EC254:

Transmission lines:

Transmission of certain physical quantities.

In this context, transmission lines (made of good conductors such as aluminium, copper) used for transmission of electrical energy

Main Agenda:

At low frequencies, electrical values of components remains same

But at high frequencies

Performance depends upon:

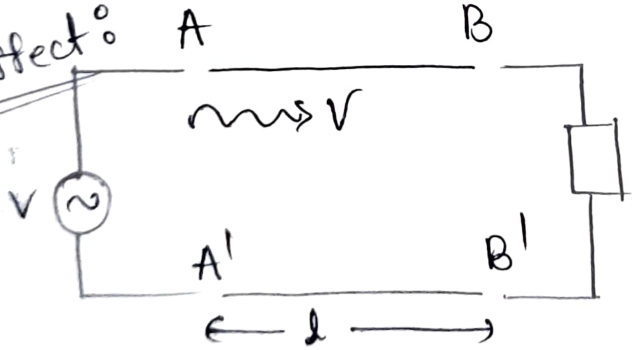
(i) Amount of power transmitted

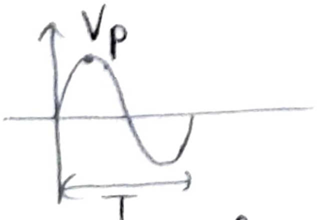
(ii) frequency involved.

Transmission lines

Power lines	Communication lines
→ transmission of large quantities of power at fixed frequency	→ transmission of small quantities of power at band of frequencies.
→ Power efficiency is high	→ Power efficiency is not so important.

Transit time effect:





frequency = f

$T = 1/f$

Assume 'Vp' voltage appears at A, A'

But same voltage doesn't occur at B, B' bcoz there might be some delay (transit time)

$$\boxed{\text{Transit time } (t_r) = \frac{l}{V}}$$

l → length of transmission line

V → Velocity with which sinusoidal voltage travels if $t_r << T$ (transit time effect can be neglected)

if $t_r \simeq T$ then transit time effect is taken into consideration.

As $f \uparrow$

$T \downarrow$

So transit time effect may come into consideration

$t_r << T$

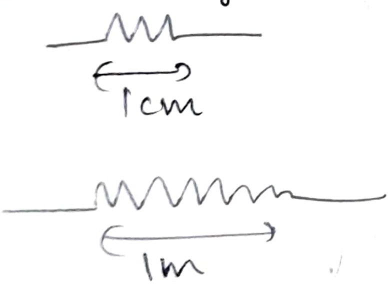
$\frac{l}{V} << T$

$\frac{l}{V} << \frac{1}{f} \rightarrow l << \frac{V}{f} \rightarrow l << \lambda$

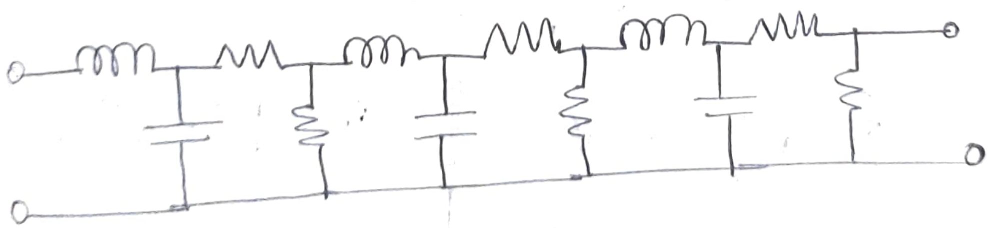
λ : wavelength

if $\lambda >> l$ Ignore X

at high frequency:



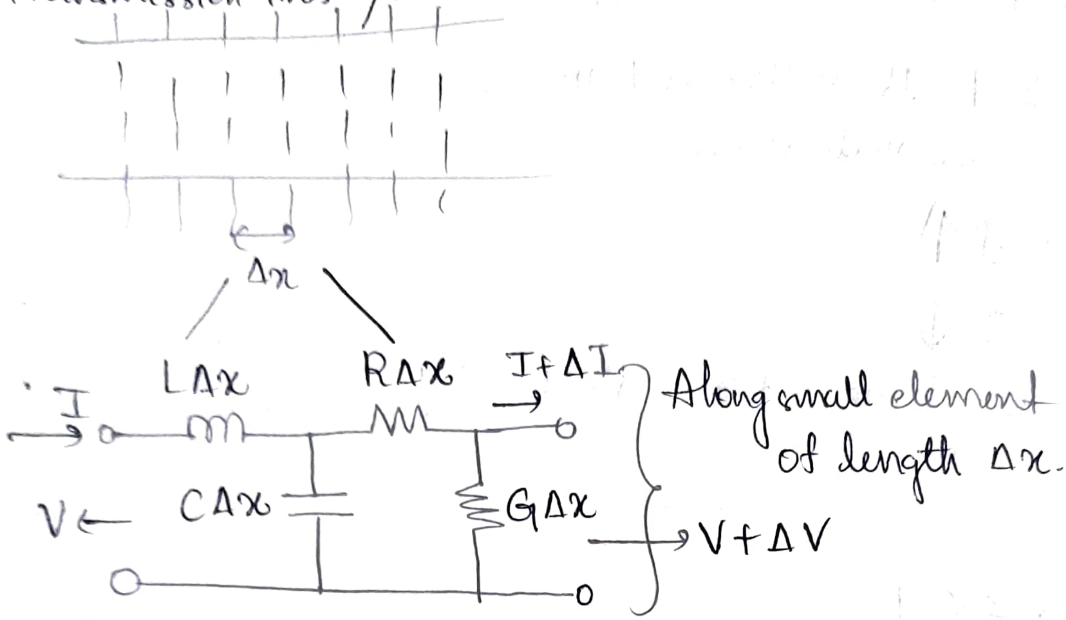
Performance changes as size of components increases



R : Ω/m
L : H/m
C : F/m
G : ℧/m

Primary constants

Transmission lines :-



change in voltage $(\Delta V) = -(R\Delta x + j\omega L\Delta x) I$

Change in current $(\Delta I) = -(G\Delta x + j\omega C\Delta x) V$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -(R + j\omega L) I$$

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = \frac{dI}{dx} = -(G + j\omega C) V$$

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx}$$

$$= +\underbrace{(R + j\omega L)(G + j\omega C)}_{\gamma^2} V$$

$$\boxed{\begin{aligned} \frac{d^2V}{dx^2} &= \gamma^2 V \\ \frac{d^2I}{dx^2} &= \gamma^2 I \end{aligned}}$$

$\gamma \to$ propagation constant.

EC254:

17/12/2021

Phasors:

$$I_1(t) = I_0 \cos(\omega t + \phi) = Re\{I_0 e^{j(\omega t + \phi)}\}$$

$$I_2(t) = I_0 \sin(\omega t + \phi) = Im[I_0 e^{j(\omega t + \phi)}\}$$

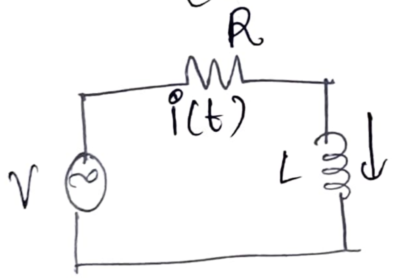
$$= Re[I_0 e^{j(\omega t + \phi)} e^{-j\pi/2}]$$

$$I_s = I_0 e^{j\theta} = I_0 \angle \theta$$

$$\frac{d}{dt}(I_1(t)) = \frac{d}{dt}\{Re(e^{j\omega t} I_s)\}$$

$$\frac{d(I_1 t)}{dt} = Re(j\omega I_s e^{j\omega t})$$

$$\frac{d^2}{dt^2}(I_1(t)) = -\omega^2 I_s$$



$$V(t) = V_0 \cos(\omega t + \theta)$$

$$V(t) = V_0 \angle \theta$$

$$V(t) = I(t) R + L \frac{d}{dt} I(t)$$

$$V_0 \angle \theta = I_1 \angle \theta R + L \cdot j\omega I_1 \angle \theta$$

$$V_0 \angle \theta = I_1 \angle \theta (R + j\omega L)$$

$$I_1 \angle \theta = \left(\frac{V_0 \angle \theta}{R + j\omega L} \right)$$

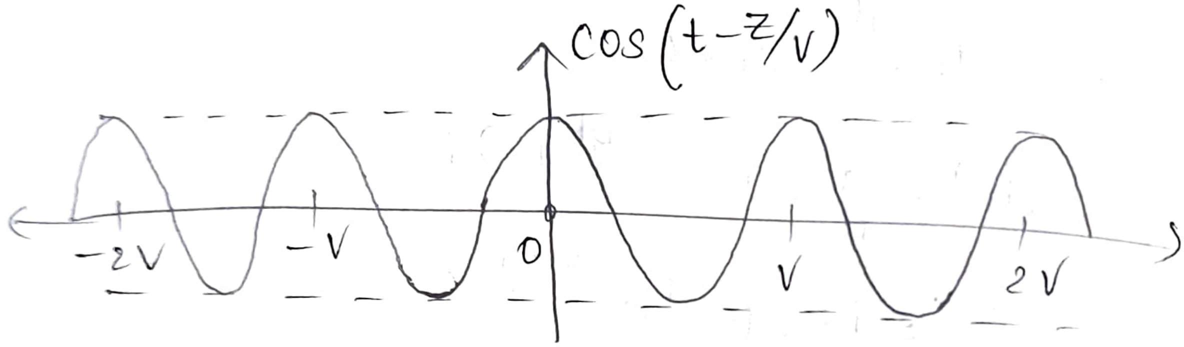
$$I(t) = Re\{I_1 \angle \theta e^{j\omega t}\}$$

- Wave function:

$f\left(t+\frac{z}{v}\right) \rightarrow$ wave travelling left by $-v, -2v, \ldots$

$f\left(t-\frac{z}{v}\right) \rightarrow$ wave travelling right by $v, 2v, \ldots$

$t \rightarrow$ time, v – velocity of wave.



$\cos(\omega t - \beta z) \longrightarrow$ travelling right

$$\text{Velocity}(v) = \frac{\omega}{\beta}$$

$$V(t) = V_0 \cos(\omega t + \beta x)$$

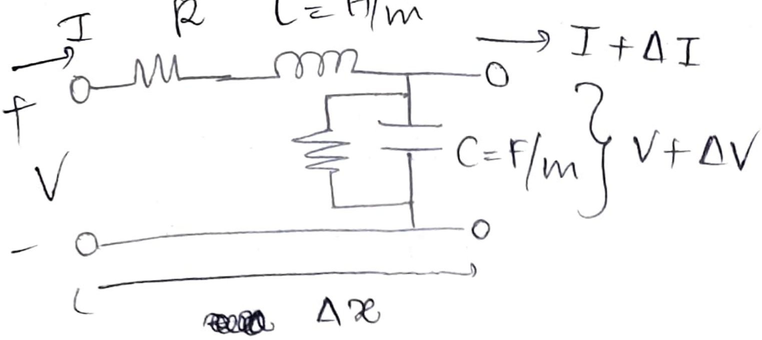
$$V_s = V_0 e^{j\beta x}$$

$$V(t) = \mathrm{Re}(V_s e^{j\omega t})$$

eg: $V_0 \sin(\omega t + \beta z)$

$$V(t) = \mathrm{Re}(V_s e^{j(\omega t - \pi/2)})$$

$$V(t) = \mathrm{Im}(V_s e^{j\omega t}) \quad \boxed{V_s = V_0 e^{j\beta z}}$$



$R \rightarrow \Omega/m \rightarrow$ Lossy wire

$G = \mho/m \rightarrow$ Lossy dielectric.

$R \neq 1/G$

$L \rightarrow H/m \rightarrow$ Time varying magnetic field.

$C \rightarrow F/m \rightarrow$ Time Varying electric field.

$$\cancel{V} = I(R\Delta x + j\omega L\Delta x) + \cancel{V} + \Delta V$$

$$\Delta V = -(R + j\omega L)\Delta x\, I$$

$$\frac{dV}{dx} = \boxed{\frac{\Delta V}{\Delta x} = -(R+j\omega L) I} \quad ✱$$

$$\cancel{I} = V\cdot G\Delta x + V\cdot j\omega C\Delta x + \cancel{I} + \Delta I$$

$$\Delta I = -(G+j\omega C)\Delta x\, \cancel{I} V$$

$$\frac{dI}{dx} = \boxed{\frac{\Delta I}{\Delta x} = -(G+j\omega C) V} \quad ✱$$

Coupled equations.

$$\frac{d^2V}{dx^2} = -(R+j\omega L)\frac{dI}{dx}$$

$$\boxed{\frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)V}$$

$$\frac{d^2V}{dx^2} = \gamma^2 V$$

$$\frac{d^2I}{dx^2} = \gamma^2 I$$

Assume, $\gamma^2 = (R+j\omega L)(G+j\omega C)$

$$V(x) = V^{+}e^{-\gamma x} + V^{-}e^{\gamma x}$$

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \alpha + j\beta$$

$$V(x) = \left(V^{+}e^{-(\alpha + j\beta)x} + V^{-}e^{(\alpha + j\beta)x}\right)e^{j\omega t}$$

$$V(x,t) = Re\left(V(x)\, e^{j\omega t}\right)$$

$$= Re\left(V^{+}e^{-(\alpha + j\beta)x}e^{j\omega t} + V^{-}e^{(\alpha + j\beta)x}e^{j\omega t}\right)$$

$$V(x,t) = V^{+}e^{-\alpha x} Re\{e^{j(\omega t - \beta x)}\}$$
$$+ V^{-}e^{\alpha x} Re\{e^{j(\omega t + \beta x)}\}$$

$$\therefore V(x,t) = V^{+}e^{-\alpha x}\cos(\omega t - \beta x) + V^{-}e^{\alpha x}\cos(\omega t + \beta x)$$

$$\therefore I(x,t) = I^{+}e^{-\alpha x}\cos(\omega t - \beta x) + I^{-}e^{\alpha x}\cos(\omega t + \beta x)$$

- Conclusion:

→ In transmission line, voltage across the lines is a function of sum of forward and backward travelling voltages.

α - Attenuation constant (Np/m)

β - phase constant (rad/m)

$$\beta\lambda = 2\pi$$

$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

$$\boxed{1\, Np/m = 8.686\, dB/m}$$

$$\beta = \frac{2\pi}{\lambda} \quad \left(\text{But } \lambda = \frac{V}{f}\right)$$

$$\beta = \frac{2\pi f}{V}$$

$$\beta = \frac{\omega}{V} \Rightarrow \boxed{V = \frac{\omega}{\beta}}$$

$$\frac{dV}{dx} = -(R+j\omega L)I$$

But, $V(x) = V^+e^{-\gamma x} + V^-e^{\gamma x}$

$I(x) = I^+e^{-\gamma x} + I^-e^{\gamma x}$

$$\frac{d}{dx}\left(V^+e^{-\gamma x} + V^-e^{\gamma x}\right) = -(R+j\omega L)\left(I^+e^{-\gamma x} + I^-e^{\gamma x}\right)$$

$$\left(V^+e^{-\gamma x} - V^-e^{\gamma x}\right) = \frac{(R+j\omega L)}{\gamma}\left(I^+e^{-\gamma x} + I^-e^{\gamma x}\right)$$

But, $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\therefore \boxed{\left(V^+e^{-\gamma x} - V^-e^{\gamma x}\right) = \sqrt{\frac{R+j\omega L}{G+j\omega C}}\left(I^+e^{-\gamma x} + I^-e^{+\gamma x}\right)}$$

$$\Downarrow$$

∘) $V^+e^{-\gamma x} = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}\left(I^+e^{-\gamma x}\right)$ —— ①

∘) $-V^-e^{\gamma x} = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}\left(I^-e^{\gamma x}\right)$ —— ②

from eq (1)

forward travelling wave

$$\frac{V^+}{I^+} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = Z_0 \rightarrow$$ characteristic Impendance quantity

$\downarrow$

Responsible for signal propagation

from eq (2)

Backward travelling wave

$$\frac{V^-}{I^-} = -\sqrt{\frac{R+j\omega L}{G+j\omega C}} = -Z_0$$

$\downarrow$

Responsible for signal reflection.

$$I(x) = I^+ e^{-\gamma x} + I^- e^{\gamma x}$$

$$I(x) = \frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{\gamma x}$$

from the load end $\boxed{l = -x}$

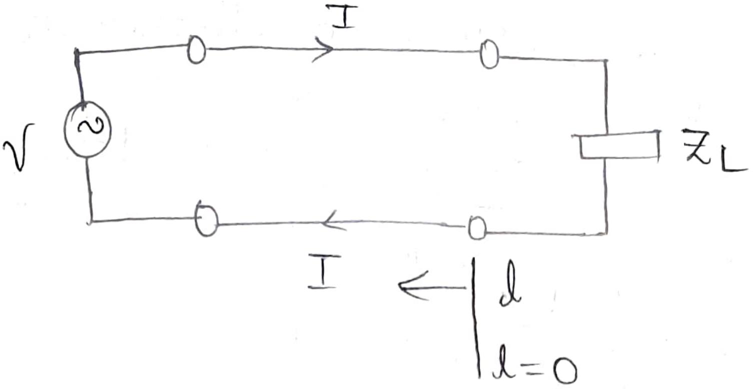
$$\longrightarrow I(l) = \frac{V^+}{Z_0} e^{\gamma l} - \frac{V^-}{Z_0} e^{-\gamma l}$$

$$\longrightarrow V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

$$\frac{V(l)}{I(l)} = Z_0 \left(\frac{V^+ e^{\gamma l} + V^- e^{-\gamma l}}{V^+ e^{\gamma l} - V^- e^{-\gamma l}} \right)$$

$$= Z_0 \cdot \frac{\cancel{V^+ e^{\gamma l}}}{\cancel{V^+ e^{\gamma l}}} \left[\frac{1 + \frac{V^-}{V^+} e^{-2\gamma l}}{1 - \frac{V^-}{V^+} e^{-2\gamma l}} \right]$$

$$\boxed{\frac{V(l)}{I(l)} = Z_0 \left[\frac{1 + \frac{V^-}{V^+} e^{-2\gamma l}}{1 - \frac{V^-}{V^+} e^{-2\gamma l}} \right]}$$



$$\frac{V(0)}{I(0)} = Z_L$$

So, at $l=0$

$$\frac{V(0)}{I(0)} = Z_L = \frac{Z_0\left[1+\frac{V^-}{V^+}\right]}{1\left[1-\frac{V^-}{V^+}\right]} \quad \text{———} \quad (*)$$

$V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$ — V^-: Backward travelling wave Amp.

V^+: Forward travelling wave Amp

$$\frac{V^-}{V^+} = \frac{\text{Reflected wave Amp}}{\text{Inc. wave Amp}} = \Gamma_L$$

Γ_L → Voltage Reflection coefficient at load end.

at $l=0$ → $V(0) = V^+ + V^-$

from $(*)$,

$$\boxed{Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right)}$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}} \Rightarrow \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$(*)$ ⇒ If $Z_L = Z_0$, $\frac{V^-}{V^+} = 0$ → $\boxed{V^- = 0}$ → No reflection at load end.

$\Rightarrow$ If $Z_L = Z_0$ there will be no reflection at the load end, this condition is said to be

<u>Impedance mismatching</u>

$$\boxed{Z(l) = Z_0 \left\{ \frac{1+\Gamma_L e^{-2\gamma l}}{1-\Gamma_L e^{-2\gamma l}} \right\}}$$

if $\Gamma_L = 0$

$\Downarrow$

$$\boxed{Z(l) = Z_0}$$

$$Z(l) = Z_0 \left\{ \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma l}}{1 + \frac{Z_0 - Z_L}{Z_L + Z_0} e^{-2\gamma l}} \right\}$$

$\to$ Towards generator

$$Z(l) = Z_0 \left\{ \frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l} \right\}$$

$\to$ Impedance transformation relation

$$Z_A = Z_0 \left\{ \frac{Z_B \cosh \gamma l - Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l - Z_B \sinh \gamma l} \right\}$$

$\to$ Towards load.

$$\bar{Z}(l) = \frac{Z(l)}{Z_0} = \left\{ \frac{\bar{Z}_L \cosh \gamma l + \sinh \gamma l}{\cosh \gamma l + \bar{Z}_L \sinh \gamma l} \right\}$$

Loss-less Transmission line:

Here, $R = G = 0$

So, $\gamma = \sqrt{(0+j\omega L)(0+j\omega C)}$

$\boxed{\gamma = j\omega\sqrt{LC}}$ But $\gamma = \alpha + j\beta$

So, $\boxed{\beta = \omega\sqrt{LC}}$

$\downarrow$

Imaginary always for loss-less Transmission.

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} \rightarrow \text{Real quantity.}$$

$$\boxed{v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}} \rightarrow \text{Speed of a loss-less transmission.}$$

low-loss Transmission line:

$R << \omega L \qquad G << \omega C$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{j\omega L\left(1-\frac{jR}{\omega L}\right)j\omega C\left(1-\frac{jG}{\omega C}\right)}$$

$$= j\omega\sqrt{LC}\left\{\left(1-\frac{jR}{\omega L}\right)\left(1-\frac{jG}{\omega C}\right)\right\}^{1/2}$$

$$= j\omega\sqrt{LC}\left\{1-\frac{jR}{\omega L}-\frac{jG}{\omega C}\right\}^{1/2}$$

$$\gamma \simeq j\omega\sqrt{LC}\left\{1-\frac{jR}{2\omega L}-\frac{jG}{2\omega C}\right\}$$

$$\boxed{\gamma \simeq \underbrace{j\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{1}{2}R\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}_{\alpha}}$$

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

$$= \frac{R}{2}\,\frac{\omega\sqrt{L}\,\sqrt{C}}{\omega L} + \frac{G}{2}\,\frac{\omega\sqrt{C}\,\sqrt{L}}{\omega C}$$

$$\alpha = \frac{R}{2} \times \frac{\beta}{\omega L} + \frac{G}{2}\,\frac{\beta}{\omega C}$$

$$\boxed{\alpha \simeq \beta\left\{\frac{R}{2\omega L} + \frac{G}{2\omega C}\right\}}$$

$\boxed{\alpha << \beta} \rightarrow$ in low-loss transmission

$$\left(\begin{array}{ll} e^{-\alpha l} & \lambda = \frac{2\pi}{\beta} \\ \downarrow & \\ e^{-\alpha \frac{2\pi}{\beta}} & = e^{-\frac{\alpha}{\beta}2\pi} \end{array}\right)$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L\left(1 - \frac{jR}{\omega L}\right)}{j\omega C\left(1 - \frac{jG}{\omega C}\right)}}$$

$$= \sqrt{\frac{L}{C}}\sqrt{\frac{1 - jR/\omega L}{1 - jG/\omega C}}$$

$$Z_0 \simeq \sqrt{\frac{L}{C}}\left\{1 - \frac{jR}{2\omega L} + \frac{jG}{2\omega C}\right\}$$

Problems:

① The characteristic impedance of Tr. line is 75Ω and load impedance is $50+j(150)\Omega$. Determine Γ_L?

Sol:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150j - 25}{150j + 125}$$

22500
2500
25000

$$= \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} = \frac{6j - 1}{6j + 5}$$

$$= 0.508 + j0.590$$

$$= 0.7788 \angle 49.27^\circ$$

$\Gamma_L = -1 \rightarrow$ short circuited $(Z_L = 0)$

$\Gamma_L = +1 \rightarrow$ open circuited. $(Z_L = \infty)$

② The primary constants $R = 17\,\Omega/m$

~~$G = 0.35$ MH/m~~

$L = 0.35$ mH/m

$G = 75\ \mu S/m$

$C = 40$ pF/m

γ, Z_0 at 2 GHz

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Sol: $\omega = 2\pi f$

$= 2\pi(2\times 10^9)$

$= 4\pi\times 10^9$

$$\gamma = \sqrt{(17 + j(4\pi\times 10^9 \times 0.35\times 10^{-3}))(75\times 10^{-6} + j(4\pi\times 10^9\times 40\times 10^{-12}))}$$

• Loss-less Transmission:

$$V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

But in loss-less $\gamma = j\beta$

So, $V(l) = V^+ e^{j\beta l} + V^- e^{-j\beta l}$

$$V(l) = V^+ e^{j\beta l}\left(1 + \frac{V^-}{V^+} e^{-j2\beta l}\right)$$

We know that, $\frac{V^-}{V^+} = \Gamma_L$

$$\frac{V^-}{V^+} = |\Gamma_L| e^{j\phi_L}$$

$$\boxed{V(l) = V^+ e^{j\beta l}\left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)}$$

if $e^{j(\phi_L - 2\beta l)} \to 0, 2\pi, 4\pi, \ldots$

$\longrightarrow 1 + |\Gamma_L|$

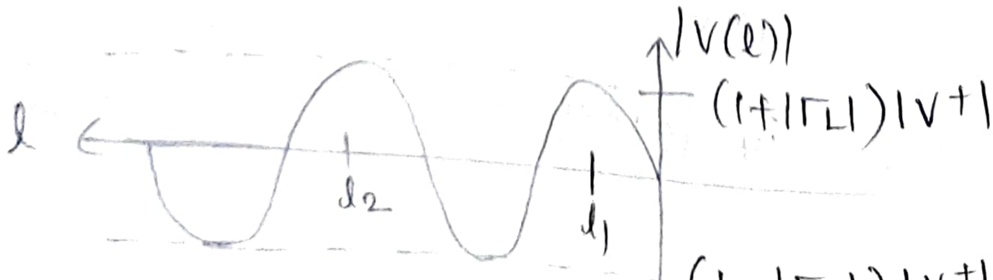
if $e^{j(\phi_L - 2\beta l)} \to \pi, 3\pi, 5\pi, \ldots$

$\longrightarrow 1 - |\Gamma_L|$

$$\Rightarrow |V(l)| = |V^+|(1 + |\Gamma_L|)$$

(or)

$$|V(l)| = |V^+|(1 - |\Gamma_L|)$$



At maxima, $\phi_L - 2\beta l = m\pi$ $m = 0, 2, 4, 6, 8, \ldots$

At minima, $\phi_L - 2\beta l = n\pi$ $n = 1, 3, 5, 7, \ldots$

$\Rightarrow$ The phase difference $\Delta\phi\Big|_{\text{max-max}} = 2\pi$

$\Delta\phi\Big|_{\text{min-min}} = 2\pi$

Successive maxima (or) Successive minima

$(\cancel{\phi_L} - 2\beta l_1 - \cancel{\phi_L} + 2\beta l_2) = 2\pi$

$2\beta(l_2 - l_1) = 2\pi$

$2 \times \frac{2\pi}{\lambda} \times (l_2 - l_1) = 2\cancel{\pi}$ $\left(\beta = \frac{2\pi}{\lambda}\right)$

Successive max or min $\leftarrow$ $\boxed{l_2 - l_1 = \lambda/2}$

$\boxed{l_2 - l_1 = \lambda/4}$ $\rightarrow$ Successive max and min.

$$\boxed{I(l) = \frac{V^+ e^{j\beta l}}{Z_0}\left(1 - |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)}$$

if $\phi_L - 2\beta l = m\pi$ $\quad m = 0, 2, 4, 6, \ldots$

then $\boxed{I(l) = \frac{V^+ e^{j\beta l}}{Z_0}(1 - |\Gamma_L|)}$

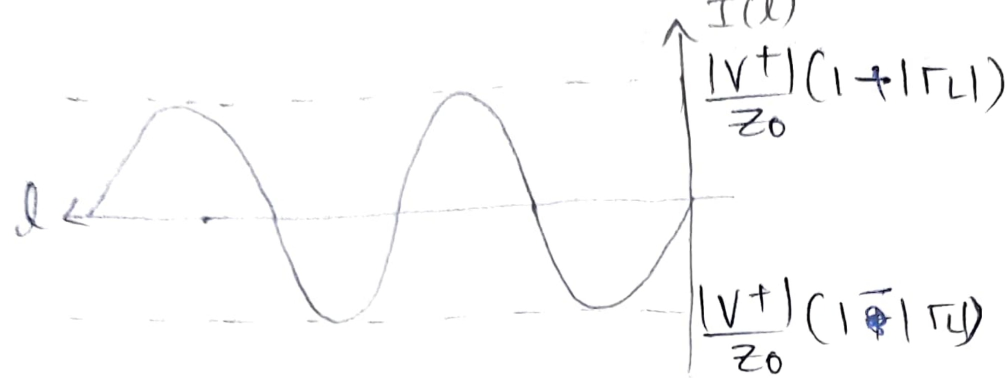
if $\phi_L - 2\beta l = n\pi$ $\quad n = 1, 3, 5, \ldots$

$$\boxed{I(l) = \frac{V^+ e^{j\beta l}}{Z_0}(1 + |\Gamma_L|)}$$

$\Gamma_L = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$

$|\Gamma_L|_{max} = 1$
- $\rightarrow Z_L = 0$ (short circuit)
- $\rightarrow Z_L =$ pure reactance
- $\rightarrow Z_L = \infty$ (open circuit)

$|\Gamma_L|_{min} = 0$ $\rightarrow Z_L = Z_0$ (Impedance mismatch)



- Voltage standing wave ratio: (ρ)

$$\rho = \frac{|V^+|(1+|\Gamma_L|)}{|V^+|(1-|\Gamma_L|)} = \frac{V_{max}}{V_{min}}$$

$\rho_{max} = \infty$

$\rho_{min} = 1$

$$\boxed{\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}}$$

$\rightarrow$ if $Z_L = Z_0$

$\Gamma_L = \frac{V^-}{V^+} = 0$

So, $\boxed{\rho = 1}$

$$\boxed{Z(l) = \frac{V(l)}{I(l)} = Z_0\left(\frac{1+|\Gamma_L|e^{j(\phi_L-2\beta l)}}{1-|\Gamma_L|e^{j(\phi_L-2\beta l)}}\right)}$$

At voltage maxima, $(\phi_L - 2\beta l = 0, 2\pi, 4\pi, \ldots)$

$$\rightarrow Z(l) = Z_0\left(\frac{1+|\Gamma_L|}{1-|\Gamma_L|}\right) = Z_0\rho$$

At voltage minima, $(\phi_L - 2\beta l = \pi, 3\pi, \ldots)$

$$\rightarrow Z(l) = Z_0\left(\frac{1-|\Gamma_L|}{1+|\Gamma_L|}\right) = \frac{Z_0}{\rho}$$

$\rightarrow$ Normalised value $(\overline{Z(l)}) = \frac{Z(l)}{Z_0}$

$$\boxed{\overline{Z(l)} = \frac{1+|\Gamma_L|e^{j(\phi_L-2\beta l)}}{1-|\Gamma_L|e^{j(\phi_L-2\beta l)}}}$$

$$\bar{Z}\left(l+\frac{\lambda}{2}\right) = \left(\frac{1+|\Gamma_L|\, e^{j(\phi_L-2\beta l)}\, e^{-j(2\beta)\lambda/2}}{1-|\Gamma_L|\, e^{j(\phi_L-2\beta l)}\, e^{-j(2\beta)\lambda/2}}\right)$$

$$\boxed{\bar{Z}\left(l+\frac{\lambda}{2}\right) = \bar{Z}(l)}$$

Impedance repeats after a distance of '$\frac{\lambda}{2}$'.

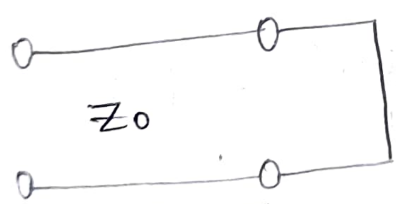
$$\bar{Z}\left(l+\frac{\lambda}{4}\right) = \left(\frac{1+|\Gamma_L|\, e^{j(\phi_L-2\beta l)}\, e^{-j(2\beta)(\lambda/4)}}{1-|\Gamma_L|\, e^{j(\phi_L-2\beta l)}\, e^{-j(2\beta)(\lambda/4)}}\right)$$

$$= \frac{1-|\Gamma_L|\, e^{j(\phi_L-2\beta l)}}{1+|\Gamma_L|\, e^{j(\phi_L-2\beta l)}} = \frac{1}{\bar{Z}(l)}$$

$$\boxed{\bar{Z}\left(l+\frac{\lambda}{4}\right) = \frac{1}{\bar{Z}(l)}}$$

Impedance reverses itself after a distance of '$\frac{\lambda}{4}$'.

Q)



$Z_L = 0\,\Omega$ $\qquad \bar{Z}_L = \frac{Z_L}{Z_0} = 0$

$\Gamma_L = -1$

$|\Gamma_L| = 1$ $\qquad \bar{Z}(\lambda/2) = \bar{Z}(l)$

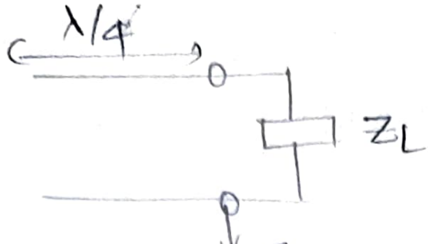
$\bar{Z}(\lambda/4) = ?$ $\qquad S = \infty$ $\qquad = 0$ //

$$\bar{Z}(\lambda/4) = \frac{1}{\bar{Z}(l)} = \frac{1}{0} = \infty \;//$$

& $Z_{min} = \frac{Z_0}{S} = 0$ //

$Z_{max} = Z_0 S = \infty$ //

Q)



$$\frac{Z_L}{Z_0}$$

$$\bar{z}(\lambda/4) = ?$$

$$\bar{z}(\lambda/4) = \frac{1}{\bar{z}(l)} = \frac{1}{\frac{Z_L}{Z_0}} = \frac{Z_0}{Z_L}$$

$$\frac{Z(\lambda/4)}{Z_0} = \frac{Z_0}{Z_L} \Rightarrow \boxed{Z(\lambda/4) = \frac{Z_0^2}{Z_L}}$$

Problem:

① A transmission line having characteristic impedance $= 50\,\Omega$, $Z_L = 150 + j(100)\,\Omega$. Determine Γ_L and S, $Z\left(l + \frac{\lambda}{4}\right)$

Sol:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j(100)}{200 + j(100)} = \frac{1+j}{2+j}$$

$$= 0.6 + 0.2j = 0.63 \angle 18.43^\circ$$

$$S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.63}{1 - 0.63} = 4.4$$

$$\bar{z}\left(l + \frac{\lambda}{4}\right) = \frac{1}{\bar{z}(l)}$$

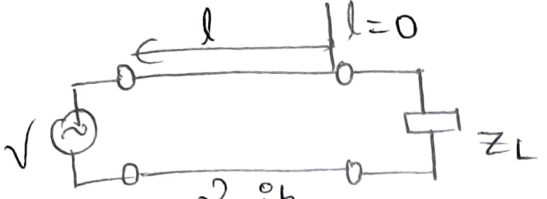
$$\frac{100}{6+4j}$$

$$Z\left(l + \frac{\lambda}{4}\right) = \frac{Z_0^2}{Z(l)} = 11.53 - j(7.69)$$

$$= 13.86 \angle -33.69^\circ$$

$$Z_{min} = \frac{Z_0}{S} = 11.36\,\Omega$$

$$Z_{max} = Z_0 S = 220\,\Omega$$



$\gamma = j\beta$

$$V(l) = V^{+} e^{j\beta l} + V^{-} e^{-j\beta l} = V^{+} e^{j\beta l}\left(1+\Gamma_L e^{-2j\beta l}\right)$$

$$I(l) = \frac{V^{+}}{Z_0} e^{j\beta l} - \frac{V^{-}}{Z_0} e^{-j\beta l} = \frac{V^{+} e^{j\beta l}}{Z_0}\left(1-\Gamma_L e^{-2j\beta l}\right)$$

$$V(l=0) = V^{+}(1+\Gamma_L)$$

$$I(l=0) = \frac{V^{+}}{Z_0}(1-\Gamma_L)$$

$$P(l=0) = \frac{1}{2} \mathrm{Re}\left(V(l=0)\, I^{*}(l=0)\right)$$

$$= \frac{1}{2} \mathrm{Re}\left(V^{+}(1+\Gamma_L) \cdot \frac{|V^{+}|^{*}}{Z_0}(1-\Gamma_L)^{*}\right)$$

$$= \frac{1}{2} \mathrm{Re}\left(V^{+}(1+|\Gamma_L| e^{j\phi_L}) \cdot \frac{|V^{+}|^{*}}{Z_0}(1-|\Gamma_L| e^{-j\phi_L})\right)$$

$$= \frac{1}{2} \mathrm{Re}\left(\frac{|V^{+}|^{2}}{Z_0}\left(1-\underbrace{|\Gamma_L| e^{-j\phi_L} + |\Gamma_L| e^{j\phi_L}}_{|\Gamma_L|\, 2j\sin\phi_L} - |\Gamma_L|^{2}\right)\right)$$

$$\boxed{P(l=0) = \frac{1}{2Z_0} |V^{+}|^{2}\left(1-|\Gamma_L|^{2}\right)} \rightarrow \text{Power at } l=0$$

$$V_f = V^{+} e^{j\beta l} \qquad I_f = \frac{V^{+} e^{j\beta l}}{Z_0}$$

$$P_f = \frac{1}{2} \mathrm{Re}\left(V_f I_f^{*}\right)$$

$$= \frac{1}{2} \mathrm{Re}\left(V^{+} \cancel{e^{j\beta l}} \cdot \frac{V^{+*}}{Z_0} \cancel{e^{-j\beta l}}\right)$$

$$\boxed{P_f = \frac{|V^{+}|^{2}}{2Z_0}} \rightarrow \text{Power of forward travelling wave.}$$

$$V_r = \Gamma_L V^+ e^{-j\beta l}$$

$$I_r = -\frac{\Gamma_L}{Z_0} V^+ e^{-j\beta l}$$

$$P_r = \frac{1}{2} \mathrm{Re}\left(V_r I_r^*\right)$$

$$= \frac{1}{2} \mathrm{Re}\left(\Gamma_L V^+ \cancel{e^{-j\beta l}} \times -\frac{\Gamma_L^*}{Z_0} V^{*+} \cancel{e^{j\beta l}}\right)$$

$$\boxed{P_r = -\frac{1}{2} \frac{|V^+|^2 |\Gamma_L|^2}{Z_0}} \rightarrow \text{Power of reflected travelling wave.}$$

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Power at arbitary location "l" in transmission line:

$$P = \frac{1}{2}(V I^*)$$

$$= \frac{1}{2}\left(V^+(1+\Gamma_L e^{-j2\beta l})\ \frac{V^{+*}}{Z_0}(1-\Gamma_L^* e^{j2\beta l})\right)$$

$$P = \frac{1}{2}\frac{|V^+|^2}{Z_0}\left(1-|\Gamma_L|^2 + Im(2\Gamma_L e^{-j2\beta l})\right)$$

$$\Rightarrow (1+\Gamma_L e^{-j2\beta l})(1-\Gamma_L^* e^{j2\beta l})$$

$$= \left(1+|\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)\left(1-|\Gamma_L| e^{-j(\phi_L - 2\beta l)}\right)$$

$$= 1+|\Gamma_L| e^{j(\phi_L - 2\beta l)} - |\Gamma_L| e^{-j(\phi_L - 2\beta l)} - |\Gamma_L|^2$$

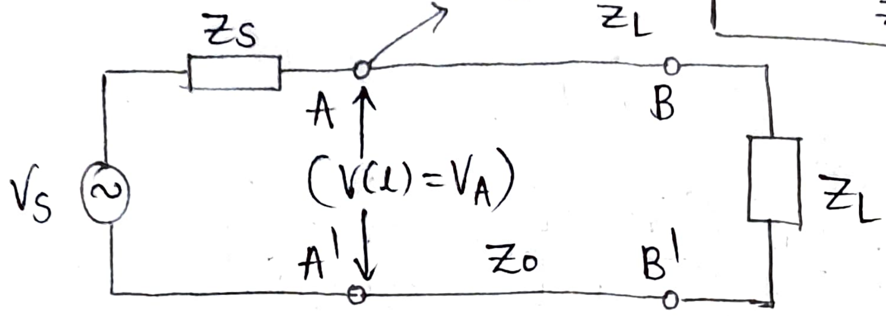
$$= 1-|\Gamma_L|^2 + \underbrace{2j\sin(\phi_L - 2\beta l)}_{\text{Imaginary part}}|\Gamma_L|$$

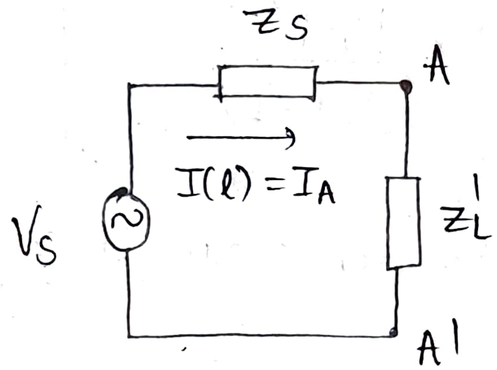
$$P_R = \frac{1}{2Z_0}|V^+|^2(1-|\Gamma_L|^2)$$ → Real power is independent of length

$$P_i = \frac{1}{2Z_0}|V^+|^2(2\Gamma_L e^{-j2\beta l})$$ → Imaginary power is dependent on length.

Evaluation of V^+:

$$\bar{Z}_A = \frac{1}{\bar{Z}_L} \qquad \boxed{\bar{Z}_A = \frac{Z_0^2}{Z_L}}$$





Z_L' = Imp. at the location A-A'

$$I_A = \frac{V_s}{Z_s + Z_L'} \qquad V_A = \left(\frac{V_s}{Z_s + Z_L'}\right) \times Z_L'$$

$$V(l) = V^+ e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right) = V_A$$

$$V(l) = V_A = \frac{V_s \cdot Z_L'}{Z_s + Z_L'} = V^+ e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right)$$

$$\boxed{V^+ = \frac{V_A e^{-j\beta l}}{\left(1 + \Gamma_L e^{-j(2\beta l)}\right)}}$$

→ Power delivered to load Z_L':

$$P = \frac{Re}{2}(V_A I_A^*)$$

$$= \frac{1}{2} Re\left(\frac{V_S \cdot Z_L'}{Z_S + Z_L'} \cdot \frac{V_S^*}{(Z_S+Z_L')^*}\right)$$

$$\boxed{P = \frac{1}{2} Re(Z_L') \times \frac{|V_S|^2}{(Z_S+Z_L')^2}}$$

→ Maximum power transfer occurs if $Z_S = Z_L'^*$

→ Max. power transfer occurs for a specific length (Z_L' is a function of length)

→ For a given length of transmission line, max. power occurs only at specific frequency ($\because l \Rightarrow \lambda \Rightarrow f$)

→ If $Z_L = Z_0$

$$P = \frac{1}{2} \cdot Z_0 \cdot \left|\frac{V_S}{Z_S + Z_0}\right|^2$$

if $Z_L = Z_0$
$Z_S = Z_0$

$$P = \frac{1}{2} \cdot Z_0 \cdot \left|\frac{V_S}{2 Z_0}\right|^2$$

$$\boxed{P = \frac{|V_S|^2}{8 Z_0}}$$

$$Z(l) = \frac{V(l)}{I(l)} = Z_0\left(\frac{1+\Gamma_L e^{-j2\beta l}}{1-\Gamma_L e^{j2\beta l}}\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z(l) = Z_0\left(\frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_L\sin\beta l}\right)$$

<u>short-circuited:</u>

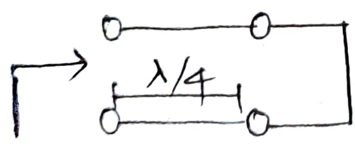
if $Z_L = 0 \Rightarrow \underline{Z(l) = jZ_0\tan\beta l}$

at $l = \frac{\lambda}{4}$, $Z(l) = jZ_0\tan\left(\frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{4}\,2}\right)$

$Z(l) = jZ_0\tan(\pi/2)$

$\rightarrow \underline{\underline{Z(l) = \infty}}$

In parallel RLC ckt, at resonance frequency ($Z = \infty$)



$Z = \infty$

} Acts as parallel resonance RLC ckt.

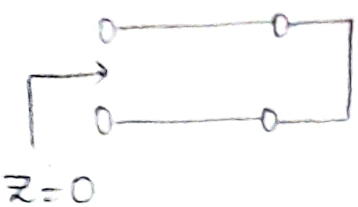
$\rightarrow$ at $l = \frac{\lambda}{4}$, it acts as short-circuited tr. line

$\underline{Z(\lambda/4) = \infty}$

→ at $l = \frac{\lambda}{2}$, $Z(l) = jZ_0 \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right)$

$$= jZ_0 \tan(\pi)$$

$$\underline{Z(l) = 0}$$



Acts as Series Resonance RLC ckt

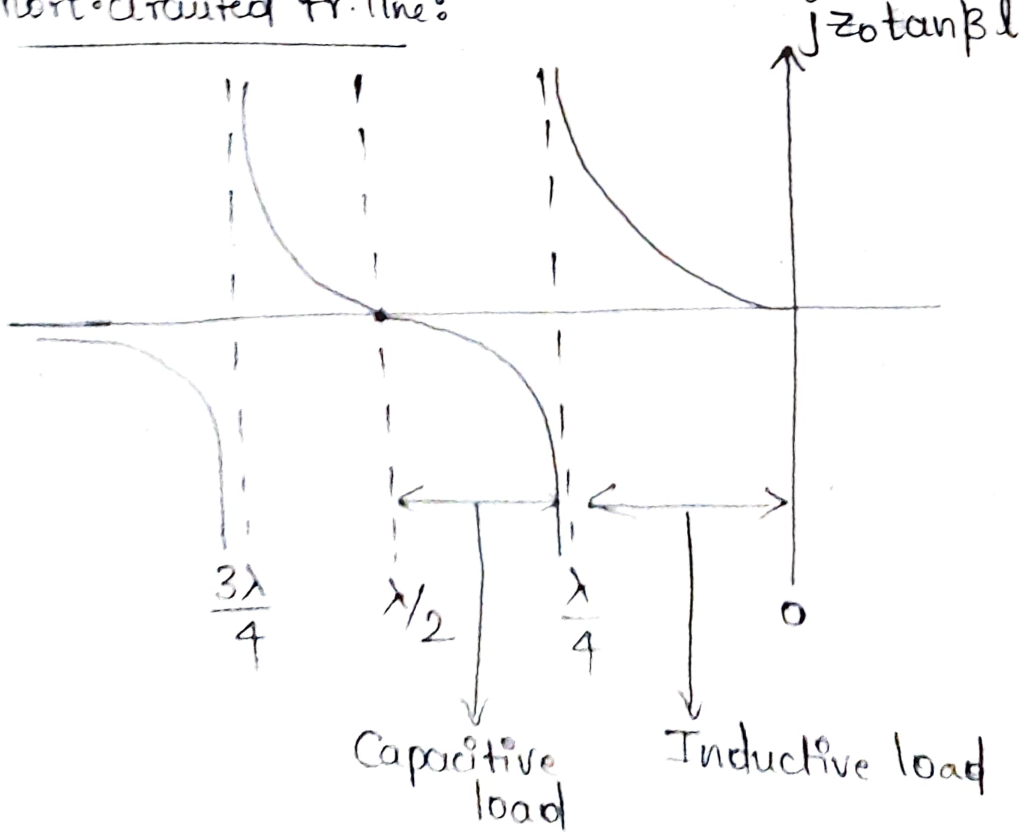
→ at $l = \frac{\lambda}{3}$, $Z(\lambda/3) = jZ_0 \tan\left(\frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{3}\right)$

$$\underline{Z(\lambda/3) = -\sqrt{3}\,jZ_0}$$

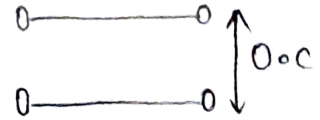
→ at $l = \frac{\lambda}{8}$, $Z(\lambda/8) = jZ_0 \tan\left(\frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{8}\,4}\right)$

$$\underline{Z(\lambda/8) = jZ_0}$$

<u>Short circuited tr. line:</u>



Open-circuit:



$$Z(l) = Z_0\left(\frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_L\sin\beta l}\right)$$

if $Z_L = \infty$, $Z(l) = -jZ_0\cot\beta l$

→ at $l = \frac{\lambda}{4}$, $Z(\lambda/4) = -jZ_0\cot\left(\frac{2\pi}{\lambda}\times\frac{\lambda}{4}\right)$

$\underline{Z(\lambda/4) = 0}$ ⇒ Series Resonance RLC ckt

→ at $l = \frac{\lambda}{2}$, $Z(\lambda/2) = -jZ_0\cot\left(\frac{2\pi}{\lambda}\times\frac{\lambda}{2}\right)$

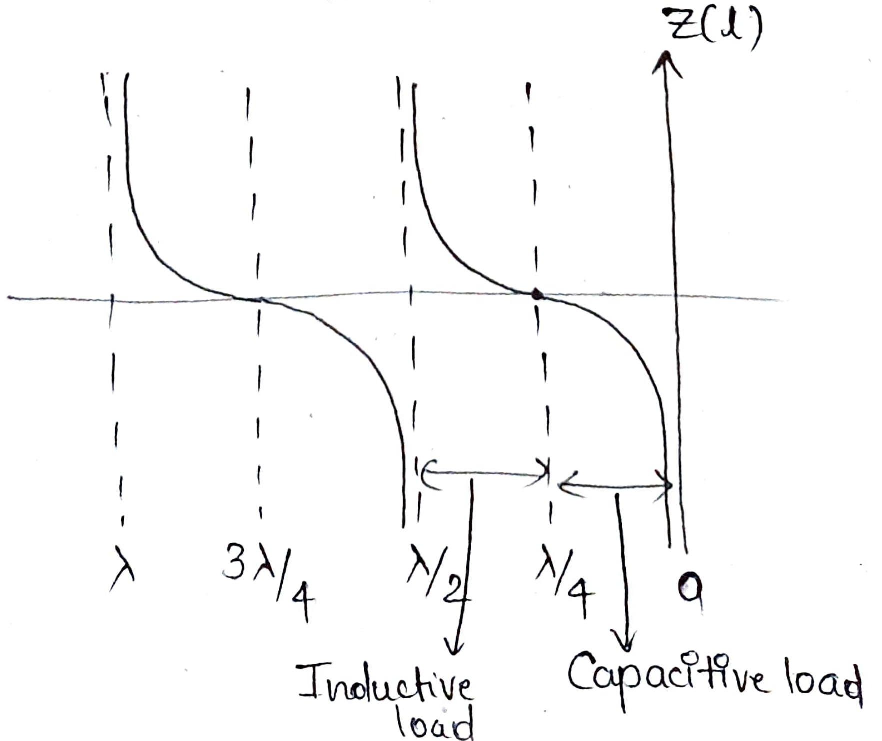
$= -jZ_0\cot(\pi)$

$\underline{Z(\lambda/2) = \infty}$ ⇒ Parallel Resonance RLC ckt.

→ at $l = \lambda/8$, $Z(\lambda/8) = -jZ_0\cot\left(\frac{2\pi}{\lambda}\times\frac{\lambda}{8}\right)$

$= -jZ_0\cot(\pi/4)$

$\underline{\underline{Z(\lambda/8) = -jZ_0}}$



$$\therefore Z(l)\Big|_{SC} = jZ_0 \tan\beta l$$

$$\therefore Z(l)\Big|_{OC} = -jZ_0 \cot\beta l$$

$$\boxed{Z(l)\Big|_{SC} \times Z(l)\Big|_{OC} = Z_0^2}$$

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Tr. line	E.M. Waves
1) $V(l,t)$, $I(l,t)$	1) $E(z,t)$, $H(z,t)$
2) Constants are R, L, G, C	2) Constants are $\mu \Rightarrow$ magnetic storage ability (permeability) $\mu \Leftrightarrow L$ (H/m) $\varepsilon \Rightarrow$ permittivity of medium $\varepsilon \Leftrightarrow$ Capacitance (F/m)
3) $\gamma = \alpha + j\beta$ $\beta = \omega\sqrt{LC}$ $Z_0 = \sqrt{\frac{L}{C}}$	3) $\gamma = \alpha + j\beta$ $\beta = \omega\sqrt{\mu\varepsilon}$ $\eta = \sqrt{\frac{\mu}{\varepsilon}}$
4) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$	4) $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{E_r}{E_i}$

How E, H propagates in a medium??

- Homogeneous
- Infinitely long

Electromagnetic Waves

- Topics going to be discussed
  - → Basics laws
    - → Coulomb's law
    - → Gauss law
    - → Ampere's law
    - → Faraday's law.
  - → Basics of Vector Calculas
    - → Dot, Cross product
    - → Gradient
    - → Flux
    - → Divergence
    - → Curl
    - → Divergence, Stoke's Theorem

Scalar: Quantity which has only magnitude

eg: distance, voltage, mass, time

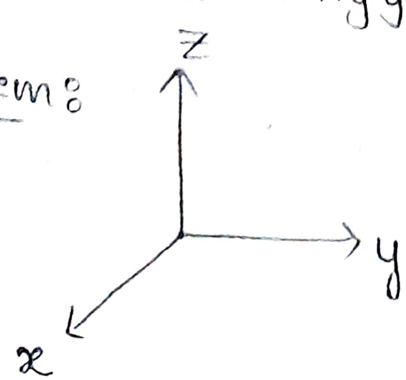
Vector: Quantity which has both magnitude and direction

eg: displacement, velocity, Force, Torque, E.f, M.f

$$\bar{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

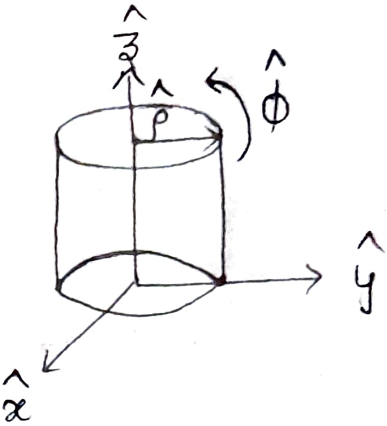
Co-ordinate System:

$(\hat{x}, \hat{y}, \hat{z})$



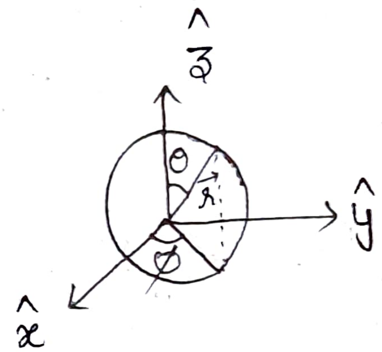
Cylindrical co-ordinate system :

$(\hat{\rho}, \hat{\phi}, \hat{z})$

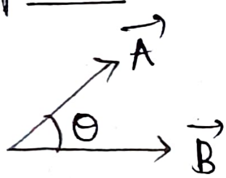


Spherical co-ordinate system :

$(\hat{r}, \hat{\theta}, \hat{\phi})$



Dot product : $\bar{A} \cdot \bar{B} = |A||B| \cos\theta$



$$\bar{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\bar{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

$\Rightarrow$ projection of $\bar{A}$ on $\bar{B}$ $\rightarrow$ physical meaning.

Cross product : $\bar{A} \times \bar{B} = |A||B| \sin\theta \, \hat{n}$

$\rightarrow$ Direction of $\bar{A} \times \bar{B}$ is $\perp$ar to both $\bar{A}, \bar{B}$.

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\rightarrow$ It tells us how well two vectors are $\perp$ar to each other.

Operator (∇) : $\bar{\nabla} = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}$

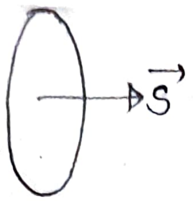
gradient operator : $\nabla\psi = \frac{\partial\psi}{\partial x}\hat{x} + \frac{\partial\psi}{\partial y}\hat{y} + \frac{\partial\psi}{\partial z}\hat{z}$

$\psi \rightarrow$ scalar

$\nabla\psi \rightarrow$ vector

→ $\nabla\psi$ is telling about the rate of change of ψ. It is in the direction where rate of change of ψ is maximum.

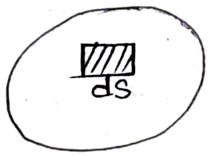
flux : The amount of vector coming out of the surface

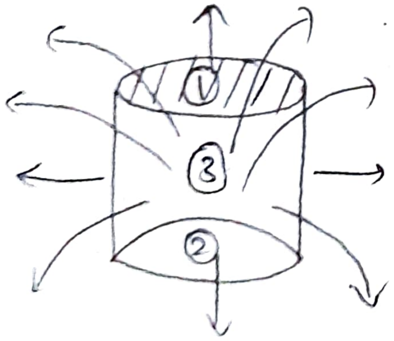


$\bar{S}$ = surface vector whose magnitude is equal to surface area, direction is ⊥ar to surface

$$\text{flux} = \bar{A} \cdot \bar{S}$$

$$\text{flux} = \iint_S \vec{A} \cdot \vec{ds}$$





$$\text{flux} = \oint_S \bar{A} \cdot \bar{ds}$$

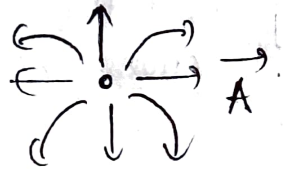
$$= \int_{①} \bar{A} \cdot \bar{ds} + \int_{②} \bar{A} \cdot \bar{ds} + \int_{③} \bar{A} \cdot \bar{ds}$$

$$\text{flux} = \oint_S \bar{A} \cdot \bar{ds}$$

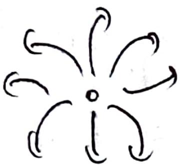
$$= \int_{①} \bar{A} \cdot \bar{ds} + \cdots + \int_{⑥} \bar{A} \cdot \bar{ds}$$

Divergence of a vector field:

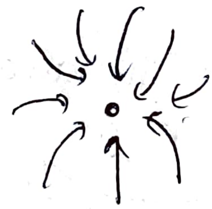
$$\nabla \cdot \vec{A} \Big|_{\text{at a point}} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \cdot \left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \right)$$



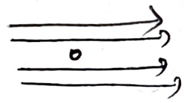
$\nabla \cdot \bar{A} > 0$



$\nabla \cdot \bar{A} < 0$



$\nabla \cdot \bar{A} = 0$



Divergence: outward flux from a closed surface per unit volume of the surface, where the volume is tending towards zero

$$\nabla \cdot \bar{A} = \frac{\text{Flux}}{\text{Volume}} \Bigg|_{\text{Volume} \to \text{zero}}$$

$$\nabla \cdot \bar{A} = \frac{\oint_S \bar{A} \cdot \overline{ds}}{\Delta V} \Bigg|_{\Delta V \to 0}$$

$$\nabla \cdot \bar{A} = \frac{\oint \bar{A} \cdot \overline{ds}}{dV}$$

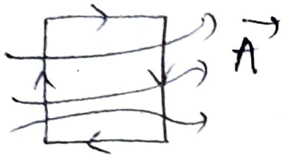
$$\boxed{\oint_S \vec{A} \cdot \vec{ds} = \int_V (\nabla \cdot \bar{A})\, dv}$$

→ Divergence theorem

Curl: $\nabla \times \vec{A}$

→ Tells us about circulation nature of field at a point.

$\nabla \times \vec{A}$ = Circulation $\bar{A}$ around the closed path per unit area of the closed path, where the area of the path tends to zero.



$$\nabla \times \bar{A} = \left.\frac{\oint \vec{A} \cdot \vec{dl}}{\Delta S}\right|_{\Delta S \to 0} = \frac{\oint \vec{A} \cdot \vec{dl}}{ds}$$

$$\boxed{\oint \vec{A} \cdot \vec{dl} = \iint_S (\nabla \times \vec{A})\, ds}$$

→ Stroke's theorem.

Laplacian (∇^2): $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Laplacian of Scalar field:

$$\nabla^2 V = \nabla \cdot (\nabla V)$$

= divergence of gradient of scalar

$$\boxed{\nabla^2 V = \frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V}$$

Laplacian of Vector field:

$$\nabla^2 \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A}$$

$$\boxed{\nabla^2 \vec{A} = \frac{\partial^2}{\partial x^2} A_x \hat{x} + \frac{\partial^2}{\partial y^2} A_y \hat{y} + \frac{\partial^2}{\partial z^2} A_z \hat{z}}$$

Solenoidal & Irrotational:

If $\nabla \cdot \bar{A} = 0$ then $\bar{A}$ = Solenoidal

→ Div(Curl of $\bar{A}$) = 0 $\qquad \nabla \cdot (\nabla \times \bar{A}) = 0$

→ Curl (gradient of scalar) = 0 $\qquad \nabla \times \nabla \phi = 0$

If $\nabla \times \bar{E} = 0$ then $\vec{E}$ = irrotational

If $\nabla \cdot \vec{B} = 0$ then $\vec{B} = \nabla \times \vec{A}$

$(\nabla \cdot (\nabla \times \bar{A} = 0)$

If $\nabla \times \vec{E} = 0$ then $\bar{E} = \nabla V$

$(\nabla \times (\nabla V) = 0)$

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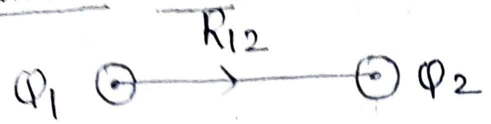
field: Variation of quantity in space.

① Coulomb's law:



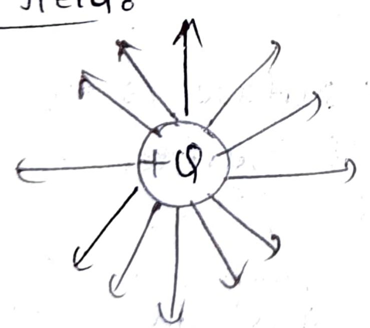
$$\overline{F_{12}} \propto \frac{Q_1 Q_2}{r^2} \hat{R}_{12}$$

$$\overline{F_{12}} = \frac{1}{4\pi\varepsilon_0 r^2} Q_1 Q_2 \hat{R}_{12}$$

ε_0 - permittivity of free space ($\varepsilon_0 = 8.854 \times 10^{-12}$ F/m)

$$\overline{F_{12}} = \frac{1}{4\pi\varepsilon_r\varepsilon_0 r^2} Q_1 Q_2 \hat{R}_{12}$$ ε_r - permittivity of medium.

• Electric field:



$\overline{E}$ = force exerted by a charge Q on a unit charge at that point.

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{R} \quad \text{V/m (or) N/c.}$$

• Electric displacement / Electric flux density:

To define $\overline{E}$, independent of medium property

$$\overline{D} = \varepsilon_r \varepsilon_0 \overline{E}$$

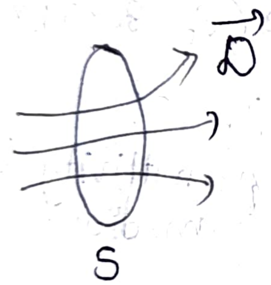
For a point charge, displacement is given by

$$\overline{D} = \frac{Q}{4\pi r^2} \hat{R} \quad C/m^2$$

$$\text{Electric flux} = \int_S \vec{D} \cdot \vec{ds} = \Psi_e$$

$\vec{D}$ = electric flux density

Unit of electric flux is coulomb

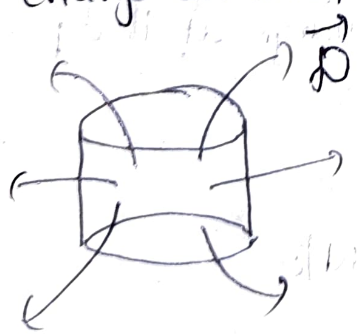


$$\int \overline{D} \cdot \overline{ds} = \Psi_e$$

Ψ_e – Electric flux crossing the surface 'S'.

• Gauss law:

Flux through any closed surface is equal to charge enclosed inside the surface.



$$\Psi_e = \oint_S \overline{D} \cdot \overline{ds}$$

$$\Psi_e = Q_{enclosed}$$

$$\overline{D} = \frac{Q}{4\pi r^2} \hat{R} \quad C/m^2$$

$$\overline{ds} = 4\pi r^2 \hat{R}$$

$$\Psi_e = \oint_S \overline{D} \cdot \overline{ds}$$

$$\psi_e = \oiint_S \bar{D} \cdot \bar{ds} = Q_{enc}$$

$$= \iiint_{Volume} \rho_v \, dV$$

Divergence theorem

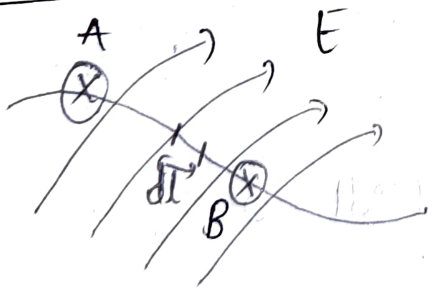
$$\iiint_V (\nabla \cdot \bar{D}) \, dv = \iiint_{Volume} \rho_v \, dV$$

$\nabla \cdot \vec{D}$ = flux of $\bar{D}$ per unit volume

$\boxed{\nabla \cdot \vec{D} = \rho_v}$ → point form of Gauss law

↳ Divergence of $\bar{D}$ at a pt = Volume charge density at that point.

- Electric potential:



Workdone in moving a charge over a distance dl, against a force field

$$dW = -\bar{F} \cdot \bar{dl}$$

Workdone in moving charge 'Q' over a distance A-B against force field.

$$W = -\int_A^B \bar{F} \cdot \bar{dl} = -Q \int_A^B \bar{E} \cdot \bar{dl}$$

- **Electric potential at point:**
Workdone in moving unit charge from infinity point to that point.

$$W = -\int_{\infty}^{B} \bar{F} \cdot d\bar{l}$$

$$= -\int_{\infty}^{B} \bar{E} \cdot d\bar{l}$$

- **Potential energy per unit charge b/w A & B:**

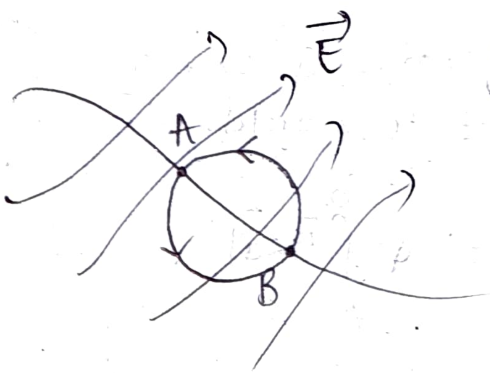
$$V_{AB} = \frac{W}{Q} = -\int_{A}^{B} \bar{E} \cdot d\bar{l}$$

$$= -\int_{\infty}^{B} \bar{E} \cdot d\bar{l} + -\int_{\infty}^{A} \bar{E} \cdot d\bar{l}$$

$$= V_B - V_A$$

$$V_{AB} = -V_{BA}$$

$$\underline{V_{AB} + V_{BA} = 0} \qquad \oint \bar{E} \cdot d\bar{l} = 0$$



$$\oint \bar{E} \cdot d\bar{l} = 0$$

$\downarrow$ Stroke's theorem

$$\int_S (\nabla \times \bar{E}) \cdot d\bar{S} = 0$$

$$\nabla \times \bar{E} = 0$$

$\bar{E}$ is irrotational

$$\boxed{\vec{E} = -\nabla V}$$

$$\nabla \cdot \bar{D} = \rho_v$$

$$\bar{D} = \varepsilon \bar{E}$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla \cdot (-\nabla V) = \frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$ → Poisson's Equation

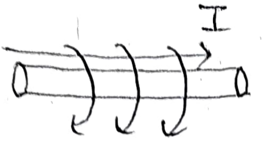
$$\boxed{\nabla^2 V = 0}$$ Since ($\rho_v = 0$)

↳ Laplace Equation

EC254:

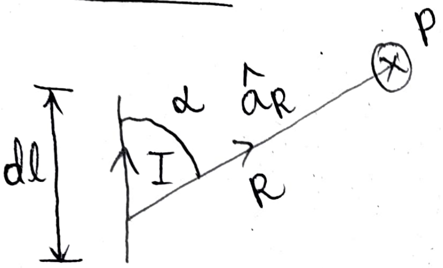
08/02/2022

Magnetostatics:



Magnetic field is generated

- **Biot-Savart's law:**



$$\overline{dH} \propto \frac{I\,dl}{R^2} \sin\alpha \; (\overline{I\,dl} \times \hat{R})$$

$$\overline{dH} \propto \frac{\overline{I\,dl} \times \hat{a}_R}{4\pi R^2}$$

$$\overline{H} = \int_R \frac{I\,\overline{dl} \times \hat{a}_R}{4\pi R^2}$$

→ current carrying wire will always have a magnetic field.

- **Ampere Circuital law:**

$$\oint \overline{H} \cdot \overline{dl} = I_{enclosed}$$

$\vec{J}$ (current density) = A/m^2

$$\vec{J} = \frac{I}{A}$$

$$I_{enclosed} = \vec{J} \cdot \vec{A}$$

$$= \int_S \overline{J} \cdot \overline{ds}$$

$$\therefore \oint \overline{H} \cdot \overline{dl} = \int_S \overline{J} \cdot \overline{ds}$$

By using Stroke's theorem,

$$\int_S (\nabla \times \overline{H}) \cdot \overline{ds} = \int_S \overline{J} \cdot \overline{ds}$$

$$\boxed{\nabla \times \overline{H} = \overline{J}}$$

$\nabla \times \overline{H} \neq 0 \Rightarrow \overline{H}$ is not conservative field

- <u>Magnetic flux density</u>:

$$\vec{B} = \mu \vec{H}$$

μ - magnetic permeability of medium

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$\downarrow$

ability to store magnetic energy

$\mu = \mu_r \mu_0$ $\quad$ μ_r - relative permittivity of medium

$\vec{B}$ is dependent on medium property.
$\vec{H}$ is independent of medium.
$\vec{E}$ is dependent on medium.
$\vec{D} = \varepsilon \vec{E}$ is independent of medium.

$$\vec{B} = Wb/m^2$$

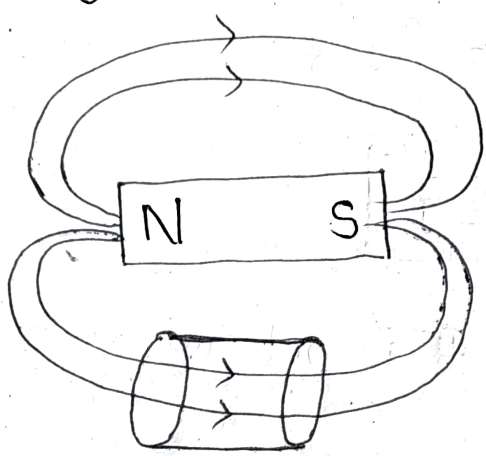
$$\int_S \overline{B} \cdot \overline{ds} = \psi_m \text{ (Magnetic flux)}$$

→ For a closed surface,

magnetic flux coming out through it is zero.

$$\oint \vec{B} \cdot \vec{ds} = 0$$

Gauss law for magnetism



$$\oint \vec{B} \cdot \vec{dS} = 0$$

$$\int_V (\nabla \cdot \vec{B})\, dV = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

$$\mu \nabla \cdot \vec{H} = 0$$

$$\boxed{\nabla \cdot \vec{H} = 0}$$

⇓

Magnetic field always lies in closed loops.

Time Varying fields:

Faraday's law:

$$EMF = -\frac{d\psi_m}{dt}$$

→ It states that the time-varying magnetic field produces electric field.

$$\oint \vec{E}\cdot\vec{dl} = -\frac{d\psi_m}{dt}$$

$$\psi_m = \int_S \vec{B}\cdot\vec{ds}$$

$$\oint \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}$$

$$\int_S (\nabla\times\vec{E})\cdot\vec{ds} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}$$

$$\int_S (\nabla\times\vec{E})\cdot\vec{ds} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

$$\boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu\frac{\partial}{\partial t}\vec{H}}$$

$$\underbrace{\oint \underbrace{\vec{E}}_{V/m}\cdot\underbrace{\vec{dl}}_{m}}_{\text{Voltage}} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot\vec{ds} = -\frac{\partial}{\partial t}\int_S \mu\bar{H}\cdot\bar{ds}$$

$$\frac{H}{m}\times\frac{A}{m}\cdot m^2$$

$$H\cdot\frac{\partial(A)}{\partial t}$$

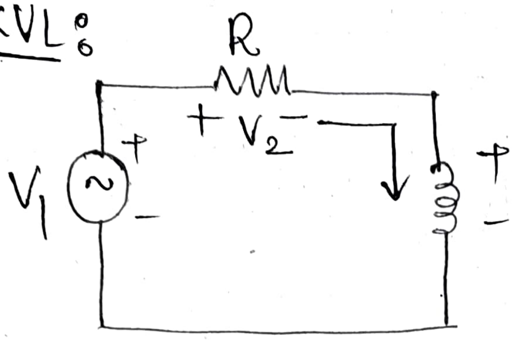
$$= \underbrace{\mu}_{H} \frac{\partial}{\partial t}A$$

$$\boxed{\text{Voltage} = -L\cdot\frac{dI}{dt}}$$

EC254:

04/02/2022

- **KVL:**



$$\oint \vec{E} \cdot \vec{dl} = -\mu \int_S \frac{\partial}{\partial t} \vec{H} \cdot \vec{ds}$$

$$\boxed{V_1 - V_2 = -L \cdot \frac{\partial I}{\partial t}}$$

Incase of DC voltage, $V_1 - V_2 = 0$

So, $\oint \vec{E} \cdot \vec{dl} = 0$

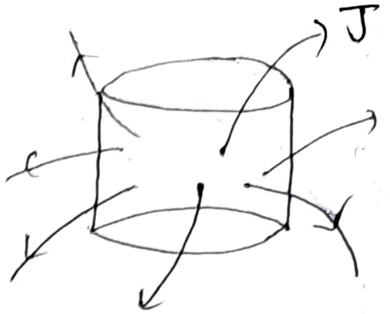
$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \psi_m}{\partial t} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot \vec{ds}$$

$$= -\mu \int_S \frac{\partial}{\partial t} \vec{H} \cdot \vec{ds}$$

$$\int_S (\nabla \times \vec{E}) \cdot ds = -\mu \int_S \frac{\partial}{\partial t} \vec{H} \cdot \vec{ds}$$

$$\boxed{\nabla \times \vec{E} = -\mu \frac{\partial H}{\partial t}}$$

• Continuity equation of current:



flux of $\vec{J}$ coming out of closed surface

$$= \oint_S \vec{J} \cdot \vec{ds}$$

$\bar{J}$ = current density = (A/m²)

$\oint \vec{J} \cdot \vec{ds}$ = current coming out of surface

→ current is motion of charges

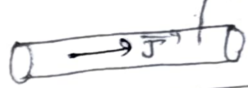
→ charges can neither be created nor destroyed

$$\boxed{\oint \vec{J} \cdot \vec{ds} = -\frac{\partial Q_{enc}}{\partial t}}$$

$$\boxed{\oint \vec{J} \cdot \vec{ds} = 0}$$

• Ohm's law:

$V = IR$ → $\sigma = S/m$ $\quad \sigma = 1/\rho_R$



$$\boxed{\vec{J}_c = \sigma \vec{E}}$$ → conduction current density.

$\vec{J}$ = current density (A/m²)

$\vec{E}$ = V/m

$$\text{Current} = \int_S \vec{J} \cdot \vec{ds} = \int_S \sigma \vec{E} \cdot \vec{ds}$$

$$\frac{S}{\cancel{m}} \cdot \frac{V}{\cancel{m}} \cdot \cancel{m^2}$$

Current = S·V → $\boxed{V = IR}$

- Ampere's law:

$$\oint_l \vec{H} \cdot \vec{dl} = I_{enc} = \int_S \vec{J_c} \cdot \vec{ds}$$

$$\int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J} \cdot \vec{ds},$$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

should be zero

$$\nabla \cdot (\nabla \times \vec{H}) = 0 \implies \boxed{\nabla \cdot \vec{J} = 0}$$

$$\oint_S \vec{J} \cdot \vec{ds} = -\frac{\partial Q_{enc}}{\partial t}$$

$$\int_V (\nabla \cdot \vec{J})\, dv = -\frac{\partial Q_{enc}}{\partial t}$$

$$\int_V (\nabla \cdot \vec{J})\, dv = -\frac{\partial}{\partial t} \int_V \rho_v \cdot dV$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial}{\partial t} \rho_v}$$

$\rightarrow \nabla \cdot \vec{J} = 0$ only when $\frac{\partial}{\partial t} \rho_v = 0$

($\rho_v =$ constant)

(Q is constant wrt time)

Ampere's law varies

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

$\Rightarrow$ When ρ_v / Q is varying with time,

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \quad \text{hence,} \quad \boxed{\nabla \times \vec{H} \neq \vec{J}}$$

(Ampere's law is not valid)

EC254:

09/02/2022

Continuity equation of current:

$$\oint \vec{J} \cdot \vec{ds} = -\frac{\partial Q_{enc}}{\partial t}$$

$$\int_V (\nabla \cdot \vec{J})\, dV = -\frac{\partial}{\partial t} \int_V \rho_v \cdot dV$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$

$$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$

$$\int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J} \cdot \vec{ds}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

should be zero

$$\nabla \cdot (\nabla \times \vec{H}) = 0 \Longrightarrow \boxed{\nabla \cdot \vec{J} = 0}$$

$$\nabla \times \vec{H} = \cancel{\vec{J}} = \vec{J_C} + \vec{J_D}$$

$$\nabla (\nabla \times \vec{H}) = 0 = \nabla \cdot \vec{J_C} + \nabla \cdot \vec{J_D}$$

$$\nabla \cdot \vec{J_D} = -\nabla \cdot \vec{J_C}$$

$$\nabla \cdot \vec{J_D} = -\left(\frac{-\partial \rho_v}{\partial t}\right)$$

$$\boxed{\nabla \cdot \vec{J_D} = \frac{\partial \rho_v}{\partial t}}$$

$$\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla \cdot \vec{J_D} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

$$\vec{J_D} = \frac{\partial \vec{D}}{\partial t} = \partial \varepsilon \vec{E}$$

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \left(\frac{\partial}{\partial t}\vec{D}\right)$$

$$\boxed{\nabla \times \vec{H} = \frac{\partial}{\partial t}\vec{D}}$$

- <u>**Modified Ampere's law:**</u>

$$\nabla \times \vec{H} = \vec{J_c} + \vec{J_D}$$

$$\nabla \times \vec{H} = \vec{J_D} \qquad \vec{J_c} = 0$$

$$= \frac{\partial}{\partial t}\vec{D}$$

$$\nabla \times \vec{H} = \varepsilon \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \varepsilon \cdot \frac{\partial \vec{E}}{\partial t} \cdot \vec{ds}$$

$$\oint_l \vec{H} \cdot \vec{dl} = \int_S \varepsilon \cdot \frac{\partial \vec{E}}{\partial t} \cdot \vec{ds}$$

$\vec{H} = A/m \quad \vec{E} = V/m \quad \varepsilon = \text{farad}/m$

$$\boxed{I = C \cdot \frac{\partial V}{\partial t}} \qquad I = j\omega C\, V$$

$$\boxed{V = \frac{1}{j\omega C} I}$$

$\vec{E}$ & $\vec{D}$:

$$\vec{D} = \varepsilon \vec{E}$$

$\downarrow$ permittivity (Farad/m)

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r}\hat{a}_r}$$

→ Homogeneous dielectric material 'ε' is not of space.

if the medium is polarised then $\varepsilon = \varepsilon_r \varepsilon_0$ $\quad \underline{\varepsilon_r > 1}$

So, $\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_r\varepsilon_0 r}\hat{a}_r}$ decreases by 'ε_r' times.

→ Incase of isotropic $\vec{D} = \varepsilon \vec{E}$

Both $\vec{D}$ and $\vec{E}$ are in same direction

→ Incase of Anisotropic,

$$\vec{D} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

ε - permittivity $= \varepsilon_r \varepsilon_0$ $\quad \varepsilon_0 = 8.854 \times 10^{-12}$ F/m

↳ Capacitance

μ - permeability $= \mu_r \mu_0$ $\quad \mu_0 = 4\pi \times 10^{-7}$ H/m

↳ Inductance

EC254:

10/02/2022

Gauss law:

$$\boxed{\nabla \cdot \vec{D} = \rho_v}$$

$$\boxed{\oint \vec{D} \cdot \vec{ds} = Q_{enc} = \iiint_V \rho_v \cdot dV}$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

$$\boxed{\oint \vec{B} \cdot \vec{dS} = 0}$$

Faraday's law:

EMF induced in a closed loop is equal to rate of change of magnetic flux.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \psi_m$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int_S \vec{B} \cdot \vec{dS}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Ampere's law: The magneto motive force around closed loop is equal to I enclosed.

$$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$

$$= \int_S (\vec{J_C} + \vec{J_D}) \cdot \vec{ds}$$

$$\nabla \times \vec{H} = \vec{J_C} + \vec{J_D}$$

$$\boxed{\nabla \times \vec{H} = \vec{J_C} + \frac{\partial \vec{D}}{\partial t}}$$

Continuity Equation:

$$\oint \vec{J} \cdot \vec{ds} = -\frac{\partial}{\partial t} Q_{enc} = -\frac{\partial}{\partial t} \iiint_V \rho_v \cdot dV$$

$$\int_V (\nabla \cdot \vec{J}) \cdot dV = -\int_V \frac{\partial \rho_v}{\partial t} dV$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$

Time varying magnetic field.

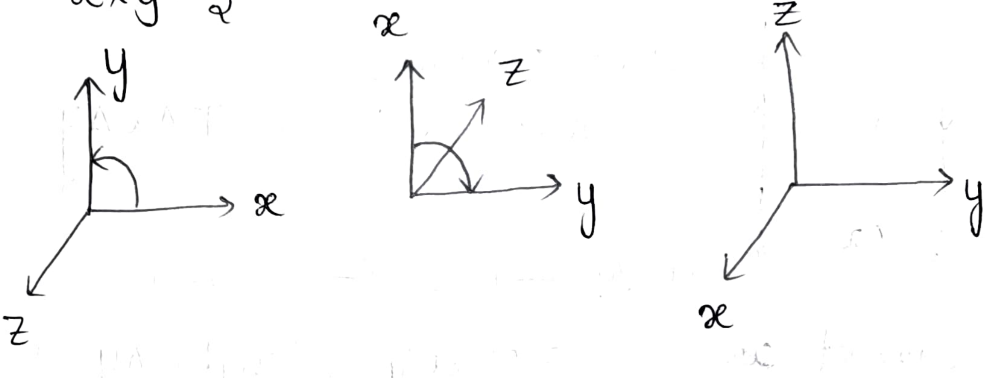
→ Source of Electric field is Electric charge (or) ↑

→ Source of Magnetic field is current carrying conductor (motion of charges)

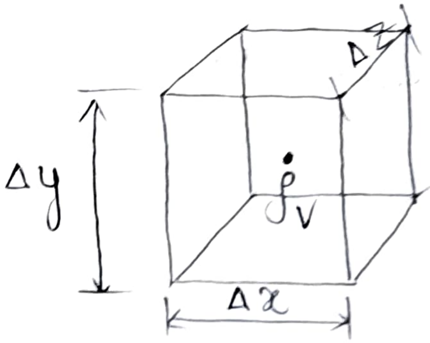
(or) Time Varying electric field.

Right-Hand Co-ordinate System:

$$\hat{x} \times \hat{y} = \hat{z}$$



- **Surface charge density: (ρ_s)**



$Q_{enclosed} = \rho_V \cdot \Delta x \, \Delta y \, \Delta z$

ρ_V – volume charge density

if $\Delta x \to 0$ and $\rho_V \to \infty$

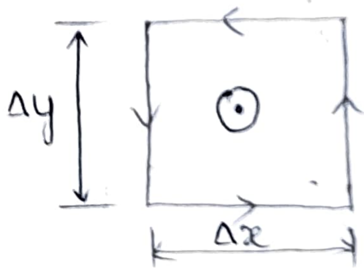
Then $Q_{enclosed} = \underbrace{\rho_S}_{\hookrightarrow \rho_V \Delta x} \Delta y \, \Delta z$

Surface charge density (ρ_s) $= \lim\limits_{\substack{\Delta x \to 0 \\ \rho_V \to \infty}} (\rho_V \, \Delta x)$

$\downarrow$

Units: C/m^2

- **Surface Current Density: (J_s)**



Current coming out of surface $= J \, \Delta x \, \Delta y$

if $\Delta x \to 0$ & $J \to \infty$ then

current coming out of strip of length Δy

$= \underbrace{J_S}_{\hookrightarrow \lim\limits_{\substack{\Delta x \to 0 \\ J \to \infty}} J \Delta x} \Delta y$

$$J_S = \lim_{\substack{\Delta x \to 0 \\ J \to \infty}} J \, \Delta x$$

$\downarrow$

Unit: A/m

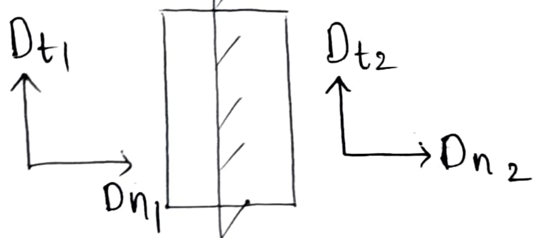
Medium-1

$\mu_1, \varepsilon_1, \sigma_1$

Medium-2

$\mu_2, \varepsilon_2, \sigma_2$

From Gauss law,



D_{t1} - Electric displacement vector along tangential

D_{n1} - along normal

D_{t2} - Electric displacement vector along tangential

D_{n2} - along normal.

- If the length of box $\rightarrow 0$
  then we will have only surface charge density.

$D_{n2} - D_{n1} = \rho_s \rightarrow$ If surface charge is present.

$D_{n2} = D_{n1} \rightarrow$ No surface charge is present.

- Boundary Conditions:

1. Normal component of $\bar{D}$ is continuous if there is No surface charge.
   In presence of surface charge,
   $$D_{n2} - D_{n1} = \rho_s$$

2. Normal component of $\bar{B}$ is continuous.