Signals and Systems

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1A) $e[n] = x[n] - y[n]$

$y[n] = e[n-1] \rightarrow ①$

$\cancel{eef}\ e[n-1] = x[n-1] - y[n-1] \rightarrow ②$

① in ②

$y[n] = x[n-1] - y[n-1]$

$y \quad x[n-1] = y[n] + y[n-1]$

2A) $g[n-26] = \cancel{u[n] - u[n-4]} \quad u[n-26] - u[n-26-4]$

a) $\cancel{Def}\ y[n] \ , \ x(n) = \delta[n-1]$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1] \left(u[n-2k] - u[n-26-4]\right)$$

$$= \sum_{k=-\infty}^{\infty} \delta[k-1] \times u[n-2k] - \sum_{k=-\infty}^{\infty} \delta[k-1] u[n-2k-4]$$

$$\left(x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)\right)$$

$$\rightarrow \quad \sum_{k=-\infty}^{\infty} u[n-2] \times \delta[k-1] - \sum_{k=-\infty}^{\infty} u[n-6]\,\delta[k-1]$$

$y[n] = u[n-2] - u[n-6]$

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b) $x[n] = \delta[n-2]$

b) $y[n] = \sum_{k=-\infty}^{\infty} \delta[n-2] \times (u[n-2k] - u[n-2k-4])$

$= \sum_{k=-\infty}^{\infty} \delta[n-2] \times u[n-2k] - \delta[n-2] u[n-2k-4]$

$= u[n-4] - u[n-8]$

c) $x[n] \xrightarrow{[n]} y[n] = x[n]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k] g[n-2k]$

$x[n-n_0] \rightarrow \sum_{k=-\infty}^{\infty} x[k-n_0] g[n-2k]$

$y[n-n_0] = x[k] g[n-n_0-2k]$

$x[n-n_0] \neq y[n-n_0]$

Not Time Invarient

d) $x[n] = u[n]$

$y[n] = \sum_{k=-\infty}^{\infty} u[k] [u[n-2k] - u[n-2k-4]]$

$= \sum_{k=0}^{\infty} u[n-2k] - u[n-2k-4]$

$y[n] = 2u[n] - \delta[n] - \delta[n-1]$

3A) a) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau$

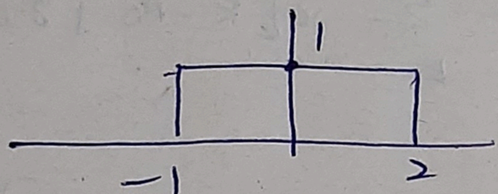
A) $x(t) = \delta(t)$

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \times \delta(\tau-2)\, d\tau$$

$$h(t) = e^{-(t-2)} \times \int_{-\infty}^{t} \delta(\tau-2)\, d\tau$$

$$\boxed{h(t) = e^{-(t-2)} \times u(\tau-2)}$$

5)



$x(t) = u(t+1) - u(t-2)$

4A) a) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$

$h(n) = 0$ for $n > 0$

so it is not causal

$$\sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^n = \infty$$

not stable

b) $h(n) = (5)^n u[3-n]$

A) $h(n) \neq 0$ for $n<0$

So not Causal

$$\sum_{n=-\infty}^{3} (5)^n = 125+25+ \cdots$$

$$= \frac{125}{1-\frac{1}{5}} = \frac{125 \times 5}{4}$$ ✓ (stable)

c) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$

A) $h[n] \neq 0$ for $n<0$

not Causal

$$\sum_{n=0}^{\infty} (-\tfrac{1}{2})^n + \sum_{\infty}^{1} (1.01)^n$$

$= 102.67$ (finite)

stable

d) $h[n] = n(\frac{1}{3})^n u[n-1]$

A) $h[n] = 0$ for $n<0$ $\therefore$ Causal

$$\sum_{n=-\infty}^{\infty} |h(n)| = 1 < \infty$$ $\therefore$ stable

5A) $h(t) = e^{2t} u(-1-t)$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{-1} e^{2t} dt$$

$$= \left(\frac{e^{2t}}{2}\right)_{-\infty}^{-1}$$

$$= \frac{1}{2e^2} \text{ (finite)}$$

$\therefore$ stable

$h(t) \neq 0$ for $t < 0$ $\therefore$ not Causal

b) $h(t) = e^{-6|t|}$

A) $h(t) \neq 0 \;\; \forall \; t < 0$

$\therefore$ not causal

$$\int_{-\infty}^{0} e^{+6t} + \int_{0}^{\infty} e^{-6t}$$

$$\left(\frac{e^{6t}}{6}\right)_{-\infty}^{0} + \left(\frac{e^{-6t}}{-6}\right)_{0}^{\infty}$$

$$= \frac{1}{3} \text{ (stable)}$$

c) $h(t) = t\, e^{-t} u(t)$

A) ~~Cost~~ Co $h(t) = 0 \;\; \forall \; t < 0$

Causal.

$$\int_{0}^{\infty} t e^{-t} = t \times (-e^{-t})_{0}^{\infty} + \int 1 \times e^{-t} dt$$

$$= 1 < \infty \text{ (Stable)}$$

d) $h(t) = \left(2e^{-t} - e^{\frac{(t-100)}{100}}\right) u(t)$

A) $h(t) = 0 \quad \forall\, t < 0$

$\therefore$ Causal

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \left(\frac{2e^{-t}}{-1}\right)_0^{\infty} - \left(\infty - \frac{e^{-1}}{100}\right)$$

$$= -\infty$$

not stable

6) a) $x(t) = e^{-\alpha t} u(t)$, $\quad y(t) = e^{-\beta t} u(t)$

<u>$\alpha \neq \beta$:</u>

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \int_0^t e^{-\alpha\tau} \times e^{-\beta(t-\tau)}\, d\tau$$

$$= e^{-\beta t} \times \int_0^t e^{-\alpha\tau} \times e^{\beta\tau}\, d\tau$$

$$= e^{-\beta t} \times \left(\frac{e^{\tau(\beta-\alpha)}}{\beta-\alpha}\right)_0^t$$

$$= e^{-\beta t} \times \left(\frac{e^{t(\beta-\alpha)}}{\beta-\alpha} - \frac{1}{\beta-\alpha}\right)$$

For $\alpha = \beta$

$$= e^{-\beta t} \times \int_0^t d\tau$$

$$= t e^{-\beta t}$$

b) $x(t) = u(t) - 2u(t-2) + u(t-5)$

$h(t) = e^{2t} u(1-t)$

A) $y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

for $t<1$: 0

for $1 \le t \le 3$

$\int_{0}^{t-1} e^{2(t-\tau)} d\tau$

for $3 \le t \le 6$

$y(t) = \int_{0}^{2} e^{2(t-\tau)} d\tau - \int_{2}^{t-1} e^{2(t-\tau)} d\tau$

for $t>6 \Rightarrow y(t) = 0$

c) $x(t) = \sin \pi t$

Let $h(t) = u(t-1) - u(t-3)$

$= \int_{0}^{2} \sin(\pi t) h(t-\tau) d\tau$

$y(t) = 0 \rightarrow t \le 1$

$y(t) = \frac{2}{\pi} [1 - \cos(\pi(t-1))]$ for $1 < t < 3$

$y(t) = 0$ for $t > 5$

d)

d) $h(t) = h_1(t) - \frac{1}{3}\delta(t-2)$

$y(t) = h^*(t) * x(t)$

$= h_1(t) * x(t) - \frac{1}{3}x(t-2)$

$h_1(t) * x(t) = \frac{4}{3}\left[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2\right]$

$+ \frac{4}{3}\left[bt - bt + b\right]$

$y(t) = \frac{4}{3}\left[\frac{1}{2}at^2 - \frac{1}{2}(a)(t-1)^2 + bt - b(t-1)\right]$

$- \frac{1}{3}\left[\left(\frac{1}{3}a(t-2) + b\right)\right]$