Q1. Sol.

$$\frac{C_R(s)}{R(s)} = \frac{\dfrac{G_c G_1 G_2 G_3}{1+G_1 G_2 H_1}}{1+\dfrac{G_c G_1 G_2 G_3 H_2}{1+G_1 G_2 H_1}} = \frac{G_c G_1 G_2 G_3}{1+G_1 G_2 H_1 + G_c G_1 G_2 G_3 H_2}$$

$$\frac{C_D(s)}{D(s)} = \frac{G_2 G_3}{1+G_2 G_3 G_1\left(G_c H_2 + \frac{H_1}{G_3}\right)} = \frac{G_2 G_3}{1+G_1 G_2 G_3 G_c H_2 + G_1 G_2 H_1}$$

Q2. Sol.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z & -1 & -p \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ z-p \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q3. Sol.

$$G(s) = \begin{bmatrix} \dfrac{1}{s^3+6s^2+4s+2} & \dfrac{s+6}{s^3+6s^2+4s+2} \\ \dfrac{s}{s^3+6s^2+4s+2} & \dfrac{s^2+6s}{s^3+6s^2+4s+2} \end{bmatrix}$$

Q4. Sol.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q5. Sol.

Rise time = 2.42 sec

Peak time = 3.63 sec

Maximum overshoot = 0.163

Settling time = 8 sec (2 % criterion)

Q6. Sol.

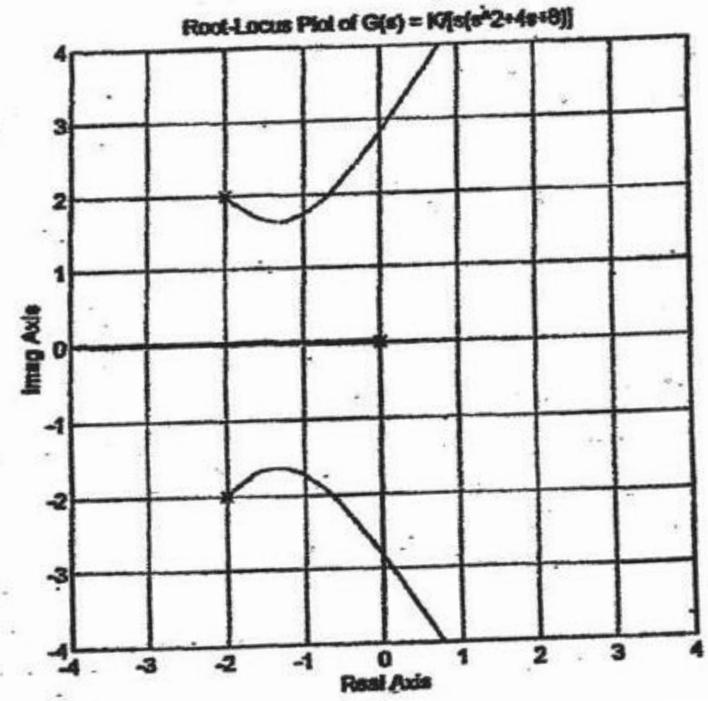
For stability, we require

$$4 + 20KK_h - 4K > 0, \qquad 20K > 0$$

or

$$5KK_h > K - 1, \qquad K > 0$$

Q7. Sol.



Q8. Sol.

Note that the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{4s+2}{s^3+3s^2+4s+2}$$

The closed-loop poles are located at $s = -1 \pm j1$ and $s = -1$.

Q9. Sol.

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{10}{s+11}$$

The steady-state outputs of the system when it is subjected to the given inputs are

(a)

$$c_{ss}(t) = 0.905 \sin(t + 24.8^\circ)$$

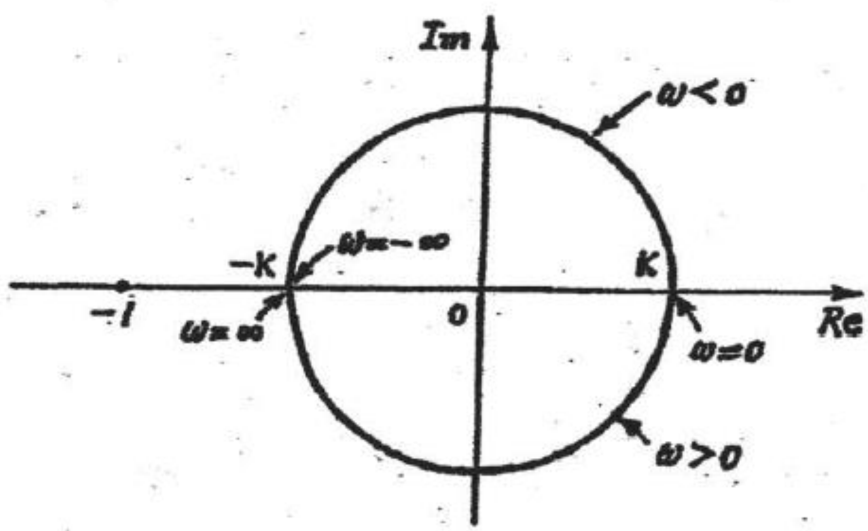
(b)

$$c_{ss}(t) = 1.79 \cos(2t - 55.3^\circ)$$

(c)

$$c_{ss}(t) = 0.905 \sin(t + 24.8^\circ) - 1.79 \cos(2t - 55.3^\circ)$$

Q10. Sol.



Q11. Sol.

a) $G(s)H(s) = \left[\frac{K_p}{s(s+p)}\right] K_D s = \frac{K_p K_D}{s+p}$

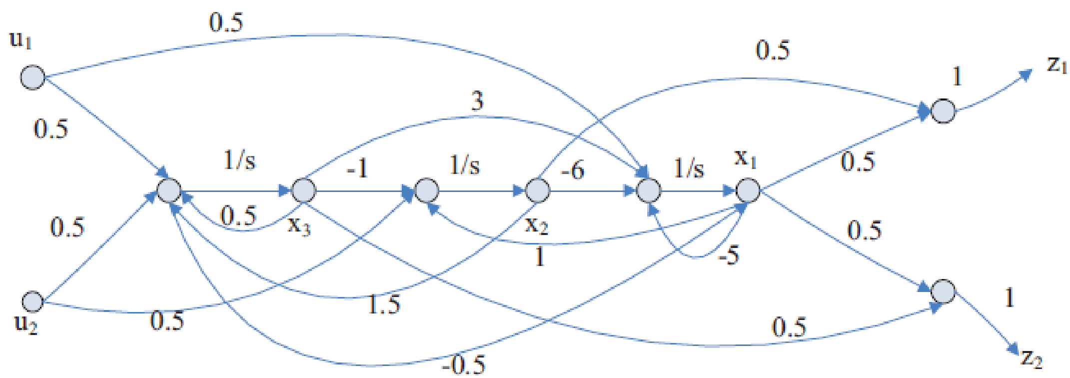
b) $G(s) = \frac{K_p}{s(s+p)}$

c) $\frac{E(s)}{R(s)} = \frac{1}{1-G(s)H(s)} = \frac{s+p}{s+p-K_p K_D}$

d) Feedback ratio $= \frac{\text{G(s)H(s)}}{1-G(s)H(s)} = \frac{K_p K_D}{s+p-K_p K_D}$

e) $\frac{Y(s)}{X(s)} = \frac{G(s)}{1-G(s)H(s)} = \frac{K_p}{s(s+p-K_D K_p)}$

Q12. Sol.



Act

Q13. Sol.

(a)

$$\frac{Y_7}{Y_1} = \frac{G_1G_2G_3G_4G_5 + G_1G_6(1+G_2H_2+G_3H_3)}{\Delta}$$

$$\frac{Y_2}{Y_1} = \frac{1+G_2H_2+G_3H_3+G_4G_5H_4+H_6+G_2G_3G_4G_5H_5+G_2H_2G_4G_5H_4+G_2H_2H_6+G_3H_3H_6}{\Delta}$$

$$\Delta = 1+G_1H_1+G_2H_2+G_3H_3+G_4G_5H_4+H_6+G_2G_3G_4G_5H_5-G_2G_6H_1H_5-G_3G_6H_1H_2H_3H_4+G_1G_2H_1H_2$$
$$+G_1G_4G_5H_1H_4+G_1H_1H_6+G_2G_4G_5H_2H_4+G_2H_2H_6+G_3H_3H_6-G_1G_5G_6H_1H_3H_5+G_1G_3H_1H_3H_6$$

(b)

$$\frac{Y_7}{Y_1} = \frac{G_1G_2G_3G_4G_5 + G_6(1+G_3H_2+G_4H_3)}{\Delta} \qquad \frac{Y_2}{Y_1} = \frac{1+G_3H_2+G_4H_3+G_2G_3G_4G_5H_4}{\Delta}$$

$$\Delta = 1+G_1G_2H_1+G_3H_2+G_4H_3+G_2G_3G_4G_5H_4-G_2G_6H_1H_4+G_1G_2G_4H_1H_3-G_2G_4G_6H_1H_3H_4$$

Q14. Sol.

a) K > 2 ⇨ system is stable

b) 0 < K < 1 and -2 < K < 0 ⇨ -2 < K < 1 ⇨system is stable

Q15. Sol.

Characteristic equation: $\Delta(s) = |s\mathbf{I} - \mathbf{A}| = s^2 + s + 2 = 0$

Eigenvalues: $s = -0.5 + j1.323, \quad -0.5 - j1.323$

State transition matrix:

$$\phi(t) = \begin{bmatrix} \cos 1.323t + 0.378 \sin 1.323t & 0.756 \sin 1.323t \\ -1.512 \sin 1.323t & -1.069 \sin\left(1.323t - 69.3^\circ\right) \end{bmatrix} e^{-0.5t}$$

Q16. Sol.

(a) Not a state transition matrix, since $\phi(0) \neq \mathbf{I}$ (identity matrix).

(b) Not a state transition matrix, since $\phi(0) \neq \mathbf{I}$ (identity matrix).

(c) $\phi(t)$ is a state transition matrix, since $\phi(0) = \mathbf{I}$ and

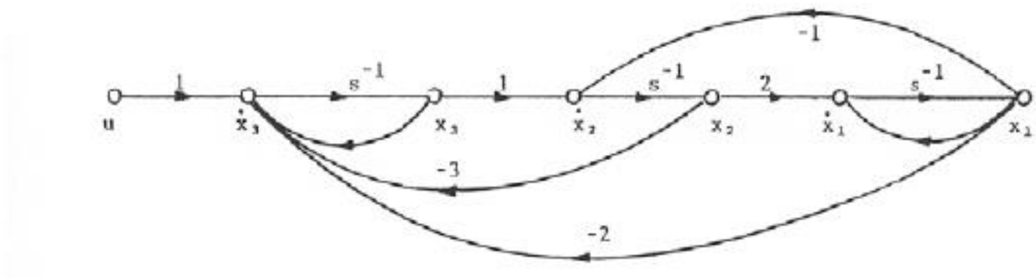
$$[\phi(t)]^{-1} = \begin{bmatrix} 1 & 0 \\ 1 - e^{-t} & e^{-t} \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 1 - e^{t} & e^{t} \end{bmatrix} = \phi(-t)$$

(d) $\phi(t)$ is a state transition matrix, since $\phi(0) = \mathbf{I}$, and

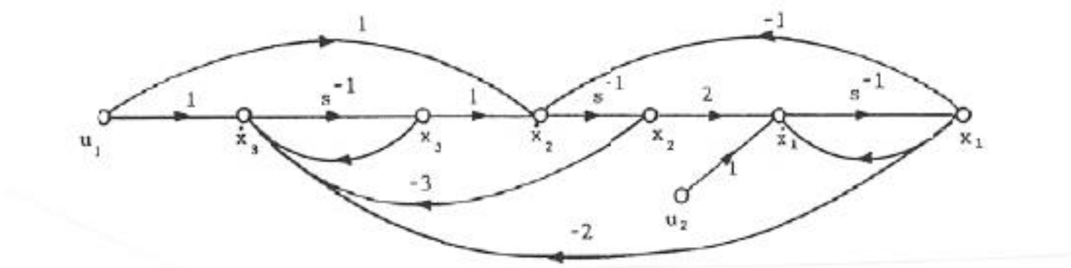
$$[\phi(t)]^{-1} = \begin{bmatrix} e^{2t} & -te^{2t} & t^2 e^{2t}/2 \\ 0 & e^{2t} & -te^{2t} \\ 0 & 0 & e^{2t} \end{bmatrix} = \phi(-t)$$

Q17. Sol.

(a) **State diagram:**



(b) **State diagram:**



Q18. Sol.

(a)

$$S = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

S is singular. The system is uncontrollable.

(b)

$$S = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & -3 & 9 \end{bmatrix}$$

S is nonsingular. The system is controllable.