Electromagnetic Waves:

23/03/2022

① Gauss law:

$$\oint \vec{D} \cdot \vec{ds} = Q_{enc} \longrightarrow \nabla \cdot \vec{D} = \rho_v$$

$$= \int_V \rho_v \, dV$$

$$\oint \vec{B} \cdot \vec{ds} = 0 \longrightarrow \nabla \cdot \vec{B} = 0$$

② Ampere's law:

- $$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$

$$= \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \vec{ds}$$

$(\vec{B} = \mu \vec{H})$

$(\vec{B} = \mu_0 \mu_r \vec{H})$

- $$\nabla \times \vec{H} = \vec{J_c} + \vec{J_D}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

③ Faraday's law:

- $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$(\vec{D} = \varepsilon \vec{E})$

$(\vec{D} = \varepsilon_0 \varepsilon_r \vec{E})$

- $$\oint \vec{E} \cdot \vec{dl} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$= -\frac{\partial}{\partial t} \int \vec{B} \cdot \vec{ds}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \psi_m}{\partial t}$$

→ Harmonics:

$$\overline{E} = \overline{E_s} \cdot e^{j\omega t}$$
$$\vec{D} = \vec{D_s} \cdot e^{j\omega t}$$
$$\vec{H} = \vec{H_s} \cdot e^{j\omega t}$$
$$\vec{B} = \vec{B_s} \cdot e^{j\omega t}$$

$$\rho_v = (\rho_v)_s \cdot e^{j\omega t}$$

From the above laws,

(i) $\nabla \cdot \vec{D_s} = \rho_{v_s}$

(ii) $\nabla \cdot \vec{B_s} = 0$

(iii) $\nabla \times \vec{H_s} = \sigma \vec{E_s} + j\omega \vec{D_s}$

(iv) $\nabla \times \vec{E_s} = -j\omega \vec{B_s}$

→ Removed the time dependance in the equations.

⇒ for a source-free medium,

↳ $\boxed{\rho_v = 0, \sigma \vec{E} = 0}$

then, (i) $\nabla \cdot \vec{D_s} = 0$

(ii) $\nabla \cdot \vec{B_s} = 0$

(iii) $\nabla \times \vec{H_s} = j\omega \vec{D_s} = j\omega \varepsilon \vec{E_s}$

(iv) $\nabla \times \vec{E_s} = -j\omega \vec{B_s} = -j\omega \mu \vec{H_s}$

- Homogeneous → μ, ε are constant
- Isotropic → μ, ε are not functions of direction.

(1)
$$\nabla \times \vec{E} = -j\omega\mu \vec{H}$$

$$\nabla \times (\nabla \times \vec{E}) = -j\omega\mu (\nabla \times \vec{H})$$

$$\nabla(\nabla \cdot (\vec{E})) - \nabla^2 \vec{E} = -j\omega\mu (\nabla \times \vec{H})$$

$$\nabla(\cancel{\nabla \cdot \vec{E}}) - \nabla^2 \vec{E} = -j\omega\mu (j\omega\varepsilon \vec{E})$$

0 (since it is source-free)

$$\therefore \boxed{\nabla^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E}}$$

(2)
$$\nabla \times (\nabla \times \vec{H}) = j\omega\varepsilon (\nabla \times \vec{E})$$

$$\nabla(\cancel{\nabla \cdot \vec{H}}) - \nabla^2 \vec{H} = j\omega\varepsilon (-j\omega\mu \vec{H})$$

0
$$\therefore \boxed{\nabla^2 \vec{H} = -\omega^2 \mu \varepsilon \vec{H}}$$

<u>Wave</u>: Any physical quantity that varies with time and space is called as wave.

$$\frac{d^2 V}{dx^2} = \frac{1}{u^2} \frac{d^2 V}{dt^2}$$

$$V = V^+ e^{j(\omega t - \beta x)} + V^- e^{j(\omega t + \beta x)}$$

$$(V = V_s e^{j\omega t})$$

$$\frac{d^2 V_s}{dx^2} = -\frac{\omega^2}{u^2} \cdot V_s$$

$$\frac{d^2 V_s}{dx^2} + \frac{\omega^2}{u^2} V_s = 0$$

$$\boxed{\frac{d^2 V_s}{dx^2} + \beta^2 V_s = 0} \longrightarrow \boxed{V_s = V^+ e^{-j\beta x} + V^- e^{j\beta x}}$$

$$\frac{\omega}{u} = \beta$$

$u \rightarrow$ velocity of wave

$\beta \rightarrow$ phase constant

$$\boxed{u = \omega/\beta}$$

$$\boxed{\beta = 2\pi/\lambda}$$

$\vec{H}, \vec{E}$ are oriented in one direction,

$$\bar{E} = E_x \hat{x} \quad (E_y = E_z = 0)$$

$$\bar{H} = H_y \hat{y} \quad (H_x = H_z = 0)$$

$\vec{E}$ is varying only in the direction of propagation

$$\therefore \left(\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \right)$$

$\rightarrow \nabla^2 E_x = -\omega^2 \mu \varepsilon E_x$

$$\therefore \boxed{\frac{\partial^2}{\partial z^2} E_x = -\omega^2 \mu \varepsilon E_x}$$

$$(\beta^2 = \omega^2 \mu \varepsilon)$$

$$\frac{d^2 E_x}{dz^2} + \beta^2 E_x = 0$$

$$\therefore \boxed{E_x = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}}$$

$$\boxed{E_x \cdot e^{j\omega t} = E_x^+ e^{j(\omega t - \beta z)} + E_x^- e^{j(\omega t + \beta z)}}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -\hat{y}\left(-\frac{\partial}{\partial z}E_x\right)$$

$$\Rightarrow \frac{\partial E_x}{\partial z}\hat{y} = -j\omega\mu\vec{H}$$

$$\frac{\partial E_x}{\partial z}\hat{y} = -j\omega\mu\left(\cancel{H_x\hat{x}}^{\,0} + H_y\hat{y} + \cancel{H_z\hat{z}}^{\,0}\right)$$

$$= -j\omega\mu H_y\hat{y}$$

$$\boxed{\frac{\partial E_x}{\partial z} = -j\omega\mu H_y}$$

$$\left.\begin{aligned} \frac{\partial}{\partial z}E_x &= -j\beta E_x^{+}\left(e^{-j\beta z}\right) + j\beta E_x^{-}\left(e^{j\beta z}\right) \\ &= -j\omega\mu H_y \end{aligned}\right\} - (1)$$

$$H_y = \frac{+j\beta}{+j\omega\mu}E_x^{+}e^{-j\beta z} + \frac{-j\beta}{+j\omega\mu}E_x^{-}e^{j\beta z}$$

$$\left.\begin{aligned} H_y &= H_y^{+}e^{-j\beta z} - H_y^{-}e^{j\beta z} \\ &= \frac{\beta}{\omega\mu}E_x^{+}e^{-j\beta z} - \frac{\beta}{\omega\mu}E_x^{-}e^{j\beta z} \end{aligned}\right\} - (2)$$

From ① & ②

$$\frac{E_x^+}{H_y^+} = \frac{\omega\mu}{\beta} = \eta_0$$ → intrinsic impedance of the medium.

$$\frac{E_x^-}{H_y^-} = -\frac{\omega\mu}{\beta} = -\eta_0$$

$$\boxed{\text{Velocity } (u) = \frac{1}{\sqrt{\mu\varepsilon}}}$$

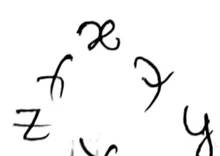
$$u = \frac{\omega}{\beta} \rightarrow \boxed{\beta = \omega\sqrt{\mu\varepsilon}}$$

$$\eta_0 = \frac{\omega\mu}{\beta} = \frac{\cancel{\omega}\mu}{\cancel{\omega}\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\boxed{\therefore \eta_0 = \sqrt{\frac{\mu}{\varepsilon}}}$$

For a free space,

$$\eta_0 = 877\,\Omega$$
$$= 120\pi\,\Omega$$



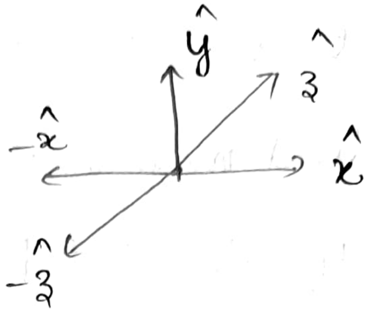
$$\left.\begin{aligned} \hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y} \end{aligned}\right\} \rightarrow \vec{E} \times \vec{H} = \text{direction}$$

The above waves are called Transverse-electromagnetic wave (TEM wave)

→ $\vec{E}$ is in y-direction and wave is in z-direction

$$\bar{E} = E_y^+ e^{j(\omega t - \beta z)} \hat{y}$$

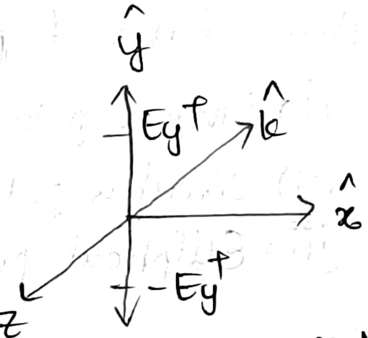
$$\bar{H} = -H_x^+ e^{j(\omega t - \beta z)}$$



$$\vec{E} = E_{y_1}^+ e^{j(\omega t + \beta z)} \hat{y}$$

$$\vec{H} = H_{x_1}^+ e^{j(\omega t + \beta z)} \hat{x}$$

$$\boxed{\frac{E_y^+}{-H_x^+} = \eta_0 = \frac{E_{y_1}^+}{H_{x_1}^+}}$$



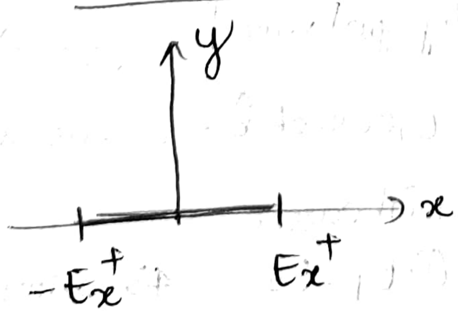
$$\bar{E_x} = E_y^+ \cos(\omega t - \beta z) \hat{y}$$

if $z = z_1 = 0$

$\bar{E} = E_y^+ \cos(\omega t) \hat{y}$ → locus of electric field in a given plane ⊥ar to direction of propagation is said to be polarisation.

$$\bar{E_x} = E_x^+ \cos(\omega t - \beta z)$$

for z=0 plane,



Generalised electric field expression is,

$$\bar{E} = E_1 \cos(\omega t - \beta z)\hat{x} + E_2 \cos(\omega t - \beta z + \delta)\hat{y}.$$

Generalised magnetic field expression is,

$$\vec{H} = H_1 \cos(\omega t - \beta z)\hat{y} - H_2 \cos(\omega t - \beta z + \delta)\hat{x}.$$

So, $\frac{E_1}{H_1} = \eta_0 = \frac{E_2}{H_2}$

→ Three states of polarisation are:

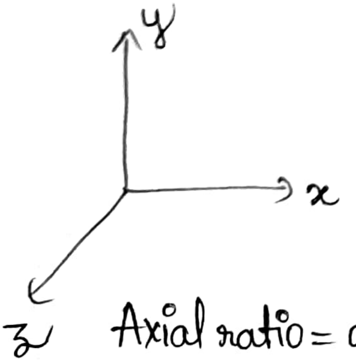
(i) Linear polarisation.
(ii) Circular polarisation.
(iii) Elliptical polarisation.

→ **Polarisation:** Locus of $\vec{E}$ in a plane perpendicular to the direction of propagation.

- The states of polarisation is differed by amplitude and phase.

(i) **Linear Polarisation:**

If locus of $\vec{E}$ is in a straight line then it is said to be linearly polarised. $(\delta = 0)$



$$\bar{E} = E_1 \cos\omega t\,\hat{x} + E_2 \cos\omega t\,\hat{y}$$

Special cases:

1. $E_1 = E_2$ 45° oriented
2. If $E_1 = 0$, Vertical polarised wave.
3. If $E_2 = 0$, Horizontal polarised wave.

Axial ratio $= \infty$

(ii) Circular polarisation:

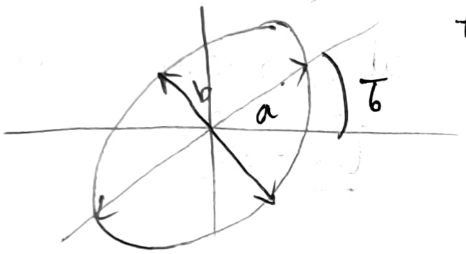
① $E_1 = E_2$ and $\delta = \pm \pi/2$

Axial ratio = 1

$0 \le \tau \le \pi$

(iii) Elliptical polarisation:

① $E_1 \ne E_2$ and $\delta = \pm$ angle



Axial ratio = a/b

$\tau = \tan^{-1}(E_2/E_1)$

eg: $\vec{E} = 10(\hat{a}_y + j\,\hat{a}_z)\, e^{-j(25)x}$

Determine the state of polarisation.

Sol:- $\vec{E} \cdot e^{j\omega t} = 10(\hat{a}_y + j a_z)\, e^{-j(25)x}\, e^{j\omega t}$

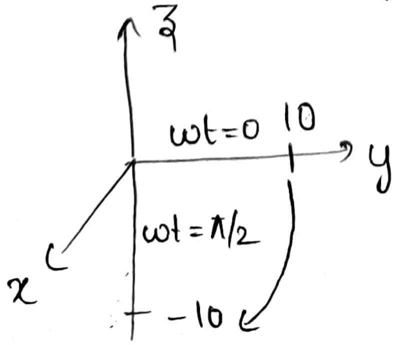
$= 10\cos(\omega t - 25x)\hat{a}_y + 10\cos(\omega t - 25x + \pi/2)\hat{a}_z$

if $x = 0$, yz-plane

$\vec{E} = 10\cos(\omega t)\,\hat{a}_y + 10\cos(\omega t + \pi/2)\,\hat{a}_z$

$\omega t = 0 \rightarrow \bar{E} = 10\hat{a}_y$

$\omega t = \pi/2 \rightarrow \bar{E} = -10\hat{a}_z$



Right-Hand Circular polarisation (R.H.C.P) eg: $\vec{E} = 5\cos(2\pi 10^9 t + \beta z)\,\hat{a}_x + 3\cos(2\pi 10^9 t + \beta z - \pi/2)\,\hat{a}_y$

Since $E_1 \neq E_2$

it is Elliptically polarised

if $z = 0 \longrightarrow$ XY plane

$\vec{E} = 5\cos(2\pi 10^9 t)\,\hat{a}_x + 3\cos(2\pi 10^9 t - \pi/2)\,\hat{a}_y$

at $t_1 = 0$

$\underline{\vec{E} = 5\hat{a}_x}$

at $t_2 \rightarrow 2\pi 10^9 t = \pi/2$

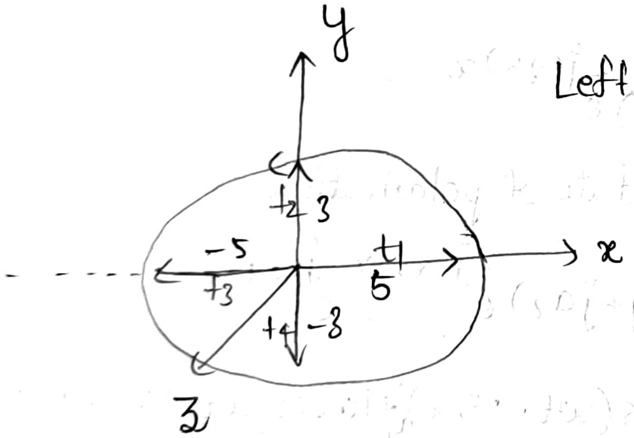
$\underline{\vec{E} = 3\hat{a}_y}$

at $t_3 \rightarrow 2\pi 10^9 t = \pi$

$\underline{\vec{E} = -5\hat{a}_x}$

at $t_4 \rightarrow 2\pi 10^9 t = 3\pi/2$

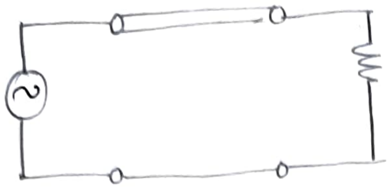
$\underline{\vec{E} = -3\hat{a}_y}$



Left-Hand Elliptical polarisation

(L.H.E.P)

Magnetic field - current



→ for the magnetic field to flow in a medium, the medium need not be highly conductive.

$$\nabla \times \vec{H} = \vec{J_c} + \frac{\partial \vec{D}}{\partial t}$$

→ For the current wave to propagate in an electrical circuit, conductivity is essential.

→ EM wave can't propagate in a highly conductive medium (it reflects from the walls of conductive materials).

$$\nabla \times \vec{H} = \vec{J_c} + \vec{J_D}$$

$$= \vec{J_c} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + j\omega\varepsilon\vec{E}$$

$$= (\sigma + j\omega\varepsilon)\vec{E}$$

ε → pamittivity of medium, $\varepsilon = \varepsilon_0 \varepsilon_r$.

$$\nabla \times \vec{H} = j\omega\varepsilon_0\left(\varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0}\right)\vec{E}$$

$\left(\varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0}\right)$ → complex dielectric constant.

→ For a highly, perfect, dielectric, imaginary part = 0

$\left(\frac{\sigma}{\omega\varepsilon_0} = 0\right)$ → no conductivity ($\sigma = 0$)

→ Medium can be classified into

(i) Pure dielectric medium

$$\frac{\sigma}{\omega \varepsilon_0} = 0$$

(ii) low loss dielectric medium

$$\varepsilon_r >> \frac{\sigma}{\omega \varepsilon_0}$$

(iii) lossy dielectric medium

$$\omega \varepsilon_0 \varepsilon_r \simeq \sigma$$

(iv) conductor medium

$$\sigma >> \omega \varepsilon_0 \varepsilon_r$$

(v) Perfect conductor

$$\sigma = \infty$$

complex dielectric constant.

$$\varepsilon_{rc} = \left(\varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right)$$

→ $$\nabla^2 \vec{E} = -\omega^2 \mu \, \varepsilon_0 \varepsilon_{rc} \vec{E}$$

$$= -\omega^2 \mu \varepsilon_0 \left(\varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right) \vec{E}$$

$$\nabla^2 \vec{E} = \gamma^2 \vec{E}$$

where, $\gamma^2 = -\omega^2 \mu \, \varepsilon_0 \left[\varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right]$

$$\therefore \gamma = j\omega \sqrt{\mu \varepsilon_0} \left[\varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right]^{1/2}$$

Assume, $\gamma = \alpha + j\beta$

* $$Re(\gamma) = \alpha = \omega \sqrt{\frac{\mu \varepsilon_0 \varepsilon_r}{2}} \left\{ \sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_0^2 \varepsilon_r^2}} - 1 \right\}^{1/2}$$

* $$Im(\gamma) = \beta = \omega \sqrt{\frac{\mu \varepsilon_0 \varepsilon_r}{2}} \left\{ \sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_0^2 \varepsilon_r^2}} + 1 \right\}^{1/2}$$

→ In low loss dielectric medium,

$$\therefore \alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon_0 \varepsilon_r}}$$

$$\beta = \omega\sqrt{\mu\varepsilon_0\varepsilon_r}\left\{1+\frac{1}{8}\frac{\sigma^2}{\omega^2\varepsilon_0^2\varepsilon_r^2}\right\}^{1/2}$$

$$\therefore \beta \simeq \omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma+j\omega\varepsilon)}}$$

$$\therefore Re(\eta) = \sqrt{\frac{\mu}{\varepsilon}}$$

→ Lossy nature of dielectric can be represented using lossy tangent

lower the tanδ, better the dielectric property of the medium.

$$\text{lossy tangent } (\tan\delta) = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r}$$

→ For a wave propagation in conductor medium,
$(\sigma >> \omega\varepsilon_0\varepsilon_r)$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad (\alpha=\beta)$$

$$\gamma = \alpha + j\beta$$

$$\therefore \gamma = \sqrt{\frac{\omega\mu\sigma}{2}}(1+j)$$

$$\therefore \eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\,e^{j\pi/4}$$

$$\therefore \frac{E_0}{H_0} = \eta = \sqrt{\frac{\omega\mu}{\sigma}}\,e^{j\pi/4}$$

→ Electric field advances magnetic field by $\pi/4$.

→ The wave propagation in a conductor is very similar to lossy transmission line.

→ Electric field gets attenuated by e^{-1}
(in case of wave propagation in conductor medium)

$$0.367E_0 = E_0 e^{-1} = E_0 e^{-\alpha z_0}$$
$$\downarrow$$
$$\boxed{z_0 = 1/\alpha}$$

- After traveling a distance of 'z_0', wave amplitude decays by e^{-1}.

$$\text{Power} = |0.367 E_0|^2$$
$$= 0.135 |E_0|^2$$

- The distance 'z_0' is called <u>skindepth</u>

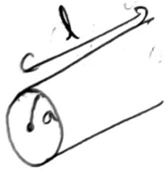
around 70% of power gets attenuated

→ Skindepth of conductor,

$$\text{skin-depth } (\delta) = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

(wave doesn't propagate in conductor medium)

→ DC resistance offered by wire at low frequency



$$R_{DC} = \frac{l}{\sigma \times (\omega \times t)}$$

a – radius
l – length

$$\boxed{\therefore R_{DC} = \frac{l}{\sigma \times (\pi a^2)}}$$

→ AC resistance offered,

$$\boxed{\therefore R_{AC} = \frac{l}{\sigma \times 2\pi a \times \delta}}$$

$\delta \to$ skin-depth.

→ At high frequencies, $\delta \downarrow$es and $R_{AC} \uparrow$es.

Q) Conductivity of copper $(\sigma) = 5.8 \times 10^{7}$ s/m

$\varepsilon_r = \mu_r = 1$

$a = 1.2$ mm, $l = 600$ m

Determine R_{DC}, R_{AC} at 100 MHz.

Sol:-

$$R_{DC} = \frac{l}{\sigma \times \pi a^2}$$

$$= \frac{600}{5.8 \times 10^{7} \times \pi \times 1.44 \times 10^{-6}}$$

$$\therefore \underline{\underline{R_{DC} = 2.287\,\Omega}}$$

$$\text{skin-depth}(\delta) = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

$$\delta = \sqrt{\frac{1}{\pi \times 100 \times 10^{6} \times 4\pi \times \cancel{10^{-7}} \times 5.8 \times \cancel{10^{7}}}}$$

$$\delta = \sqrt{\frac{10^{-8}}{4\pi^2 \times 5.8}}$$

$$\therefore \underline{\underline{\delta = 6.6 \times 10^{-6}\ \text{m}}}$$

$$R_{AC} = \frac{l}{\sigma \times 2\pi a \times \delta}$$

$$= \frac{600}{5.8 \times 10^{7} \times 2\pi \times 1.2 \times 10^{-3} \times 6.6 \times 10^{-6}}$$

$$\therefore \underline{\underline{R_{AC} = 207.61\,\Omega}}$$

Poynting Theorem:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \rightarrow \text{Faraday's law}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \rightarrow \text{Ampere's law}$$

$$\vec{E} \cdot (\nabla \times \vec{H}) = \sigma |\vec{E}|^2 + \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

(By property,
$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$)

$$\vec{H} \cdot (\nabla \times \vec{E}) + \nabla \cdot (\vec{H} \times \vec{E}) = \sigma |E|^2 + \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} \quad \text{--- (1)}$$

$$\Rightarrow \frac{\partial |E|^2}{\partial t} = \frac{\partial}{\partial t} \vec{E} \cdot \vec{E}$$

$$= \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$= 2 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial |E|^2}{\partial t}$$

$$\rightarrow \vec{H} \cdot (\nabla \times \vec{E}) = -\vec{H} \cdot \mu \frac{\partial \vec{H}}{\partial t}$$

$$= -\frac{1}{2} \mu \frac{\partial}{\partial t} |\vec{H}|^2$$

So, the equation (1) is written as,

$$-\frac{\mu}{2}\frac{\partial}{\partial t}|\vec{H}|^2 - \nabla\cdot(\vec{E}\times\vec{H}) = \sigma|E|^2 + \frac{\varepsilon}{2}\frac{\partial}{\partial t}|\vec{E}|^2$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\sigma|E|^2 - \frac{\varepsilon}{2}\frac{\partial}{\partial t}|\vec{E}|^2 - \frac{\mu}{2}\frac{\partial}{\partial t}|\vec{H}|^2$$

Applying volume integral,

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv = \int_V -\sigma|E|^2\,dv - \int_V \frac{\mu}{2}\frac{\partial}{\partial t}|\vec{E}|^2\,dv - \int_V \frac{\varepsilon}{2}\frac{\partial}{\partial t}|\vec{E}|^2\,dv$$

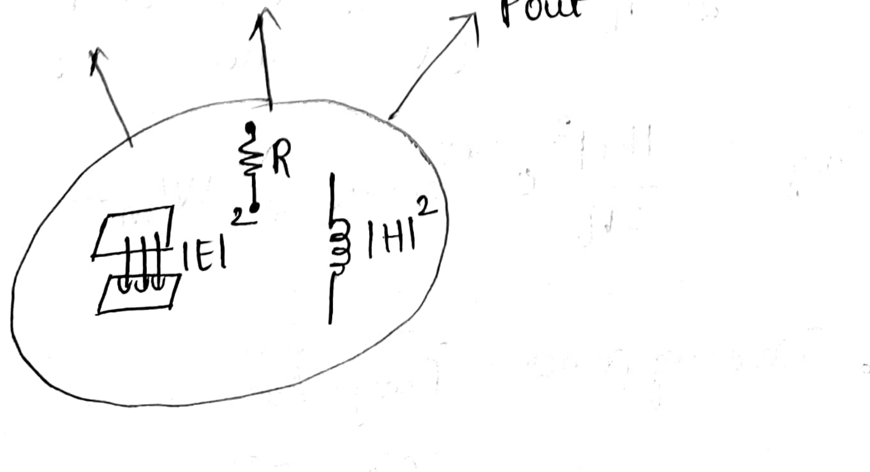
Divergence theorem.

ohmic power loss

Rate of reduction of magnetic energy

$$\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds} = \int_V -\sigma|E|^2\,dv - \frac{\mu}{2}\int_V \frac{\partial}{\partial t}|\vec{H}|^2\,dv - \frac{\varepsilon}{2}\int_V \frac{\partial}{\partial t}|E|^2\,dv$$

→ For a outward flow of power, there must be reduction in magnetic energy in such a way that difference b/w them is equal to ohmic losses.



$$\vec{P} = \vec{E} \times \vec{H}$$

$$\vec{E} = V/m \qquad E \times \vec{H} = Watt/m^2$$

Poynting vector.

$$\vec{H} = A/m$$

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{x}$$

$$\vec{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z - \theta_n)\, \hat{y}$$

$$\left(\frac{E_0}{H_0} = |\eta| \right)$$

$$\vec{P} = E_0 H_0 e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_n) \cdot \hat{z}$$

$$\therefore \vec{P} = \frac{|E_0|^2}{\eta} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_n)\, \hat{z}$$

$$\vec{P}_{avg} = \frac{1}{T} \int_0^T \vec{P} \cdot dt$$

$$= \frac{1}{T} \int_0^T \frac{|E_0|^2}{\eta} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_n)\, \hat{z}\, dt$$

$$= \frac{1}{2T} \int_0^T \frac{|E_0|^2}{\eta} e^{-2\alpha z} \left[\cos(2\omega t - 2\beta z - \theta_n) + \cos(\theta_n) \right] dt\, \hat{z}$$

$$= \frac{1}{2T} \frac{|E_0|^2}{\eta} e^{-2\alpha z} \left[\cancel{\left(\frac{\sin(2\omega t - 2\beta z - \theta_n)}{2\omega} \right)_0^T}^{\,0} + T \cos\theta_n \right] \hat{z}$$

$$\therefore \vec{P}_{avg} = \frac{|E_0|^2}{2\eta} e^{-2\alpha z} \cos\theta_n\, \hat{z} \quad W/m^2$$

$$\therefore \text{Time avg power} = \vec{P}_{avg} \cdot \vec{Area}$$

$$= \int_S \vec{P}_{avg} \cdot \vec{ds} \;//$$

Generalised equation for time-average power is,

$$\vec{P}_{avg} = \frac{1}{2} \mathrm{Re}(\vec{E} \times \vec{H}^*)$$

eg: Magnetic field of EM wave is given by

$$\vec{H} = \frac{5\sqrt{3}}{\eta} \cos(\omega t - \beta z)\hat{a}_x + \frac{5}{\eta}\sin(\omega t - \beta z + \pi/2)\hat{a}_y \text{ A/m}$$

Determine the time-average power density.

Sol: ~~$\vec{E} = \eta \cdot \vec{H}$~~

~~$= 5\sqrt{3}\cos(\omega t - \beta z)\hat{a}_x + 5\sin(\omega t - \beta z + \pi/2)\hat{a}_y$~~

$$\vec{E} = \eta \cdot \vec{H}$$

$$\vec{E} = 5\sqrt{3}\cos(\omega t - \beta z)(-\hat{a}_y) + 5\sin(\omega t - \beta z + \pi/2)\hat{a}_x$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= \left(\frac{(5\sqrt{3})^2}{\eta}\hat{a}_z + \frac{(5)^2}{\eta}\right) \times \frac{1}{2} \cdot \hat{a}_z$$