

① (a) The system is not memoryless because  $y[n]$  depends on past values of  $x[n]$

(b) The o/p of the system will be  $y[n] = \delta(n) \delta(n-2) = 0$ .

The system o/p is always zero for inputs of the form  $\delta(n-k)$ ,  $k \in \mathbb{Z}$ . Therefore, the system is not invertible.

② (a)

The system is not causal because the o/p  $y(t)$  at some time may depend on future values of  $x(t)$ .

For instance  $y(-\pi) = x(0)$ .

(b) Consider two arbitrary inputs  $x_1(t)$  and  $x_2(t)$ .

$$x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \rightarrow y_2(t) = x_2(\sin(t))$$

Additivity:

$$\begin{aligned} x_1(t) + x_2(t) &\rightarrow x_1(\sin(t)) + x_2(\sin(t)) \\ &= y_1(t) + y_2(t) \end{aligned}$$

Homogeneity

$$\alpha x(t) \rightarrow \alpha x(\sin(t)) = \alpha y(t)$$

Therefore, the system is linear.

③ Consider two arbitrary inputs  $x_1[n]$  and  $x_2[n]$

$$(a) \quad x_1[n] \rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \rightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Additive Property

$$\begin{aligned} x_1[n] + x_2[n] &\rightarrow \sum_{k=n-n_0}^{n+n_0} [x_1[k] + x_2[k]] \\ &= \sum_{k=n-n_0}^{n+n_0} x_1[k] + \sum_{k=n-n_0}^{n+n_0} x_2[k] \\ &= y_1[n] + y_2[n] \end{aligned}$$

Homogeneity

$$\alpha x[n] \rightarrow \sum_{k=n-n_0}^{n+n_0} (\cancel{\alpha} x[k]) = \cancel{\alpha} \sum_{k=n-n_0}^{n+n_0} x[k] = \alpha y[n]$$

Therefore, the system is linear

$$(b) \quad y[n] = x[n-n_0] + x[n-n_0+1] + \dots + x[n+n_0-1] + x[n+n_0]$$

If the  $x[n]$  is delayed by  $n_1$  units then

$$\begin{aligned} x[n-n_1] &\rightarrow x[n-n_1-n_0] + x[n-n_1-n_0+1] + \dots + \\ &\quad x[n-n_1+n_0-1] + x[n-n_1+n_0] \\ &= y[n-n_1] \end{aligned}$$

∴ Therefore, the system is ~~invariant~~ time-invariant.

3 (c) If  $|x(n)| < B$ , then

$$y(n) \leq (2n_0 + 1) B$$

$$\text{Therefore } C \leq (2n_0 + 1) B.$$

(4)

(a) Consider two arbitrary inputs  $x_1(t)$  and  $x_2(t)$

Additivity

$$x_1(t) \rightarrow y_1(t) = t^r x_1(t-1)$$

$$x_2(t) \rightarrow y_2(t) = t^r x_2(t-1)$$

$$x_1(t) + x_2(t) \rightarrow t^r [x_1(t-1) + x_2(t-1)]$$

$$= t^r x_1(t-1) + t^r x_2(t-1)$$

$$= y_1(t) + y_2(t)$$

Homogeneity

$$\alpha x(t) \rightarrow t^r [\alpha x(t-1)] = \alpha t^r x(t-1)$$

$$= \alpha y(t)$$

∴ The system is linear.

(b) Consider ~~two~~  
If the IIP is delayed by  $t_0$  units

$$x(t-t_0) \rightarrow t^r x(t-t_0) \neq y(t-t_0)$$

∴ The system is Time-Invariant

4

(b)

$$y[n] = x^r[n-2]$$

$$x_1[n] \rightarrow y_1[n] = x_1^r[n-2]$$

$$x_2[n] \rightarrow y_2[n] = x_2^r[n-2]$$

$$x_1[n] + x_2[n] \rightarrow [x_1[n-2] + x_2[n-2]]^r$$

$$\neq y_1[n] + y_2[n]$$

$\therefore$  The system is Non-Linear

$$x[n-n_0] \rightarrow x^r[n-n_0-2]$$

$$= y[n-n_0]$$

$\therefore$  The system is time-invariant

4  
(c)

$$x_1[n] \rightarrow y_1[n] = x_1[n+1] - x_1[n-1]$$

$$x_2[n] \rightarrow y_2[n] = x_2[n+1] - x_2[n-1]$$

Additive

$$x_1[n] + x_2[n] \rightarrow [x_1[n+1] + x_2[n+1]] - [x_1[n-1] + x_2[n-1]]$$

$$= [x_1[n+1] - x_1[n-1]] + [x_2[n+1] - x_2[n-1]]$$

$$= y_1[n] + y_2[n]$$

Homogeneity

~~$$\alpha x[n] \rightarrow \alpha [x_1[n+1] - x_1[n-1]]$$~~

$$\alpha x[n] \rightarrow \alpha y[n]$$

$\therefore$  The system is Linear

$$x[n-n_0] \rightarrow x[n-n_0+1] - x[n-n_0-1] \\ = y[n-n_0]$$

The System is time-invariant

4  
(d)

$$y(t) = \text{odd}(x(t)) \\ = \frac{x(t) + x(-t)}{2}$$

$$x_1(t) \rightarrow y_1(t) = \frac{1}{2} (x_1(t) + x_1(-t))$$

$$x_2(t) \rightarrow y_2(t) = \frac{1}{2} (x_2(t) + x_2(-t))$$

$$x_1(t) + x_2(t) \rightarrow \frac{1}{2} \left[ (x_1(t) + x_2(t)) + (x_1(-t) + x_2(-t)) \right] \\ = y_1(t) + y_2(t)$$

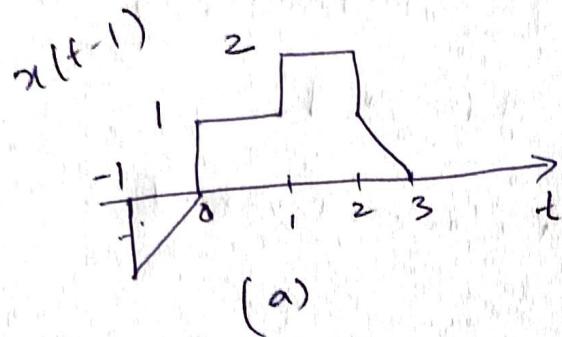
$$\alpha x(t) \rightarrow \frac{1}{2} \left[ \alpha x(t) + \alpha x(-t) \right] \\ = \alpha \frac{1}{2} \left[ x(t) + x(-t) \right] \\ = \alpha y(t)$$

The System is linear

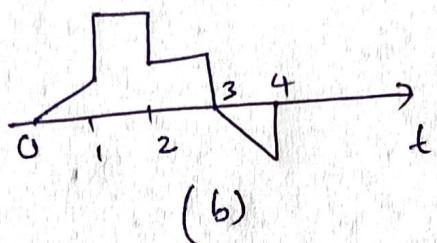
$$x(t-t_0) \rightarrow \frac{1}{2} \left[ x(t-t_0) + x(-t-t_0) \right] \\ \neq y(t-t_0)$$

The System is Time-variant.

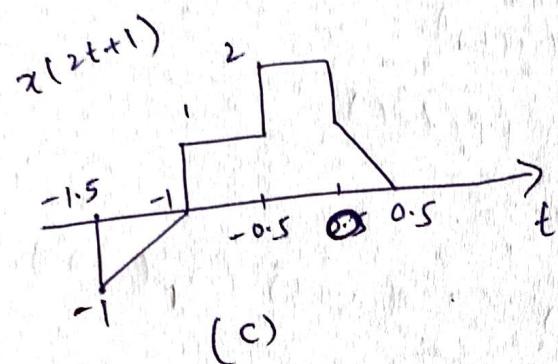
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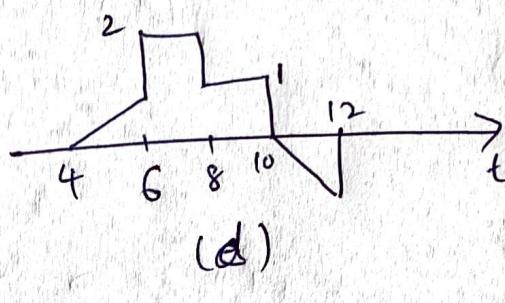
(a)

 $x(2-t)$ 

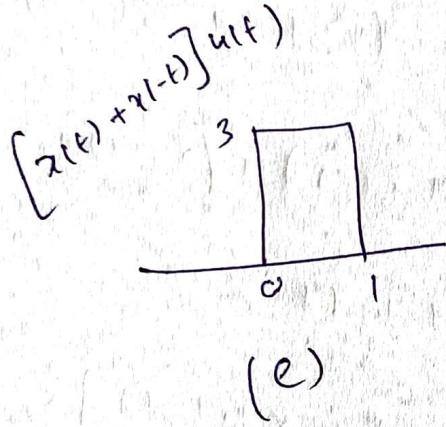
(b)



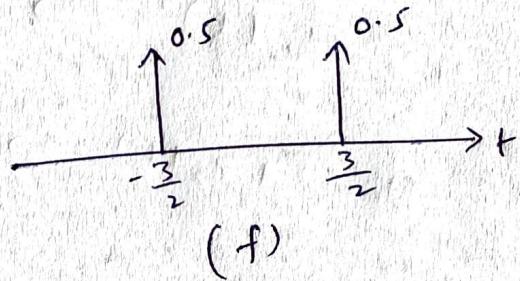
(c)

 $x(4-\frac{t}{2})$ 

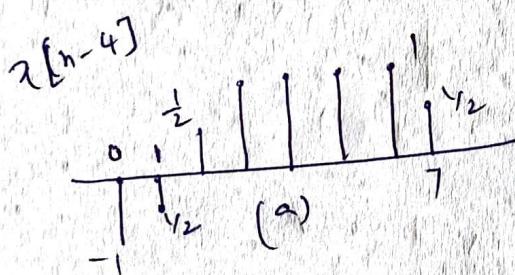
(d)



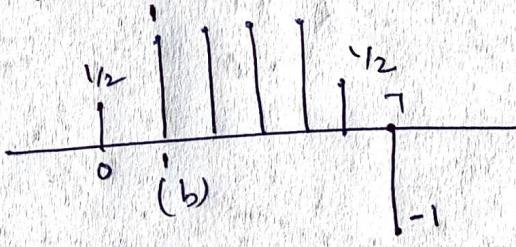
(e)



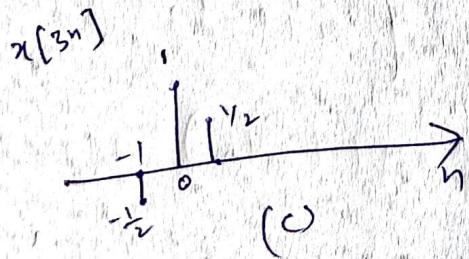
(6)



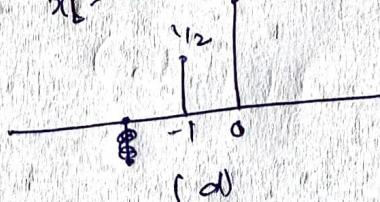
(a)

 $x(3^n)$ 

(b)

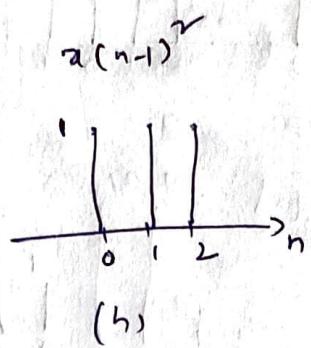
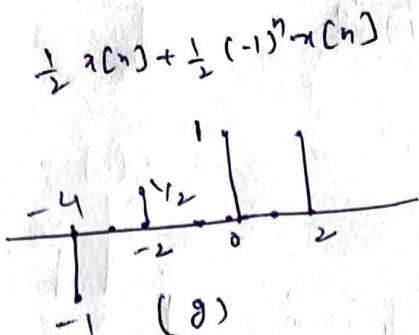
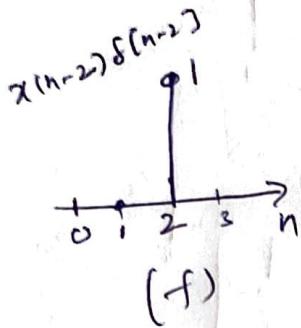


(c)

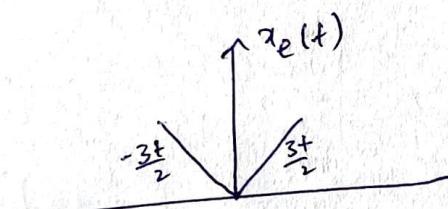
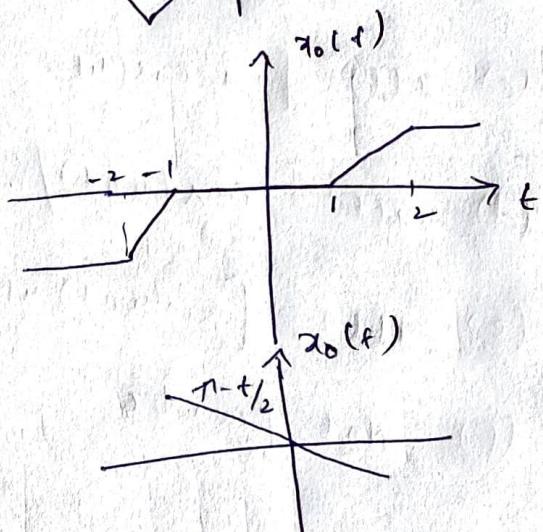
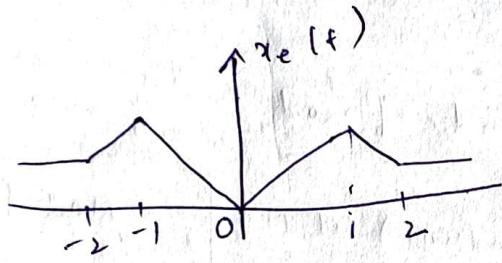
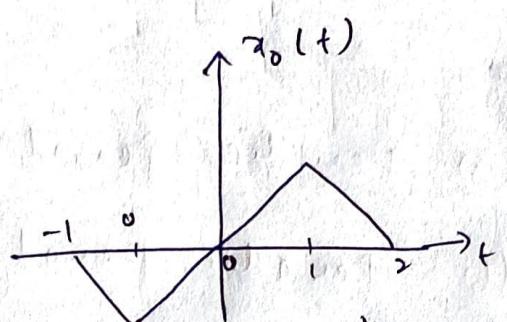
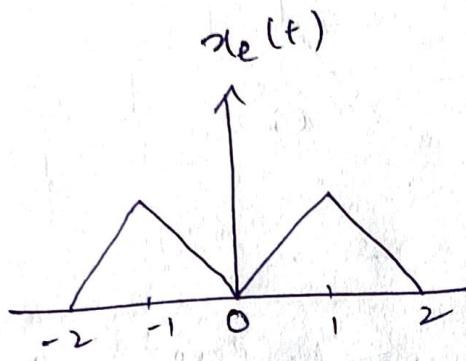
 $x(3^n+1)$ 

(d)

$$(e) \quad x(n) \cdot x(3-n) = x(n)$$

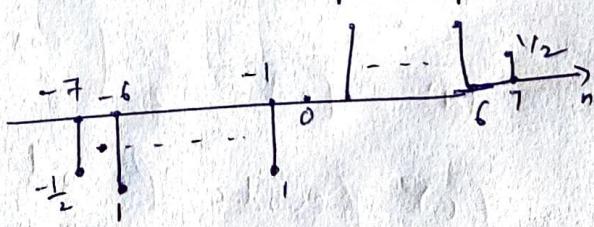
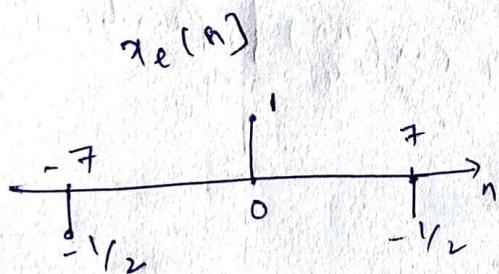


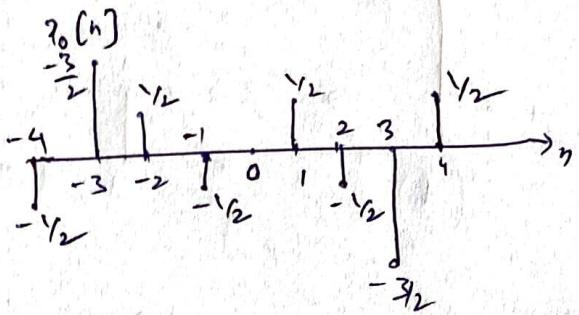
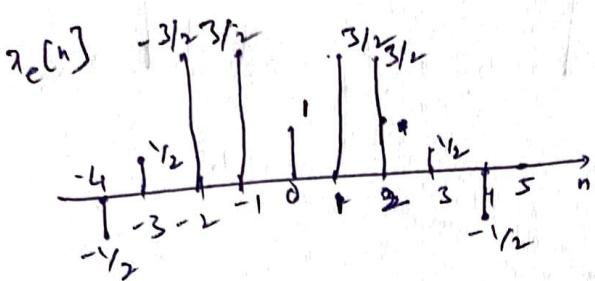
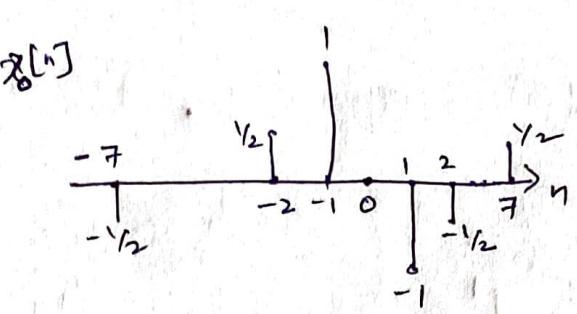
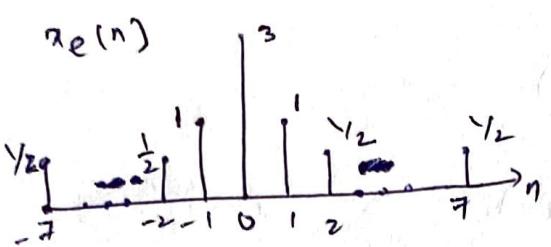
7



(c)

8





9 (a) Periodic, Period =  $\frac{2\pi}{4} = \frac{\pi}{2}$ .

(b) Periodic, Period =  $\frac{2\pi}{\pi} = 2$ .

(c)  $x(t) = [1 + \cos(4t - 2\pi/3)]/2$ . Periodic, Period =  $\frac{2\pi}{4} = \frac{\pi}{2}$ .

(d)  $x(t) = \cos(4\pi t)/2$ . Periodic, Period =  $\frac{2\pi}{4\pi} = \frac{1}{2}$

(e)  $x(t) = [\sin(4\pi t) \cdot 4(t) - \sin(4\pi t) \cdot 4(-t)]/2$ . Not periodic

(f) Not periodic

10 (a) Periodic, Period = 7

(b) Not periodic

(c) Periodic, Period = 8

(d)  $x[n] = \frac{1}{2} \left[ \cos\left(\frac{3\pi n}{8}\right) + \cos\left(\frac{\pi n}{4}\right) \right]$ . Periodic, Period = 8.

(e) Periodic, Period = 16.

11) a) Linear, Stable

b) Memoryless, linear, causal, stable

c) Linear

d) Linear, causal, stable

e) Time invariant, linear, causal, stable

f) Linear, stable

g) Time-Invariant, linear, causal.

12) a) Linear, stable

b) Time-Invariant, linear, causal, stable

c) Memoryless, linear, causal

d) Linear, stable

e) Linear, stable

f) Memoryless, linear, causal, stable

g) Linear, stable

13) a) Invertible, Inverse system;  $y(t) = x(t+4)$

b) Non-Invertible. The signals  $x(t)$ ,  $x(t+2\pi)$  give the same o/p.

c) Non-Invertible.  $\delta(n)$  and  $2 \delta(n)$  give the same o/p.

d) Invertible. Inverse system:  $y(t) = \frac{d}{dt} x(t)$

r) Invertible. Inverse system:  $y[n] = x[n+1]$  for  $n \geq 0$   
and  $y[n] = x[n]$  for  $n < 0$ .

f) Non-Invertible  $x[n]$  and  $-x[n]$  give the same  
result.

g) Invertible. Inverse system:  $y[n] = x[1-n]$

h) Invertible. Inverse system  $y(t) = x(t) + \frac{dx}{dt}$

i) Invertible. Inverse system  $y[n] = x[n] - \frac{1}{2}x[n-1]$

j) Non invertible.  $f(n)$  and  $2f(n)$  result in  $y[n] = 0$

k) Invertible. Inverse system:  $y(t) = x(t/2)$ .

l) Non-Invertible.  $x_1[n] = \delta(n) + \delta(n-1)$  and  $x_2[n] = \delta(n)$   
give  $y[n] = \delta(n)$

m) Invertible. Inverse system:  $y[n] = x[2n]$ .