EC302 23/08/2022

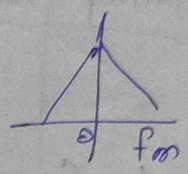
Analog & Digital Communications

→ What is Modulation?

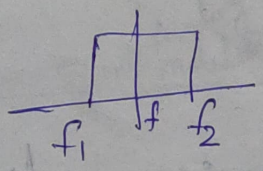
- Change

→ Base band Signal - centre frequency is '0'

- Low frequency signal
- Sensitive, cannot travel for long distances



→ Band Pass Signal :



- The frequency conversion from low to high frequency is called Modulation.

- Message Signal / Information Signal - Low frequency Signal

→ Why there is a need of Modulation?

- In order to pass information signal for long distances.

Message (or) base band Signal + Carrier signal (H.F) → Modulated Wave Signal

(or) Low freq. (or) Modulating Signal

- Carrier signal does not have any information.
- There is a need to assign more power to carrier, such that it fullfills its aim (to make M reach destination)
- Combining Message & Carrier signals is Modulation

$$M + C = \text{Modulated Wave} \Rightarrow M + C - C$$

$$\Rightarrow M \quad \boxed{\text{Demodulation}}$$

1. Coherent Demodulation: generating same Carrier signal
   - Need of carrier regeneration & remove it from Modulated signal
   - More accuracy
2. Non-Coherent Demodulation:

Modulation:

→ Changing charcteristics (freq., Phase) of Carrier signal (C) w.r.t Amplitude of Message Signal (M).

- AM: Change of A in Carrier signal w.r.t A of M signal

M	A	A	A	
C	Amplitude ⇓ AM	Freq. ⇓ FM	Phase ⇓ PM	

SCHEMES

	Analog Modulation Schemes (or) Continuous Wave Modulation Schemes	Pulse Modulation		Digital
		Analog	Digital	
M	✓	✓	✓	✓
C	✓	Pulse	Pulse	✓
M+C	✓	Pulse Analog modulation	Pulse Digital Modulation	✓

- Analog signal with Digital representation ← Pulse
- Intermediate level between Analog & Digital

- Pulse Analog Modulation – PAM (Amplitude)/Sampling
  PWM (width)
  PPM (Position)

- Pulse Digital Modulation – PCM
  DM
  ADM

- Digital Modulation – ASK – Amplitude Shift key
  FSK
  PSK – (180° in last of phase)
  MSK

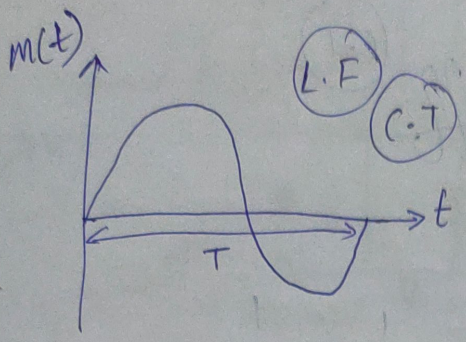
  ex: it is on & off (1,0)

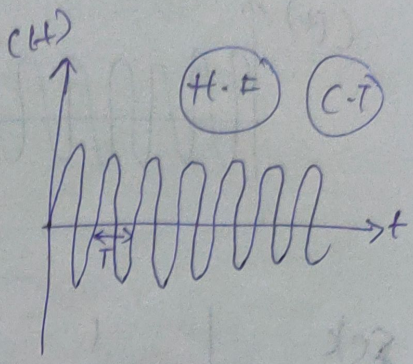
Modulation	M	C	M + C
Analog/ Continuous	C.T	C.T	C.T
Pulse Analog	C.T	Pulse	pulse
Pulse Digital	Digital	Pulse	Pulse
Digital	Digital (1's & 0's)	Continuous (C.T)	C.T (following Amplitude of M signal)

- Digital Signals are less prone to noise than Analog signal.
- Re-construction is difficult in Analog than in Digital
- In Analog & Digital Modulation, Carrier signal is Continuous, difference w.r.t Message Signal.

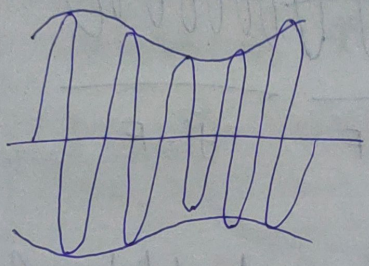
ANALOG MODULATION

A.M. : Amplitude of C(t) changing w.r.t Amp. of m(t)

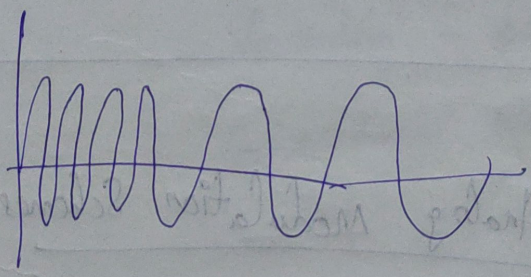




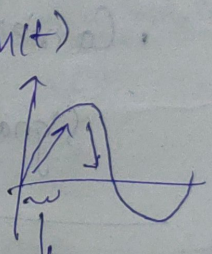
A.M :



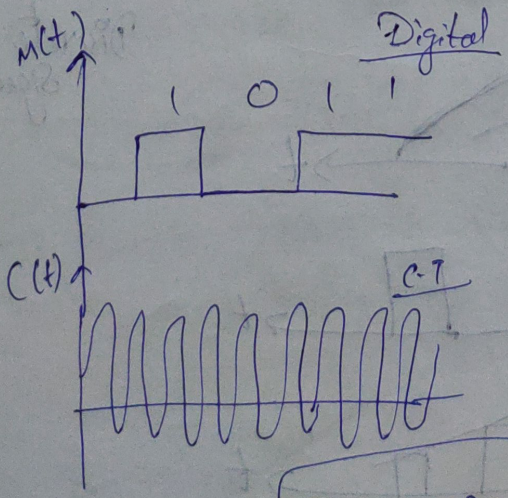
F.M :



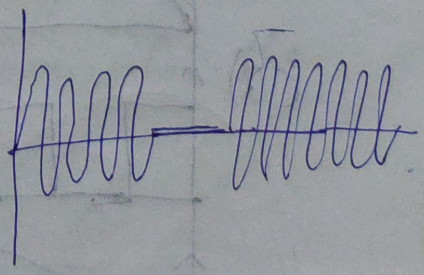
∵



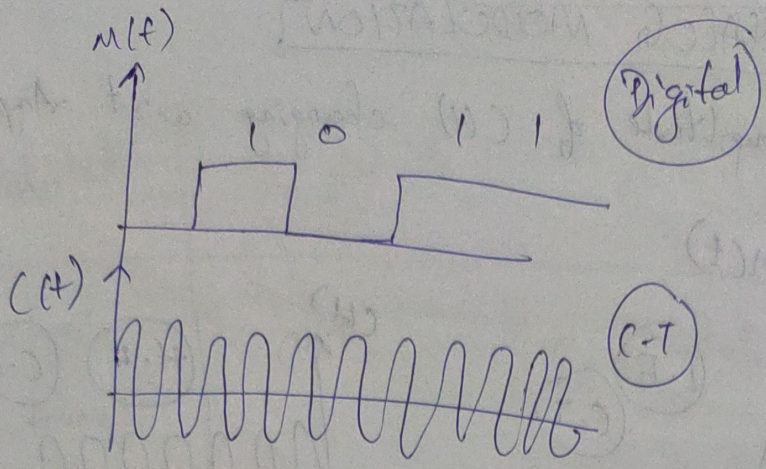
Amplitude ↑
freq. high
⇒ Small Time period



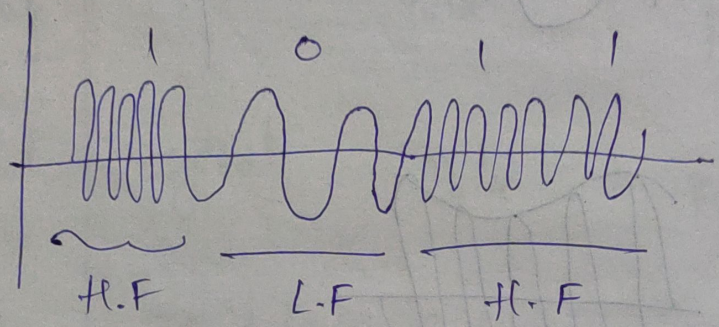
ASK - Amplitude Shift key



DIGITAL MODULATION



FSK



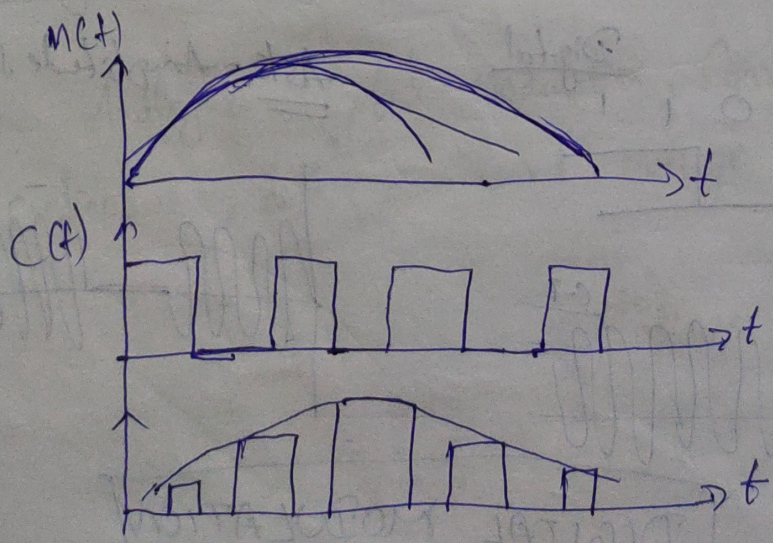
- Carrier signal follows Amplitude of Message Signal.

24/08/2022

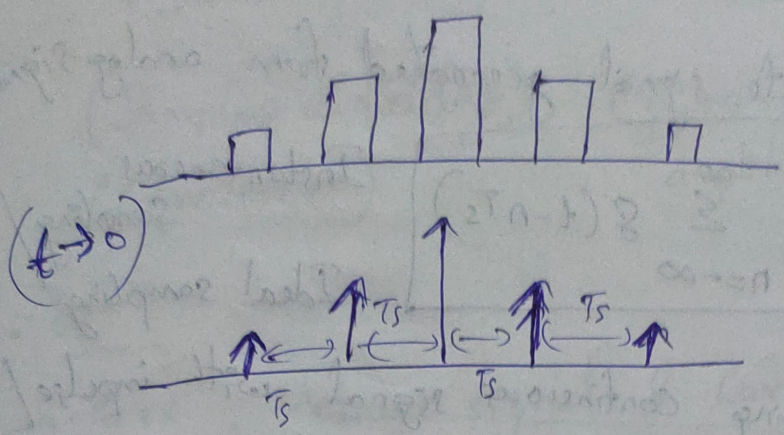
Pulse Analog Modulation Schemes

(1) PAM – Pulse Amplitude Modulation (Sampled Signal)

Discrete Signal



- Compress each pulse ⇒ $t \to 0$ ⇒ Impulse



- Sampled version of message signal – PAM (Sampled signal)

- Nyquiste rate : $f_s > 2f_m$ f_s always greater $2f_m$

  $f_s \geq 2f_m$ – better case

  $f_s < 2f_m$ – Worst case

- <u>Sampling Theorem:</u>

→ Determines the reconstruction of original (analog signal) signal from discrete signal

→ Reconstruction is achieved max. for the condition $f_s > 2f_m$

(Sampling Theorem :)

- How discrete signal generated from analog signal

$$\boxed{g(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)}$$

Instantaneous Sampling / Ideal sampling

→ By Multiplying continuous signal with impulse / pulse train

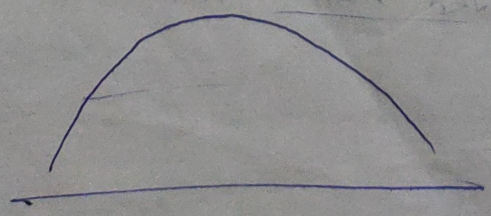
→ Considering every instance after every T_s.

→ In real time, there is no physical presence of impulse

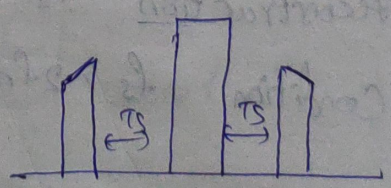
→ when impulse train replaced by pulse train then it is <u>PAM</u>.

① IdealSampling : $g(t) \times$ impulse train

② Natural Sampling [PAM] : $g(t) \times$ pulse train



Message Signal



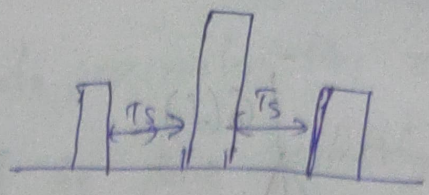
PAM Signal

③ Flat-top Sampling:

(S&H)

(Sampling & Holding)

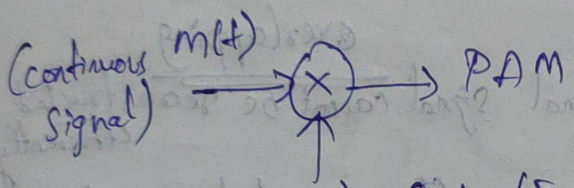
(Capacitor ✓)



(pulses with constant amplitude)

• PAM:

→ Here, Message signal & Carrier signal are combined using productive modulator



Multiplier

Product Modulator

Sampling theorem:

• $$g_s(t) = g(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

(in Time-Domain)

$$G_s(f) = G(f) \circledast f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

(in freq.-Domain)

$$= f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

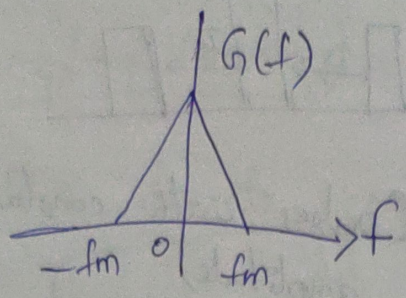
{G(f) – Fourier Transform of g(t)}

Conditions for sampling Theorem:

• Always band-limit the signal

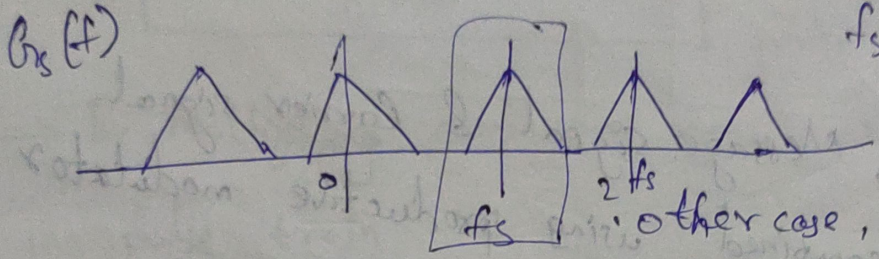
(∵ to satisfy $f_s > 2f_m$)

Spectrum of Sampled Signal



→ Periodic repetition of same signal for infinite times.

- Advantageous only when $f_s > 2f_m$



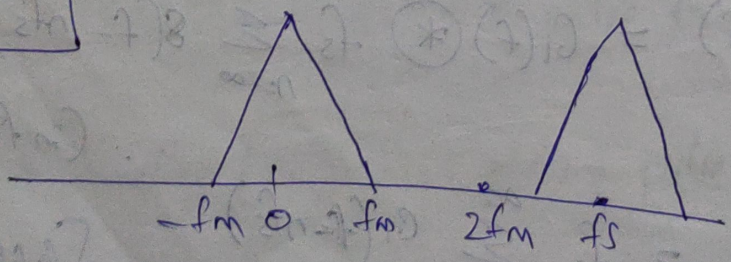
- other case, its not advantageous due to overlapping

- Due to overlapping, original signal cannot be reconstructed efficiently

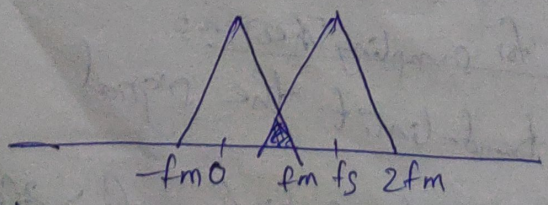
Reconstruction: By passing the sampled signal through a LPF. (LPF → Demodulator)

→ Only one copy will pass out & rest all copies rejected.

$f_s > 2f_m$:



$f_s < 2f_m$:



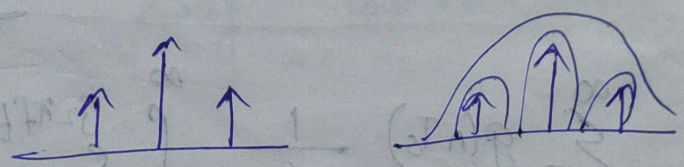
Aliasing Effect: (high freq. components act as low freq. components)

- Due to Aliasing effect, reconstruction of original signal cannot be done perfectly, some part of information is lost.
  - $G(f) \rightarrow g(t)$

Interpolator:

- Process of creating new samples based on the previous knowledge of samples is called Interpolation.

$g(t)$:



26/08/2022

Reconstruction of Analog Signal with the knowledge of discrete samples:

- $G_s(f) = f_s \sum_{n=-\infty}^{\infty} G(f - n f_s)$

o/p after passing through LPF, one copy passed out at the origin

$$G_s(f) = f_s \, G(f) \rightarrow (1)$$

$\boxed{f_s = 2f_m}$

DFT

$$G_s(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \, e^{-j\omega n T_s} \rightarrow (2)$$

$(\omega = 2\pi f)$

$g(t)$ – Continuous

$g(nT_s)$ – discrete

Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$$ (Inverse F.T)

from eq (1)

$$= \int_{-\infty}^{\infty} \frac{G_\delta(f)}{f_s}\, e^{j2\pi f t}\, df$$

$$\cancel{= \int_{-\infty}^{\infty} \frac{1}{f_s} f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)\, e^{j2\pi f t}\, df}$$

from eq (2)

$$= \sum_{n=-\infty}^{\infty} g(nT_s) \frac{1}{2f_m} \int_{-\infty}^{\infty} e^{j2\pi f t}\, e^{-j2\pi n T_s f}\, df$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s) \frac{1}{2f_m} \int_{-\infty}^{\infty} e^{j2\pi f t}\, e^{-j\frac{2\pi n f}{2f_m}}\, df$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s) \frac{1}{2f_m} \int_{-\infty}^{\infty} e^{j2\pi f\left(t - \frac{n}{2f_m}\right)}\, df$$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \mathrm{Sinc}(2f_m t - n)}$$

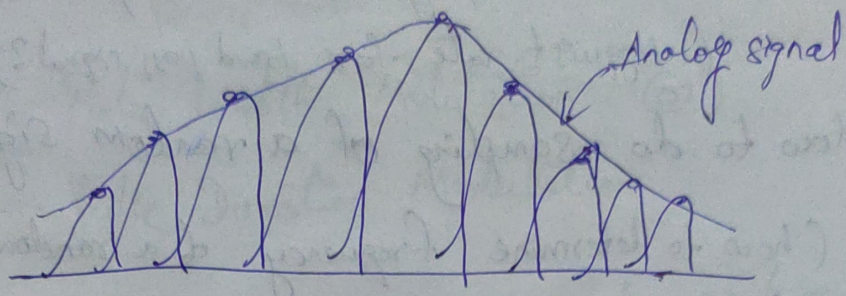
Analog signal

Discrete sampled signal

Sinc function (interpolating function)

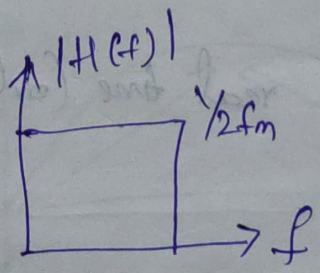
Important Derivation

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\ \mathrm{sinc}(2f_m t - n)}$$



- $S_{PAM}(t) = \underset{\text{Analog}}{m(t)} \times \underset{\text{Pulse train}}{c(t)}$

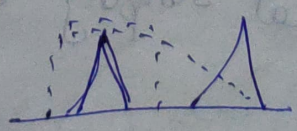
- $|G_s(f)| = f_s\,|G(f)| \Rightarrow G(f) = \frac{1}{2f_m}|G_s(f)|$



- Interpolation

Limitation:

1. Sharp edge is difficult
2. f_s always far greater than $2f_m$

or else  - some unnecessary information too passed out

- portion of other pattern is also passed

(6) B.W requirement (8) Limitations
(7) Power requirement (9) Application 02/09/2022

Assignment: (Communication System by Simon Haykin) (1978) M. Jonaler.

→ How to do sampling of a band pass signal?
(Nyquist rate for band pass signal?)

→ How to do sampling of a random signal?
(how to determine frequency of a random signal?)
(Nyquist rate for random signal?)

→ Compare PAM, PWM and PPM w.r.t the following
(1) Definition (2) Waveform representation
(3) generation & modulation (4) Demodulation (5) B.W (Spectrum)

Limitation

(3) Low frequency signals are not physically realizable

$\because \downarrow f \propto \frac{1}{T} \uparrow$

(1) No impulse signal in real time (only pulse signal)

(2) No sharp edge

Purpose of Sampling Theorem:

- Reconstruction of original signal

- $f_s = 2f_m$ – Critical Sampling rate

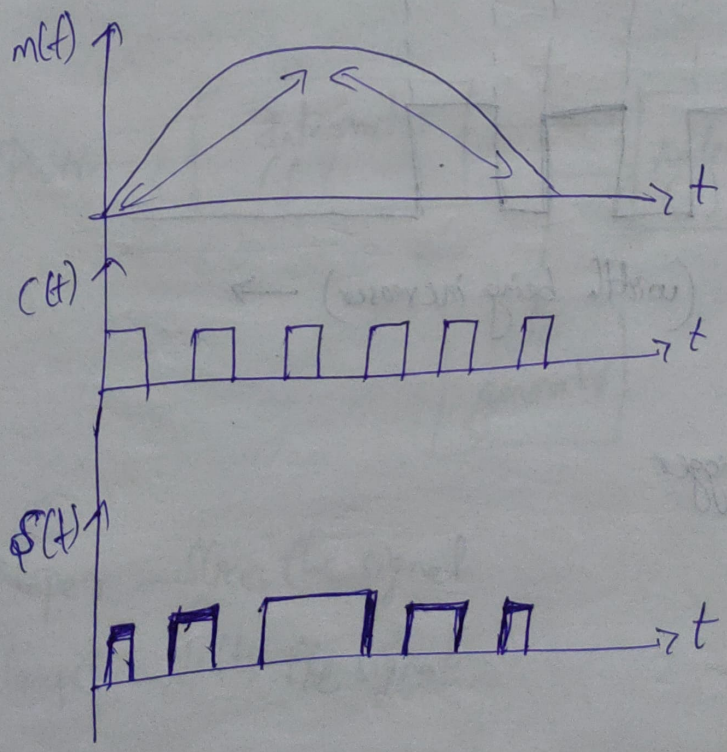
Pulse Time Modulation: ① PAM (Amplitude)

② PWM (width) ③ PPM (Position)

PWM – Pulse Width Modulation (or)
Pulse Duration Modulation

- Width of carrier signal changed w.r.t amplitude of message signal.
- m(t) – Analog & c(t) – Pulse

$$\boxed{\text{width of pulse} \propto |m(t)|}$$

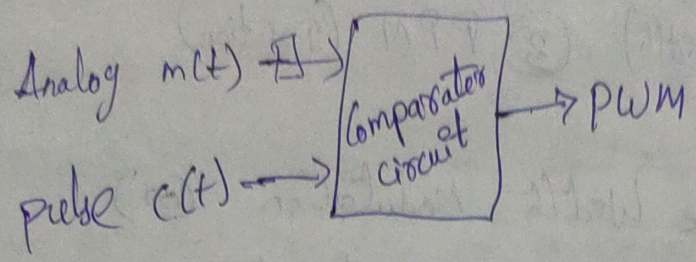


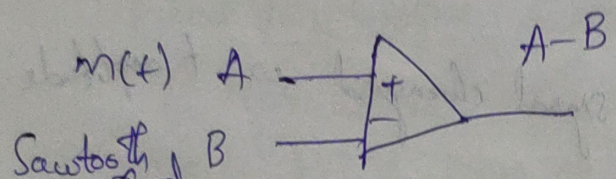
(Amplitude is constant)

(Difference in width)

- Change in width of c(t) w.r.t m(t) is PWM

Generation of PWM : Pass through a Comparator ckt.

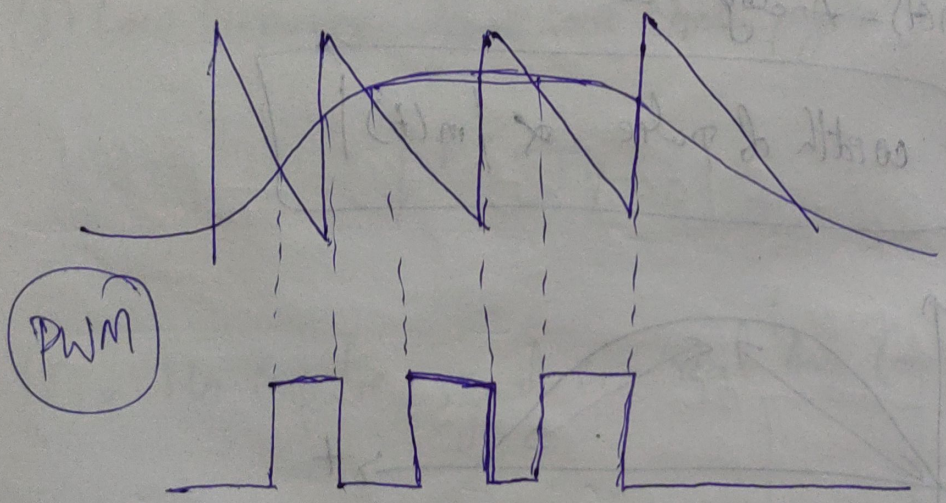




~~A<B~~

A > B, pulse on

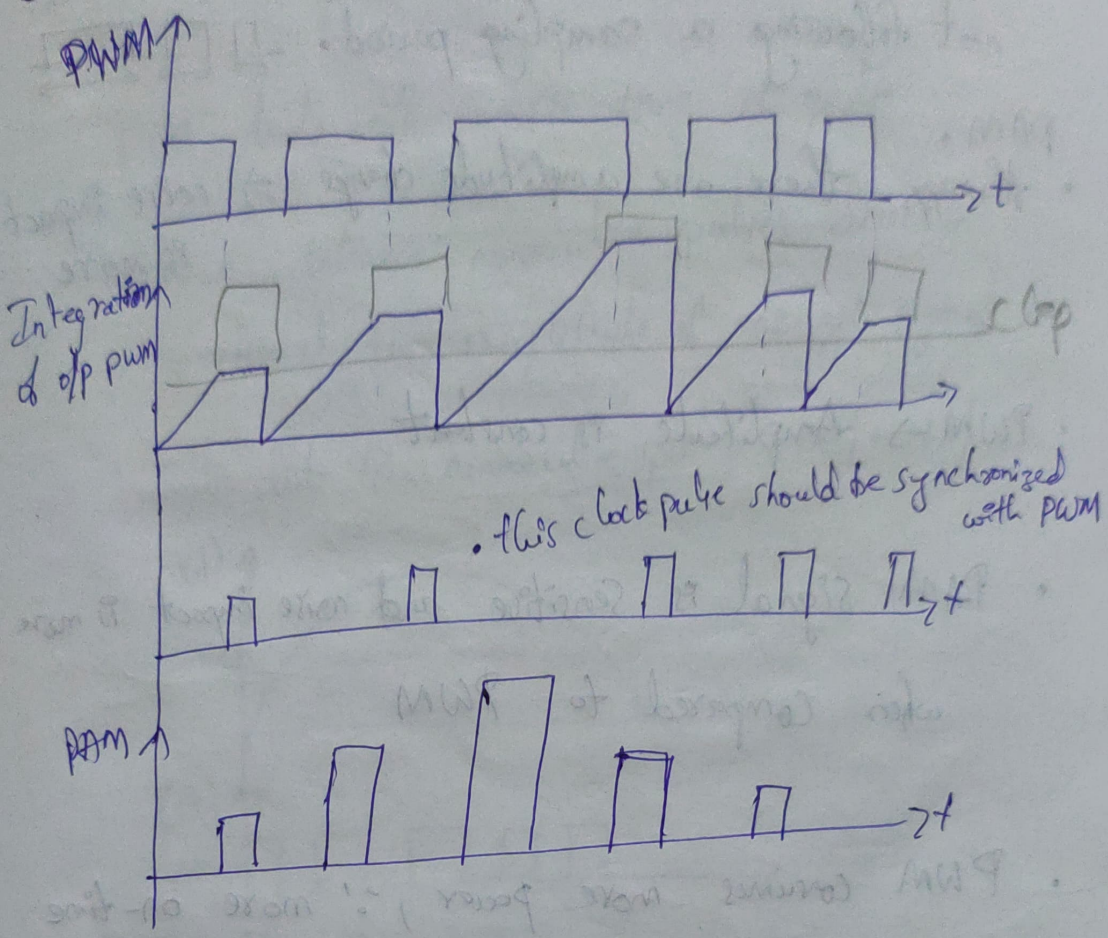
A < B, pulse off

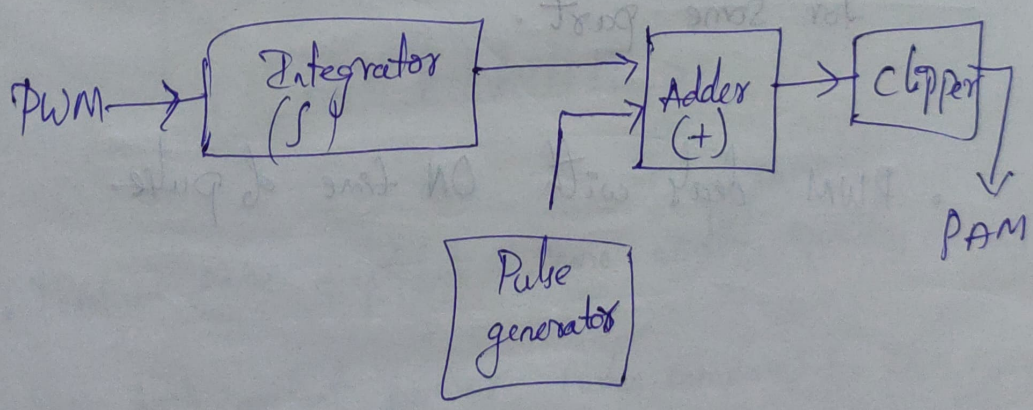


(width being increases) →

- Scmitt trigger

Demodulation of PWM :





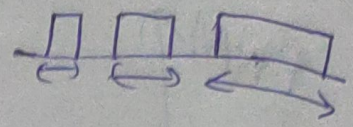
Cliper – Slices the signal

Clamper – shifts the signal

• Step signal → [Integrator] → Ramp signal

Analysis

- Spectrum of PWM is complex, since it is not following a sampling period.



- PAM, Always there are amplitude change ⇒ noise impact is more

- PWM → Amplitude is constant

- PAM signal is sensitive and noise impact is more when compared to PWM

- PWM consumes more power, ∵ more on-time for some part.

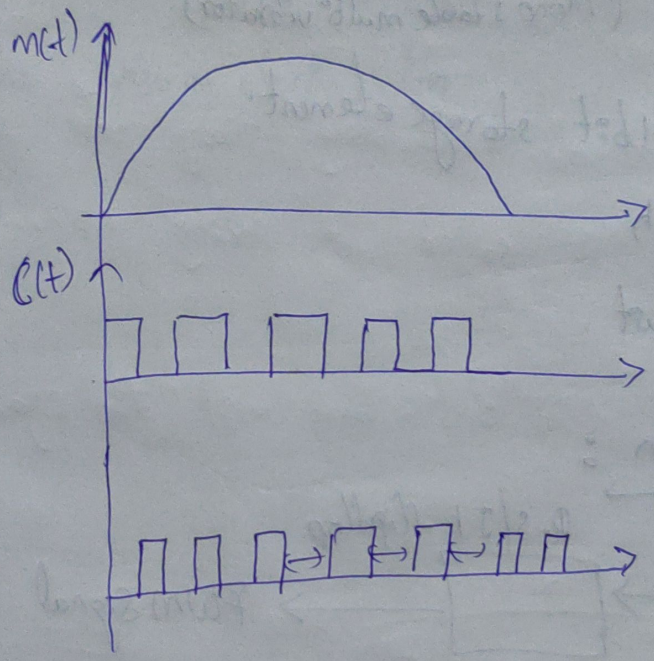
- PWM deals with ON time of pulse.

06/09/2022

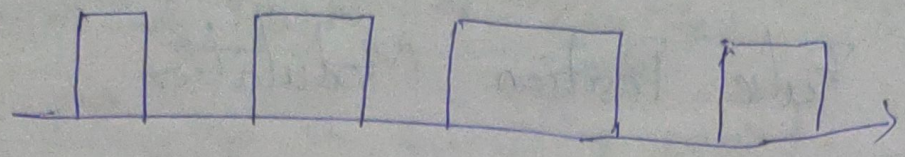
PPM : Pulse Position Modulation

- PPM deals with OFF time of pulse.
- Distance between consecutive pulses directly proportional to amplitude of message signal

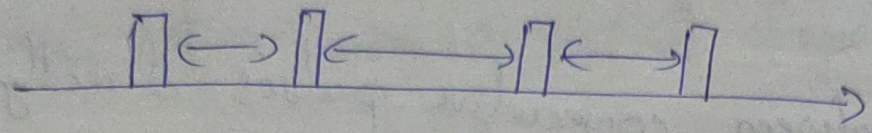
dist. b/w consecutive pulses $\propto |m(t)|$



- PPM – same amplitude, same width/duration, position changes
- PPM ⇒ constant power (when compared to PAM, PWM)
- Synchronize communication required for PPM.
- (Vibrator circuit – produces same pulses at o/p)
- Change in position of pulses w.r.t amplitude of message signal.



(PWM)
(deals with duration)
(ON TIME)



(PPM)
(deals with OFF time)

PWM signal → [M m.stable Vibrator] → PPM
(Mono stable multi vibrator)

- flipflops / latch – 1bit storage element.
- Buffer circuit

Demodulation :

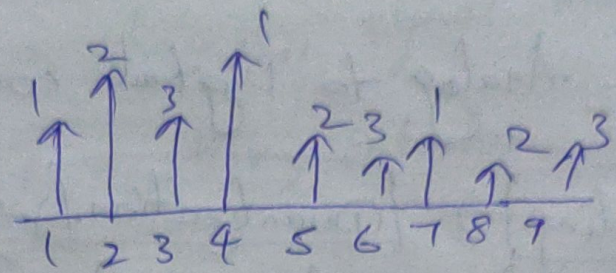
R-S/JK flipflop

PPM → [ ] → PWM signal

Applications

- PAM – Multiplexing, broad casting (on Time base)

PAM – Ex:



- PWM – clock pulses for operating multiple signals at a time
- PPM – Also as clock pulses.

Pulse Digital Modulation

(i) PCM – Pulse code modulation

- Nothing but Analog to Digital conversion

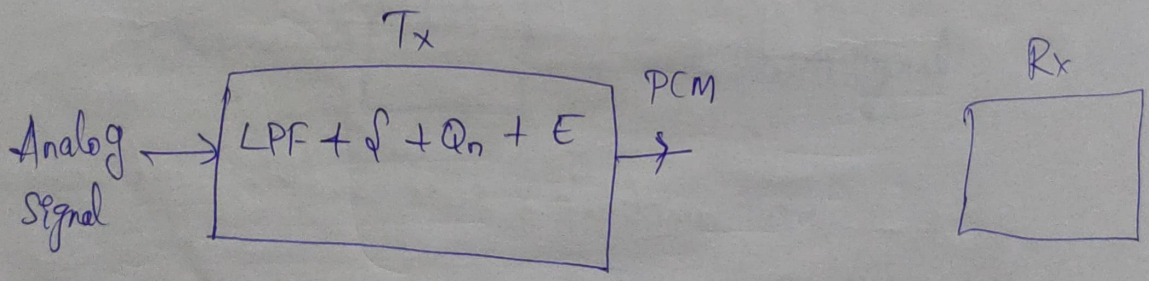
$$\boxed{\text{Sampling}} \rightarrow \boxed{\text{Quantization}} \rightarrow \boxed{\text{Encoding}}$$

(CT → DT) (CA → DA) (representation in terms of binary)

- Any communication System: Transmitter, Channel, Receiver

- The process of representing binary codes in terms of pulses is called line coding (PCM)

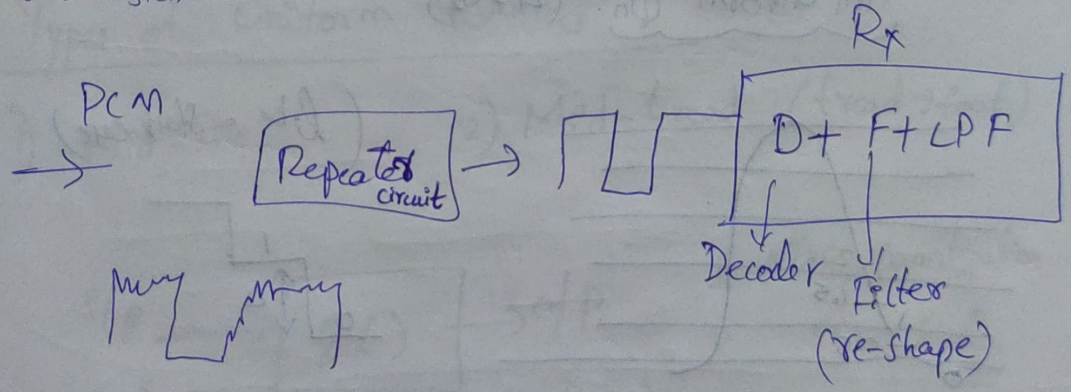
- Purpose of LPF: To band limit the signal (f_m)



- Sampling successful when $f_s > 2f_m$
- Sampling: CT to DT (Discrete Time)
- Q_n: CA to DA (Discrete Amplitude) (or) Approximation
- E_n: Finite amplitudes to binary representation.

- As dist. b/w Tx & Rx increases, PCM cannot be travelled with the required information.
- Becoz, based on channel, signal may be absorbed, reflect, diffract, scatter, interfere.
- And so, Repeater circuits are required.
- If dist b/w Tx & Rx is more, then more no. of repeater circuits are required.

- Polishes or enhances the signal : Repeater Circuit
- Decision circuit



Limitation:

1. B.W requirement is large. $\boxed{B.W \geq n f_m}$

no. of bits required to represent levels

$L = 2^n$

$8 = 2^{3}$ → 8 levels required 3 bit representation

(000, 001, 010, 011, 100, 101, 110, 111)

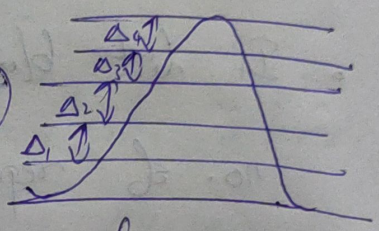
2. Reliability of PCM is low, since no. of bits/levels increasing and so performance is poor

07/09/2022

Quantization: → (Process in conversion of A. to Digital)

- Uniform Qn. and Non-Uniform Qn.
- The difference between any 2 amplitude levels is called as Step Size - (Δ)

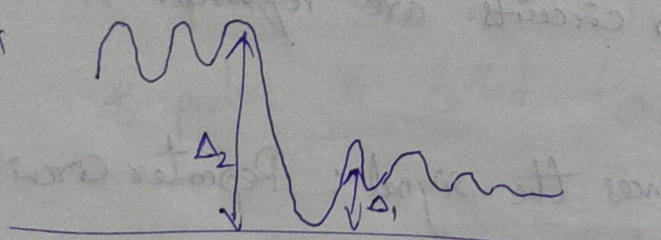
Ex



Uniform Qn.

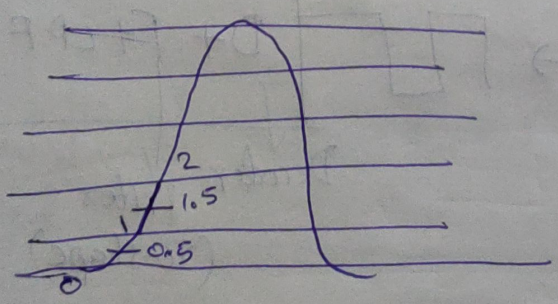
($\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4$)

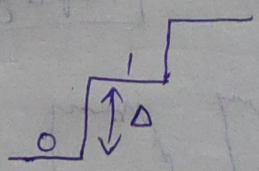
Ex



Non Uniform Qn. ($\Delta_1 \neq \Delta_2$)

(like a stair case)





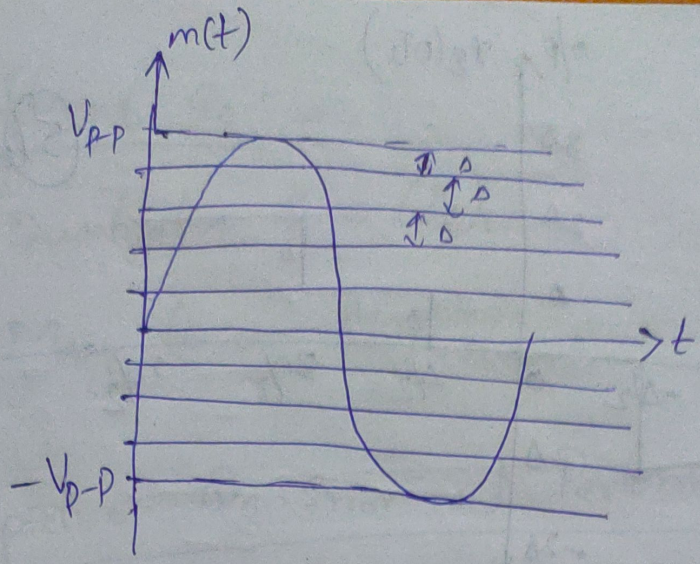
Original value		Approximating to
0.75	→	1
0.35	→	0

above threshold → high level value

below threshold → low level value

- Its many to one mapping. ⇒ Many to one Mapping

⇒ Lossy Coding



L - levels

(V_{p-p} ⇒ Max. Peak Voltage)

$$\therefore \Delta = \frac{2V_{p-p}}{L}$$

- No. of bits required to represent each sample amplitude - n. $\boxed{L = 2^n}$
- n - length of codeword

Types of Uniform Quantization:

1) Mid-Rise 2) Mid-tread (tread → foot)

i/p → [Q( )] → o/P

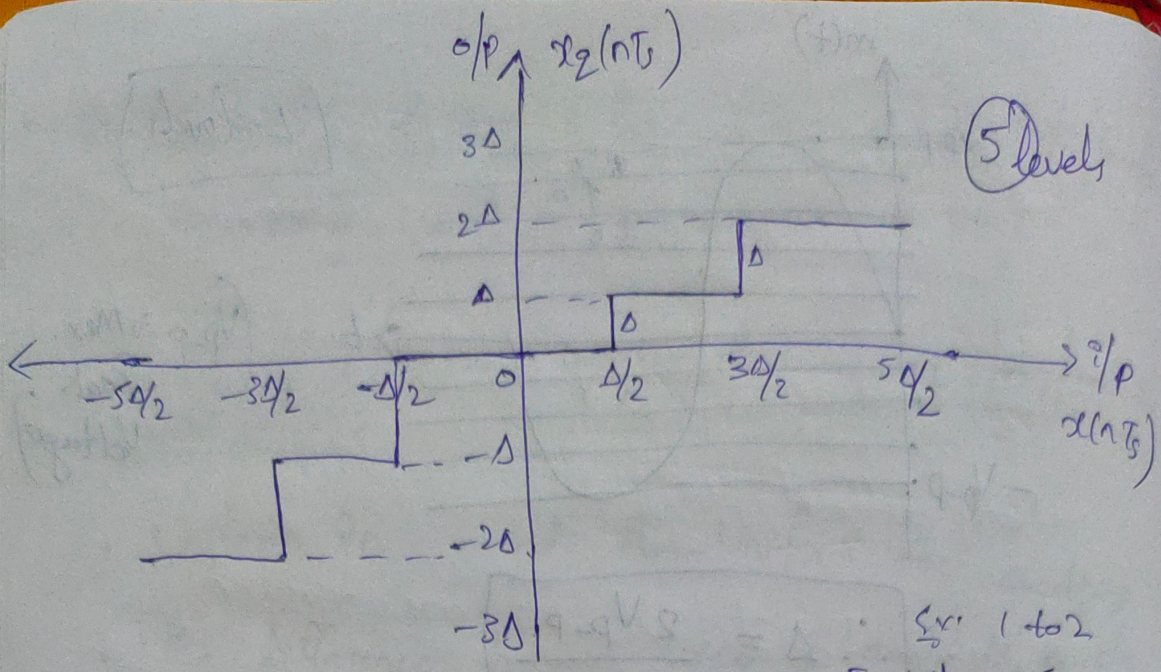
$x(nT)$ $x_q(nT)$

Ex: 0.75 + Qe 1

0.35 − Qe 0

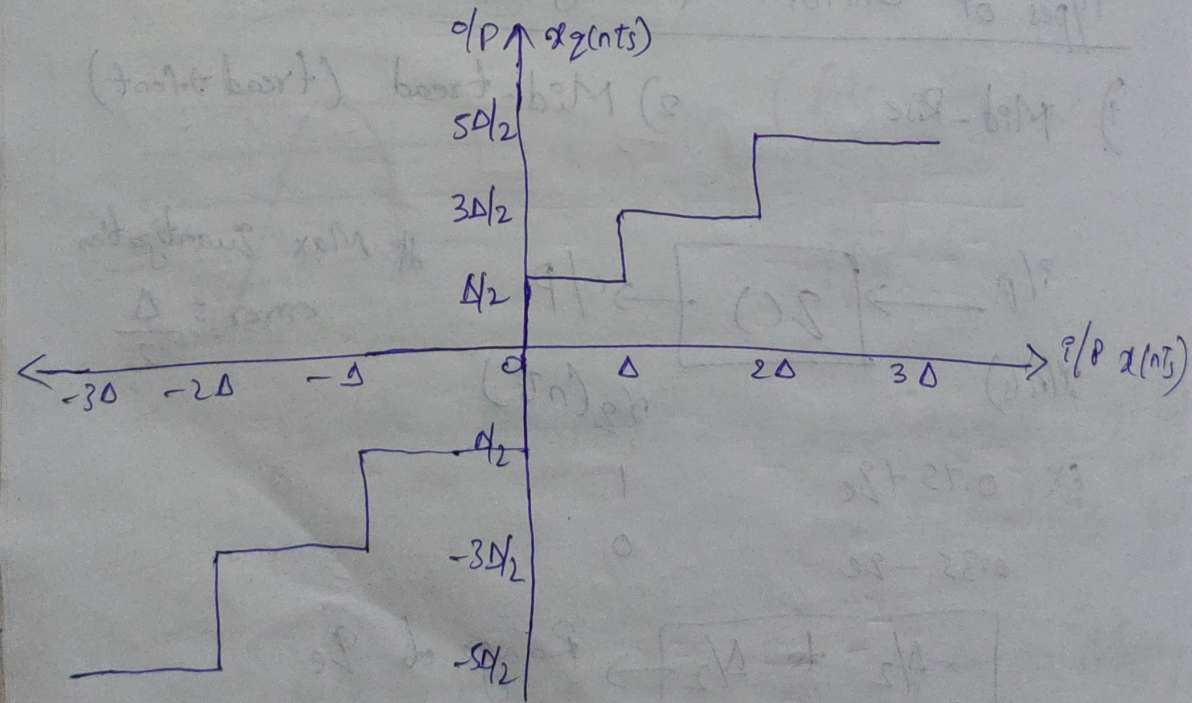
Max. Quantization error: $\frac{\Delta}{2}$

$\boxed{-\Delta/2 \text{ to } \Delta/2}$ → Range of Qe



Ex: 1 to 2
Threshold → 1.5

- For Mid-tread type, no. of levels is an odd number.
- The foot is parallel to ground, foot-half in +ve side & other half in -ve side.



- For Mid-rise type, no. of levels is an even number.

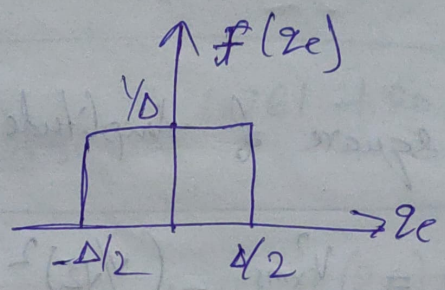
Limitation of Qn:

1) Quantization Error (q_e).

2) Many to one Mapping.

Impact of Qn. Error and Performance of PCM:

- q_e follows uniform distribution.



Probability Distribution Function (Pdf of q_e)

- 1st Expectation - Mean/Average → gives nature of the signal
- 2nd Expectation - Variance → gives Power
- SNR
- Noise Power $= \sigma_N^2 = E[x^2]$ ($\because \mu = 0$ mean is zero)

$$= E[x^2] - (E[x])^2$$

$$= \int_{-\Delta/2}^{\Delta/2} f(q_e)\, q_e^2 \, dq_e$$

$$= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq = \frac{1}{\Delta} \left. \frac{q_e^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

Noise Power $= \frac{1}{3\Delta} \cdot 2 \cdot \frac{\Delta^3}{8} = \frac{\Delta^2}{12}$ //

$$\boxed{\therefore \text{ Noise Power} = \sigma_N^2 = \frac{\Delta^2}{12}} \rightarrow (1)$$

in PCM

- Signal Power: $A_m \sin \omega t$ (let $A_m \rightarrow V_{p-p}$)
  $A_m \cos \omega t$

- It is the square of Amplitude

Signal Power $= \frac{V_{rms}^2}{R} = \frac{(V/\sqrt{2})^2}{R} = \frac{V_{p-p}^2}{2R}$

$\left(\because \Delta = \frac{2V_{pp}}{L}\right) \rightarrow$ Put this in eq (1)

Noise Power: $\sigma_N^2 = \left(\frac{2V_{p-p}}{L}\right)^2 \times \frac{1}{12} = \frac{V_{p-p}^2}{3L^2}$

- $(SNR)_{PCM} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{V_{p-p}^2/2R}{V_{p-p}^2/3L^2}$

(Ideal case: $R=1$) $\Rightarrow$ $SNR = \frac{3}{2}L^2$
(or)
(for Normalized Condition: $R=1$)

$$\boxed{\therefore (SNR)_{PCM} = 1.5\,L^2}$$

Put $L = 2^n$

$$\Rightarrow \boxed{(SNR)_{PCM} = 1.5\, 2^{2n}} \rightarrow \text{unitless}$$

- Ratio's represented in terms of "dB"

$$(SNR)_{in\,dB} = 10\log(1.5) + 10\log L^2$$
$$\simeq (0.176)10 + 20\log L$$

$$\# \quad \boxed{(SNR)_{dB} \simeq 1.761 + 20\log L}$$

$$(SNR)_{dB} \simeq 1.761 + (20)\log 2^n$$

$(\log 2 \simeq 0.3)$

$$\# \Rightarrow \boxed{(SNR)_{dB} \simeq 1.761 + 6n}$$

- Speciality of PCM: Power is independent of Amplitude.

$(SNR)_{PCM} \rightarrow$ depends on L, n

Disadvantage:

- With high requirement of B.W, SNR is large.
- Large B.W $\rightarrow$ SNR large $\rightarrow$ low Noise
- For low noise, need to pay a penalty of large B.W.

- $\Delta = \frac{2V_{p-p}}{L}$; $L = 2^n$; n - no. of bits required to represent each sample amplitude

- Range of Stepsize : $-\Delta/2$ to $\Delta/2$

- Noise Power : $\sigma_N^2 = \frac{\Delta^2}{12} = \frac{V_{p-p}^2}{3L}$

- Signal Power : $\frac{V_{p-p}^2}{2R}$

- SNR

Q. For a PCM system of 40dB (SNR), determine no. of bits & levels required to represent the analog signal in terms of digital.

Sol $(SNR)_{dB} \simeq 1.76 + 20\log L$

$\Rightarrow 40 \simeq 1.76 + 20\log L$

$\Rightarrow \log L \simeq \frac{38.24}{20} = 1.912 \Rightarrow \boxed{L \doteq 82}$

(or)

$\Rightarrow n = \frac{38.24}{6} \simeq 6.3 \Rightarrow n = 6$

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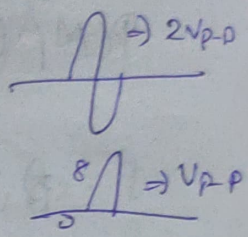
Problems

Q. An analog voltage in the range 0 to 8V is divided into 16 levels (equal intervals) for conversion to 4 bit digital output. Find the max. q_e?

Sol $\quad$ Max $q_e = \frac{\Delta}{2}$ $\qquad$ $\left(L = 2^n = 2^4 = 16 \right)$

$$\Delta = \frac{2V_{p-p}}{L} = \frac{1 \times 8}{16} = \frac{1}{2}$$



$$\boxed{\therefore \text{ Max. } q_e = 1/4 = 0.25}$$

Q. $SNR = \frac{3}{2}L^2$ $\qquad$ $(SNR)_{dB} = 1.76 + 6n = 1.76 + 20\log L$

Q. If the no. of bits per sample in a PCM system is increased from n to n+1 then the improvement in signal to noise ratio is?

Sol $\quad (SNR)_{dB} = 1.76 + 6n$

$$(SNR)'_{dB} = 1.76 + 6(n+1) = 1.76 + 6n + 6$$

Improvement is "6"

Q The B.W requirement for transmission of a PCM signal increases when the no. of qn. levels are increased from 4 to 64 is given by ?

Sol:

$L_1 = 4$ & $L_2 = 64$

$n_1 = 2$ $\quad$ $n_2 = 6$

$PCM_1 \ (B.W_1) \geq n_1 f_m$	$PCM_2 \ (B.W_2) \geq n_2 f_m$
$\Rightarrow BW_1 \geq 2 f_m$	$\Rightarrow (B.W_2) \geq 6 f_m$

$\rightarrow$ increased by a factor of "3"

B.W requirement is always greater than or equal to nf_m. $\boxed{B.W \geq n f_m}$

Q The no. of bits in a binary PCM system is increased from n to n+1. Then the signal to noise power increased factor?

Sol $SNR_1 = \frac{3}{2} L^2 = \frac{3}{2} 2^{2n}$

$SNR_2 = \frac{3}{2} 2^{2n+2} = \left(\frac{3}{2} 2^{2n}\right) 4$

$\rightarrow$ Increased by a factor of "4"

Q In a PCM system with uniform Qn, increase of bits from 8 to 9 will reduce the Qn Noise power by a factor of ?

Sol $\sigma_N = \frac{\Delta^2}{12} = \frac{1}{12}\left(\frac{2V_{p-p}}{L}\right)^2 = \frac{V_{p-p}^2}{3L^2}$

$$\boxed{\sigma_N \propto \frac{1}{L^2}} \Rightarrow$$

$n_1 = 8$, $n_2 = 9$

$$\boxed{\sigma_N \propto \frac{1}{2^{2n}}} \Rightarrow \frac{\sigma_{N_2}}{\sigma_{N_1}} = \frac{2^{2(8)}}{2^{2(9)}} = \frac{2^{16}}{2^{18}}$$

$= 1/2^2 = 1/4$

→ Reduce by a factor of "4"

Q The P-P input to an 8-bit PCM, the signal power to noise power ratio in dB for an input of $0.5\cos(\omega t)$ is given by ?

Sol: $n = 8$; $(SNR)_{dB} = 1.76 + 6(8) = 1.76 + 48$

$= 49.76$

Bit rate / Data Rate:

- For a PCM; Data Rate $= n f_s$

- $B.W \geq n f_m$ (or) $B.W \geq \frac{\text{Bit rate}}{2}$

Q. In a PCM system, if code word length increased from 6 to 8 bits, the signal to Qn. noise power ratio will be improved by a factor of?

Sol $(SNR)_{dB} = 1.76 + 6n$ $\quad n_1 = 6$ & $n_2 = 8$

Linear case $(SNR) = \frac{3}{2} 2^{2n}$; $(SNR_1) = \frac{3}{2} 2^{12}$

$(SNR_2) = \frac{3}{2} \cdot 2^{16}$

→ Improved by a factor of "16".

dB case

$(SNR_1) = 1.76 + 6(6) = 1.76 + 36$

$(SNR_2) = 1.76 + 6(8) = 1.76 + 48$

→ Increased by a factor of "12".

Q A sinusoidal signal with P-P value of 1.536 V is Quantized into 128 Level, using a mid-rise uniform quantized, then determine Q_n noise power?

sol $\sigma_N = \frac{\Delta^2}{12} = \frac{V_{pp}^2}{3L^2}$ $\left(\Delta = \frac{2V_{p-p}}{L}\right)$

$\Rightarrow \sigma_N = \frac{(1.536)^2}{3(128)} = 0.006144$

$\boxed{\sigma_N = 0.006}$

Sampling frequency of sin, cos, sinc, $(\text{sinc})^2$

Modern A & D C's by B.P. Lathi — Book

- Solve problems
- How to find Nyquist rate of following triangular, square

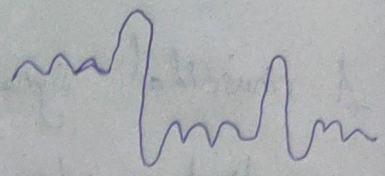
- Genesis Library —

gen.lib.rec → Website : A&D C's books

Simon Haykin books (old copy - 2010)

- Shyam Sharma

Non-Uniform Quantization :

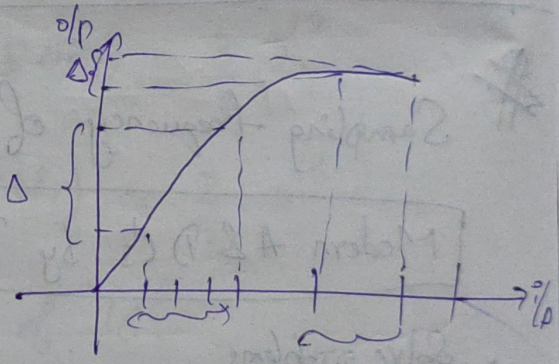


- Companding – other name of Non-Uniform Qn.
  ↳ Compressing and Expanding.

- To convert Non-uniform to uniform : Compress higher amplitudes and Expand lower amplitudes.
- Since, Δ-step size is varying, make it constant.
- Compander (Compresser) ckt : ckt which relates o/p to i/p a logarithmic form.

$$\boxed{C + U \cdot Q_n + E}$$

↳ Compander



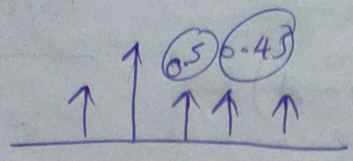
- Compressing + Uniform Qn + Expanding → Compander
- Logarithmic is considered over Linear relation.

Limitation:

1. Redundancy : some samples having same amplitude ⇒ on encoding these samples have same code.

→ To avoid that : encode the difference

Ex



For Q_n : 0.5 & 0.45 has same value

- So encode difference : 0.05 → encode 0.05

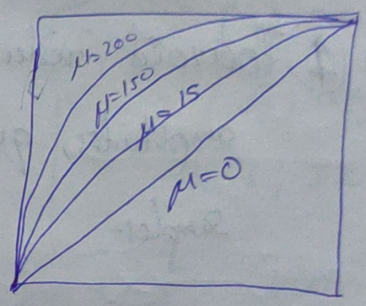
13/09/2022

- Non-linear relation b/w input and output.

(1) Logarthmic relation.

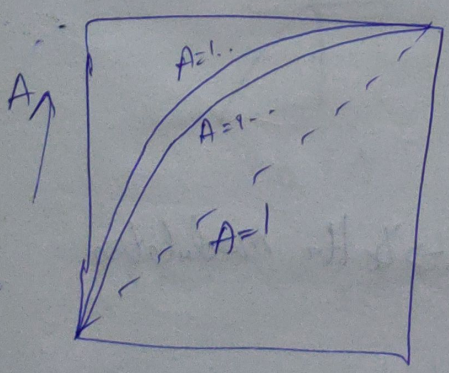
<u>μ-Law:</u>

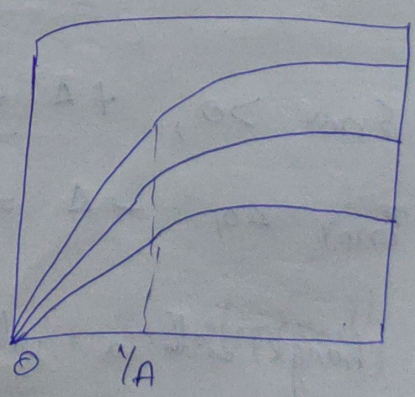
$$V_2 = \frac{\log(1+\mu|V_1|)}{\log(1+\mu)}$$



<u>A-Law:</u>

$$V_2 = \begin{cases} \dfrac{A|V_1|}{1+\log A} & ; 0 \le |V_1| \le \dfrac{1}{A} \quad \text{(linear relation)} \\[2ex] \dfrac{1+\log A|V_1|}{1+\log A} & ; \dfrac{1}{A} < |V_1| \le 1 \quad \text{(non-linear relation)} \end{cases}$$





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Delta Modulation: (Step size - Δ)

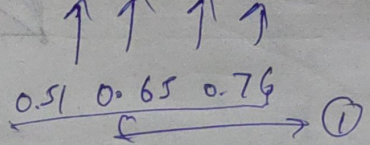
- Limitation of PCM: ① Repetition of bits. – Redundancy

  ② B.W depends on 'n' : $BW \geq nf_m$

  More n ⇒ large B.W required

  ↳ large SNR → good performance

⇒ ① To avoid redundancy, instead of quantising sample amplitudes, quantise the error b/w any two samples.



⇒ "Levels" → "$\frac{L}{2}$" levels

DM: Quantizing the error b/w samples.

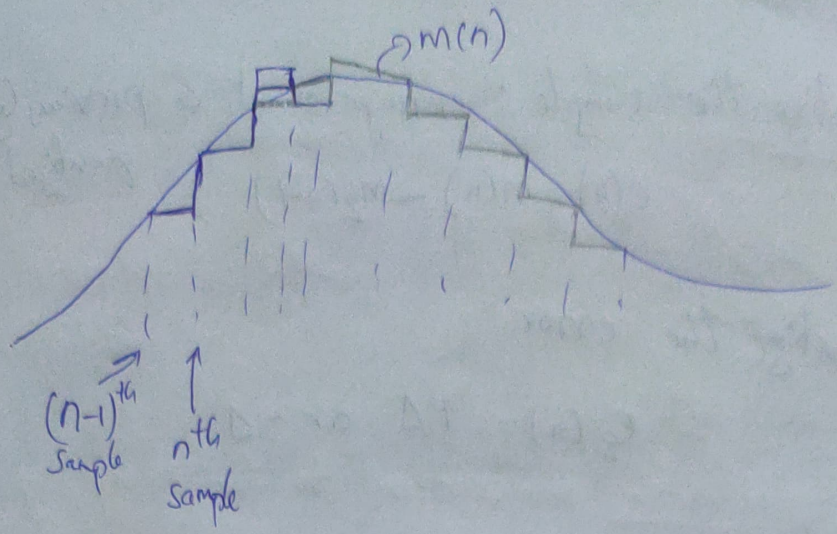
→ Unique.

→ In DM, 1 bit is enough to represent each sample & so it is also called as 1-bit quantizer.

- Error > 0, $+\Delta \Rightarrow \underset{\text{bit}}{1}$

  Error < 0, $-\Delta \Rightarrow 0$

→ Change with a value of Δ → Delta Modulation



$$e(n) = m(n) - m_q(n-1) \rightarrow (1)$$

$$\boxed{Sgn(x) = \begin{cases} 1; & x>0 \\ 0; & x=0 \\ -1; & x<0 \end{cases}}$$

$$e_q(n) = \Delta\, Sgn(e(n))$$

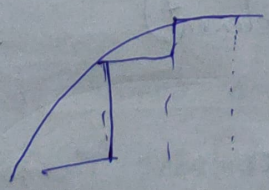
$$e_q(n) = \underset{e(n)>0}{+\Delta} \text{ (or) } \underset{e(n)<0}{-\Delta} \rightarrow (2)$$

- Compare the error always.

- $$m_q(n) = m_q(n-1) + e_q(n) \rightarrow (3)$$

Staircase formation

Ex



$+\Delta$ $+\Delta$

($+\Delta$: since, analog signal greater than stepsignal)

(1). Quantize the sample : message signal & previously quantized signal

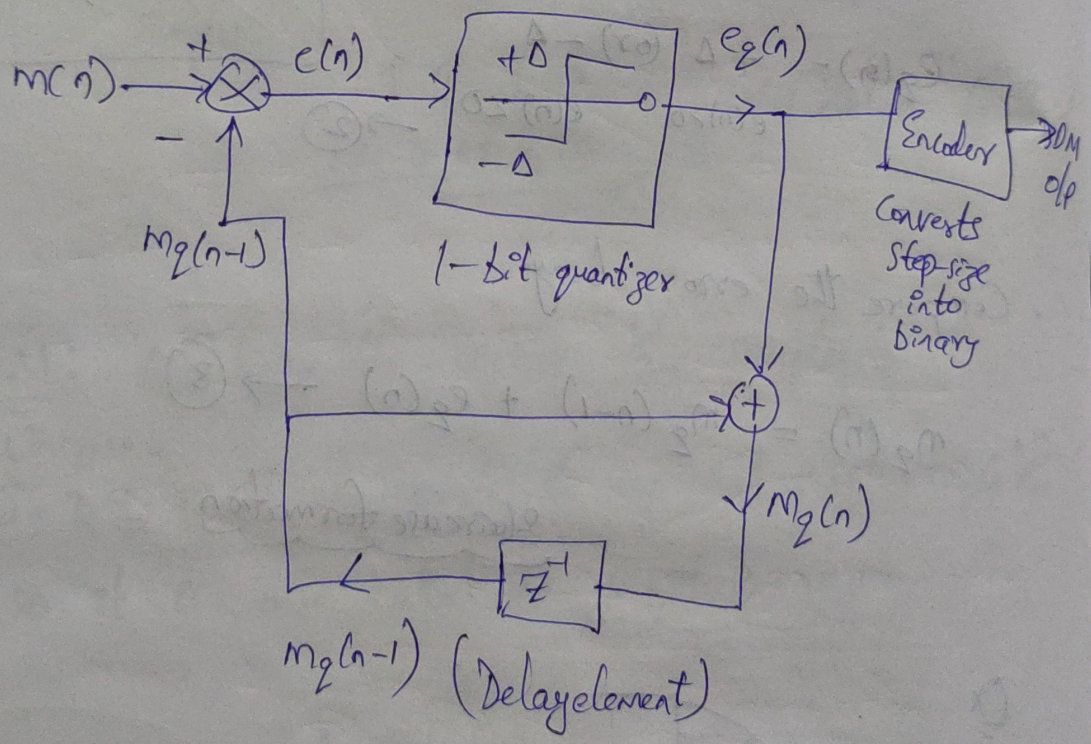
$$e(n) = m(n) - m_q(n-1)$$

(2). Quantize the error

$$e_q(n) = +\Delta \text{ or } -\Delta$$

(3). $m_q(n) = m_q(n-1) + e_q(n)$

<u>Block Diagram</u> <u>(Generation)</u>



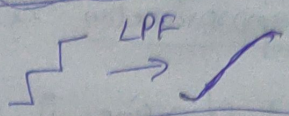
→ Required things : 1-bit quantizer

Delay circuit

Summer circuit

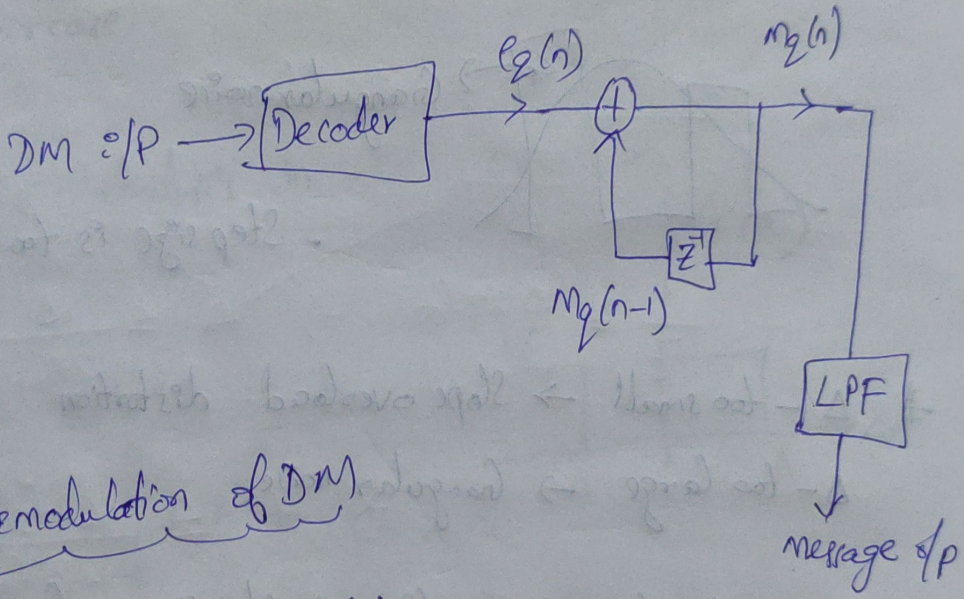
- In PCM, Range : $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$ 2e
- In DM, Range : $(-\Delta, \Delta)$ 2e

<u>Demodulation</u>: (1) Pass through "Accumulator then LPF"



- DM o/p of encoder – given as DM i/p of decoder

$\boxed{\text{LPF}: f_m \le f_M \le f_s}$



<u>Demodulation of DM</u>

$M_q = \underbrace{m_q(n-1)}_{\text{previous value}} + \underbrace{e_q(n)}_{\text{error}}$ ← Accumulator Circuit

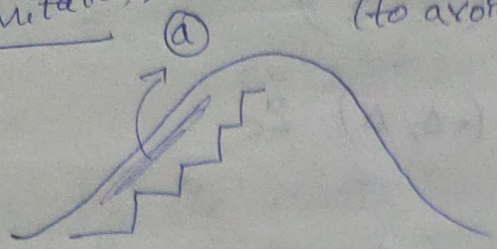
- Staircase formed along with m(t) – message signal (with constant step size Δ)

f_m – max. freq. component of m(t)
f_M – max freq. component passed by LPF

Limitation:

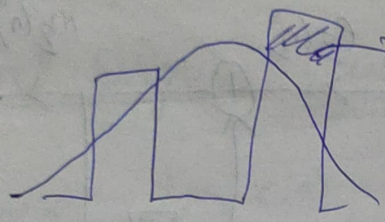
2. DM suffers with more noise than PCM. (to avoid this noise power is multiplied with a scaling factor)

1.



ⓐ → Slope over load distortion

- The approximation never equal to the message signal.
  (∵ step size is too small)
  → never meet the requirement of $m(t)$



→ Gangular noise

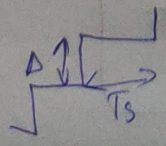
- Step size is too large.

- Δ - too small → Slope overload distortion
  Δ - too large → Gangular noise

- To avoid this, tune the Δ-step size: for low amplitudes decrease Δ and for high amplitudes enhance Δ.
  ⇒ **Adaptive delta modulation**

- In Adaptive delta modulation, 'δ' is variable

- Δ - too small ⇒ slope of analog signal is > slope of staircase



$$\frac{d}{dt} m(t) > \frac{\Delta}{T_s}$$

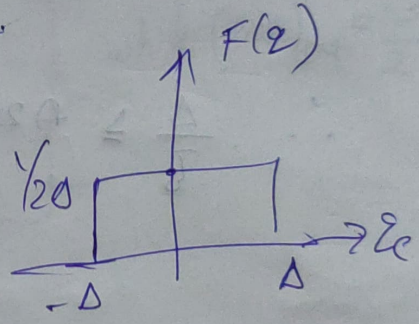
- For No slope overload distortion: Condition is

$$\boxed{\frac{\Delta}{T_s} \geq \left|\frac{d}{dt} m(t)\right|_{max}}$$

(Slope of staircase $\geq$ slope of message signal)

- To avoid Granular noise, reduce the slope of staircase.

Q. Noise Analysis in DM :



- Noise variance

$$\sigma_N^2 = \int_{-\Delta}^{\Delta} F(e) \cdot e^2 \, de$$

$$= \int_{-\Delta}^{\Delta} \frac{1}{2\Delta} \cdot e^2 \, de$$

$$= \frac{1}{2\Delta} \cdot \frac{e^3}{3}\Big|_{-\Delta}^{\Delta} = \frac{\Delta^2}{3}$$

$$\boxed{\sigma_N^2 = \frac{\Delta^2}{3} \text{ in DM}}$$

Scaling factor to tune Noise Power

$$\sigma_N^2 = \left(\frac{\Delta^2}{3}\right) \times \frac{f_M}{f_s}$$

Signal Power in DM

$$m(t) = A\cos 2\pi f_m t$$

Condition: $\frac{\Delta}{T_s} \geq \left| \frac{d\,m(t)}{dt} \right|_{max}$

$$\Rightarrow \frac{d\,m(t)}{dt} = -A\,2\pi f_m \sin 2\pi f_m t$$

$$\Rightarrow \left| \frac{d\,m(t)}{dt} \right|_{max} = A\,2\pi f_m$$

$$\therefore \frac{\Delta}{T_s} \geq A\,2\pi f_m \Rightarrow A \leq \frac{\Delta}{2\pi f_m T_s}$$

$$\Rightarrow A \leq \frac{\Delta f_s}{2\pi f_m}$$

Signal Power $= \frac{(\overset{\text{rms value}}{A/\sqrt{2}})^2}{R} = \frac{A^2}{2R}$ (Normalized condition $R=1$)

$$= \frac{A^2}{2} = \frac{1}{2} \times \frac{\Delta^2 f_s^2}{4\pi^2 f_m^2}$$

$$= \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \text{ in DM}$$

SNR in DM:

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \times \frac{3}{\Delta^2} \times \frac{f_s}{f_M}$$

$$= \frac{3 f_s^2}{8\pi^2 f_m^2} \times \frac{f_s}{f_M}$$

$$= \frac{3 f_s^3}{8\pi f_m^2 f_M}$$

f_M - max. freq of LPF

f_m - message freq.

$$\boxed{\therefore (SNR)_{DM} = \frac{3 f_s^3}{8\pi f_m^2 f_M}}$$

while $(SNR)_{PCM} = \frac{3}{2} L^2$

- Independent of step-size.
- Only parameter is frequency.
- ~~(SNR)DM~~ $f_s \uparrow \Rightarrow SNR \uparrow \rightarrow$ good performance
- # DM can be preferred for over-sampled signals. ($f_s > 2f_m$)
  (f_s far greater than $2f_m$)

Disadvantages:

1. Cannot be preferred for low frequencies, under-sampled signals. ($\because f_s < 2f_m \Rightarrow$ bad performance)

Advantage of DM over PCM :

1. Less B.W → B.W = f_m while in PCM B.W = nf_m (more B.W) #3
2. Independent of Δ → SNR
3. Noise power can be tuned.
4. 1 bit required
5. SNR independent of Δ, n, L and dependent on frequency.
   (frequency → very sensitive parameter
   → Many factors affect frequency)

	DM	PCM
Bits	1	n – no. of bits
BW	f_m	nf_m
Limitations	More noise power	less noise power

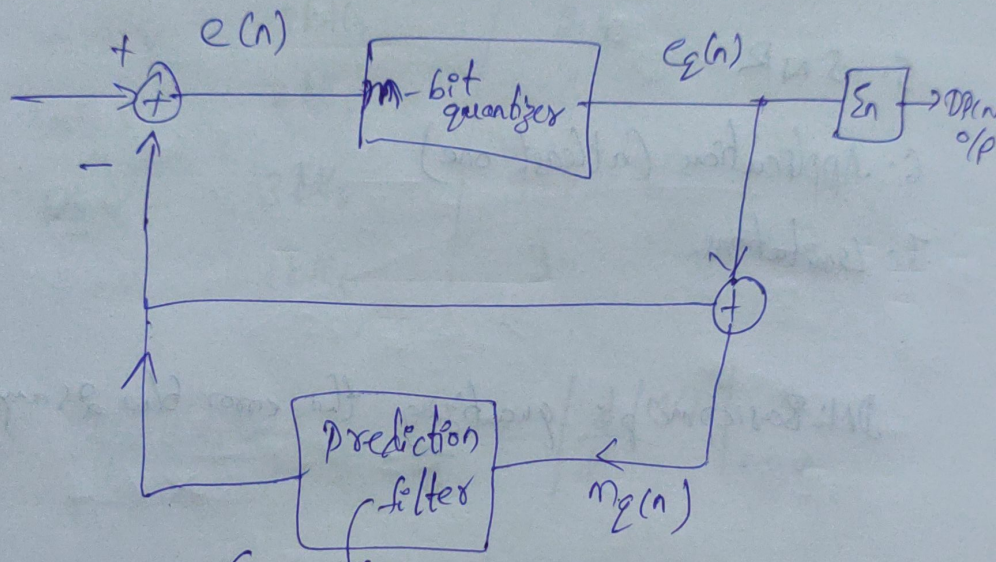
#3 DPCM : Differential Pulse Code Modulation

- 'm' no. of bits : $1 < m < n$

  $(m \simeq n/2)$

- Basic principle same as DM

Block Diagram



(Determines next samples based on past knowledge of samples)

Why DPCM :

- Signal recovery is difficult even in DM (even though its more preffable than PCM)
- There is a chance of losing complete information in DM, since in DM → it requires 1 bit → there is a huge load on single bit.
- Hence, DPCM is required.

Q Compare Pulse Digital Modulation Schemes
(PCM, DM, ADM, DPCM)

ASSIGNMENT
~~Before~~
Mid

1. Definition: Basic principle
2. Block Diagram
for generation & Demodulation
3. B.W (DPCM – intermediate: $f_m < B.W < 2f_m$)
4. Noise power
5. SNR
6. Applications (atleast one)
7. Limitations.

DM: Basic principle: quantizing the error b/w 2 samples

Problems

14/09/2022

1. The i/p to the Linear Delta modulator having stepsize $\Delta = 0.628$ is a sine wave with freq. f_m and the peak amplitude E_m, if sampling freq. $f_s = 40\,kHz$. The combination of sine wave freq. and peak amplitude where the slope overload will takes place is given as (2m adt)

	E_m	f_m	$E_m f_m$
(a)	0.3 V	8 kHz	2.4
(b)	1.5 V	4 kHz	6
(c)	1.5 V	3 kHz	4.5
(d)	3.0 V	1 kHz	3

Sol for Slope overload $\frac{\Delta}{T_s} \le \left| \frac{d}{dt} |m(t)| \right|_{max}$

$m(t) = E_m \sin 2\pi f_m t$

$m'(t) = 2\pi f_m E_m \cos 2\pi f_m t$

$T_s = \frac{1}{f_s}$

$\frac{0.628}{1} \times 40 \times 10^3 \le 2\pi f_m E_m$

$\Rightarrow f_m E_m \ge 3.94 \times 10^3$

Ans. (b), $\because f_m = 4\,kHz > 3\,kHz$

(28) The min. Δ required for a DM operating at 32 k samples per sec. to track the signal $x(t)$, where $x(t) = 125t\{u(t) - u(t-1)\}$

$$+ (120 - 125t)\{u(t-1) - u(t-2)\}$$

so that the slope overload is avoided would be

(a) 2^{-10} (b) 2^{-6} (c) 2^{-8} (d) 2^{-4}

Sol: $f_s = 32 \times 2^{10}$

To avoid slope overload: $\frac{\Delta}{T_s} \geq \left|\frac{d}{dt}(m(t))\right|_{max}$

32k $\Rightarrow$ in Bit

$\Rightarrow 32 \times 1024$

$\Rightarrow 2^5 \times 2^{10}$

$\Rightarrow \frac{\Delta}{T_s} \geq 125$

$\Rightarrow \Delta \geq 125 \times \frac{1}{32 \times 1024}$

$\Delta \geq \frac{125}{2^{15}}$

$\Delta \geq 2^{-8}$

---

(3) Consider a sinusoidal signal $m(t)$ equal to $A\cos\omega_m t$ applied to a delta modulator with step size 'Δ', then show that, the slope overload distortion will occur if

$$A > \frac{\Delta}{\omega_m T_s} = \frac{\Delta}{2\pi}\left(\frac{f_s}{f_m}\right)$$

Sol Condition. $\frac{\Delta}{T_s} \leq \left| \frac{d}{dt}(m(t)) \right|_{max}$

To occur slope overload

$$\frac{\Delta}{T_s} \leq A\omega_m$$

$$\Rightarrow \quad A > \frac{\Delta}{T_s \omega_m} = \frac{\Delta}{2\pi f_m} \times \frac{f_s}{1}$$

$$\Rightarrow \quad \boxed{A > \frac{\Delta}{T_s \omega_m} = \frac{\Delta}{2\pi}\left(\frac{f_s}{f_m}\right)}$$

---

4 Q For a sinusoidal signal $m(t) = A\cos\omega_m t$ then show that max. o/p signal power to qn. noise ratio in DM system under the assumption of no slope over load is given by ?

Sol: $$(SNR)_{DM} = \frac{3 f_s^3}{8\pi^2 f_m^2 f_M}$$

. Determine the o/p (SNR) in a DM system for 1 kHz sinusoidal sampled at 32 kHz without slope overload followed by 4 kHz post reconstruction LPF.

Sol $f_m = 1\,kHz$ $f_s = 32\,kHz$, $f_M = 4\,kHz$

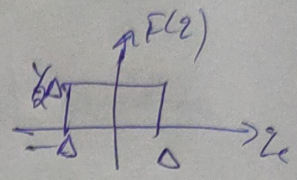
$$(SNR)_{DM} = \frac{3(32k)^3}{8\pi^2 (1k)^2 (4k)} = \frac{3 \times 32^3}{8\pi^2 \times 4}$$

$$= 311.258$$

$\rightarrow (SNR)_{DM} = 311.258$

$(SNR)_{in(dB)} = 24.93$

<u>Derving</u>: $(SNR)_{DM} = \dfrac{3 f_s^3}{8\pi^2 f_m^2 f_M}$



<u>Noise Power</u>:

$$\sigma_N^2 = \int_{-\Delta}^{\Delta} F(e) \cdot e^2 \, de$$

$$= \frac{1}{2\Delta} \times \frac{e^3}{3} \Big|_{-\Delta}^{\Delta} = \frac{\Delta^3}{3}$$

$\therefore \sigma_N^2 = \dfrac{\Delta^3}{3}$; to tune nois power $\Rightarrow$ $\boxed{\sigma_N^2 = \dfrac{\Delta^3}{3} \times \dfrac{f_M}{f_s}}$

<u>Signal Power</u>: <u>Condition</u>: $\dfrac{\Delta}{T_s} \geq \left| \dfrac{d}{dt} m(t) \right|_{max}$ $\left(\because m(t) = A\cos 2\pi f_m t\right)$

$$\Rightarrow \frac{\Delta}{T_s} \geq A\, 2\pi f_m$$

$$\Rightarrow A \leq \frac{\Delta}{2\pi f_m T_s} = \frac{\Delta f_s}{2\pi f_m}$$

Signal Power $= \dfrac{(A/\sqrt{2})^2}{R} = \dfrac{A^2}{2} = \dfrac{1}{2} \times \dfrac{\Delta^2 f_s^2}{(2\pi f_m)^2}$ $(\because R = 1)$

$$= \Delta^2 f_s^2 / 8\pi^2 f_m^2$$

$\therefore SNR = \dfrac{\text{Signal Power}}{\text{Noise Power}} = \dfrac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \times \dfrac{3}{\Delta^2} \times \dfrac{f_s}{f_M} = \dfrac{3 f_s^3}{8\pi^2 f_m^2 f_M}$

Hence Derived

5Q A DM system is designed to operate 3 times the Nyquist rate for a signal with 3kHz B.W. The quantising step size is 250mV.

(i) Determine max. amplitude of a 1kHz input sinusoid for which the Delta Modulator does not show slope overload.

(ii) Determine the post filter o/p signal to qn. noise ratio

Sol (i) $\Delta = 250mV$, $f_m = 3kHz$, ~~$f_s = 2f_m = 6kHz$~~

$$f_s \geq 3 \times N.R$$
$$f_s = 3 \times 2f_m$$
$$= 3 \times 2 \times 3 \times 10^3$$
$$= 18 \times 10^3$$

$$\frac{\Delta}{T_s} \geq A_m \omega_m$$

$$\Rightarrow A_m < \frac{\Delta}{T_s} \times \frac{1}{2\pi f_m}$$

$$\Rightarrow A_m < \frac{250 \times 10^{-3}}{2\pi \times 3 \times 10^3} \times 18 \times 10^3$$

$$\Rightarrow \boxed{A_m < 7.16mV}$$

16/09/2022

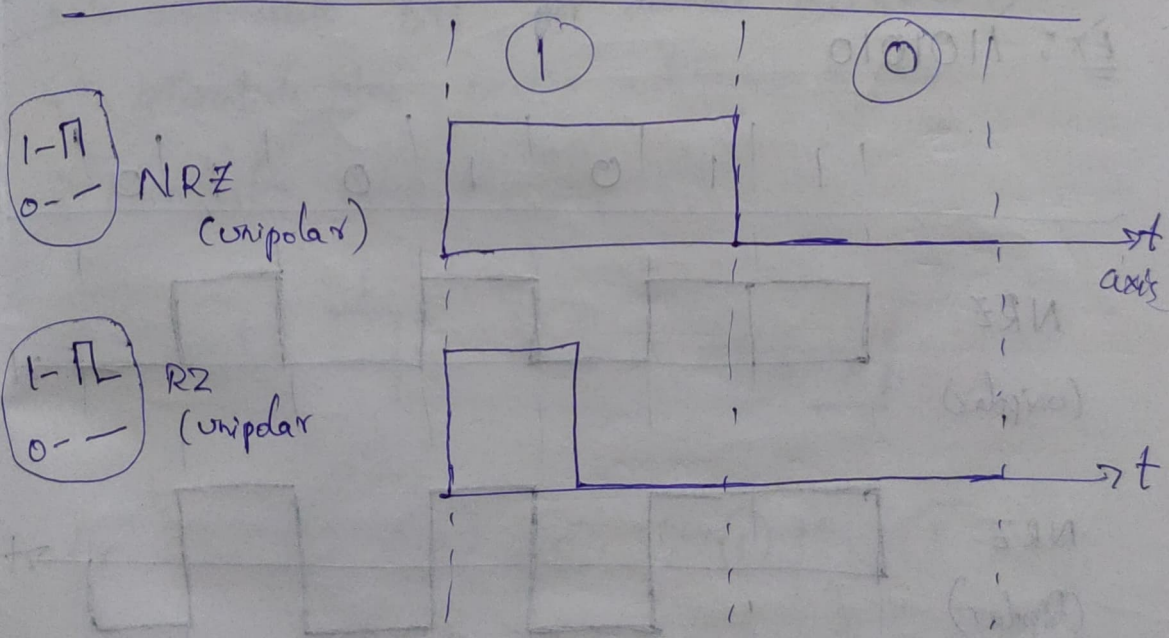
Line Coding:

- Process of representing binary data (modulated data - PCM or DM) in terms of pulses (waveforms) is called Line Coding.

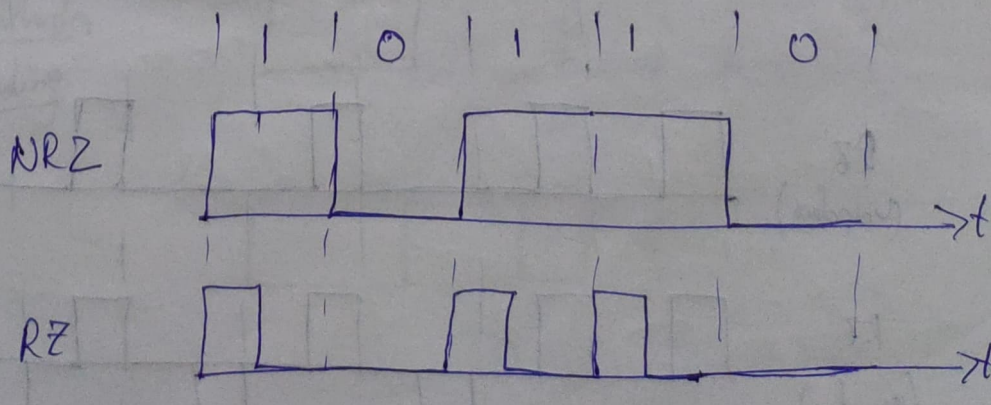
Types:

(2) RZ – Return to zero → Unipolar / Bipolar

(1) NRZ – Non return to zero (never comes to zero) → Unipolar / Bipolar

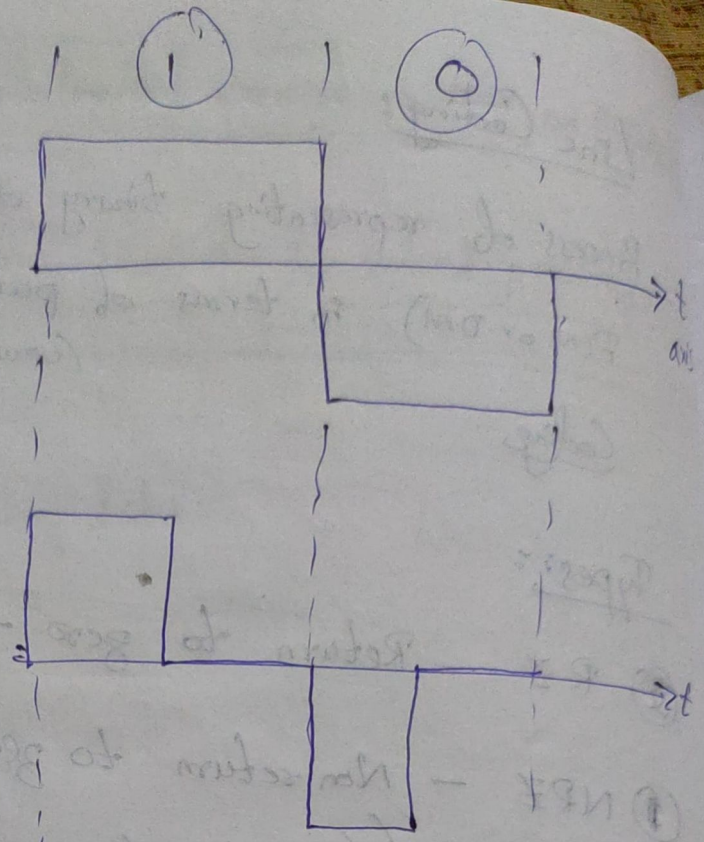


Ex.

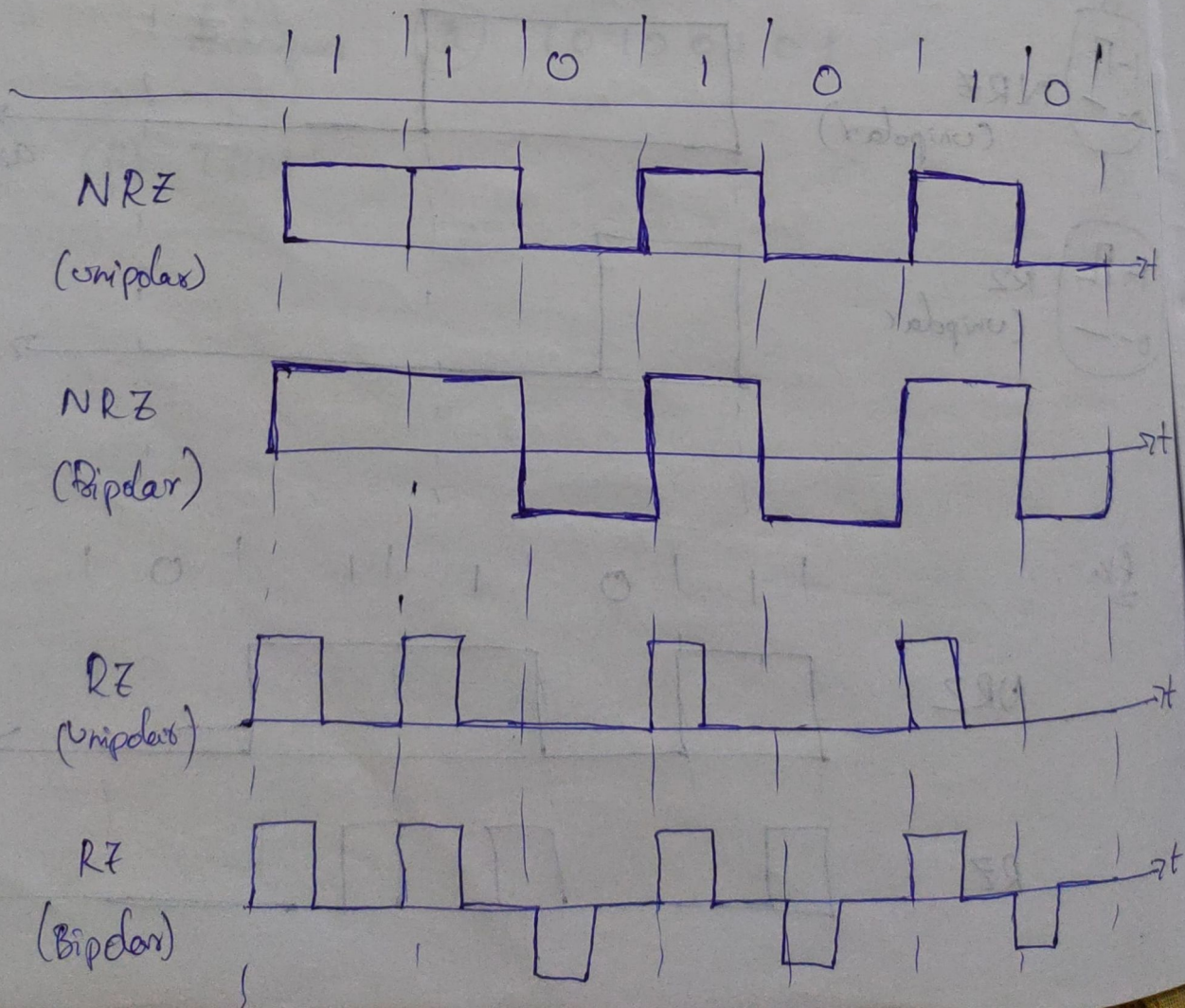


1 — ⊓ NRZ
0 — ⊔ (Bipolar)

1 — ⊓ RZ
0 — ⊔ (Bipolar)



Ex: 1101010



Special cases:

1. AMI → A
(RZ) for
(or) (NRZ) both

RZ

AMI:

↳ invert

• To different

• To differentia

2. Manchester

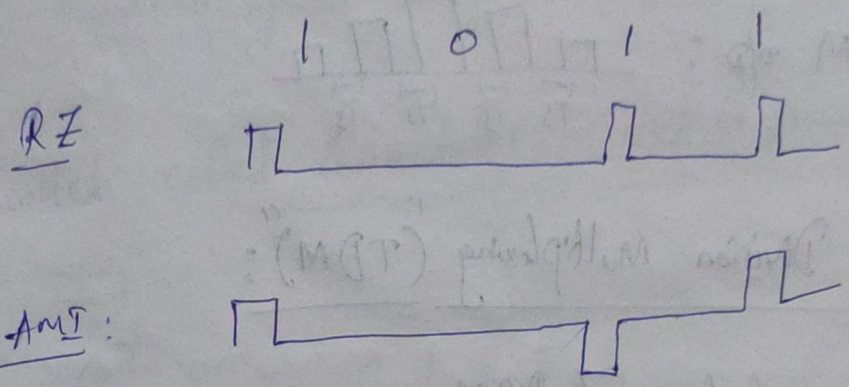
1

Ex

Manchester
Coding

Special cases:

1. AMI → Alternative Mark Inversion
   (RZ) for
   (or) (NRZ) both

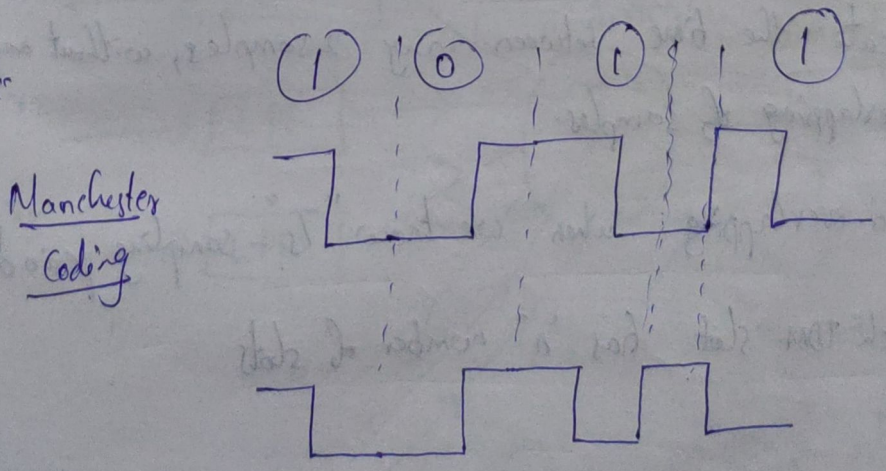


↳ invert for Alternative one's. (∵ 1's are more)

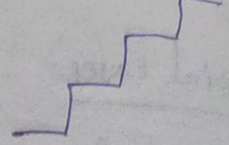
- To differentiate bits for dominant bits (0 or 1)
- To differentiate phase, creating a difference in phase.

2. Manchester coding (or Split-phase)

Ex:

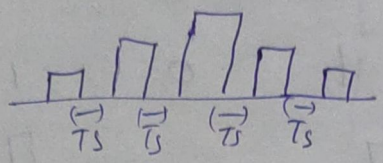


- Staircase : Quantization o/p



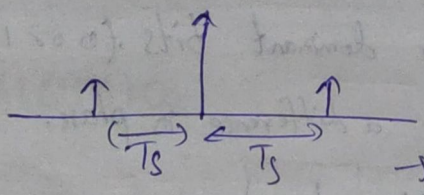
- ↑↑↑ - Sampled o/p

- PAM o/p :

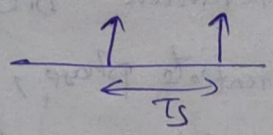


Time Division Multiplexing "(TDM)":

- One application of PAM.

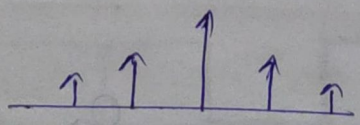


→ (1)



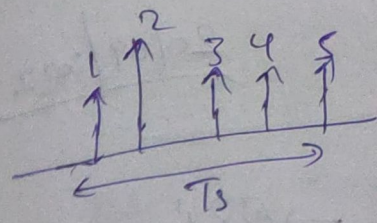
→ (2)

(1) + (2)



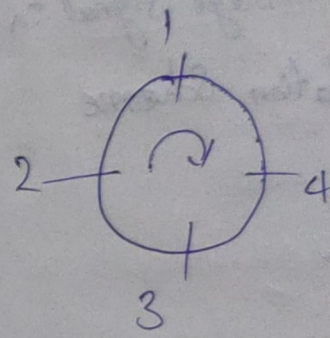
⇒ without any overlapping of samples

- Allocate the time between any 2 samples, without any overlapping of samples.
- No overlapping when we know "Ts" - sampling period
- Each TDM slot has 'n' number of slots



Ex: Broadcast sign

- Switch called



- The process o

  - Commu

Signals

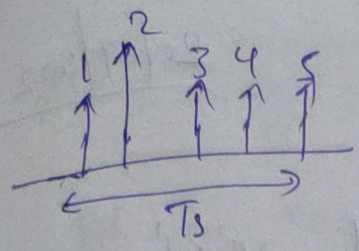
x_1 ⟶

x_2 ⟶

x_3 ⟶

⋮

x_n ⟶

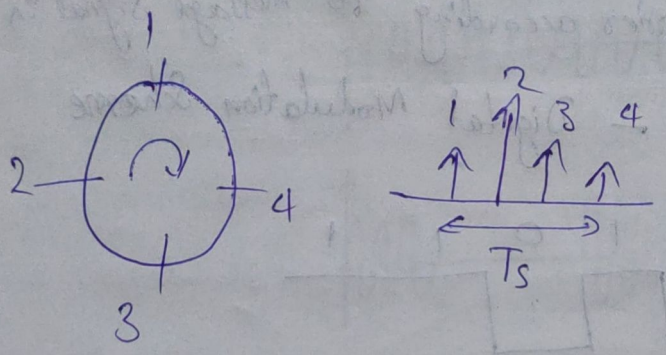


$\boxed{\frac{T_s}{n}}$ – space between each sample

(n – no. of samples)

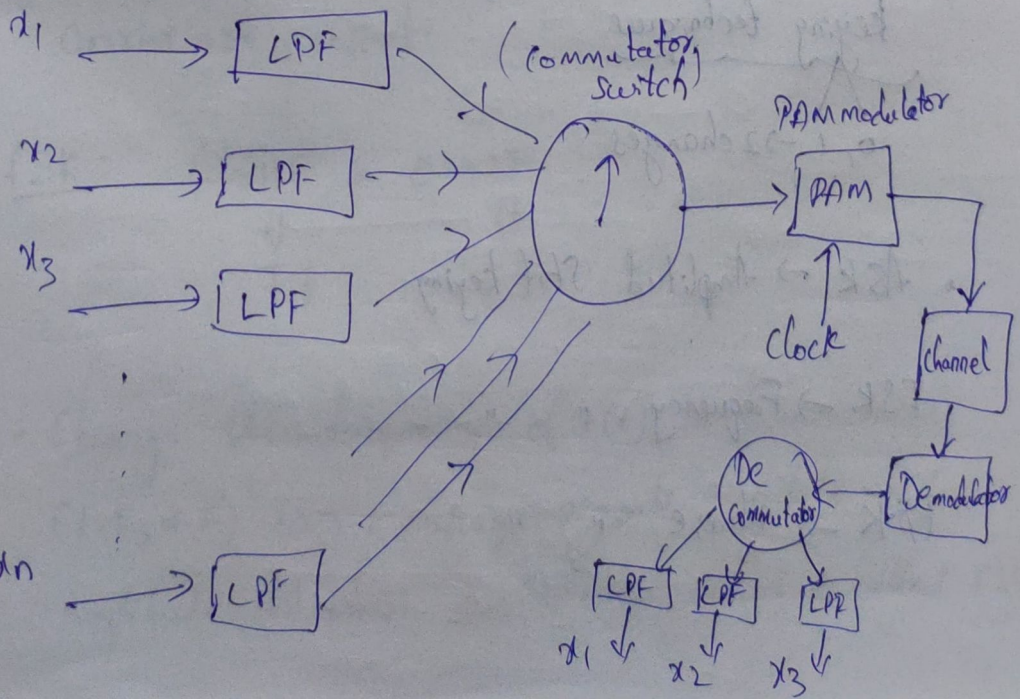
Ex: Broadcast signals are TDM signals.

∴ Switch called Commutator – used to process TDM.



- The process of interleaving the samples in TDM frame – Commutator work.

Signals

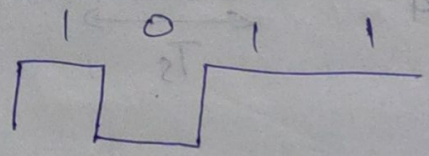


Digital Modulation Schemes :

20/09/2022

- The message signal $m(t)$ is digital → 0's & 1's
- The carrier signal $c(t)$ is CT (or) Analog
- Use line code to represent 0's & 1's interms of pulses.
- Change the carrier according to message signal :
  two instances – Digital Modulation Scheme

- $m(t)$ →



$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

–

Another name for Digital Modulation Scheme is keying techniques.

0, 1 → 2 changes

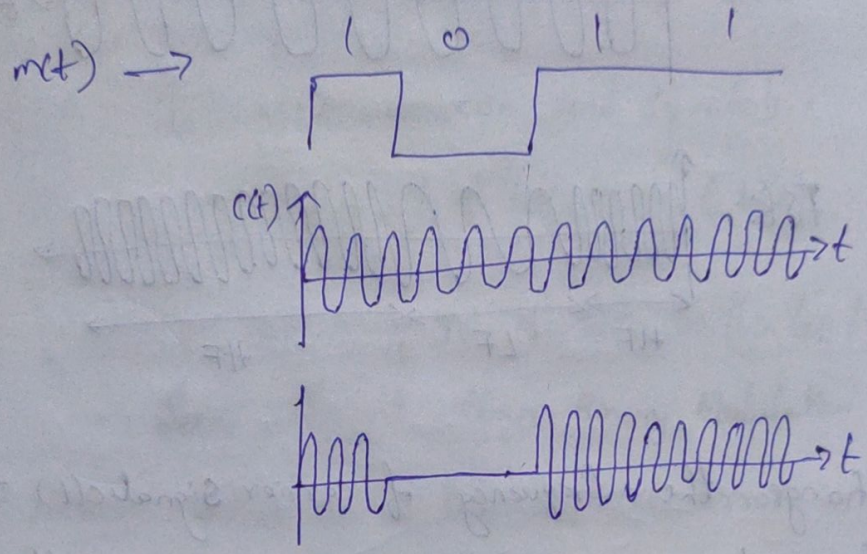
- ASK → Amplitud Shif keying

FSK → Frequency " "

PSK → Phase " "

- ASK : Change in amplitude of carrier key in 2 instances

Bit '1' – Max. amplitude. ⇒ Signal ON
Bit '0' – Min. amplitude. ⇒ Signal OFF
} ON-OFF Keying

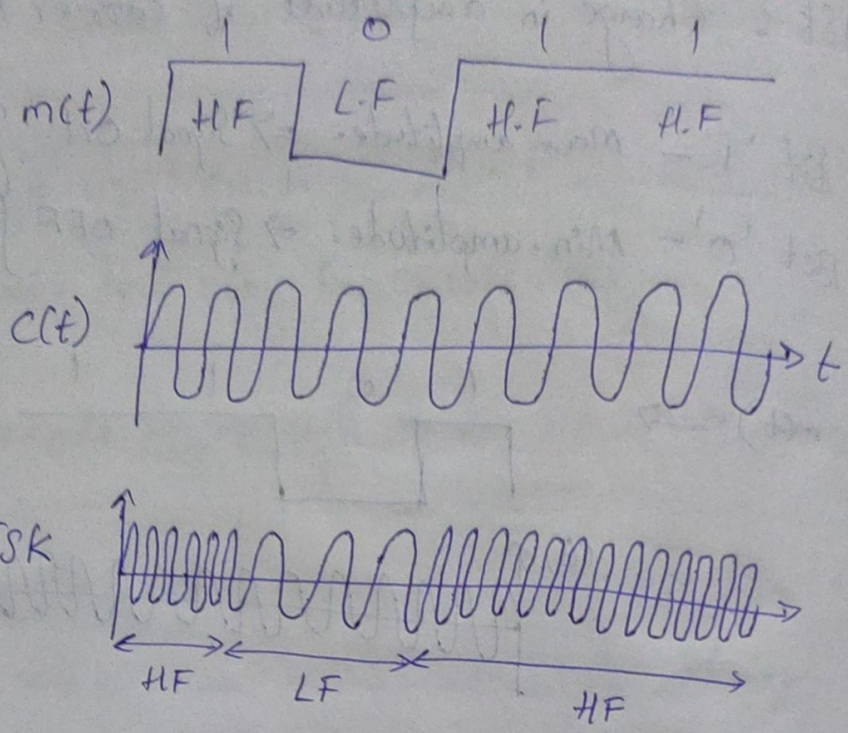


- ASK : Change the amplitude of carrier signal in two instances. (minimum & maximum). (or ON & OFF) w.r.t amplitude of message signal, where frequency and phase of carrier are constant.

- FSK : 1 → HF & 0 → LF
⇓ ⇓
T↓ T↑

- Change the frequency of c(t) in two instances (L.F, H.F) w.r.t message signals amplitude, where amplitude and phase of c(t) are constant is called FSK

Ex:



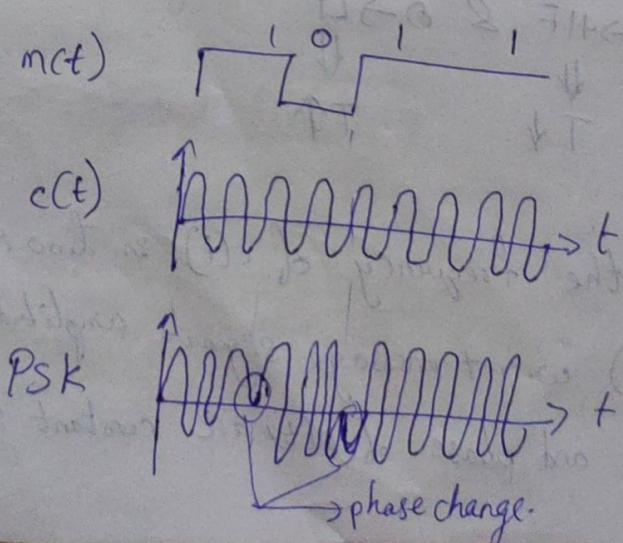
- Change the frequency of carrier signal c(t) in two instances (L.F, H.F) w.r.t message signal's amplitude where Amplitude & phase of c(t) are constant is called FSK.

③ PSK:

'1' → $\phi_c + 0°$

'0' → $\phi_c + 180°$

eg: m(t)



Schemes

- Binary Modulation - deals with 2 instances (1&0)

- MLA - deals with 'M' no. of bits.

→ only 1 bit at a time. possibilities either '0' or '1'

→ Higher B.W required

- MLA: In MLA, we can send symbols.

symbols ⇒ set of bits (M no. of bits)

→ Can send many no. of bits within less time.

→ Less efficient than Binary Modulation Scheme

Limitation: More no. of bits ⇒ more error

- MLA preferred when dealing with high no. of data

- Binary modulation scheme preferred for to send efficient data.

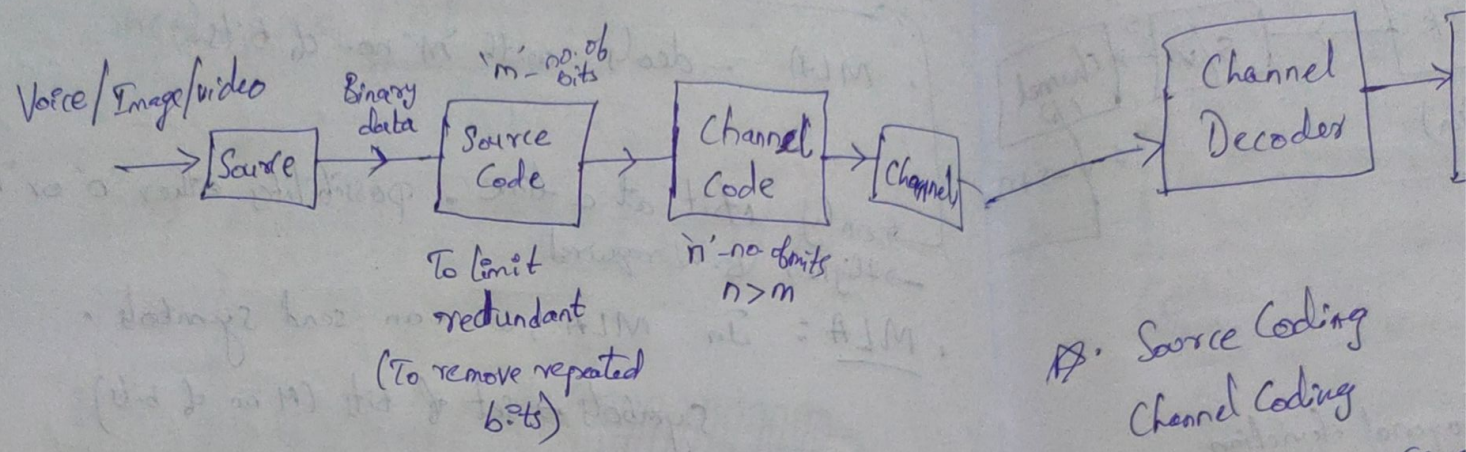
Angle Modulation

1 → $\cos\omega t$

0 → $\cos\omega t + \phi$

$\cos\omega t \propto (A\alpha)^2$

Block Diagram of Digital Communication System :



- Channel code : To identify the error and to correct the error
  - Error detection and correction
  - By adding some no. of bits
  - To get more efficient data.
  - To avoid noise/channel impact. (or) To save data from noise

Drawback : B.W requirement is more (∵ more no of bits)

Channel Coding : The process of adding extra bits and separating it.

Drawback : Dealing with large bits data ; it has more complexity.

- Source code : To make data more efficient, to limit redundancy.
- Channel code : To overcome effect / impact of noise in channel
  ↳ purpose of adding bits (extra bits).

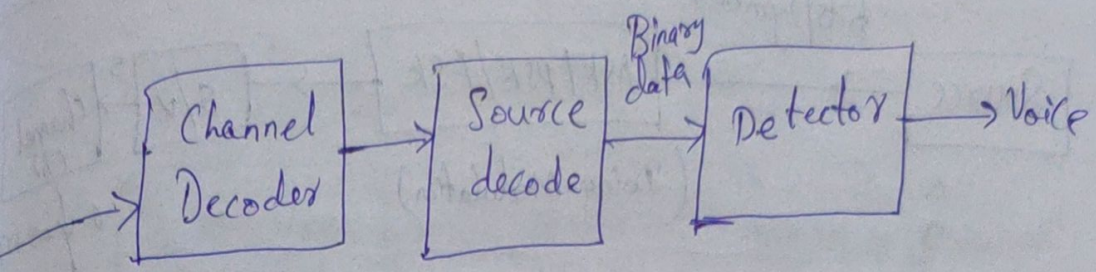
- Source Coding
  Channel Coding
  Detection Theory (& E

- Waveforms are
  vector forms are
- As, for instance
  in channel
  if y-value
- Hence Vector

Signal Space

Constellation

- Distance <
  Distance > 0.
- Representati

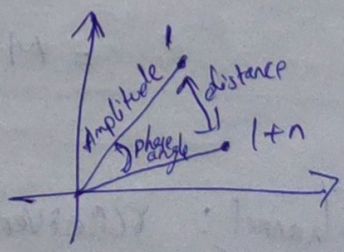


Source Coding
Channel Coding
Detection Theory (& Estimation Theory)

- Waveforms are not preferred in DCS, while vector forms are preferred.
- As, for instance '1' is the input data, if noise added in channel and at Detector: $y = 1+n$
  if y-value $< 0.5 \Rightarrow 0$ - So desired o/p is not observed.
- Hence Vector forms preferred over waveforms.

Signal Space Representation :

Constellation diagram



- Distance < 0.5 = Considered as '0'.
  Distance > 0.5 = Considered as '1'.
- Representation of waveforms in terms of vector is called "Constellation Diagram"

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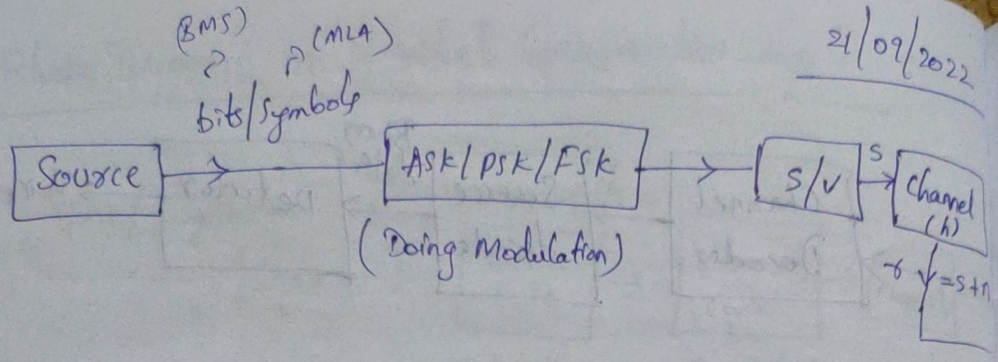


- $S_i(t) = \sum_{j=1}^{N} S_{ij}\, \phi_j(t)$

  $S_i(t)$ → Signal w.r.t 't'

  S_{ij} → Coefficients – vector form of signal → set of values/coefficients

  $\phi_j(t)$ → orthogonal function

Representing

- Any waveforms interms of orthogonal functions, where we donot have any intervals

- S/V ⇒ Signal to vector conversion

- Demodulation → Coherent & Non Coherent

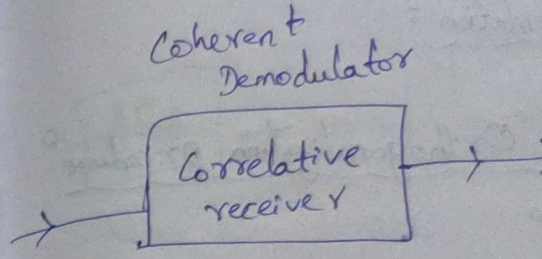
  M+C−C

  = M

- Channel: received signal is message signal + noise

Ex: $S_i(t) = \sum S_{ij}\, \phi_j(t) \times \phi_j(t)$

$\cos\theta \times \cos\theta = \cos^2\theta = \frac{1+\cos 2\theta}{2}$ → excluding as it is high frequency component



(Obtain message signal information)

S_{ij} or $S_i(t)$

(Do some Detection & Estimation)

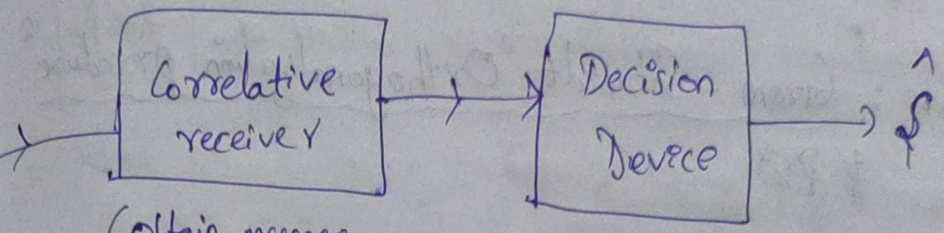
- Detect & estimate t
  give it to De

- Why need of conver

→ As Detection, Estima
easier for Vector f

- Analysis of the ex

Coherent Demodulator



(Obtain message signal information)

S_{ij} or $S_i(t)$

(Do some Detection & Estimation)

Put a Threshold and decide, then produce it

$>0.5 \Rightarrow 1$

$<0.5 \Rightarrow 0$

- Detect & estimate the values & give it to Decision Device

- Why need of converting signal to vector form?

→ As Detection, Estimation & Decision become much easier for Vector form.

- Analysis of the error is easy in case of vector form.

Representation of Signal waveforms into vectors:

(or) Signal Space Representation:

Procedure: "Gram - Schmidt Orthogonalization procedure"

- Dot product of $\vec{a}, \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

  ↓ in continuous waveform $(x(t), y(t))$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} x(t) \cdot y^*(t)\, dt}$$

- For orthogonality; Dot product is "zero"

- Orthonormality: Means signals having unit energy.

$$\int_{-\infty}^{\infty} x(t)\, x^*(t)\, dt = \underbrace{\left| \int_{-\infty}^{\infty} x^2(t)\, dt \right|}_{\text{Energy of } x(t)} = 1$$

Need of Orthogonality for waveforms!

→ Signal must be constructive (same phase) with itself and must be destructive (out of phase) with others ⇒ results no interfaces

- Carrier signal $c(t) = \underbrace{A_c}_{\sqrt{2}\,A_{crms}} \cos(2\pi f_c t + \phi_c)$ $\left(\because A_{crms} = \frac{A_c}{\sqrt{2}}\right)$

$\boxed{\text{ASK}}$

$$\sqrt{2\,A_{c(rms)}^2}$$

$$= \sqrt{2\,\frac{E_b}{T_b}}\cos(2\pi f_c t + \phi_c)$$

T_b - bit duration
$E_b \rightarrow$ energy per bit

$$c(1) \Rightarrow \quad = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \times \cos(2\pi f_c t + \phi_c)$$

$$c(0) \Rightarrow \quad 0 \times \sqrt{\frac{2}{T_b}} \times \cos(2\pi f_c t + \phi_c)$$

Coefficients $\longleftrightarrow$ Orthogonal function $\longleftrightarrow$

$\therefore s_i = \{\sqrt{E_b}, 0\}$ & $\phi_j(t) = \cos(2\pi f_c t + \phi_c)$

- Amplitudes as Coefficients
- ~~Orthogonal function as carrier~~
- Carrier as Orthogonal function

$\rightarrow$ For ASK, PSK $\Rightarrow$ Same orthogonal functions

$\rightarrow$ For FSK $\Rightarrow$ Different orthogonal functions

$\cos(2\pi f_{c_1} t + \phi)$, $\cos(2\pi f_{c_2} t + \phi)$

- $S_i(t) = \sum_{j=1}^{N} S_{ij}\, \phi_j(t)$

- Representing any signal waveform in terms of orthogonal functions in 'N' Number with S_{ij} as coefficients, by multiplying it. — Gram-Schmidt Orthogonalization

- $N \leq M$: How many no. of signal representing are greater than orthogonal functions.

ASK

Ex. (1) $\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$ (constant)

(0) 0

} 2 signals representing with 1 orthogonal function.

PSK

Ex '1' $\leftarrow \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

'0' $\leftarrow -\sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

} 2 signals.

One orthogonal function

FSK

Ex: '1' $\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_{c_1} t$

'0' $\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_{c_2} t$

} f_{c_1} – HF, f_{c_2} – LF

→ FSK : Constant Amplitude, 2 orthogonal function

→ ASK & PSK : 1 orthogonal function.

Determination / Ge

from set of

Binary ASK : 1 ← S

0 ← S_2

A ⇒ $\sqrt{E_b}$;

Constellation Diag

- if error is sm

- if error is La

Binary PSK : (or) BPS

'1' ⇐ $\sqrt{}$

'0' ⇐ $\sqrt{}$

$(-\sqrt{E_b}, +\sqrt{E_b})$

$\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

Determination / Generation of orthogonal functions from set of signals $[S_1, S_2, S_3, \ldots S_m(t)]$

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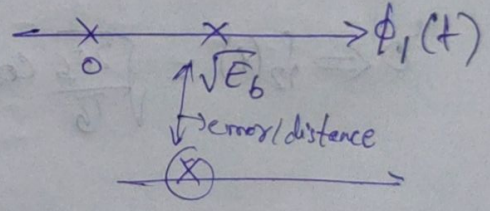
Binary

ASK: $1 \leftarrow S_1(t) = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

$0 \leftarrow S_2(t) = 0 \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

$A \Rightarrow (\sqrt{E_b}, 0)$ & ϕ is constant, $\sqrt{\frac{2}{b}} \cos 2\pi f_c t$

Constellation Diagram of ASK:



- if error is small $\Rightarrow$ approximate to '1'
- if error is Large $\Rightarrow$ approximate to '0'

Binary

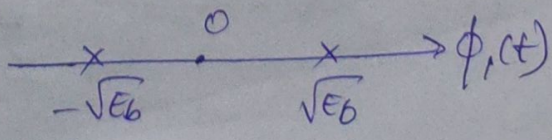
PSK: (or) BPSK

$1 \Leftarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + 0^\circ) \Rightarrow S_1(t) = +\sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$

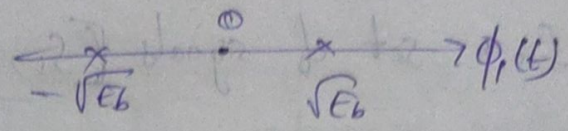
$0 \Leftarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + 180^\circ) \Rightarrow S_2(t) = -\sqrt{E_b} \sqrt{\frac{2}{b}} \cos 2\pi f_c t$

$(-\sqrt{E_b}, +\sqrt{E_b})$

$\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$



Constellation Diagram of BPSK:



- Orthogonal function with coefficients representing interms of vector → Constellation Diagram.

BFSK:

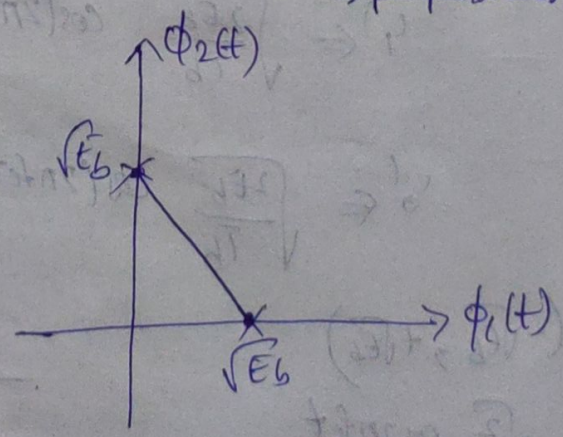
'1' ⇐ $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t$ ⇒ $\sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t$ → $\phi_1(t)$ (Max. frequency)

'0' ⇐ $s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t$ (min frequency) ⇒ $\sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$ → $\phi_2(t)$

Assumption: $f_1 > f_2$
(H.F) (L.F)

- 2 Orthogonal functions & $(\sqrt{E_b}, \sqrt{E_b})$
  ↳ relation b/w two functions is angle b/w them is 90° ⇒ perpendicular

Constellation Diagram of BFSK:



. QPSK ⇒ combination of 2 BPSK
↳ sends 1 bit at a time

↳ Can send 2 bits at a time

⇒ Data Rate increased but paying B.W penality.

→ Quadarature Phase Shift keying. - QPSK

Representing 'M' no. of signals using 'N' orthogonal functions ($N \leq M$):

$$\boxed{S_i(t) = \sum_{j=1}^{N} S_{ij}\, \phi_j(t)}$$

. Coefficient : S_{ij}

S_{ij} ⇒ Projection of $S_i(t)$ on orthogonal function $\phi_j(t)$

$S_i(t)$: ith waveform

$\phi_j(t)$: jth orthogonal function

. S_{ij}
- ↳ orthogonal function
- ↳ Waveform

. S_{ij} = Projection of $S_i(t)$ on $\phi_j(t)$

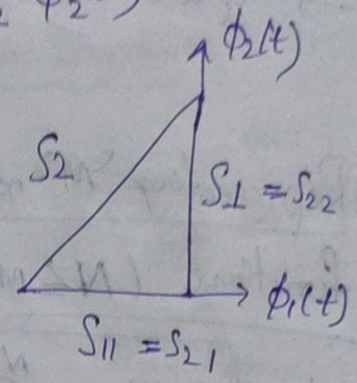
$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}} \; ; \quad \text{Energy} \Rightarrow \int_{-\infty}^{\infty} \phi_1^2(t)\,dt = \frac{\int_{-\infty}^{\infty} S_1^2(t)\,dt}{E_1} = \frac{E_1}{E_1} = 1$$

↳ Representation of Orthonormal Signal

, Orthonormal Signal : $\boxed{\phi_1(t) = \dfrac{S_1(t)}{\sqrt{E_1}}}$

$$\Rightarrow \boxed{S_1(t) = \sqrt{E_1}\ \phi_1(t)}$$

$$S_2(t) = S_{21}\ \phi_1(t) + S_{22}\ \phi_2(t)$$



$$\Rightarrow \phi_2(t) = \frac{S_2(t) - S_{21}\ \phi_1(t)}{S_{22}}$$

S_{22} = Projection of S_2 on ϕ_2

∴ Calculation of $\phi_2(t)$ is difficult as S_{22} is depending $\phi_2(t)$

- $S_{21} = \int_{-\infty}^{\infty} S_2(t)\ \phi_1(t)\ dt$
- $S_{22} = \int_{-\infty}^{\infty} S_2(t)\ \phi_2(t)\ dt$ → Infinite loop of S_{22} & $\phi_2(t)$

- So, new determination : don't consider denominator

$$\boxed{g_2(t) = S_2(t) - S_{21}\ \phi_1(t)}$$

∴ $\phi_2(t) =$

- All orthogonal
other, should

$\Rightarrow \phi_2(t)$ –
$g_2(t) \perp$

Orthogonality :

Proof :

$\int_{-\infty}^{\infty} g_2(t)$

$\Rightarrow \int_{-\infty}^{\infty} g_2(t) \cdot$

$\Rightarrow \int_{-\infty}^{\infty} [g_2(t)$ –

$\Rightarrow \int_{-\infty}^{\infty} S_2(t)$

$\Rightarrow S_{21}$ –

∴ g_2

$$\therefore \phi_2(t) = \frac{g_2(t)}{\sqrt{\int_{-\infty}^{\infty} g_2^2(t)\,dt}} \qquad \text{like } \phi_1(t) = \frac{S_1(t)}{\sqrt{E}}$$

- All orthogonal functions should be $\perp$r to each other, should have orthogonality.

$$\Rightarrow \phi_2(t) \perp \phi_1(t)$$

$$g_2(t) \perp \phi_1(t)$$

Orthogonality: Dot product is zero (in vector form)

Integral of its product is zero (in wave form)

(conjugate)

<u>Proof</u>:

$$\int_{-\infty}^{\infty} g_2(t) \cdot \phi_1^*(t)\,dt$$

$$\Rightarrow \int_{-\infty}^{\infty} g_2(t) \cdot \phi_1(t)\,dt$$

$$\Rightarrow \int_{-\infty}^{\infty} \left[S_2(t) - S_{21}\phi_1(t)\right] \phi_1(t)\,dt$$

$$\Rightarrow \int_{-\infty}^{\infty} S_2(t) \times \phi_1(t)\,dt - S_{21} \underbrace{\int_{-\infty}^{\infty} \phi_1^2(t)\,dt}_{\text{orthonormal}}$$

$$\Rightarrow S_{21} - S_{21}(1) = 0$$

$\therefore g_2(t)$ & $\phi_1(t)$ are Orthogonal to each other

[Hence, proved]

Energy of $g_2(t)$:

$$\int_{-\infty}^{\infty} g_2^2(t)\, dt = \int_{-\infty}^{\infty} \left[s_2(t) - s_{21}\phi_1(t) \right]^2 dt$$

$$= \int_{-\infty}^{\infty} \left[s_2^2(t) - 2 s_2(t)\, s_{21}\, \phi_1(t) + s_{21}^2\, \phi_1^2(t) \right] dt$$

$$= \int_{-\infty}^{\infty} s_2^2(t)\, dt - 2 s_{21} \int_{-\infty}^{\infty} s_2(t)\, \phi_1(t) + \int_{-\infty}^{\infty} s_{21}^2\, \phi_1^2(t)\, dt$$

$$= E_2 - 2 s_{21}^2 + s_{21}^2$$

$$= E_2 - s_{21}^2$$

$$\therefore \phi_2(t) = \frac{g(t)}{\sqrt{\int_{-\infty}^{\infty} g_2^2(t)\, dt}}$$

$$\boxed{\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}}$$

Generalization:

$$\phi_j(t) = \frac{s_j(t) - \sum_{i=1}^{j-1} s_{ij}\, \phi_i(t)}{\sqrt{E_j - \sum_{i=1}^{j-1} s_{ij}}}$$

① $S_i(t) = \sum_{j=1}^{N} S_{ij}\, \phi_j(t)$; $1 \le i \le M$

$0 \le t \le T_b$

$N \le M$

② • $S_{ij} = \int_{-\infty}^{\infty} S_i(t) \times \phi_j(t)\, dt$

③ • $\int_{-\infty}^{\infty} \phi_i(t)\, \phi_j(t)\, dt = 0$; $i \ne j$ Orthogonality

$= 1$; $i = j$ Orthonormality

① Representation of waveform interms of vector form

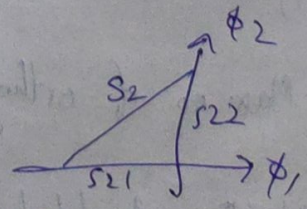
② Generation of coefficients

③ Orthogonality & Orthonormality

} Gram-Schmidt Orthogonalization Equations

$\phi_1(t) = \dfrac{S_1(t)}{\sqrt{E_1}}$; $\phi_2(t) = \dfrac{S_2(t) - S_{21}\phi_1(t)}{\sqrt{E_2 - S_{21}^2}}$

$\because S_2 = S_{21}\phi_1 + S_{22}\phi_2$



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Ly

$$S_3 = S_{31}\phi_1(t) + S_{32}\phi_2(t) + S_{33}\phi_3(t)$$

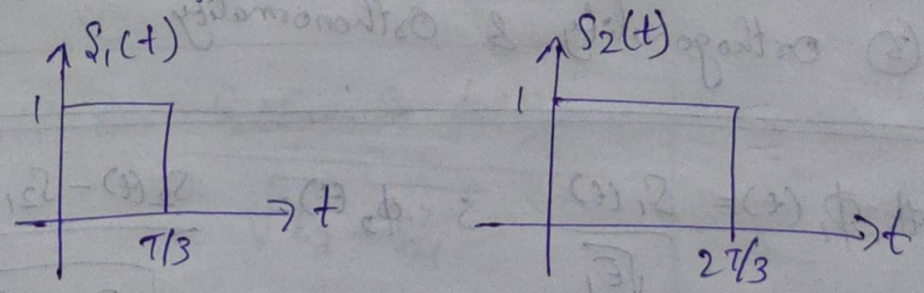
$$\Rightarrow \phi_3(t) = \frac{S_3(t) - S_{31}\phi_1(t) - S_{32}\phi_2(t)}{S_{33}}$$

generalization

$$\boxed{\phi_i(t) = \frac{S_i(t) - \sum_{j=1}^{i-1} S_{ij}\,\phi_j(t)}{\sqrt{E_i - \sum_{j=1}^{i-1} S_{ij}^2}}}$$

Problem

Using Gram-Schmidt orthogonalization procedure, determine Orthogonal functions for the following input waveforms.



Sol

no. of Signals $= M = 2$

- Max. no. of orthogonal function (possibility) $N = 2$

NOTE: Amplitudes of $S_1(t)$ & $S_2(t)$ are same $\Rightarrow$ in FSK

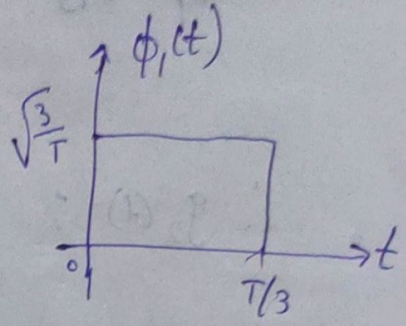
$\Rightarrow N = M \Rightarrow$ 2 Orthogonal functions

Sol W.K.T

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

$$E_1 = \int_{-\infty}^{\infty} s_1^2(t)\, dt = \int_0^{T/3} (1)^2\, dt = \frac{T}{3}$$

$$\boxed{\therefore \phi_1(t) = \sqrt{\frac{3}{T}}\, S_1(t)}$$



$$\phi_2(t) = \frac{S_2(t) - S_{21}\,\phi_1(t)}{\sqrt{E_2 - S_{21}^2}}$$

Assume, $\boxed{g_2(t) = S_2(t) - S_{21}\,\phi_1(t)}$

$$S_{21} = \int_{-\infty}^{\infty} S_2(t) \cdot \phi_1(t)\, dt$$

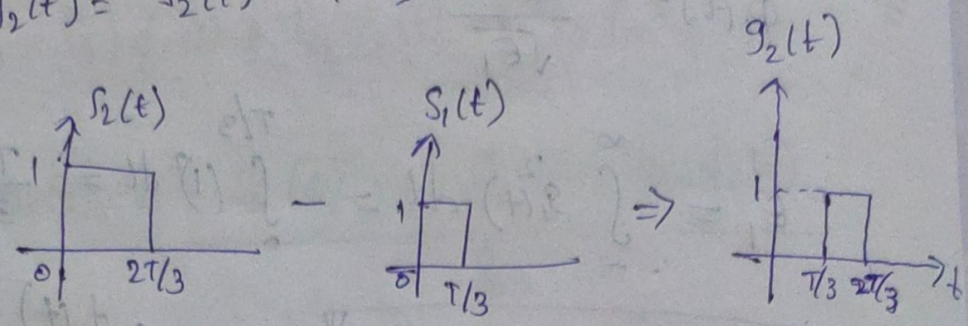
$$= \int_0^{T/3} (1)\left(\sqrt{\frac{3}{T}}\right) dt$$

$$= \sqrt{\frac{3}{T}} \times \frac{T}{3} = \sqrt{\frac{T}{3}}$$

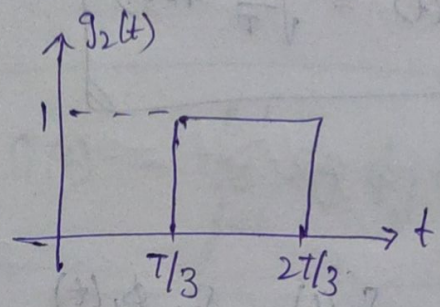
$$\therefore S_{21} = \sqrt{\frac{T}{3}}$$

$$\Rightarrow S_{21}\,\phi_1(t) = \sqrt{\frac{T}{3}} \times \sqrt{\frac{3}{T}}\, S_1(t) = S_1(t)$$

$\therefore g_2(t) = s_2(t) - s_1(t)$



$\therefore g_2(t)$:

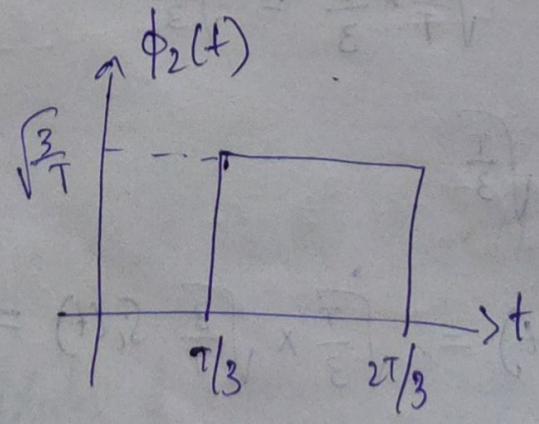


$$\text{Energy of } g_2(t) = \int_{T/3}^{2T/3} 1^2\,dt = \frac{2T}{3} - \frac{T}{3} = \frac{T}{3}$$

$$\therefore \phi_2(t) = \frac{s_2(t) - s_1(t)}{\sqrt{\frac{T}{3}}} = \sqrt{\frac{3}{T}}\left(s_2(t) - s_1(t)\right)$$

$$\left(\because \phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2(t)}}}\right) \quad \boxed{\therefore \phi_2(t) = \sqrt{\frac{3}{T}}\, g_2(t)}$$

$\phi_2(t)$:



Generalization

$\phi_i(t) =$

$g_i(t$

Correlation

- $s_i(t) =$

waveform

$(T \times$

s_{i1} —

s_{i2} —

s_{i3} —

s_{iN} —

- $\boxed{\int s\,dt}$

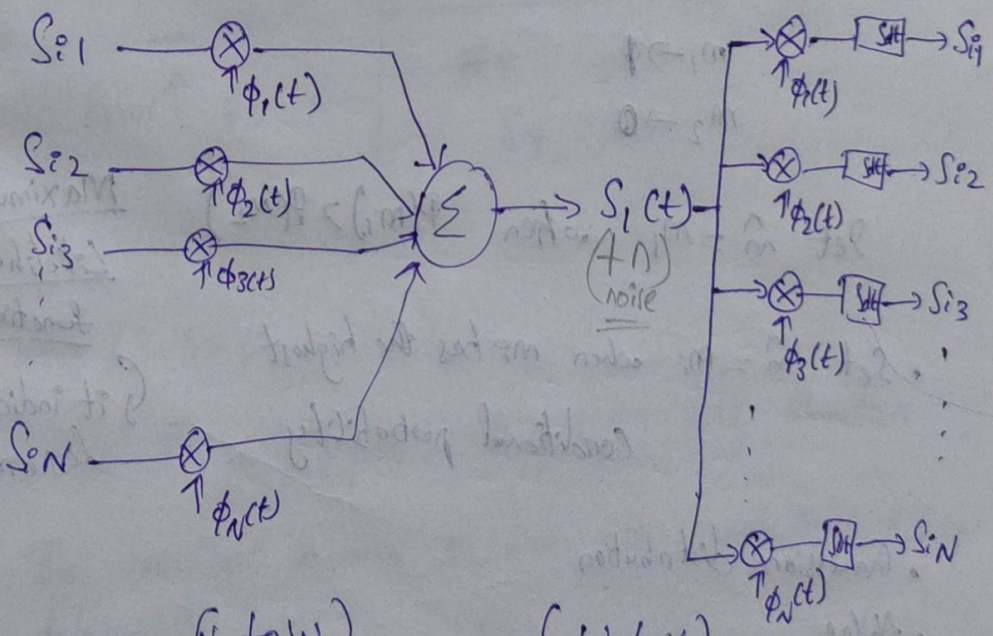
Generalization :

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_{-\infty}^{\infty} g_i^2(t)\, dt}} \; ; \text{ where}$$

$$g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij}\, \phi_j(t)$$

Correlation Receivers : Coherent demodulate type

- $S_i(t) = \sum_{j=1}^{N} S_{ij}\, \phi_j(t)$ — waveform to vector (Tx/Rx)
- $S_{ij} = \int_{-\infty}^{\infty} S_i(t)\, \phi_j(t)\, dt$ — v (coefficients of vector) to w (waveform) (Rx/Tx)



(N to W) (W to v)

- $\boxed{\text{Sdt}}$ – Integrator
  nothing but L.P.F

$\boxed{\text{Matched filter}}$

Limitation :

- Original $S_i(t)$ is not received at receiver, it is $S_i(t)$ + noise

$$x = \underset{\text{Random signal}}{\underset{D}{S_i}} + \underset{\text{Deterministic}}{\underset{R}{\eta}}$$

- Conditional Probability : $P(X/Y)$: determining uncertain X by knowing deterministic Y.

- $P(x/S_i)$ $\{0, \sqrt{E_b}\}$ $\{-\sqrt{E_b}, \sqrt{E_b}\}$ $\{\sqrt{E_b}, \sqrt{E_b}\}$

$m_1 \rightarrow 1$

$m_2 \rightarrow 0$

Set $\hat{m} = m_1$ when $P(m_1) > P(m_2)$ — Maximum Likelihood function

- Set $\hat{m} = m_i$ when m_i has the highest conditional probability — it indicates less error

- Gaussian distribution
- AWGN

- White – variance is same irrespective of change in frequency.

- $\frac{1}{\sqrt{2\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\boxed{\text{ML detector}}$

(Maximum Likelihood detector)

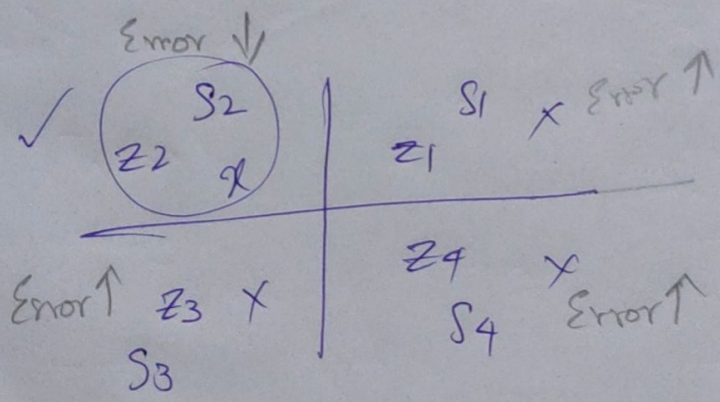
- To find probability; apply log.

$$-\frac{(x-S_i)^2}{2\sigma^2}$$; to get high probability

$(x-S_i)^2$ should be small. ⇒ less error ⇒ max. probability ⇒ ~~max~~

- $\underset{r}{X} = \underset{D}{S_i} + \underset{r}{W}$; W_noise ⇒ mean = 0

⇒ Variance $= \frac{N_0}{2}$



- <u>ML detector</u>:
- Conditional probability nothing but ML function
- The resultant becomes random even signal transmitted is deterministic.

→ Compare the transmitted signal (S_i) with the received signal (x) in vector space using likelihood function, given as ⟶

ML detector :

(σ) $1/\sqrt{2\pi N_0}$

$$f(x/s_i) = \frac{1}{\sqrt{\pi \sigma_n^2}} e^{-\frac{(x-s_i)^2}{2\sigma^2}}$$

(N_0) Noise Power

$\sigma_n^2 \rightarrow$ Variance

$\rightarrow$ Likelihood function.

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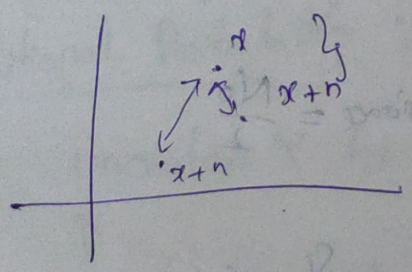
- Less error $\rightarrow$ More probability
- More error $\rightarrow$ Less probability



Matched filter : (O/P SNR Maximizer)

- $\underbrace{g(t)}_{\text{original signal}} + \underbrace{w(t)}_{\text{Noise signal}} = x(t)$

Filter's impulse response matched with incoming signal with a delay of 'T' sec and Time reversal

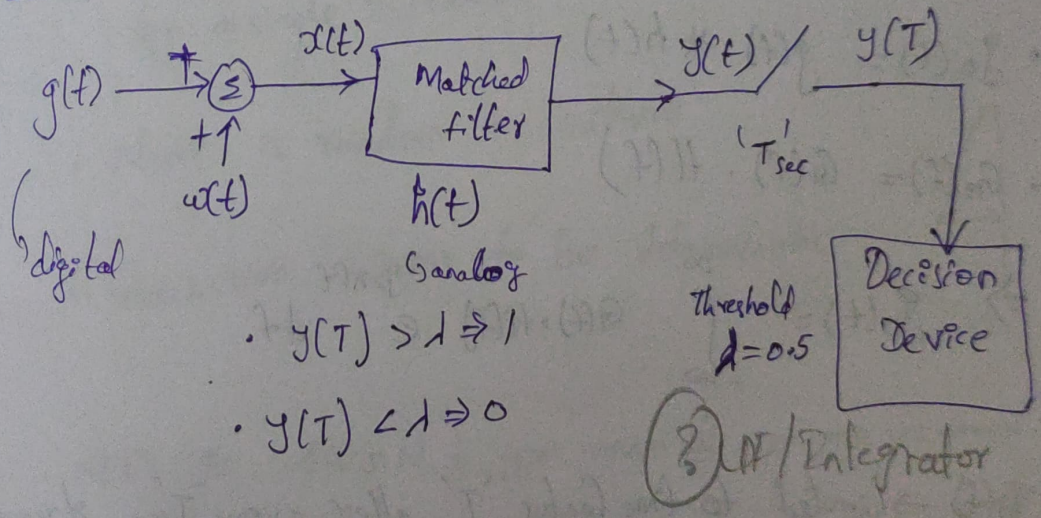
- obtaining $g(t)$ from $x(t)$

SNR is doubled

- Reducing impact due to $w(t)$
- Possible, when $g(t) >> w(t)$
- $\Rightarrow$ $SNR = \frac{S}{N}$ should be high.

- A filter which maximizes SNR then we can easily obtain $g(t)$ from $x(t)$.
- Matched filter also called as Optimum filter.
- Purpose of Matched filter : o/p SNR Maximization.
- Helpful in detecting signal without error.
- Detecting Capability of original signal.

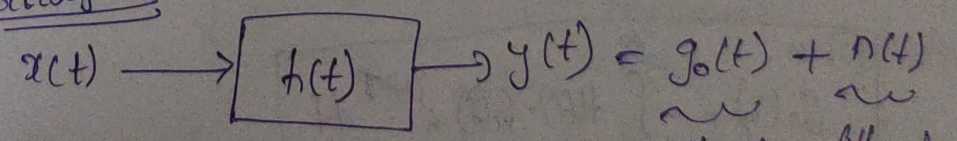
Impulse response of Matched filter :



- $y(T) > \lambda \Rightarrow 1$
- $y(T) < \lambda \Rightarrow 0$

- $x(t) \rightarrow$ original signal + noise i.e. $x(t) = g(t) + w(t)$

Blockdiagram :



(Transfer function of)

filtered version of $g(t)$ — filtered version of $n(t)$

- $g_0(t) = T[g(t)]$
- $n(t) = T[w(t)]$

- $SNR = \dfrac{\text{Signal Power}}{\text{Noise Power}} = \dfrac{|g_o(t)|^2}{E[n^2(t)]} \rightarrow (1)$

- $n(t)$ ~~has~~ zero mean & non-zero variance.

- To maximize SNR: $|g_o(t)|^2 > |E[n^2(t)]|$

- $g_o(t) = \int_{-\infty}^{\infty} G_o(f)\, e^{j2\pi ft}\, df$

- $g_o(t) = g(t) * h(t)$

- $G_o(f) = G(f) \cdot H(f)$

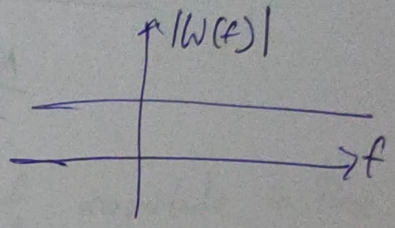
$\Rightarrow g_o(t) = \int_{-\infty}^{\infty} G(f) \cdot H(f) \cdot e^{j2\pi ft}\, df$

- $g_o(t) \rightarrow$ Limited to time factor 'T' after every T sec, it repeat

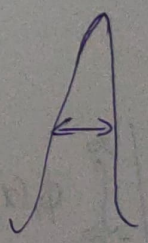
$\Rightarrow g_o(T) = \int_{-\infty}^{\infty} H(f) \cdot G(f)\, e^{j2\pi fT}\, df$

$$\boxed{\therefore g_o(T) = \int_{-\infty}^{\infty} H(f) \cdot G(f)\, e^{j2\pi fT}\, df}$$

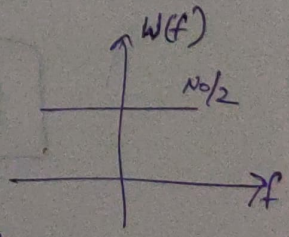
- **AWGN : Additive White Gaussian Noise**
- Ideal noise, in real-time not possible.
- White indicates constant, uniform, all
- At any freq., magnitude is same - White noise



- Gaussian Distribution : as noise cannot be determined by a single instance.
- Noise is random in nature
- Gaussian seems to be deterministic



- n(t) is AWGN : zero mean & non-zero variance
  has ↳ Constant Power Spectral density.



(?) $S_n(f) = \sqrt{\frac{N_0}{2} \times (H(f))^2}$ (in freq. domain : multiplication)

• $E[n^2(t)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df$

$$\therefore SNR = \frac{|g_0(T)|^2}{E[n^2(t)]}$$

$$= \frac{\left| \int_{-\infty}^{\infty} H(f)\, G(f)\, e^{j2\pi f T}\, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

- Schwarz's inequality: $\phi_1(x)$ & $\phi_2(x)$

$$\left| \int_{-\infty}^{\infty} \phi_1(x) \cdot \phi_2(x)\, dx \right|^2 \leq \left| \int_{-\infty}^{\infty} \phi_1(x)\, dx \right|^2 \left| \int_{-\infty}^{\infty} \phi_2 \right.$$

$$\# \left| \int_{-\infty}^{\infty} \phi_1(x) \cdot \phi_2(x)\, dx \right|^2 \leq \left| \int_{-\infty}^{\infty} \phi_1(x)\, dx \right|^2 \cdot \left| \int_{-\infty}^{\infty} \phi_2(x)\, dx \right|^2$$

Condition: There should be a relation between $\phi_1(x)$ & $\phi_2(x)$

$$\boxed{\phi_1(x) = k\, \phi_2^*(x)}$$

- Assume: $\phi_1(x)$ is $H(f)$

$\phi_2(x)$ is $G(f)\, e^{j2\pi f T}$

$$\Rightarrow SNR \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |G(f)|^2 df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$\leq \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df$$

. Parsvelle's Theorem: square of magnitude in Time & frequency domain, both represent energy

$$\# \; |g(t)|^2 = |G(f)|^2 = E$$

$$\therefore SNR \leq \frac{2E}{N_0}$$

$$\boxed{\therefore SNR = \frac{2E}{N_0}}$$

<u>Condition</u>: $\phi_1(x) = k\, \phi_2^*(x)$

$$H(f) = K\, G^*(f)\, e^{-j2\pi f T}$$

$$\Rightarrow H(f) = K\, G(-f)\, e^{-j2\pi f T}$$

<u>W.K.T</u>:

$$\Rightarrow h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi f t}\, df$$

$$\Rightarrow h(t) = \int_{-\infty}^{\infty} H(f) \cdot e^{j2\pi ft} \, df$$

$$= \int_{-\infty}^{\infty} k\, G(-f)\, e^{-j2\pi fT} \cdot e^{j2\pi ft} \, df$$

$$= \int_{-\infty}^{\infty} k\, G(-f)\, e^{-j2\pi f(T-t)} \, df$$

$$= \int_{-\infty}^{\infty} k\, G(f) \cdot e^{j2\pi f(T-t)} \, df$$

$$= k \int_{-\infty}^{\infty} G(f) \cdot e^{j2\pi f(T-t)} \, df$$

$$\# \quad \boxed{h(t) = k\, g(T-t)}$$

- If $h(t)$ matching the input signal, SNR is doubled.

- <u>Matched filter</u> : A filter whose impulse response is matching to incoming signal / information signal which can maximize the o/p SNR is called <u>Matched filter</u>

<u>Properties</u>

- Spectrum of matched filter is same as $\left\{ \begin{array}{l} \text{spectrum of} \\ \text{input signal} \end{array} \right.$ by two factors :
  1. T sec time delay
  2. Time reversal
- SNR is doubled.
-

- Significance of "T" sec:
- Limit Response to T sec
- What if T sec delay is not there

Ex: 1 0 1 1
T sec

- To minimize interference.

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Probability of Error:

1. Representation of each Modulation scheme in mathematical expression
2. Constellation Diagram of ASK, BPSK, BFSK
3. Probability of Error (or) Bit Error Rate (BER)
   Symbol Error rate

- Order of Modulation: no. of bits sending at a time
- M-ary modulation: M no. of bits are sent
- Collection of bits: Symbol
- Symbol error rate (M-ary schemes)
- Bit Error rate for (Binary schemes - ASK, PSK, FSK)
- Quadrature: between Binary & M-ary.
- Set of 2 bits - dibit

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Probability of Errors

	received (or) assumed	
Transmitted bit: 1	0	Error
0	1	

- Transmitted : $S_1(t)$
  Assumed : $S_2(t)$

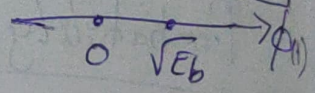
- Probability of assuming 1 as 0 & 0 as 1

ASK :

$$S_1(t) = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

$$S_2(t) = 0$$

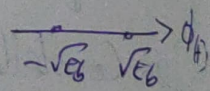
$$\boxed{d_{min} = \sqrt{E_b}}$$



PSK :

$$S_1(t) = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

$$S_2(t) = -\sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$



$$\boxed{d_{min} = 2\sqrt{E_b}}$$

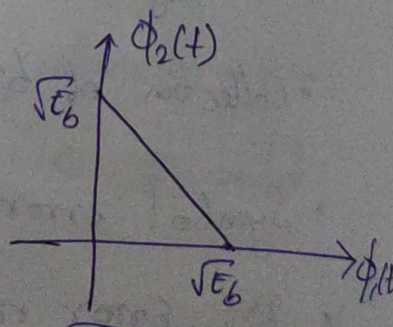
FSK :

$$S_1(t) = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t$$

$$(f_1 > f_2)$$

$$S_2(t) = \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$$

dmin : minimum distance between 2 points in Constellation diagram

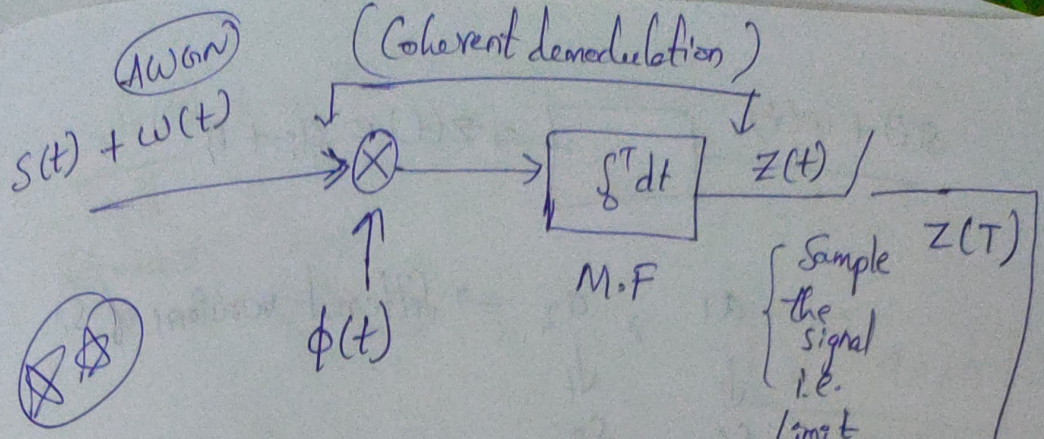


$$\boxed{d_{min} = \sqrt{2E_b}}$$

BPSK generation } $m(t) \longrightarrow \otimes \longrightarrow s(t) = m(t)\phi_1(t)$ BPSK

$\phi_1(t)$

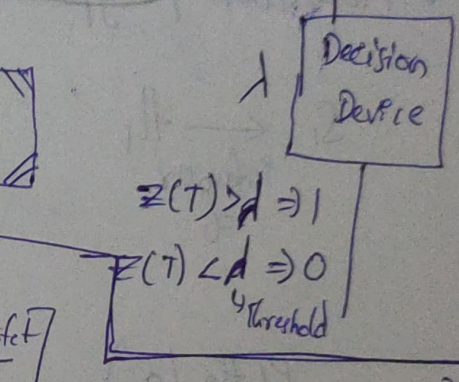
At Transmitting side



- Regenerating same carrier & passing through LPF
  - Coherent demodulation

Receiver circuit

RECEIVING Block diagram



Ex:

- $s_1 \times \underbrace{\phi_1(t)}_{=\cos 2\pi f_c t} \Rightarrow \sqrt{E_b} \times \sqrt{\frac{2}{T_b}} \left[\frac{1+\cos 4\pi f_c t}{2}\right]$

$\left(\cos^2 2\pi f_c t = \frac{1+\cos 4\pi f_c t}{2}\right)$

$\Rightarrow \frac{m(t)}{2} +$

↓ LPF (→ high frequency / carrier component is removed)

$m(t)$

- To separate carrier, we are multiplying the signal with the same carrier.

- If we don't have knowledge of $c(t)$ → square it

  ⇒ Square Law (non-linear device)

$$s(t) + \omega(t) \longrightarrow \boxed{} \longrightarrow z(t) = a_i + n$$

a_1 , a_2 → filtered versions of S_i

$a_1 \downarrow_1 S_1$ $\quad$ $a_2 \downarrow_1 S_2$

- Detection & Estimation Theory
- Proposition / Hypothesis

$S_1 \xleftarrow[\text{infavour of}]{} H_1$ $\qquad$ $H_2 \xrightarrow{\text{infavour of}} S_2$

$P(H_2/S_1)$ → Transmitted S_1 but assumption taken as H_2

$P(H_1/S_2)$ → Transmitted S_2 but assumed as H_1 (S_1)

$$\boxed{\therefore P_e = \underbrace{P(H_2/S_1)\,P(S_1)}_{S_1 \Rightarrow S_2} + \underbrace{P(H_1/S_2)\,P(S_2)}_{S_2 \Rightarrow S_1}}$$

Conditional probability concept

P_e - Probability of Error

- $Z = a_i + n$

S_1: $Z = a_1 + n \Rightarrow n = Z - a_1$

S_2: $Z = a_2 + n \Rightarrow n = Z - a_2$

$$\underbrace{S_i(t)}_{d} + \underbrace{w(t)}_{n} \Rightarrow n$$

deterministic

noise/ random in nature

$$\boxed{Z = a_i + n}$$

. Assume 'Z' has Gaussian distribution as signal added with noise

$$f(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(x-\mu)^2/2\sigma^2}$$

Assume : $(x-\mu) = n_0$

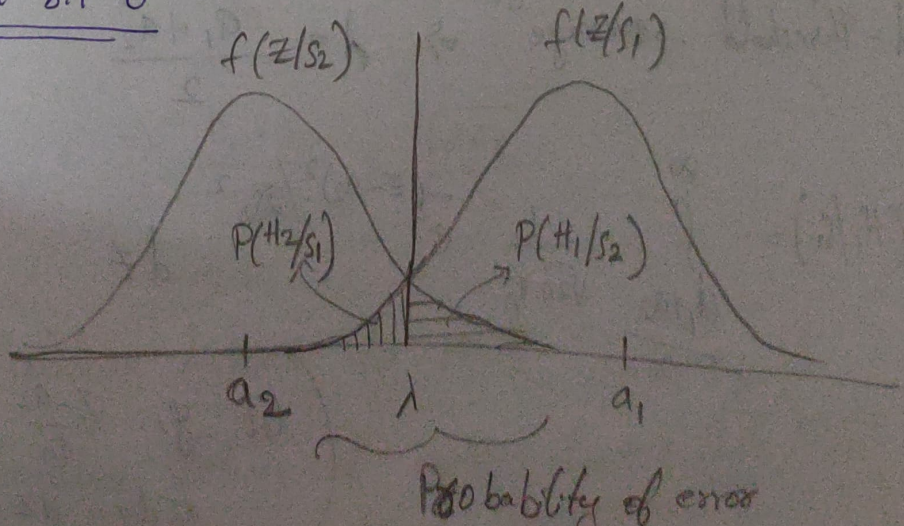
$$f(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-n_0^2/2\sigma^2}$$

S_1. $f(Z/S_1) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-(Z-a_1)^2/2\sigma^2}$ $\quad (\because n = Z - a_1)$

wrt bit '1'

S_2. $f(Z/S_2) = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-(Z-a_2)^2/2\sigma^2}$ $\quad (\because n = Z - a_2)$

wrt bit '0'



- In case of binary case (0,1)

Probability of getting '1' = Probability of getting '0'

- As area under Gaussian distribution is same
- Both $f(Z/s_2)$ & $f(Z/s_1)$ are symmetric

$$\therefore P(H_2/s_1) = P(H_1/s_2) \Rightarrow P_e$$

- $$P(H_2/s_1) = \int_{-\infty}^{\lambda} f(Z/s_1)\, dZ$$
- $$P(H_1/s_2) = \int_{\lambda}^{\infty} f(Z/s_2)\, dZ$$
- Ideal case: $P(H_2/s_1) = P(H_1/s_2)$
- Approximation: Complementary error functions factors

  Q( ) (or) erfn( )

- λ - Threshold (optimum) → average $\Rightarrow \lambda = \dfrac{a_1 + a_2}{2}$
- $$P(H_1/s_2) = \int_{\frac{a_1+a_2}{2}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-(Z-a_2)^2/2\sigma_n^2}\, dZ$$

$$\left(\text{let } y = \frac{Z-a_2}{\sigma_n} \quad \sigma_n\, dy = dZ \right)$$

$$\Rightarrow P(H_1/S_2) = \int_{\frac{a_1+a_2}{2}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} \times e^{-y^2/2} \times \sigma_n \, dy$$

assume $Z = \frac{a_1+a_2}{2}$

$$\cdot\ \lambda_0 = \frac{\frac{a_1+a_2}{2} - a_2}{\sigma_n} = \frac{a_1-a_2}{2\sigma_n}$$ # Consider Mean Time

$$\therefore P_e = 2 \times \int_{\frac{a_1-a_2}{2\sigma_n}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \, dy$$

$\left(P_e = \frac{1}{2} + \frac{1}{2} = 1\right)$

$$\# Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \, dy$$ # $Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$

$$\cdot\ P_e = Q\left(\frac{a_1-a_2}{2\sigma_n}\right) = Q\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$\cdot\ SNR = \frac{[a_1-a_2]^2}{\sigma_n^2} = \frac{2E_b}{N_0}$$

$$\Rightarrow \frac{a_1-a_2}{\sigma_n} = \sqrt{\frac{2E_b}{N_0}}$$

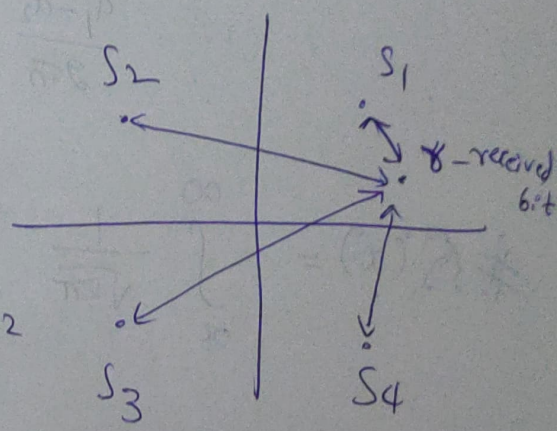
- Euclidean distance b/w points
- Euclidean distance of energy factor $\sqrt{E_b} \Rightarrow |Z-a_1|^2$

$$\cdot\ P_e = Q\left[\frac{a_1 - a_2}{2\sigma_n}\right] = Q\left[\frac{\sqrt{E_b}}{\sqrt{2N_0}}\right]$$

$$= Q\left[\frac{[S_1(t) - S_2(t)]^2}{\sqrt{2N_0}}\right] = Q\left[\frac{d_{min}}{\sqrt{2N_0}}\right]$$

$$\boxed{P_e = Q\left[\frac{d_{min}}{\sqrt{2N_0}}\right]}$$

- received bit close to which bit, that bit is the required bit



ML function

$$f(x/s_i) = \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-(x-s_i)^2/2\sigma_n^2}$$

$$\Rightarrow = \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-(z-a_i)^2/2\sigma_n^2}$$

- P_e = Q function of $\frac{d_{min}}{\sqrt{2N_0}}$; N_0: Noise Power

ASK : $d_{min} = \sqrt{E_b}$; $\therefore P_e = Q\left[\sqrt{\frac{E_b}{2N_0}}\right]$

BPSK : $d_{min} = 2\sqrt{E_b}$; $\therefore P_e = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$

FSK : $d_{min} = \sqrt{2E_b}$; $\therefore P_e = Q\left[\sqrt{\frac{E_b}{N_0}}\right]$

Ask: SNR is low SNR ↓ ⇒ Pe ↑

Psk: SNR is high SNR ↑ ⇒ Pe ↓ ✓

Fsk: SNR is neutral; SNR = ⇒ Pe medium

- Ask is not preffered as SNR is low (or) noise power is doubled.

- Psk is preferred ⇒ Pe ↓ as SNR ↑

- Fsk requires heavy bandwidth → Drawback
  ↳ for audio signals

- Psk & Ask for video signals.

- $Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$

- $SNR = \frac{\text{Signal Power}}{\text{Noise Power}}$

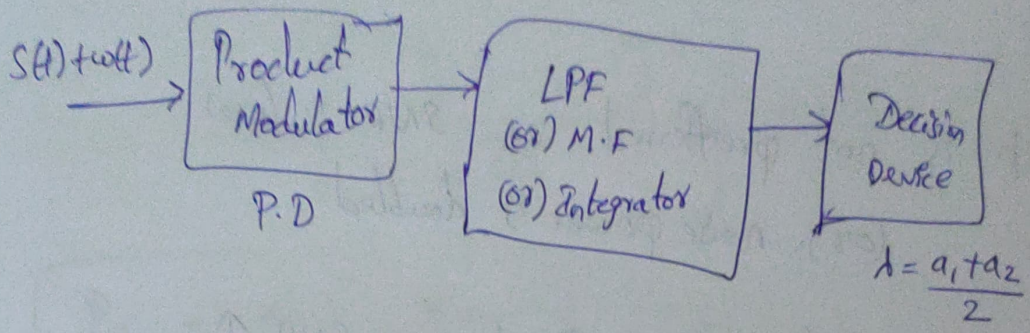
- N_0 → Noise power

Among Ask, Psk, Fsk : Psk is preferred.

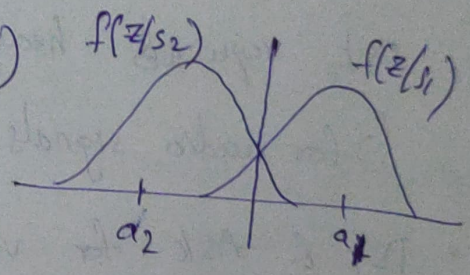
- Limitation of Psk : Sensitive to phase change

STEPS

1. Applying the Receiver block diagram



2. Determine '2' Gaussian distributions of Z w.r.t S_1 & S_2



3. $P_e \rightarrow$ is the overlapping area

$$P(H_2/S_1) = \int_{-\infty}^{\lambda} f(z/S_1)\, dz$$

$$P(H_1/S_2) = \int_{\lambda}^{\infty} f(z/S_2)\, dz$$

4. $$P_e = Q\left[\frac{a_1 - a_2}{2\sigma_n}\right] = Q\left[\frac{d_{min}}{\sqrt{2N_0}}\right]$$

QPSK : Quadrature Phase Shift keying

: Combination of 2 BPSK signals

- 2 bits at a time
- BW effeciency increases : Benefit
- Bit rate is doubled i.e. Data rate increases
- 2 bits → dibit
- General Representation :

$$S(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\pi/4\right]$$

; $i = 1, 2, 3, 4$

; $0 \le t \le T$

$$\Rightarrow S(t) = \underbrace{\sqrt{\frac{2E}{T}}}_{m(t)} \left[\underbrace{\cos(2\pi f_c t)}_{c(t)} \underbrace{\cos((2i-1)\pi/4)}_{\text{Scaling factor}} - \underbrace{\sin(2\pi f_c t)}_{c(t)} \underbrace{\sin((2i-1)\pi/4)}_{\text{Scaling factor}}\right]$$

$$\Rightarrow \phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t \quad ; \quad \phi_2 = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$$

$$m_1(t) = \sqrt{E} \cos(2i-1)\pi/4 \quad ; \quad m_2(t) = -\sqrt{E} \sin(2i-1)\pi/4$$

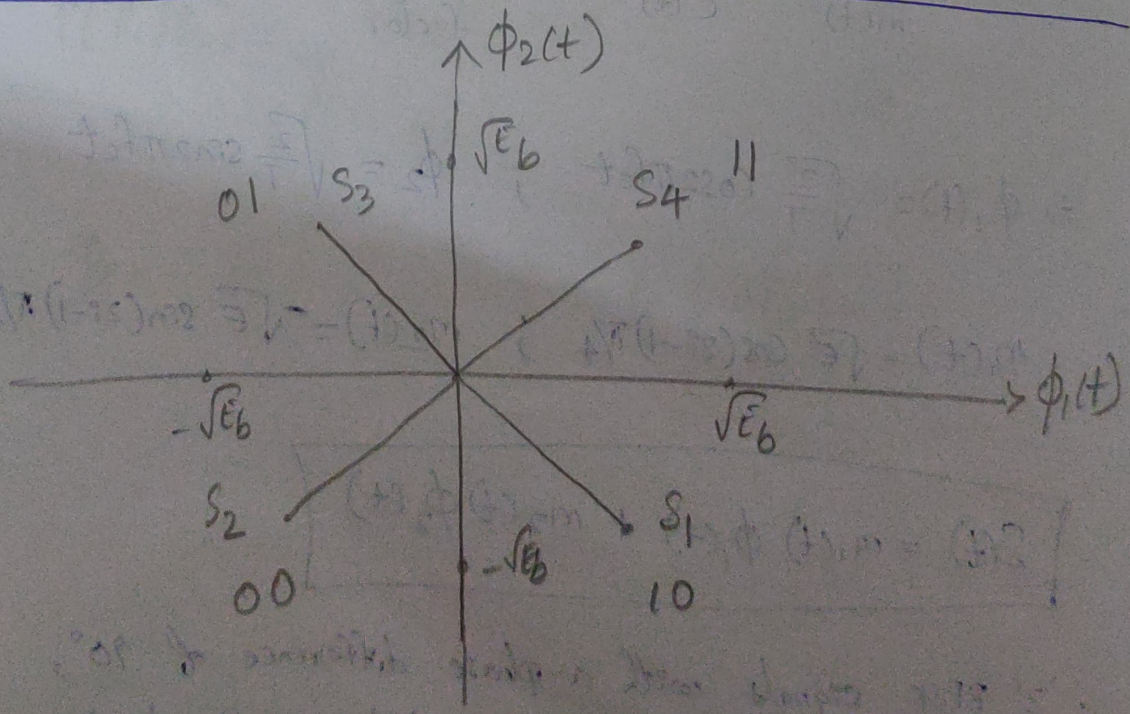
$$\boxed{S(t) = m_1(t)\,\phi_1(t) + m_2(t)\,\phi_2(t)}$$

- 2 BPSK signals with a phase difference of 90°, thats why called as Quadrature.

- Generated signals in QPSK

[Gray code combination: 1 0
0 0
0 1
1 1]

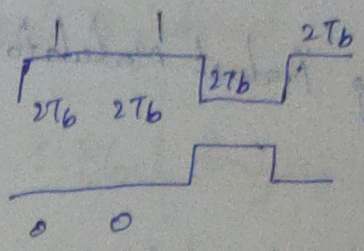
					$m_1(t)$	$m_2(t)$
S_1	$\Rightarrow$	1	0	$\Rightarrow \pi/4$	$+\sqrt{E}$	$-\sqrt{E}$
S_2	$\Rightarrow$	0	0	$\Rightarrow 3\pi/4$	$-\sqrt{E}$	$-\sqrt{E}$
S_3	$\Rightarrow$	0	1	$\Rightarrow 5\pi/4$	$-\sqrt{E}$	$+\sqrt{E}$
S_4	$\Rightarrow$	1	1	$\Rightarrow 7\pi/4$	$+\sqrt{E}$	$+\sqrt{E}$



Ex: (Even places) 1 0 1 0 0 1 1 0

even no. of bits
$m_1(t)$: 1 1 0 1

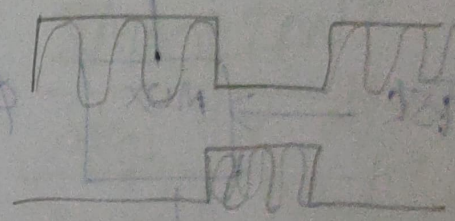
odd no. of bits
$m_2(t)$: 0 0 1 0



- To have same as in BPSK:
- Compression in freq. domain ⇒ expanding in Time domain

- $\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$

- $\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

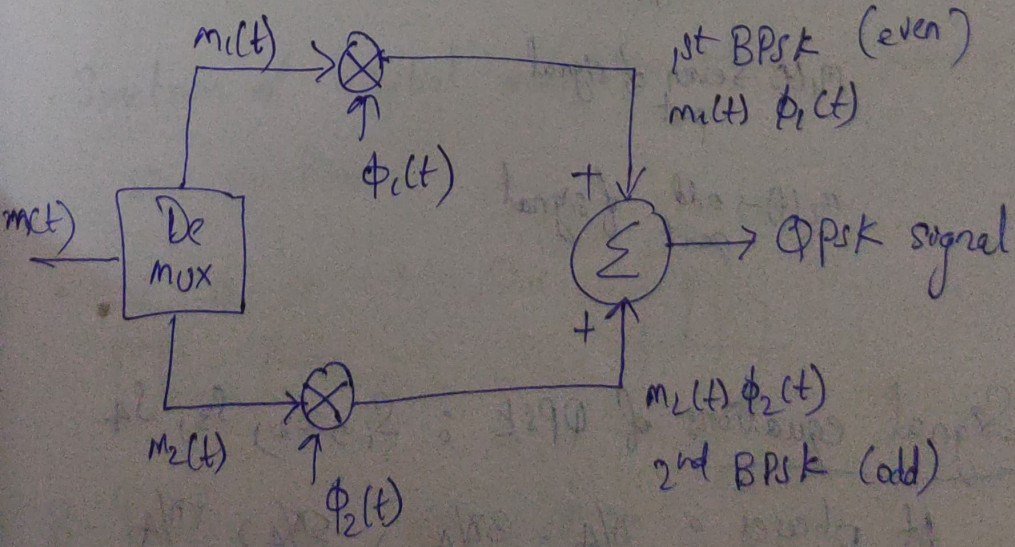


- BPSK : 0 → 1 (Pe error)
- QPSK : 00 → Y2 ⇒ Condition: one of the 2 bits is assumed correctly
- Locus of QPSK: Circle

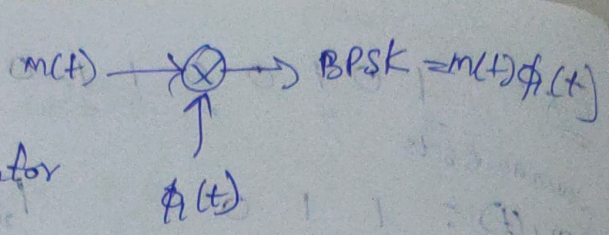
→ for this SNR should be high

Generation of QPSK:

- $S(t) = m_1(t)\,\phi_1(t) + m_2(t)\,\phi_2(t)$

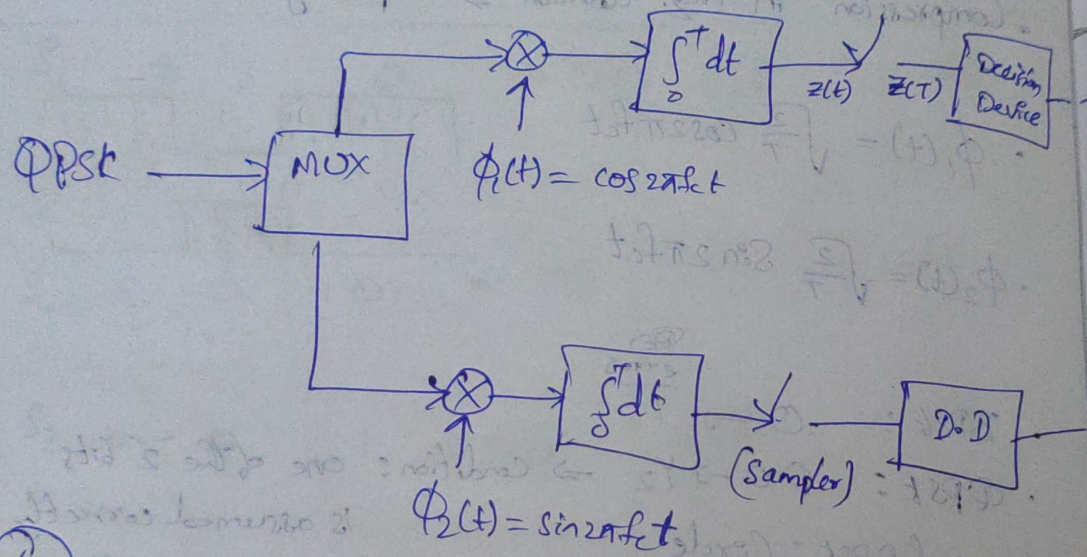


Generation of BPSK:



- Using Product Modulator

Demodulation of QPSK



1. Representation of QPSK signals mathematically

$$S(t) = m_1(t)\,\phi_1(t) + m_2(t)\,\phi_2(t)$$

$m_1(t) \rightarrow$ even part of signal

$m_2(t) \rightarrow$ odd part of signal

2. Signal equations of QPSK : S_1, S_2, S_3, S_4

At phases : $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

3. Constellation di

→ located each

4. Generation of

5. Demodulation of

6. Probability of

7. Spectrum of

- PSD : freq. v

- Spectrum of Di

sine functions

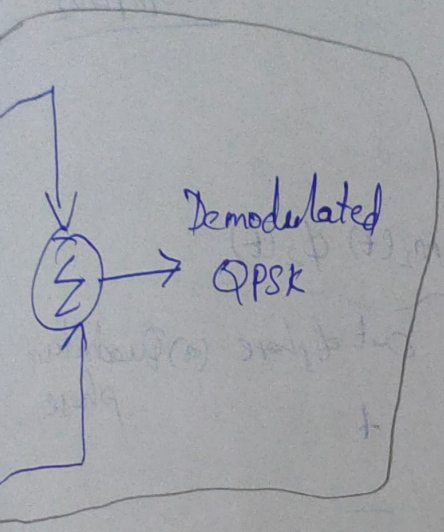
- 1 0 1

8. Preferrable

3. Constellation diagram for QPSK : 8 Circle
→ located each in ~~each~~ Quadrant

4. Generation of QPSK : Block diagram

5. Demodulation of QPSK : Block diagram

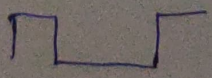


IMPORTANT
Steps for all
Digital Modulation
Schemes

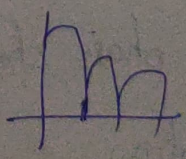
6. Probability of error of QPSK

7. Spectrum of QPSK

- PSD : freq. Vs magnitude w.r.t time too
- Spectrum of Digitial modulation schemes : interms of sine functions.
- 1 0 1

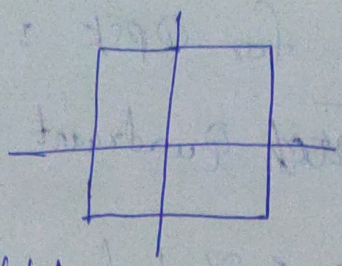


- Sinc



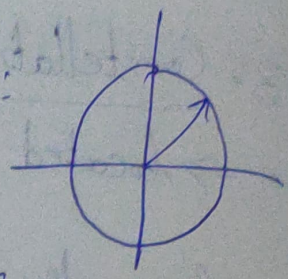
8. Preferrable ~~or~~ not

QAM:



(Amplitude change)

QPSK:



(phase change)
(Same amplitude)

18/11/2022

Offset - QPSK:

Limitation of QPSK:

$$S(t) = \underbrace{m_1(t)\,\phi_1(t)}_{\text{Inphase component}} + \underbrace{m_2(t)\,\phi_2(t)}_{\text{Out of phase (or) Quadrature phase}}$$

+	+
+	−
−	+
−	−

Ex. 01, 10 → 2 bit change → 180° phase shift required

10, 11 → 1 bit change → 90° phase shift required

Change in phase $\xrightarrow{\text{leads to}}$ change in amplitudes

Limitation:

- Along with phase change, QPSK has amplitude changes also

12/11/2020

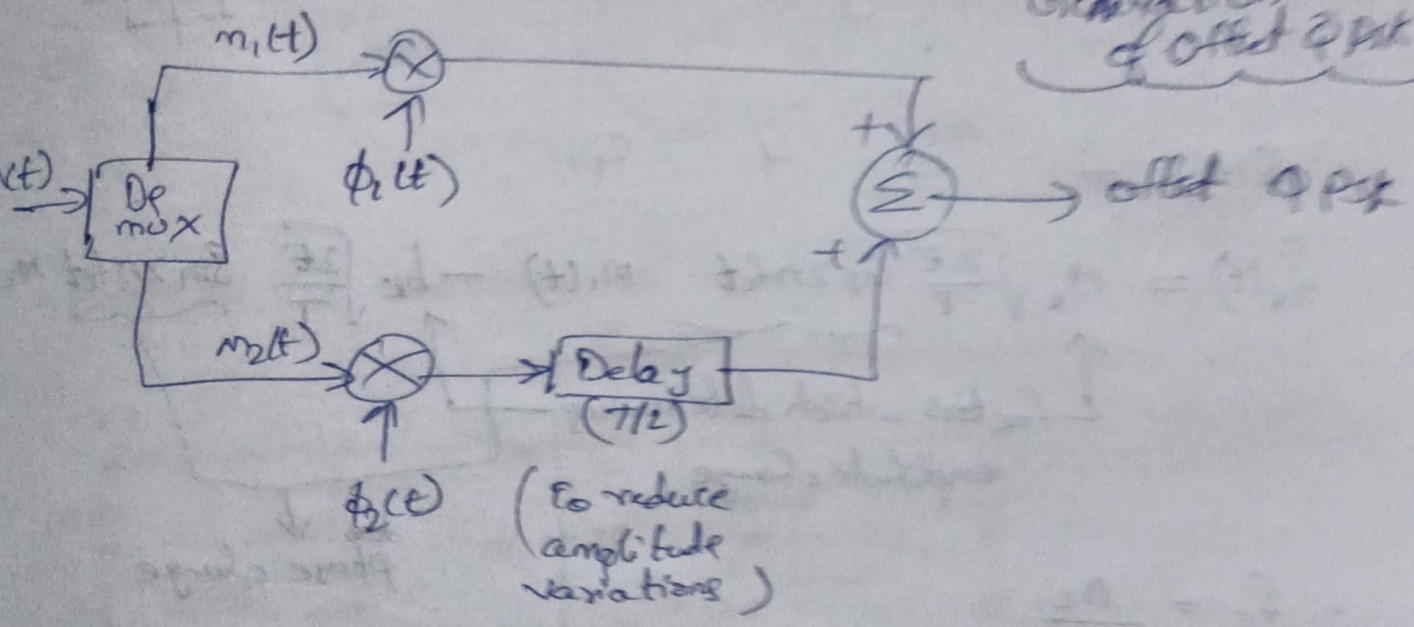
Solution: <u>Offset QPSK</u> : To limit amplitude change in QPSK

- Produce some delay either in Inphase component or in Quadrature phase component

- Hold 1st BPSK component for some time or hold 2nd BPSK " " " "

$$S(t) = m_1(t)\,\phi_1(t) + m_2(t)\,\phi_2(t)$$

Generation of offset QPSK



Limitation: Interface may occur.

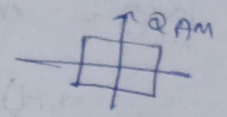
- We are making synchronous to asynchronous
- Proper Time shifter are required at receiver end (as we are giving Delay)

QAM: Quadrature Amplitude Modulation

- Hybrid modulation, which have amplitude changes & phase change also

- Combination of two modulations: Phase Modulation (PM) & Amplitude Modulation (AM)

- And so Constellation diagram is Square (not Circle) for QAM signal.



$$S(t) = I + Q$$

$$S_k(t) = a_k \sqrt{\frac{2E}{T}} \cos 2\pi f_c t \; m_1(t) - b_k \sqrt{\frac{2E}{T}} \sin 2\pi f_c t \; m_2(t)$$

factors which determines amplitude change

for phase change

- $f_c = \frac{n_c}{T}$

$; \; 0 \le t \le T$

- $S_k(t) = a_k \sqrt{\frac{2E}{T}} \cos 2\pi f_c t \; m_1(t) - b_k \sqrt{\frac{2E}{T}} \sin 2\pi f_c t \; m_2(t)$

'T' → Symbol duration

- For 4 symbol case: QPSK and QAM are same

QAM Conste

Mary → M no.

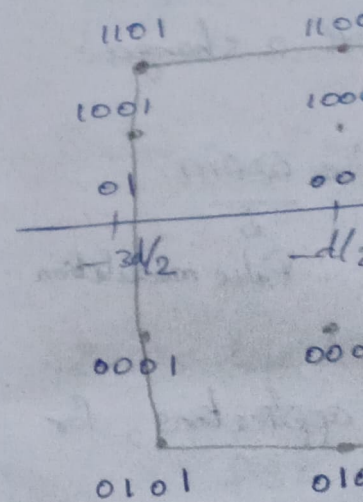
- Symbol M

(Binary) ← M =

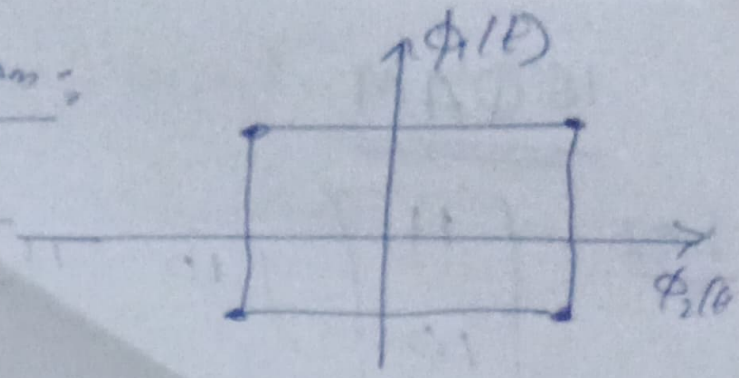
m no. of

M-ary QA

- Each symbol



QAM Constellation Diagram:



Mary → M no. of symbols

- Symbol $M = 4 \Rightarrow 2^n = 2^2 \rightarrow$ 2 bits at a time

(Binary) ← $M = 2 \Rightarrow 2^n = 2^1 \Rightarrow$ 1 bit at a time

→ m no. of bits needs to be transmitted.

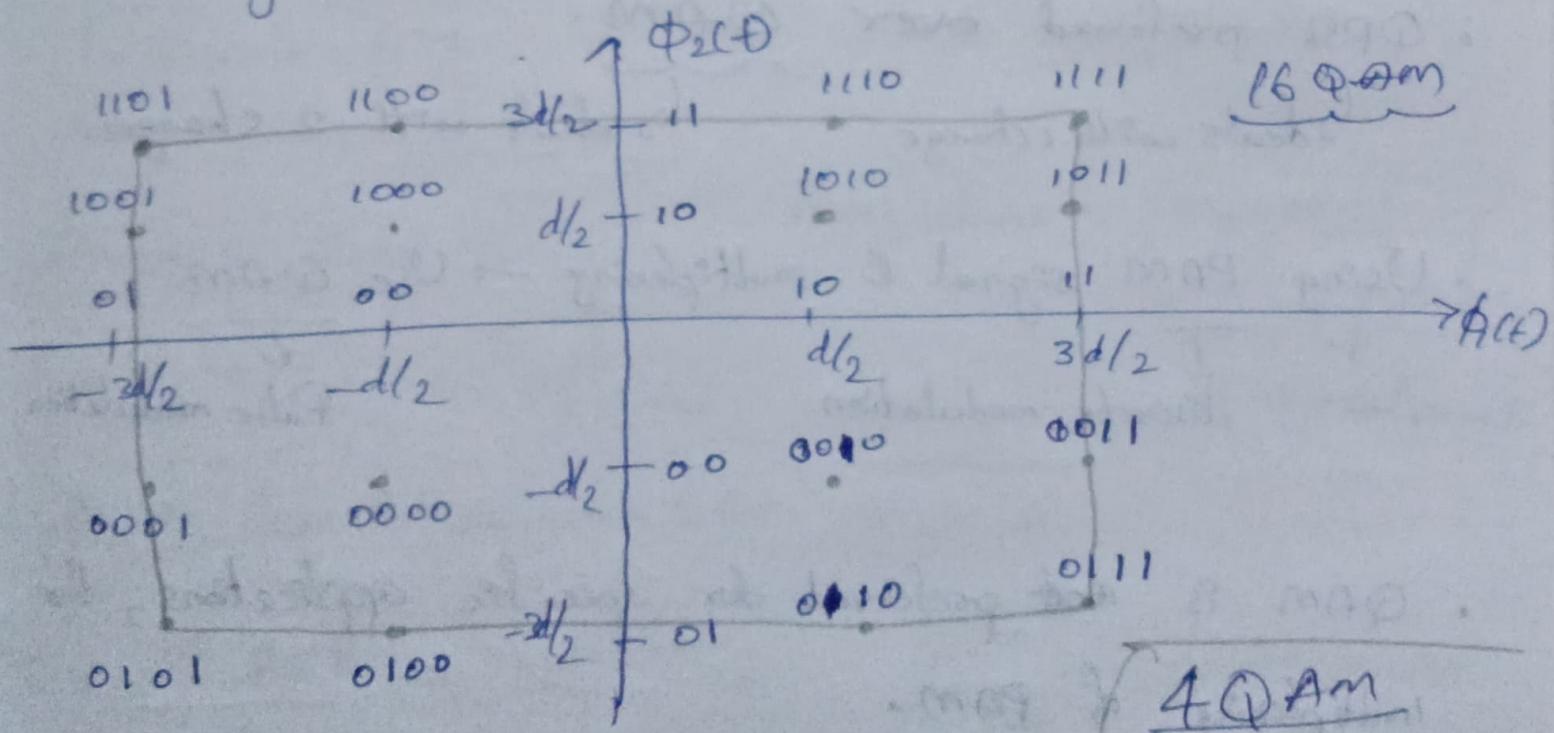
M-ary QAM: $M = 2^n \Rightarrow 16 - QAM$

$16 = 2^4$

↳ 4 no. of bits at a time

↳ 16 no. of bits are required points

- Each symbol consists of 4 bits.



4QAM

4 symbols

↳ each required 2 bits representation

16 QAM

In QI

$$\begin{bmatrix} 11 \\ 10 \end{bmatrix} \begin{bmatrix} 10 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1110 & 1111 \\ 1010 & 1011 \end{bmatrix}$$ – 4 Points in QI

In QII:

$$\begin{bmatrix} 11 \\ 10 \end{bmatrix} \begin{bmatrix} 00 & 01 \end{bmatrix} = \begin{bmatrix} 1100 & 1101 \\ 1000 & 1001 \end{bmatrix}$$

- QPSK preferred over QAM.
  - QPSK → deals with 1 change
  - QAM → deals with 2 changes.
- Using PAM signal & multiplexing → Use QAM.
  - PAM → discrete modulation
  - QAM → Pulse modulation
- QAM is ~~not~~ preferred for specific applications, for multiplexing of PAM.

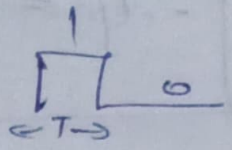
. Bit rate : no. of bits can be sent at a time.

. Baud rate : no. of symbols sent at a time.

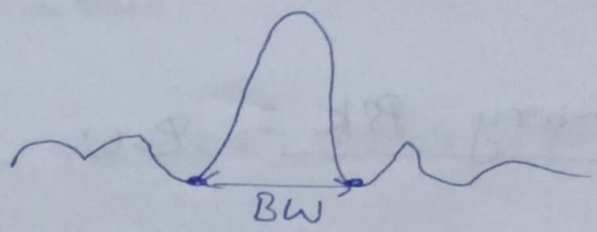
. When dealing with M-ary → Baud rate
↳ ∵ deals with symbols.

Bandwidth efficiency:

In Time-Domain: presence of pulse — 1 ; absence — 0



In Freq-Domain: Sinc function



B.W: Null to NULL
(Time occupancy)

$$\boxed{\text{B.W} \propto \frac{1}{T}}$$

Binary: $\text{B.W} \propto \frac{1}{T_b}$ (T_b → T - bit period)

Mary scheme: $\boxed{\text{B.W} \propto \frac{1}{T}}$ (T → T-symbol period)

. B.W: depends on main lobe

FSK	PSK	ASK
More B.W		Less BW

. ASK – poor performance . FSK – require large BW

So, $\boxed{\text{PSK - Preferred}}$ //

(• The no. of bits / symbols to be accomodated within the spectrum.)

The no. of bits / no. of symbols to be accomodated in the available spectrum / B.W → B.W efficiency (η)

$$\boxed{\therefore \eta = \frac{R_b}{B.W}}$$; R_b – rate of information (no. of bits per second)
B.W → available bandwidth

<u>M-ary PSK</u> : $B.W = \frac{2}{T}$; T – symbol period
↳ collection of bits

• $T = T_b \log_2 M$

<u>For Binary case</u> : $T = T_b \log_2 2^1 \Rightarrow T = T_b$

<u>for QPSK case</u> : $T = T_b \log_2 2^2 \Rightarrow T = 2T_b$

Each symbol → $\overset{T_b}{0}\ \overset{T_b}{0} \rightarrow 2T_b$
0 1
1 0
1 1

• $B.W = \frac{2}{T_b \log_2 M}$

$$\Rightarrow B.W = \frac{2R_b}{\log_2 M} \Rightarrow \boxed{\frac{R_b}{B.W} = \frac{\log_2 M}{2}}$$

For M-ary scheme

M 2

η 0.5

• Increasing th

• B.W efficiency

Ex: $16 = 2^4$

• For most

• Digital mod

• M-ary o

• Lower order

Binary

Minimum

no need of any carrier signal ← Differentia

Binary

For M-ary scheme: $\eta = \frac{R_b}{B.W} = \frac{\log_2 M}{2}$

M	2	4	8	16
η	0.5	1	1.5	2

- Increasing the value of $n \uparrow \Rightarrow \eta$ increasing
- B.W efficiency $\uparrow \Rightarrow$ Performance decreases as P_e increases

Ex: $16 = 2^{4} \rightarrow$ 4 bits at a time

- For most of the cases: (4 – 64) order is preferred.
- Digital modulation scheme B.W is func. of Time period (Ts (symbol))
- M-ary or Higher order : M - indicates order of Modulation.
- Lower order : Binary scheme : $M = 2^1$

22/11/2022

Binary Shift key } Coherent demodulation
Minimum Shift keying }

no need of any carrier signal ← Differential Phase shift keying } → Non-coherent demodulation (Carrier re-generation is not required)
Binary Frequency shift keying }

- DPSK → purely depends on how the phase changes b/w 2 consecutive bits. (So no need of carrier signal)

Ex: 101

Assignment

1. QPSK, QAM
   generate & demodulate QPSK signal and
   Compare QPSK with QAM

2. Compare all ~~MARY~~ M-ary modulation schemes (Higher order schemes)

   (MPSK, M-FSK, MSK)

   → Mathematical representation / expression of each.

   → B.W efficiency of each.

   → Constellation Diagram.

   → Probability of Error.

ANALOG MODULATION

22/11/2022

Continuous wave modulation

Amplitude Modulation: Change in the amplitude of carrier signal according to the amplitude of message signal.

$c(t) = A_c \cos 2\pi f_c t$; $m(t) = A_m \cos 2\pi f_m t$

- $A_c \propto |m(t)|$ $(A_c \propto A_m)$
- Let: $A = A_c + m(t)$ [the change in amplitude of $c(t)$]

$$s_{AM}(t) = A \cos 2\pi f_c t$$

$$= [A_c + m(t)] \cos 2\pi f_c t$$

$$= A_c \left[1 + \frac{1}{A_c} m(t)\right] \cos 2\pi f_c t$$; $\frac{1}{A_c} \rightarrow$ Amplitude Sensitivity

$$= A_c [1 + k_a m(t)] \cos 2\pi f_c t$$

$k_a = \frac{1}{A_c}$

Unit: V^{-1} (volt inverse)

$$\therefore \boxed{s_{AM}(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t}$$

Mathematical expression for AM signal

- **Single Tone Amplitude Modulation:** Considering $m(t)$ at a one particular frequency (f_m)

$$m(t) = A_m \cos 2\pi f_m t$$

$$s_{AM}(t) = A_c [1 + k_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

$\longrightarrow$

Single Tone AM :

$$s_{AM}(t) = A_c [1 + k_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

- The value of change

Modulation Index: $\mu = k_a A_m$ – level of index

↳ The level of change in the carrier holds

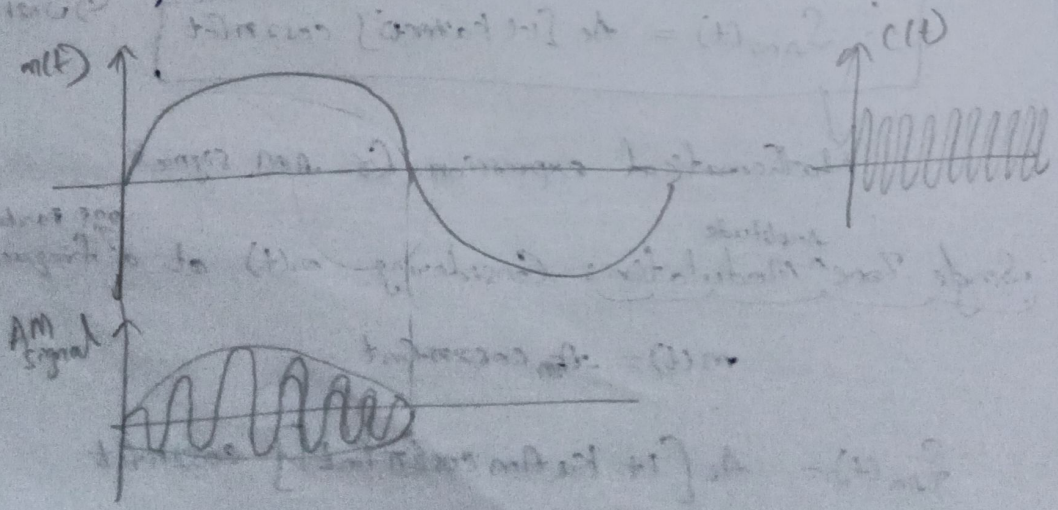
$$\boxed{s_{AM}(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t}$$

- Based on value of μ: $\mu < 1 \rightarrow$ under modulated

$\mu = 1 \rightarrow$ 100% modulation

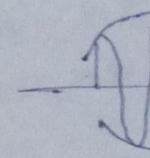
$\mu > 1 \rightarrow$ Over modulation

- Over modulation should not be preferred.
- $\mu < 1$ & $\mu = 1$ are preferrable.



- Envelope of AM s

when we trace



- $\mu > 1 \rightarrow$ leads

revers

$\mu > 1$ – not preferred:

- Due to an

↳ because of P

↳ $\mu >$

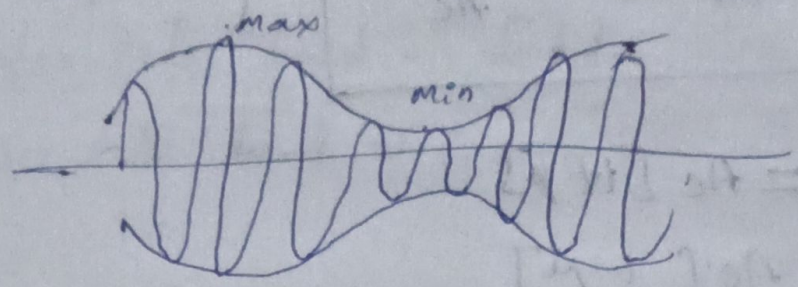
Case 1 : $s_{AM}(t$

$\mu < 1$: $\mu = 1/2$

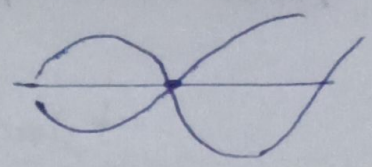
Case 2 :

$\mu > 1$: $\mu = 3/2$

- Envelope of AM signal looks as message signal when we trace the peaks of AM signal



- $\mu > 1$ → leads to envelope distortion ∵ phase reversal occurs



$\mu > 1$ – not preferred: envelope
- Due to analog distortion
  ↳ because of phase reversal
  ↳ $\mu > 1$

Case 1: $s_{AM}(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$

$\mu < 1$: $\mu = 1/2$

$A_{max} = 3/2$

$A_{min} = 1/2$

[∵ cos → max = 1, min = −1]

Case 2:

$\mu > 1$: $\mu = 3/2$

$A_{max} = 1 + 3/2 \Rightarrow 5/2$

$A_{min} = 1 - 3/2 \Rightarrow -1/2$ → Phase reversal (phase inversion)

⇓

leads to overlap

⇓

leads to distortion

~~(Spectrum of AM signal)~~: $S_{AM} = A_c[1+\mu\cos 2\pi f_m t]\cos 2\pi f_c t$

$$\boxed{\mu = k_a A_m = \frac{A_m}{A_c}}$$ (Modulation Index)

$A_{max} = A_c[1+\mu]$

$A_{min} = A_c[1-\mu]$

$$\frac{A_{max}}{A_{min}} = \frac{1+\mu}{1-\mu} \Rightarrow A_{max} - \mu A_{max} = A_{min} + \mu A_{min}$$

$$\Rightarrow \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$\boxed{\therefore \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

Time Domain Analysis: $S_{AM}(t)$ ✓ → in terms of A_c ✓ A_m ✓ k_a ✓ μ ✓

Frequency Domain Analysis of AM Signal / Spectrum of AM Signal:

$$S_{AM}(t) = A_c[1+\mu\cos 2\pi f_m t]\cos 2\pi f_c t$$

$$= A_c\cos 2\pi f_c t + \mu A_c\cos(2\pi f_m t)\cos(2\pi f_c t)$$

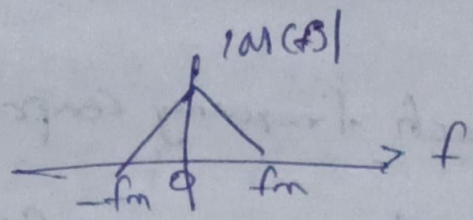
$$= A_c\cos 2\pi f_c t + \frac{A_c\mu}{2}[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t]$$

- $s_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\left[\cos[2\pi(f_c+f_m)t] + \cos 2\pi(f_c-f_m)t\right]$

- Carrier Component
- USB - Upper side band → $f_c + f_m$
- LSB - Lower side band → $f_c - f_m$

$m(t)$ → ~~Base~~ Band signal

Magnitude of message signal - Spectrum →



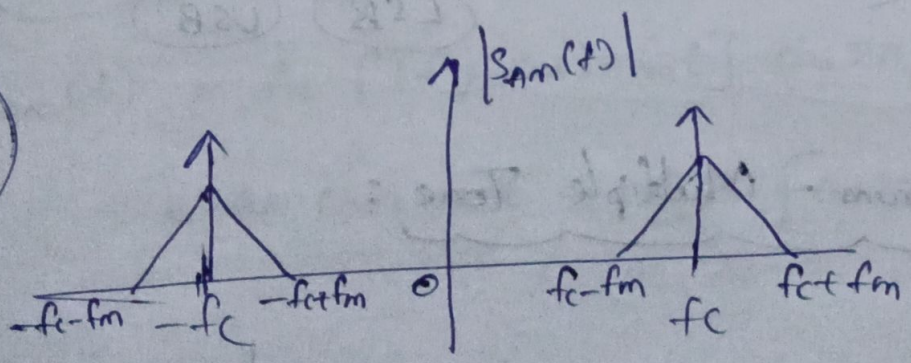
$$s_{AM}(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

$$S_{AM}(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] \quad (1)$$

$$+ A_c k_a M(f) * \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$(2) \quad \left\{\frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]\right\}$$

AM multiple Tone



Spectrum of AM Signal

$$s_{AM}(t) = A_c\,[1 + \mu\cos 2\pi f_m t]\,\cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi[f_c - f_m]t\right]$$

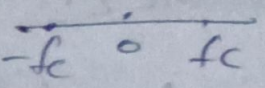
- Each frequency component → 2 pulses (+, −)

1st Component : $\pm f_c$

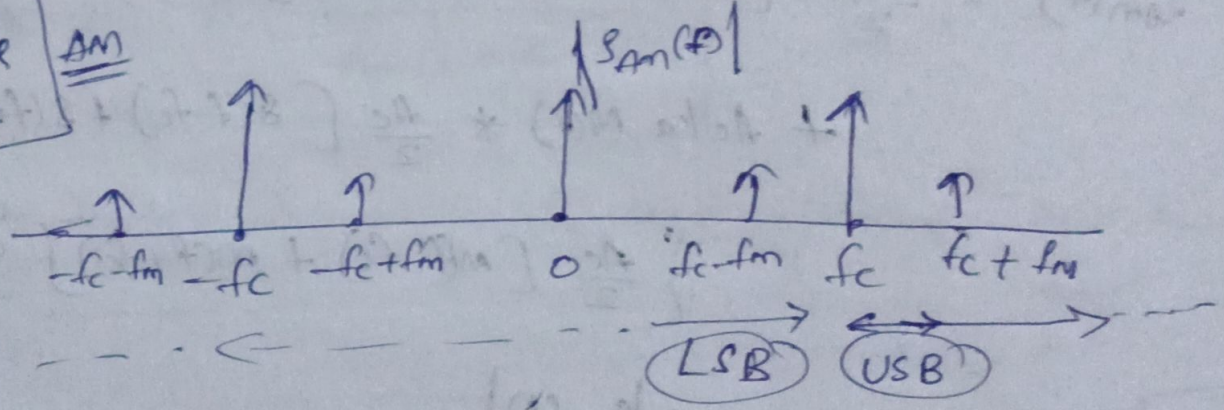
2nd component : $\pm(f_c + f_m)$

3rd component : $\pm(f_c - f_m)$

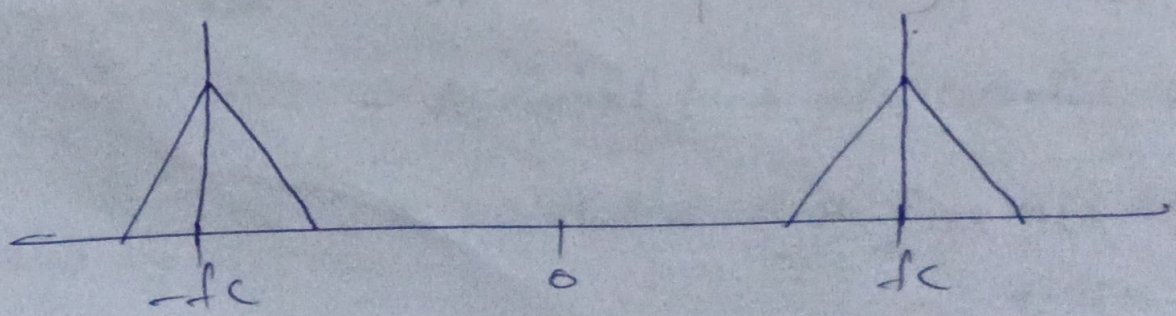
Total 6 frequency Components Single Tone AM



Single Tone AM



AM Spectrum - Multiple Tone :



$$s_{AM}(t) = A_c\,[1 + k_a\, m(t)]\cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \underbrace{A_c k_a\, m(t) \cos 2\pi f_c t}_{\text{Copy of } m(t) \text{ at } +f_c \,\&\, -f_c}$$

. Frequency components in Multi tone AM : ∞ (finite (∵ it is a range))

. Frequency components in Single Tone AM : 6

$$[\pm f_c,\ \pm(f_c + f_m),\ \pm(f_c - f_m)]$$

Bandwidth : $(f_c + f_m) - (-f_c - f_m)$

$$\boxed{B.W = 2 f_m}$$

Power relations in AM signal :

. $s_{AM}(t) = A_c\,[1 + \mu \cos 2\pi f_m t]\cos 2\pi f_c t$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\left[\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t\right]$$

. Total Power $(P_T) = P_c + P_{USB} + P_{LSB}$ ☆

$$\text{Power} = \frac{(A_c/\sqrt{2})^2}{R} = \frac{A_c^2}{2R} = P_c \longrightarrow$$

$$P_c = \frac{A_c^2}{2R} = \frac{A_c^2}{2} \; ; \; R=1$$

$$P_{USB} = P_{LSB} = \frac{(A_c \mu/2)^2}{R} = \frac{A_c^2 \mu^2}{8R} = \frac{A_c^2 \mu^2}{8} \; ; \; R=1$$

$$= P_c \frac{\mu^2}{4}$$

$$\therefore P_T = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{8} + \frac{A_c^2 \mu}{8} \; ; \; R=1 \text{ for Normalized Condition}$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{\therefore P_t = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

Transmission efficiency of AM signal (η)

- Carrier does not carry any information.
- Only side bands carry information.

$$\eta = \frac{\text{Total side band Power}}{\text{Total power}} = \frac{P_{USB} + P_{LSB}}{P_T}$$

$$\Rightarrow \eta = \frac{\left(\frac{A_c^2\mu^2}{8} + \frac{A_c^2\mu^2}{8}\right)}{P_{c\times}\left(\frac{\mu^2}{2}+1\right)} = \frac{\left(\frac{A_c^2\mu^2}{4}\right)}{\left\{\frac{A_c^2}{2}\left(1+\frac{\mu^2}{2}\right)\right\}}$$

$$= \frac{\mu^2}{2\left(1+\frac{\mu^2}{2}\right)}$$

$$= \frac{\mu^2}{2+\mu^2} \qquad \boxed{\therefore \eta = \frac{\mu^2}{2+\mu^2}}$$

<u>For 100% Modulation</u> : $\mu = 1 \Rightarrow \eta = \frac{1}{2+1} = \frac{1}{3}$

$$\Rightarrow \eta\% = 33.33\%$$

- Carrier signal consumes more than 60% while message signal only 33.33%.

<u>Limitation</u> : • High Power consumption by Carrier signal, which does not carry any information.

<u>Generation of AM Signal</u> <u>23/11/2022</u>

① Square Law Modulation

② Switching Modulation

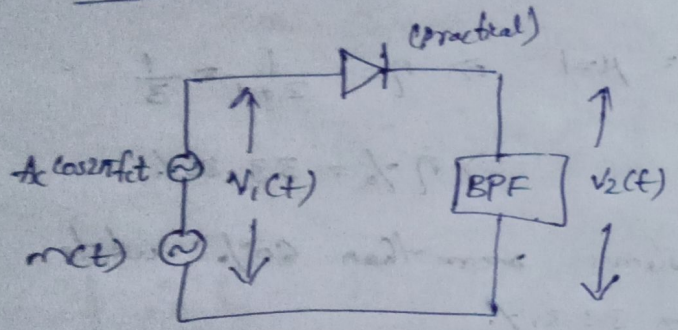
23/11/2022

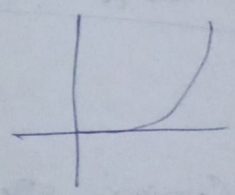
Components: Summer (Adder), non-linear device (for square), BPF, Diode (practical)

Linear device → (for switching) : ~~practical~~ (Ideal diode)

- Square Law : Practial diode
- Switching Modulation : Piecewise is required, so Ideal diode is required

Square Law : Non Linear relation b/w $V_1(t)$ & $V_2(t)$





∴ **Ideal case :** $V_2(t) \propto V_1(t)$

$$V_2(t) = a_1 V_1(t)$$

- We need non-linear relation : So, $V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$

Square Law : $\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)}$

$\Rightarrow V_1(t) = m(t) + A_c \cos 2\pi f_c t$

$\Rightarrow V_2(t) = a$

$\Rightarrow V_2(t) = a_1 m$

$\Rightarrow V_2(t) = a_1 m(t) +$

$= a_1 m(t) + a$

$= a_1 m(t) +$

- BPF rejects

$\therefore V_2(t) =$

$=$

$=$

$\boxed{\therefore V_2(t}$

$$\Rightarrow V_2(t) = a_1[m(t) + A_c \cos 2\pi f_c t] + a_2[m(t) + A_c \cos 2\pi f_c t]^2$$

$$\Rightarrow V_2(t) = a_1 m(t) + a_1 A_c \cos 2\pi f_c t + a_2[m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2 m(t) A_c \cos 2\pi f_c t]$$

$$\Rightarrow V_2(t) = a_1 m(t) + a_1 A_c \cos 2\pi f_c t + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t + 2 a_2 m(t) A_c \cos 2\pi f_c t$$

$$= a_1 m(t) + a_1 A_c \cos 2\pi f_c t + a_2 m^2(t) + \frac{a_2 A_c^2}{2}[1 + \cos 4\pi f_c t] + 2 a_2 m(t) A_c \cos 2\pi f_c t$$

$$= a_1 m(t) + a_1 A_c \cos 2\pi f_c t + a_2 m^2(t) + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos 4\pi f_c t + 2 a_2 m(t) A_c \cos(2\pi f_c t)$$

- BPF rejects frequencies other than f_c; after passing through BPF.

$$\therefore V_2(t) = a_1 A_c \cos 2\pi f_c t + 2 a_2 m(t) A_c \cos 2\pi f_c t$$

$$= A_c[a_1 + 2 a_2 m(t)] \cos 2\pi f_c t$$

$$= A_c a_1 \left[1 + \frac{2 a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

$$\boxed{\therefore V_2(t) = A_c a_1 \left[1 + \frac{2 a_2}{a_1} m(t)\right] \cos 2\pi f_c t} \longrightarrow$$

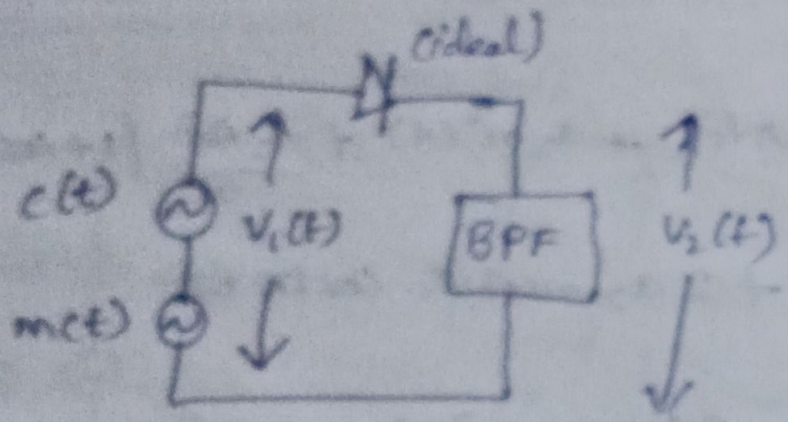
$\therefore V_2(t) = A_c a_1 \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t$

Amplitude Sensitivity : $k = \frac{2a_2}{a_1}$

<u>Limitation of Square Law device</u> : $\frac{2a_2}{a_1} \times A_m < 1$

(i.e. Modulation index $(\mu) < 1$)

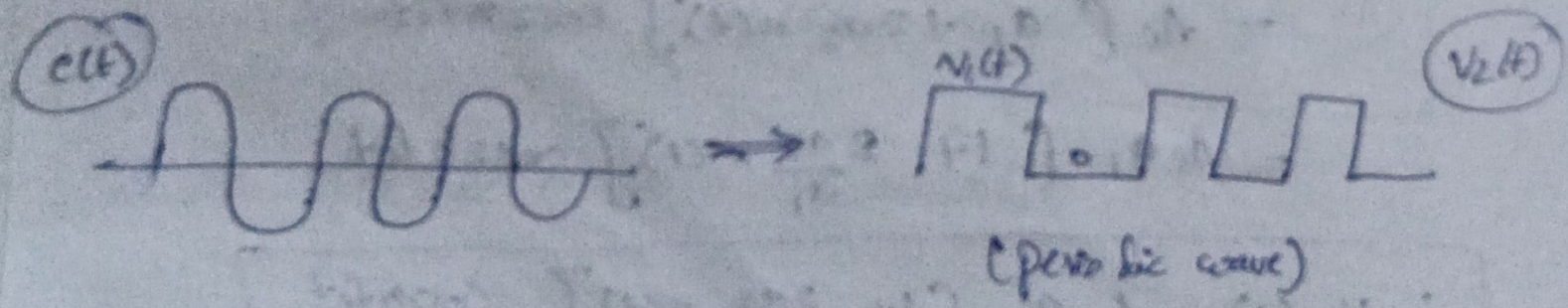
<u>Switching Modulation device</u> : Linear relation



- $V_1(t) = m(t) + A_c \cos 2\pi f_c t$

Assumption : $|c(t)| >> |m(t)|$

- Diode is ON : $V_2(t) = V_1(t)$; $|c(t)| > 0$
- Diode is OFF : $V_2(t) = 0$; $|c(t)| < 0$



- $V_2(t) = V_1(t) \times g_p(t)$

$$g_p(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{2n-1} \cos 2\pi f_c t(2n-1)$$

$$= \left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t\right]$$; other frequencies are rejected

$$\Rightarrow V_2(t) = V_1(t) \times \left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t\right]$$

$$= \left[m(t) + A_c \cos 2\pi f_c t\right]\left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t\right]$$

$$= \frac{m(t)}{2} + \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{2A_c}{\pi} \cos^2 2\pi f_c t$$

$$= \frac{A_c}{2} \cos 2\pi f_c t + \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{m(t)}{2} + \frac{2A_c}{\pi} \cos^2 2\pi f_c t$$

<u>Limitation</u>:

- Practical diode is required
- $|c(t)| >> |m(t)|$ → power consumption

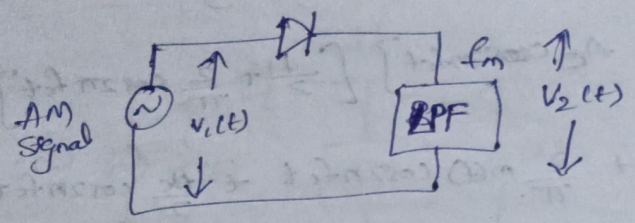
Demodulation of AM Signal :-

① Square Law Demodulator / detector

② Envelope detector

① Square Law Detector :-

- $V_2(t) = a_1 v_i(t) + a_2 v_i^2(t)$



(LPF used : ∵ it is decoding / demodulating)

- $v_i(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$

$$\Rightarrow V_2(t) = a_1 A_c [1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + k_a m(t)]^2 \cos^2 2\pi f_c t$$

$$= a_1 A_c \cos 2\pi f_c t + a_1 A_c k_a m(t) \cos 2\pi f_c t + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 A_c^2 k_a^2 m^2(t) \cos^2 2\pi f_c t + 2 a_2 A_c^2 k_a m(t) \underbrace{\cos^2 2\pi f_c t}_{\left(\frac{1 + \cos 4\pi f_c t}{2}\right)}$$

$= \frac{2 a_2 A_c^2 k_a}{2} m(t)$ is only passed through LPF

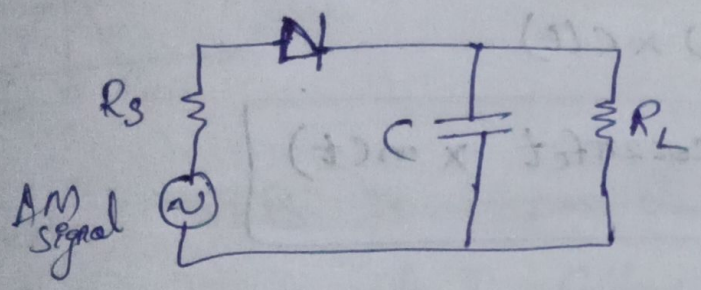
- Reject all high freq. components & pass through only low freq. component w.r.t f_m.

$$V_2(t) = a_2 A_c^2 k_a m(t)$$

<u>Limitation:</u> No ideal filters.

(2) Envelope Detector: (Diode + RC filter) (LPF)

<u>Diode (D)</u>



<u>Limitation:</u>
- Time constant should not be too small and too large.

- Put limits of τ b/w $1/f_c$ & $1/f_m$; $\tau = R_L C$

<u>Condition:</u>

$$\boxed{\frac{1}{f_c} << R_L C << \frac{1}{f_m}} \quad (f_c > f_m)$$

Why?

- Exact value of $R_L C$ to avoid distortion ?

DSB-SC :- Double Side Band (with) Suppressed Carrier

- Saving the Power. (Benefit)
- Power consumption is reduced

$$s_{am}(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

$$= \underbrace{A_c \cos 2\pi f_c t}_{\text{(suppressed carrier)}} + m(t) \cos 2\pi f_c t$$

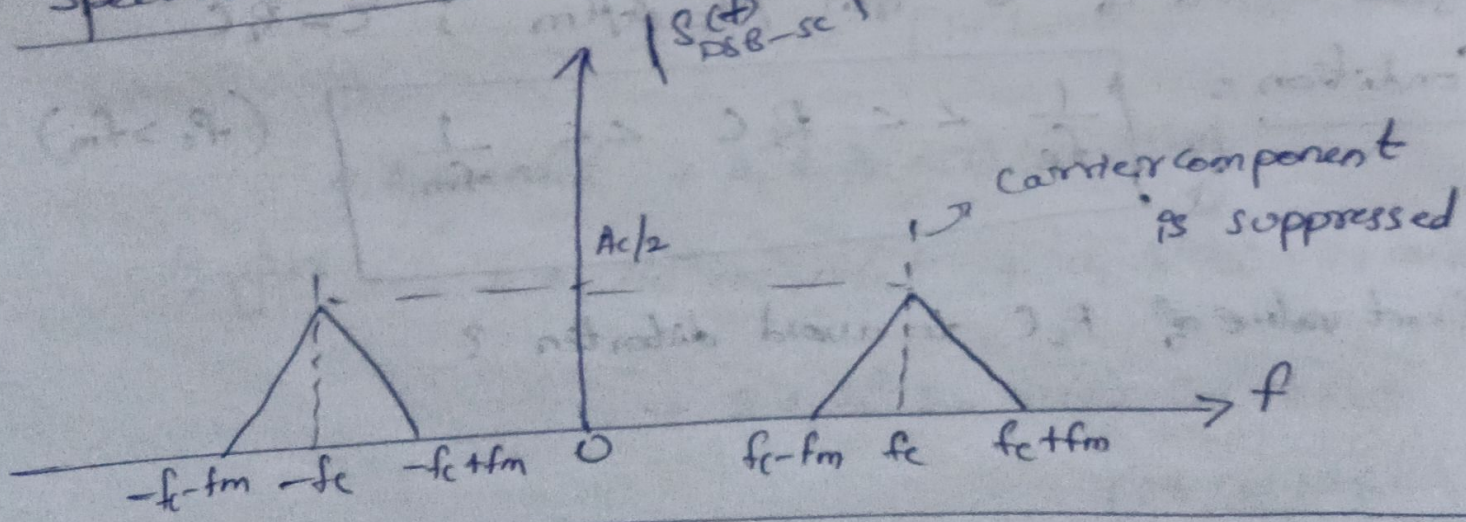
DSB-SC : Product of message signal & Carrier signal

$$s_{DSB}(t) = m(t) \times c(t)$$

$$\boxed{s_{DSB-SC}(t) = A_c \cos 2\pi f_c t \times m(t)}$$

Generation : Using multiplier (or) Product Modulator

Spectrum of DSB-SC :



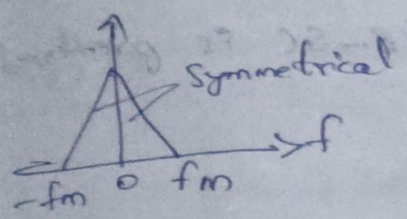
- B.W = $2f_m$

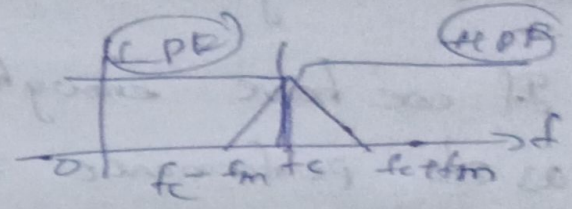
$$\boxed{S_{DSB-SC}(f) = \frac{A_c}{2}[M(f+f_c) + M(f-f_c)]}$$

- Total available Power is shared b/w LSB & USB.

SSB - Single Side Band Transmission.

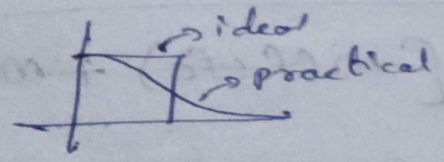
- Advantage : If BW needs to be saved i.e. Narrow B.W required SSB is preferred.





- Perfect SSB is impossible in practical case, as there are no ideal filters.

Filter: (LPF)



- generation of SSB is complex as ideal filter is difficult.

VSB : Vestigal Side band

- 1 side band - complete and other side band - partial
- When practical filter is used.

Performance of SSB < Performance of DSB-SC (But a penality of B.W)

Comparison: CAM → 2fm (Conventional AM)

DSB → 2fm

SSB → fm

VSB → fm < B.W < 2fm

- Overall VSB is preferred. (over all AM schemes)
- For video signals → VSB
- If we have enough B.W then DSB-SC is preferred as its performance is good.

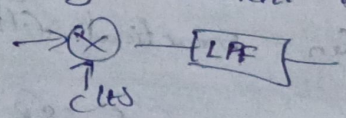
Spectrum of DSB-SC: $m(t) \times A_c \cos 2\pi f_c t$

- 4 frequency components

$$= M(f) * \frac{A_c}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$

$$\boxed{S_{DSB-SC}(f) = \frac{A_c}{2}\left[M(f+f_c) + M(f-f_c)\right]}$$

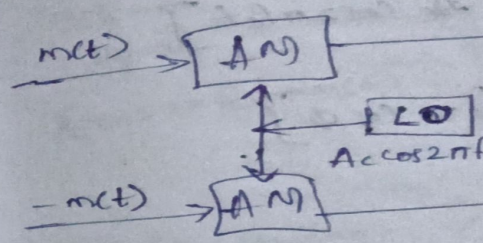
- Benefit: Generation and Demodulation of DSB-SC is easy.
  - Product Modulator
  - Using Coherent Demodulation



Generating DSB-SC

- Consider 2 AM signals

Balanced Modulator



Ring modulator:

- At a time, 2 d
  other 2 a
  generates
- And after
  are OFF
- Add and

~~Coherent Demodulator~~

DSB-SC Generation

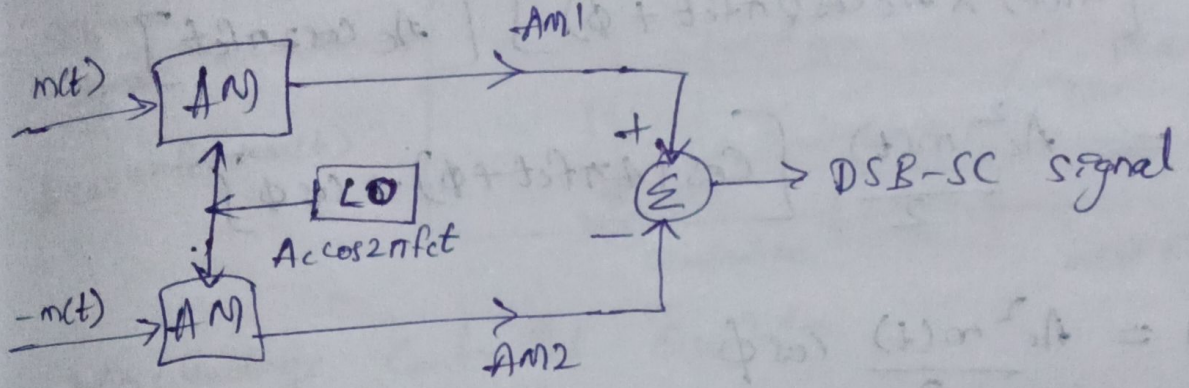
Generating DSB-SC from AM signal :

. Consider 2 AM signals : $A_c[1+k_a m(t)]\cos(2\pi f_c t)$

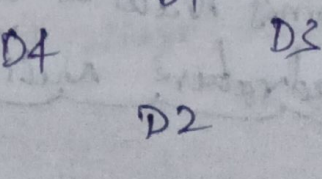
$\ominus$ $A_c[1-k_a m(t)]\cos 2\pi f_c t$

$2A_c \cos 2\pi f_c t \times m(t)$

Balanced Modulator

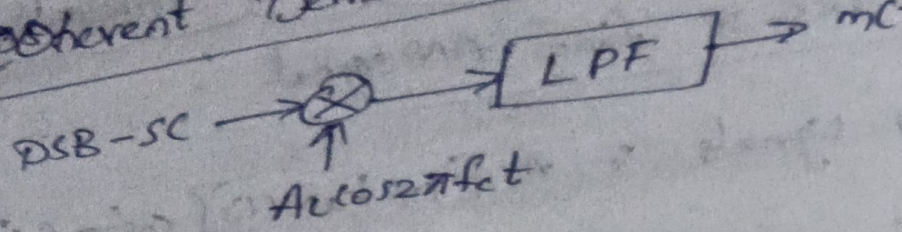


Ring modulator : Using 4 Diodes : connected in a ring



- At a time, 2 diodes are ON & other 2 are OFF - which generates m(t)
- And after that, other 2 diodes are ON & 1st two diodes are OFF which generates −m(t)
- Add and DSB-SC signal is produced

Coherent Demodulation of DSB-SC



- Random phase ~~noise~~ due to noise is present:

$$m(t) \times A_c \cos(2\pi f_c t + \phi)$$

$$[m(t) \times A_c \cos(2\pi f_c t + \phi)][A_c \cos 2\pi f_c t]$$

$$= \frac{A_c^2 m(t)}{2}[\cos(4\pi f_c t + \phi) + \cos\phi]$$

$$V_o(t) = \frac{A_c^2 m(t)}{2} \cos\phi$$

Limitation of DSB-SC (in demodulation)

- Due to random phase by noise output of Demodulator becomes Null — Quadrature Null effect — when random phase becomes $\frac{\pi}{2}$ i.e. $\phi = \pi/2$

- Perfect Synchronization is needed.

Solution: make random phase $\phi = 0°$ → form a costas loop

$\phi > 0 \downarrow$

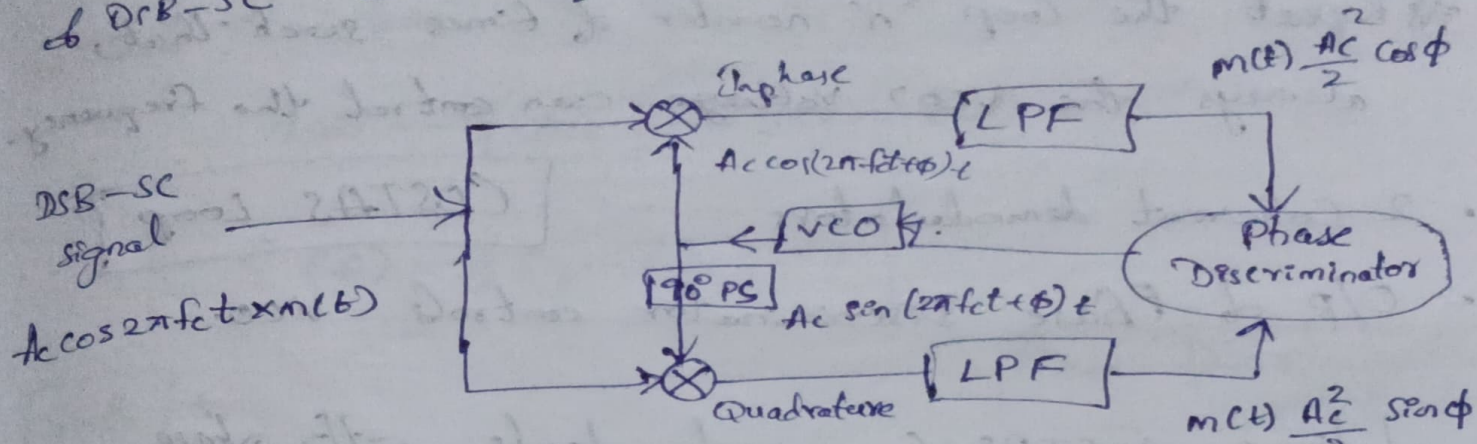
$\phi < 0 \uparrow$

$\phi = 0$ ✓ (desired one)

24/11/2022

Costas Loop :

- Purpose : is to make the random phase (ϕ zero), which is present due to noise.
- Maximum demodulated signal achieved in the demodulation of DSB-SC when $\phi = 0$



- VCO : Voltage Controlled Oscillator.
- PS : Phase shifter.
- Oscillator : without any i/p, ∿ analog signal is observed at o/p
- VCO : $f \propto V_{in}$; by controlling input voltage, frequency can be controlled.

$\Rightarrow m(t)\, A_c \cos 2\pi f_c t \; A_c \cos(2\pi f_c t+\phi)$

$\Rightarrow m(t)\, \frac{A_c^2}{2}\left[\cos(4\pi f_c t+\phi) + \cos\phi\right]$

$\Rightarrow m(t)\, \frac{A_c^2}{2}\cos\phi$

Ex. $\cos A \cos B \longrightarrow \cos(A+B) + \cos(A-B)$

Phase discriminator : if 2 freq. components given separate them into 2 or give a change in phase

Phase discriminator : $m^2(t) \frac{A_c^4}{4} \sin\phi \cos\phi$

- Increase phase if $\phi < 0$
- Decrease phase if $\phi > 0$

Repeat the loop 'n' number of times such that, always the VCO voltage can control the frequency.

$\boxed{\text{COSTAS Loop}}$

- 2 Coherent demodulators.
- O/P of Phase Discriminator controls VCO.

Limitation: COSTAS Loop only deals with phase.

- Does not deal with Amplitude changes & frequency changes.

SSB - Transmition : B.W = f_m

- SSB-SC / SSB Generation

1. Frequency Discrimination Method
2. Phase Discrimination Method

Phase Discrimination Method :

Hilbert Transform: Transform which changes the phase of the signal.

(90° phase shift)

$$\cos \rightarrow \boxed{\hat{H}(f)} \rightarrow \sin$$

Hilbert Transform : in time domain : $1/\pi t$

$$x(t) \rightarrow \boxed{1/\pi t} \rightarrow y(t) = \hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f)}$$

SSB:

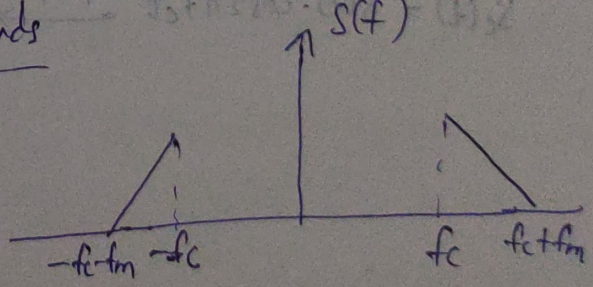
Phase Discrimination Method

USB: $s(t) = \underbrace{s_c(t)\cos 2\pi f_c t}_{I} - \underbrace{s_s(t)\sin 2\pi f_c t}_{Q}$

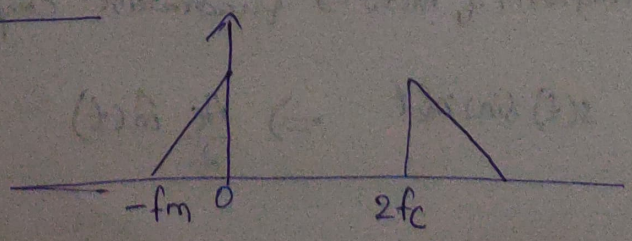
$$s_c(t) = s(t)\cos 2\pi f_c t \xrightarrow{FoT} \frac{1}{2}\left[S(f-f_c) + S(f+f_c)\right]$$

$$s_s(t) = s(t)\sin 2\pi f_c t$$

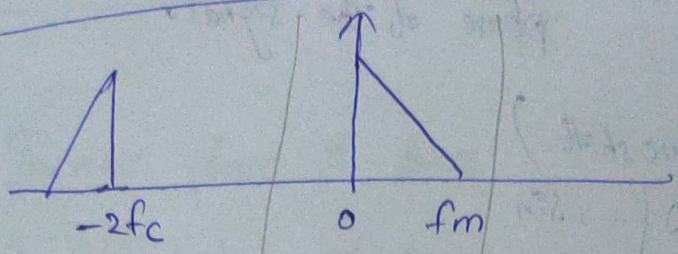
SSB with Upper Side Bands



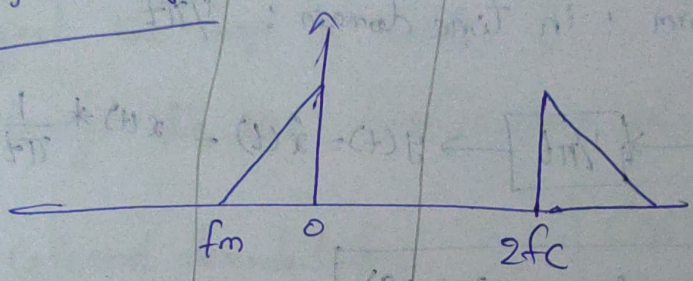
Providing a shift of $+f_c$



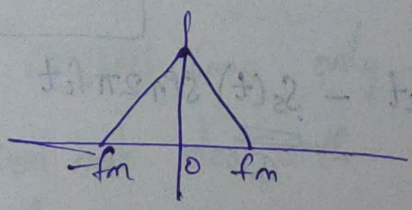
providing a shift of $-f_c$



Providing a shift of f_c



Combining & Passing through LPF



$$S(t) = S_c(t)\cos 2\pi f_c t - S_s(t)\sin 2\pi f_c t$$

$$S_c(t) = s(t)\cdot\cos 2\pi f_c t \xrightarrow{F.T} \frac{1}{2}\underbrace{[s(f-f_c)+s(f+f_c)]}_{M(f)}$$

$\Downarrow$ $m(t)$

(Inphase component)

Hilbert transform of $m(t) \rightarrow$ quadrature component

$$S_s(t) = s(t)\sin 2\pi f_c t \Rightarrow \frac{A_c}{2}\hat{m}(t)$$

$$\boxed{s(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t}$$

; where $\hat{m}(t) = m(t) * \frac{1}{\pi t}$

= Hilbert transform of $m(t)$

= provide 90° phase shift to $m(t)$

for

<u>Single tone</u>: $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = \frac{A_c}{2} A_m \cos 2\pi f_m t \cos 2\pi f_c t$$

$$- \frac{A_c}{2} A_m \sin 2\pi f_m t \sin 2\pi f_c t$$

$$= \frac{A_c A_m}{2} \left[\cos 2\pi f_m t \cos 2\pi f_c t - \sin 2\pi f_m t \sin 2\pi f_c t \right]$$

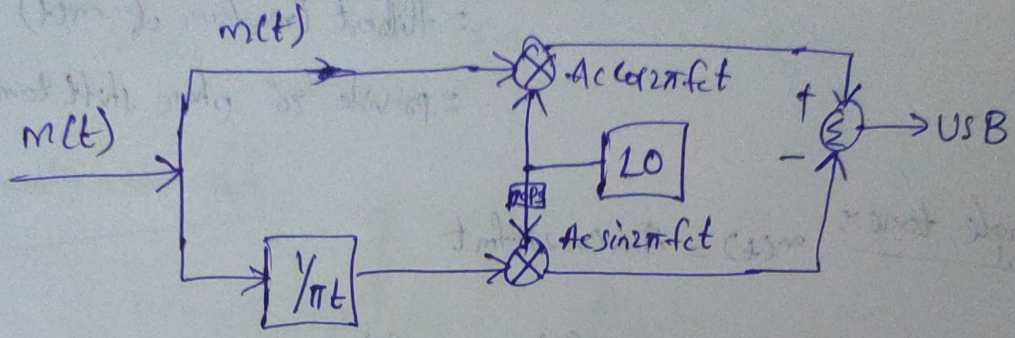
$$= \frac{A_c A_m}{2} \left[\cos 2\pi (f_c + f_m) t \right]$$

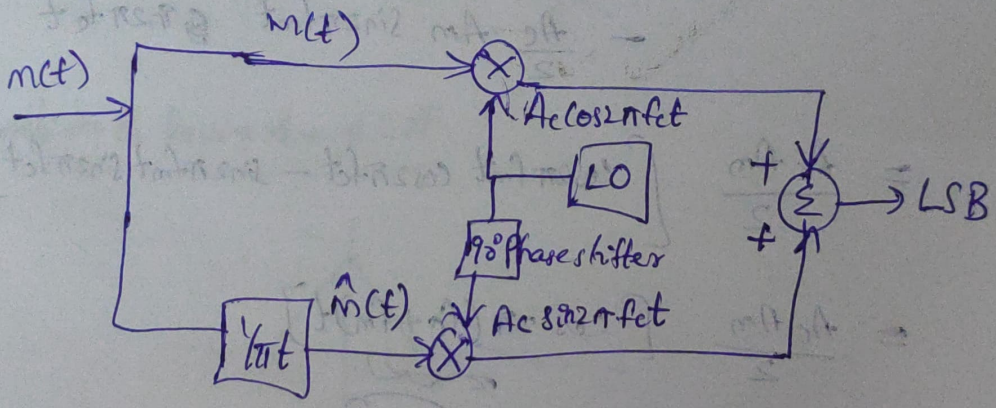
USB

LSB: $s(t) = s_c(t) \cos 2\pi f_c t + s_s(t) \sin 2\pi f_c t$

Block diagram for Phase Discrimination Method

(SSB - generation)





- SSB : Combination of 2 components, one inphase and the other is Quadrature.

- Pass through LPF → LSB
- Pass through HPF → USB

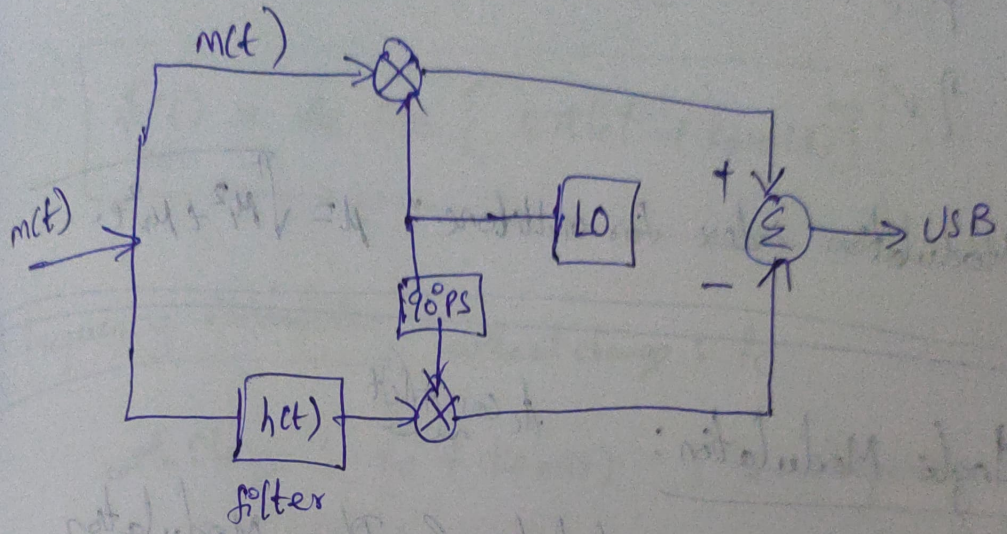
Frequency discrimination Method

AM Frequency Range

(88-108) MHz - Broadcasting

VSB :

Generation :



Comparison b/w AM Schemes

	CAM	DSB-SC	SSB-SC	VSB
Expression:	$A_c(1+k_a m(t))\cos 2\pi f_c t$	$A_c m(t)\cos 2\pi f_c t$ (product modulator)	$\frac{A_c}{2} m(t)\cos 2\pi f_c t - \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t$	
BW =	$2f_m$	$2f_m$	f_m ✓	$f_m \le B.W \le 2f_m$

Complex to operate

preferable: DSB ✓

, AM frequency range ?

- Do some problems on AM modulation scheme

P_T ✓

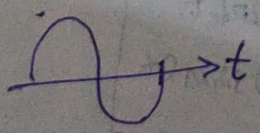
μ ✓

η ✓

Modulation index for multitone : $\mu = \sqrt{\mu_1^2 + \mu_2^2 + \dots}$

Angle Modulation:

$A_c \cos 2\pi f_c t$

- Both frequency Modulation & Phase Modulation
- Combination of FM & PM is called as Angle Modulation.
- Change the angle of carrier signal according to instantaneous amplitude of message signal.
- $\theta_i(t) \propto |m(t)|$
- The rate of change of phase — Frequency — $\frac{\text{Change in phase}}{\text{Change in time}}$
- 

$$\boxed{f = \frac{1}{2\pi}\frac{d\theta_i}{dt}}$$

Phase Modulation: Change the phase of carrier according to amplitude of $m(t)$.

Phase Modulation:

without change : $2\pi f_c t$

with change : $2\pi f_c t + k_p m(t)$; k_p - phase sensitivity

$$\theta_i = 2\pi f_c t + k_p m(t)$$

$$\boxed{\phi_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]}$$

Frequency Modulation:

without change : f_c

with Change : $f_c + k_f m(t)$; k_f = frequency sensitivity

$$f_i = f_c + k_f m(t)$$

$$\boxed{\therefore f = \frac{1}{2\pi}\frac{d\theta_i}{dt}}$$

$$\Rightarrow \theta_i = 2\pi f_c t + k_f \int_0^t m(t)\,dt$$

$$\boxed{\phi_{FM}(t) = A_c \cos\left[2\pi f_c t + k_f \int_0^t m(t)\,dt\right]}$$

Conversion from FM to PM & PM to FM:

- FM + Differentiator = PM
- PM + Integrator = FM

FM & PM :

1. Why FM preferred over PM for broadcasting

Phase is more sensitive to noise

2. Why high frequency components has no effect of noise ?

FM signal < Narrow band FM ≈ AM (similar)
Wide band FM → no upper limit (∞)

(Modulation Index (β))

less prone to noise (with penalty of B.W)

AM signal prone to noise

generation, Demodulation, Spectrum, B.W.

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Time domain Analysis of Frequency modulation

$$c(t) = A_c \cos 2\pi f_c t$$

$$f_i = f_c + k_f\, m(t) \quad ; \quad f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$\Rightarrow \theta_i(t) = 2\pi \int_0^t f_i(t)\,dt$$

$$s_{FM}(t) \Rightarrow A_c \cos\theta_i(t)$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

For singletone Modulation ; $m(t) = A_m \cos 2\pi f_m t$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{2\pi k_f A_m}{2\pi f_m} \sin 2\pi f_m t\right]$$

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$$\Rightarrow s_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{\Delta f = k_f A_m} \rightarrow \text{Frequency Deviation (variation)}$$

$$\boxed{s_{FM}(t) = A_c\left[\cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]\right]}$$

- Modulation Index in FM :

$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}}$$

$$\boxed{s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]}$$

dependent on β

① Wide band FM
$\beta > 1$
$\Delta f \uparrow$

② Narrow Band FM
$\beta < 1$
$\Delta f \downarrow$

Types of FM Signals

Narrow Band FM : Δf is small $\Rightarrow \beta << 1$
($\Delta f \downarrow$)

Wide Band FM : Δf is high $\Rightarrow \beta >> 1$
($\Delta f \uparrow$)
- B.W = ∞ (ideaticare)

Narrow Band FM : Generation $(\beta < 1)$

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$= A_c \cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - A_c \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t)$$

$\forall\ x \to 0$; $\cos x \to 1$, $\sin x \to x$ $\quad (\because \beta << 1)$

$$\Rightarrow s_{NBFM}(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t\ (\beta \sin 2\pi f_m t)$$

$$\boxed{\begin{aligned} \therefore s_{NBFM}(t) &= A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t\ (\beta \sin 2\pi f_m t) \\ &= A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \sin 2\pi f_m t \end{aligned}}$$

$$\Rightarrow s_{NBFM}(t) = A_c \cos 2\pi f_c t - \frac{A_c \beta}{2}\left[\cos 2\pi (f_c - f_m)t - \cos 2\pi (f_c + f_m)t\right]$$

$$= A_c \cos 2\pi f_c t - \frac{A_c \beta}{2} \cos 2\pi (f_c - f_m)t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m)t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m)t - \frac{A_c \beta}{2} \cos 2\pi (f_c - f_m)t$$

- NBFM is similar to CAM signal.

- LSB in NBFM is out of phase when compared with CAM signal.

- NBFM → B.W = $2f_m$ = B.W of CAM signal.

<u>Similarities</u> b/w $\boxed{\text{NBFM \& CAM}}$

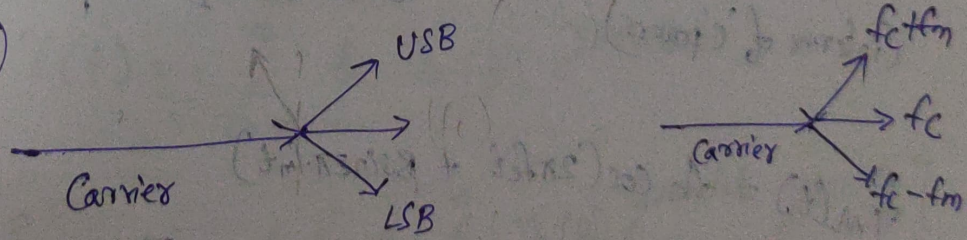
1. No. of freq. components are same
2. Same B.W

<u>Difference</u> : LSB out of phase in NBFM when compared to CAM.

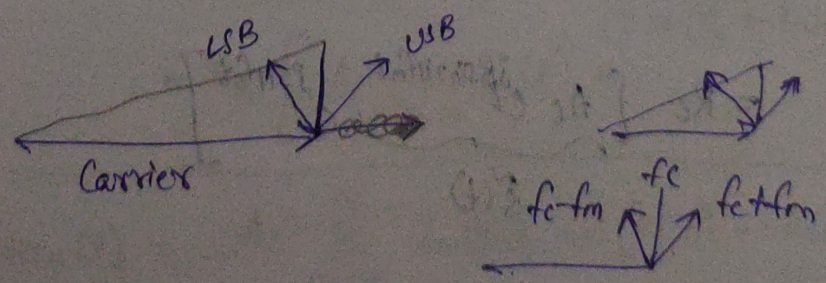
- Digital Modulation → Constellation Diagrams.
- Analog Modulation → Phasor Diagrams

Phasor Diagrams for FM & AM :

AM

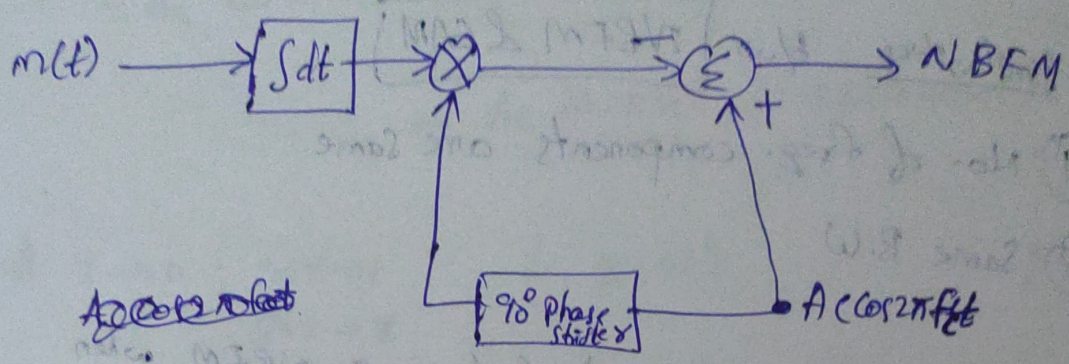


FM



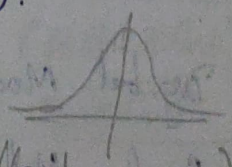
Block Diagram for generation of NBFM

(Summer, 90° PS, Integrator, Multiplier)



Wide Band F. Modulation: (Consists of infinite no. of frequency components).

Application: Broadcasting.



(monotonically decreasing)

- $\Delta f \to \infty$, $\beta >> 1$
- Complex Envelope: Representing High Freq. B.P. signals with Low freq. components.

(in terms of 'e' powers)

$$s_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$= Re\left[A_c e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\right]$$

$$= Re\left[\underbrace{A_c e^{j\beta \sin 2\pi f_m t}}_{\tilde{s}(t)} e^{j2\pi f_c t}\right]$$

$$\tilde{s}(t) = A_c e^{j\beta \sin 2\pi f_m t} \quad \text{(periodic signal)}$$

$$\Rightarrow \tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t} \quad \text{(in terms of fourier series)}$$

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j\beta \sin 2\pi f_m t} e^{-j(2\pi f_m t n)} dt$$

$$\Rightarrow C_n = f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j(\beta \sin 2\pi f_m t - 2\pi n f_m t)} dt$$

$x = 2\pi f_m t$

$dx = 2\pi f_m dt$

$$= \frac{f_m}{2\pi f_m} \int_{-\pi}^{\pi} A_c e^{j(\beta \sin x - nx)} dx$$

$$= \frac{1}{2\pi} \underbrace{\int_{-\pi}^{\pi} A_c e^{j(\beta \sin x - nx)} dx}_{\text{(Bessel's function)}}$$

$$\therefore \tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t}$$

$$s_{WBFM}(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t} \right]$$

$$= Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi (f_c + n f_m) t} \right]$$

$$\boxed{s_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi (f_c + n f_m) t]}$$

Wide Band Equation (FM) (Expression)

$$\boxed{S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]}$$

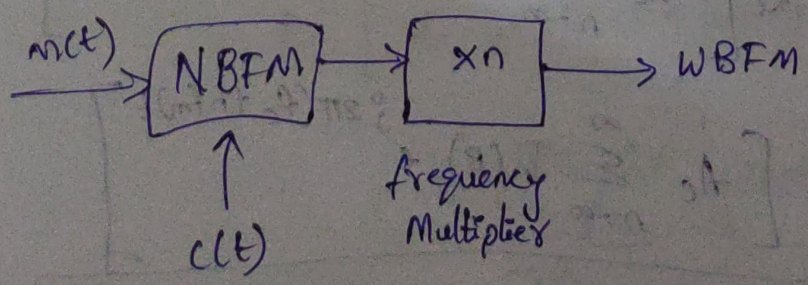
for $(n=1)$:
$$\begin{cases} f_c - f_m \Leftarrow A_c/2 \\ f_c \Leftarrow A_c \\ f_c + f_m \Leftarrow A_c/2 \end{cases}$$

→ Amplitude is indicated/determined by Bessel's function.

- Frequency components extend from $-\infty$ to ∞.
- B.W of WBFM → ∞ (Ideal case)
  → higher B.W (practical case)

Generation of WBFM:
- Direct Method
- Indirect Method (NBFM → WBFM) (using Frequency Multiplier)

- **Indirect Method** of generating WBFM



Direct Method of generating WBFM

- Take any Oscillator (Frequency generator) $f_c = \frac{1}{2\pi\sqrt{LC}}$
- Change any component L or C, WBFM ✓

Practical B.W of WBFM: Carson's Rule

$$B.W = 2(f_m + \Delta f) \quad ; \quad \underset{(\text{new freq})}{f} = f_m + \underset{\text{Freq. deviation}}{\Delta f}$$

$$= 2 f_m \left(1 + \frac{\Delta f}{f_m}\right)$$

$$= 2 f_m (1+\beta) = 2(\beta+1) f_m$$

$$\boxed{\therefore B.W_{(WBFM)} = 2(f_m + \Delta f) = 2 f_m (1+\beta) = 2(\beta+1) f_m}$$

Demodulation of FM signals:

- Indirect Method (FM → AM) LPD
- Direct Method (using PLL)

Indirect method of demodulating FM

$$s_{FM}(t) = \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$= \left[2\pi f_c + 2\pi k_f m(t)\right]\left(-\sin(\quad)\right)$$

$$= 2\pi f_c \left[1 + \frac{k_f}{f_c} m(t)\right] \sin(\quad)$$

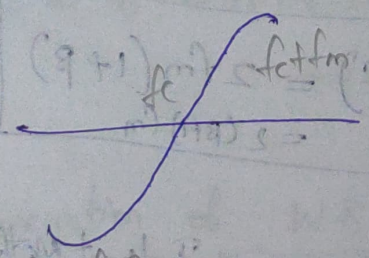
- Using Differentiator or slope detector: FM → AM

- One time differentiating FM signal → there are amplitude changes
  ↓
  AM signal

- Slope detector: $f_c + f_m \rightarrow \uparrow$ High freq → high Amplitude
  $f_c \rightarrow 0$
  $f_c - f_m \rightarrow \downarrow$ low freq → lower Amplitude

Indirect Demodulation } : Slope detector + Envelope Detector

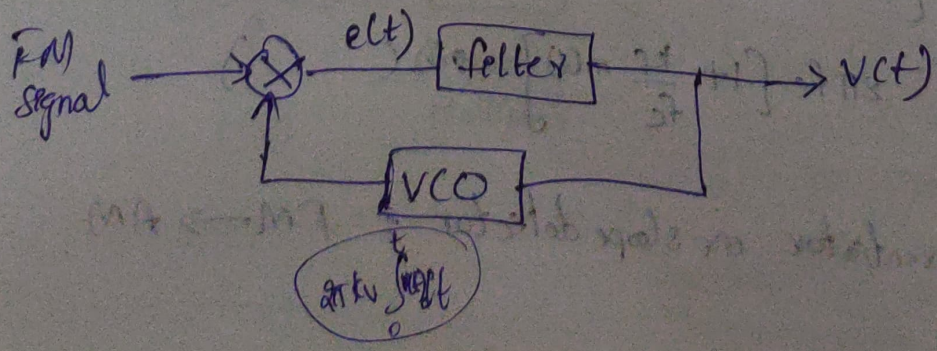
Frequency Discrimination method for demodulation of FM :



- Combination of 2 differentiator
- Combination of 2 sets

- **Direct Method** of demodulating FM signal: PLL method

Components:
- VCO (Heart of PLL) - Voltage Controlled Oscillator
- Phase Detector / Multiplier
- Filter



Conditions Required : ① Local Oscillator (VCO) always tuned to (f_c) Carrier freq. Component - - before starting PLL

② 90° Phase shift output of VCO to the FM signal

FM signal i/p

$$s_{FM}(t) = A_c \sin\Big[2\pi f_c t + \underbrace{2\pi k_f \int_0^t m(t)\,dt}_{\phi_1(t)}\Big] \rightarrow ①$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt$$

VCO o/p

$$\Rightarrow A_c \cos\Big[2\pi f_c t + \underbrace{2\pi k_v \int_0^t v(t)\,dt}_{\phi_2(t)}\Big] \rightarrow ②$$

$$\phi_2(t) = 2\pi k_v \int_0^t v(t)\,dt$$

Multiplying ① & ② eqns :

$$c(t) = A_c \sin\Big[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\Big] \; A_c \cos\Big[2\pi f_c t + 2\pi k_v \int_0^t v(t)\,dt\Big]$$

$$e(t) = \frac{A_c^2}{2} \sin\Big[\underbrace{\phi_1(t) - \phi_2(t)}_{\phi_e \text{ - phase error}}\Big]$$; after passing through LPF (rejecting high freq. Component)

$$v(t) = e(t) * h(t)$$; filter o/p

- $\phi_e(t) = \phi_1(t) - \phi_2(t)$

$$= \phi_1(t) - 2\pi k_v \int_0^t v(t)\,dt$$

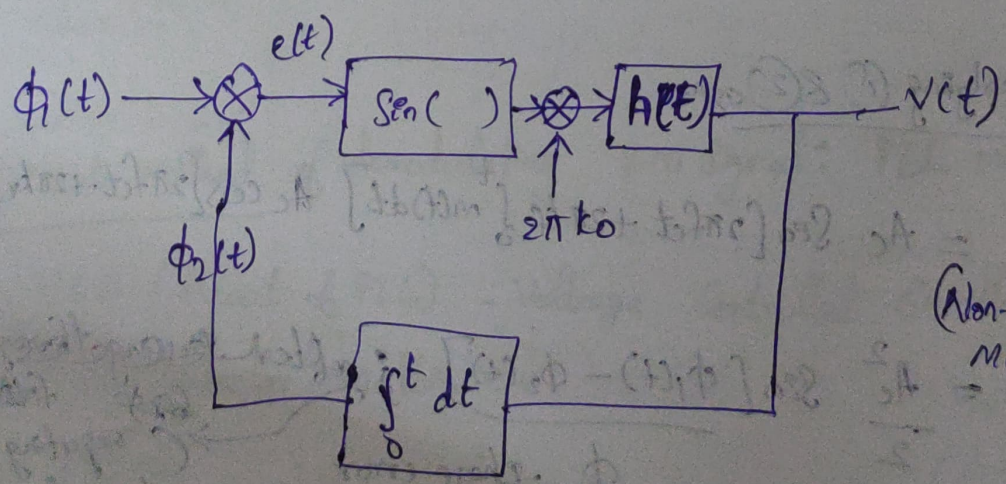
$$= \phi_1(t) - 2\pi k_v \int_0^t \left(e(t) * h(t)\right) dt$$

$$= \phi_1(t) - 2\pi k_v \int_0^t k_m A_c \sin(\phi_e(t))\, h(t-\tau)\, d\tau$$

$$= \phi_1(t) - 2\pi \underbrace{k_v k_m A_c}_{k_0} \int_0^t \sin(\phi_e(t))\, h(t-\tau)\, d\tau$$

$$\Rightarrow \phi_e(t) = \phi_1(t) - 2\pi k_0 \int_0^t \sin(\phi_e(t))\, h(t-\tau)\, d\tau$$

$$\Rightarrow \frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi_1(t) - 2\pi k_0 \sin\phi_e(t) * h(t-\tau)$$



(Non-linear Model)

$\boxed{\text{It becomes Linear}}$ when $\sin x \to x$ As $x \to 0 \Rightarrow \phi_e(t) \to 0$

$\Rightarrow \phi_1(t) = \phi_2(t)$

PLL - Linear model

- Phase of i/p FM signal same as phase of VCO o/p

- o/p voltage directly proportional to incoming frequency.

$$\phi_1(t) = \phi_2(t) \quad \text{(Phase locked)}$$

$$\Rightarrow 2\pi k_f \int_0^t m(t)\,dt = 2\pi k_v \int_0^t v(t)\,dt$$

$$\Rightarrow \boxed{v(t) = \frac{k_f}{k_v} m(t)}$$; message signal is obtained back.

Condition:

- Incoming phase should be locked with VCO o/p
- 90° Phase shift b/w FM & VCO o/p
- VCO always tuned to Carrier freq. component

FM Drawback:

- Requires More Bandwidth

FAQ Questions: on AM, FM, Pulse Modulations

AM – Types
Generation
Demodulation
Freq. Domain analysis / Spectrum
Time Domain Analysis → BW

FM – Types
generation
Demodulation
B.W