

Practice Problems - 2

EC252 PTSP

1. X and Y are uniformly on the triangular region $0 < x \leq y \leq x+y < 2$. Find the pdf of $X+Y$ and $X-Y$. Assume X and Y are independent.

2. X and Y independent uniformly distributed random variables on $(0,1)$. Find the joint pdf of $X+Y$ and $X-Y$.

3. The random variables X and Y are independent and Y is uniform in the interval $(0,1)$. Show that if $Z = X+Y$, then

$$f_Z(z) = F_X(z) - F_X(z-1).$$

4. X and Y are independent uniformly distributed random variables in $(0,1)$. Let $W = \max(X,Y)$, $Z = \min(X,Y)$.

Find the pdf of $S = W+Z$.

5. Let the joint pdf of X and Y be given by

$$f_{XY}(x,y) = \begin{cases} e^{-x}, & 0 < y \leq x < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Define $Z = X+Y$, $W = X-Y$. Find the joint pdf of Z and W .

6. Suppose that random variables x and y have joint pdf:

$$f_{xy}(x,y) = \begin{cases} 4x^2, & 0 < y < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find $E[xy]$

b) Find $f_y(y)$

c) Find $f_{x|y}(x|y)$

d) Find $E[x^2 | y=y]$ for $0 < y < 1$.

7. Let U and V be independent random variables such that U is uniformly distributed over the interval $[0,1]$ and V has the exponential probability density function with parameter λ .

Calculate a) $E[V^2 | 1+U]$ b) $P(U \leq V)$.

8. Let X and Y be statistically independent Gaussian-distributed random variables, each with zero mean and unit variance.

Define the Random process

$$Z(t) = X \cos(2\pi t) + Y \sin(2\pi t)$$

a) Determine the joint pdf of the random variables $Z(t_1)$ and $Z(t_2)$ obtained by $Z(t)$ at times t_1 and t_2 respectively.

b) Is the process $Z(t)$ stationary?

9. Consider a pair of stationary processes $X(t)$ and $Y(t)$. Show that the cross-correlations $R_{xy}(\tau)$ and $R_{yx}(\tau)$ have the following properties:

a) $R_{xy}(\tau) = R_{yx}(-\tau)$

b) $|R_{xy}(\tau)| \leq \frac{1}{2} [R_{xx}(0) + R_{yy}(0)]$