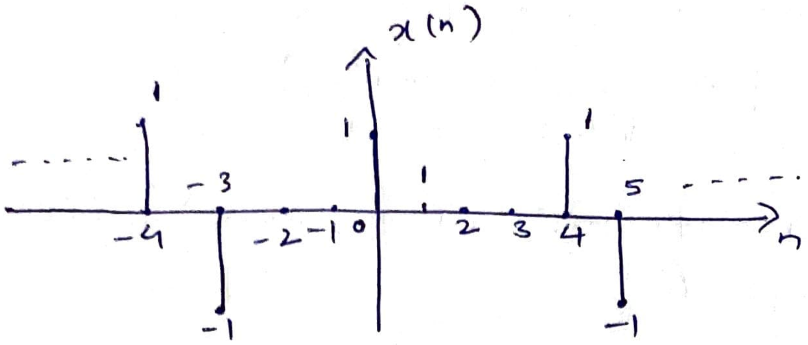
Assignment - I Solutions

① (a)



Therefore, it is Periodic with a fundamental Period of 4.

① (b)

Period of the first term $N_1 = 1$

Period of the second term $N_2 = \left(\frac{2\pi}{4\pi/7}\right)m = 7$ (where $m=2$)

Period of the third term $N_3 = \left(\frac{2\pi}{2\pi/5}\right)m = 5$ (where $m=1$)

$\frac{N_1}{N_2}, \frac{N_1}{N_3} \rightarrow$ Rational number

overall period of the signal $\Big\}$ $N = N_1 \times \text{Lcm}(N_2, N_3)$
$= 35$

① (c)

Period of the first term $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$

Period of the Second term $T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$

$\frac{T_1}{T_2} \rightarrow$ Rational number

$\therefore$ $x(t)$ is Periodic with fundamental Period $T = \text{Lcm}(T_1, T_2) = \pi$

(2)

$$x(n) = 1 - \sum_{k=3}^{\infty} \delta(n-1-k)$$

$$= 1 - [\delta(n-4) + \delta(n-5) + \delta(n-6) + \cdots]$$

$$= 1 - [u(n-3)]$$

$$= u(-n+3)$$

This implies that $M=-1$, $n_0=-3$

(3)

(a) $E = \int_0^{\infty} e^{-4t} dt = \frac{1}{4}$; $P=0$, because $E<\infty$

(b) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$; $|x_2(t)| = 1$, $E = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \infty$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_2(t)|^2 dt = 1$$

(c) $E = \int_{-\infty}^{\infty} \cos^2(t)\, dt = \infty$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} \cos^2(t)\, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1+\cos(2t)}{2} dt = \frac{1}{2}$$

(d) $E = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{4}{3}$, $P = \infty$

(e) $E = \infty$, $P = 0$

(f) $E = \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{\pi}{4}n\right) = \infty$, $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4}n\right)$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1}{2} + \frac{\cos(\frac{\pi}{2}n)}{2}\right)$$

$$= \frac{1}{2}$$

(1) (a) The system is not memoryless because $y[n]$ depends on past values of $x[n]$

(b) The o/p of the system will be $y[n] = \delta(n)\,\delta[n-2] = 0$.

The system o/p is always zero for inputs of the form $\delta(n-k)$, $k \in \mathbb{Z}$. Therefore, the system is not Invertible.

(2) (a) The system is not causal because the o/p $y(t)$ at some time may depend on future values of $x(t)$.

For instance $y(-\pi) = x(0)$.

(b) Consider two arbitarary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \longrightarrow y_2(t) = x_2(\sin(t))$$

Additivity:

$$x_1(t) + x_2(t) \longrightarrow x_1(\sin(t)) + x_2(\sin(t))$$

$$= y_1(t) + y_2(t)$$

Homoginity

$$\alpha\, x(t) \longrightarrow \alpha\, x(\sin(t)) = \alpha\, y(t)$$

Therefore, the system is Linear.

(3) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$

(a) $$x_1[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Additive Property

$$x_1[n] + x_2[n] \longrightarrow \sum_{k=n-n_0}^{n+n_0} \left[x_1[k] + x_2[k]\right]$$

$$= \sum_{k=n-n_0}^{n+n_0} x_1[k] + \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= y_1[n] + y_2[n]$$

Homoginity

$$\alpha\, x[n] \longrightarrow \sum_{k=n-n_0}^{n+n_0} \left(\alpha\, x[k]\right) = \alpha \sum_{k=n-n_0}^{n+n_0} x[k] = \alpha\, y[n]$$

Therefore, the system is linear

(b) $$y[n] = x[n-n_0] + x[n-n_0+1] + \dots + x[n+n_0-1] + x(n+n_0]$$

If the $x[n]$ is delayed by n_1 units then

$$x[n-n_1] \longrightarrow x[n-n_1-n_0] + x[n-n_1-n_0+1] + \dots +$$
$$x[n-n_1+n_0-1] + x[n-n_1+n_0]$$
$$= y[n-n_1]$$

$\therefore$ Therefore, the system is ~~Invariant~~ time-Invariant.

3 (c) If $|x(n)| < B$, then

$$y(n) \le (2n_0+1)B$$

Therefore $C \le (2n_0+1)B$.

(4)

(a) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$

Additivity

$$x_1(t) \longrightarrow y_1(t) = t^2 x_1(t-1)$$

$$x_2(t) \longrightarrow y_2(t) = t^2 x_2(t-1)$$

$$x_1(t) + x_2(t) \longrightarrow t^2[x_1(t-1) + x_2(t-1)]$$

$$= t^2 x_1(t-1) + t^2 x_2(t-1)$$

$$= y_1(t) + y_2(t)$$

Homoginity

$$\alpha x(t) \longrightarrow t^2[\alpha x(t-1)] = \alpha t^2 x(t-1)$$

$$= \alpha y(t)$$

∴ The System is linear.

(b) ~~Consider two~~ If the I/p is delayed by t_0 units

$$x(t-t_0) \longrightarrow t^2 x(t-t_0) \neq y(t-t_0)$$

∴ The system is Time-Invariant

4 (b)

$y[n] = x^2[n-2]$

$x_1[n] \longrightarrow y_1[n] = x_1^2[n-2]$

$x_2[n] \longrightarrow y_2[n] = x_2^2[n-2]$

$x_1[n] + x_2[n] \longrightarrow [x_1[n-2] + x_2[n-2]]^2$

$\neq y_1[n] + y_2[n]$

$\therefore$ The system is Non-Linear

$x[n-n_0] \longrightarrow x^2[n-n_0-2]$

$= y[n-n_0]$

$\therefore$ The system is time-Invariant

4 (c)

$x_1[n] \longrightarrow y_1[n] = x_1[n+1] - x_1[n-1]$

$x_2[n] \longrightarrow y_2[n] = x_2[n+1] - x_2[n-1]$

Additive

$x_1[n] + x_2[n] \longrightarrow [x_1[n+1] + x_2[n+1]] - [x_1[n-1] + x_2[n-1]]$

$= [x_1[n+1] - x_1[n-1]] + [x_2[n+1] - x_2[n-1]]$

$= y_1[n] + y_2[n]$

Homoginity

$\alpha x(n) \longrightarrow \alpha x_1(n+1) - \alpha x(n-1)$

$= \alpha y[n]$

$\therefore$ The System is Linear

$$x[n-n_0] \longrightarrow x[n-n_0+1] - x[n-n_0-1]$$
$$= y[n-n_0]$$

$\therefore$ The System is time-Invariant

4 (d)

$$y(t) = odd(x(t))$$
$$= \frac{x(t) + x(-t)}{2}$$

$$x_1(t) \longrightarrow y_1(t) = \frac{1}{2}(x_1(t) + x_1(-t))$$
$$x_2(t) \longrightarrow y_2(t) = \frac{1}{2}(x_2(t) + x_2(-t))$$
$$x_1(t) + x_2(t) \longrightarrow \frac{1}{2}\left[(x_1(t) + x_2(t)) + (x_1(-t) + x_2(-t))\right]$$
$$= y_1(t) + y_2(t)$$

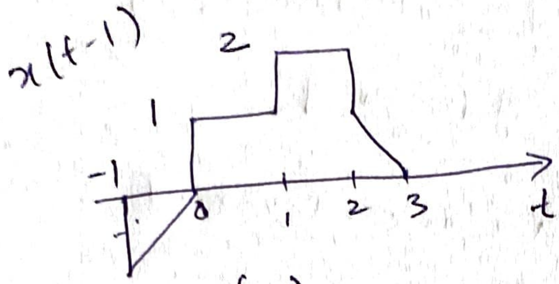
$$\alpha x(t) \longrightarrow \frac{1}{2}\left[\alpha x(t) + \alpha x(-t)\right]$$
$$= \alpha \frac{1}{2}\left[x(t) + x(-t)\right]$$
$$= \alpha y(t)$$

$\therefore$ The System is Linear

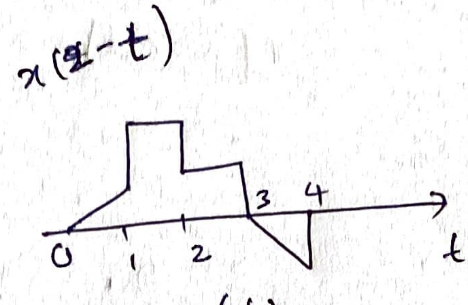
$$x(t-t_0) \longrightarrow \frac{1}{2}\left[x(t-t_0) + x(-t-t_0)\right]$$
$$\neq y(t-t_0)$$

$\therefore$ The system is Time-Variant

5



(a)



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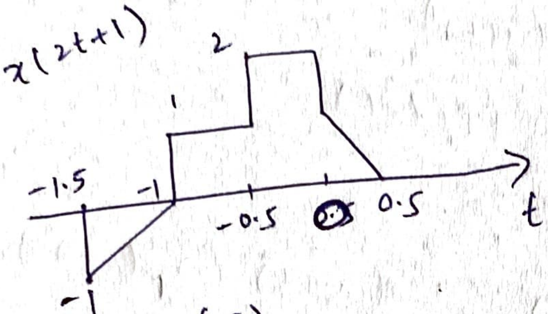

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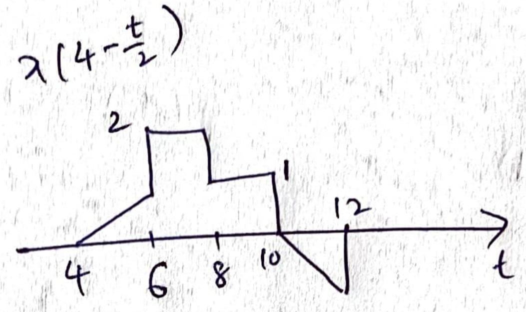

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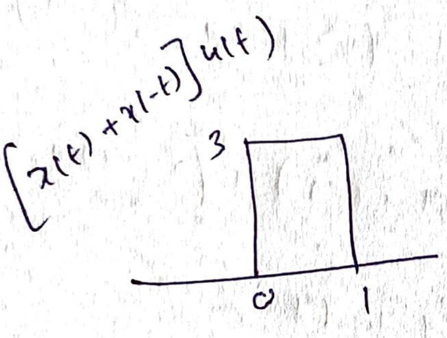

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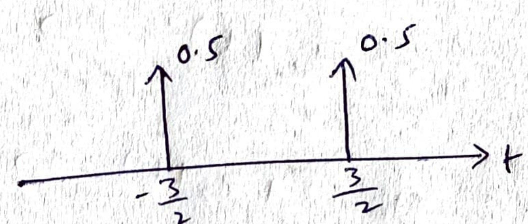

(f)

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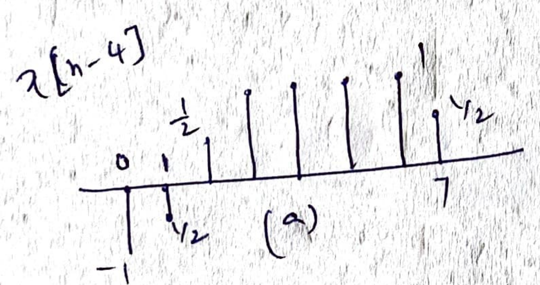
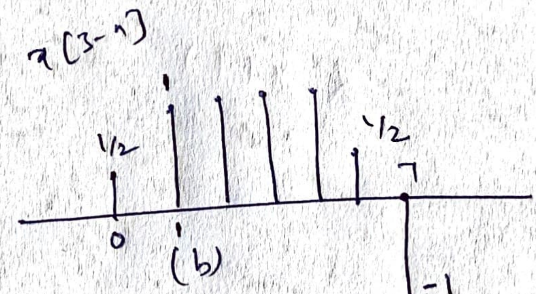


(a)



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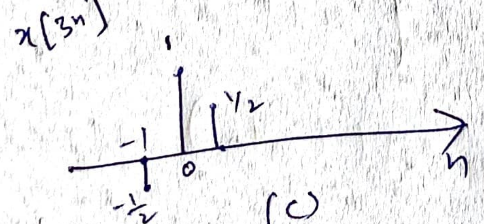

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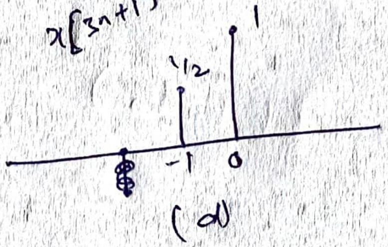
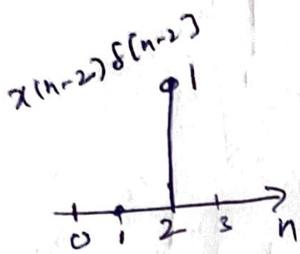


(d)

(e) $x[n]\, x[3-n] = x(n)$



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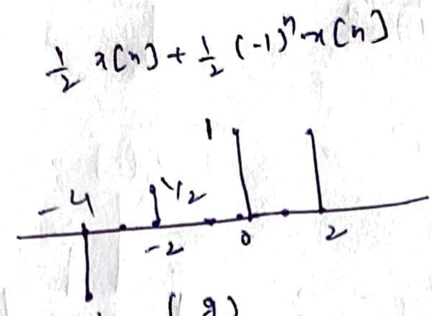
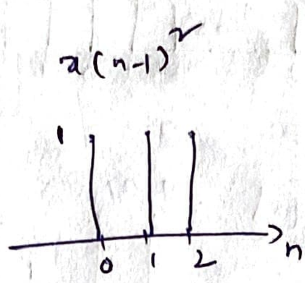


(g)



(h)

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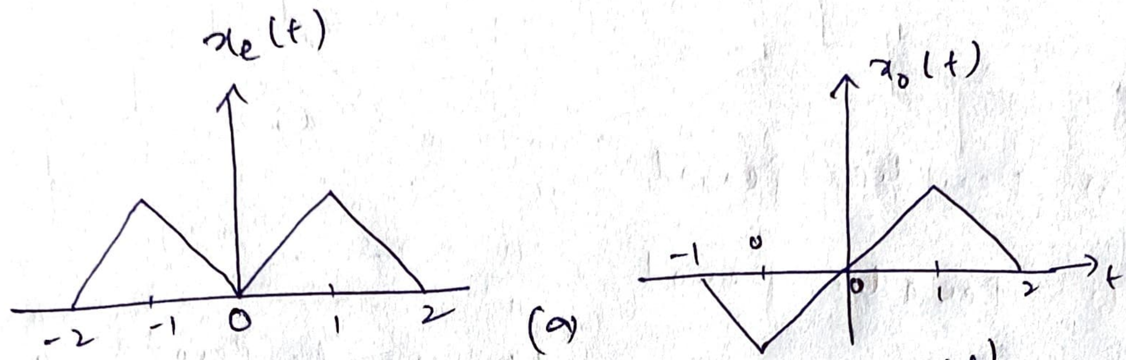
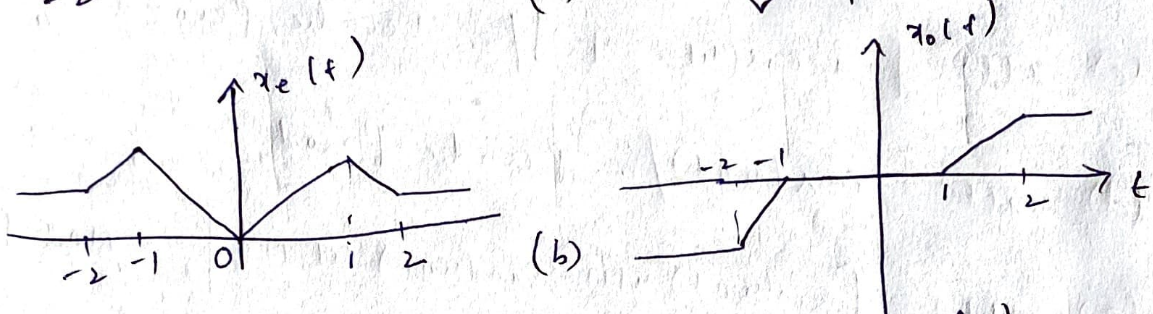
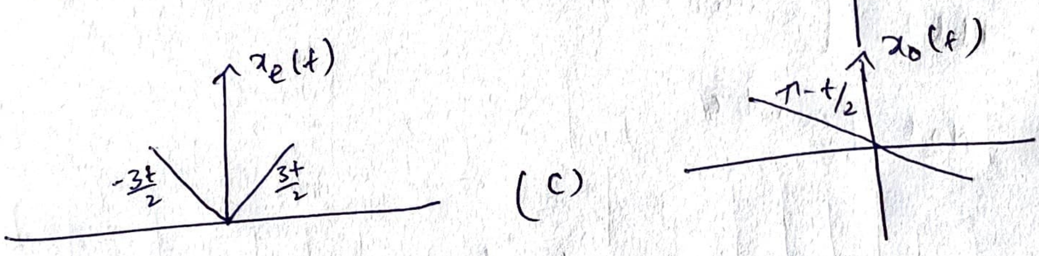


(a)



(b)



(c)

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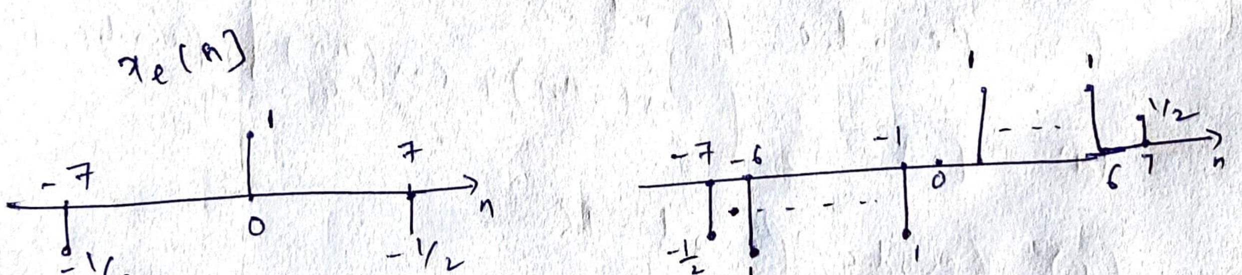
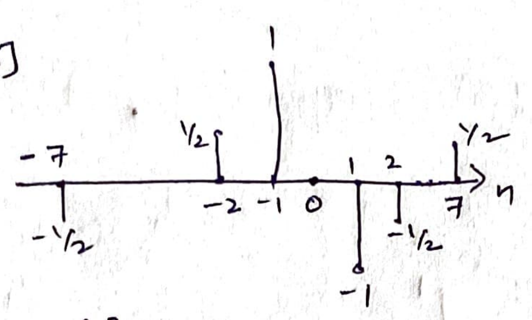
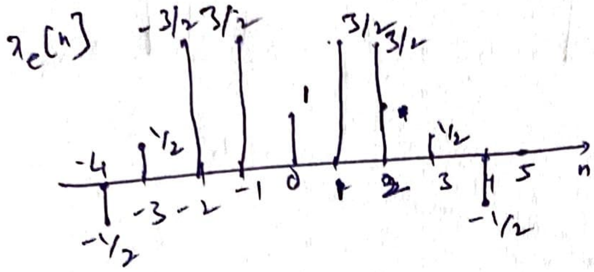
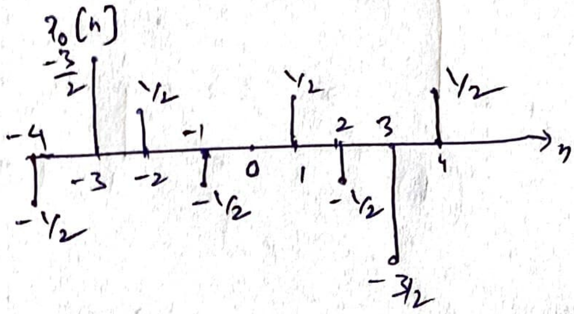


(a)







9

(a) Periodic, Period $= \frac{2\pi}{4} = \frac{\pi}{2}$.

(b) Periodic, Period $= \frac{2\pi}{\pi} = 2$.

(c) $x(t) = [1 + \cos(4t - 2\pi/3)]/2$. Periodic, Period $= \frac{2\pi}{4} = \frac{\pi}{2}$.

(d) $x(t) = \cos(4\pi t)/2$. Periodic, Period $= \frac{2\pi}{4\pi} = \frac{1}{2}$

(e) $x(t) = [\sin(4\pi t)\,u(t) - \sin(4\pi t)\,u(-t)]/2$. Not Periodic

(f) Not Periodic

10 (a) Periodic, Period = 7

(b) Not Periodic

(c) Periodic, Period = 8

(d) $x[n] = \frac{1}{2}\left[\cos\left(\frac{3\pi n}{4}\right) + \cos\left(\frac{\pi n}{4}\right)\right]$. Periodic, Period = 8.

(e) Periodic, Period = 16.

11) a) Linear, stable

b) Memoryless, linear, causal, stable

c) Linear

d) Linear, causal, stable

e) Time invariant, linear, causal, stable

f) Linear, stable

g) Time-Invariant, linear, causal.

12) a) Linear, stable

b) Time-Invariant, linear, causal, stable

c) Memoryless, linear, causal

d) Linear, stable

e) Linear, stable

f) Memoryless, linear, causal, stable

g) Linear, stable

13) a) Invertible, Inverse system; $y(t) = x(t+4)$

b) Non-Invertible. The signals $x(t)$, $x(t+2\pi)$ give the same o/p.

c) Non-Invertible. $\delta(n)$ and $2\delta[n]$ give the same o/p.

d) Invertible. Inverse system: $y(t) = \frac{d}{dt} x(t)$

e) Invertible. Inverse system: $y[n] = x[n+1]$ for $n \geq 0$

and $y[n] = x[n]$ for $n < 0$.

f) Non-Invertible $x[n]$ and $-x[n]$ give the same result.

g) Invertible. Inverse system: $y[n] = x[1-n]$

h) Invertible. Inverse system $y(t) = x(t) + \frac{d}{dt}x(t)$

(i) Invertible. Inverse system $y[n] = x[n] - \frac{1}{2}x[n-1]$

(j) Non Invertible. $\delta[n]$ and $2\delta[n]$ result in $y[n] = 0$

(k) Invertible. Inverse system: $y(t) = x(t/2)$.

(l) Non-Invertible. $x_1[n] = \delta[n] + \delta[n-1]$ and $x_2[n] = \delta[n]$

give $y[n] = \delta[n]$

(m) Invertible. Inverse system: $y[n] = x[2n]$.

① $e(n) = x(n) - y(n) \rightarrow (1)$

$y(n) = e(n-1) \rightarrow (2)$

Replace n by $n-1$ in (1)

$e(n-1) = x(n-1) - y(n-1) \rightarrow (3)$

From (2), (3)

$y[n] = x[n-1] - y[n-1]$

$y[n] + y[n-1] = x[n-1]$

① (a) If $n=0$

$y[0] + \cancel{y[-1]}^{\,0} = \cancel{\delta(-1)}^{\,0}$

$y[0] = 0$

If $n=1$ $\quad y[1] + y[0] = \delta(0)$

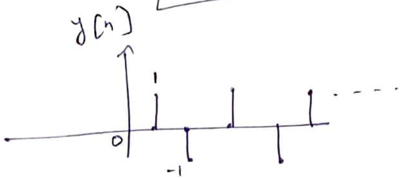
$\boxed{y(1) = 1}$

If $n=2$ $\quad y[2] + y[1] = \cancel{\delta(1)}^{\,0}$

$\boxed{y[-2] = -1}$

If $n=3$ $\quad y[3] + y[2] = \cancel{\delta(2)}^{\,0}$

$\boxed{y[3] = 1}$



1 (b)

$$y[n] + y[n-1] = u[n-1]$$

$n=0 \rightarrow$ $y[0] + \cancel{y[-1]}^{\,0} = \cancel{u[-1]}^{\,0}$

$$\boxed{y[0] = 0}$$

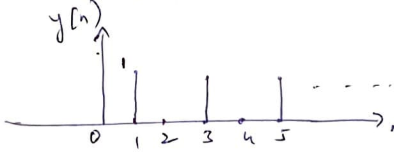
$n=1 \rightarrow$ $y[1] + y[0] = u[0]$

$$y[1] + 0 = 1$$

$$\boxed{y[1] = 1}$$

$n=2 \rightarrow$ $y[2] + y[1] = u[1]$

$$y[2] = 0$$



(2) (a)

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-1]\, g[n-2k]$$

$$= g[n-2]$$

$$= u[n-2] - u[n-6]$$

2 (b)

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k-2]\, g[n-2k]$$

$$= g[n-4]$$

$$= u[n-4] - u[n-8]$$

2 (c) Using 2(a) & 2(b) we conclude that the system is not LTI.

Because the I/P in 2(b) shifted right by 1 unit for the I/P ~~in~~ in 2(a).

However, the O/P ~~in~~ 2(b) is not shifted right by 1 unit for the O/P obtained in 2(a).

2 (d)

$$y[n] = \sum_{k=0}^{\infty} g[n-2k]$$

$$y[n] = g[n] + g[n-2] + g[n-4] + \cdots$$

$$= \left[u[n] - u[n-4]\right] + \left[u(n-2) - u(n-6)\right] + \left[u(n-4) - u(n-8)\right] + \cdots$$

$$y[n] = \begin{cases} 1, & n = 0, 1 \\ 2, & n \geq 2 \\ 0, & \text{otherwise} \end{cases} = 2u(n) - \delta(n) - \delta(n-1)$$

3

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$$

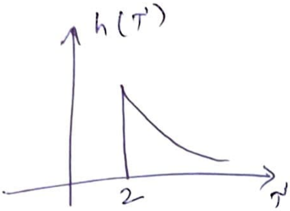
Let $\tau' = \tau - 2$

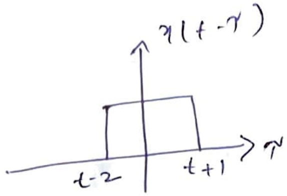
$d\tau' = d\tau$

$$y(t) = \int_{-\infty}^{t-2} e^{-(t-2-\tau')} x(\tau')\, d\tau'$$

Therefore $h(t) = e^{-(t-2)} u(t-2)$

3 (b)





$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{2}^{\infty}$$

$$y(t) = \begin{cases} 0, & t < 1 \\ \int_{2}^{t+1} e^{-(\tau-2)}\, d\tau = 1 - e^{-(t-1)}, & 1 < t < 4 \\ \int_{t-2}^{t+1} e^{-(\tau-2)}\, d\tau = e^{-(t-4)}\left[1 - e^{-3}\right], & t > 4 \end{cases}$$

4

(a) Anti-Causal, because $h[n] = 0$ for $n > 0$

Stable, because $\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{5}{4} < \infty$

(b) Not causal, because $h[n] \neq 0$ for $n < 0$

Stable, because $\sum_{n=-\infty}^{3} 5^n = \frac{625}{4} < \infty$

(c) Not causal because $h[n] \neq 0$, $n < 0$

Unstable because second term become infinite as $n \to \infty$.

4 (g) Causal because $h[n]=0$ for $n<0$.

Stable because $\sum_{n=-\infty}^{\infty} |h(n)| = 1 < \infty$

5

(a) Not causal because $h(t) \neq 0,\ t<0$

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = \frac{e^{-2}}{2} < \infty$

(b) Not causal

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = \frac{1}{3} < \infty$

(c) Causal

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = 1 < \infty$

(d) causal

unstable, $\int_{-\infty}^{\infty} |h(t)|\, dt = \infty$

6

(a)

$$y(t) = \begin{cases} \dfrac{e^{-\beta t}\left(e^{-(\alpha-\beta)t} - 1\right)}{\beta - \alpha} \ ; & \alpha \neq \beta \\ t\, e^{-\beta t} u(t) \ ; & \alpha = \beta \end{cases}$$

6 (b)

$$y(t) = \begin{cases} \int_0^2 e^{2(t-\tau)} d\tau - \int_2^5 e^{2(t-\tau)} d\tau & ; t \le 1 \\ \int_{t-1}^2 e^{2(t-\tau)} d\tau - \int_2^5 e^{2(t-\tau)} d\tau ; & 1 \le t \le 3 \\ -\int_{t-1}^5 e^{2(t-\tau)} d\tau, & 3 \le t \le 6 \\ 0, & t > 6 \end{cases}$$

$$y(t) = \begin{cases} \frac{1}{2}\left[e^{2t} - 2e^{2(t-2)} + 2e^{2(t-5)}\right. & ; t \le 1 \\ \frac{1}{2}\left[e^{2} + e^{2(t-5)} - 2e^{2(t-5)}\right. ; & 1 \le t < 3 \\ \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right], & 3 \le t \le 6 \\ 0 & ; t > 6 \end{cases}$$

6 (c)

$$y(t) = \begin{cases} 0 & ; t < 1 \\ \frac{2}{\pi}\left[1 - \cos\pi(t-1)\right] & ; 1 < t < 3 \\ \frac{2}{\pi}\left[\cos\{\pi(t-3)\} - 1\right] & ; 3 < t < 5 \\ 0 & ; t > 5 \end{cases}$$

6 (d)

$$y(t) = at + b$$

6 (e)

$$y(t) = \frac{1}{4} + t - t^2 ; \quad -\frac{1}{2} < t < \frac{1}{2}$$

$$= t^2 - 3t + \frac{7}{4} ; \quad \frac{1}{2} < t < \frac{3}{2}$$

The period of $y(t) = 2$

Solutions

(1)

$$f_2(t) = f(t-1) + f_1(t-1)$$

$$f_3(t) = f(t-1) + f_1(t+1)$$

$$f_4(t) = f(t-0.5) + f_1(t+0.5)$$

$$f_5(t) = 1.5\, f\left(\frac{t}{2} - 1\right)$$

(2)

$$\omega_0 = \frac{\pi}{6}, \quad T_0 = \frac{2\pi}{\omega_0} = 12 \text{ sec}$$

(3) The system is linear

Additive

$$\frac{d^n y_1}{dt^n} + \cdots + a_0 y_1 = b_m \frac{d^m x_1}{dt} + \cdots + b_0 x_1 \rightarrow (1)$$

$$\frac{d^n y_2}{dt^n} + \cdots + a_0 y_2 = b_m \frac{d^m x_2}{dt} + \cdots + b_0 x_2 \rightarrow (2)$$

(1) + (2)

$$\frac{d^n (y_1 + y_2)}{dt^n} + \cdots + a_0 (y_1 + y_2) = b_m \frac{d^n}{dt}(x_1 + x_2) + \cdots + b_0 (x_1 + x_2)$$

Above equation becomes $y(t) = y_1(t) + y_2(t)$, $x(t) = x_1(t) + x_2(t)$ } when the i/p is $x_1(t) + x_2(t)$ the o/p is $y_1(t) + y_2(t)$. ∴ The system is Linear.

(4)

$$E_{x(t)} = \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} 4 e^{-t} dt = 4 + 4 = 8$$

$$P_{x(t)} = 0$$

$$E_{y(t)} = \infty$$

$$P_{y(t)} = \frac{1}{2} \int_{-1}^{1} y^2(t)\, dt$$

$$= \frac{1}{2} \int_{-1}^{1} t^2 dt = \frac{1}{3}$$

(5) Even Part of the Signal

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{e^{-at}u(t) + e^{+at}u(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{e^{-at}u(t) - e^{at}u(-t)}{2}$$

$x_e(t)$