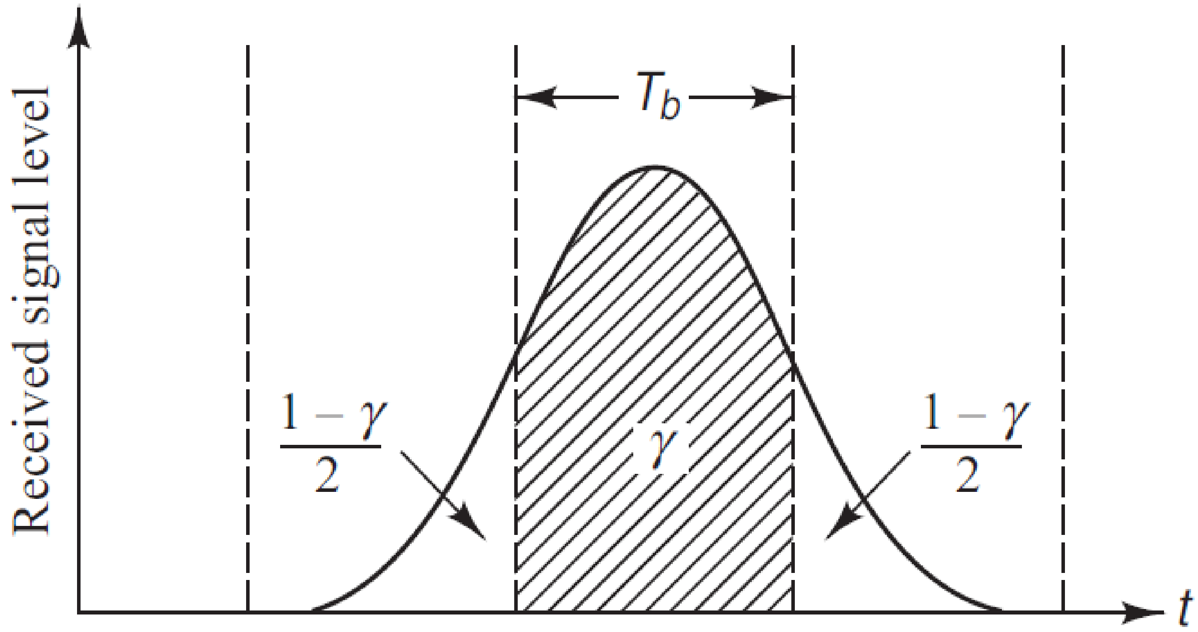
- For a detector with a mean avalanche gain M and an ionization rate ratio k, the excess noise factor F(M) for electron injection is

$$F(M) = kM + \left(2 - \frac{1}{M}\right)(1 - k)$$

$= M^x$; approximated empirical expression

ERROR SOURCES: INTER-SYMBOL INTERFERENCE (ISI)

- Which results from pulse spreading in the optical fiber.
- When a pulse is transmitted in a given time slot, most of the pulse energy will arrive in the corresponding time slot at the receiver



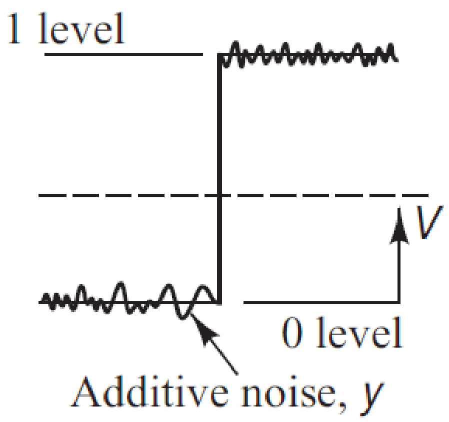
PROBABILITY OF ERROR/ BIT ERROR RATE (BER)

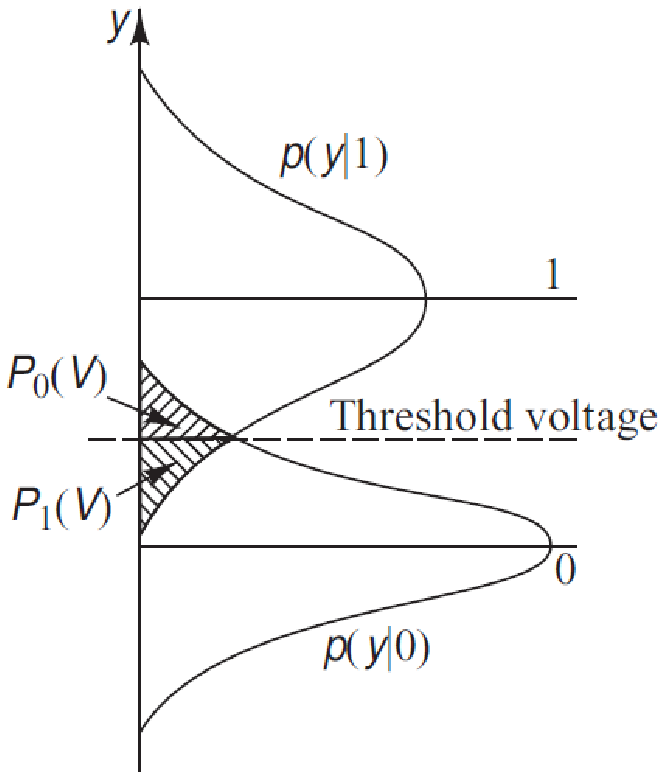
- The performance of the optical receiver with digital data (in the form of bits binary 1 and 0) can defined in terms of BER
- One of the simplest techniques for sending binary data is amplitude-shift keying (ASK) or on-off keying (OOK), wherein a voltage level is switched between two values, which are usually on or off.
- That is a voltage pulse of amplitude V relative to the zero voltage level when a binary 1 occurs and a zero-voltage-level space when a binary 0 occurs.
- The BER is defined as

$$BER = \frac{number\ of\ errors\ (N_e)}{Total\ number\ of\ bits\ (N_T)}$$

PROBABILITY OF ERROR/ BIT ERROR RATE (BER)

- Assume that the overall noise of the system be Gaussian noise which is additive in nature.
- The introduction of noise into the detected signal causes the photo-current to fluctuate about its mean level in the two logic levels (0 and 1)
- The variance for the logic '0' is narrower than that for the logic '1' because the logic '1' level involves shot noise which is Poisson distribution but the logic '0' involves only dark-current noise and thermal noise which are Gaussian in nature.





PROBABILITY OF ERROR/ BIT ERROR RATE (BER)

- Let us assume the photo-current (or voltage) corresponding to the threshold level be denoted by 'I_{th}' (or 'V_{th}').
- Bit error occurs, if a '0' was transmitted by the source but, due to noise and other factors, the detected bit would be '1' (the detected signal level exceeds the threshold level). Similarly, if a '1' was transmitted by the source and the detected signal level drops below threshold level, the detected bit is assigned '0'.
- BER is the probability of occurrence of bit errors in the transmission

$$BER = P_e = P(1/0)P(0) + P(0/1)P(1)$$

If the probabilities of 0 and 1 pulses are equally likely P(0) = P(1) = 1/2

$$BER = \frac{1}{2}(P(1/0) + P(0/1))$$

Where

$$P(1/0) = \frac{1}{\sqrt{2\pi}\sigma_0}\int_{I_{th}}^{\infty} e^{-\frac{(I-I_0)^2}{2\sigma_0{}^2}} dI \text{ and } P(0/1) = \frac{1}{\sqrt{2\pi}\sigma_1}\int_{-\infty}^{I_{th}} e^{-\frac{(I_1-I)^2}{2\sigma_1{}^2}} dI$$

PROBABILITY OF ERROR/ BIT ERROR RATE (BER)

Where

$$P(1/0) = \frac{1}{\sqrt{2\pi}\sigma_0} \int_{I_{th}}^{\infty} e^{-\frac{(I-I_0)^2}{2\sigma_0{}^2}} dI$$

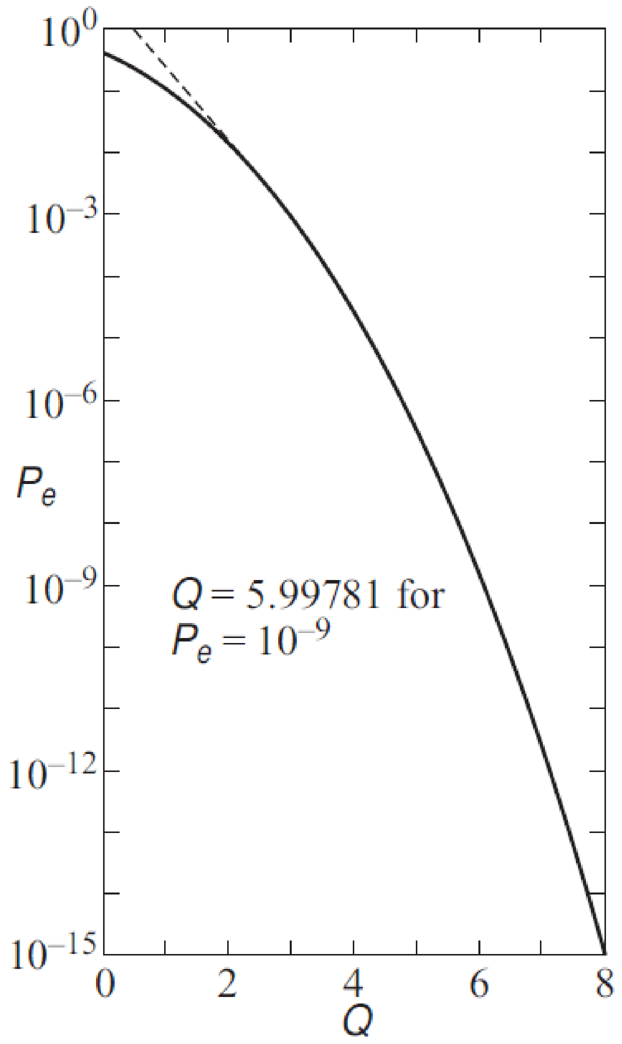
$$= \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} e^{-x^2} dx\,; \quad where\ Q = \frac{I_{th} - I_0}{\sigma_0} = \frac{I_1 - I_{th}}{\sigma_1} = \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$$

The above expression is like a complementary error function

The error function is defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dx$$

$$BER = \frac{1}{2} \mathrm{erfc}\big(Q/\sqrt{2}\big)$$



10^0
10^{-3}
10^{-6}
P_e
10^{-9}
$Q = 5.99781$ for
$P_e = 10^{-9}$
10^{-12}
10^{-15}
0
2
4
6
8
Q

Receiver Sensitivity

- Optical communication systems use a BER value to specify the performance requirements for a particular transmission link application
- To achieve a desired BER at a given data rate, a specific minimum average optical power level must arrive at the photodetector. The value of this minimum power level is called the receiver sensitivity (i.e., an average optical power (P_{avg}) in dBm incident on the photodetector).
- The average power received is

$$P_{avg} = \frac{P_1 + P_0}{2}$$

Where P_1 is the average incident power onto the photo-detector when logic ‘1’ is received and ‘P_0’ is the average power incident onto the detector when logic ‘0’ is received.

RECEIVER SENSITIVITY

- For the logic '0' reception, there is no incident power and the thermal noise component dominates the noise component in the output signal of the photo-detector. So, the total variance in the '0' level (denoted by σ_0^2) is given by: $\sigma_0{}^2 = \sigma_T{}^2 = \frac{4k_B TB}{R_L}$
- For the logic '1', the noise component is composed of shot noise and thermal noise. Therefore, the total noise variance (σ_1^2) is $\sigma_1{}^2 = \sigma_s{}^2 + \sigma_T{}^2$

$$\sigma_s{}^2 = 2qI_1B = 2q\mathcal{R}P_1B$$

- The Q-parameter for the photo-detector is $\mathrm{Q} = \frac{I_1 - I_0}{\sigma_0 + \sigma_1}$
- Assume there is no optical power in a zero pulse, $P_{avg} = \frac{P_1}{2}$ and $\mathrm{Q} = \frac{I_1}{\sigma_0 + \sigma_1}$

$$\mathrm{Q} = \frac{2\mathcal{R}P_{avg}}{\sigma_T + (\sigma_s{}^2 + \sigma_T{}^2)^{1/2}}$$

$$P_{sensivity} = P_{avg} = \frac{Q}{2\mathcal{R}}\left(\sigma_T + (\sigma_s{}^2 + \sigma_T{}^2)^{\frac{1}{2}}\right)$$

Receiver Sensitivity

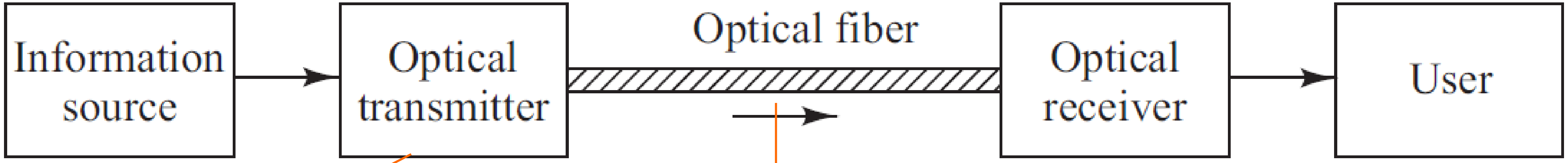
- Substitute ${\sigma_s}^2 = 4q\mathcal{R}BP_{sensitivity}$ and ${\sigma_T}^2 = \frac{4k_BTB}{R_L}$; and solve for $P_{sensitivity}$

$$P_{sensivity} = \frac{Q}{2\mathcal{R}}\left(\frac{qBQ}{2} + \left(\frac{4k_BTB}{R_L}\right)^{1/2}\right)$$

Point-point optical link

The following key system requirements are needed in analyzing a link:

- The desired (or possible) transmission distance
- The data rate or channel bandwidth
- The bit-error rate (BER)



LED or laser diode optical source

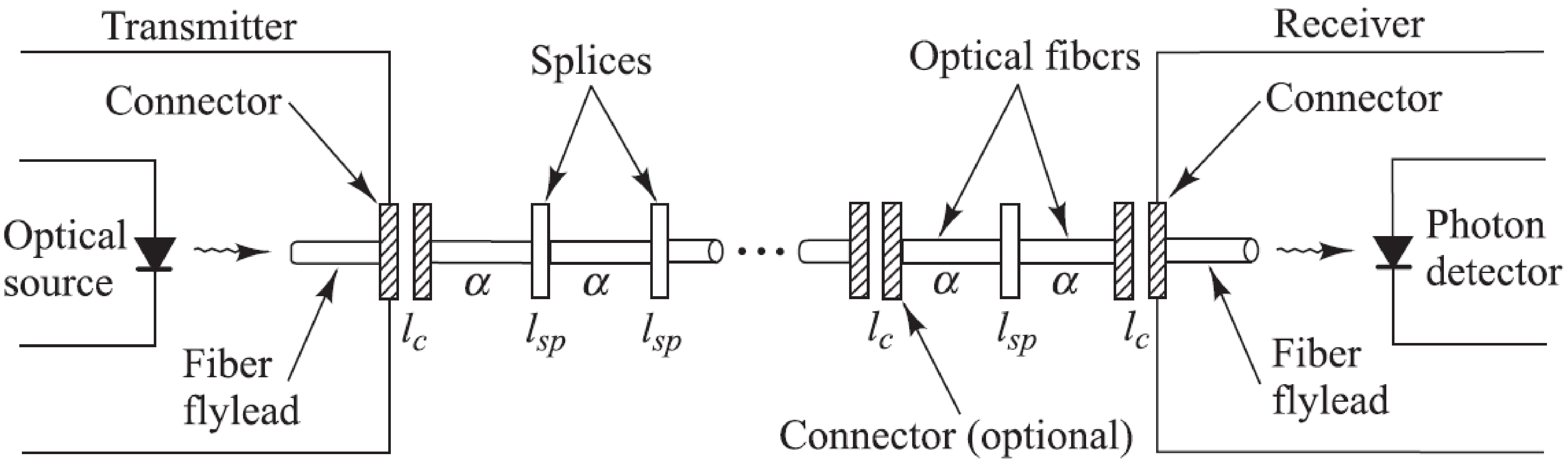
- Emission wavelength
- Spectral line width
- Output power
- Effective radiating area
- Emission pattern
- Number of emitting modes

- Multimode or single-mode
- Core size, refractive index profile
- Numerical aperture or mode-field diameter
- Bandwidth or dispersion, Attenuation

PIN or Avalanche photodiode

- Responsivity
- Operating wavelength
- Speed
- Sensitivity

Link Power Budget



Optical power loss model for a point-to-point link. The losses occur at connectors (l_c), at splices (l_{sp}), and in the fiber (α).

The maximum allowable loss $\alpha_{max} = P_t - P_r = \alpha_{fiber} + \alpha_{splices} + \alpha_{connectors} + \alpha_{system}$

POWER LAUNCHING AND COUPLING

- A measure of the amount of optical power emitted from a source that can be coupled into a fiber is usually given by the coupling efficiency η_C defined as

$$\eta_C = \frac{P_{CF}}{P_{ES}}$$

where, P_{CF} is the power coupled into the fiber and P_{ES} is the power emitted from the light source.

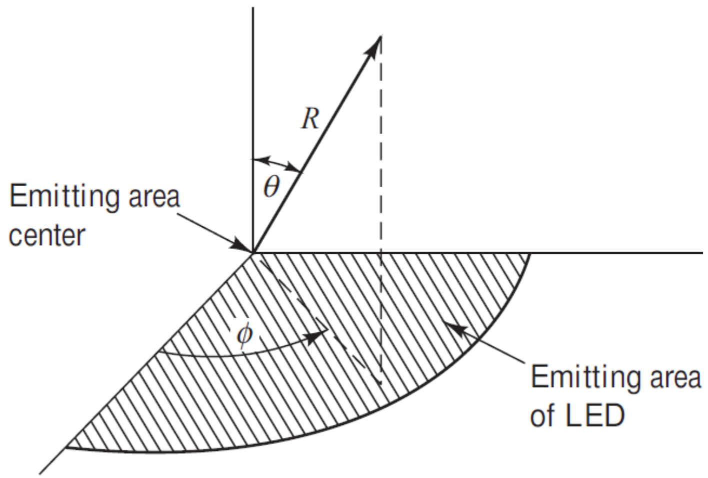
- The launching or coupling efficiency depends on the type of fiber that is attached to the source and on the coupling process

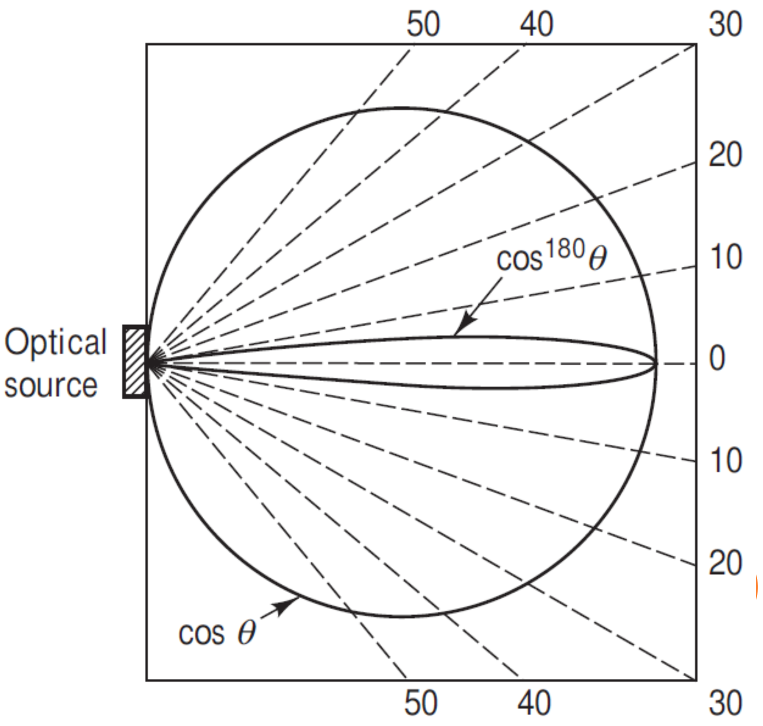
SOURCE-TO-FIBER POWER LAUNCHING

- The optical power that can be coupled into a fiber depends on the radiance (i.e., on the spatial distribution of the optical power).
- Radiance is the **optical power radiated into a unit solid angle per unit emitting surface area** (watts per square centimeter per steradian).
- The radiance may be a function of both θ and ϕ, and can also vary from point to point on the emitting surface.
- The power delivered at an angle θ, measured relative to a normal to the emitting surface

$$B(\theta, \phi) = B_0 \cos\theta$$

where B_0 is the radiance along the normal to the radiating surface.





TOTAL OPTICAL POWER EMITTED FROM SOURCE

the total optical power P_s that is emitted from the source of area A_s

$$P_s = A_s \int_0^{2\pi} \int_0^{\pi/2} B(\theta, \phi) \sin\theta \ d\theta \ d\phi$$

$$= (\pi r_s{}^2) 2\pi B_0 \int_0^{\pi/2} \cos\theta \sin\theta \ d\theta$$

$$= \pi^2 r_s{}^2 B_0 [sin^2(\theta)]_0^{\pi/2}$$

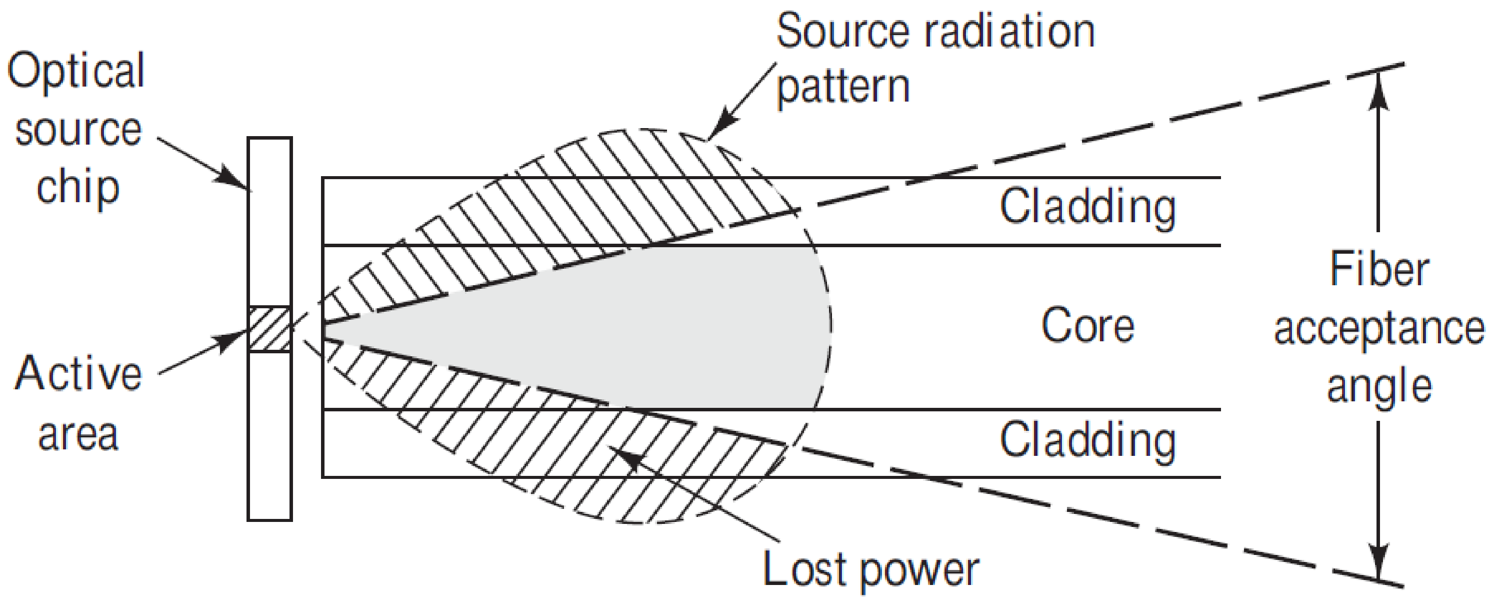
$$P_s = \pi^2 r_s{}^2 B_0$$

Power-Coupling Calculation

- Consider the fiber end face is centered over the emitting surface of the source and is positioned as close to it as possible.
- The coupled power can be found using the relationship

$$P_{C,LED} = \iint B(A_s, \Omega_s)\, dA_s \; d\Omega_s$$

where A_s and Ω_s are the area and solid emission angle of the source, respectively



POWER-COUPLING CALCULATION

$$P_{C,LED} = \int_0^{r_m} \int_0^{2\pi} \left(\int_0^{2\pi} \int_0^{\theta_A} B(\theta,\phi) \sin\theta \; d\theta \; d\phi \right) r \; d\theta_s \; dr$$

where θ_A is the acceptance angle of the fiber, $rdr \; d\theta_s$ is the emitting-point source of incremental area.

- If the source radius $r_s < a$ (fiber-core radius), then the upper integration limit $r_m = r_s$; for source areas larger than the fiber-core area, $r_m = a$.

$$P_{C,LED} = \int_0^{r_s} \int_0^{2\pi} 2\pi B_0 \left(\int_0^{\theta_A} \cos\theta \sin\theta \; d\theta \right) r \; d\theta_s \; dr$$

$$= \pi B_0 sin^2(\theta_A) \int_0^{r_s} \int_0^{2\pi} r \; d\theta_s \; dr = 2\pi^2 B_0 NA^2 \int_0^{r_s} r \; dr$$

$$P_{C,LED} = \pi^2 {r_s}^2 B_0 NA^2 = 2\pi^2 {r_s}^2 B_0 {n_1}^2 \Delta$$

$$P_{C,LED} = P_s \; NA^2$$

Coupled Power

- The power coupled from a surface LED source into step-index fiber

$$P_{LED,step} = \begin{cases} P_s\, NA^2;\, for\ r_s \le a \\ \left(\frac{a}{r_s}\right)^2 P_s\, NA^2;\, for\ r_s > a \end{cases}$$

- The power coupled from a surface-emitting LED into a graded-index fiber becomes (for $r_s < a$)

$$P_{LED,grad} = 2\pi^2 B_0 \int_0^{r_s} [n^2(r) - {n_2}^2]\, r\, dr$$

$$= 2\pi^2 {r_s}^2 B_0 {n_1}^2 \Delta \left(1 - \frac{2}{\alpha + 2}\left(\frac{r_s}{a}\right)^{\alpha}\right)$$

$$= 2\, P_s {n_1}^2 \Delta \left(1 - \frac{2}{\alpha + 2}\left(\frac{r_s}{a}\right)^{\alpha}\right)$$

Coupled Power for perfect coupling conditions

- For perfect coupling conditions between the source and the fiber, the power coupled into the fiber depends on Fresnel reflection or the reflectivity at the fiber-core end face (R)

$$P_{coupled} = (1 - R)P_{emitted}$$

Where

$$R = \left(\frac{n_1 - n}{n_1 + n}\right)^2$$

Here, n and n_1 are the refractive indices of the medium and fiber respectively

$$\text{Coupling loss} = -10\ \log_{10}\left(\frac{P_{Coupled}}{P_{Emitted}}\right)$$

PROBLEMS

- Consider an LED that has a circular emitting area of radius 35 mm and a Lambertian emission pattern with 150 W/(cm^2/sr) axial radiance at a given drive current. Compare the optical powers coupled into two step-index fibers, one of which has a core radius of 25 mm with NA = 0.20 and the other which has a core radius of 50 mm with NA = 0.20.
- A GaAs optical source with a refractive index of 3.6 is coupled to a silica fiber that has a refractive index of 1.48. What is the power loss between the source and the fiber? (Assume perfectly coupled)

OPTICAL FIBER CONNECTIONS AND THEIR LOSSES

- Any fiber optic system installation is requirement to interconnect fibers in a low loss manner.
- Interconnections occur
    - At the optical source
    - At the photo-detector
    - At the intermediate points within a cable where two fibers are joined
- The fiber to fiber connections may be achieved in two ways
- Splice: permanent joints b/w two fibers (it is analogous to the electrical soldering of two metallic wires)
- Connectors: demountable joints (analogous to a plug in socket arrangement)

- The losses due to fiber connections depends on parameters
  - The input power distribution to the joints
  - The length of the fiber b/w the source and joints
  - The geometrical and waveguide characteristics of the two fiber ends at the joint
  - the fiber end face qualities
- The optical power that can be coupled from one fiber to other is limited by the number of modes that can propagate in each fiber.

- For a G.I fiber , the total number of modes is estimated by

$$M = k^2 \int_0^a [n^2(r) - {n_2}^2]\, r\, dr$$

$$M = k^2 \int_0^a NA^2(r)\, r\, dr = k^2 NA^2(0) \int_0^a \left(1 - \left(\frac{r}{a}\right)^{\alpha}\right) r\, dr$$

$$M = \frac{\alpha}{\alpha + 2} \left(\frac{2\pi a n_1}{\lambda}\right)^2 \Delta$$

- The two fibers that are joined will have
  - Varying degree of differences in their radii ‘a’
  - Axial numerical aperture (NA(0))
  - Index profiles ‘α’

- The fraction of power coupled from one fiber to other is proportional to the common mode volume
- The fiber to fiber coupling efficiency is given by

$$\eta_F = \frac{M_{comm}}{M_E}$$

Where M_E is the number of modes in the emitting fiber

- The fiber to fiber coupling loss is given by

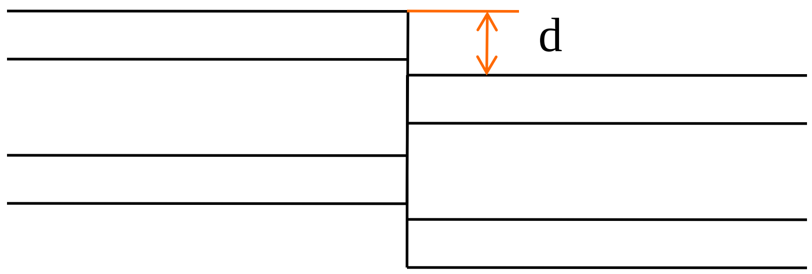
$$L_F = -10\log(\eta_F)$$

- The loss at the joint b/w multimode fibers depends on the power distribution among the modes in the fiber

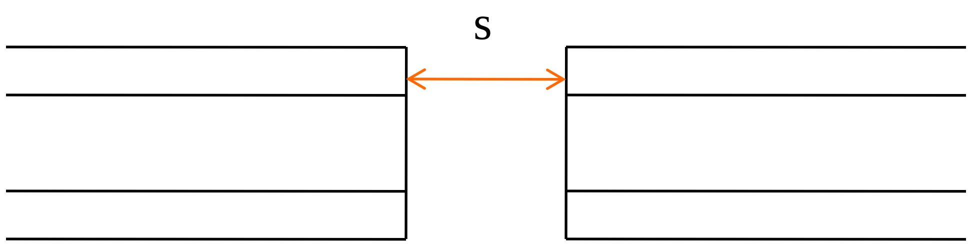
- When all modes in a fiber are equally excited, in this case for receiving fiber to accept all the optical power emitted by the first fiber
  - There must be perfect mechanical alignment b/w the two optical fibers
  - Their geometric and waveguide characteristics must match precisely
- If steady state modal equilibrium has been established in emitting fiber, most of the energy is concentrated in the lower order fiber modes (the optical power is concentrated near the center of the fiber core)
- In this case, NA of the receiving fiber is larger than the equilibrium NA of the emitting fiber
- Hence, slight mechanical misalignments do not contribute significantly to joint loss.

Mechanical misalignment

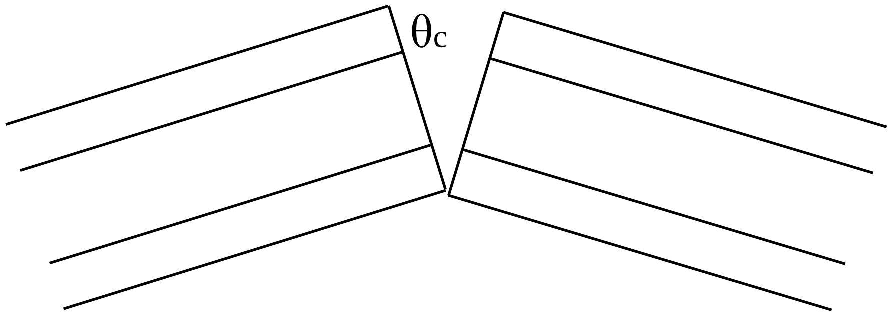
- Radiation losses results from mechanical misalignments because the radiation cone of the emitting fiber does not match the acceptance cone of the receiving fiber.
- There are three fundamental types of misalignment b/w the fibers
  - Axial displacement or lateral displacement occurs when the axes of the two fibers are separated by some distance
  - Longitudinal separation occurs when the fibers have the same axis but have a gap between their end faces
  - Angular misalignment occurs when the two axes form an angle so that the fiber end faces are no longer parallel



Lateral (axial) displacement



Longitudinal separation

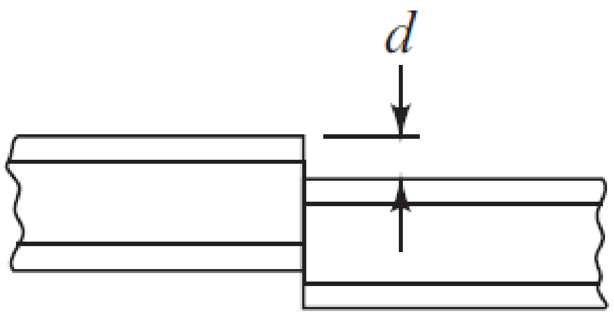


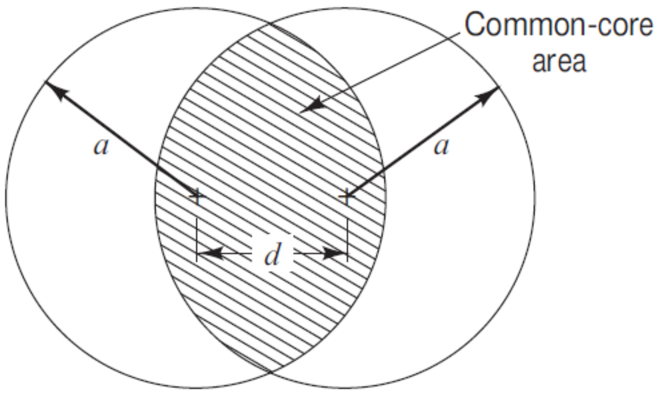
Angular misalignment

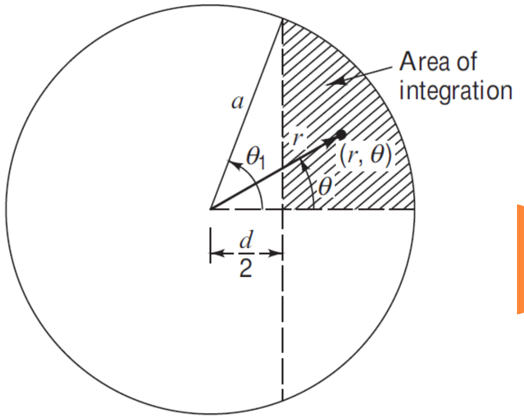
AXIAL DISPLACEMENT

- In practice, the most common misalignment occurring is axial displacement. Which causes the greatest power loss.
- It reduces the overlap area of the two fiber core end faces
- Reduces the amount of optical power that can be coupled from one fiber into other.
- Let us consider two identical S.I fibers of radii ‘a’ and assume there is uniform modal power distribution in the emitting fiber.
- In this case the optical power coupled is simply proportional to the common area of the two fibers

$$\eta_L = \frac{A_{comm}}{\pi a^2} = \frac{2}{\pi}\cos^{-1}\left(\frac{d}{2a}\right) - \frac{d}{2\pi}\sqrt{1-\left(\frac{d}{2a}\right)^2}$$







AXIAL DISPLACEMENT(CONT…)

- In G.I fiber the NA varies across the fiber end face
- The total power coupled into the receiving fiber is limited by the NA of the transmitting or receiving fiber
- The optical power density p(r) at a point ‘r’ on the fiber end is proportional to the square of the local NA at that point

$$p(r) = p(0)\frac{NA^2\,(r)}{NA^2\,(0)}$$

- For an arbitrary index profile

$$p(r) = p(0)\left(1 - \left(\frac{r}{a}\right)^{\alpha}\right)$$

- The total power P in the fiber is

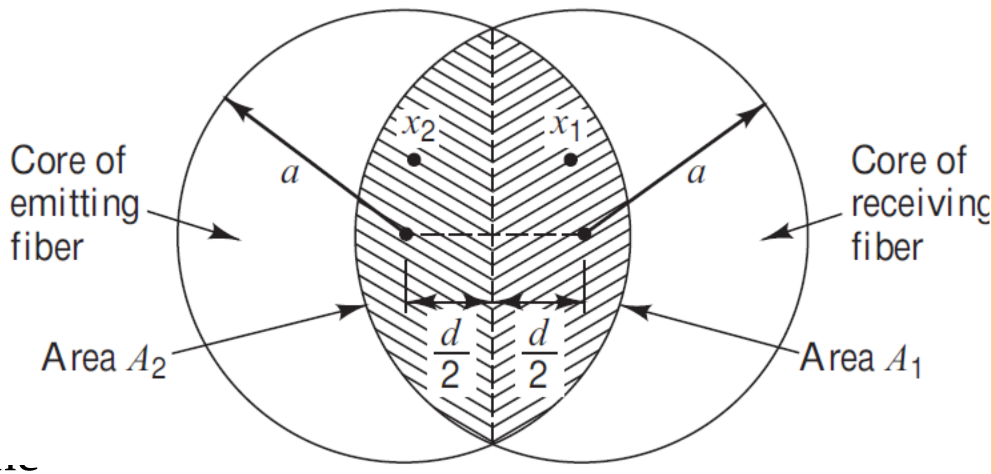
$$P = \int_0^{2\pi}\int_0^{a} p(r)\, r\, d\theta dr$$

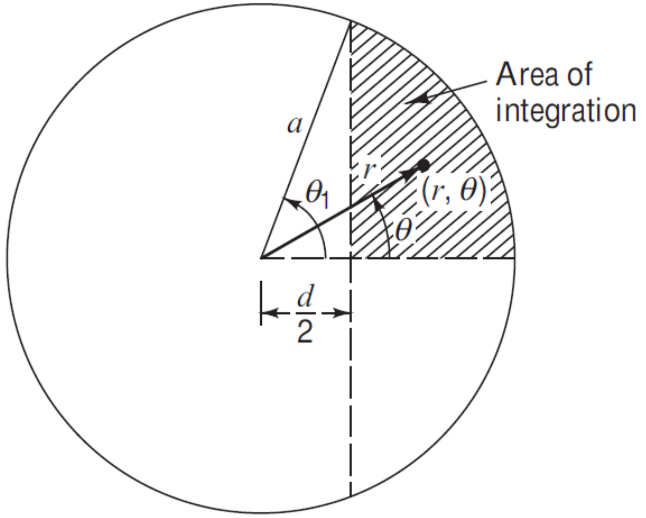
POWER COUPLED INTO THE RECEIVING FIBER

- Consider the power transmitted across the butt joint of the tw parabolic graded-index fibers with an axial offset d,
- The overlap region must be considered separately for the area A_1 and A_2.
- In area A_1 the numerical aperture is limited by that of the emitting fiber, whereas in area A_2 the numerical aperture of the receiving fiber is smaller than that of the emitting fiber.
- The received power P_1 in area A_1 is

$$P_1 = p(0) \int_{r_1}^{a} \int_{0}^{\theta_1} \left(1 - \left(\frac{r}{a}\right)^2\right) r \, d\theta dr$$

$$= \frac{a^2}{2} p(0) \left(\cos^{-1}\left(\frac{d}{2a}\right) - \frac{d}{2\pi} \sqrt{1 - \left(\frac{d}{2a}\right)^2} \right) \frac{d}{6a} \left(5 - \frac{d^2}{2a^2}\right)$$





AXIAL DISPLACEMENT(CONT…)

- The total power P in the emitting fiber is

$$P = \frac{\pi a^2}{2}\, p(0)$$

- When the axial misalignment 'd' is small compared with the core radius 'a', then the total power accepted by the receiving fiber is

$$P_T \cong P\left(1 - \frac{8d}{3\pi a}\right)$$

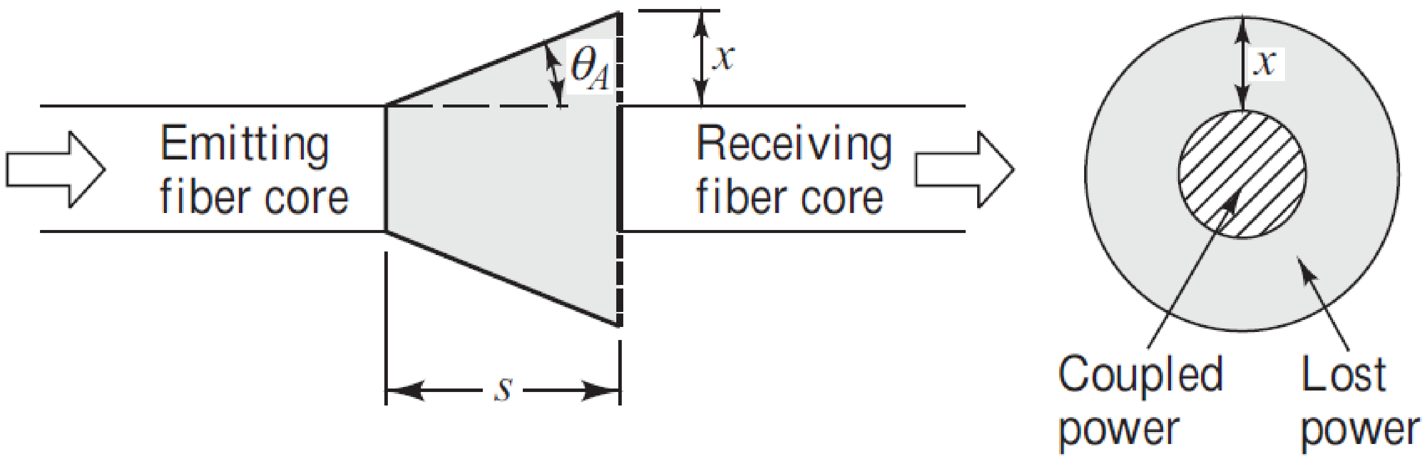
- The coupling loss is

$$L_F = -10\log\left(\frac{P_T}{P}\right)$$

LONGITUDINAL SEPARATION

- All the higher mode optical power emitted in the ring of the width ‘x’ will not be intercepted by the receiving fiber
- For S.I fiber, the loss is

$$L_F = -10\log\left(\frac{a}{a+s\tan\theta_A}\right)^2$$



ANGULAR MISALIGNMENT

- In this case, the optical power that leaves the emitting fiber outside of the solid acceptance angle of the receiving power
- For two S.I fibers that have an angular misalignment ‘θ’ , the power loss at the joint is

$$L_F = -10\log\left(\cos\theta\left\{\frac{1}{2}-\frac{1}{\pi}\left(p(1-p^2)^{1/2}-\sin^{-1}(p)\right)-q\left[\frac{1}{\pi}\left(y(1-y^2)^{1/2}+\sin^{-1}(y)\right)+\frac{1}{2}\right]\right\}\right)$$

$$p = \frac{\cos\theta_c(1-\cos\theta)}{\sin\theta_c\sin\theta}$$

$$q = \frac{\cos^3\theta_c}{(\cos^2\theta_c-\sin^2\theta)^{3/2}}$$

$$y = \frac{\cos^2\theta_c(1-\cos\theta)-\sin^2\theta}{\sin\theta_c\cos\theta_c\sin\theta}$$

Fiber related losses

- Difference in the geometrical and waveguide characteristics will effect on fiber to fiber coupling losses
- These are variations in **core diameter, numerical aperture** and **refractive index profile**
- If the radii are not equal but axial NA and index profiles are equal, the coupling loss is

$$L_F(a) = \begin{cases} -10\ \log_{10}\left(\dfrac{{a_R}^2}{{a_E}^2}\right); & for\ a_R < a_E \\ 0; & for\ a_R \geq a_E \end{cases}$$

- If the radii and the index profiles of the two coupled fibers are identical but their axial numerical apertures are different, then

$$L_F(NA) = \begin{cases} -10\ \log_{10}\left(\dfrac{{NA_R}^2(0)}{{NA_E}^2\ (0)}\right); & for\ NA_R(0) < NA_E(0) \\ 0; & for NA_R(0) \geq NA_E(0) \end{cases}$$

FIBER RELATED LOSSES

- If the radii and axial NAs are equal but index profiles are different, the coupling loss is

$$L_F(a) = \begin{cases} -10\ \log_{10}\left(\dfrac{\alpha_R(\alpha_E+2)}{\alpha_E(\alpha_R+2)}\right); & for\ \alpha_R < \alpha_E \\ 0; & for\ \alpha_R \geq \alpha_E \end{cases}$$

TEXT BOOKS

1. Gerd Keiser, Optical Fiber Communications, TMH India, Fourth Edition, 2010.
2. Senior John M., Optical Fiber Communications, Pearson Education India, Third Edition, 2009.