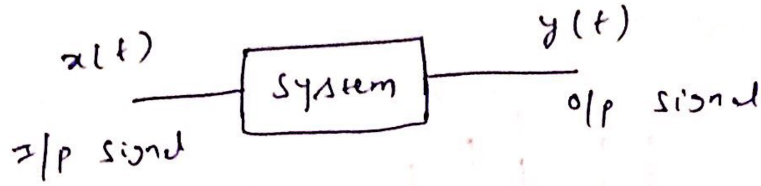
Signals and Systems



Applications:

1) Electrical Engg – Power systems, Smart grid
2) Electronic systems – TV, mobile phones
3) Communication systems – 3G, 4G, wireless systems, wifi – 802.11
4) Instrumentation & control – Aircraft

Signal: physical quantity that conveys information about phenomenon.

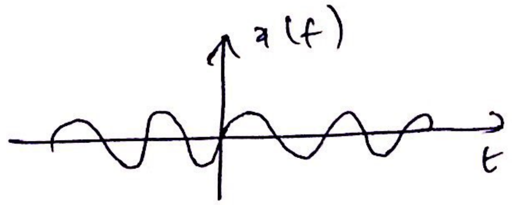
Ex: EM wave, typically exhibits variation in time or space, Image (space), video (space & time)

$x(t)$, $y(t)$ → Time varying signals

Classification of Signals:

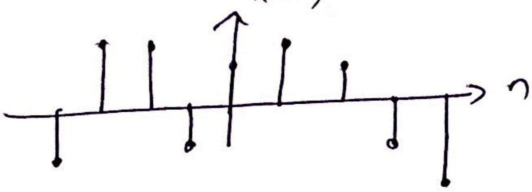
Continuous time (CT) signal: Defined for all time events in an interval $[t_1, t_2]$

Ex: $x(t) = \sin(2\pi t)$



Discrete time (DT) Signals: Defined at a discrete set of time instants.

Ex :



$x(-2), x(-1), x(0), x(1), x(2)$

Sequence or series of numbers

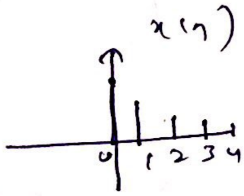
→ Discrete time signal can be obtained from continuous time signal using Sampling.

Applications — Continuous - FM Radio, TV broadcast

Discrete - Modem systems (3G/4G) GSM | wifi | TV settop

Example → DT signal.

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n ; & n \geq 0 \\ 0 ; & \text{else} \end{cases} \rightarrow$$

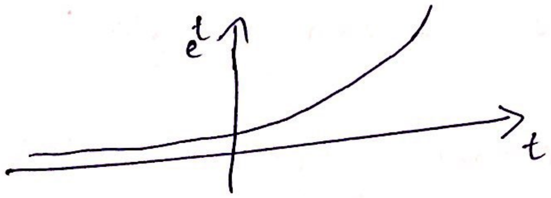


Analog and digital Signals :

→ **Analog Signal :** Continious time signal $x(t)$ can take values belonging to an interval $[a, b]$

Ex : 1) $\sin(2\pi f t) \in [-1, 1]$ → Infinitely possible values

2) $e^t \in (0, \infty)$



→ **Digital signal :** Discrete time signal $x(n)$ that can take discrete set of values or finite set of values.

Ex : $x(n) \in \{-1, 1\}$ → $x(n)$ either equals -1 or $+1$.
↳ can take only one of two values

Real and Complex Signals :

→ $x(t), x(n) \in R$ → set of Real numbers

Signals that take values belonging to set of real numbers — Real Signals.

Ex : 1) $\sin(2\pi f t)$
2) e^t } Real Signals

$\rightarrow x(t), x(n) \longrightarrow \mathbb{C}$ - complex numbers

$\searrow$ Signals that take values belonging to the set of complex numbers — complex signals

<u>Ex</u>: $x(t) = e^{j2\pi f t}$

$= \cos(2\pi f t) + j\sin(2\pi f t) \; ; \; j = \sqrt{-1}$

<u>Deterministic and Random Signals</u>:

$\rightarrow$ Determinstic Signal: $x(t)$ or $x(n)$ that is completely and specified at any given time instant.

<u>Ex</u>: $\sin(2\pi f t), \; e^{t}$

$\rightarrow$ Random Signal: Takes random values at different time instants.

<u>Ex</u>: ① $x(n) = \begin{cases} +1, & \text{If outcome} = \text{head} \\ -1, & \text{If outcome} = \text{Tails} \end{cases}$

$\uparrow$
Representing a coin toss experiment
(Discrete time random signal)

② 

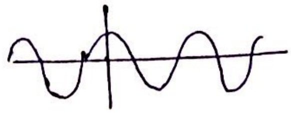
Random in nature

It is fundamentally imp to understand noise to characterize and analyse behaviour of system.

Noise limits the system performance.

Even and odd signals

→ Even signal : $x(t) = x(-t)$
$x(n) = x(-n)$

Ex : $\cos(2\pi f t)$ — 

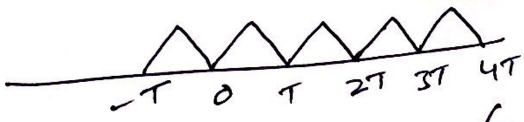
→ odd signal : $\cancel{x(t) = x(-t)}$
$\cancel{x(n) =}$

$x(-t) = -x(t)$
$x(-n) = -x(n)$

Ex : $x(t) = \sin(2\pi f t)$

Periodic & Aperiodic Signals :

→ $x(t)$ is periodic if there exists a time period T such that $x(t+T) = x(t)$ $\forall$ t.

Ex : ① 

② $\sin(2\pi t) = \sin(2\pi(t+1))$
$= \sin(2\pi t + 2\pi)$
$= \sin(2\pi t)$

$T = 1$ is period of $\sin(2\pi t)$

Fundamental Period : If 'T' is a Period of the Periodic signal, then mT is also a period for any integer m.

$x(t+mT) = x(t) \quad \forall t$

Fundemental Period = smallest ~~positi~~ time Period such that $x(t+T) = x(t)$

All other time Periods are mutiple of fundemental Period.

Ex: $\sin(2\pi t)$, $T=1$ is a fundemental Period.

$mT : \underbrace{2T, 3T, -T, -2T}_{\text{Periods}}$

→ For a discrete signal $x(n+N) = x(n) \quad \forall n$

Period of discrete time signal.

Smallest 'N' for which this holds = fundemental period N_0.

Energy & power Signals

→ Energy of the CT signal $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

→ Energy of the DT signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

→ If energy is finite ⇒ energy signal - $0 < E < \infty$

Ex: $e^{-t} u(t)$, $e^{-n} u(n)$

→ power $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$ - CT signal

$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$ - DT signal

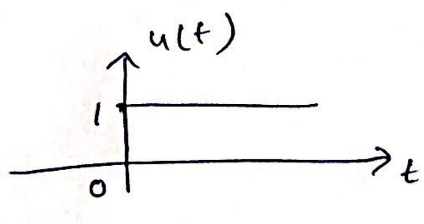
→ If Power is finite i.e $0 < P < \infty$ ⇒ power signal.

Ex: $\sin(2\pi f t)$

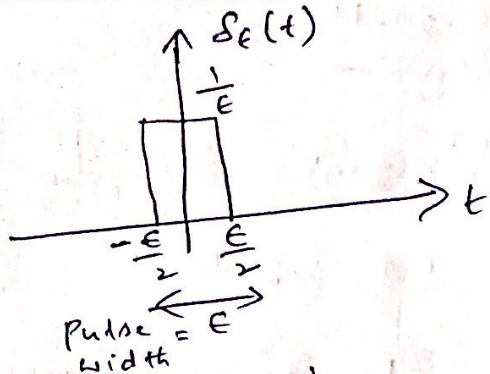
Important CT Signals

→ Unit-step signal:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



→ Unit impulse signal :



Height = $\frac{1}{\epsilon}$

Area under pulse $= \epsilon \times \frac{1}{\epsilon} = 1$

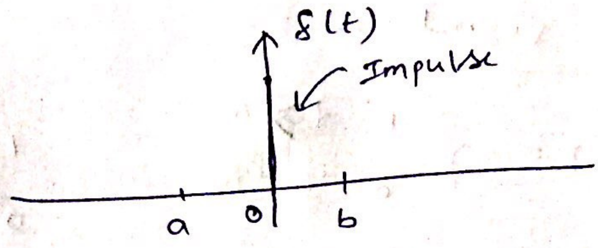
$$\int_{-\infty}^{\infty} \delta_\epsilon(t)\, dt = 1$$

Impulse signal $\quad \delta(t) = \lim_{\epsilon \to 0} \delta_\epsilon(t)$

As $\epsilon \to 0, \Rightarrow$ width $\epsilon = 0$, Height $\frac{1}{\epsilon} \to \infty$

But, Area = 1
= constant

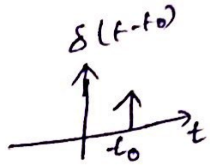
$$\int_a^b \delta(t)\, dt = 1 \; ; \; a < 0 < b$$
$$= 0 \; ; \; \text{otherwise}$$



Properties:

$\rightarrow \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$

$\rightarrow \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$



Impulse shifted to t_0

$\rightarrow \delta(at) = \frac{1}{|a|}\delta(t)$

$\rightarrow \delta(-t) = \delta(t)$

$\rightarrow \delta(t)\,x(t) = x(0)\,\delta(t) \rightarrow$ Shifting property of the impulse functn.

$\rightarrow \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)$

Proof: $\tau - t = \bar{t} \Rightarrow \tau = \bar{t} + t$

$d\tau = d\bar{t}$

LHS

$$\int_{-\infty}^{\infty} x(\bar{t}+t)\,\delta(-\bar{t})\,d\bar{t}$$

$$= \int_{-\infty}^{\infty} x(\bar{t}+t)\,\delta(\bar{t})\,d\bar{t}$$

$$= x(\bar{t}+t)\Big|_{\bar{t}=0}$$

$$= x(t)$$

Complex exponential

$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$

$|x(t)| = \sqrt{\cos^2(\omega_0 t) + \sin^2(\omega_0 t)} = 1$

$$e^{j2\pi f_0 t} = e^{j\omega_0 t}$$

$$\boxed{f_0 = \frac{\omega_0}{2\pi}}$$

(Hz) Rad/sec

(Angular frequency)

Period $T = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$

If $f_0 = 5\,Hz \Rightarrow T = \frac{1}{5} = 0.2$ Sec.

General Complex exponential:

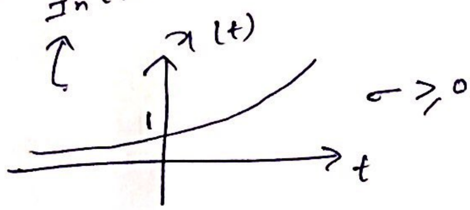
$$s = \sigma + j\omega$$

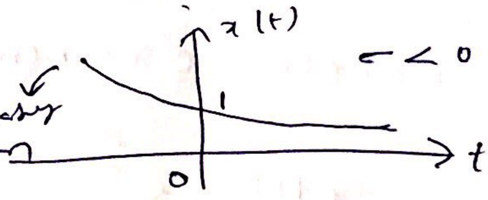
$$x(t) = e^{st}$$
$$= e^{(\sigma + j\omega)t}$$
$$= e^{\sigma t}\left[\cos(\omega t) + j\sin(\omega t)\right]$$

Real Exponential signals

$$x(t) = e^{\sigma t}$$

Increasing function





Decreasing function

Sinusoidal Signal.

$$x(t) = A\cos(\omega_0 t + \theta)$$

A = Amplitude

ω_0 = Angular frequency

θ = phase

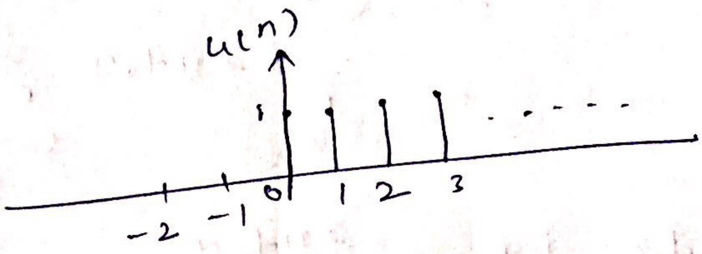
$f_0 = \frac{\omega_0}{2\pi}$ = Fundamental frequency

$x(t)$ is a periodic signal.

Basic discrete time (DT) signals

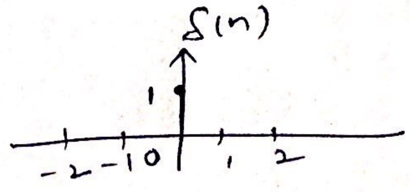
Unit step function:

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Unit impulse function:

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



Properties :

$\rightarrow \quad \sum_{n=-\infty}^{\infty} \delta(n) = 1$

$\rightarrow \quad \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) = x(n) \rightarrow$ shifting property

$\rightarrow \quad \delta(n) = u(n) - u(n-1)$

$\rightarrow \quad u(n) = \sum_{k=-\infty}^{n} \delta(k)$

Complex exponential DT Signal

$$x(n) = e^{j\Omega_0 n}$$
$$= \cos(\Omega_0 n) + j \sin(\Omega_0 n)$$

Is $x(n)$ periodic ?

$$e^{j2\pi f_0 n} = e^{j2\pi f_0 (n+N)}$$
$$= e^{j2\pi f_0 n} \underbrace{e^{j2\pi f_0 N}}_{1}$$

Periodic if $e^{j2\pi f_0 N} = 1$

$f_0 N = K \rightarrow$ Integer

$$\boxed{f_0 = \frac{K}{N}}$$

Rational number

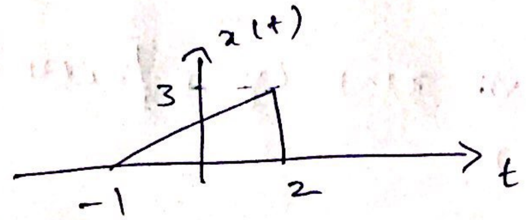
Therefore, $e^{j\Omega_0 n}$ is periodic only if

$\frac{\Omega_0}{2\pi} = \frac{m}{n} \rightarrow$ Rational number

Basic Signal operations :

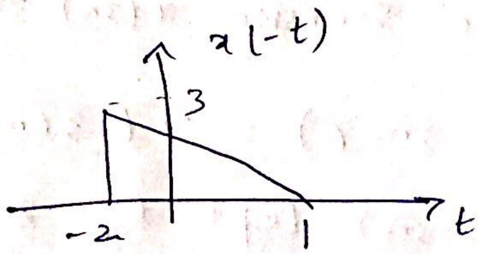
1) Shifting
2) Scaling
3) Time - Reversal

Ex



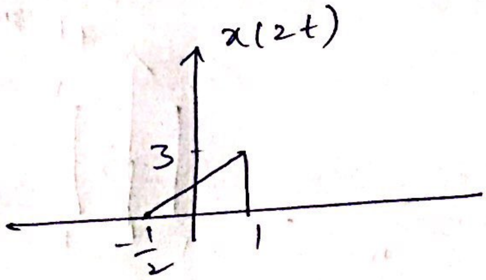
$$x(t) = \begin{cases} t+1 \; ; & -1<t<2 \\ 0 \; ; & \text{else} \end{cases}$$

Time reversed →



To get $x(-t)$ reflect $x(t)$ about $t=0$

Time Scaling →

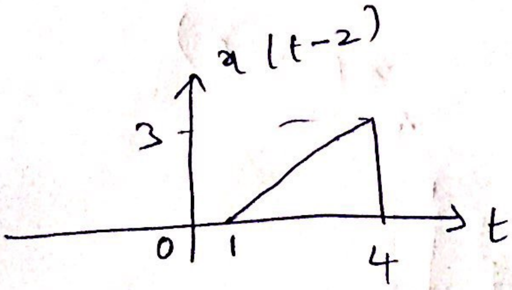


$x(at) \rightarrow$ Time scaling

$|a|>1 \rightarrow$ compression of $x(t)$

$|a|<1 \rightarrow$ Expansion of $x(t)$

Shifting

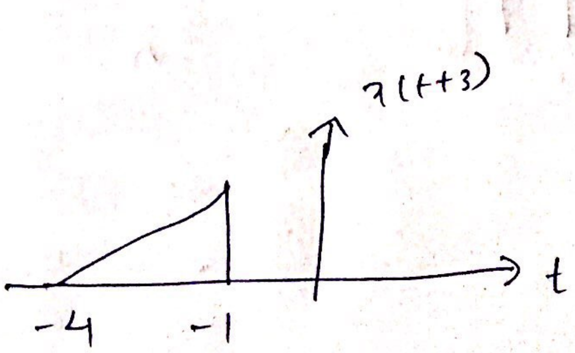


$x(t-t_0)$

$t_0>0 \rightarrow$ delay by t_0

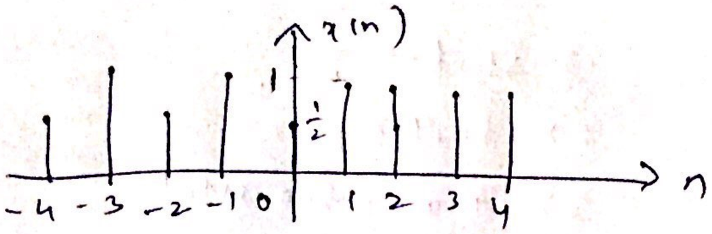
$t_0<0 \rightarrow$ Advance by t_0

$x(t-2) \rightarrow$ Shift $x(t)$ by 2



$x(t+3)$ Shift $x(t)$ left by 3

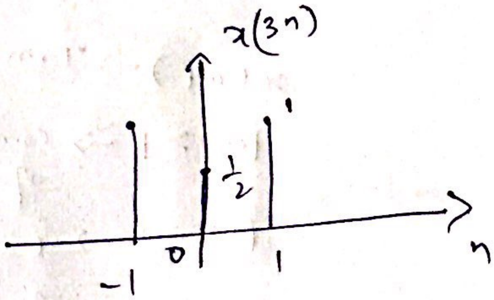
Ex : Sketch the waveform $x(-6-3n)$

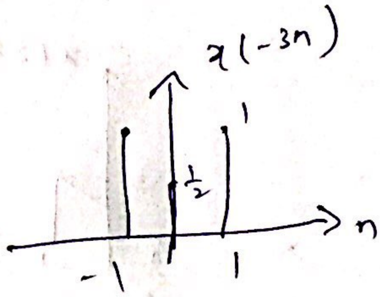


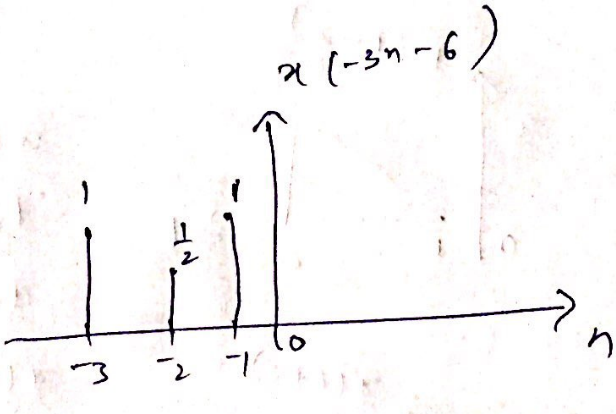
(i) compress $x(n)$ by a factor 3 to obtain $x(3n)$

(ii) Time - reverse $x(3n)$ to obtain $x(-3n)$

(iii) $x(-3n-6) = x(-3(n+2))$. Shift $x(-3n)$ left by 2 units.







→ Check $x(t) = e^{-t} u(t)$ is a energy (or) power signal ?

$$E = \int_0^{\infty} |x(t)|^2 dt$$

$$= \int_0^{\infty} e^{-2t} dt$$

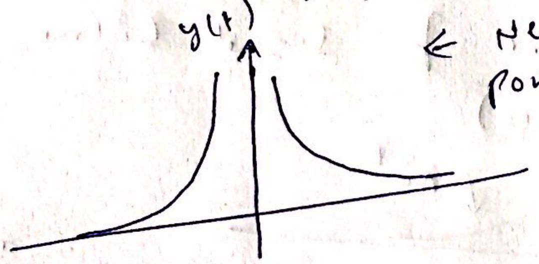
$$= 1/2 \text{ (finite)}$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-2t} dt = \frac{1}{\infty} = 0$$

∴ since $0 < E < \infty$, $x(t)$ is a energy signal.

For energy signal power $= 0$

→ $y(t) = \left|\frac{1}{t}\right|$



← Neither energy nor power signal

→ $y(t) = A\,u(t)$

$$P_{avg} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 dt = \frac{A^2}{2}$$

$$E = \int_{-\infty}^{\infty} |y(t)|^2 dt = \infty$$

so $y(t)$ is a power signal. The signal can't be both energy and power signal

→

→ All periodic signals are power signals because they maintain constant amplitude over infinite time

→ Ex: $x(t) = A\cos(\omega_0 t + \theta)$

$$P_{avg} = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

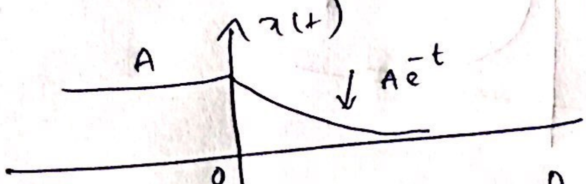
→ $x(t) = A e^{j\omega_0 t}$

$$P_{avg} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left|A e^{j\omega_0 t}\right|^2 dt$$

$$= A^2$$

→ Complex sinusoidal is also a power signal.

→ What is nature of signal below.



$$P_{avg} = \lim_{T\to\infty} \frac{1}{T}\left[\int_{-T/2}^{0} A^2\, dt + \int_{0}^{T/2} A^2 e^{-2t}\, dt\right]$$

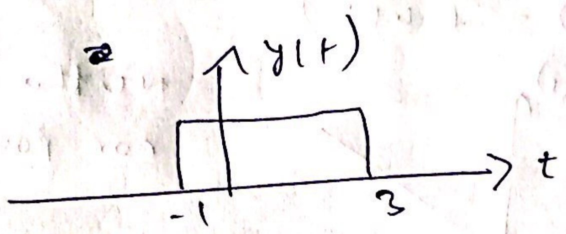
$$= A^2/2$$

So Power + Energy = power

→ If $x(t) = \delta(t+1) - \delta(t-3)$. Find energy in $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$$y(t) = \int_{-\infty}^{t} \left(\delta(t+1) - \delta(t-3)\right) d\tau$$

$$= u(t+1) - u(t-3)$$



$$E_{y(t)} = \int_{-1}^{3} (1)^2\, dt = 4$$

→ All finite duration signal of finite amplitude are energy signals.

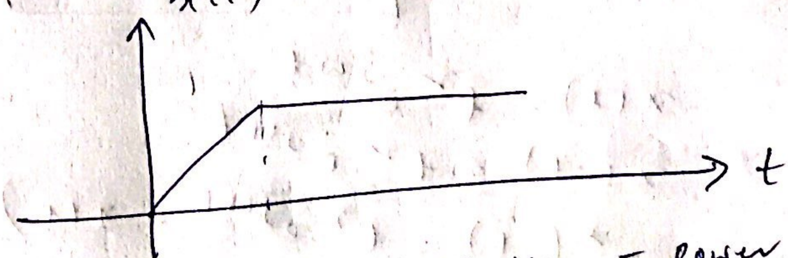
→ For above signal

$$E_{y(2t)} = \int_{-1/2}^{3/2} (1)^2\, dt = \frac{4}{2} = 2$$

$$\boxed{E_{y(\alpha t)} = \frac{1}{|\alpha|}\, y(t)}$$

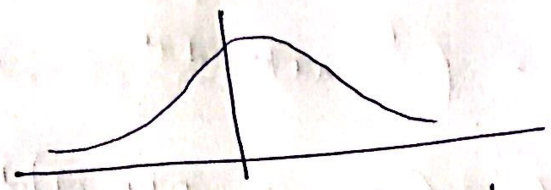
$$x(t) = r(t) - r(t-1)$$

→



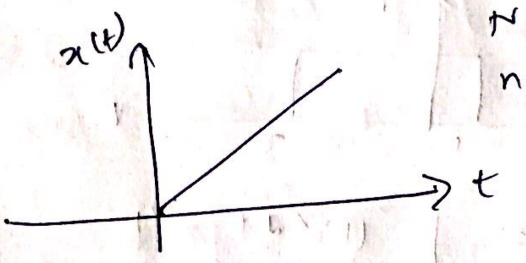
Energy + Power = Power

→ $x(t) = e^{-\pi t^2}$



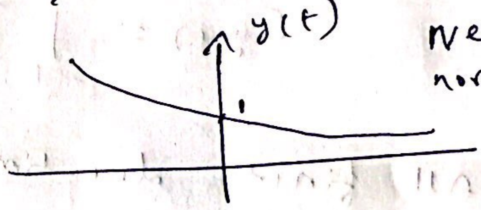
Energy signal

→ $x(t) = t\,u(t)$



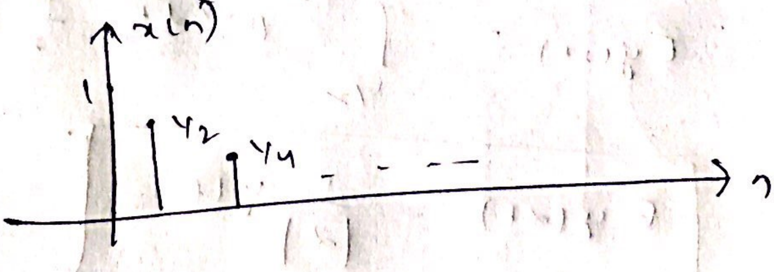
Neither energy nor power signal

→ $y(t) = e^{-3t}$



Neither Energy nor power signal

→ $x(n) = \left(\frac{1}{2}\right)^n u(n)$ → Energy signal



Even conjugate : $\boxed{x(t) = x^*(-t)}$

Ex : $x(t) = t^2 e^{j\omega t}$

$x(-t) = t^2 e^{-j\omega t}$

$x^*(-t) = t^2 e^{j\omega t} = x(t)$

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→ **Odd Conjugate**

$$\boxed{x(t) = -x^*(-t)}$$

Ex: $x(t) = t\,e^{j\omega t}$

→ $x(t) = x_e(t) + x_o(t) \rightarrow (1)$

$x(-t) = x_e(-t) + x_o(-t)$

$x(-t) = x_e(t) - x_o(-t) \rightarrow (2)$

Solving (1), (2)

$$\boxed{\begin{aligned} x_e(t) &= \frac{x(t)+x(-t)}{2} \\ x_o(t) &= \frac{x(t)-x(-t)}{2} \end{aligned}}$$

→ Steps for finding time Period of $x_1(t)+x_2(t)+x_3(t)$

1. $T_1, T_2, T_3, \ldots$
2. $T_1/T_2,\ T_1/T_3,\ T_1/T_4, \ldots$
3. If all Ratios of second step is Rational number then the overall signal is periodic
4. LCM of Denominators of 2nd Step.
5. $T = (LCM)\,T_1$

(a)

$\longrightarrow$ $y(t) = \cos(50\pi t) + \sin(60\pi t)$

(1) $\omega_1 = 50\pi$ | $T_2 = \frac{1}{30}$

$T_1 = \frac{2\pi}{\omega_1} = \frac{1}{25}$

(2) $\frac{T_1}{T_2} = \frac{30}{25} = \frac{6}{5}$

(3) Valid ✓

(4) LCM = 5

(5) $T = (LCM) \times T_1$

$= 5 \times \frac{1}{25}$

$$\boxed{T = \frac{1}{5}}$$

$\longrightarrow$ $y(t) = e^{j\frac{5\pi}{6}t} + e^{j\frac{\pi t}{3}}$

$$\boxed{T = 12}$$

$\longrightarrow$ $x(t) = \cos(13\pi t) + \sin(17t)$

$T_1 = \frac{2}{13}$, $T_2 = \frac{2\pi}{17}$

So, $\frac{T_1}{T_2} = \frac{17}{13\pi}$ (irrational)

So, $x(t)$ is a non-periodic signal.

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→ $y(t) = \cos 50\pi t + \sin 60\pi t$

(1) $\omega_1 = 50\pi$ | $T_2 = \frac{1}{30}$

$T_1 = \frac{2\pi}{\omega_1} = \frac{1}{25}$

(2) $T_1 / T_2 = \frac{30}{25} = 6/5$

(3) valid

(4) L.C.M = 5

(5) $T = (LCM) \times T_1 = 5 \times \frac{1}{25} = \frac{1}{5}$

Alternative method using GCD

$\omega_1 = 50\pi = 5(10\pi)$

$\omega_2 = 60\pi = 6(10\pi)$

$\omega_0 = 10\pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = \frac{1}{5}$

→ $y(t) = e^{j\frac{5\pi t}{6}} + e^{j\frac{\pi t}{3}}$

$\omega = GCD\left(\frac{5\pi}{6}, \frac{2\pi}{6}\right) = \frac{\pi}{6}$

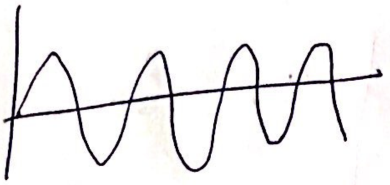
$T = \frac{2\pi}{(\pi/6)} = 12$

→ $y(t) = j\, e^{j10t}$

sol $y(t) = e^{j\pi/2} e^{j10t} = e^{j\left(\frac{\pi}{2} + 10t\right)}$

$T = \frac{2\pi}{10} = \frac{\pi}{5}$

$\rightarrow \quad y(t) = \sin\left(\frac{\pi t}{6}\right) u(t)$

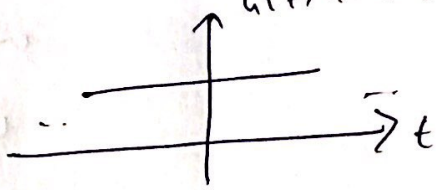

$\rightarrow$ Non-Periodic

$\rightarrow$ All Periodic signals are everlasting signals

$\rightarrow \quad x(t) = Ev\left[\cos(3\pi t)\, u(t)\right]$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{1}{2} \cos 3\pi t \left[u(t) + u(-t)\right]$$

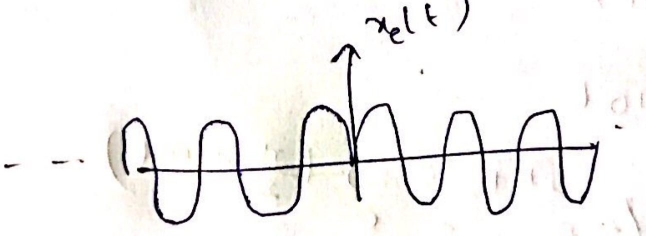


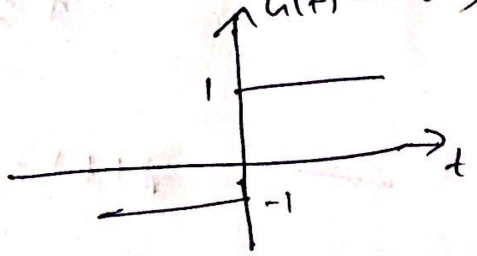
$$\omega = \frac{2\pi}{T} = 3\pi$$

$$T = \frac{2\pi}{3\pi} = \frac{2}{3}$$

$\rightarrow \quad x(t) = Ev\left[\sin(3\pi t)\, u(t)\right]$

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{\sin(3\pi t)}{2}\left[u(t) - u(-t)\right]$$

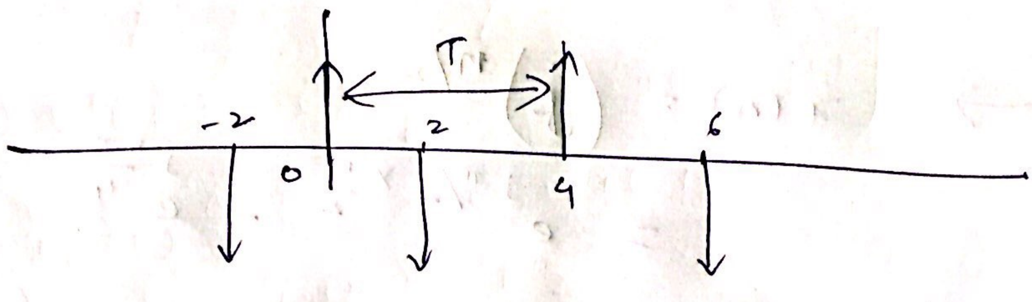




Non-Periodic

$$\rightarrow \quad y(t) = \sum_{k=-\infty}^{\infty} (-1)^k \, \delta(t-2k)$$

$$y(t) = \cdots - \delta(t+2) + \delta(t) - \delta(t-2) + \delta(t-4) + \cdots$$



$$\boxed{T = 4s}$$

$$\rightarrow \quad x(n) = \sin\left(\frac{3\pi}{5} n\right)$$

$$\frac{\omega_0}{2\pi} = \frac{m}{N}$$

$$\frac{(3\pi/5)}{2\pi} = \frac{m}{N} \Rightarrow \frac{3}{10} = \frac{m}{N} \Rightarrow \boxed{N = 10}$$

$$\rightarrow \quad x(n) = \cos\left(\frac{n}{6} + \frac{\pi}{4}\right)$$

$$\frac{\omega_0}{2\pi} = \frac{m}{N} \Rightarrow \frac{1}{12\pi} = \frac{m}{N} \quad \text{(irrational)}$$

Non-Periodic

$$\rightarrow \quad x(n) = \sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{4}\right)$$

$$\omega_1 = \frac{\pi}{3}, \quad \omega_2 = \frac{\pi}{4}$$

$N_1 = 6, \quad N_2 = 8$

$N = LCM(N_1, N_2) = LCM(6, 8) = 24$

$\boxed{N = 24}$

$\rightarrow \quad x(n) = (j)^{n/2}$

$= e^{j\pi/2 \times \frac{n}{2}} = e^{j\frac{n\pi}{4}}$

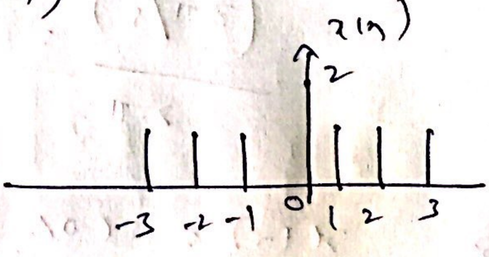
$\frac{\omega_0}{2\pi} = \frac{m}{N} \Rightarrow$

$\frac{(\pi/4)}{2\pi} = \frac{m}{N} \Rightarrow \frac{1}{8} = \frac{m}{N}$

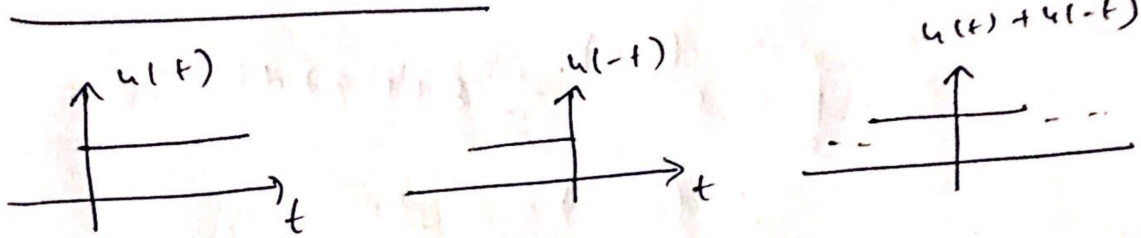
$\boxed{N = 8}$

$\rightarrow \quad x(n) = u(n) + u(-n)$

Aperiodic



→ Unit step function



$$u(t) + u(-t) = 1$$

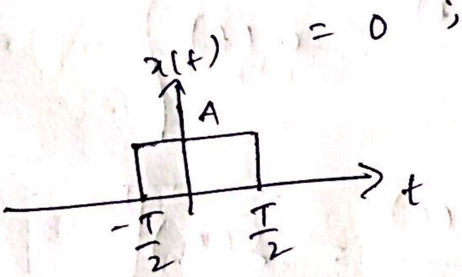
$$u(0) + u(0) = 1$$

$$\boxed{u(0) = \frac{1}{2}}$$

→ Rectangular function:

$$x(t) = A\,\mathrm{rect}(t/T) = A \quad ; \quad -\frac{T}{2} < t < \frac{T}{2}$$

$$= 0 \quad ; \quad \text{else}$$



→ Examples:

① $\delta(t)\cos(t) = \cos(\pi) = -1$

② $t\,\delta(t) = 0$

③ $\int_0^{\infty} (t + \cos\pi t)\,\delta(t-1)\,dt = 1 + \cos\pi$
$= 1 - 1$
$= 0$

④ $\int_0^{\infty} \underbrace{\cos(t)\,u(t-3)}_{x(t)}\,\delta(t-1)\,dt = 0$

$\downarrow$

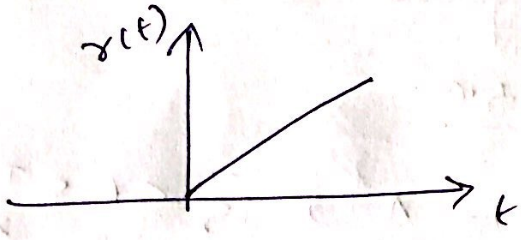
$\cos(1)\,\underbrace{u(-2)}_{0}$

(5) $\int_0^{\infty} e^{(t-3)}\,\delta(3t-9)\,dt$

$$= \int_0^{\infty} e^{(t-3)}\,\frac{1}{3}\,\delta(t-3)\,dt$$

$$= \frac{1}{3}\,e^{(3-3)} = \frac{1}{3}$$

→ **Unit Ramp function**



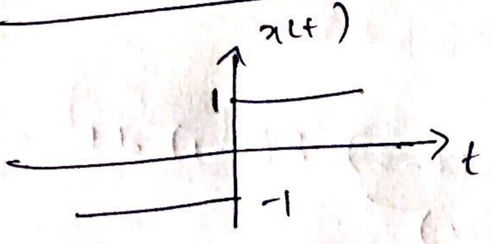
$$r(t) = t\,;\ t > 0$$
$$= 0\,;\ t < 0$$

$$\boxed{\frac{d\,r(t)}{dt} = u(t)}$$

$$\boxed{\frac{d\,u(t)}{dt} = \delta(t)}$$

$$\boxed{u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau}$$

→ **Signum function :**



$$\text{sign}(t) = u(t) - u(-t)$$
$$= 2u(t) - 1$$

$u(t) + u(-t) = 1$

$u(-t) = 1 - u(t)$

Scanned with CamScanner

→ Causal and Non-causal Signals

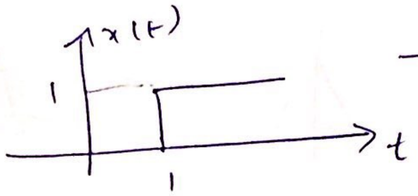
Causal : Defined for positive time only

⇒ $t > 0$ (or) $n > 0$
(CT) (DT)

⇒ $x(t) = 0$; $t < 0$

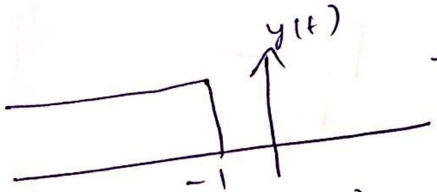
$x(n) = 0$; $n < 0$

Ex : ① $x(t) = u(t-1)$



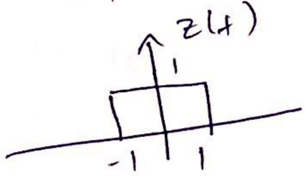
→ causal

② $y(t) = u(-t-1) = u[-(t+1)]$



→ Anti-causal

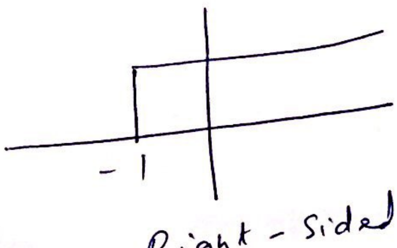
③



→ Non-causal

Note : All causal signals are right sided signal, but all right sided signal may or may not be causal signal.

Ex :



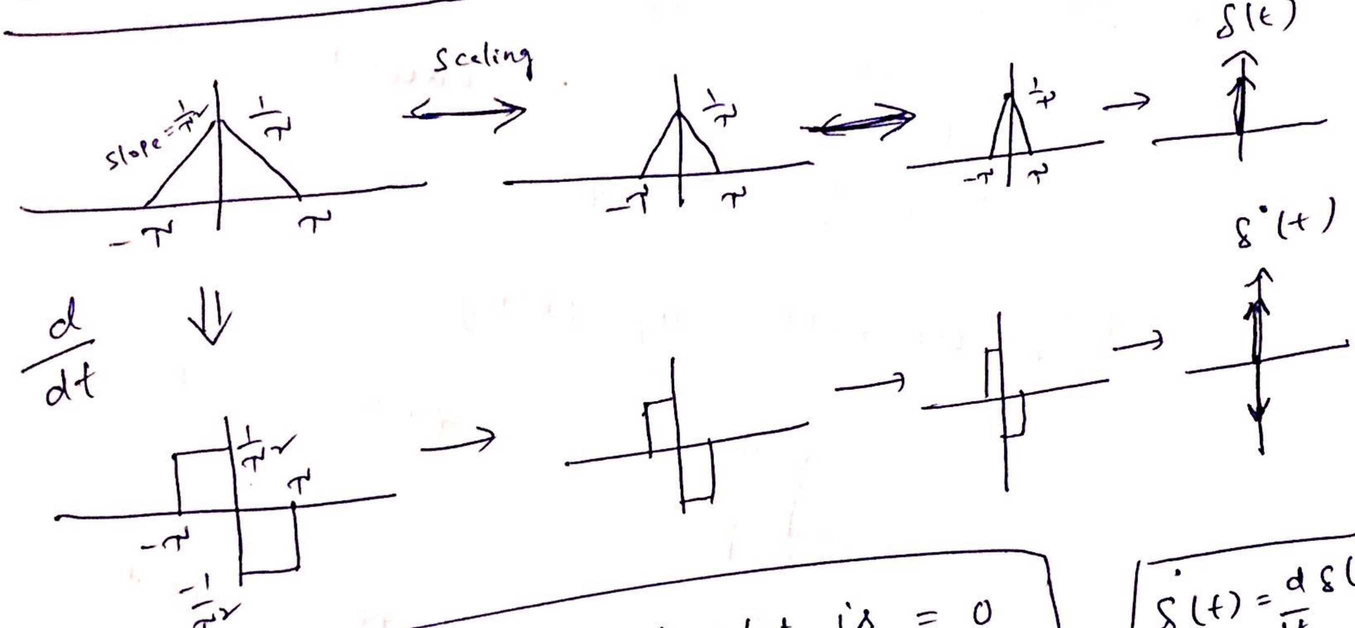
Right-sided
Non-causal

→ Periodic and Random signals are power signal.

→ But aperiodic and Deterministic signal may (or) may not be energy signal.

Ex : u(t) → is non-periodic and it is power signal

Doublet function



$\boxed{\text{Area under doublet is } = 0}$

$\boxed{\dot{\delta}(t) = \frac{d\,\delta(t)}{dt}}$

Shifting property

① $\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = x(t_0)\;;\quad t_1 \le t_0 \le t_2$

② $\int_{t_1}^{t_2} x(t)\,\delta^{(n)}(t-t_0)\,dt = (-1)^n \frac{d^n x(t)}{dt^n}\Big|_{t=t_0}$

Ex:

$$\int_0^{\infty} e^{-2t^2}\, \delta^{\cdot}(t-10)\, dt$$

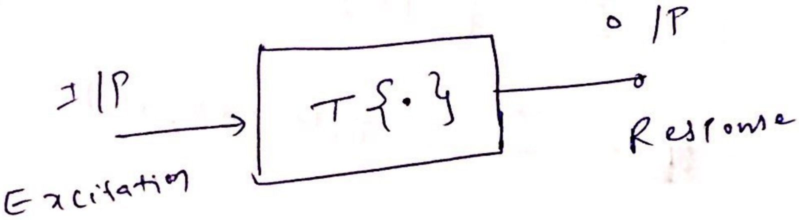
$t_0 = 10$, $\quad 0 < 10 < \infty$, $n = 1$

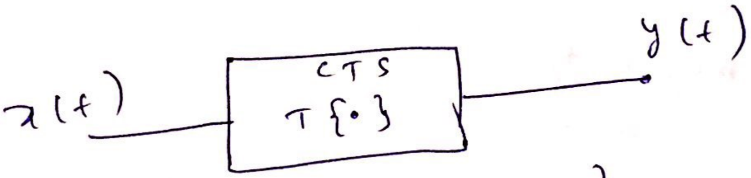
$$\int_0^{\infty} e^{-2t^2}\, \delta'(t-10)\, dt = (-1)^1 \frac{d}{dt}\left(e^{-2t^2}\right)\Big|_{t=10}$$

$$= -e^{-2t^2}(-4t)\Big|_{t=10}$$

$$= 40e^{-200}$$

Systems

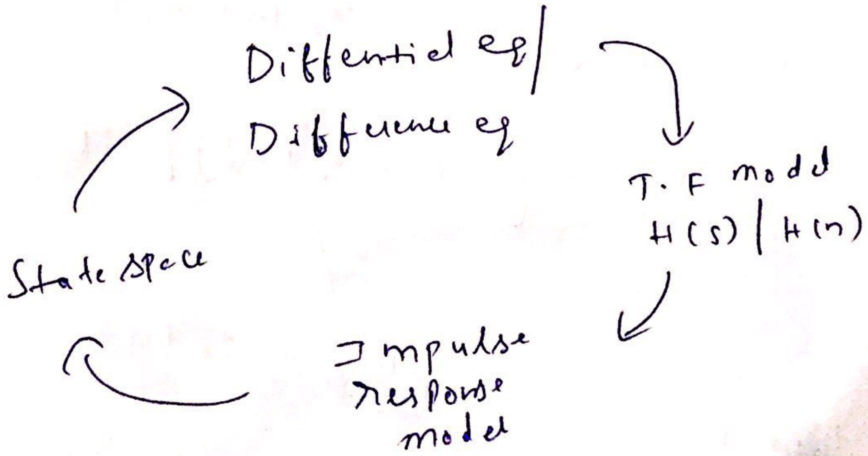




$$y(t) = T\{x(t)\}$$

Ex: $y(t) = \ln x(t)$

$y(t) = x^2(t)$



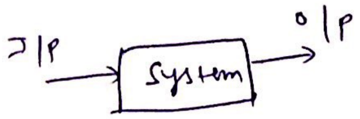
Classification of systems

- → Linear, Stable, Invertible → considering amplitude
- → Time inveriant, static, causal → time

→ Linear & Non-linear system

Linear ⟹ superposition = Additivity + Homoginity

① **Additivity :**



$$x_1(t) \xrightarrow{T} y_1(t)$$

$$x_2(t) \xrightarrow{T} y_2(t)$$

$$x_1(t) + x_2(t) \xrightarrow{T} y_1(t) + y_2(t)$$

② **Homoginity**

$$\alpha\, x(t) \longrightarrow \alpha\, y(t)$$

Ex :

$$y(t) = x(t)\, x(t-3)$$

$$x_1(t) \longrightarrow y_1(t) = x_1(t)\, x_1(t-3)$$

$$x_2(t) \longrightarrow y_2(t) = x_2(t)\, x_2(t-3)$$

$$(x_1(t) + x_2(t)) \rightarrow [x_1(t) + x_2(t)]\,[x_1(t-3) + x_2(t-3)]$$

$$\neq$$

$$y_1(t) + y_2(t)$$

Non-linear

$\rightarrow \quad y(t) = x(t) \cos 5t$

$x_1(t) \xrightarrow{T} y_1(t) = x_1(t) \cdot \cos(5t)$ $\quad \xrightarrow{i/p} \boxed{\text{System}} \xrightarrow{o/p}$

$x_2(t) \xrightarrow{T} y_2(t) = x_2(t) \cos(5t)$

$x_1(t) + x_2(t) \xrightarrow{T} [x_1(t) + x_2(t)] \cos 5t$

$= x_1(t) \cos 5t + x_2(t) \cos 5t$

$= y_1(t) + y_2(t)$

Linear

$\rightarrow \quad y(t) = \sin[x(t)]$

$y_1(t) = \sin[x_1(t)] \quad ; \quad y_2(t) = \sin(x_2(t))$

$x_1(t) + x_2(t) \xrightarrow{T} \sin[x_1(t) + x_2(t)]$

$\neq y_1(t) + y_2(t)$

Non-linear

$\rightarrow \quad y(t) = |m(t)|$

Non-linear

$\rightarrow \quad y(t) = 3x(t) + 5$

~~o/p due to~~ $\alpha y(t) = \alpha[x(t) + 5]$

$= \alpha x(t) + 5\alpha$

o/p due to $\alpha x(t) = 3\alpha x(t) + 5$

$\neq \alpha y(t) \rightarrow$ Non-linear

$\rightarrow \quad y(n) = 2^{x(n)} \rightarrow$ Non-linear

$\rightarrow \quad y(n) = x^*(n)$

$\alpha\, y(n) = \alpha\, x^*(n)$

$\alpha\, x(n) \xleftrightarrow{T} [\alpha\, x(n)]^* = \alpha^* x^*(n) \neq y(n)$

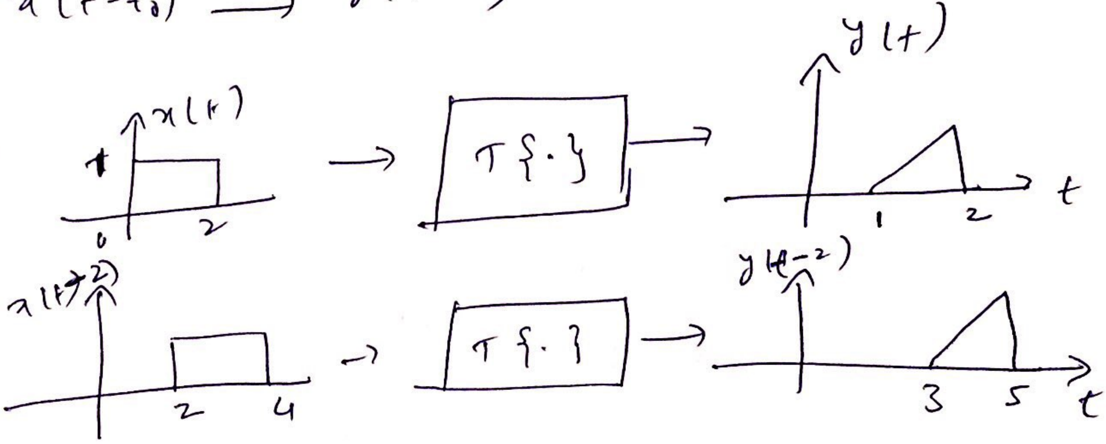
2) Time invariant / shift invariant :

$\rightarrow$ A system is T.I if I/P and O/P characteristics do not change with time

If $\quad x(t) \xrightarrow{T} y(t)$

$\quad\quad x(t-t_0) \xrightarrow{T} y(t-t_0)$

Ex : (1)



(2) $\quad y(t) = t\, x(t)$

O/P due to delay input $\quad y_1(t) = t\, x(t-t_0)$

$y(t-t_0) = (t-t_0)\, x(t-t_0) \neq y_1(t)$

Time variant.

3. $y(t) = e^{-x(t)}$

o/p due to delayed I/p $y_1(t) = e^{-x(t-t_0)}$

$y(t-t_0) = e^{-x(t-t_0)} = y_1(t)$

Time invariant

4. $y(t) = x^2(t)$ — Time invariant

5. $y(t) = x(2t)$

o/p due to delayed I/p $y_1(t) = x(2t-t_0)$

$y(t-t_0) = x(2(t-t_0)) = y(2t-2t_0) \neq y_1(t)$

Time Variant

6. $y(t) = x(-t)$ — Time variant

7. $y(n) = g(n)\, x(n)$ — Time invariant

8. $y(n) = 3x(n)$ — Time invariant

→ Causal and Non- Causal System

A system is causal If the present o/p depends only on present I/p and past values of the I/p but not on future values i.e Causal systems are non- anticipative.

Ex

① $y(t) = (3t+1)\,x(t)$

② $y(t) = x(t)\cos\omega_0(t)$ — Causal

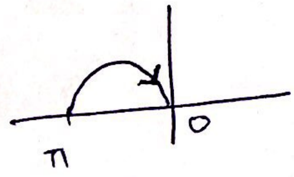
③ $y(t) = \sin[x(t)]$

④ $y(t) = x(\sin(t))$ — Non causal

$\sin t = 0$ for $t = \pm m\pi$, $m = 0, 1, 2, \ldots$

$y(-\pi) = x(\sin(-\pi)) = x(0)$

o/p is expecting future values



⑤ $y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$ — Non causal

⑥ $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ — causal

⑦ $y(n) = 2x(n) + 3u(n+1)$ — causal

⑧ $y(n) = \sum_{k=-\infty}^{n} x(k)$ — causal

It is a accumulator

$y(1) = \ldots + x(-2) + x(-1) + x(0) + x(1)$

→ Static | memory less & Dynamic | with memory

Static system is that in which o/p at Particular instant is depend only on I/p at that instant only.

Ex: ① $y(t) = e^{-(t+3)}$
② $y(t) = 2x(t) + 3$
③ $y(n) = g(n+3) \; x(n)$
} Static

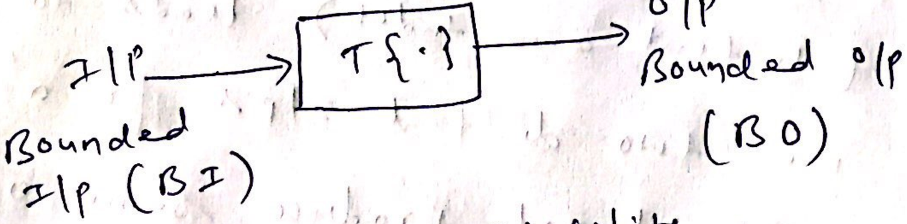
④ $y(n) = x(3)$ — Dynamic

→ Note: All Static Systems are Causal Systems.

→ Stable and Un Stable Systems

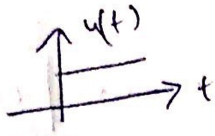
It is a magnitude concept

For a finite I/P, o/p should be finite

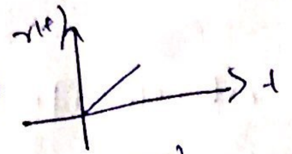


BIBO - Stability

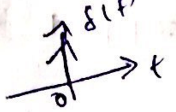
If $|x(t)| \leq M_x < \infty$ then
$|y(t)| \leq M_y < \infty$
} Stable

Ex: (1) 

→ stable

(2) 

→ un stable

(3) 

→ un stable

(4) $y(t) = \frac{d}{dt} x(t)$

Let $x(t) = u(t)$

$y(t) = \delta(t)$ → un stable

(5) $y(t) = x(t) \cos(\omega t)$

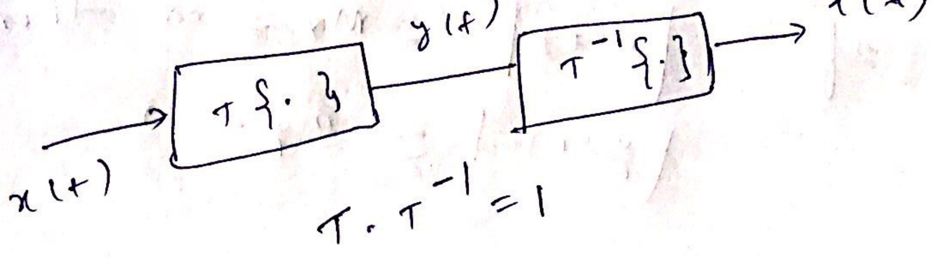
$|y(t)| = |x(t) \cos(\omega t)|$

$= |x(t)| \cos(\omega t)$

⇒ stable

→ <u>Invertible & inverse system</u>

A system is an invertible if different I/ps (different amplitudes) leads to different o/ps i.e two different I/ps for a given system should not produce same o/p.

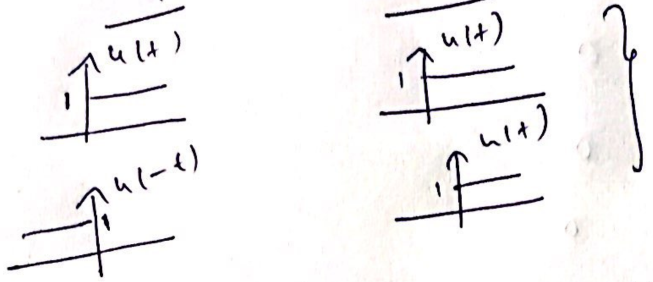


$T \cdot T^{-1} = 1$

Ex:

① $y(t) = x^2(t)$

I/P   O/P



Two diffnt I/Ps leads to same O/P

So, Non-invertible

② $y(t) = x(t-4)$

I/P		O/P
$\delta(t)$	$\rightarrow$	$\delta(t-4)$
$-\delta(t)$	$\rightarrow$	$-\delta(t-4)$

Two diffnt I/Ps leads to two diffnt O/Ps

Inverse is $y(t+4)$

③ $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

I/P	O/P
$\delta(t)$	$u(t)$
$-\delta(t)$	$-u(t)$
$u(t)$	$t\,u(t)$
$-u(t)$	$-t\,u(t)$

So Invertible

Inverse is $\frac{d\,y(t)}{dt}$

(3) $y(t) = \frac{d}{dt} x(t)$

I/P	O/P
2	0
3	0
4	0

Non invertible

(4) $y(n) = n\,x(n)$

I/P	O/P
$\delta(n)$	$n\,\delta(n) = 0$
$-\delta(n)$	$-n\,\delta(n) = 0$

Non Invertible

(5) $y(n) = x(n)\,x(n-3)$

I/P	O/P
$\delta(n)$	$\delta(n) \cdot \delta(n-3) = 0$
$-\delta(n)$	$-\delta(n)\,\delta(n-3) = 0$

Non-invertible

(6) $y(n) = \sum_{k=-\infty}^{n} x(k)$ — Invertible

I/P	O/P
$\delta(n)$	$u(n)$
$-\delta(n)$	$-u(n)$

Continous Convolution

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ \text{system} \\ h(t)\end{matrix}} \longrightarrow y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

Steps:

1. $x(t) \longrightarrow x(\tau)$
   $h(t) \longrightarrow h(\tau)$
2. Folding $\begin{cases} x(-\tau) \\ h(-\tau) \end{cases}$
3. Shifting $\begin{cases} x(t-\tau) \\ h(t-\tau) \end{cases}$
4. multiplication $\begin{cases} x(t-\tau)\, h(\tau) \\ h(t-\tau)\, x(\tau) \end{cases}$
5. Integration

Discrete convolution

$$x(n) \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ \text{system}\end{matrix}} \longrightarrow y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)\, h(k)$$

Steps :

1. $x(n) \longrightarrow x(k)$
   $h(n) \longrightarrow h(k)$

2. Folding $\left\langle \begin{array}{l} x(-k) \\ h(-k) \end{array} \right.$

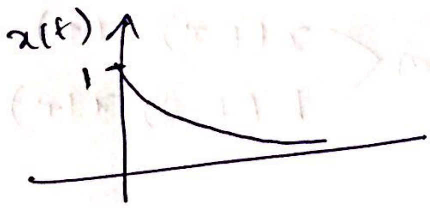
3. Shifting $\left\langle \begin{array}{l} x(n-k) \\ h(n-k) \end{array} \right.$

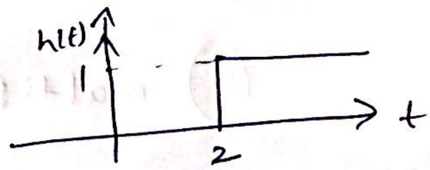
4. Multiplication $\left\langle \begin{array}{l} x(k)\, h(n-k) \\ x(n-k)\, h(k) \end{array} \right.$

5. Summation

→ 1. Find the convolution of the two signals

$x(t) = e^{-3t} u(t)$, $h(t) = u(t-2)$





Steps

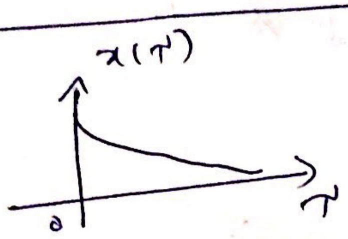
1. Obtaining the limits of $y(t)$:

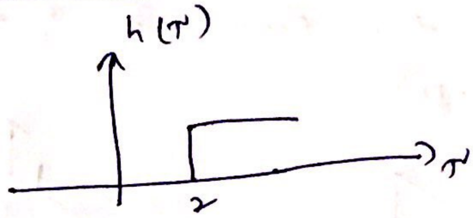
Sum of lower limits < t < Sum of upper limits

$0 + 2 < t < \infty + \infty$

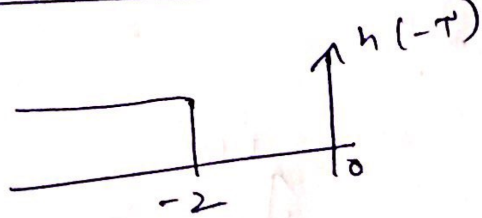
$$\boxed{2 < t < \infty}$$

(2) Change of axis from 't' to 'τ'





(3) Folding | Flipping

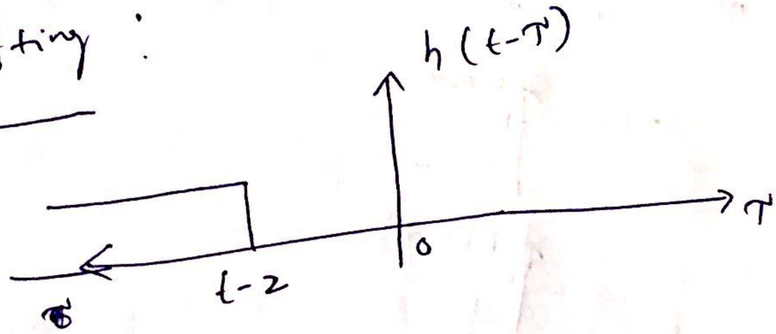


obtain $h(t-\tau)$ from $h(\tau)$

$$h(\tau) = \begin{cases} 1; & \tau > 2 \\ 0; & \text{else} \end{cases}$$

$$h(t-\tau) = 1; \quad t-\tau > 2, \quad \tau < t-2$$
$$= 0; \quad \text{else}$$

(4) Shifting :



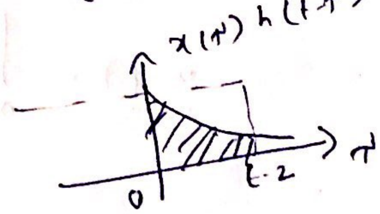
(5) Multiplication

Case 1 : $t-2 < 0 \Rightarrow t < 2$

$y(t) = 0 \quad ; t < 2$

No overlapping

Case 2 : $t-2 > 0$



$$y(t) = \int_0^{t-2} e^{-3\tau} u(\tau)\, d\tau$$

$$= \frac{1 - e^{-3(t-2)}}{3} \quad ; t > 2$$

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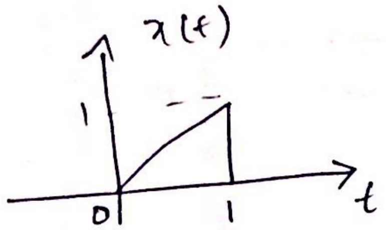
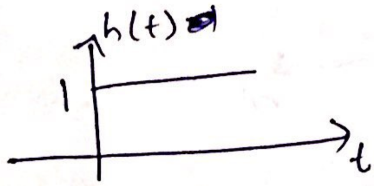
Note: Convolution of two Causal signals is Causal.

→ Find the Convolution of the two signals

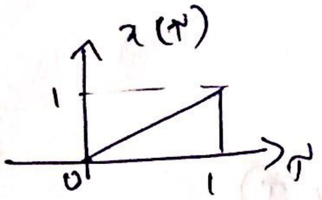


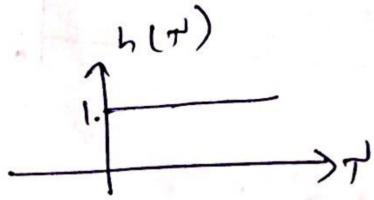


Step 1: $0+0 < t < 1+\infty$

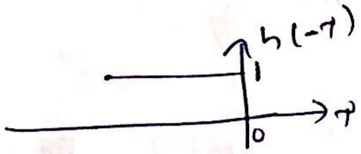
$$\boxed{0 < t < \infty}$$

Step 2:

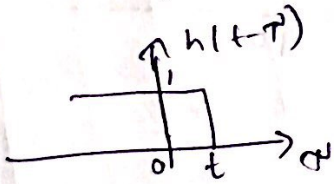




Step 3:

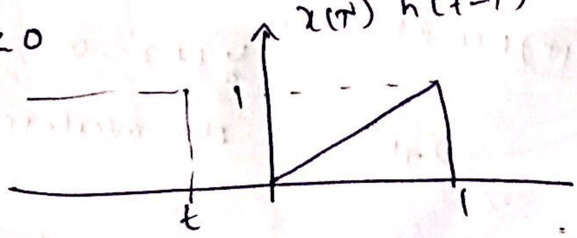


Step 4:



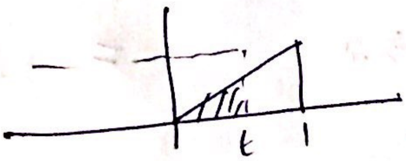
Step 5: Multiplication & Integration

Case 1: $t<0$



$y(t) = 0$

Case 2: $0<t<1$



$$y(t) = \int_0^t (\tau).(1)\,d\tau$$

$$= t^2/2 \,;\, 0<t<1$$

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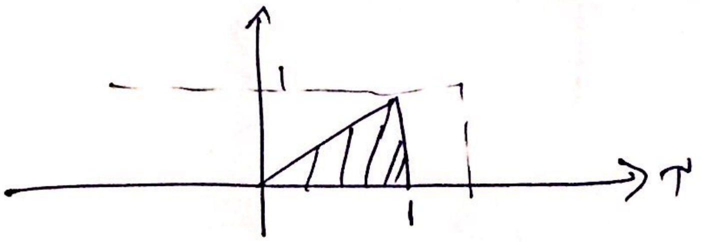
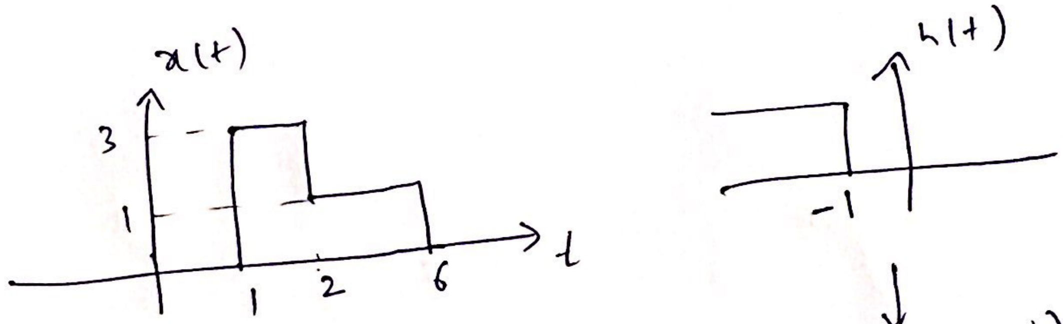
Case 3 : when $t > 1$



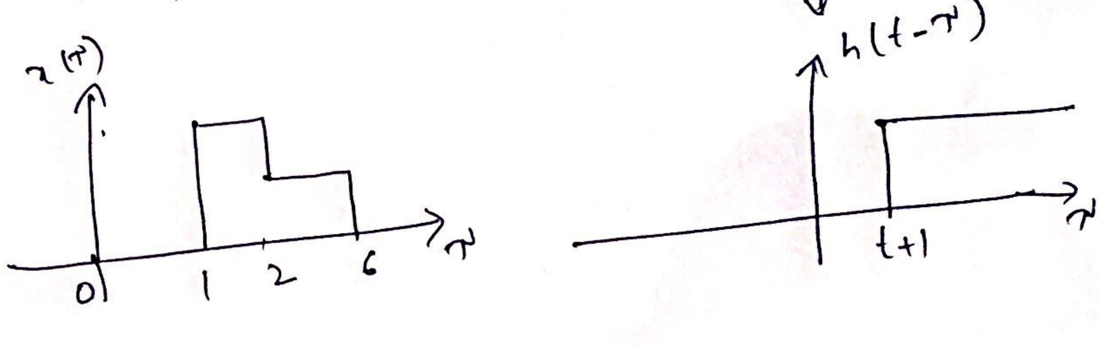
$$y(t) = \int_0^1 (\tau) \cdot 1 \, d\tau = \frac{1}{2}$$

$$y(t) = \begin{cases} 0 \ ; & t < 0 \\ t^2/2 \ ; & 0 < t < 1 \\ \frac{1}{2} \ ; & t > 1 \end{cases}$$

→



Sol



Homework