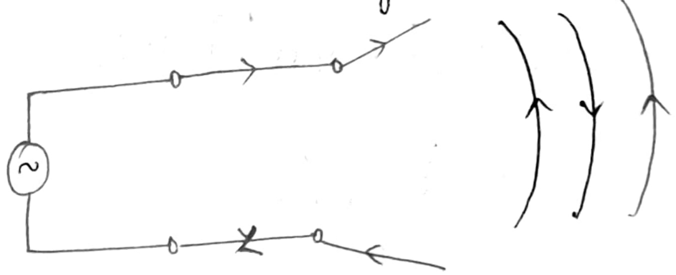
23/08/22

EC305

- **Antenna:** It is a transducer which converts electric signals into electromagnetic waves.

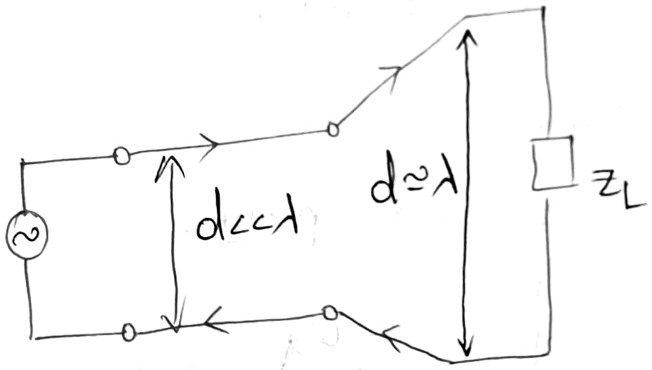


→ Radiation is evolved by two processes.

(i) Time varying charge.

(ii) Spacial Imbalance

(It is nothing but distance b/w the lines is equal to wavelength of radiation $d \simeq \lambda$.)



- **Transverse Electromagnetic wave:** (travels in the same direction as the wave)

$$\vec{E} = E_0 e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{j\omega t} \hat{x} \ V/m$$

$$\vec{H} = \frac{E_0}{\eta_0} e^{-\alpha z} e^{-j\beta z} e^{j\omega t} \hat{y} \ A/m$$

$$\boxed{\vec{E} \times \vec{H} = \text{direction}} \qquad \boxed{\eta_0 = \frac{|\vec{E}|}{|\vec{H}|}}$$

$$\vec{P}(t) = \vec{E} \times \vec{H} \quad W/m^2$$

$$\text{Power} = \int_S \vec{P}_{avg} \cdot \vec{ds}$$

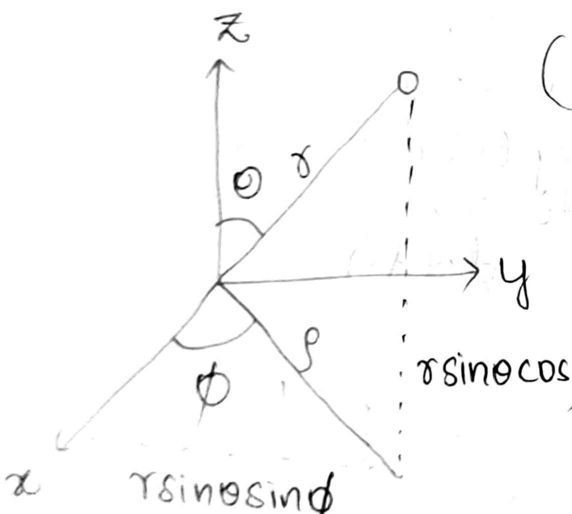
$$\vec{P}_{avg} = \frac{1}{T}\int_0^T \vec{P}(t)\, dt$$

power density

$$= \frac{1}{2} \operatorname{Re}\left[\vec{E} \times \vec{H}^*\right] \quad W/m^2$$

$$= \frac{1}{2} \frac{|\vec{E}|^2}{\eta_0} \cdot \hat{a}_z$$

- Spherical co-ordinate system:



(r, θ, ϕ)

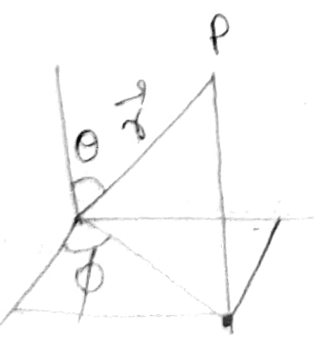
r — radius

θ — 0 to π

ϕ — 0 to 2π

$\rho = r\sin\theta$

- Projection of r in XY-plane:



Differential Area

$ds_1 = r\sin\theta\, d\phi\, dr$

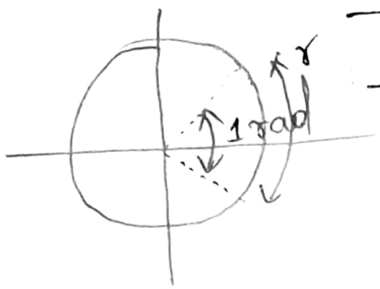
$ds_2 = (dr)\, r\, d\theta$

$ds_3 = r\sin\theta\, (d\phi)\, r\, d\theta$

$\boxed{r\,d\theta \; r\sin\theta\, d\phi}$

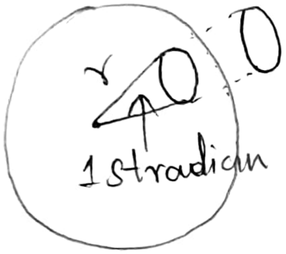
$$\boxed{\vec{ds} = \hat{r} \cdot r\,d\theta \cdot r\sin\theta\, d\phi}$$

Radian & Steradian:



→ Circle has a total angle of 2π rad
→ 1 radian is angle created by an arc of length 'r'.

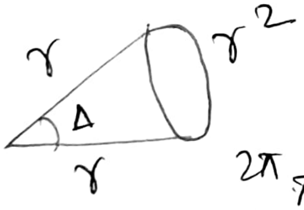
1 radian = Arclength $\hat{r}$.



$A = r^2$

→ Sphere has a total solid angle of 4π rad
→ 1 steradian is square of angle (solid angle) created by cone of surface area "r^2"

Unit of solid angle is Steradian



Surface area of sphere $= 4\pi r^2$

$$ds = \int_0^{2\pi}\int_0^{\pi} (r\,d\theta)(r\sin\theta)\,d\phi$$

$$\int ds = 4\pi r^2$$

$$2\pi r^2 \int_0^{\pi} \sin d\theta = \underline{4\pi r^2}$$

$4\pi r^2$ = Total surface area of sphere.

4π = Total solid angle of sphere

$$d\Omega = \frac{ds}{r^2} = \frac{(\cancel{r}\sin\theta)(\cancel{r}d\theta)\,d\phi}{\cancel{r^2}}$$

$$d\Omega = \frac{ds}{r^2} = \sin\theta\, d\theta\, d\phi$$

$$\int d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta\, d\theta\, d\phi$$

$$= 2\pi \int_0^{\pi} \sin\theta\, d\theta$$

$$d\Omega = 4\pi$$

$$\vec{P}_{avg} = \frac{|E|^2}{2\eta_0}\,\hat{a}_r$$

$$\boxed{\text{Power} = \frac{|E|^2}{2\eta_0}\cdot 4\pi r^2}$$

$$\text{Power} = \int \vec{P}_{avg}\cdot \vec{ds}$$

$$\boxed{\text{Power} = P_{avg}\cdot(\text{area})}$$

$$\boxed{\text{Power} = |P_{avg}|\cdot(4\pi r^2)}$$

$$P_{avg} = \frac{\text{Power}}{4\pi r^2}$$

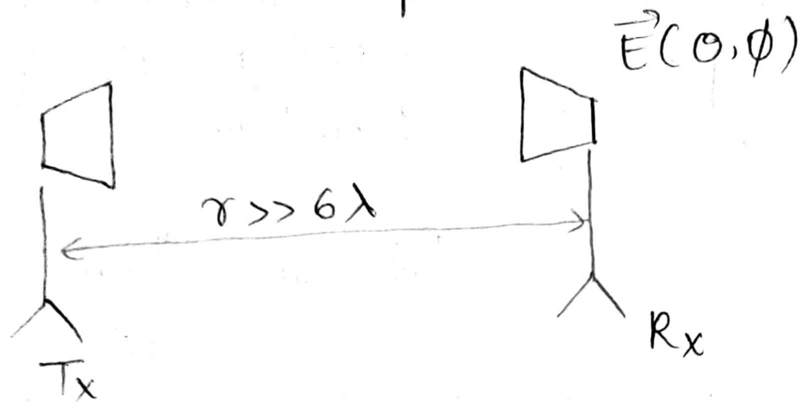
$$\boxed{\text{Radiation Intensity }(U) = P_{avg}(r^2) = \frac{\text{Power}}{4\pi}}$$

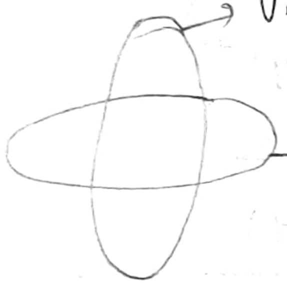
↓

Power per solid angle

$$\boxed{\vec{E} = \frac{E_0}{4\pi r}\cdot e^{-j(\omega t-\beta r)}\,\hat{\theta}}$$

- Radiation pattern: ~~data~~ plot of electromagnetic field intensity in a space.





→ Vertical Plane (Elevation plane) (YZ-plane) → $\phi = 90^\circ$, θ varies

→ Horizontal plane (Azimuthal plane) (XY-plane) → $\theta = 90^\circ$, ϕ varies

→ We can understand the radiation pattern of an antenna by finding electromagnetic field along Azimuthal and Elevation planes.

→ Isotropic radiation: Radiation is same along the whole surface.

- Directional pattern
- Omnidirectional pattern
- Radiation pattern

$$U_{rad} = \frac{\text{Power}}{4\pi}$$

→ The following antenna is radiating uniformly with Prad Watts.

$$P_{density} = \frac{P_{rad}}{4\pi r^2}$$

→ If antenna is not radiating uniformly then,

$$P_{density} = \frac{\iint_S P_{density} \cdot \vec{ds}}{4\pi r^2}$$

→ $$\text{Directivity}(\theta, \phi) = \frac{U(\theta, \phi)}{[\text{Avg radiation Intensity}]}$$

$$\boxed{\text{Directivity}(\theta, \phi) = \frac{U(\theta, \phi)}{U_{avg}}}$$

→ $$P_{density} = \frac{1}{2} \frac{|\vec{E}(\theta, \phi)|^2}{\eta_0}$$

$$U(\theta, \phi) = \frac{1}{2} \frac{|\vec{E}(\theta, \phi)|^2 \cdot r^2}{\eta}$$

$$\text{Space avg } P_{density} = \frac{\iint_S P_{density} \cdot \vec{ds}}{4\pi r^2}$$

$$U_{avg} = \frac{\iint P_{density} \cdot ds}{4\pi}$$

$$\boxed{D(\theta, \phi) = \frac{\frac{1}{2\eta} |\vec{E}(\theta, \phi)|^2 \cdot r^2}{\iint P_{den} \cdot \vec{ds}} \cdot 4\pi}$$

$$D(\theta,\phi) \Big| = \frac{\frac{1}{2\pi}|E(\theta,\phi)|^2 \, r^2 \cdot 4\pi}{\iint_S \frac{|E(\theta,\phi)|^2}{2\pi} \cdot r^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\iint_S \left(\frac{|E(\theta,\phi)|}{|E(\theta,\phi)|_{max}}\right)^2 \cdot \sin\theta \, d\theta \, d\phi}$$

$$D(\theta,\phi) = \frac{4\pi}{\iint_S |E(\theta,\phi)|_n^2 \sin\theta \, d\theta \, d\phi}$$

Examples:

1) $\vec{E} = \frac{E_0}{4\pi\varepsilon r} e^{j(\omega t - \beta r)} \hat{a}_\theta$

(i) isotropic antenna:

wave propagation = +ve r-direction

$$U(\theta,\phi) = \frac{P_{rad}}{d\Omega} = \left(\frac{P_{rad}}{dA}\right) \cdot r^2$$

$$= \text{Power density} \cdot r^2$$

$$\text{Power density} = \vec{P}_{avg} = \frac{|E|^2}{2\eta_0} \cdot \hat{a}_r$$

$$\vec{P}_{avg} = \frac{E_0^2}{(4\pi\varepsilon r)^2} \cdot \frac{1}{2\eta_0} \hat{a}_r$$

$$\boxed{U(\theta,\phi) = \frac{E_0^2}{(4\pi\varepsilon)^2} \cdot \frac{1}{2\eta_0}}$$

$$U_{avg} = \frac{\text{Power}}{4\pi} \quad \text{where power} = \iint_S \vec{P}_{avg} \cdot \vec{ds}$$

$$\vec{ds} = (r d\theta) \cdot (r \sin\theta \, d\phi) \cdot \hat{a}_r$$

$$\text{Power} = \iint_S \frac{E_0^2}{(4\pi\varepsilon r)^2} \cdot \frac{1}{2\eta_0} \hat{a}_r \, \vec{ds}$$

$$= \frac{E_0^2}{(4\pi\varepsilon)^2 2\eta_0} \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{E_0^2}{(4\pi\varepsilon)^2 \cdot 2\eta_0} \cdot (4\pi)$$

$$\text{Power} = \frac{E_0^2}{4\pi\varepsilon^2 \cdot (2\eta_0)}$$

$$\boxed{U_{avg} = \frac{\text{Power}}{4\pi} = \frac{E_0^2}{(4\pi\varepsilon)^2} \cdot \frac{1}{2\eta_0}}$$

$$U(\theta,\phi) = U_{avg}$$

for all values of θ, ϕ.

(ii) directional antenna:

const $= E_0$

$$\vec{E} = j \frac{\qquad}{4\pi\omega\varepsilon_0 r} \beta^2 \sin\theta \cdot e^{j(\omega t - \beta x)} \hat{a}_\theta$$

wave $=$ +ve r-direction

$$\vec{E} = j \frac{E_0 \sin\theta}{r} e^{j(\omega t - \beta r)} \hat{a}_\theta$$

$$\left(\text{Power density } \vec{P_{avg}} = \frac{|E_0|^2}{2\eta_0}\right)$$

$$\vec{P_{avg}} = \frac{E_0^2}{r^2} \cdot \frac{1}{2\eta_0} \sin^2\theta \ \hat{a_r}$$

$$U(\theta,\phi) = \frac{E_0^2}{2\eta_0} \sin^2\theta \qquad \left(U(\theta,\phi) = \vec{P_{avg}} \cdot r^2\right)$$

$$U_{avg} = \frac{\text{Power}}{4\pi}$$

$$\text{Power} = \iint_S \vec{P_{avg}} \cdot \vec{ds}$$

$$\text{Power} = \iint_S \frac{E_0^2}{2\eta_0} \sin^2\theta \ \hat{a_r} \cdot \vec{ds} \qquad \vec{ds} = (r\,d\theta)(r\sin\theta\,d\phi)\,\hat{r}$$

$$= \frac{E_0^2}{2\eta_0} \iint \sin^2\theta \cdot \sin\theta\,d\theta\,d\phi$$

$$= \frac{E_0^2}{2\eta_0} \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \cdot \sin\theta\,d\theta\,d\phi$$

$$= \frac{E_0^2}{2\eta_0} \int_0^{2\pi} \int_0^{\pi} (1-\cos^2\theta) \sin\theta\,d\theta\,d\phi$$

$$t = \cos\theta$$

$$= \frac{E_0^2}{2\eta_0} \int_0^{2\pi} d\phi \int_1^{-1} (1-t^2)\,dt$$

$$= \frac{E_0^2}{2\eta_0} \cdot (2\pi) \cdot \frac{4}{3}$$

$$= \frac{4E_0^2\pi}{3\eta_0} \qquad (\eta_0 = 120\pi)$$

$$\boxed{\text{Power} = \frac{E_0^2}{90}}$$

$$U_{avg} = \frac{Power}{4\pi} = \frac{E_0^2}{90 \times 4\pi}$$

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{E_0^2 \sin^2\theta}{2 \times 120\pi} \times \frac{90 \times 4\pi}{E_0^2}$$

$$\boxed{D(\theta,\phi) = \frac{3}{2}\sin^2\theta}$$

→ if $D(\theta = 90^\circ) \Rightarrow D(\theta,\phi) = 3/2$
↓
performs better compared to isotropic.

- Half Power Beam Width :

$$D(\theta,\phi) = \frac{3}{2}\sin^2\theta \qquad 0 \leq \theta \leq \pi$$

$$U(\theta,\phi) = \frac{E_0^2}{2\eta_0}\sin^2\theta$$

$$\left(\frac{U(\theta,\phi)_{max}}{2} = \frac{E_0^2}{2\eta_0} \times \frac{1}{2}\right)$$

if $U(\theta,\phi) = \frac{U(\theta,\phi)_{max}}{2}$

then $\Rightarrow \frac{\cancel{E_0^2}}{\cancel{2\eta_0}}\sin^2\theta_{HP} = \frac{\cancel{E_0^2}}{\cancel{2\eta_0}} \cdot \frac{1}{2}$

$$\sin\theta_{H.P} = \pm\frac{1}{\sqrt{2}}$$

$$\boxed{\theta_{H.P} = 45^\circ, 135^\circ}$$

$$\therefore \boxed{\theta_{H.P.B.W} = 90^\circ}$$

2) $U(\theta,\phi) = \cos^2(2\theta) \quad 0 \le \theta \le 90^\circ$

Determine HPBW.

Sol: $U = \cos^2(2\theta) \longrightarrow \frac{U_{max}}{2} = \frac{1}{2}$

$$\frac{1}{2} = \cos^2(2\theta) \Rightarrow \cos(2\theta) = \pm\frac{1}{\sqrt{2}}$$

$$2\theta = 45^\circ \text{ (or) } 135^\circ$$

$$\theta = 22.5^\circ \text{ (or) } 67.5^\circ$$

$$\therefore \boxed{\theta_{HPBW} = 45^\circ}$$

3) Prad $= 10\,W$; max power density at 1000 m from antenna

Sol: $U(\theta,\phi) = B_0 \cos^2\theta$

$$P_{density} = \frac{U(\theta,\phi)}{r^2} \quad \Bigg| \quad \begin{array}{l} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array}$$

$$= \frac{\max(U(\theta,\phi))}{r^2}$$

$$P_{density} = \frac{B_0}{r^2}$$

$$P_{rad} = \iint U \cdot d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} B_0 \cos^2\theta \, \sin\theta \, d\theta \, d\phi$$

$$10\,W = \frac{4\pi}{3} B_0 \longrightarrow \boxed{B_0 = \frac{30}{4\pi}}$$ ✓

max power density,

$$\frac{B_0}{r^2} = \frac{30}{4\pi} \times \frac{1}{(1000)^2}$$

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}}$$

$$\left(U_{avg} = \frac{P_{rad}}{4\pi} \right)$$

$$D(\theta,\phi) = \frac{B_0 \cos^2\theta}{10/4\pi}$$

$$= \frac{\frac{30}{\cancel{4\pi}} \cos^2\theta}{10/\cancel{4\pi}}$$

$$\boxed{D(\theta,\phi) = 3\cos^2\theta}$$

→ In the direction of antenna,
max directivity is $\underline{\underline{3}}$

$$\text{HPBW} \Rightarrow \frac{\max(U(\theta,\phi))}{2} = U(\theta,\phi)$$

$$\frac{\cancel{B_0}}{2} = \cancel{B_0} \cos^2\theta$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ \text{ (or) } 135^\circ$$

$$\boxed{\theta_{HPBW} = 90^\circ}$$

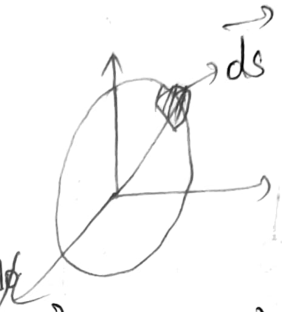
Radiation Resistance: (R_r)

Given, $\vec{E} = \dfrac{j I_o dl \beta^2 \sin\theta}{4\pi \varepsilon \omega r} e^{j(\omega t - \beta r)} \hat{a}_\theta$

$$\vec{P}_{avg} = \frac{1}{2\eta_o} |E|^2 \hat{a}_r$$

$$\vec{P}_{avg} = \frac{1}{2\eta_o} \frac{(I_o dl)^2 \beta^4}{(4\pi\omega\varepsilon r)^2} \sin^2\theta \, \hat{a}_r \quad W/m^2$$

$$P_{rad} = \iint \vec{P}_{avg} \cdot \vec{ds}$$



$$P_{rad} = \frac{1}{2\eta_o} \frac{I_o^2 dl^2 \beta^4}{(4\pi\omega\varepsilon r)^2} \int_0^{2\pi}\int_0^{\pi} r^2 \sin^3\theta \, d\theta \, d\phi$$

$$\vec{ds} = (r\sin\theta \, d\phi)(r \, d\theta) \, \hat{a}_r$$

$$P_{rad} = \frac{1}{2\eta_o} \frac{I_o^2 dl^2 \beta^4}{(4\pi\omega\varepsilon)^2} \left[\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\phi \right]$$

$$= (I_o dl)^2 .$$

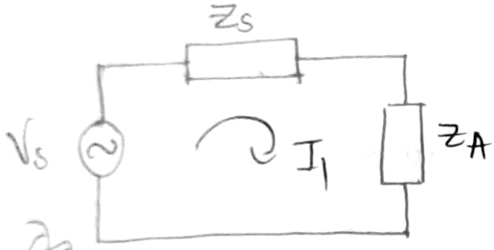
$\beta = \omega\sqrt{\mu\varepsilon}$

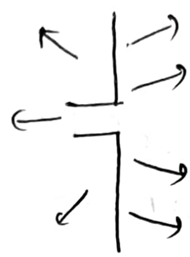
$\beta = 2\pi/\lambda$

$$\bigstar \quad R_r = \frac{2P_{rad}}{I_0^2} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

dl - length of antenna.

$Z_A = R_r + jX_A$

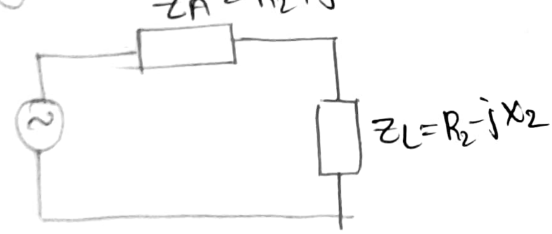


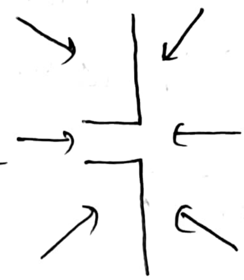


Equivalent circuit of transmitter antenna.

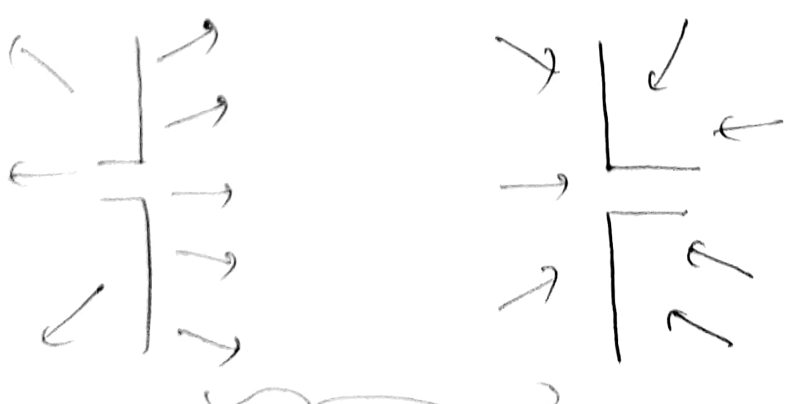
Z-mutual

$Z_A = R_2 + jX_2$





Equivalent circuit of Receiver antenna.



Z_{mut}
↓
for coupling b/w antenna's

→ for transmitter - directivity.

for Receiver - Effective appearance (Effective apperature)

(*) <u>Effective Apperature</u>: Power capturing ability of the antenna.
(θ, ϕ)

→ Pden of the incident wave.

→ It is directional dependent.

$$\underset{(\theta,\phi)}{\text{Power density}} = \frac{P_r}{4\pi r^2} \cdot D(\theta,\phi)$$

$$\text{Power radiated} (P_{rad}) = \frac{1}{2} I_2^2 R_2$$

$$I_2 = \frac{V_{oc}}{2R_2} \; ; \; V_{oc} = I_1 (Z_{mut})$$

$$\boxed{P_{load} = \frac{1}{2} \frac{V_{oc}^2}{(4R_2)} = \frac{1}{8R_2} |Z_{mut}|^2 |I_1|^2}$$

$$\text{Power transmitted} : \frac{1}{2} I_1^2 R_1$$

$$\text{Pdensity of radiating signal} = \frac{P_t}{4\pi r^2} \cdot D_r(\theta,\phi)$$

Power received by antenna at a distance 'r' from tx is $P_{den} * A_{E_2}(\theta,\phi)$

$$\boxed{P_{received} \frac{\frac{1}{2} I_1^2 R_1}{4\pi r^2} D_1 \times A_{e_2}}$$

A_e - effective aperature

$$\frac{1}{\cancel{8}R_2} |Z_{mut}|^2 |\cancel{I_1}|^2 = \frac{1}{\cancel{2}} \frac{\cancel{I_1^2} R_1}{\cancel{4}\pi r^2} D_1 A_{e_2}$$

$$|Z_{mut}|^2 = \frac{R_1 R_2}{\lambda r^2} D_1 A_{e2}$$

$$\boxed{|Z_{mut}|^2_{1\to 2} = \frac{R_1 R_2 D_1 A_{e2}}{\lambda r^2}}$$

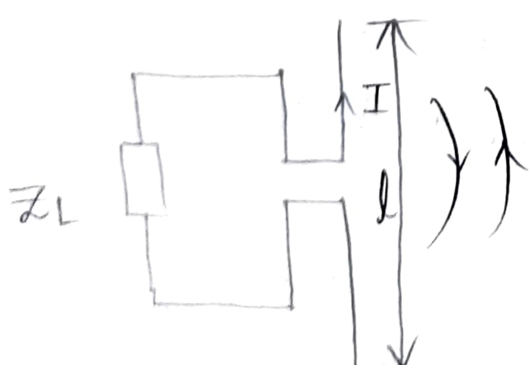
$$\boxed{|Z_{mut}|^2_{2\to 1} = \frac{R_2 R_1 D_2 A_{e1}}{\lambda r^2}}$$

$$\boxed{\frac{D_1}{A_{e1}} = \frac{D_2}{A_{e2}}} \Rightarrow \boxed{D_1 \propto A_{e1}}$$

$$\boxed{A_e = \frac{\text{Power captured } (\theta, \phi)}{\text{incident EM wave power density}}}$$

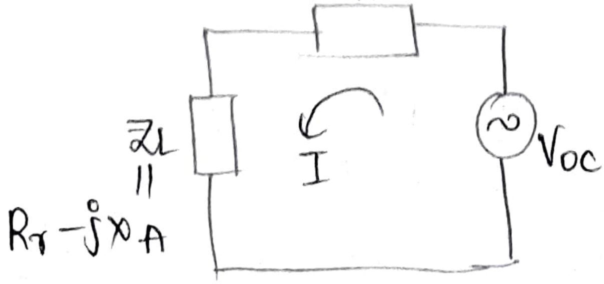
→ The ratio of directivity to the effective aperture is always constant.

$$\boxed{\frac{D}{A_e} = \text{const}}$$



$$A_e = \frac{\text{Power captured}}{\text{Power density}}$$

$\Downarrow$ $Z_A = R_r + jX_A$



$$P_{load} = \frac{1}{2}|I|^2 \cdot R_r$$

$$= \frac{1}{2}\frac{|V_{oc}|^2}{(2R_r)^2} \cdot R_r$$

$$\boxed{P_{load} = \frac{1}{8}\frac{|V_{oc}|^2}{R_r}}$$

$$\left(|V_{oc}| = |E| \times |l| \right)$$

So, $P_{load} = \frac{1}{8R_r} |E|^2 \cdot |l|^2$

$$\left(\text{But, } R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \right)$$

$$A_e = \frac{1}{8} \cdot \frac{\cancel{|E|^2} \cdot \cancel{|l|^2}}{\cancel{|E|^2} \cdot 80\pi^2 \left(\frac{\cancel{l}}{\lambda}\right)^2} \cdot 2\eta_0$$

$$= \frac{1}{8} \times \frac{2 \times \cancel{120}\,\cancel{\pi}}{\frac{\cancel{80}\,\pi^{\cancel{2}}}{\lambda^2}}$$

$$\boxed{A_e = \frac{3\lambda^2}{8\pi}}$$ ⊛

→ $\frac{D_0}{A_e} = \frac{\cancel{3}/2}{\frac{\cancel{3}\lambda^2}{\cancel{8}_4\pi}} = \frac{4\pi}{\lambda^2}$ is always constant.

→ Power density (isotropic antenna) $= \frac{P_r}{4\pi r^2}$

→ Power density (directional antenna) $= \frac{P_r}{4\pi r^2} \cdot D(\theta, \phi)$

→ Effectively we have 3 major losses in an antenna:

① Conductor loss } can't be controlled.
② Dielectric loss }
③ Reflection loss → can be controlled.

- Antenna efficiency:

$$\text{Antenna efficiency } (e_0) = e_{cd} \cdot e_r$$

e_{cd} → conductor + dielectric losses.

e_r → reflection loss

$\boxed{e_0 \leq 1}$

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

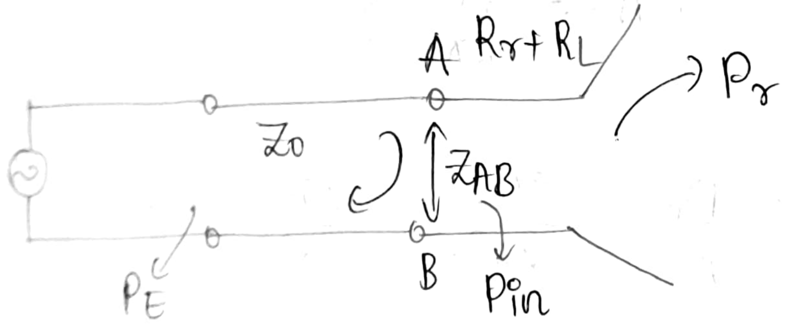
R_r → radiation resistance

R_L → loss resistance

$$|\Gamma_L| = \frac{Z_{AB} - Z_0}{Z_{AB} + Z_0}$$

$\boxed{e_r \leq 1}$

$\boxed{e_r = 1 - |\Gamma_L|^2}$ ✻



$\boxed{P_r = P_{in} \cdot e_{cd}}$

$\boxed{P_r = P_E \cdot e_{cd} \cdot e_r}$

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{U(\theta,\phi)}{P_r/4\pi}$$

$$\text{Gain}(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}\big|_{(P_{in})}} = \frac{U(\theta,\phi)}{P_{in}/4\pi}$$

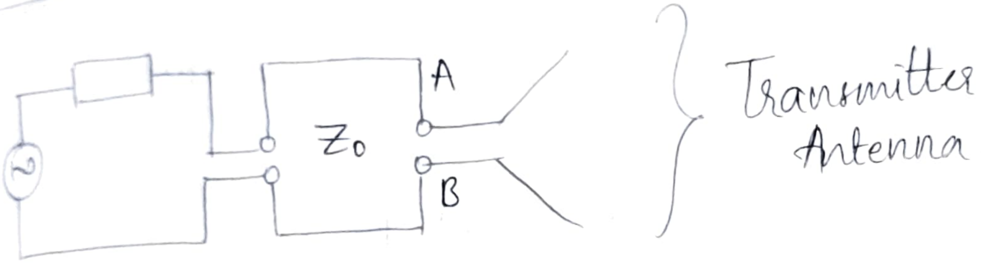
$$\text{Gain}(\theta,\phi) = \frac{U(\theta,\phi)}{\frac{P_r}{e_{cd}} \times \frac{1}{4\pi}} = e_{cd} \times \frac{U(\theta,\phi)}{(P_r/4\pi)}$$

$$\boxed{\text{Gain}(\theta,\phi) = e_{cd} \times D(\theta,\phi)}$$ ✻

$$\boxed{\text{Absolute Gain} = e_r \cdot e_{cd} \times D(\theta,\phi)} \; (*)$$

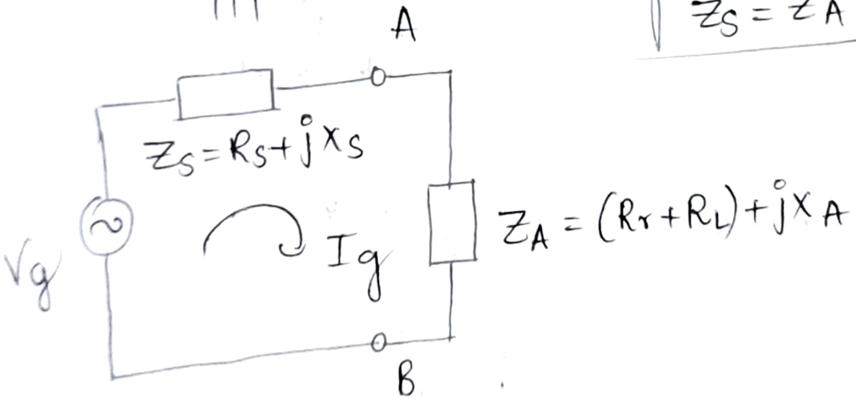
$$\boxed{\text{Gain}\Big|_{\text{decibels}} = 10\log(\text{Gain}) \,\Big|\, \text{dB}}$$

$$\boxed{\text{Gain} = \frac{4\pi}{\theta_{HPBW} \cdot \phi_{HPBW}}} \rightarrow \text{directional beam.}$$



|||

$$\boxed{Z_S = Z_A^*}$$



$$I_g = \frac{V_g}{Z_S + Z_A}$$

$$|I_g| = \frac{|V_g|}{\left[(R_S + R_r + R_L)^2 + (X_S + X_A)^2\right]^{1/2}}$$

$$\boxed{Z_S = Z_A^*} \rightarrow R_S = R_r + R_L \checkmark$$

$$X_S = -X_A \checkmark$$

Power lost in $Z_s = \frac{1}{2} |I_g|^2 \cdot R_s$

$$= \frac{1}{2} \frac{|V_g|^2}{(2R_s)^2} \cdot R_s$$

$$= \frac{1}{8} \cdot \frac{|V_g|^2}{R_s}$$

Power radiated by antenna $= \frac{1}{2} |I_g|^2 \cdot R_r$

$$= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)^2} \cdot R_r$$

Power lost by antenna $= \frac{1}{2} |I_g|^2 \cdot R_L$

$$= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)^2} \cdot R_L$$

Power supplied $P_s = \frac{1}{2} |V_g| |I_g|$

$$= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)}$$

Power delivered $= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)^2} \times (R_r+R_L)$

Radiation pattern → Variation of $\vec{E}$ wrt direction

- Polarization: Locus of $\vec{E}$ in given place as a function of time.

→ state of Antenna for which state of polarization responds maximaly.

$$\vec{E} = E_{x0}\cos(\omega t - \beta z)\hat{a}_x + E_{y0}\cos(\omega t - \beta z + \phi)\hat{a}_y$$

↳ generalised form

+z-direction

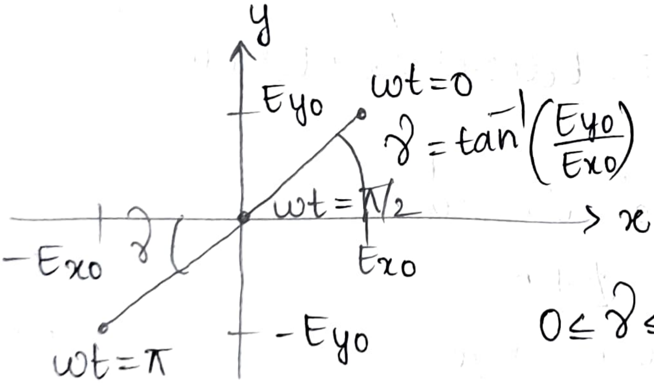
↳ propagation of wave.

- Linear Polarized

At $z=0$,

$$\vec{E} = E_{x0}\cos\omega t\,\hat{a}_x + E_{y0}\cos(\omega t+\phi)\,\hat{a}_y$$

At $\phi = 0^\circ \rightarrow \vec{E} = E_{x0}\cos\omega t\,\hat{a}_x + E_{y0}\cos(\omega t)\,\hat{a}_y$



$\gamma = \tan^{-1}\left(\frac{E_{y0}}{E_{x0}}\right)$

$0 \le \gamma \le \pi/2$

if $\gamma = 0$ → horizontal polarization

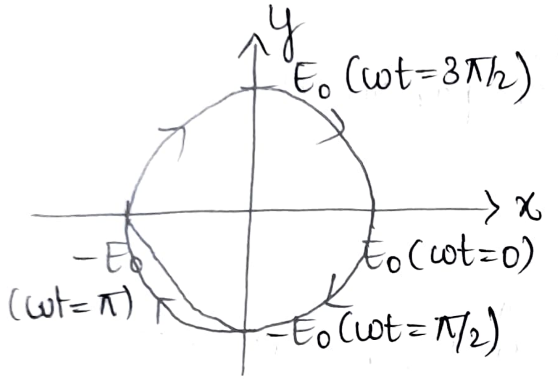
if $\gamma = \pi/2$ → Vertical polarization

- Circular polarized

$$\phi = \pm \pi/2$$
$$E_{x0} = E_{y0} = E_0$$
$$E = E_0 \cos\omega t \, \hat{a}_x + E_0 \cos(\omega t + \phi) \, \hat{a}_y$$

at $\phi = \pi/2$,

$$E = E_0 \cos\omega t \, \hat{a}_x - E_0 \sin\omega t \, \hat{a}_y$$



Left Hand Circular Polarization

at $\phi = -\pi/2$, $\longrightarrow$ Right Hand Circular Polarization.

$$E = E_0 \cos\omega t \, \hat{a}_x + E_0 \sin\omega t \, \hat{a}_y$$

- Elliptical Polarized

$$\phi \neq 0$$
$$E_{x0} \neq E_{y0}$$

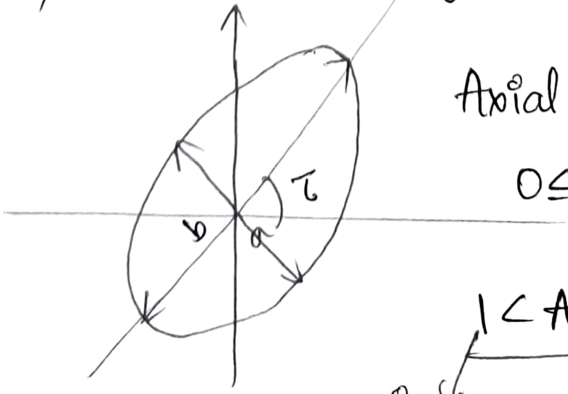
$+AR \rightarrow LH$

$-AR \rightarrow RH$

if $\phi = +ve \rightarrow$ LHEP

if $\phi = -ve \rightarrow$ RHEP

(+z-direction)



$$\text{Axial Ratio} = \frac{a}{b}$$

$$0 \leq \tau \leq 180^\circ$$

$$1 < AR < \infty$$

circular polarization

linear polarization

Point Care Sphere:

- $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$ → Electric field parameters.
- $0 \le \tau \le 180^\circ$

$1 \le \left(\frac{a}{b}\right) \le \infty$

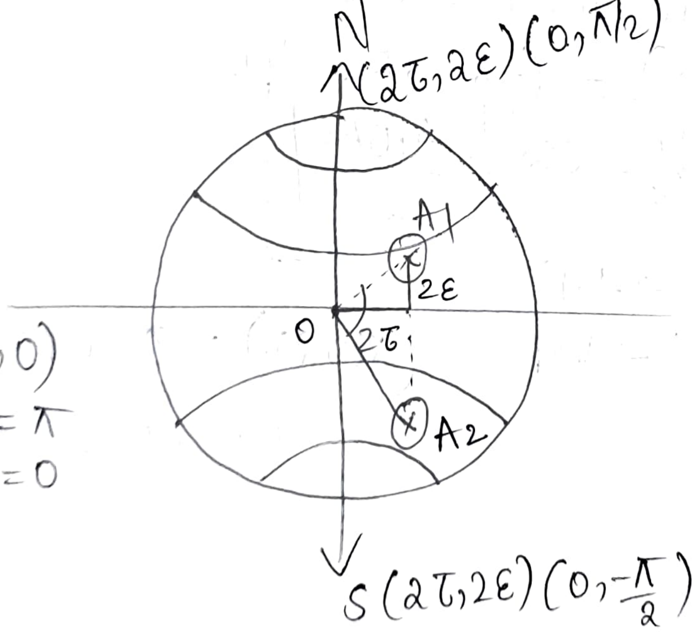
AR

$\varepsilon = \cot^{-1}(\pm AR)$

- $-\frac{\pi}{4} < \varepsilon < \pi/4$

$\gamma = \tan^{-1}\left(\frac{E_{y0}}{E_{x0}}\right)$

- $0 \le \gamma \le \pi/2$



$(2\tau, 0)$
$2\tau = \pi$
$2\varepsilon = 0$

Northern Hemisphere (NH) → 2ε positive
Southern Hemisphere (SH) → 2ε negative

if $\varepsilon > 0$ → $AR > 0$ → LH
if $\varepsilon < 0$ → $AR < 0$ → RH

Equator, $\varepsilon = 0$
$AR = \infty$ → linear polarisation

North pole,

$\tau = 0$

$\epsilon = \pi/4 \rightarrow$ LHCP

$(0,0) \rightarrow$ Horizontal polarisation

$(\pi,0) \rightarrow$ Vertical polarisation

South pole,

$\tau = 0$

$\epsilon = -\pi/4 \rightarrow$ RHCP

- Orthogonal state of Polarization:

The state for which response of Antenna is minimum is called Orthogonal Polarization.

$(2\epsilon, 2\tau) = (-2\epsilon, 2\tau + 180^\circ)$

$$\boxed{\eta = \cos^2 \left| \frac{\angle A_1 O A_2}{2} \right|} \rightarrow$$ Efficiency in which two states share information.

$A_1, A_2 \rightarrow$ States

$(0,0) \rightarrow$ H.P

$(0,\pi) \rightarrow$ V.P

$(2\epsilon, 2\tau) = \left(\frac{\pi}{2}, 0\right) \rightarrow$ LHCP

$\rightarrow \left(\frac{-\pi}{2}, 0\right) \rightarrow$ RHCP

→ The efficiency is max b/w same state of polarization

→ The efficiency is zero, b/w orthogonal state of polarisation

eg: $(0,0) \rightarrow$ H.P $(\pi,0) \rightarrow$ V.P

$$\eta = \cos^2 \left| \frac{\pi}{2} \right| = 0$$

$\left(\frac{\pi}{2}, 0\right) \rightarrow$ LHCP $\left(\frac{-\pi}{2}, 0\right) \rightarrow$ RHCP

$\eta =$

→ ~~[illegible]~~

→ In satellite communication we use, Circular polarisation (not HP or VP)

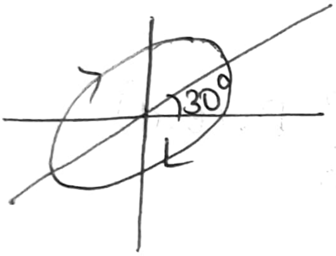
- Since we can increase rate by '2' times
  information sharing
  (but need to design 2 antennas for two states)

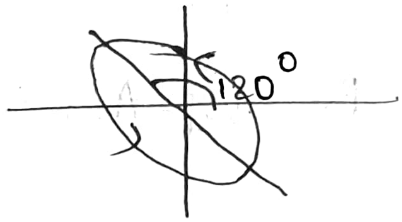
eg: Orthogonal state of (120, 60) ?

$(2\varepsilon, 2\tau) = (120, 60)$ → LHEP

orthogonal states

$(-2\varepsilon, 2\tau + 180) = (-120, 240)$





→ $\eta = 1$ (no loss)

$\eta = 0$ → (Severe loss)

→ Polarization state of Antenna :

$$\hat{\rho}_a = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$$

$$E_{x0} = E_{y0} \ ; \ \phi = \pi/2$$

LHCP

→ Incident EM wave :

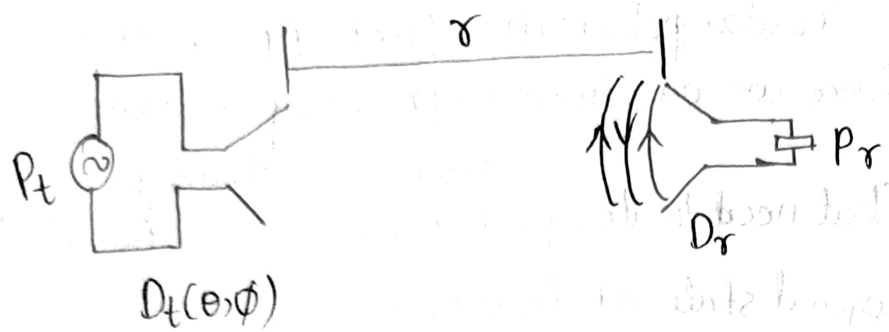
$$\hat{\rho}_w = \hat{a}_x$$

~~[illegible]~~ : $\eta = |\hat{\rho}_a \cdot \hat{\rho}_w|^2$

Polarization loss factor

$$\eta = |\cos\psi|^2$$

(*) Friss Transmission Relation:



$$A_e = \frac{\lambda^2}{4\pi} D(\theta,\phi)$$

$$W_i = \frac{P_t}{4\pi r^2} \cdot D_t(\theta,\phi)$$

$$P_r = W_i \cdot A_{er} = W_i \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)$$

$$P_r = \frac{P_t}{4\pi r^2} D_t(\theta,\phi) \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)$$

↓

Power received by antenna. (lossless)

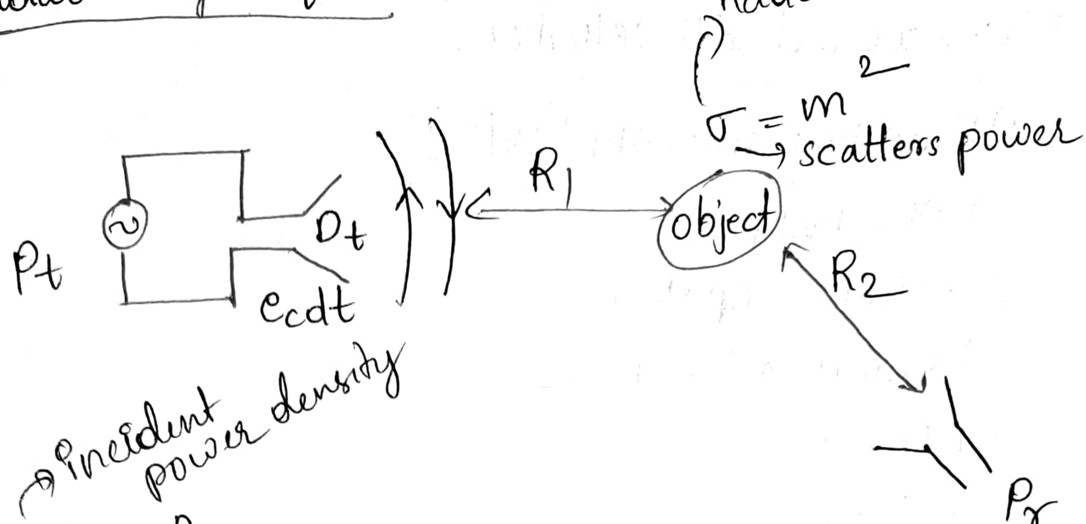
→ Power received by antenna (including losses)

$$W_i = \frac{P_t}{4\pi r^2} \cdot D_t(\theta,\phi) \cdot e_{cdt} \cdot (1-|\Gamma|^2)_t$$

$$P_r = W_i \cdot A_{er} = W_i \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)\, e_{cdr} (1-|\Gamma|^2)_r$$

$$P_r = \left[\frac{P_t}{4\pi r^2} D_t(\theta,\phi) \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)\right] \cdot e_{cdt} \cdot e_{cdr} \cdot (1-|\Gamma_L|^2)_t \cdot (1-|\Gamma_L|^2)_r \, |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

• Radar Range Equation:



incident power density

$$W_i = \frac{P_t}{4\pi R_1^2} \cdot D_t(\theta,\phi) \cdot e_{cdt}$$

Power captured by the object located at a distance R_1 from transmitter $= W_i \cdot \sigma$

At location R_2,

Scattered power density

$$W_s = \lim_{R_2 \to \infty} \frac{P_{cap}}{4\pi R_2^2}$$

$$W_s = \lim_{R_2 \to \infty} \frac{W_i \sigma}{4\pi R_2^2}$$

$$\sigma = \lim_{R_2 \to \infty} 4\pi R_2^2 \cdot \frac{W_s}{W_i}$$

→ $\sigma = 0$ if $W_s = 0$
(In this case we can't track object)

→ It tells about ability of object to scatter power.

Power received $(P_r) = W_s \cdot A_{er}$

$$\boxed{P_r = \frac{P_t \cdot D_t(\theta,\phi)}{4\pi R_1^2} \cdot e_{cdt} \cdot \frac{\sigma}{4\pi R_2^2} \cdot \frac{\lambda^2}{4\pi} \cdot D_r(\theta,\phi)\, e_{cdr}}$$

1. An elliptically polarized wave travelling in $-ve$ z direction is received by circularly polarized antenna whose main lobe is along $\theta = 0$ direction.
The unit vector describing the polarization of the incident wave is given by,

$$\hat{\rho}_w = \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}}$$

find polarization loss factor (PLF) when the wave that would transmitted by,

(a) RHCP
(b) LHCP

$\xrightarrow{+z}$ $\xleftarrow{-z}$

Sol:-

$$\begin{cases} \vec{A}_1 = \hat{a}_x + j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y) \Rightarrow \text{LHCP} \\ \vec{A}_2 = a_x - j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x - j\hat{a}_y) \Rightarrow \text{RHCP} \end{cases}$$

if wave is travelling $+z$ direction.

$$\begin{cases} \vec{A}_1 = \hat{a}_x - j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x - j\hat{a}_y) \Rightarrow \text{LHCP} \\ \vec{A}_2 = \hat{a}_x + j\hat{a}_y \rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y) \Rightarrow \text{RHCP} \end{cases}$$

if wave is travelling $-z$ direction

$$\hat{\rho}_w = \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}}$$

~~LHCP~~ $\cancel{\hat{\rho}_w \cdot \hat{\rho}_a = \sqrt{\frac{2}{5}} \ldots \frac{j}{\sqrt{10}}}$

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

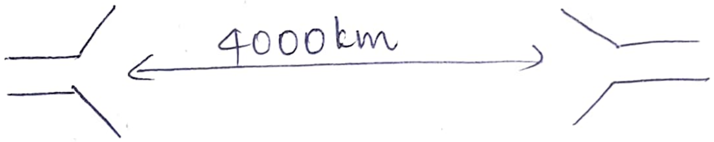
$$= \left| \frac{2}{\sqrt{10}} + \frac{1}{\sqrt{10}} \right|^2 = \frac{9}{10} \rightarrow \text{LHCP}$$

$$PLF = \left| \frac{1\times 2}{\sqrt{10}} - \frac{1}{\sqrt{10}} \right|^2 = \frac{1}{10} \rightarrow \text{RHCP}$$

~~RHCP~~ $\cancel{\hat{\rho}_w \cdot \hat{\rho}_a = \sqrt{\frac{2}{5}} + \frac{j}{\sqrt{10}}}$

2. A 30dB, RHCP antenna in a radio link radiates 5W power at 2GHz. The receiving antenna has a impedance mismatch at terminals which leads to VSWR of 2. The receiving antenna is about 95% efficient and has a field pattern of near the beam maximum given by $\vec{E}_r = (2\hat{a}_x + j\hat{a}_y) F_r(\theta,\phi)$. The distance b/w two antennas is 4,000km and receiving antenna is required to deliver 10^{-14}W to the receiver. Determine max effective aperture of receiving antenna.

Sol:-



Given

$G = 30dB$

$f = 2GHz$

$P_t = 5W$

RHCP

$VSWR = 2$

$\eta = 95\%$

$P_r = 10^{-14}$ W

$A_{er} = ?$

$|\Gamma_L| = 1/3$

$(1-|\Gamma_L|^2) = 8/9$

for receiving antenna,

$$W = \frac{P_t}{4\pi r^2} \cdot D \ W/m^2$$

$$P_r = \frac{P_t}{4\pi r^2} \cdot D \cdot A_{er} \cdot (0.95) \cdot (8/9) \cdot 10^3$$

$$\hat{\rho}_r = \frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}}$$

$$\hat{\rho}_t = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}$$

$$PLF = |\hat{\rho}_r \cdot \hat{\rho}_t|^2$$

$$= \left|\frac{2}{\sqrt{10}} + \frac{1}{\sqrt{10}}\right|^2$$

$$= 9/10$$

$$10^{-14} = \frac{5}{4\times\pi\times(4000\times10^3)^2} \times (0.95)\times(8/9)\times10^3\times\frac{9}{10}\times A_{er}$$

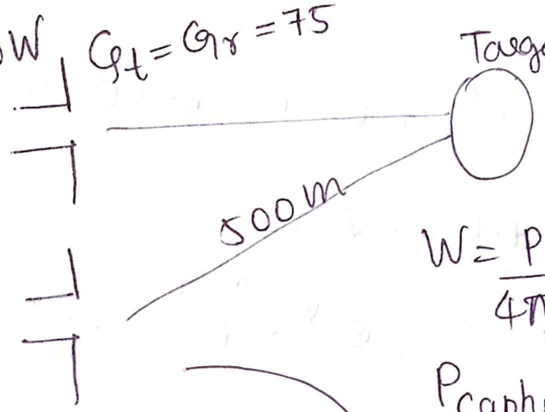
$$A_{er} = \frac{10^{-14}\times4\pi\times16\times10^{12}\times9\times10}{5\times(0.95)\times8\times10^3} = \frac{1808.64\times10^{-2}}{3800}$$

$$\underline{\underline{A_{er} = 0.475\times10^{-2}\ m^2}}$$

$$\underline{A_{er} = 0.00476\ m^2}$$

3. <u>Problem-6 in Assignment-1,</u>

$P_t = 1000W$, $G_t = G_r = 75$



$$W = \frac{P_t}{4\pi r^2}\cdot D\ W/m^2$$

$$P_{captured} = \sigma\cdot W$$

$P_r = 0.1425 mW$

$$W_s = \frac{P_c}{4\pi R^2}$$

$$P_r = W_s\cdot A_{er}.$$

$$P_r = \frac{P_c}{4\pi R^2}\cdot\frac{\lambda^2}{4\pi}\cdot D_r$$

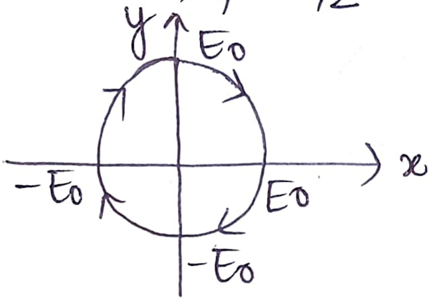
$$P_r = \frac{\sigma\cdot P_t}{(4\pi R^2)^2}\cdot D_t\cdot\frac{\lambda^2}{4\pi}\cdot D_r$$

$$\underline{\sigma = 0.01257\ m^2}$$

$$\Rightarrow \vec{E} = E_{01}\cos(\omega t - \beta z)\,\hat{a}_x + E_{02}\cos(\omega t + \phi - \beta z)\,\hat{a}_y$$

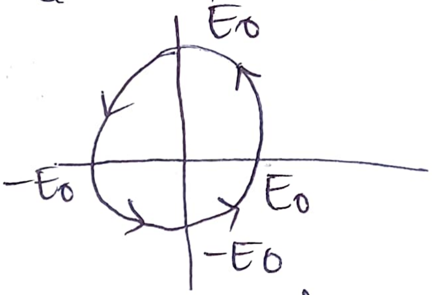
$$E_{01} = E_{02} \qquad \phi = \pm \pi/2$$

if $z = 0$, $\phi = \pi/2$



LHCP(+z-direction)

if $z = 0$; $\phi = -\pi/2$



RHCP(+z-direction)

if $z = 0 \Rightarrow \vec{E} = E_0\cos(\omega t)\,\hat{a}_x + E_0\cos(\omega t + \phi)\,\hat{a}_y$

$$\vec{E}_p = E_{01}\,\hat{a}_x + E_{02}\angle\phi\,\hat{a}_y$$

$$\boxed{\vec{E}_p = E_{01}\,\hat{a}_x + E_{02}\,e^{j\phi}\,\hat{a}_y} \rightarrow (*)$$

$$\left(e^{j\phi} = \cos\phi + j\sin\phi\right)$$

<u>for +z-direction,</u>

$\phi = \pi/2 \rightarrow$ LHCP

$\phi = -\pi/2 \rightarrow$ RHCP

<u>for -z-direction,</u>

$\phi = \pi/2 \rightarrow$ RHCP

$\phi = -\pi/2 \rightarrow$ LHCP

→ <u>for elliptical polarization;</u>

<u>for +z-direction,</u>

$\phi = +ve \rightarrow$ LHEP

$\phi = -ve \rightarrow$ RHEP

<u>for -z-direction,</u>

$\phi = +ve \rightarrow$ RHEP

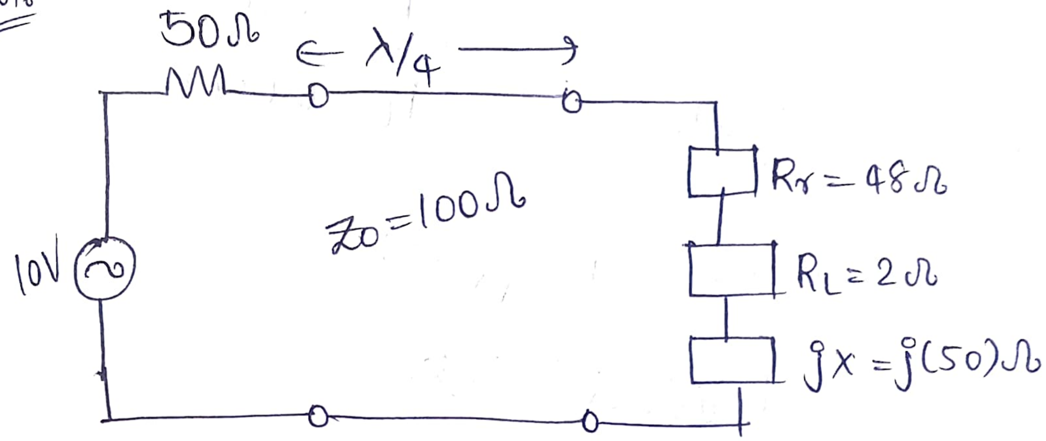
$\phi = -ve \rightarrow$ LHEP.

Problem :

Radiation ~~resch~~ resistance $= 48\,\Omega$
loss resistance $= 2\,\Omega$
Reactance $= 50\,\Omega$
} of antenna.

An antenna is connected to generator with open circuit voltage of 10V & internal impedance of $50\,\Omega$ via $\lambda/4$ long transmission line with characteristic impedance of $100\,\Omega$.

Sol :-



$$Z_L = R_L + jX_L$$

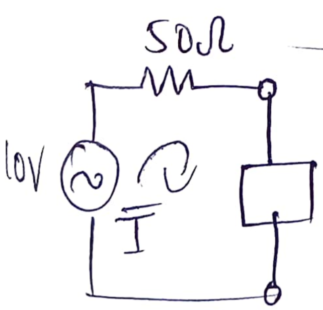
$$Z_L = 50 + j(50)$$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = 0.5 + j(0.5)$$

$$\bar{Z}_{in} = \frac{1}{\bar{Z}_L} = \frac{1}{0.5 + j(0.5)} = (1 - j)$$

$$Z_{in} = (1 - j)\,Z_0$$

$$\boxed{Z_{in} = 100 - j(100)}$$



$Z_{in} = 100 - j(100)$

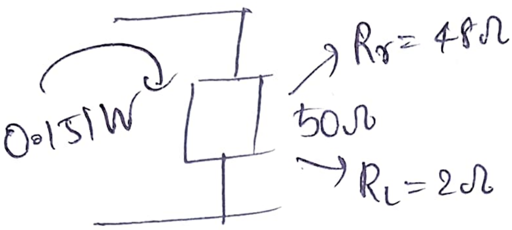
$$\bar{I} = \frac{V}{150 - j(100)} \Rightarrow \bar{I} = \frac{1}{15 - j(10)}$$

$$\bar{I} = \frac{15 + j(10)}{\sqrt{325}} \longrightarrow \boxed{\bar{I} = 0.0055\angle 33.69^\circ}$$

Power delivered to load $= \frac{1}{2}|I|^2 \cdot Z_0$

$= \frac{1}{2}|I|^2 \times 100$

$$\boxed{P_{delivered} = 0.151\,W}$$



$P_r = P_D \cdot \frac{48}{50}$

$$\boxed{P_r = 0.145\,W}$$

$$\boxed{P_L = 0.006\,W}$$

$\frac{1}{2}|I|^2 \times 50 = 0.075\,W$

Power supplied $= \frac{Re(V \times I^*)}{2}$

$P_{supplied} = 0.231\,W$

Problem:

$$E = \begin{cases} 1 & 0^\circ \le \theta \le 45^\circ \\ 0 & 45^\circ \le \theta \le 90^\circ \\ 1/2 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

(a) What is directivity of this antenna.

(b) What is radiation resistance of antenna at 200m from it if the field is equal to 10 V/m (rms) for $\theta = 0^\circ$ at that distance and terminal current (5A) rms.

Sol: Directivity $= \dfrac{U_D}{U_{avg}}$

But, $U_{avg} = \dfrac{P_{rad}}{4\pi}$

$$U = \frac{\text{Power}}{\text{Solid angle}}$$

$$r^2 \cdot W = \frac{\text{Power}}{\text{Area}} \cdot r^2$$

$$W = \frac{1}{2} \cdot \frac{|E_0|^2}{\eta_0} = \frac{|E_{rms}|^2}{\eta_0}$$

$$U = \frac{|E_{rms}|^2}{\eta_0} \cdot r^2$$

$$U = W \cdot r^2 = \begin{cases} \dfrac{r^2}{\eta_0} & 0 \le \theta \le 45^\circ \\ 0 & 45^\circ \le \theta \le 90^\circ \\ \dfrac{r^2}{4\eta_0} & 90^\circ \le \theta \le 180^\circ \end{cases}$$

$$U_0 = \frac{r^2}{\eta_0}$$

$$P_r = \int W \cdot dA$$

$$P_r = \int_{\phi=0}^{2\pi}\int_0^{2\pi} U \cdot \sin\theta \, d\theta \, d\phi$$

$$= 2\pi\left[\int_0^{\pi/4}\left(\frac{r^2}{\eta_0}\right)\sin\theta \, d\theta + \int_{\pi/2}^{\pi}\left(\frac{r^2}{4\eta_0}\right)\sin\theta \, d\theta\right]$$

$$= 2\pi\left[\frac{r^2}{\eta_0}(-\cos\theta)\Big|_0^{\pi/4} + \frac{r^2}{4\eta_0}(-\cos\theta)\Big|_{\pi/2}^{\pi}\right]$$

$$= \frac{2\pi r^2}{\eta_0}\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{4}\left[1 - 0\right]\right)$$

$$\begin{array}{r} 1.2\cancel{5}0 \\ -0.707 \\ \hline 543 \end{array}$$

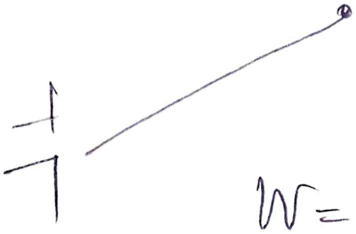
$$P_r = \frac{2\pi r^2}{\eta_0}(0.543)$$

$$U_{avg} = \frac{P_r}{4\pi} = \frac{\cancel{2\pi} r^2}{\eta_0} \times (0.543) \times \frac{1}{\cancel{4\pi}_{\,2}}$$

$$\boxed{U_{avg} = \frac{0.271 r^2}{\eta_0}}$$

$$\text{Directivity} = \frac{U_0}{U_{avg}} = \frac{1}{0.271} = 3.69$$

$$= 5.67 \text{ dB}$$



$E_{rms} = 10\ V/m$

$I_{rms} = 5\ A$

$$W = \frac{|E_{rms}|^2}{\eta_0} = \frac{|10|^2}{\eta_0} = \frac{100}{\eta_0}$$

Power radiated $= \iint_S W \cdot ds$

$$P_r = \frac{2\pi}{\eta_0} \times (0.543) \times 100 \times (200)^2$$

$$= \frac{2\pi}{120\pi} \times (0.543) \times 100 \times (200)^2$$

$$P_r = 36{,}193\ W$$

$$P_r = I_{rms}^2 \cdot R_r$$

$$36{,}193 = (25) \cdot (R_r)$$

$$\underline{\underline{R_r = 1447.72\ \Omega}}$$

Problem:

Normalized radiation intensity of antenna is,

$U = \sin\theta \cdot \sin^3\phi$

The intensity exists only in $0 \le \theta \le \pi$, $0 \le \phi \le \pi$ region, and it is zero elsewhere. Find

(a) Exact directivity

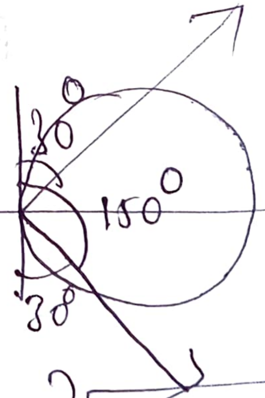
(b) Azimuthal & elevation plane half power Beamwidths.

Sol:
$$U_{avg} = \frac{P_r}{4\pi} \qquad D = \frac{U_0}{U_{avg}}$$

$$P_r = \int_{\Omega} U \, d\Omega$$

$$= \int_0^{\pi}\int_0^{\pi} \sin\theta \sin^3\phi \, d\theta \, d\phi$$

$$\boxed{P_r = 2\pi/3}$$



$U = \sin\theta \sin^3\phi$

$\rightarrow \phi = \pi/2$

$U = \sin\theta$

$\frac{1}{2} = \sin\theta \rightarrow \boxed{\theta_{HP} = 30^\circ}$

$\boxed{\theta = 150^\circ}$ $\boxed{\theta_{HPBW} = 120^\circ}$

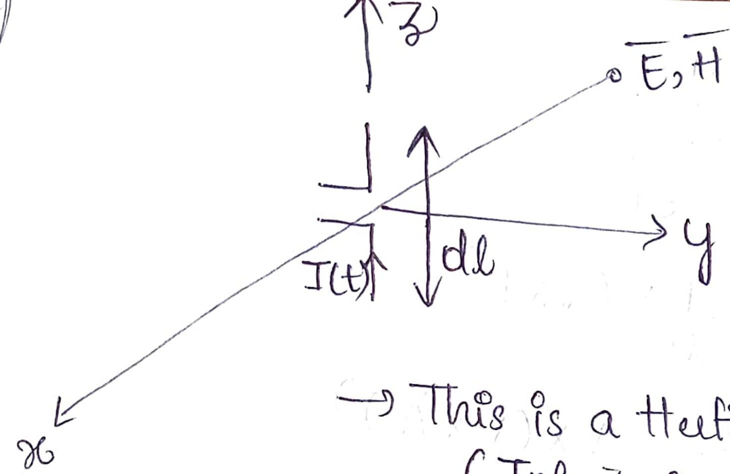
$\rightarrow \theta = \pi/2$

$U = \sin^3\phi$

$\frac{1}{2} = \sin^3\phi \rightarrow \sin\phi = \sqrt[3]{\frac{1}{2}} \rightarrow \phi = 52.53^\circ$

$\phi = 127.46^\circ$

$\boxed{\phi_{HPBW} = 74.94^\circ}$



→ This is a Hertzian dipole Antenna (Infinity Small Antenna).

→ Maxwell's equations,

(i) $\nabla \cdot \overline{D} = \rho_v$

(ii) $\nabla \cdot \overline{B} = 0$

(iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\nabla \times \vec{H} = \vec{J_c} + \frac{\partial \vec{D}}{\partial t}$

→ $\boxed{\vec{B} = \mu \vec{H}}$

→ $\boxed{\vec{D} = \varepsilon \vec{E}}$

→ $\nabla \cdot \vec{B} = 0$

But $\nabla \cdot (\nabla \times \vec{A}) = 0$

So, $\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ → vector magnetic potential.

→ $\nabla \times \vec{E} = -\dot{\overline{B}}$

$= -(\nabla \times \dot{\vec{A}})$

$\boxed{\nabla \times \vec{E} = -(\nabla \times \dot{\vec{A}})}$

$\boxed{\nabla \times (\overline{E} + \dot{\overline{A}}) = 0}$

$\boxed{\overline{E} + \dot{\overline{A}} = -\nabla V}$ → $\boxed{V \rightarrow \text{scalar electric potential}}$

$\nabla \times (-\nabla V) = 0$

$$\rightarrow \quad \nabla \times \vec{H} = J_c + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J_c} + \varepsilon \dot{\bar{E}}$$

$$\nabla \times \frac{1}{\mu}\vec{B} = \frac{1}{\mu} \times (\nabla \times \vec{B})$$

$$= \frac{1}{\mu}\left(\nabla \times (\nabla \times \vec{A})\right)$$

$$= \vec{J_c} + \varepsilon \dot{\bar{E}}$$

$$\frac{1}{\mu}\left(\nabla \times (\nabla \times \bar{A})\right) = \bar{J_c} + \varepsilon\left(\dot{\overline{-\nabla V - \bar{A}}}\right)$$

$$\boxed{\nabla \times (\nabla \times \bar{A}) = \bar{J_c}\mu + \mu\varepsilon\left(-\nabla\dot{V} - \ddot{\bar{A}}\right)}$$

$$\text{But } \nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J_c} + \mu\varepsilon\left(-\nabla\dot{V} - \ddot{\bar{A}}\right)$$

$$\nabla^2 \bar{A} - \mu\varepsilon\ddot{A} = -\mu\vec{J_c} + \mu\varepsilon\nabla\dot{V} + \nabla(\nabla \cdot \bar{A})$$

$$\nabla^2 \bar{A} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\bar{A} = -\mu\vec{J_c} + \underbrace{\mu\varepsilon\nabla\dot{V} + \nabla(\nabla \cdot \bar{A})}$$

$$\downarrow$$

$$\nabla(\mu\varepsilon\dot{V} + \nabla \cdot \bar{A}) = 0$$

$$\mu\varepsilon\dot{V} + \nabla \cdot \bar{A} = 0$$

$$\boxed{\nabla \cdot \bar{A} = -\mu\varepsilon\dot{V}}$$

$$\downarrow$$

Lourentz Gauge

$$\boxed{\nabla^2 \bar{A} - \mu\varepsilon\ddot{\bar{A}} = -\mu\vec{J}} \quad \text{—— (1)}$$

$$\bar{E} + \mathring{\bar{A}} = -\nabla V$$

$$\nabla \cdot \bar{E} + \nabla \cdot \mathring{\bar{A}} = -\nabla \cdot \nabla V$$

$$\frac{\delta_v}{\varepsilon} + (-\mu \varepsilon \ddot{V}) = -\nabla^2 V$$

$$\boxed{\nabla^2 V - \mu \varepsilon \ddot{V} = \frac{-\delta_v}{\varepsilon}} \quad (2)$$

from (1) & (2),

- $\nabla^2 \bar{A_p} + \omega^2 \mu \varepsilon \bar{A_p} = -\mu \vec{J_p}$
- $\nabla^2 V_p + \omega^2 \mu \varepsilon V_p = -\frac{\delta_{vp}}{\varepsilon}$

Writing in terms of phasors.

$$\rightarrow \nabla^2 \bar{A_p} + \beta^2 \bar{A_p} = -\mu \vec{J_p} \quad (\beta = \omega\sqrt{\mu\varepsilon})$$

$$\nabla^2 G + \beta^2 G = \delta(r)$$

$G \rightarrow$ spatial Impulse response

$$\int \delta(r)\, dr = 1$$

$$\lim_{r \to 0} r$$

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial G}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \cancel{\frac{\partial}{\partial \theta}}^{0}\left(\sin\theta \frac{\partial G}{\partial \theta}\right) +$$

$$\frac{1}{r^2 \sin^2\phi} \cancel{\frac{\partial^2 G}{\partial \phi^2}}^{0} + \beta^2 G = \delta$$

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial G}{\partial r}\right) + \beta^2 G = \delta \qquad (\psi = rG)$$

$$\boxed{\frac{d^2\psi}{dr^2} + \beta^2 \psi = \delta}$$

$$\hookrightarrow \boxed{\psi = C_1 e^{j\beta r} + C_2 e^{-j\beta r}}$$

$$\psi = \cancel{c_1 e^{j\beta r}}^{\,0} + c_2 e^{-j\beta r}$$

$$\frac{d^2y}{dt^2} + ay = 0 \qquad m = \pm j\sqrt{a}$$

$$y(t) = c_1 e^{m_1 t} + c_2 e^{m_2(t)}$$

$$\psi = c e^{-j\beta r}$$

But, $(\psi = rG)$

$$c e^{-j\beta r} = r G$$

$$\boxed{G = \frac{c}{r} e^{-j\beta r}}$$

Substituting G in $\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dG}{dr}\right) + \beta^2 G = \delta$

$$\int_{\lim r\to 0} \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr}\left(\frac{c}{r} e^{-j\beta r}\right)\right) + \beta^2\left(\frac{c}{r} e^{-j\beta r}\right) = \int_{\substack{r \\ \lim r\to 0}} \delta(v)\,d$$

On solving, we get $\boxed{c = \frac{-1}{4\pi}}$

$$\boxed{\therefore G = \frac{-1}{4\pi r} e^{-j\beta r}} \to \text{Spatial Impulse Response.}$$

from, $\nabla^2 G + \beta^2 G = \delta(r)$

$$G \leftarrow \delta$$

$$\vec{A_p} \leftarrow -\mu \vec{J_p}$$

$$\underbrace{\overline{A_p}}_{y(t)} = \underbrace{G}_{r(t)} * \underbrace{-\mu \vec{J_p}}_{x(t)}$$

$$= \int G(\bar{r} - \bar{r}') \cdot \mu \bar{J}(r') \, dv'$$

$$y(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

$$\vec{A_p} = \int \frac{1}{4\pi|\bar{r}-\bar{r}'|} \cdot e^{-j\beta(\bar{r}-\bar{r}')} \cdot \mu \bar{J}(r')\, dv'$$

$$A_p e^{j\omega t} = \int_v \frac{1}{4\pi|\bar{r}-\bar{r}'|}\, e^{j\omega t} \cdot e^{-j\beta(\bar{r}-\bar{r}')} \cdot \mu \bar{J}(r')\, dv'$$

$$\boxed{A_p e^{j\omega t} = \int_v \frac{1}{4\pi|\bar{r}-\bar{r}'|} \cdot e^{j\omega\left(t-\frac{1}{c}(\bar{r}-\bar{r}')\right)} \cdot \mu \bar{J}(r')\, dv'}$$

$$\nabla \times \bar{A} = \bar{B}$$

$$\nabla \times \bar{H} = \bar{J_c} + \frac{\partial \vec{D}}{\partial t}$$

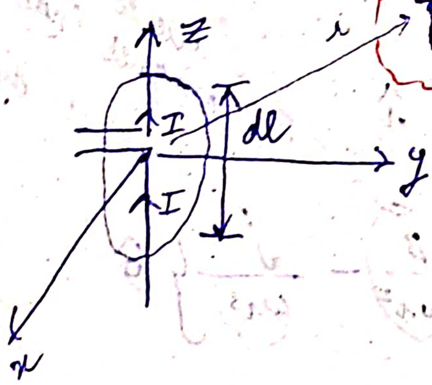
$$V(r) = \int_v \frac{\rho_v(r')}{4\pi|\bar{r}-\bar{r}'|\varepsilon}\, dv' \cdot e^{-j\beta|\bar{r}-\bar{r}'|}$$

14/10/

$$\vec{A}(r) = \int_V \frac{\mu \bar{J}(r')}{4\pi|\bar{r}-\bar{r}'|} e^{-j\beta \cdot |\bar{r}-\bar{r}'|} dr$$

$$= \int \frac{\mu J(0)}{4\pi r} e^{-j\beta r} dr$$

$\int \bar{J} dV$ $A/m^2 \cdot m^3 = A \cdot m \Rightarrow$ current in some length.



$\therefore \int \bar{J} \cdot dr = \underline{I \cdot \vec{dl}}$

current element.

$\vec{A}, V$

$$\vec{A} = \frac{\mu I \vec{dl}}{4\pi r} e^{-j\beta r} = \frac{\mu I dl}{4\pi r} \cdot e^{-j\beta r} \hat{a}_z$$

$$\vec{B} = \nabla \times \vec{A}$$

at a dist. r from antenna, it is a <u>source free region.</u>

there is no maxwell's sources.

maxwell's

$\nabla \cdot \bar{D} = \rho_v$

$\nabla \times \bar{E} = \frac{\partial \bar{B}}{\partial t}$

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{B}}{\partial t}$

$\nabla \cdot \bar{B} = 0$

$\vec{A_r} = A_z \cos\theta \cdot \hat{a_r}$

$\vec{A_\theta} = A_z \cos(90-\theta) \cdot \hat{a_\theta}$

$\hat{A_\phi} = 0.$

$\bar{B} = \nabla \times \bar{A}$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r.\hat{\theta} & r\sin\theta.\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & \underset{(0)}{r\sin\theta A_\phi} \end{vmatrix}$$

$$\bar{B} = \hat{r}(0) + \hat{\theta}(0) + \frac{r\sin\theta}{r^2\sin\theta} \cdot \hat{\phi}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]$$

$$\bar{B} = \frac{\mu I_0 dl}{4\pi} e^{-j\beta r} \sin\theta\left[\frac{j\beta}{r} + \frac{1}{r^2}\right]\hat{\phi}$$

$$\bar{H} = \frac{I_0 dl}{4\pi} e^{-j\beta r} \sin\theta\left[\frac{j\beta}{r} + \frac{1}{r^2}\right]\hat{\phi}$$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$$

$$\Rightarrow \nabla \times \bar{H_r} = j\omega\varepsilon \bar{E_D}$$

$$\boxed{\bar{E} = \frac{\nabla \times \bar{H}}{j\omega\varepsilon}}$$

$$\bar{E} = \frac{1}{j\omega\varepsilon}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & \phi H_\phi \end{vmatrix} \frac{1}{r^2}\cdot\sin\theta$$

$$\bar{E} = \frac{I_0 dl \cos\theta}{4\pi\omega\varepsilon} e^{-j\beta r}\left\{\frac{\beta}{r^2} - \frac{j}{r^3}\right\}\cdot\hat{r}$$

$$+ \frac{I_0 dl \sin\theta}{4\pi\varepsilon} e^{-j\beta r}\left\{\frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right\}\hat{\theta}$$

we ignore these fields due to electro magnet induct. elec. st. dipole

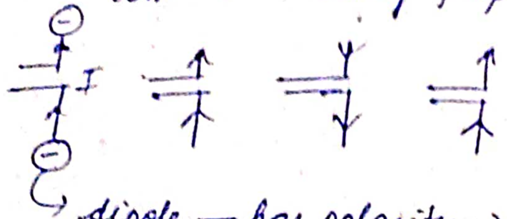
$\beta = \omega\sqrt{\mu\varepsilon}$

$\left(\frac{\beta^2}{\omega r}\right) \Rightarrow \frac{\omega\mu\varepsilon}{r} \longrightarrow$ freq term

~~w~~ near to 0. then this term can be ignored

$\frac{\beta}{\omega r^2} \longrightarrow$ independent of freq.

$\frac{j}{w r^3}$ ⇒ inversely proportional to frequency.



→ I direcⁿ changes.

dipole — has polarity ⇒ i changes direcⁿ then this current keeps on changing ⇒ called diffusion current.

$\frac{j}{w r^3}$ → due to electrostatic dipole nature.

$\frac{j\beta^2}{w r}$ → part of radiating field / far field.

$\bar{E}$ due to $\bar{r}$ will be negligibly small & will be only due to $\vec{\theta}$ → radiating / far field.

$$\Rightarrow \bar{E} \simeq \frac{I_0 dl \sin\theta}{4\pi \epsilon r} \cdot j\beta^2 \cdot e^{-j\beta r} \hat{a}_\theta \rightarrow \text{in far field region.}$$

$$\bar{H} \simeq \frac{I_0 dl \cancel{\cos\theta} \sin\theta}{4\pi r} \cdot j\beta \cdot e^{-j\beta r} \hat{a}_\phi$$

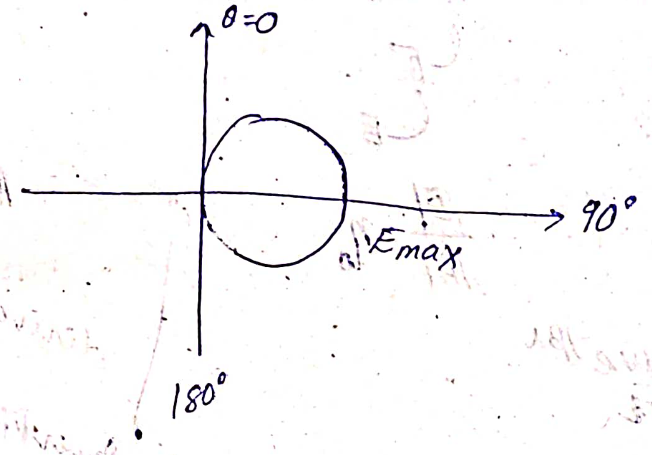
$$\frac{j\beta^2}{w r} = \frac{\beta}{w r^2} \Rightarrow r = \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

$$\boxed{r = \lambda/2}$$ to check whether antenna is in far field or not.

TEM

In far field region, $\bar{E}$ & $\vec{H}$ fields are transverse in direction & hence acts as Transverse Electromagnetic Wave ⇒ TEMW

$$\frac{|E|}{|H|} = \sqrt{\frac{\mu}{\epsilon}} = \eta_0$$



$$E_{max} = \frac{I_0 \, dl \, \beta^2}{4\pi \varepsilon \omega}$$

$$\bar{E} = \frac{E_{max}}{r} \cdot \sin\theta \cdot \hat{a}_\theta$$

$$P_{avg} = \frac{1}{2} \frac{|E|^2}{\eta_0}$$

$$= \frac{1}{2\eta_0} \cdot \frac{\sin^2\theta}{r^2} E_{max}^2 \, \hat{a}_r$$

$$W = \int \bar{P}_{avg} \cdot \vec{ds}$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{2\eta_0} \cdot \frac{\sin^2\theta}{r^2} E_{max}^2 \cdot r^2 \sin\theta \, d\theta \cdot d\phi$$

$$= \frac{E_{max}^2}{2\eta_0} \times 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$= \frac{E_{max}^2}{2 \times 120\pi} \cdot 2\pi \cdot \frac{4}{3} = \frac{E_{max}^2}{90}$$

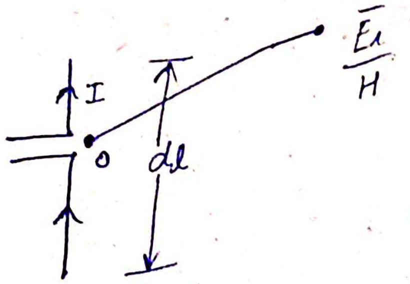
$$W = \left(\frac{I_0 \, dl}{4\pi \varepsilon \omega}\right)^2 \frac{\beta^4}{90} = \boxed{40\pi^2 \cdot I_0^2 \left(dl/\lambda\right)^2 = W}$$

power radiated by antenna =

$$W = 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2$$

$$= \frac{1}{2} I_0^2 (R_r)$$

$$R_r = 80\pi^2 (dl/\lambda)^2$$



$\bar{A} \propto \bar{J}$

$\bar{A} = \bar{J}$

↳ B

↳ E

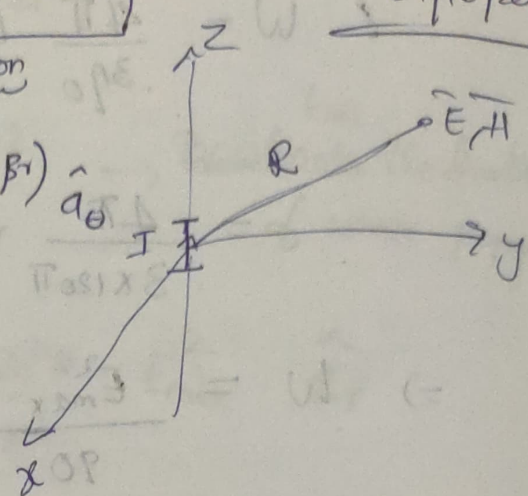
$$\frac{|E|}{|H|} = \eta_0$$

$$\bar{A} = \int \frac{\bar{J} \, dv \, e^{-j\beta r}}{4\pi r}$$

Revision : Hertzian Dipole Antenna
Infar-field Region

31/10/2022

$\bullet\ \bar{E} = \dfrac{j\, I_o dl\, \beta^2 \sin\theta}{4\pi\omega\varepsilon r} e^{j(\omega t - \beta r)}\, \hat{a}_\theta$



$\bullet\ \bar{H} = \dfrac{|E|}{\eta}\, \hat{a}_\phi$

$\bullet\ R_r = 80\pi^2 \left(\dfrac{dl}{\lambda}\right)^2$

$\bullet\ dl = 0.01\,\lambda$

$\Rightarrow R_r = 80\pi^2 (10^{-2})^2 \quad [\pi^2 \simeq 10]$

$= 800 \times 10^{-4}$

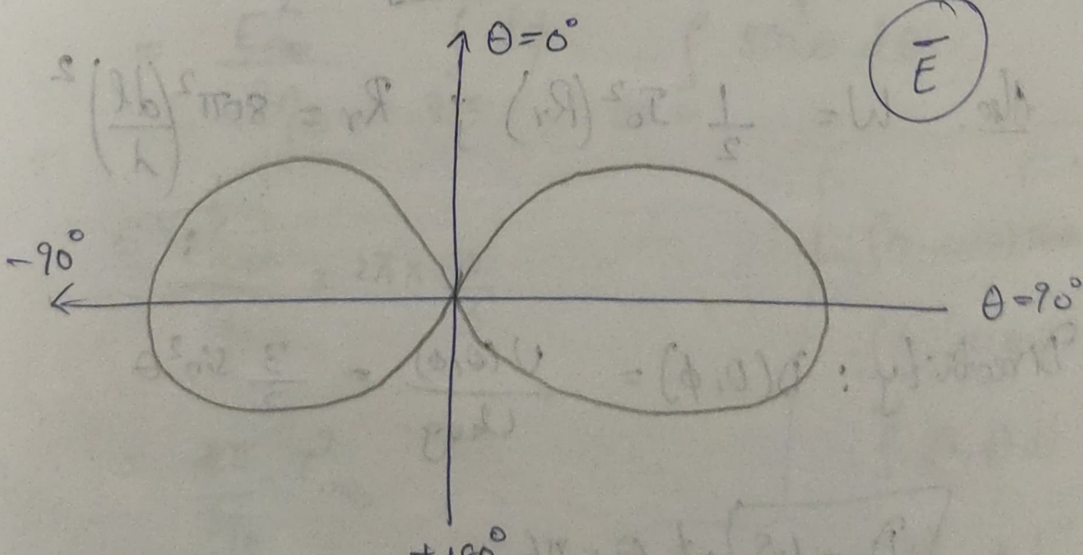
$R_r = 0.08\ \Omega$

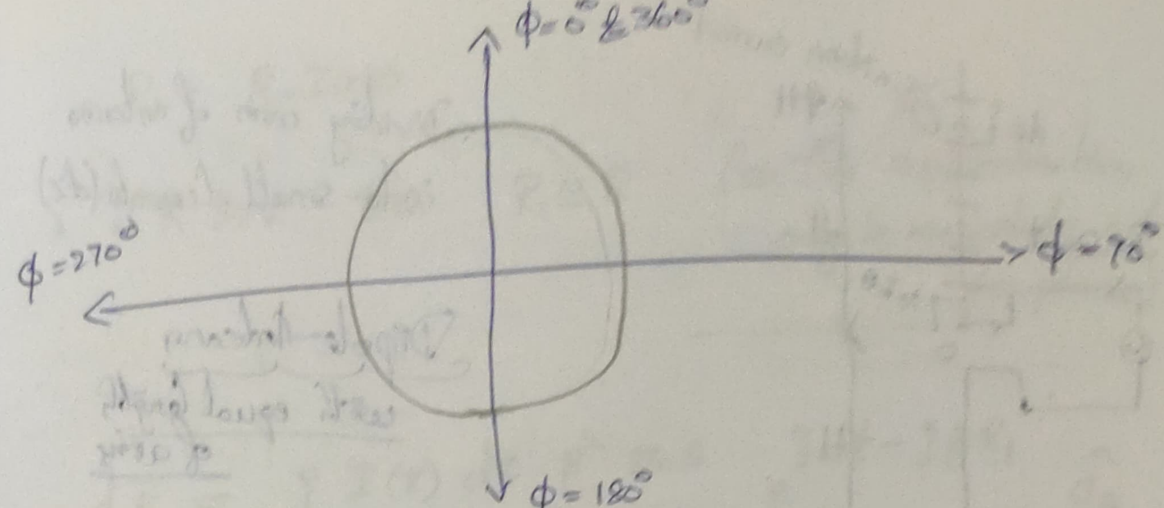
$\bullet\ D_o = 3/2$

• R_r – is low, antenna is not radiating efficiently.

• $\bar{E}$ is LP in the direction of $\hat{\theta}$.

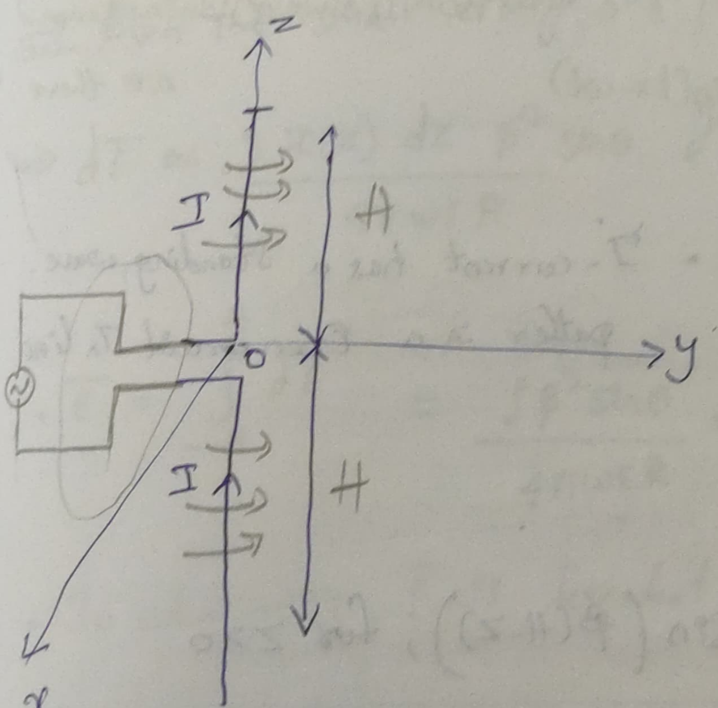
• When $\theta = 90^\circ$, $\bar{E}$ is going to be maximum.



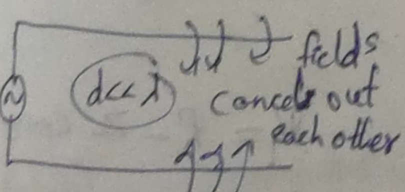


Hertzian Dipole Antenna is

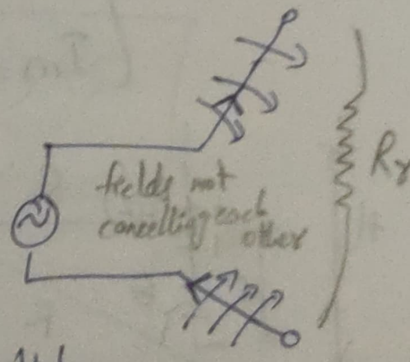
- Omnidirectional & uniform in ϕ direction.



- Only condition is "I" should be same (Current)



if we bend the wire



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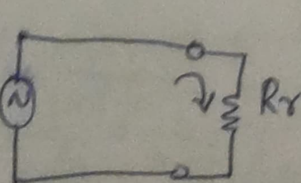
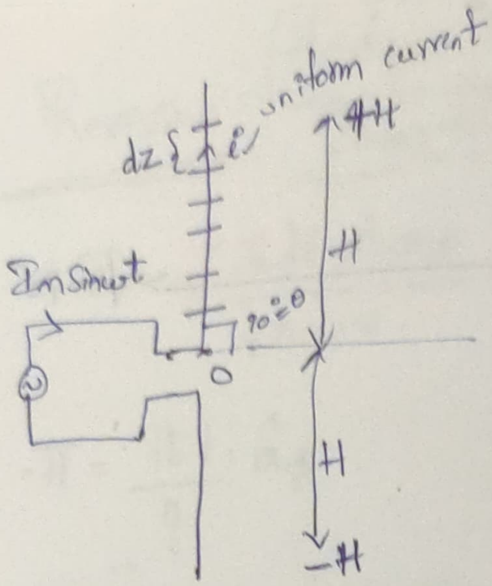


Circuit P.O.V

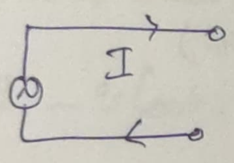


• Dividing arm of antenna into small elements (dz)

Dipole Antenna with equal length of arms

• Standing Wave : $e^{-\gamma} + e^{\gamma}$ (or) $A\sin(kx-\omega t) + A\sin(kx+\omega t)$ → both Forward & Backward waves are there

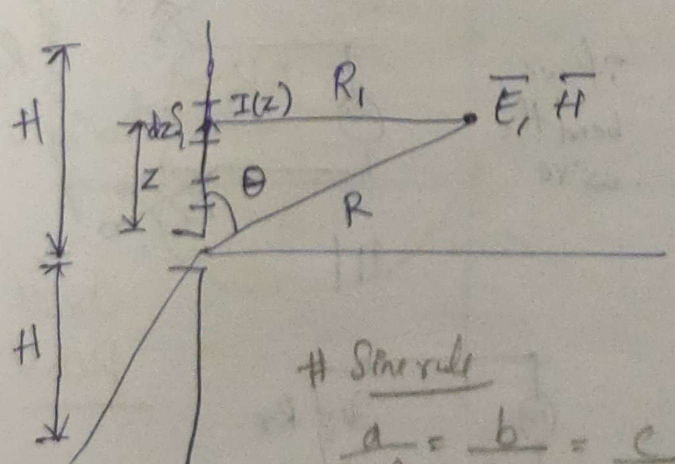
• Travelling Wave : $A\sin(kx-\omega t)$



• I - current has a standing wave patter in a Open circuit Tx line

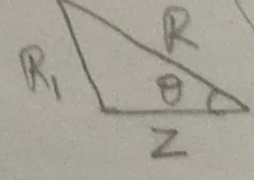
$I = I_m \sin \beta l$

$I(z) = \begin{cases} I_m \sin(\beta(H-z)); & \text{for } z>0 \\ I_m \sin(\beta(H+z)); & \text{for } z<0 \end{cases}$



$R_1 = R - z\cos\theta$

$R_1^2 = R^2 + z^2 - 2Rz\cos\theta$



Sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule : $a^2 = b^2 + c^2 - 2bc\cos A$

- $R_1 = R - Z\cos\theta$

- if $R >> Z$ then $R_1 \simeq R$ (use this approximation only in magnitudes not for phases)

- $$d\bar{E} = \frac{j\, I(z)\, dz\, \beta^2 \sin\theta}{4\pi\omega\varepsilon R_1} e^{j\omega t - j\beta R_1}\, \hat{a}_\theta$$

→ Even the small change in phase, should be retained

⇒ $$d\bar{E} \simeq \frac{j\, I(z)\, dz\, \beta^2 \sin\theta}{4\pi\omega\varepsilon R} e^{j\omega t} \cdot e^{-j\beta R} \cdot e^{+j\beta Z\cos\theta}\, \hat{a}_\theta$$

- $$\bar{E} = \int_{-H}^{H} d\bar{E} = \frac{j\beta^2 \sin\theta}{4\pi\omega\varepsilon R} e^{j\omega t}\, e^{-j\omega R} \int_{-H}^{H} I(z)\, e^{+j\beta Z\cos\theta}\, dz$$

- In what way $\bar{E}$ is dependent on θ?

$$\therefore \bar{E} = \frac{j\, 60\, I_m}{r} e^{-j\beta r} \cdot e^{j\omega t}\, F(\theta) \;;$$

$$F(\theta) = \frac{\cos(\beta H\cos\theta) - \cos(\beta H)}{\sin\theta}$$

- $\bar{E}$ → function of only θ, uniform in ϕ direction.
  → Omni directional antenna → depends on 'H'-Height of antenna

$$\therefore \bar{E} = \frac{j\,60\,I_m}{r}\, e^{-j\beta r} \cdot e^{j\omega t} \cdot \left\{ \frac{\cos(\beta H \cos\theta) - \cos(\beta H)}{\sin\theta} \right\} \hat{a}_\theta$$

Dipole Antenna (with equal Arms)

→ $\bar{E}$ varies with $1/r$

→ $\bar{E}$ travelling in 'r' direction

→ $\bar{E}$ is interms of 'θ' alone

→ Omnidirectional antenna, uniform in φ direction.

→ depends on Height of the dipole antenna

$$\bar{H} = \frac{|E|}{\eta} \cdot \hat{a}_\phi$$

Applications: FM stations, Radio Stations (widely used in)

---

① For $H = \lambda/4$, $F(\theta) = ?$

Sol

$$F(\theta) = \frac{\cos(\beta H \cos\theta) - \cos(\beta H)}{\sin\theta}$$

- $H = \lambda/4 \rightarrow$ Total Length of antenna $= \lambda/2 = 2(\lambda/4) = \lambda/2$

  $(\lambda/2)$ Dipole antenna – widely used antenna

$$F(\theta) = \frac{\cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4} \cos\theta\right) - \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}\right)}{\sin\theta}$$

$$\therefore F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$\therefore \bar{E} = \frac{j60 I_m}{r} e^{-j\beta r} \cdot e^{j\omega t} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \hat{a}_\theta$$

$$\bar{H} = \frac{|E|}{\eta} \hat{a}_\phi$$

(W/rad)

- $P_{den} = \frac{1}{2}\frac{|E|^2}{\eta_0} = \frac{1}{2\eta_0} \times \left(\frac{60 I_m}{r}\right)^2 \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \cdot \vec{a}_r$

- $P_{rad} = \iint_S P_{den} \cdot ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{(60)^2}{2\eta_0} \frac{I_m^2}{r^2} \times \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \times r^2 \sin\theta \times d\theta\, d\phi$

$$= \int_{\theta=0}^{\pi} \frac{1800}{120\pi} \times 2\pi \times I_m^2 \times \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$= \int_0^{\pi} 30 I_m^2 \frac{\cos^2\left(\left(\frac{\pi}{2}\right)\cos\theta\right)}{\sin\theta} d\theta$$

$$= 30 I_m^2 \times \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$= 30 I_m^2 \times 1.218$$

$= 36.54 I_m^2$; I_m - max. value of Time-varying current

$I = I_m \sin\omega t$

01/11/20

$\Rightarrow P_r = 36.54\, I_m^2\ (W)$

W.K.T : $P_r = \frac{1}{2} I_m^2 R_r$

$\lambda/2$ Dipole Antenna

$\Rightarrow R_r = 2 \times 36.54$

$= 73.08\ \Omega$

$$\boxed{\therefore R_r \simeq 73.08\ \Omega}$$

• Directivity : $D = \frac{U(\theta, \phi)}{U_{avg}}$ & $D_0 = \frac{U_0}{U_{avg}}$

• $U_{avg} = \frac{P_{rad}}{4\pi}$.

$\Rightarrow U(\theta) = \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \times \frac{1800 I_m^2}{\eta_0} \rightarrow$ max at $\theta = \pi/2$

As :

• $\overline{W}_r = \frac{1}{2}\frac{|E|^2}{\eta_0} = \frac{1800}{\eta_0}\frac{I_m^2}{r^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\ \hat{a}_r\ (W/m^2)$

• $U(\theta) = \overline{W}_r \times r^2 = \frac{1800}{\eta_0} I_m^2 \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

$$\therefore U(\theta) = \frac{1800}{\eta_0} I_m^2 \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$\cdot\ U_0 = \frac{1800}{\eta_0} I_m^2$ at $\theta = \pi/2$

$\cdot\ U_{avg} = \frac{P_{rad}}{4\pi} = \frac{36.54\, I_m^2}{4\pi}$

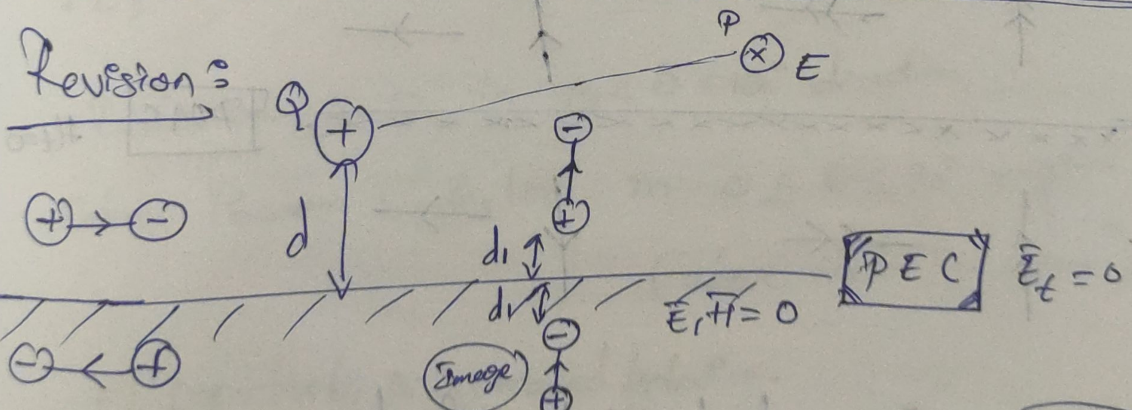
$\therefore D_0 = \frac{U_0}{U_{avg}} = \frac{1800}{120\pi} I_m^2 \times \frac{4\pi}{36.54\, I_m^2}$

$= \frac{180}{3 \times 36.54} = \frac{1000}{609} = 1.642$

$\boxed{D_0 = 1.642}$ $\boxed{D_0 (\text{in dB}) = 2.1538 \text{ dB}}$

- While Hertzian dipole antenna has, $D_0 = 1.5$

Revision:



Introduction of imaginary charge

Image Theory

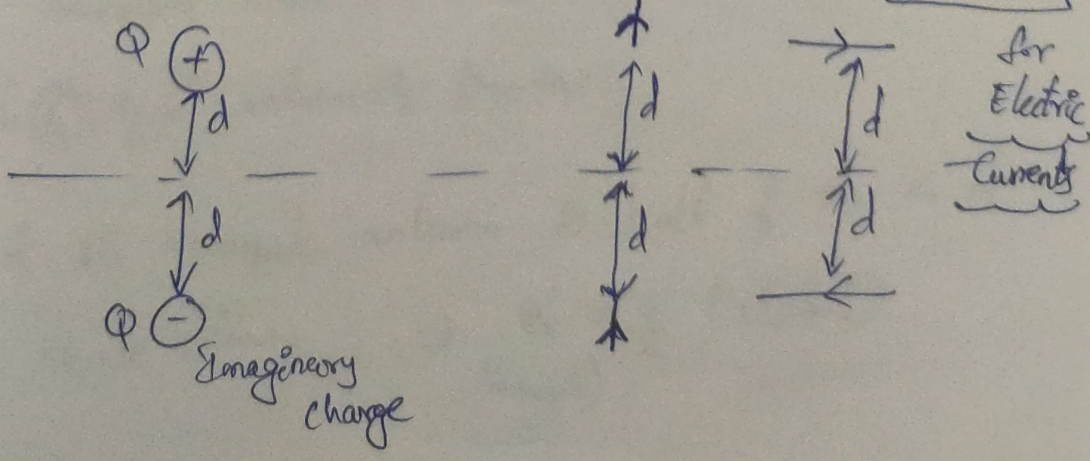
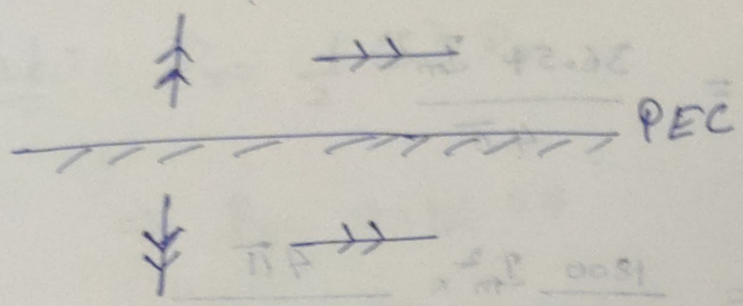


Image Theory for Magnetic Currents w.r.t PEC

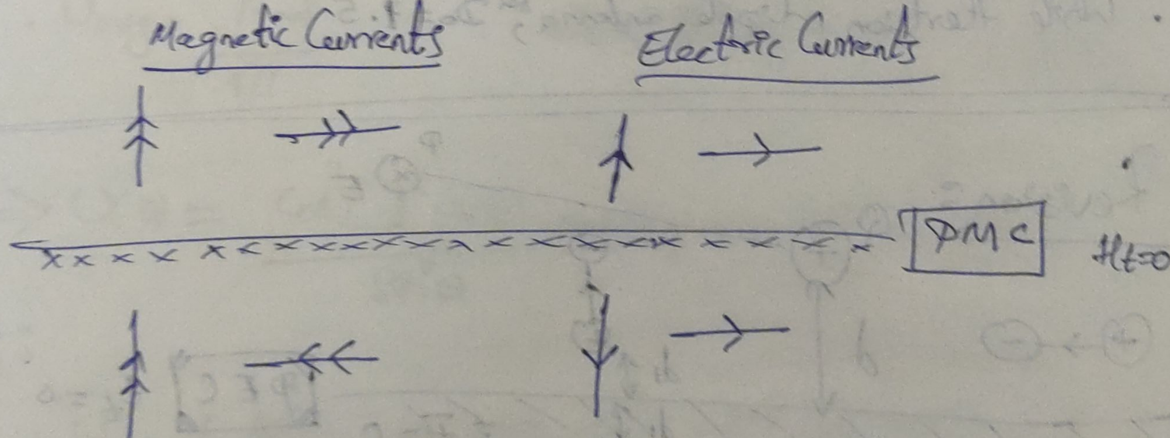


• PEC : Perfect Electric Conductor

Inside : $\bar{E} = 0$, $\bar{H} = 0$ & $\bar{E_t} = 0$ across the boundary

• PMC : Perfect Magnetic Conductor

$H_t = 0$ across the boundary



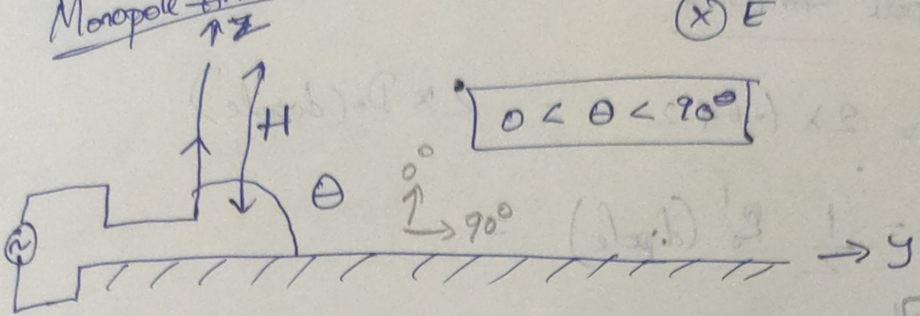
• Removing the conductor plate & solving

In cars

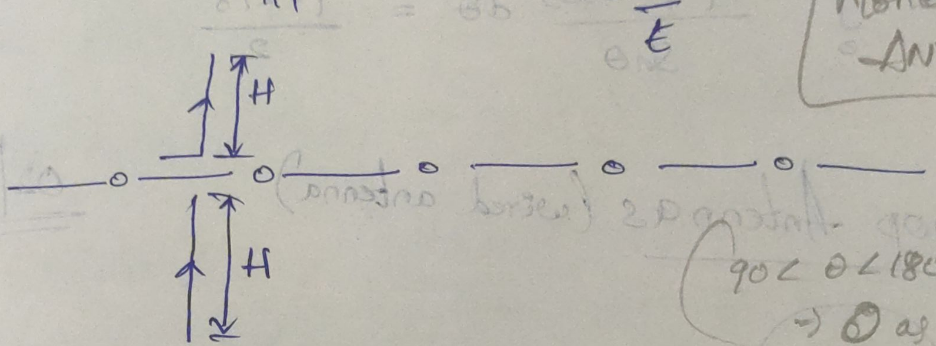
Monopole Antenna

• Total electric field at a point P

$\boxed{0 < \theta < 90^\circ}$



λ/4 monopole ANTENNA



$90 < \theta < 180^\circ$ ⇒ 0 as inside PEC

• $H = \lambda/4$ – Monopole Antenna. Varying in region: $0 \le \theta \le 90^\circ$

• E, $\bar{H}$

• Radiation is '0' in $90 < \theta < 180^\circ$ direction

• Entire Power radiates in $0 \le \theta \le 90^\circ$ region (above metallic plane)

⇒ Directivity is increased by twice.

• $\lambda/4$ – Monopole antenna's directivity is double that of $\lambda/2$ Dipole antenna's directivity.

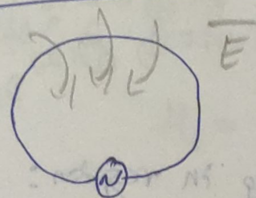
• R_r' of $\lambda/4$ monopole antenna is half of the R_r' of $\lambda/2$ dipole antenna ⇒ $\underset{(\text{Monopole})}{R_r'} = \frac{1}{2} R_r(\text{dipole})$

Monopole Antenna

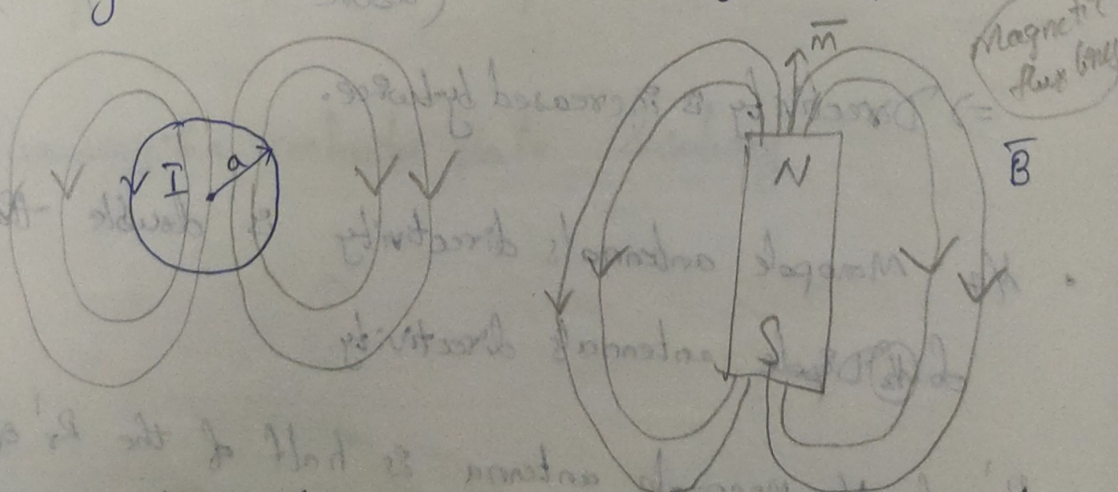
- $D_0 = 2 \times 1.642 = 2 \times D_0(\text{dipole})$
- $R_r' = \frac{1}{2} R_r' (\text{dipole}) =$
- $\int_0^{\pi/2} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} \, d\theta = \frac{1.218}{2}$

Loop Antennas (wired antenna)

02/11/2022



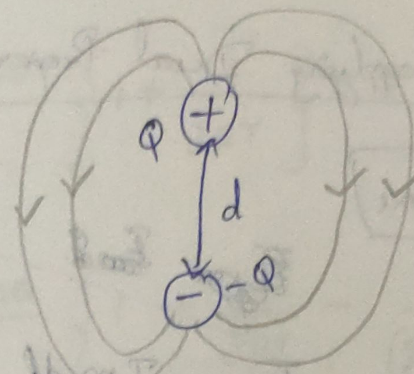
- Current carrying loops have magnetic flux densities around it.
- Current carrying loop will behave similar as magnet → can consider as magnetic dipole



- $\bar{B} = \frac{\mu m}{4\pi r^3} (2\cos\theta \, \hat{a}_r + \sin\theta \, \hat{a}_\theta)$

Magnetic Dipole

• Electric Dipole:



$\bar{E}$

• $\bar{E} = \dfrac{Qd}{4\pi \varepsilon_0 r^3}(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta)$

• Electric dipole moment : $p = Qd$

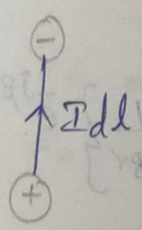
• Magnetic dipole moment : $m = S \cdot I = \pi a^2 I$

• ε – Permitivitty
$\varepsilon_0 = 8.854 \times 10^{-12}$ H/m

• μ = permeability
$\mu_0 = 4\pi \times 10^{-7}$ F/m

• Radio receivers, old age phones : Loop Antenna Applications

• They act as inductance also.



(Electric dipole)

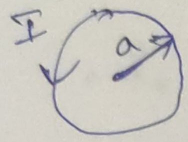
$$\bar{E}_r = \frac{I_0\, dl \cos\theta}{4\pi \omega \varepsilon}\left\{\frac{\beta}{r^2} - \frac{j}{r^3}\right\} e^{-j\beta r}$$

$$\bar{E}_\theta = \frac{I_0\, dl \sin\theta}{4\pi \varepsilon}\left\{\frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right\}$$

$$\bar{H}_\phi = \frac{I_0\, dl}{4\pi} \sin\theta \left\{\frac{j\beta}{r^2} + \frac{1}{r^2}\right\} e^{-j\beta r}$$

By using / applying Dual Property / principle

Loop Antenna



similar to (Magnetic dipole)

physically does not exist that's why (-ve) sign.

$$E_\phi = \frac{-I_m dl}{4\pi} \sin\theta \left\{ \frac{j\beta}{r} + \frac{1}{r^2} \right\} e^{-j\beta r}$$

$$\bar{B}_r = \frac{I_m dl}{4\pi\omega} \cos\theta \left\{ \frac{\beta}{r^2} - \frac{j}{r^3} \right\} e^{-j\beta r}$$

$$\bar{B}_\theta = \frac{I_m dl \sin\theta}{4\pi} \left\{ \frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3} \right\} e^{-j\beta r}$$

- $I_m dl = j I (\pi a^2) \omega\mu$; m – Magnetic dipole moment. $j\pi a^2$ – area of the loop.

- $$\vec{E}_\phi = \frac{-j I (\pi a^2) \omega\mu}{4\pi} \sin\theta \, \frac{j\beta}{r} \left\{ 1 + \frac{1}{j\beta r} \right\} e^{-j\beta r}$$

Also: $\omega\mu = \beta\eta$; as $\beta = \omega\sqrt{\mu\varepsilon}$ & $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$$\Rightarrow \vec{E}_\phi = -\frac{j I (\pi a^2) \beta\eta}{4\pi} \sin\theta \, \frac{j\beta}{r} \left\{ 1 + \frac{1}{j\beta r} \right\} e^{-j\beta r}$$

$$\therefore \vec{E}_\phi = \frac{I (\pi a^2) \beta^2 \eta}{4\pi r} \left\{ 1 + \frac{1}{j\beta r} \right\} \sin\theta \, e^{-j\beta r}$$

$$\bar{H}_r = \frac{j\beta a^2 I \cos\theta}{4r^2}\left\{1+\frac{1}{j\beta r}\right\} e^{-j\beta r}$$

$$\bar{H}_\theta = \frac{-(\beta a)^2 I \sin\theta}{4r}\left\{1+\frac{1}{j\beta r}-\frac{1}{(\beta r)^2}\right\} e^{-j\beta r}$$

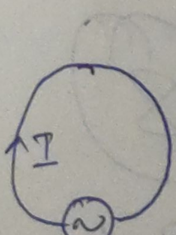
$$\bar{E}_\phi = \frac{I(\pi a^2)\beta^2 \eta \sin\theta}{4\pi r}\left\{1+\frac{1}{j\beta r}\right\} e^{-j\beta r}$$

In a far-field region: $r >> \lambda$

$$E_\phi \cong \frac{I(\pi a^2)\beta^2 \eta \sin\theta}{4\pi r} e^{-j\beta r} = \frac{\beta^2 a^2 I \eta \sin\theta}{4r} e^{-j\beta r}$$

$$\bar{H}_\theta \cong \frac{-(\beta a)^2 I \sin\theta}{4r} e^{-j\beta r}$$

$$\frac{\bar{E}_\phi}{\bar{H}_\theta} = \eta$$



$$P_r = \iint \bar{W}_{rad} \cdot d\bar{s}$$

$$\vec{W}_{rad} = \frac{1}{2}\frac{|E|^2}{\eta}\hat{a}_r$$

$$= \frac{1}{2}\left(\frac{I\beta^2 a^2 \eta}{4r}\right)^2 \frac{\sin^2\theta}{\eta}\hat{a}_r$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{(I\,\beta^2 a^2)^2\, \eta}{32\, r^2} \sin^2\theta \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{\eta (I\beta^2 a^2)^2}{32} \times 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$= \frac{\eta (I\beta^2 a^2)^2}{16} \pi \times \frac{4}{3}$$

$$= \frac{\pi}{12} \eta (I\beta^2 a^2)^2 = \frac{\pi}{12} \eta I^2 (\beta a)^4$$

$$\boxed{\therefore P_r = \frac{\pi}{12} \eta I^2 (\beta a)^4}$$

$\eta = 120\pi$

$\beta = 2\pi/\lambda$

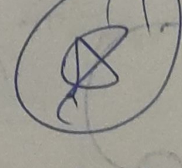
- $P_r = 40\pi^2 \left(\frac{C}{\lambda}\right)^4 \cdot I^2$; C – Circumference of the loop.

$C = 2\pi a$

W.K.T $P_r = \frac{1}{2} I^2 R_r$

$$\boxed{\therefore R_r = 20\pi^2 \left(\frac{C}{\lambda}\right)^4}$$

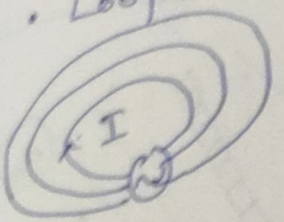
→ Radiation Resistance due to Single loop Antenna



LOOP ANTENNA

- Uniformly radiating in ϕ direction
- Omnidirectional . $\overline{E}$ is fn. of $\sin\theta$.
- Max. radiation at $\theta = \pi/2$. Radiation pattern { Similar to Dipole antenna

· Loop of 'N' turns:



$$\rightarrow R_r = 20\pi^2 \left(\frac{c}{\lambda}\right)^4 (N)^2$$

Q. For a loop of radius 'a' = $\lambda/25$. $R_r' = ?$

Sol: $R_r = 20\pi^2 \left(\frac{c}{\lambda}\right)^4 (N)^2$

$$= 20\pi^2 \frac{1}{\lambda^4} \times (2\pi a)^4 N^2$$

$$= 20\pi^2 \times \left(2\pi \times \frac{\lambda}{25} \times \frac{1}{\lambda}\right)^4 N^2$$

$$= 20\pi^6 \left(\frac{2}{25}\right)^4 N^2$$

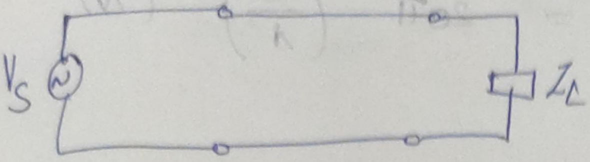
$$= 20\pi^6 (0.4096) \times 10^{-4} \times N^2$$

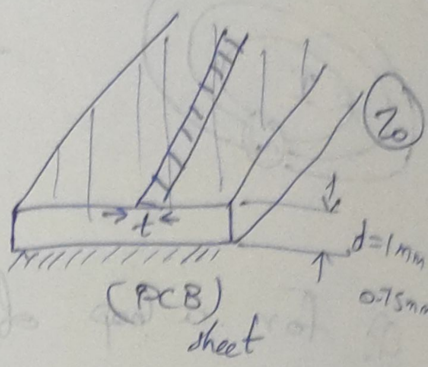
$$= 0.787 N^2 \Omega$$

· if $N = 8$ turns:

$$R_r = 50.4 \Omega$$

07/11/2022

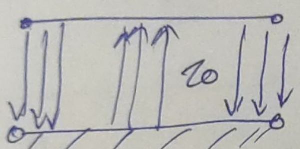


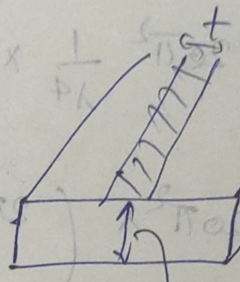


- $Z_0 \big|_{f=1GHz} = 50\Omega$

- Bottom layer is untouched (GND)

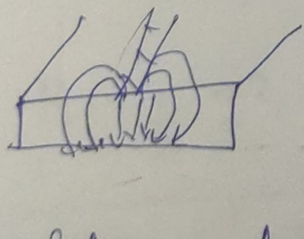
- Microstrip line : planar





- Field is confined to dielectric region for filter (BSF), Coupler

- For Microstrip antenna : • Field should not be confined
  • Fringes should be there



- Dielectric medium should be ideal (less losses)

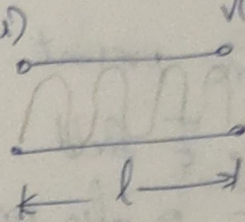
- Mostly used dielectric medium FR-4 for (<10 GHz)
  Laminates / Laminate sheets

  $\epsilon_r = 4.4$

- FR-4 Lossy nature increases as freq. increases
  ↳ Hence, not preferred for higher freq.s

- Rogers : for freq > 10 GHz
  ↳ $\epsilon_r = 2.2$, $\epsilon_r = 9.1$

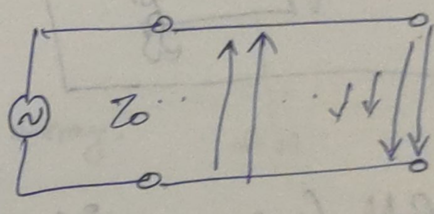
- $\epsilon_r \rightarrow$ velocity 'C' $\rightarrow \lambda$ wavelength
- length → Phase changes



- considering only forward wave

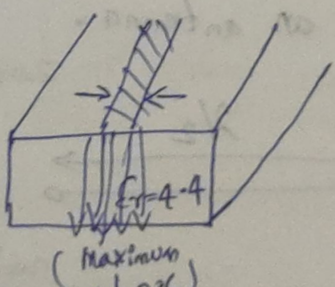
Microstrip Antenna :

O.C. Tx line :



Field max. lines at open Circuit

In Planar :

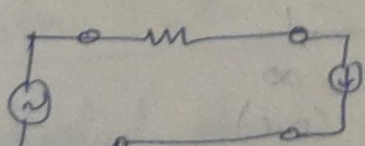


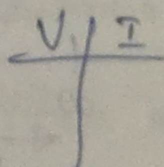
$W \propto (Z_0, f, \epsilon_r)$
↓
width of Tx. line

- Imperical Relation

- Imperical Relation & Mathematical relation/analysis - difference?

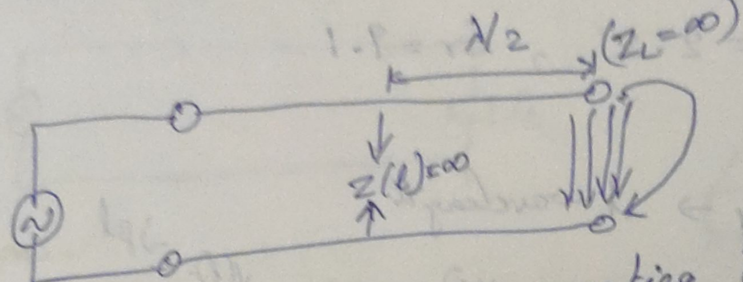
Ex :





- Relation b/w V & I arrived, such relation called as Imperical analysis : no mathematical proof is needed.

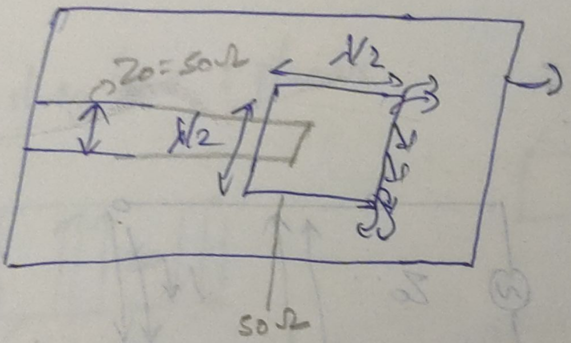
• Tx line : O.C at load



• Fringes are present i.e. making $\bar{E}$ field lines to come out (forcibly)

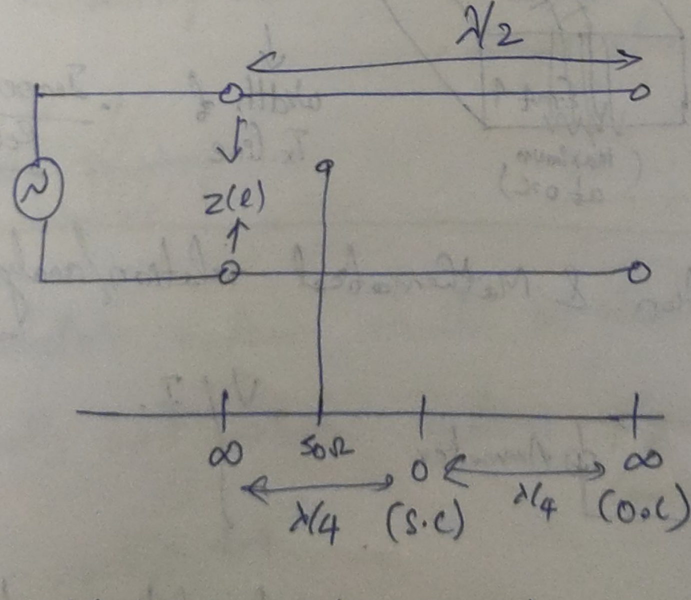
• Time varying $\bar{E}$ field occurs AS source is time varying

• And so $\bar{B}$ is also present



• Fringing field lines are increasing then Microstrip line behaves like an antenna.

• We deliberately make $\bar{E}$ field lines to come out of the confined region to radiate



• Match the impedance at source end.

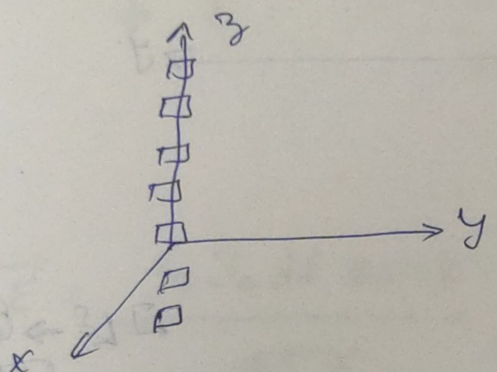
• Fringes ✓

• So, power radiated sufficiently by the antenna

- Microstrip patch antennas
  ↳ Refer websites

Array Antenna

- Gain ~~upto~~ 40dB can be achieved.
  i.e. compared to Isotropic antenna, performs 10^4 times better

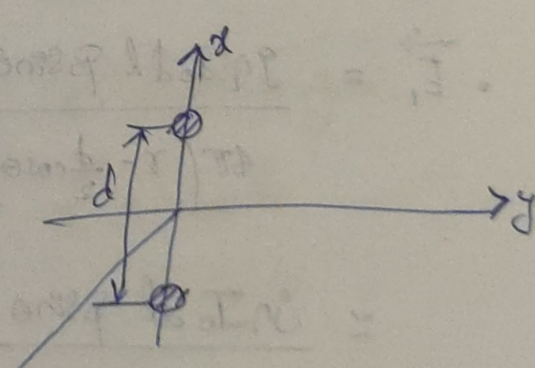


- group of antennas like $\lambda/2$ dipole antennas

- 5G → Beam forming array.

09/11/2022

- Choosing antennas
- Properties
- Distance b/w them
- Phase of excitation



Radiation Pattern of an Array Antenna: depends on

1) Individual element
2) Spacing elements
3) Amplitude of Excitation
4) Phase of excitation

→ By controlling ③, ④, we can control radiation pattern of an array antenna.

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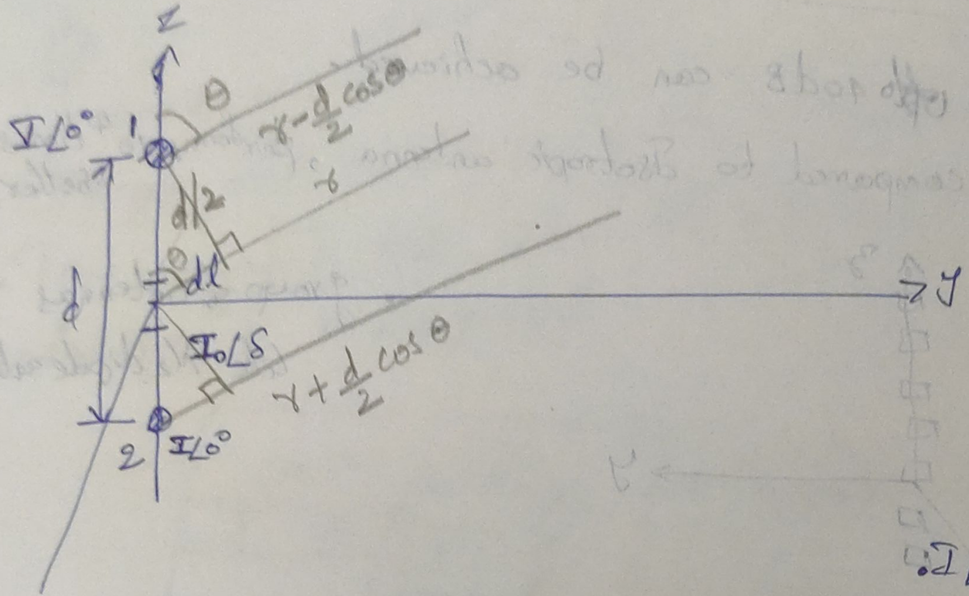
Radiation Pattern of Array Antenna

Dipole antenna : $\bar{E}_\theta = j\eta I_o dl \frac{\beta \sin\theta}{4\pi r} e^{-j\beta r} \hat{a}_\theta$



$\therefore I\angle\delta \rightarrow$ Current Phase excitation

- $\bar{E}_\theta = \frac{j\eta I_o dl \beta \sin\theta}{4\pi r} e^{j\delta} e^{-j\beta r} \hat{a}_\theta$ (located at origin)

- $\vec{E}_1 = \frac{j\eta I_o dl \beta \sin\theta}{4\pi(r - \frac{d}{2}\cos\theta)} e^{-j\beta(r - \frac{d}{2}\cos\theta)} \hat{a}_\theta$

$\simeq \frac{j\eta I_o dl \beta \sin\theta}{4\pi r} e^{-j\beta(r - \frac{d}{2}\cos\theta)} \hat{a}_\theta$ (For field region)

- Element 1 excited with $\angle 0°$ phase

Ily. • Element 2 excited with same amplitude & $\angle 0°$ phase

- $\vec{E}_2 = \frac{j\eta I dl \beta \sin\theta}{4\pi(r + \frac{d}{2}\cos\theta)} e^{-j\beta(r + \frac{d}{2}\cos\theta)} \hat{a}_\theta$

$$\cdot E_2 \simeq \frac{j\eta I_0 dl\, \beta \sin\theta}{4\pi r} e^{-j\beta r} e^{-j\beta\frac{d}{2}\cos\theta} \hat{a}_\theta$$

(Far field region)

· Total Electric field $(\vec{E}) = \vec{E_1} + \vec{E_2}$

$$\Rightarrow \vec{E} = \frac{j\eta I_0 dl\, \beta \sin\theta}{4\pi r} e^{-j\beta r} \left[e^{+j\beta\frac{d}{2}\cos\theta} + e^{-j\beta\frac{d}{2}\cos\theta} \right] \hat{a}_\theta$$

$$\Rightarrow \vec{E} = \underbrace{\frac{j\eta I_0 dl\, \beta \sin\theta}{4\pi r}}_{①} \underbrace{\left[2\cos\left(\frac{\beta d}{2}\cos\theta\right)\right]}_{②} \hat{a}_\theta$$

$$\boxed{\because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}}$$

$\therefore E = \vec{E}$ due to individual element $\times$ (A.F) (Array Factor)

① → due to dipole antenna located at origin.

② → due to arrangement of elements

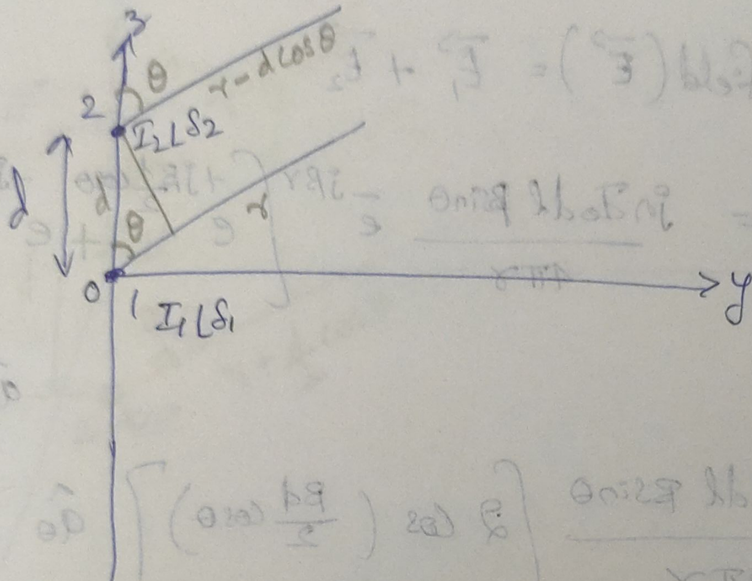
· Pattern multiplication.

· (A.F) depends on spacing (d) and relative phase excitations and relative amplitude excitations.

(Here, $I_1 = I_2$ & $\delta_1 = \delta_2 = 0$)

For Isotropic Antenna:

$$\overline{E} = \frac{I_0}{4\pi r} e^{-j\beta r} e^{j\delta}$$



- 1st element excited at $I_1 \angle \delta_1$
- 2nd element excited at $I_2 \angle \delta_2$

- $$\vec{E_1} = \frac{I_1}{4\pi r} e^{-j\beta r} e^{j\delta_1}$$

- $$\vec{E_2} = \frac{I_2}{4\pi(r - d\cos\theta)} e^{-j\beta(r - d\cos\theta)} e^{j\delta_2}$$

$$\simeq \frac{I_2}{4\pi r} e^{-j\beta(r - d\cos\theta)} e^{j\delta_2}$$

- $$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{I_1}{4\pi r} e^{-j\beta r} e^{j\delta_1} + \frac{I_2}{4\pi r} e^{-j\beta(r - d\cos\theta)} e^{j\delta_2}$$

$$\Rightarrow \bar{E} = \frac{e^{-j\beta r}}{4\pi r}\left[I_1 e^{j\delta_1} + I_2 e^{j\beta d\cos\theta} e^{j\delta_2} \right]$$

$$= \underbrace{\frac{I_1 e^{-j\beta r}}{4\pi r} e^{j\delta_1}}_{\text{Individual element pattern}} \underbrace{\left[1 + \frac{I_2}{I_1} e^{j(\delta_2 - \delta_1)} e^{j\beta d\cos\theta} \right]}_{\text{Array Factor}}$$

Array Factor (depending on spacing, relative amplitude excitation, relative phase excitation) difference

$\boxed{\text{Pattern multiplication}}$

- Assuming $I_1 = I_2$, $\delta_1 = \delta_2 = 0$

$$\Rightarrow \bar{E} = \frac{I_1 e^{-j\beta r}}{4\pi r}\left[1 + e^{j\beta d\cos\theta} \right]$$

$$= \frac{I_1}{4\pi r} e^{\frac{j\beta d\cos\theta}{2}} e^{-j\beta r}\left[e^{-\frac{j\beta d\cos\theta}{2}} + e^{\frac{j\beta d\cos\theta}{2}} \right]$$

$$= \frac{I_1}{4\pi r} e^{\frac{j\beta d\cos\theta}{2}} e^{-j\beta r}\left[2\cos\left(\frac{\beta d\cos\theta}{2}\right) \right]$$

- By keeping array, we made Isotropic antenna as a directional antenna (θ dependent) and so Gain ↑.
- Power fed is same : Isotropic like light bulb
  Directional like torch

- As Isotropic antenna made to behave as Directional antenna, Gain of the antenna is increased.

- $\vec{E} = 0$, at when $\cos\left(\frac{\beta d \cos\theta}{2}\right) = 0$

$$\Rightarrow \frac{\beta d \cos\theta}{2} = \pm \frac{m\pi}{2} \quad ; m = 1, 3, 7, \ldots$$

$$\Rightarrow \cos\theta = \pm \frac{m\pi}{\beta d} \quad ; m = 1, 3, 7, \ldots$$

$\left(\beta = \frac{2\pi}{\lambda}\right)$

$$\Rightarrow \cos\theta = \pm \frac{m\pi}{\left(\frac{2\pi}{\lambda}\right) d}$$

$$\Rightarrow \cos\theta = \pm \frac{m\lambda}{2d} \quad ; m = 1, 3, 7, \ldots$$

- for $m = 1 \Rightarrow \cos\theta = \pm \frac{\lambda}{2d}$

if $d < \lambda/2 \Rightarrow \cos\theta > 1 \rightarrow$ We won't get any nulls

at $d = \lambda/2 \Rightarrow \cos\theta = \pm 1 \Rightarrow \theta = 0$ or π

$\therefore$ Nulls at $\theta = 0$ or

- (2 nulls) for d=

- For $d = \lambda/2 \Rightarrow$ 2 nulls at $\theta = 0, \pi$

$$\Rightarrow \cos\theta = \pm m \quad ; m = 1$$

$$\Rightarrow \cos\theta = \pm 1 \Rightarrow \theta = 0, \pi \text{ (2 nulls)}$$

- For $d = \lambda \Rightarrow \cos\theta = \pm \frac{m\lambda}{2d}$

$$\Rightarrow \cos\theta = \pm \frac{m}{2} \; ; \; m = 1$$

m=3 → not included as cos θ cannot be greater than 1

$$\Rightarrow \cos\theta = \pm 1/2$$

→ 2 Nulls at $\Rightarrow \theta = 60^\circ, 120^\circ$

- For $d = 2\lambda \Rightarrow \cos\theta = \frac{\pm m\lambda}{2d}$

$$\Rightarrow \cos\theta = \frac{\pm m}{4} \; ; \; m = 1, 3$$

$$\cos\theta = \pm\frac{1}{4} \quad \& \quad \cos\theta = \pm\frac{3}{4}$$

→ 4 Nulls

- Spacing b/w the array elements will controls the no. of nulls and location of nulls.
- Space increased then nulls also increased.

14/11/2022

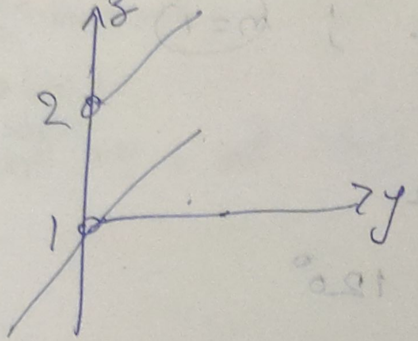
Radiation - Pattern Multiplication

- $\vec{E}_t = \vec{E}_{ind} \times (A \cdot F)$

$\hookrightarrow \left(\frac{I_2}{I_1}, \delta, d\right)$

2-element Isotropic array antenna

14/11/2022



$$\vec{E}_t = \frac{I_1 e^{-j\beta r} e^{j\delta_1}}{4\pi r}\left[1 + \frac{I_2}{I_1} e^{j(\delta_2-\delta_1)} e^{j\beta d\cos\theta}\right]$$

$$= K_0 e^{j\delta_1}\left[1 + \underbrace{\frac{I_2}{I_1}}_{\text{relative amplitude excitation}} \underbrace{e^{j(\delta_2-\delta_1)}}_{\text{relative phase difference excitation}} \underbrace{e^{j\beta d\cos\theta}}_{\text{spacing}}\right]$$

→ Spacing controls the no. of nulls in the pattern

- (Let $d=\lambda$ & $\delta_1-\delta_2=0$ & $\delta_1=0$)
- $E_t = k_0\left\{1 + \frac{I_2}{I_1} e^{j\beta d\cos\theta}\right\}$

→ Relative amplitude controls the depth of the nulls.

→ Relative phase controls the Max. direction.

- Direction of Max. is controlled by Progressive Phase of the elements (δ)

Beam forming Technique

- let $I_1 = I_2 = 1$ & $\delta_2 - \delta_1 = \delta$, $\delta_1 = 0$

$$\Rightarrow \vec{E}_t = k_0 \left\{ 1 + e^{j\delta} e^{j\beta d\cos\theta} \right\}$$

$$= k_0\, e^{j\frac{(\beta d\cos\theta + \delta)}{2}} \left[e^{-j\frac{(\beta d\cos\theta + \delta)}{2}} + e^{+j\frac{(\beta d\cos\theta + \delta)}{2}} \right]$$

$$= k_0\, e^{j\frac{(\beta d\cos\theta + \delta)}{2}} \left[2 \times \cos\left(\frac{\beta d\cos\theta + \delta}{2} \right) \right]$$

- Maxima occurs at $\cos\left(\frac{\beta d\cos\theta + \delta}{2} \right) = \pm 1$

$$\Rightarrow \left. \frac{\beta d\cos\theta + \delta}{2} \right|_{\theta = \theta_m} = \pm m\pi \; ; m = 0, 1, 2, 3 \ldots$$

$$\Rightarrow \beta d\cos\theta_m + \delta = \pm 2m\pi \; ; m = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \cos\theta_m = \frac{\pm 2m\pi - \delta}{\beta d} \; ; m = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \theta_m = \cos^{-1}\left[\frac{\pm 2m\pi - \delta}{\beta d} \right] \; ; m = 0, 1, 2, 3, \ldots$$

$\therefore$ We can tune/control the direction of maxima, by tuning the relative phase excitations.

At $m=0$: $\theta_m = \cos^{-1}\left[-\delta/\beta d\right]$

- If we want the maxima to lie at $0^\circ \Rightarrow \theta_m = 0^\circ$

$$\Rightarrow \cos \theta_m = -\delta/\beta d$$

$$\Rightarrow 1 = -\delta/\beta d$$

$$\Rightarrow \boxed{\delta = -\beta d}$$

- If we want maxima at 90°

$$\Rightarrow \cos 90^\circ = -\delta/\beta d$$

$$\Rightarrow \boxed{\delta = 0^\circ}$$

Beam forming Technique

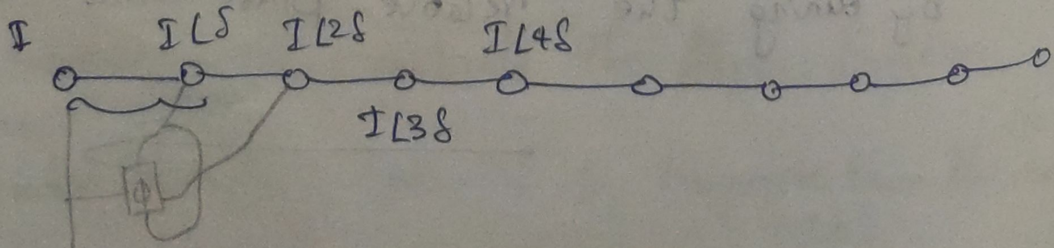
- If we want maxima at 45°

$$\Rightarrow \cos 45^\circ = -\delta/\beta d$$

$$\Rightarrow \boxed{\delta = -\frac{\beta d}{\sqrt{2}}}$$

- This type of array is called Phased array.

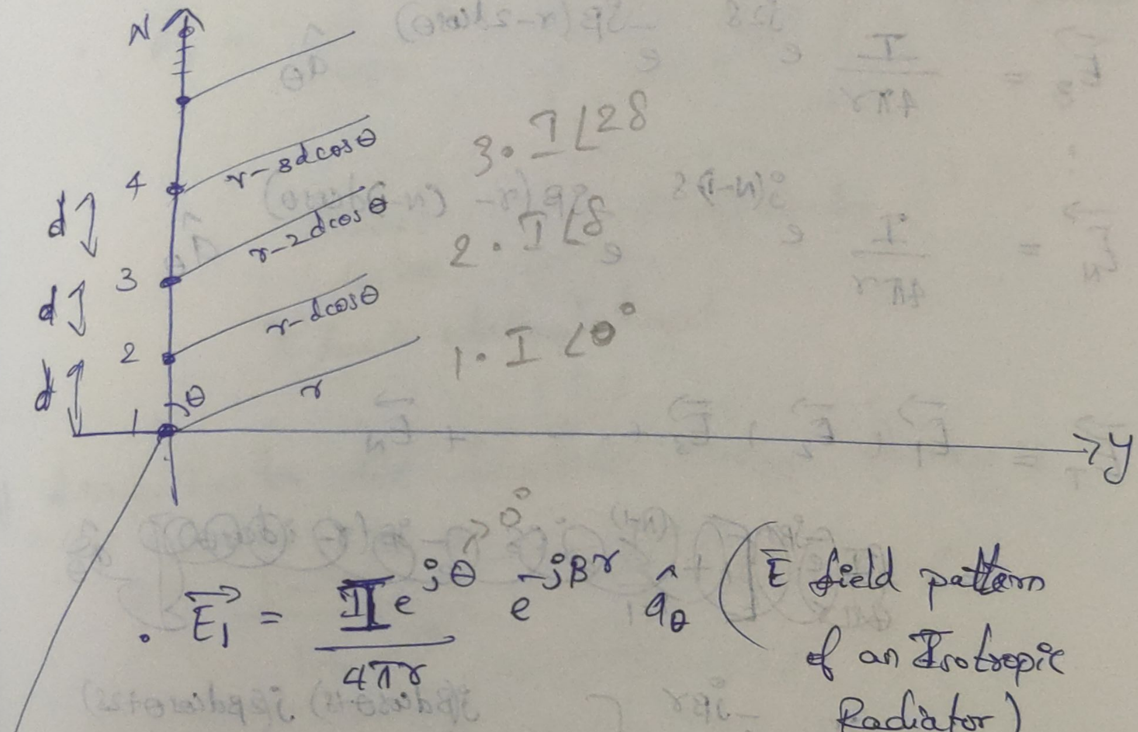
Uniform Linear Array:



Uniform Linear Array:

- phase difference b/w any successive elements is same & amplitude is also same

- Phase shifter.



- $\vec{E_1} = \dfrac{I e^{j0}}{4\pi r} e^{-j\beta r}\,\hat{a}_\theta$ ($\bar{E}$ field pattern of an Isotropic Radiator)

- $\vec{E_2} = \dfrac{I e^{j\delta}}{4\pi(r-d\cos\theta)} e^{-j\beta(r-d\cos\theta)}\,\hat{a}_\theta$

$$\vec{E_2} \cong \frac{I e^{j\delta}}{4\pi r} e^{-j\beta r} e^{j\beta d\cos\theta}\,\hat{a}_\theta$$

- $\vec{E_3} \Rightarrow I\angle 2\delta$

- $\vec{E_4} \Rightarrow I\angle 3\delta$

- $\vec{E_N} \Rightarrow I\angle (N-1)\delta$

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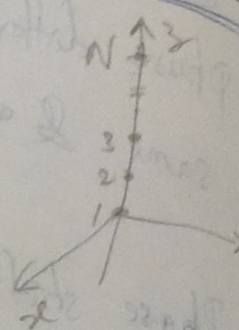


$$\vec{E_1} = \frac{I}{4\pi r} e^{-j\beta r} \hat{a}_\theta$$

$$\vec{E_2} = \frac{I}{4\pi r} e^{+j\delta} e^{-j\beta(r - d\cos\theta)} \hat{a}_\theta$$

$$\vec{E_3} = \frac{I}{4\pi r} e^{j2\delta} e^{-j\beta(r - 2d\cos\theta)} \hat{a}_\theta$$

$$\vdots$$

$$\vec{E_N} = \frac{I}{4\pi r} e^{j(N-1)\delta} e^{-j\beta(r - (N-1)d\cos\theta)} \hat{a}_\theta$$

$$\vec{E_T} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \cdots + \vec{E_N}$$

$$= \frac{I}{4\pi r} e^{-j\beta r} \left[1 + e^{j(\beta d\cos\theta + \delta)} + e^{j(2\beta d\cos\theta + 2\delta)} + \cdots + e^{j\{(N-1)\beta d\cos\theta + (N-1)\delta\}} \right]$$

Geometric Series : $a = e^{j(\beta d\cos\theta + \delta)}$

$$1 + a + a^2 + a^3 + \cdots + a^{N-1} = \frac{1 - a^N}{1 - a}$$

$$\vec{E_T} = \frac{I}{4\pi r} e^{-j\beta r} \left(\frac{1 - e^{jN(\beta d\cos\theta + \delta)}}{1 - e^{j(\beta d\cos\theta + \delta)}} \right)$$

- $\delta \rightarrow$ phase difference b/w 2 elements.
- progressive phase

- $\psi = \beta d \cos\theta + \delta$

$$\therefore \vec{E}_T = \underbrace{\frac{I}{4\pi r} e^{-j\beta r}}_{\substack{\text{Element}\\ \text{factor}}} \underbrace{\left[\frac{1-e^{jN\psi}}{1-e^{j\psi}}\right]}_{\text{Array Factor}} ; \psi = \beta d\cos\theta + \delta$$

$\vec{E}$ due to individual element

Array factor for Uniform Linear Array is

$$AF = \frac{1-e^{jN\psi}}{1-e^{j\psi}} ; \psi = \beta d\cos\theta + \delta$$

Simplyfying A.F :

$$A.F = \frac{1-e^{jN\psi}}{1-e^{j\psi}} = \frac{e^{j\frac{N}{2}\psi}\left[e^{-j\frac{N}{2}\psi} - e^{j\frac{N}{2}\psi}\right]}{1-e^{j\psi}}$$

$$= \frac{e^{j\frac{N}{2}\psi}\left[-2j\sin\frac{N}{2}\psi\right]}{e^{\frac{j\psi}{2}}\left[-2j\sin\frac{\psi}{2}\right]}$$

$$\boxed{\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}} \quad \boxed{\because \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}}$$

- $A.F = e^{j\frac{\varphi}{2}(N-1)} \dfrac{\sin N\varphi/2}{\sin \varphi/2}$

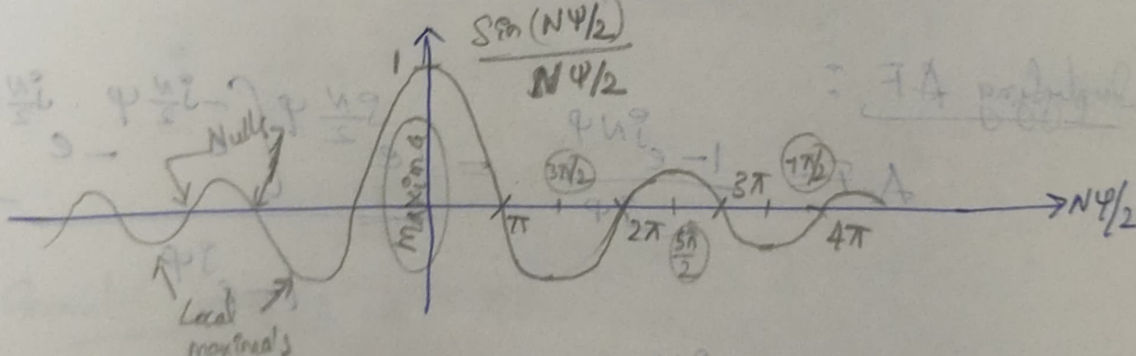
- $A.F|_{max} = N$ (the no. of elements taken)

Normalizing A.F :

$$(A.F)_{normalized} = \frac{\sin(N\varphi/2)}{N\sin(\varphi/2)}$$

$$\cong \frac{\sin(N\varphi/2)}{N\varphi/2} \quad ; \varphi << \text{too small}$$

$$\cong \operatorname{Sinc}(N\varphi/2) \quad \left(\because \operatorname{Sinc} x = \frac{\sin x}{x}\right)$$



$$\therefore (A.F)_n \cong \frac{\sin(N\varphi/2)}{(N\varphi/2)}$$

Maxima of $(A.F)_n$ occurs at: (from graph)

$$\frac{N\psi}{2} = 0 \Rightarrow \psi = 0 \quad ; \psi = \beta d\cos\theta + \delta$$

$$\Rightarrow \beta d\cos\theta + \delta = 0$$

$$\Rightarrow \delta = -\beta d\cos\theta_m \Rightarrow \theta_m = \cos^{-1}(-\delta/\beta d)$$

$$\Rightarrow \boxed{\theta_m = \cos^{-1}\left[-\frac{\delta}{\beta d}\right]}$$

→ Always maxima direction is controlled by (δ) Progressive phase b/w the elements.

Nulls of $(A.F)_n$ occurs at:

$$\sin(N\psi/2) = 0$$

$$N\psi/2 = \pm m\pi \quad ; m = 1, 2, 3, \ldots$$

$$\neq N, 2N, 3N, 4N \ldots$$

If $m = 2N$: $\frac{N\psi}{2} = \pm 2N\pi$ (So $n \neq N, 2N, 3N, \ldots$)

$$\Rightarrow \frac{\psi}{2} = \pm 2\pi$$

$$\Rightarrow \frac{\beta d\cos\theta + \delta}{2} = \pm 2\pi$$

$$\Rightarrow \cos\theta = \frac{\pm 4\pi - \delta}{\beta d}$$

$$\Rightarrow \psi = \pm 4\pi \text{ (maxima's locations)}$$

Nulls At:

$$\frac{N\psi}{2} = \pm m\pi$$

$$\frac{N(\beta d\cos\theta_n + \delta)}{2} = \pm m\pi$$

$$\Rightarrow \cos\theta_n = \frac{\left(\pm\frac{2m\pi}{N} - \delta\right)}{\beta d} = \frac{1}{\beta d}\left[\pm\frac{2m\pi}{N} - \delta\right]$$

$$\Rightarrow \theta_n = \cos^{-1}\left(\frac{\pm\frac{2m\pi}{N} - \delta}{\beta d}\right)$$

$$\Rightarrow \boxed{\theta_n = \cos^{-1}\left[\frac{1}{\beta d}\left(\pm\frac{2m\pi}{N} - \delta\right)\right]} \; ; \; m = 1, 2, 3, \ldots \quad \neq N, 2N, 3N, \ldots$$

16/11/2022

Uniform Linear Array

$$(A.F)_n = \frac{\sin(N\psi/2)}{N\sin(\psi/2)}$$

$$\simeq \frac{\sin(N\psi/2)}{(N\psi/2)}$$

1

$$\therefore \theta_m = \cos^{-1}(-\delta/\beta d)$$

[Maxima at $\psi = 0$

$\beta d\cos\theta_m + \delta = 0$]

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(2) Minima's / Nulls at $\left[\sin \frac{N\psi}{2} = 0\right]$:

$$\frac{N\psi}{2} = \pm m\pi$$

$$(\beta d\cos\theta_n + \delta)\frac{N}{2} = \pm m\pi$$

$$\therefore \theta_n = \cos^{-1}\left[\frac{1}{\beta d}\left[\pm\frac{2m\pi}{N} - \delta\right]\right];$$

$$\left(m = 1,2,3,\ldots \ \neq N, 2N, 3N, \ldots\right)$$

(3) Local Maxima's :

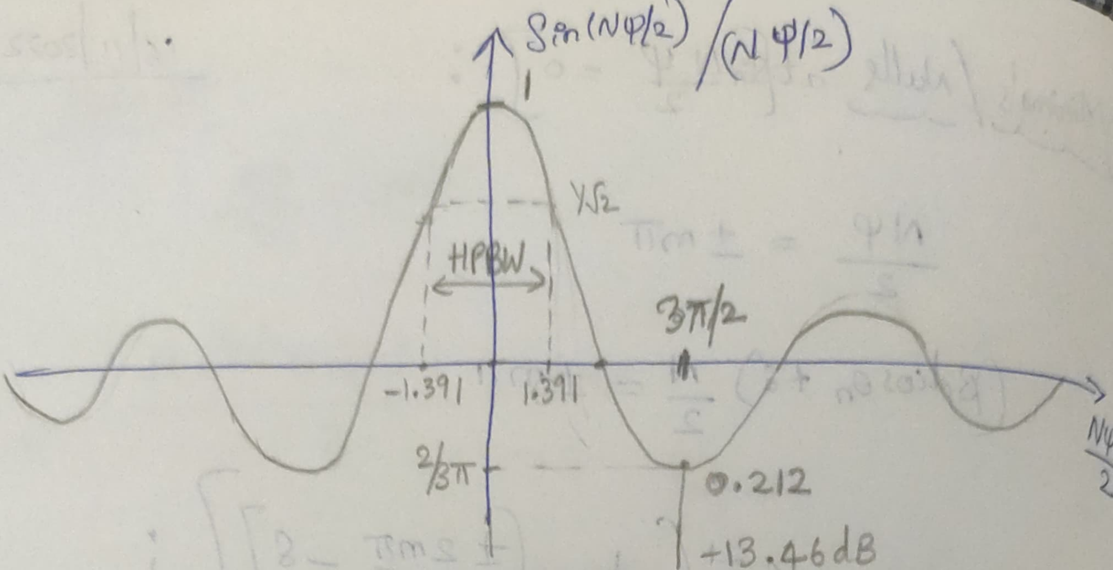
$$\frac{N\psi}{2} = \pm(2s+1)\frac{\pi}{2} \ ; \ s = 1,2,3,\ldots$$

$$\frac{N}{2}(\beta d\cos\theta_s + \delta) = \pm(2s+1)\frac{\pi}{2}$$

$$\Rightarrow \beta d\cos\theta_s = \frac{\pm(2s+1)\pi}{N} - \delta$$

$$\theta_s = \cos^{-1}\left[\frac{1}{\beta d}\left(\frac{\pm(2s+1)\pi}{N} - \delta\right)\right]$$

$$\therefore \theta_s = \cos^{-1}\left[\frac{1}{\beta d}\left(\frac{\pm(2s+1)\pi}{N} - \delta\right)\right]; \ s = 1,2,3,\ldots$$



• First Local maxima at $\frac{N\psi}{2} = \pm \frac{3\pi}{2}$

$$\theta_s = \cos^{-1}\left[\frac{1}{\beta d}\left[\frac{\pm 3\pi}{N} - \delta\right]\right]$$

$$(A.F)_n = \frac{1}{(3\pi/2)} = \frac{2}{3\pi}$$

• $20 \log_{10}(2/3\pi) = -13.465$ dB

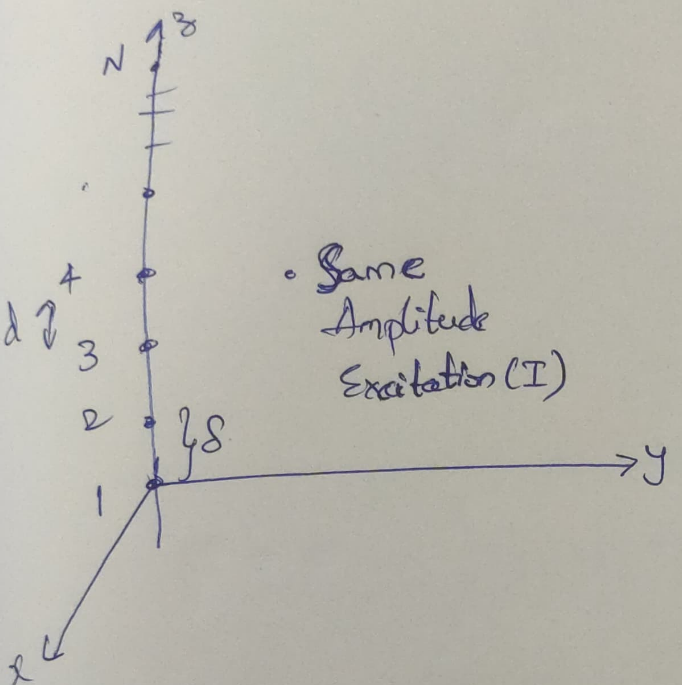
When $\frac{N\psi}{2} = \pm 1.391$ (in radians) then $(A.F)_n = \frac{1}{\sqrt{2}}$

$$\frac{N}{2}(\beta d \cos\theta + \delta)\Big|_{\theta = \theta_h} = \pm 1.391$$

4 (Half Powerangle) $\Rightarrow$ i.e. $\theta_h = \cos^{-1}\left[\frac{1}{\beta d}\left(\frac{\pm 2.782}{N} - \delta\right)\right]$

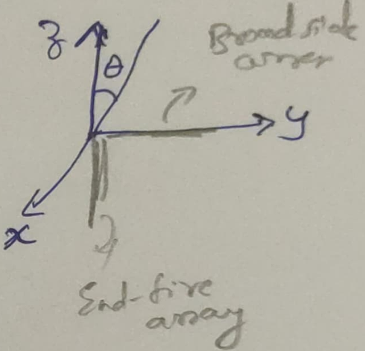
- $HPBW = 2|\theta_m - \theta_h|$

- Beam Width between the First Nulls (BWFN)
  - $BWFN = 2|\theta_m - \theta_n|$



- Same Amplitude Excitation (I)

$$\theta_m = \cos^{-1}\left[-\delta/\beta d\right]$$



- $\theta_m = 0$, $\delta = -\beta d$
- $\theta_m = \pi$, $\delta = \beta d$
- Where maxima's occur ~~are~~ at ends → such arrays are called as End-fire Array, i.e. along the z-axis: ($\theta_m = 0$ & $\theta_m = \pi$)
- $\theta_m = \frac{\pi}{2}$, $\delta = 0$ → such array called as Broad side array where maxima direction is $\perp^r$ to the elements line Broad-Side array ($\theta_m = \frac{\pi}{2}$)

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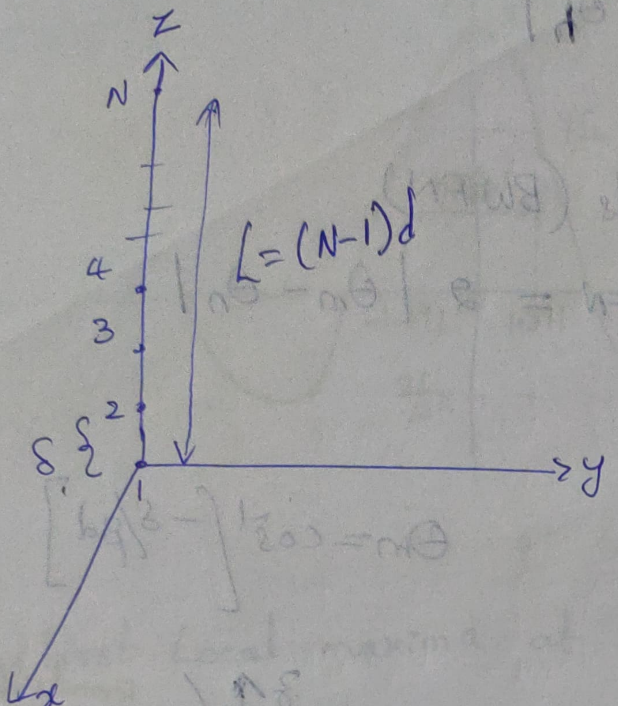


if $\theta_{max} = 0$ or π

$\delta = -\beta d$ or βd

(End-fire array)

if $\theta_{max} = \pi/2$

$\delta = 0$

(Broad side array)

→ $A.F \cong \dfrac{\sin(N\psi/2)}{(N\psi/2)}$; $\psi = \beta d\cos\theta + \delta$

For Broad Side array: $A.F = \dfrac{\sin(N\beta d\cos\theta/2)}{(N\beta d\cos\theta/2)}$; $\left(\begin{array}{c}\theta_m = \pi/2 \\ \delta = 0\end{array}\right)$

- Directivity in the direction of maxima:

$$D_o = \frac{U_o}{U_{avg}} \quad ; \quad U = \text{Power density} \times r^2$$

- For Isotropic antenna: $D_o\Big|_{Iso.} = 1$

- $$\bar{E}_T = \left(\frac{I}{4\pi r} e^{-j\beta r}\right) \times (A.F.)$$

- Power density $(W_r) = \dfrac{1}{2}\dfrac{|E|^2}{2}$

. Power density : $W_r = \frac{1}{2\eta_0}\left(\frac{I}{4\pi r} A.F.\right)^2$

. Radiation Intensity : $U_r = \frac{1}{2\eta_0}\left(\frac{I}{4\pi} . A.F.\right)^2$

for Broad side array :

$$= \frac{1}{2\eta_0} \frac{I^2}{(4\pi)^2} \frac{\sin^2(N\psi/2)}{(N\psi/2)^2}$$

$$= \frac{1}{2\eta_0} \frac{I^2}{(4\pi)^2} \left[\frac{\sin(N\beta d\cos\theta/2)}{N\beta d\cos\theta/2}\right]^2$$

. Maximum radiation Intensity : $U_0 = \frac{I^2}{2\eta_0 (4\pi)^2}$

. $U_{avg} = \frac{P_{rad}}{4\pi}$

. $P_{rad} = \iint U_{rad}\, d\Omega$

$ds = r^2 \sin\theta\, d\theta\, d\phi$

$d\Omega = \sin\theta\, d\theta\, d\phi$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{I^2}{2\eta_0(4\pi)^2} \left[\frac{\sin(N\beta d\cos\theta/2)}{N\beta d\cos\theta/2}\right]^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{I^2}{2\eta_0 (4\pi)^2} \times 2\pi \int_{\theta=0}^{\pi} \frac{\sin^2(N\beta d\cos\theta/2)}{(N\beta d\cos\theta/2)^2} \sin\theta\, d\theta$$

let $z = \frac{N\beta d\cos\theta}{2}$

$dz = -\frac{N\beta d\sin\theta}{2} d\theta$

$\Rightarrow \sin\theta\, d\theta = -\frac{2}{N\beta d} dz$

$$\Rightarrow P_r = \frac{I^2}{2\eta_0 (4\pi)^2} \times 2\pi \int_{\theta=0 \,\to\, z=\frac{N\beta d}{2}}^{\pi \to z=-N\beta d/2} \frac{\sin^2 z}{z^2} \times \frac{-2}{N\beta d}\, dz$$

$$\boxed{\because z = \frac{N\beta d \cos\theta}{2}}$$

$$= \frac{I^2}{16\pi\eta_0} \int_{-\frac{N\beta d}{2}}^{\frac{N\beta d}{2}} \left(\frac{\sin z}{z}\right)^2 \times \frac{2}{N\beta d}\, dz$$

$$\Rightarrow P_r = \frac{I^2}{8\pi\eta_0 N\beta d} \int_{-\frac{N\beta d}{2}}^{\frac{N\beta d}{2}} \left(\frac{\sin z}{z}\right)^2 dz$$

$$\Rightarrow P_r = \frac{I^2}{8\pi\eta_0 N\beta d} \int_{-\infty}^{\infty} \left(\frac{\sin z}{z}\right)^2 dz$$

Considering

$N >>$ (large)

$\frac{N\beta d}{2} \to \infty$

then $\int_{-\infty}^{\infty} \left(\frac{\sin z}{z}\right)^2 dz = \pi$

$$\Rightarrow P_r = \frac{I^2}{8\pi\eta_0 N\beta d} \times (\pi)$$

$$\Rightarrow P_r = \frac{I^2}{8\eta_0 N\beta d} \qquad \therefore P_r = \frac{I^2}{8\eta_0 N\beta d}$$

- $U_{avg} = \frac{P_r}{4\pi} = \frac{I^2}{32\pi\eta_0 N\beta d}$

- $U_0 = \frac{I^2}{2\eta_0 (4\pi)^2}$

$$\therefore D_0 = \frac{U_0}{U_{avg}} = \frac{I^2}{2\eta_0 \times 16\pi^2} \times \frac{32\pi\eta_0 N\beta d}{I^2}$$

$$\boxed{D_0 = \frac{N\beta d}{\pi}}$$

$$D_0 = \frac{N\beta d}{\pi} \quad ; \quad \beta - \text{phase constant} = \frac{2\pi}{\lambda}$$

$$= \frac{N d \times 2\pi}{\pi \lambda} = \frac{2Nd}{\lambda} = 2N\left(\frac{d}{\lambda}\right)$$

$$\boxed{D_0 = \frac{2Nd}{\lambda}}$$ ← Directivity for Broad-Side Array

- Length of the array: $L = (N-1)d \Rightarrow N = \left(\frac{L}{d} + 1\right)$

$$\Rightarrow D_0 = 2N\left(\frac{d}{\lambda}\right) \simeq 2\left(\frac{L}{d} + 1\right)\left(\frac{d}{\lambda}\right)$$

$$\Rightarrow D_0 \simeq 2\left(\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \quad ; \quad L >> d$$

$$D_0 \simeq 2(L/\lambda)$$

- Approximated Directivity for Broad-side array is $\boxed{D_0 \simeq 2(L/\lambda)}$

For End-fire array: $\theta_{max} = 0$ & $\delta = -\beta d$

$$\bar{E}_T = \left(\frac{I}{4\pi r} e^{-j\beta r}\right)(A.F.) \quad ; \quad A.F. = \frac{\sin(N\psi/2)}{N\psi/2}$$

- $\psi = \beta d \cos\theta + \delta = \beta d \cos\theta - \beta d = \beta d(\cos\theta - 1)$

$\longrightarrow$

For End-fire array:

$$A.F = \frac{\sin\left(\frac{N\beta d(\cos\theta - 1)}{2}\right)}{\frac{N}{2}\beta d(\cos\theta-1)}$$

$U_0 = 4Nd/\lambda$

$\simeq 4(4_\lambda)$

$P_{rad} \to \int_0^\infty \left(\frac{\sin z}{z}\right)^2 dz = \frac{\pi}{2}$

- Power density $= \frac{1}{2\eta_0}\left(\frac{I}{4\pi r} A.F\right)^2$

$$U_{rad} = \frac{1}{2\eta_0}\left(\frac{I}{4\pi}\right)^2 \left[\frac{\sin \frac{N\beta d(\cos\theta-1)}{2}}{\frac{N}{2}\beta d(\cos\theta-1)}\right]^2$$

- $U_0 = \frac{I^2}{2\eta_0(4\pi)^2}$

- $U_{avg} = \frac{P_{rad}}{4\pi}$

- $P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U_r \, d\Omega$

$$= \frac{I^2}{2\eta_0(4\pi)^2} \times 2\pi \int_{\theta=0}^{\pi} \left[\frac{\sin \frac{N\beta d(\cos\theta-1)}{2}}{\frac{N}{2}\beta d(\cos\theta-1)}\right]^2 \times \sin\theta \, d\theta$$

$$= \frac{I^2}{16\pi\eta_0} \int_{Z=0}^{-N\beta d} \left(\frac{\sin Z}{Z}\right)^2 \times \frac{-2}{N\beta d} dZ$$

$$= \frac{I^2}{(N\beta d)8\pi\eta_0} \int_{-N\beta d}^{0} \left(\frac{\sin z}{z}\right)^2 dz$$

$$\boxed{\begin{aligned} Z &= \frac{N\beta d}{2}(\cos\theta - 1) \\ dz &= -\frac{N\beta d \sin\theta}{2} d\theta \\ \sin\theta \, d\theta &= -\frac{2}{N\beta d} dZ \end{aligned}}$$

$$\Rightarrow P_{rad} = \frac{I^2}{(N\beta d)(8\pi\eta_0)} \times \int_{-\infty}^{0} \left(\frac{\sin z}{z}\right)^2 dz \ ; \left(\text{when } N>>1\right)$$

$$= \frac{I^2}{(8\pi\eta_0)\,N\beta d} \times \frac{\pi}{2}$$

$$P_{rad} = \frac{I^2}{16\eta_0 N\beta d}$$

$$\cdot\ U_{avg} = \frac{P_{rad}}{4\pi} = \frac{I^2}{64\pi\eta_0 N\beta d}$$

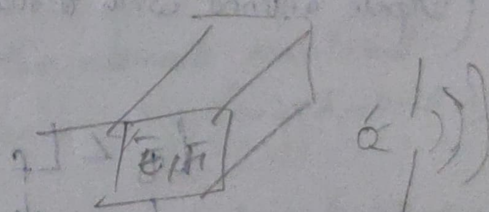
$$\cdot\ U_0 = \frac{I^2}{2\eta_0 (4\pi)^2}$$

$$\cdot\ D_0 = \frac{U_0}{U_{avg}} = \frac{I^2}{2\eta_0(4\pi)^2} \times \frac{64\pi\eta_0 N\beta d}{I^2}$$

$$\Rightarrow D_0 = \frac{2N\beta d}{\pi} \ ; \ \beta = \frac{2\pi}{\lambda}$$

→ Aperture antennas

→ Horn antennas



$$\Rightarrow \boxed{D_0 = 4N\left(\frac{d}{\lambda}\right)}$$

$$\because L = (N-1)d \Rightarrow N = \frac{L}{d} + 1$$

$$D_0 \simeq 4\left(\frac{L}{d}+1\right)\left(\frac{d}{\lambda}\right)$$

$$D_0 \simeq 4\left(\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \ ; \ L >> d$$

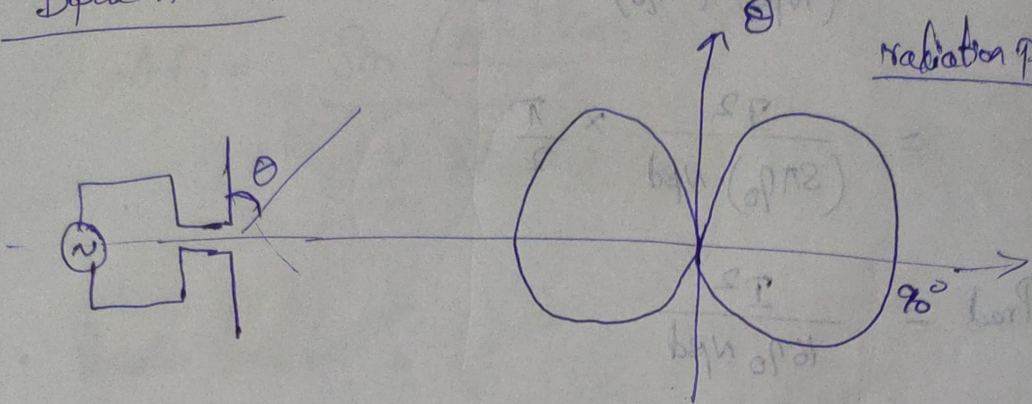
$$\boxed{D_0 \simeq 4(L/\lambda)}$$

for End-fire Array

22/11/2022

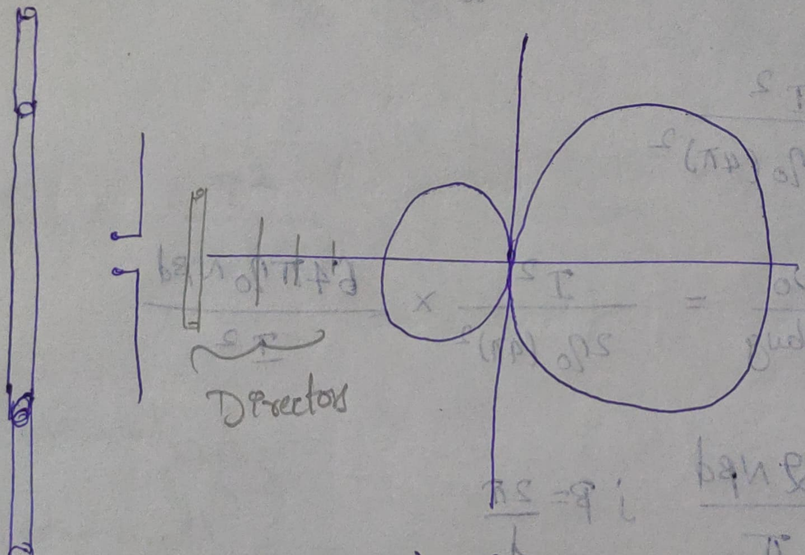
Dipole Antenna

radiation pattern



• In order to increase Directivity, keeping a reflector

$D_0 = 5\,dB$

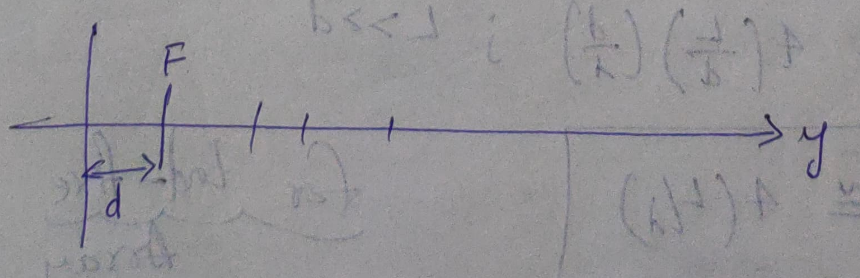


(Delivering max. gain in the axes direction)

(Dipole antenna with reflector) (Length of reflector > Length of Dipole antenna)

$L_D < L_f < L_R$

↓ Directors ↓ field ↓ Reflector

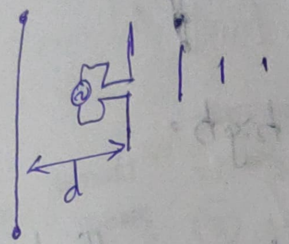


(like End-fire array)

$\theta_m = 0$, $\delta = -\beta d$

$\hookrightarrow d = \lambda/4 \Rightarrow \delta = -\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{-\pi}{2}$

- This type of antenna is called as "Yagi-Uda-Antenna"



$\longrightarrow$ deliver max. gain in the direction of axis

Challenges : Useful to only a specific frequency.

- Practically D (gain) = 7dB | ideally gain is 8 dB.

- Typically L_f - field length is kept to $\lambda/2$

$\longrightarrow L_R : (0.5\cancel{1} - 0.52)\lambda$

$\longrightarrow L_D : (0.\cancel{4}8 - 0.49)\lambda$

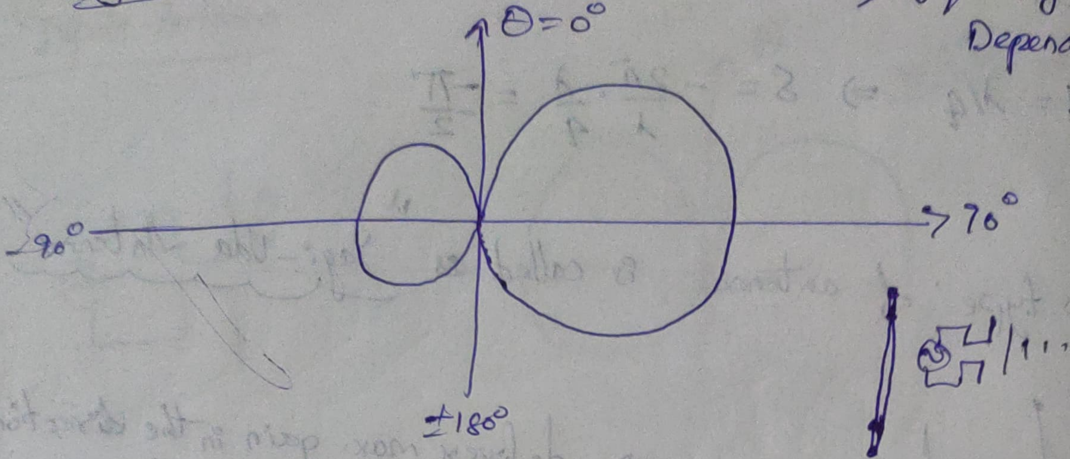
$\longrightarrow d : 0.25\lambda - 0.28\lambda \rightarrow$ Spacing b/w the elements maintained as $(0.25\lambda - 0.28\lambda)$

$\longrightarrow L_f \simeq 0.5\lambda$; $L_r \simeq (0.5\cancel{1} - 52)\lambda$; $L_D \simeq (0.48 - 0.49)\lambda$

- Practical case, no. of directors ; $N \simeq 10$

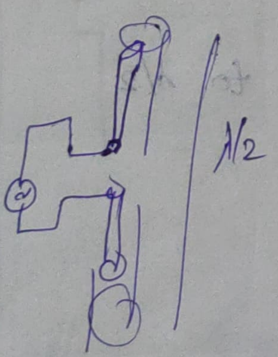
- If more no. of directors are used, losses come into picture & so not preferred.

Yagi-Uda Antenna : Radiation Pattern → Frequency Dependent.



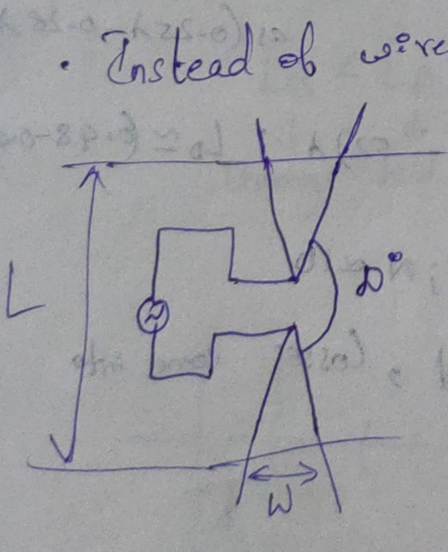
- Front to Back ratio is very very high.

- Increase in B.W can be achieved by using thick wired antenna's over thin wired antenna



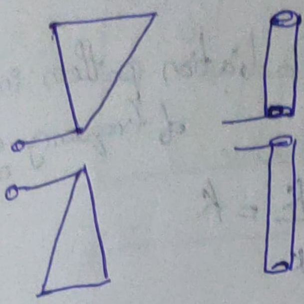
→ A decent increase in B.W by increasing the volume of wire of antenna.

- Instead of wires, using sheets (2 Triangular sheets)



- ~~Bold~~
- Broader frequency range
  → Broad/Wide B.W achieved
- frequency independent of Ω°
- offers greater B.W
- More lossy in nature

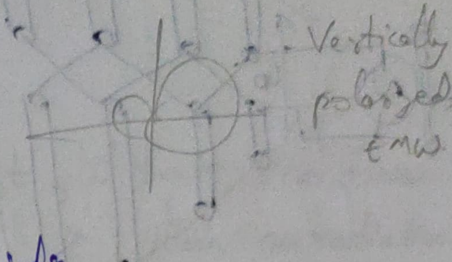
Bow-Tie Antenna (Independent of frequency)



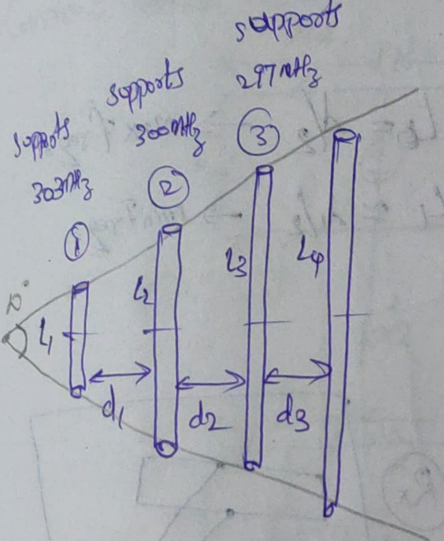
- Applications: Printed . . . (PCB)?
- Offers greater B.W

  B.W → close to 83%.

- At the same time more lossy in nature



Log-Periodic Antenna



$f = 300\,MHz$

$\lambda = 1m$

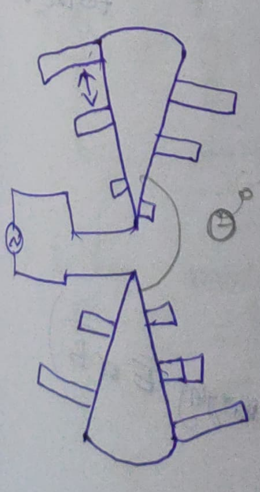
(299–301) MHz $\lambda/2 = 0.5m$

$f = 303\,MHz$, $\lambda/2 = 0.495m$

- $\frac{L_n}{d_n} = k$ (if this can be maintained) then it can cover broad frequency range. i.e. greater B.W.

  . then can maintain at Broader B.W.

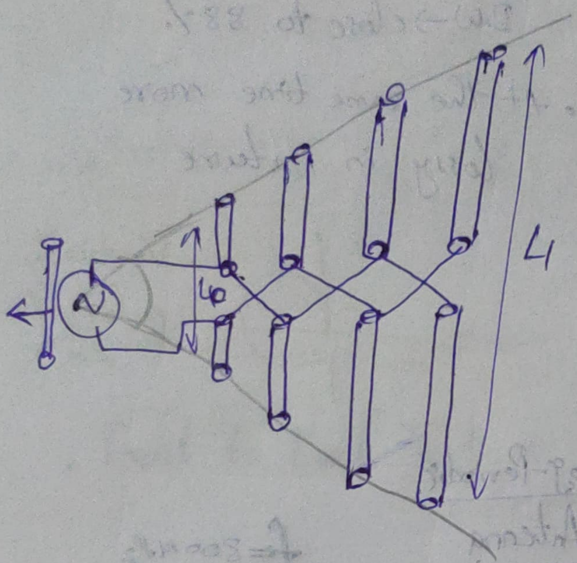
Log-Periodic Tooth Antenna



- $\frac{L}{d} = k$ (constant)
- Larger dimensions determine lower frequencies
- Shorter dimensions determine higher frequencies

Log-Periodic Dipole Array (LPD)

23/11/2022



- Radiation pattern independent of frequency over a range of frequencies

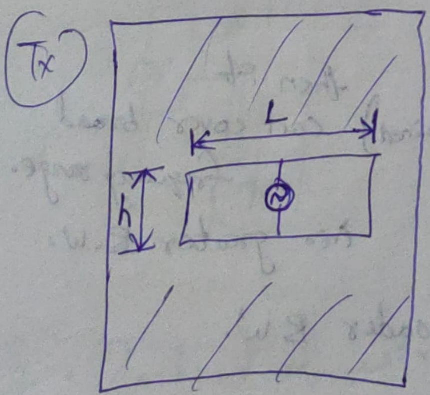
$$\frac{L_k}{d_k} = k$$

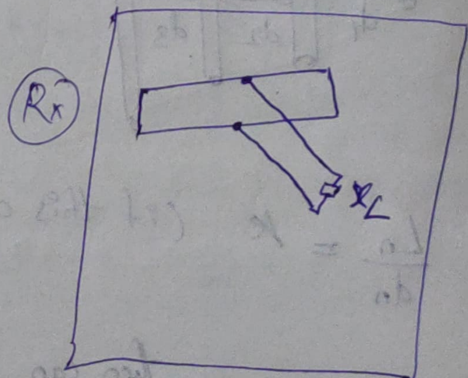
- Wide Band Antenna's

- $L_0 = \lambda/2 \rightarrow$ max freq.

$L_1 \simeq \lambda_1/2 \rightarrow$ min freq.

Slot Antenna:

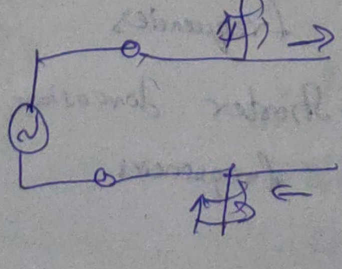




- Energy is coupled to the load.

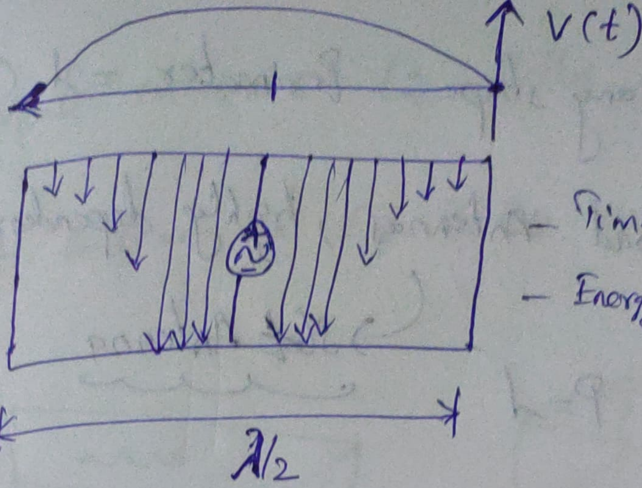
- Height of the slot: $h << \lambda$.
- Length of the slot: $L \simeq \frac{\lambda}{2}$

Reason:



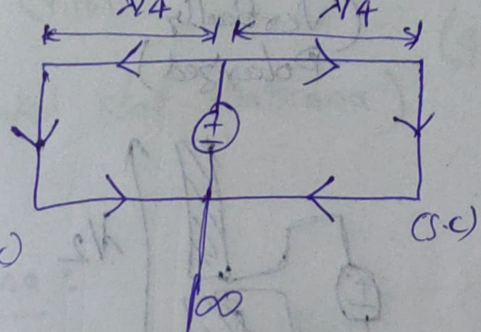
(Time varying $\vec{E}$ or $\vec{B}$)

slot :

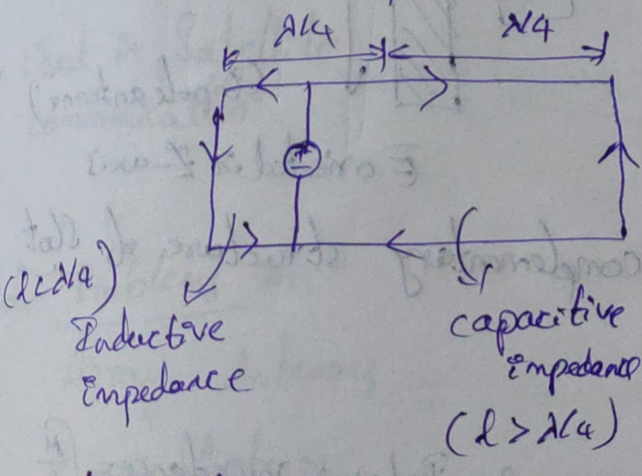


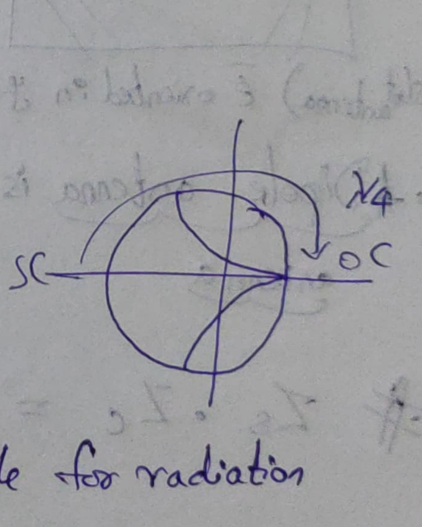
- Time varying $\bar{E}$ is present
- Energy starts to propagate

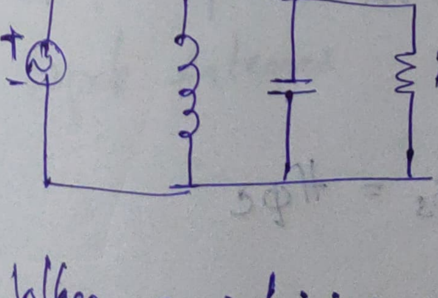
• Current must flow across the slot, if there is no flow then no radiation.



• Circuit fed not exactly at centre
• But at half centre.







R_r → responsible for radiation

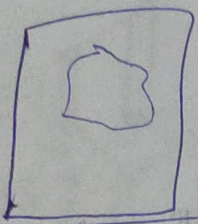
Equivalent circuit model of slot antenna

• When maintaining resonance, max. radiation occurs

Slot can take any shape : Perimeter = λ. (condition)

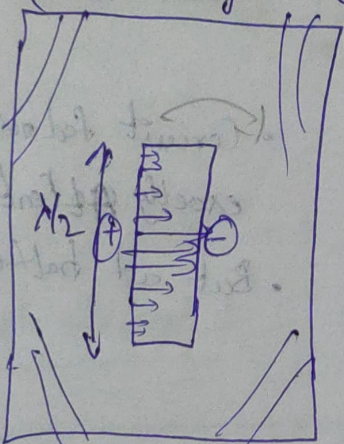
→ Narrow Band Antennas, highly dependent on Perimeter

→ Slot Antenna



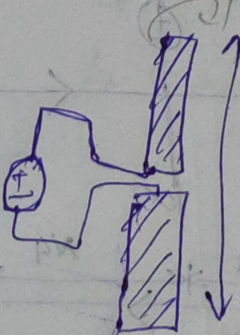
$P = \lambda$

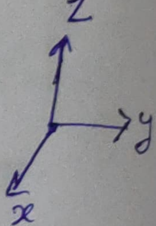
(Horizontally Polarized) (V.P)



(slot antenna) E oriented in y axis

(Vertically Polarized) (H.P)





(Dipole antenna)

E oriented in Z-axis

- A Dipole antenna is a complementary structure of Slot antenna

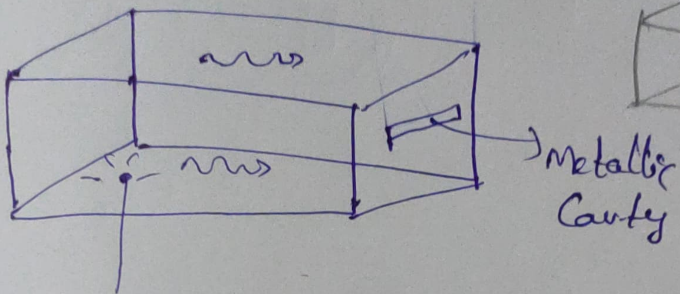
$Z_s \cdot Z_c = \frac{\eta^2}{4}$; η = Intrinsic impedance = $\sqrt{\frac{\mu}{\varepsilon}}$

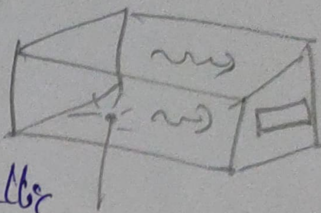
$E_{\theta s} = H_{\theta c}$ & $E_{\phi s} = H_{\phi c}$

$H_{\theta s} = \frac{-E_{\theta c}}{\eta^2}$ & $H_{\phi s} = \frac{-E_{\phi c}}{\eta^2}$

- For a Dipole Antenna: E_θ & H_ϕ exist. (in far-field region)

- For a Slot Antenna: E_ϕ & H_θ exist.

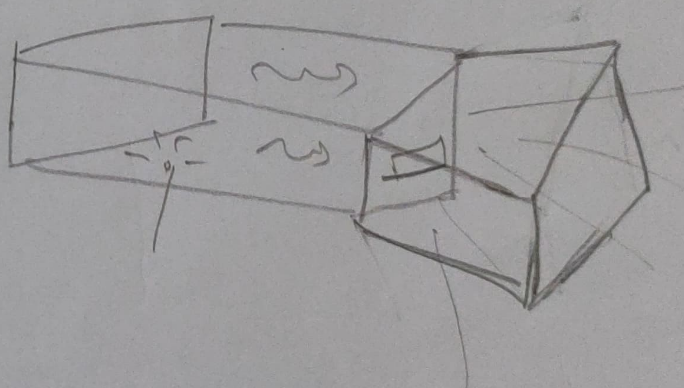




(Cavity backed slot Antenna)

Horn Antenna:

- Used in Satellite communication



Solve Problems on
  Array Antennas
  Loop Antennas
  Dipole Antennas