Solutions

(1) $f_2(t) = f(t-1) + f_1(t-1)$

$f_3(t) = f(t-1) + f_1(t+1)$

$f_4(t) = f(t-0.5) + f_1(t+0.5)$

$f_5(t) = 1.5\, f\left(\frac{t}{2} - 1\right)$

(2) $\omega_0 = \frac{\pi}{6}$, $T_0 = \frac{2\pi}{\omega_0} = 12$ sec

(3) The system is Linear

Additive $\frac{d^n y_1}{dt^n} + \cdots + a_0 y_1 = b_m \frac{d^m x_1}{dt} + \cdots + b_0 x_1 \rightarrow (1)$

$\frac{d^n y_2}{dt^n} + \cdots + a_0 y_2 = b_m \frac{d^m x_2}{dt} + \cdots + b_0 x_2 \rightarrow (2)$

(1)+(2) $\frac{d^n}{dt^n}(y_1+y_2) + \cdots + a_0(y_1+y_2) = b_m \frac{d^m}{dt}(x_1+x_2) + \cdots + b_0(x_1+x_2)$

Above equation becomes $y(t) = y_1(t) + y_2(t)$, $x(t) = x_1(t) + x_2(t)$ } when the I/P is $x_1(t)+x_2(t)$ the O/P is $y_1(t)+y_2(t)$.

∴ The system is Linear.

(4) $E_{x(t)} = \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} 4e^{-t} dt = 4 + 4 = 8$

$P_{x(t)} = 0$

$E_{y(t)} = \infty$

$P_{y(t)} = \frac{1}{2} \int_{-1}^{1} y^2(t)\, dt$

$= \frac{1}{2} \int_{-1}^{1} t^2 dt = \frac{1}{3}$

(5) Even Part of the signal

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{e^{-at}u(t) + e^{+at}u(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{e^{-at}u(t) - e^{at}u(-t)}{2}$$