(1)

Active filters

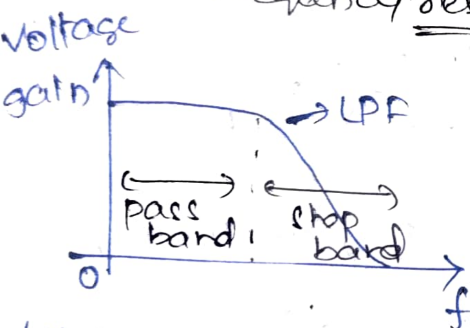
filters (2 types)

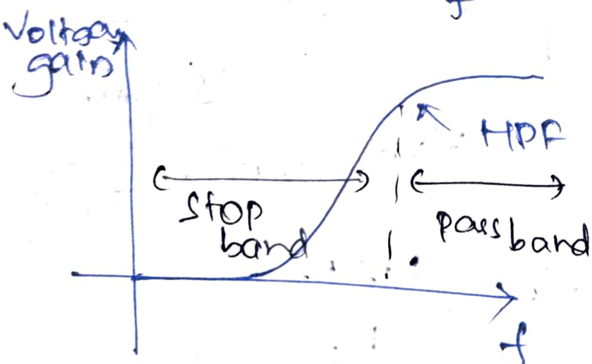
→ combination of both active and passive components (OPAMP) → (generati energy)

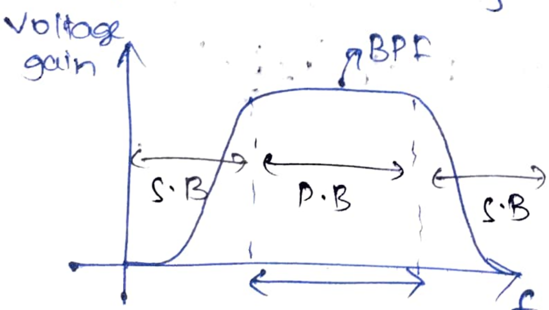
⇒ A filter is a frequency selective circuit, which allows signals of certain range of frequencies to pass through it & rejecting other frequencies.

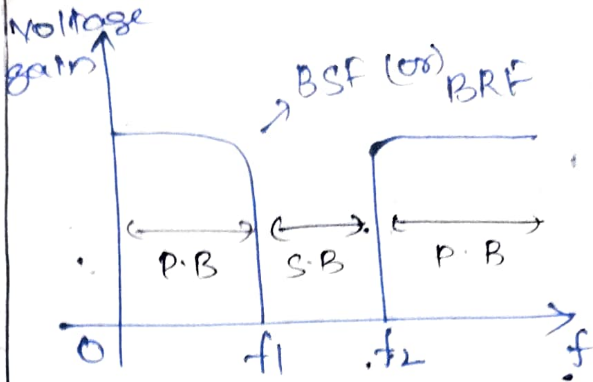
⇒ Active filters
1. Lowpass filter
2. High pass filter
3. Band pass filter
4. Band reject filter / Band stop filter.

frequency responses



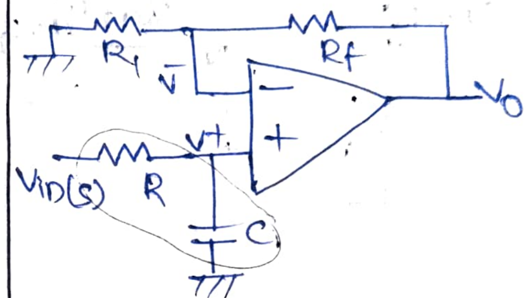






* Low Pass filter

first order LPF



$$V^+ = \frac{V_{in}(S)}{R + 1/SC} \times 1/SC$$

$$V^+(S) = \frac{V_{in}(S)}{RSC + 1}$$

$$V_o(S) = \left(1 + \frac{R_F}{R_1}\right)V^+(S) = \left(1 + \frac{R_F}{R_1}\right)\frac{V_{in}(S)}{1+SRC}$$

$$A_{vf}(S) = \frac{V_o(S)}{V_{in}(S)} = \left(1 + \frac{R_f}{R_1}\right)\frac{1}{1+SRC}$$

Let $S = j\omega$

$$A_{vf}(j\omega) = \left(1 + \frac{R_f}{R_1}\right) \times \frac{1}{1 + j\omega RC}$$

$$|A_{vf}(j\omega)| = \frac{\left(1 + \frac{R_f}{R_1}\right)}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\boxed{X = \frac{a+jb}{c+jd} \; ; \; |X| = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}}$$

$$\left|A_{vf}\right|_{max} = \left(1+\frac{R_f}{R_1}\right)\Bigg|_{\omega=0}$$

$$\Rightarrow |A_{vf}(j\omega)| = \frac{A_{vf\,max}}{\sqrt{1+\omega^2R^2C^2}}$$

cutt-off frequency = f_o.

corresponding angular frequency = ω_o

At this frequency, the voltage gain is $\frac{1}{\sqrt{2}}$ times the maximum gain.

At $\omega = \omega_o$ or $f = f_o$

$$\frac{|A_{vf}|_{max}}{\sqrt{2}} = \frac{A_{vf\,max}}{\sqrt{1+\omega_o^2R^2C^2}}$$

$$\omega_o = 1/RC \Rightarrow \boxed{f_o = \frac{1}{2\pi RC}}$$

when 'ω' is very large;

$$|A_{vf}(j\omega)| \simeq 0$$

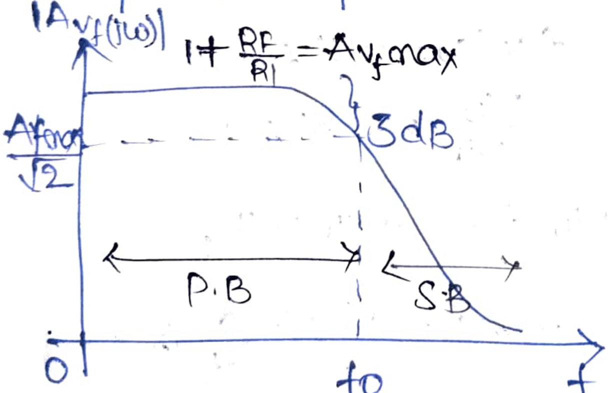


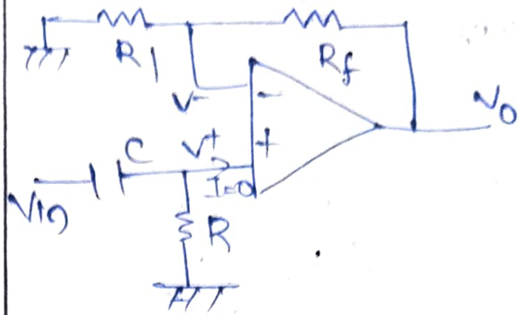
fig: frequency responce of LPF

$$A_{vf}(S) = \frac{1+\frac{R_F}{R_1}}{1+SRC}$$

1st order filter

only one RC section

* 1st order High Pass Filter (Butterworth)



$$V^+(S) = \frac{V_{in}(S)\times(R)}{\frac{1}{SC}+R}$$

$$V^+(S) = \frac{V_{in}(S)\cdot SRC}{1+SRC}$$

for non-inverting opamp,

$$V_o(S) = V^+(S)\left[1+\frac{R_F}{R_1}\right]$$

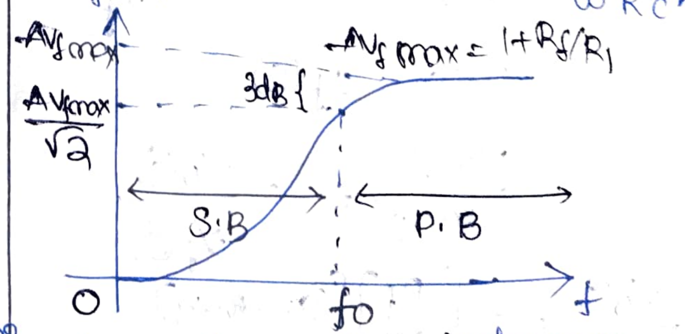
$$= \frac{V_{in}(S)SRC}{1+SRC}\left[1+\frac{R_F}{R_1}\right]$$

$$A_{vf}(S) = \frac{V_o(S)}{V_{in}(S)} = \frac{SRC\left[1+\frac{R_f}{R_1}\right]}{1+SRC}$$

$$A_{vf}(j\omega) = \frac{j\omega RC\left[1+\frac{R_f}{R_1}\right]}{1+j\omega RC}$$

$$|A_{vf}(j\omega)| = \frac{\omega RC\left[1+\frac{R_f}{R_1}\right]}{\sqrt{1+(\omega RC)^2}}$$

$$|A_{vf}(j\omega)| = \frac{(1+R_f/R_1)}{\sqrt{1+\frac{1}{\omega^2R^2C^2}}}$$



$A_{vf\,max} = 1+R_f/R_1$ when $\omega=\infty$

when $\omega=0$: gain = 0

At $f = f_0$,

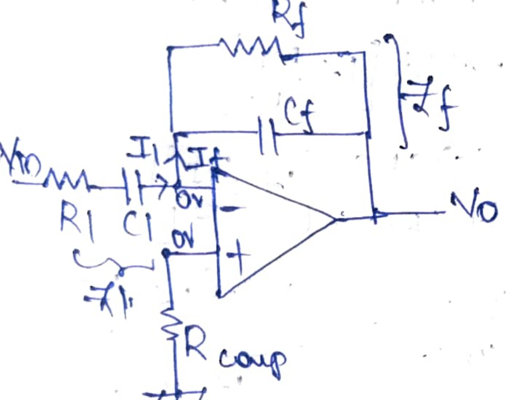
$$|A_{Vf}(j\omega)| = \frac{A_{Vf\max}}{\sqrt{2}}$$

$$\frac{A_{Vf\max}}{\sqrt{2}} = \frac{A_{Vf\max}}{\sqrt{1 + \frac{1}{\omega_0^2 R^2 C^2}}}$$

$$\omega_0 = \frac{1}{RC}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi RC}}$$

II. Practical Differentiator using OPAMP



$$Z_1(S) = R_1 + \frac{1}{SC_1} = \frac{SR_1C_1 + 1}{SC_1}$$

$$Z_f(S) = \frac{R_f \times \frac{1}{SC_f}}{R_f + \frac{1}{SC_f}} = \frac{R_f}{SC_fR_f + 1}$$

$$I_1(S) = \frac{V_{in}(S) - 0}{Z_1(S)} \; ; \; I_f(S) = \frac{0 - V_o(S)}{Z_f}$$

$$I_1(S) = I_f(S)$$

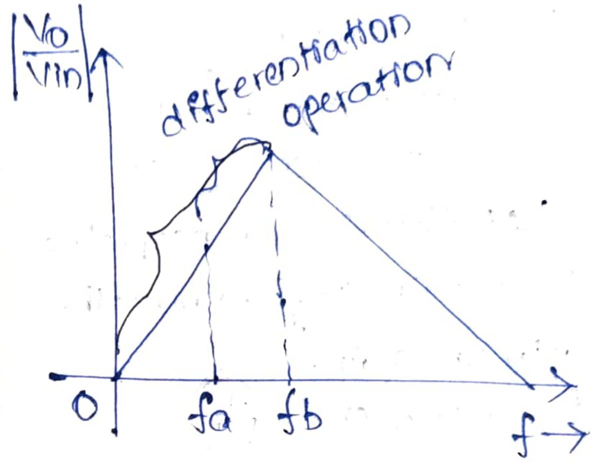
$$\frac{V_{in}(S)}{Z_1(S)} = -\frac{V_o(S)}{Z_f(S)}$$

$$\frac{V_o(S)}{V_{in}(S)} = \frac{-[R_f][SC_1]}{[SC_fR_f + 1][SC_1R_1 + 1]}$$

$$|A_V(S)| = \frac{SR_fC_1}{(1 + SC_fR_f)(1 + SR_1C_1)}$$

If $R_1C_1 = C_fR_f$

$$\therefore |A_V(S)| = \frac{SR_fC_1}{(1 + SR_1C_1)^2} = \frac{\omega R_f C_1}{}$$



f_a = frequency at which gain = 1

f_{max}

$f_b = 10 f_a$

$$f_a = \frac{1}{2\pi R_f C_1} \; , \; f_b = \frac{1}{2\pi R_1 C_1}$$

For proper differentiation;

If $R_fC_1 \gg R_fC_f$

$R_fC_1 \gg R_1C_1$, then

$$A_V(S) = -SR_fC_1$$

$$\frac{V_o}{V_{in}} = -SR_fC_1$$

$$V_o(S) = -SV_{in}(S)R_fC_1$$

$$\therefore V_o(S) = +R_fC_1 \frac{d}{dt} V_{in}(t)$$

Now;

$$|A_V(S)| = \frac{\omega R_f C_1}{1 + (\omega C_1 R_1)^2}$$

$$|A_V(S)| = \frac{2\pi f R_f C_1}{1 + (2\pi f C_1 R_1)^2}$$

where,

$$f_a = \frac{1}{2\pi R_f C_1} \; ; \; f_b = \frac{1}{2\pi R_1 C_1}$$

$$|A_v(s)| = 1$$

$$1 = \frac{f/f_a}{1 + (f/f_b)^2}$$

$$1 + (f/f_b)^2 = f/f_a$$

The above eqn satisfies if $\left(\frac{f_a}{f_b}\right)^2 << 1$

for, at $f = f_a$; Both LHS & RHS equals to 1

$\left(\frac{f_a}{f_b}\right)^2 << 1$ (or) from graph

$\frac{f_a}{f_b} < 1$ | $f_b > f_a$

$\Rightarrow \boxed{f = f_a}$ | $\frac{f_a}{f_b} < 1$

Q] Design a differentiator using OPAMP to differentiate an i/p signal with $f_{max} = 200Hz$. Draw the output waveform for sine wave with 1V peak at 200 Hz.

$$\boxed{\begin{array}{l} f_{min} = f_a \\ f_{max} = f_b \end{array}}$$

$$f_{min} = f_a = 200Hz$$

$$\frac{1}{2\pi R_f C_1} = 200$$

Let, $\boxed{C_1 = 0.1\mu F}$

$$R_f = \frac{1}{2\pi \times 200 \times 0.1 \times 10^{-6}}$$

$$\boxed{\therefore R_f = 7.96k\Omega}$$

Let, $\boxed{f_b = 10 f_a}$ (for designing purpose)

$$\frac{1}{2\pi R_f C_1} = 10 \times 200$$

$$C_1 = 0.1\mu F$$

$$\Rightarrow \boxed{R_1 = 0.796k\Omega}$$

WKT

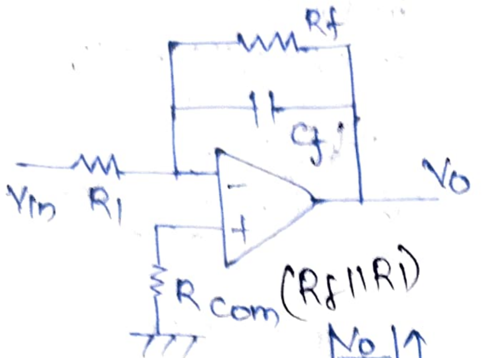
$$\Rightarrow R_1 C_1 = R_f C_f$$

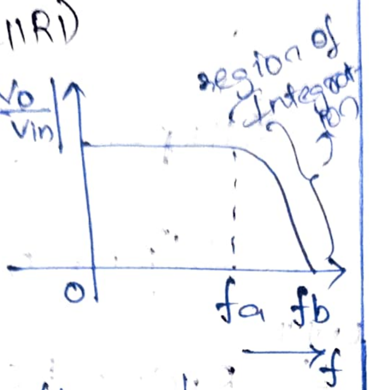
$$\frac{(0.796 \times 10^3)(0.1 \times 10^{-6})}{(7.962 \times 10^3)} = C_f$$

$$\boxed{\therefore C_f = 0.01\mu F}$$

O/p wave form:

Practical Integrator:





$f_a = \frac{1}{2\pi R_f C_f}$; $f_b = \frac{1}{2\pi R_1 C_f}$

$\boxed{f_b = 10 f_a}$ (always)

Q] Design an practical Integrator with lower limit of frequency is 160Hz.

$f_a = 160Hz \Rightarrow f_b = 1600Hz$

$\frac{1}{2\pi R_f C_f} = 160$

Let $C_f = 0.01nF$

$\Rightarrow \boxed{R_f = \quad}$

$f_b = 1600Hz$

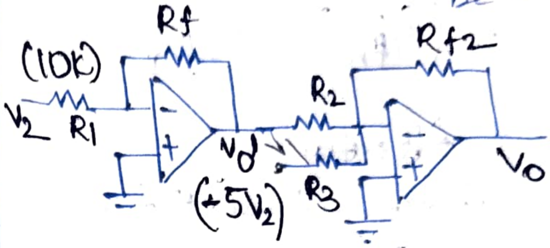
$\frac{1}{2\pi R_1 C_f} = 1600$

$\Rightarrow \boxed{R_1 = \quad}$

Q] Design a circuit using OPAMP which gives output $V_o = 5V_2 - 8V_1$, where V_2 and V_1 are inputs.

O/p: $V_o = 5V_2 - 8V_1$ → as two diff signals (2 OPAMP should be used)

i/p: V_2 and V_1



$\frac{V_o'}{V_2} = -5 \Rightarrow \frac{-R_f}{R_1} = -5$

$\Rightarrow R_f = 5R_1$

Let $R_1 = 10K$

$\Rightarrow R_f = 50k\,\Omega$

$V_o = -\frac{R_{f2}}{R_2}(V_o') - \frac{R_{f2}}{R_3}(V_1)$

$= -\frac{R_{f2}}{R_2}(-5V_2) - \frac{R_{f2}}{R_3}(V_1)$

$\Rightarrow R_{f2} = R_2 = 10k$ (assumption)

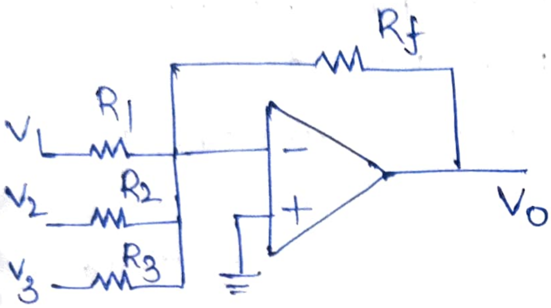
$\Rightarrow R_{f2} = 8R_3$

$\Rightarrow R_3 = \frac{10k}{8} = 125\Omega$

Q) Design an OPAMP circuit which performs the following operation

$$V_o = \underbrace{-V_1 - 2V_2 - 3V_3}_{\text{as all have same sign}}$$

→ 1 OPAM is enough



$$V_o = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right]$$

$$\frac{R_f}{R_1} = 1; \quad \frac{R_f}{R_2} = 2; \quad \frac{R_f}{R_3} = 3$$

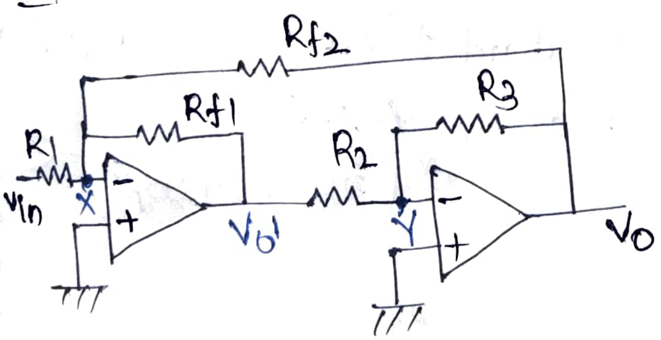
let, $R_f = 100K$

$\Rightarrow R_1 = 100k\Omega$

$R_2 = 50\ k\Omega$

$R_3 = 33.3K\Omega$

Q) find the output.



KCL at node X

$$\frac{V_{in}-0}{R_1} = \frac{0-V_o'}{R_{f1}} + \frac{0-V_o}{R_{f2}} \quad \text{—①}$$

KCL at node Y

$$\frac{V_o'-0}{R_2} = \frac{0-V_o}{R_3}$$

$$V_o' = -V_o\left[\frac{R_2}{R_3}\right] \quad \text{—②}$$

① & ②

$$\frac{V_{in}}{R_1} = +\frac{1}{R_{f1}}\left[\frac{V_oR_2}{R_3}\right] - \frac{V_o}{R_{f2}}$$

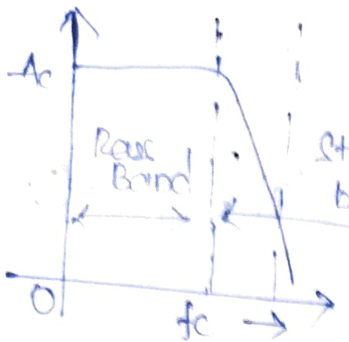
$$\frac{V_{in}}{R_1} = V_o\left[\frac{R_2}{R_3R_{f1}} - \frac{1}{R_{f2}}\right]$$

$$\frac{V_{in}}{R_1} = V_o\left[\frac{R_2R_{f2} - R_3R_{f1}}{R_2R_3R_{f1}}\right]$$

$$\therefore V_o = V_{in}\left[\frac{R_2R_3R_{f1}}{R_1(R_2R_{f2} - R_3R_{f1})}\right]$$

→ problem on Analog Computer.

Butterworth LPF (vu) → flat-flat filter

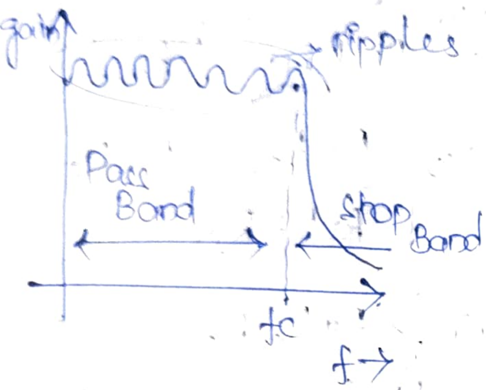


- As both PB and SB are flat
- Require high order filter to have fast transition from PB to SB.

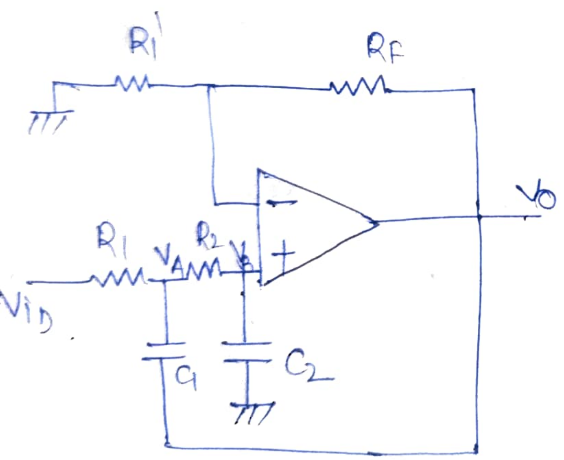
Flat-flat filter

Chebyshev filter →

- Ripples are present in PassBand
- Stop Band is flat
- Has a sharp transition PB to SB
- Require Low order filter to have fast transition from PB to SB.



Butterworth second order LPF :-



Apply KCL at node A :

$$\frac{V_{in}(S)-V_A(S)}{R_1} = \frac{V_A(S)-V_B(S)}{R_2} + \frac{V_A(S)-V_o(S)}{1/SC_1}$$

$$V_B(S) = \frac{V_A(S)}{R_2 + 1/SC_2} \times \frac{1}{SC_2} \quad \text{—(1)}$$

$$\therefore V_B(S) = \frac{V_A(S)}{1+SR_2C_2} \quad \text{—(2)}$$

and, as non-inverting Amp.

$$V_o = \left[1+\frac{R_f}{R_1}\right] V_B(S) \quad \text{—(3)}$$

(2) & (3) in (1)

$$\frac{V_o(S)}{V_{in}(S)} = \frac{A_o/R_1R_2C_1C_2}{S^2 + S\frac{(C_2(R_1+R_2)+(1-A_o)R_1C_1)}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}} \quad \text{—(4)}$$

standard 2nd OE

$$H(S) = \frac{A_o\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} \quad \text{—(5)}$$

$$Q = \frac{1}{2\xi}$$

Comparing eq (4) & (5)

$$2\pi f_n = \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1R_2 + R_2C_2 + R_1C_1(1-A_0)}$$

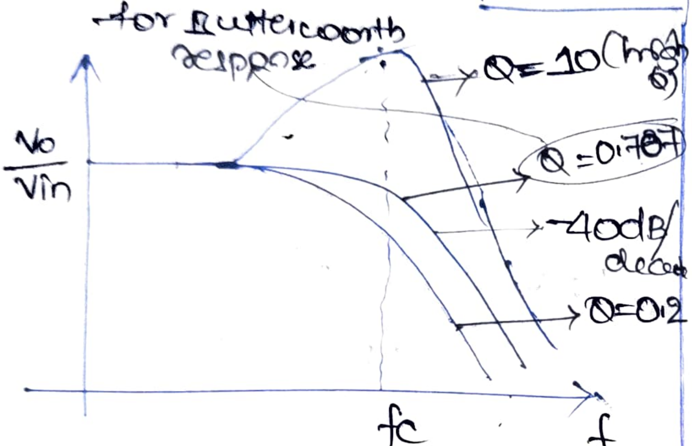
$$A_0 = 1 + \frac{R_f}{R_1}$$

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$

then

$$f_c = f_n = \frac{1}{2\pi RC} \qquad \boxed{Q = \frac{1}{3-A_0}}$$

for Butterworth response



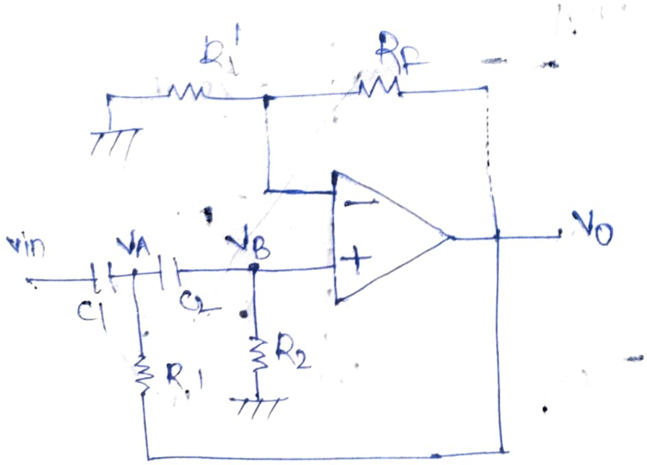
-20N dB/decade → LPF

B.W.F

$$0.707 = \frac{1}{3-A_0}$$

$\therefore A_0 = 1.586$

Butterworth second order HPF :



KCL at node A:

$$\frac{V_{in}(S) - V_A(S)}{1/SC_1} = \frac{V_A(S) - V_B(S)}{1/SC_2} + \frac{V_A(S) - V_0(S)}{R_1} \quad — (1)$$

$$V_B(S) = \frac{V_A(S) \times R_2}{1/SC_2 + R_2} \quad — (2)$$

$$V_0(S) = \left[1 + \frac{R_F}{R_1'}\right] V_B(S) \quad — (3)$$

(1) & (2) & (3).

$$\frac{V_0(S)}{V_{in}(S)} = \frac{A_0 S^2}{S^2 + \frac{S(R_1C_1 + R_1C_2 + R_2C_2(1-A_0))}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}} \quad — (4)$$

$$A_0 = 1 + \frac{R_F}{R_1'}$$

$$\frac{V_0(S)}{V_{in}(S)} = \frac{A_0 S^2}{S^2 + \alpha\omega_0 S + \omega_0^2} \quad — (5)$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\frac{\alpha}{\sqrt{R_1R_2C_1C_2}} = \frac{R_1C_1 + R_1C_2 + R_2C_2(1-A_0)}{R_1R_2C_1C_2}$$

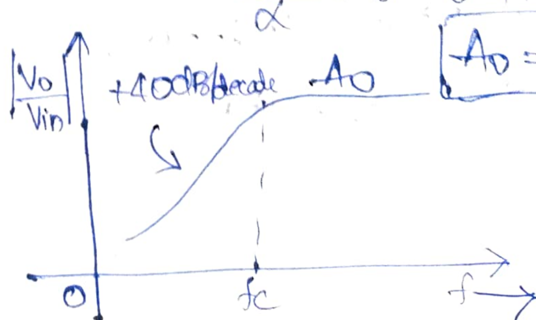
If, $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$f_c = f_0 = \frac{1}{2\pi RC} \; ; \; \alpha = 3 - A_0$$

for Butterworth response

$$\frac{1}{\alpha} = 0.707,$$

$\boxed{A_0 = 1.586}$



Q] Design a butterworth second order LPF with cutt-off frequency = 1KHz.

Ans: ① Select value of capacitor $C \le 1\mu F$

procedure ② Assume $R_1 = R_2 = R$ and $C_1 = C_2 = C$; using $f_c = \frac{1}{2\pi RC} \to$ find (R)

③ PassBand gain = 1.586 ; $1 + \frac{R_F}{R_1'} = 1.586$

⇒

1. $C = 0.1\mu F$, $f_c = 1KHz$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{2\pi(10^3)(0.1\times10^{-6})} \approx 1.6K\Omega$$

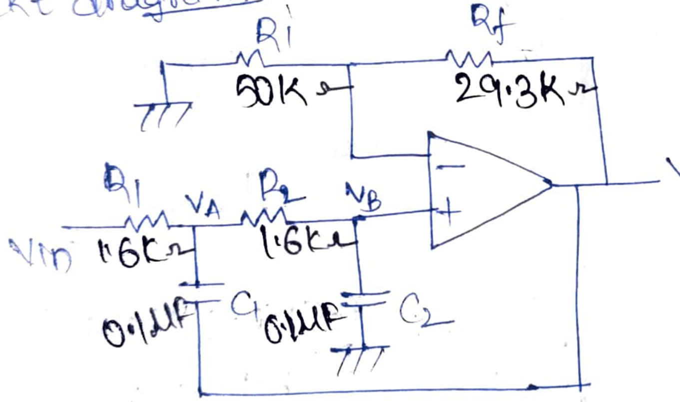
2. $$1 + \frac{R_F}{R_1'} = 1.586$$

$$\frac{R_F}{R_1'} = 0.586$$

Assume, $R_1' = 50K \Rightarrow R_F = 0.586 \times R_1'$

$\therefore R_F = 29.3K$

Ckt diagram:



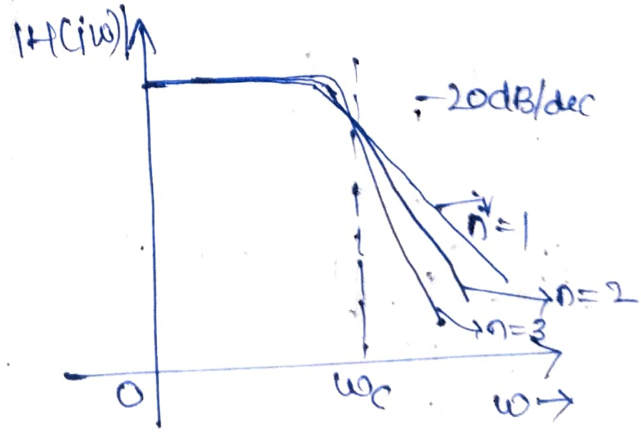
Butterworth filter (LPF)

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}}$$

ω_c is the filter cuttoff freq.

'n' - order of filter

Butterworth filters.



Normalised Butterworth LPF

$$|H_n(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}} \quad ; \omega_c = 1 \text{ rad/sec}$$

$$|H_n(j\omega)|^2 = H_n(j\omega)\,H_n(-j\omega) = \frac{1}{1+\omega^{2n}}$$

let $\omega = \frac{s}{j}$

$$|H_n(j\omega)|^2 = H_n(j\omega)\,H_n(-j\omega)$$
$$= H_n(s)\,H_n(-s)$$

$$|H_n(j\omega)|^2 = \frac{1}{1+\left(\frac{s}{j}\right)^{2n}}$$

The $2n$ poles of $H_n(s)H_n(-s)$ occur when $\left(\frac{s}{j}\right)^{2n} = -1$

$$s^{2n} = -1$$

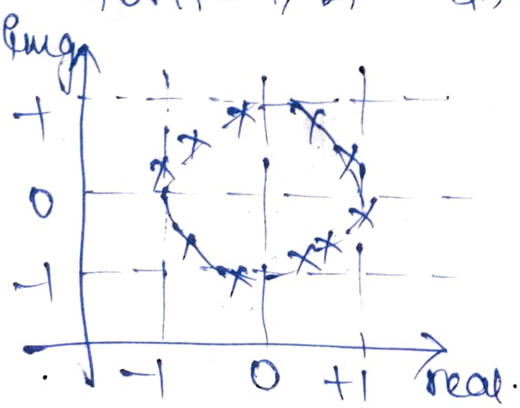
since, $-1 = e^{j\pi(2k-1)}$ for k integer & $j = e^{j\pi/2}$

we have, $s^{2n} = e^{j(\pi)(2k-1+n)}$

$$s_k = e^{j\pi/2n(2k+n-1)}$$

$$s_k = \cos\left(\frac{\pi}{2n}(2k+n-1)\right) + j\sin\left(\frac{\pi}{2n}(2k+n-1)\right)$$

for $k = 1, 2, \cdots 2n$



n poles in LHP correspond to $H_n(s)$
n poles in RHP " " $H_n(-s)$

For obtaining stable & causal filter we set

$$H_n(s) = \frac{1}{(s-s_1)(s-s_2)\cdots(s-s_n)}$$
$$= \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1}$$
$$= \frac{1}{B_n(s)}$$

order n	factors of polynomial $B_n(s)$
1	$s+1$
2	$s^2+1.414s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$

scalling for normalized filter,

① Replace 'ω' with $\frac{\omega}{\omega_c}$ in $H_n(\omega)$

② Replace 's' with $\frac{s}{\omega_c}$ in $H_n(s)$

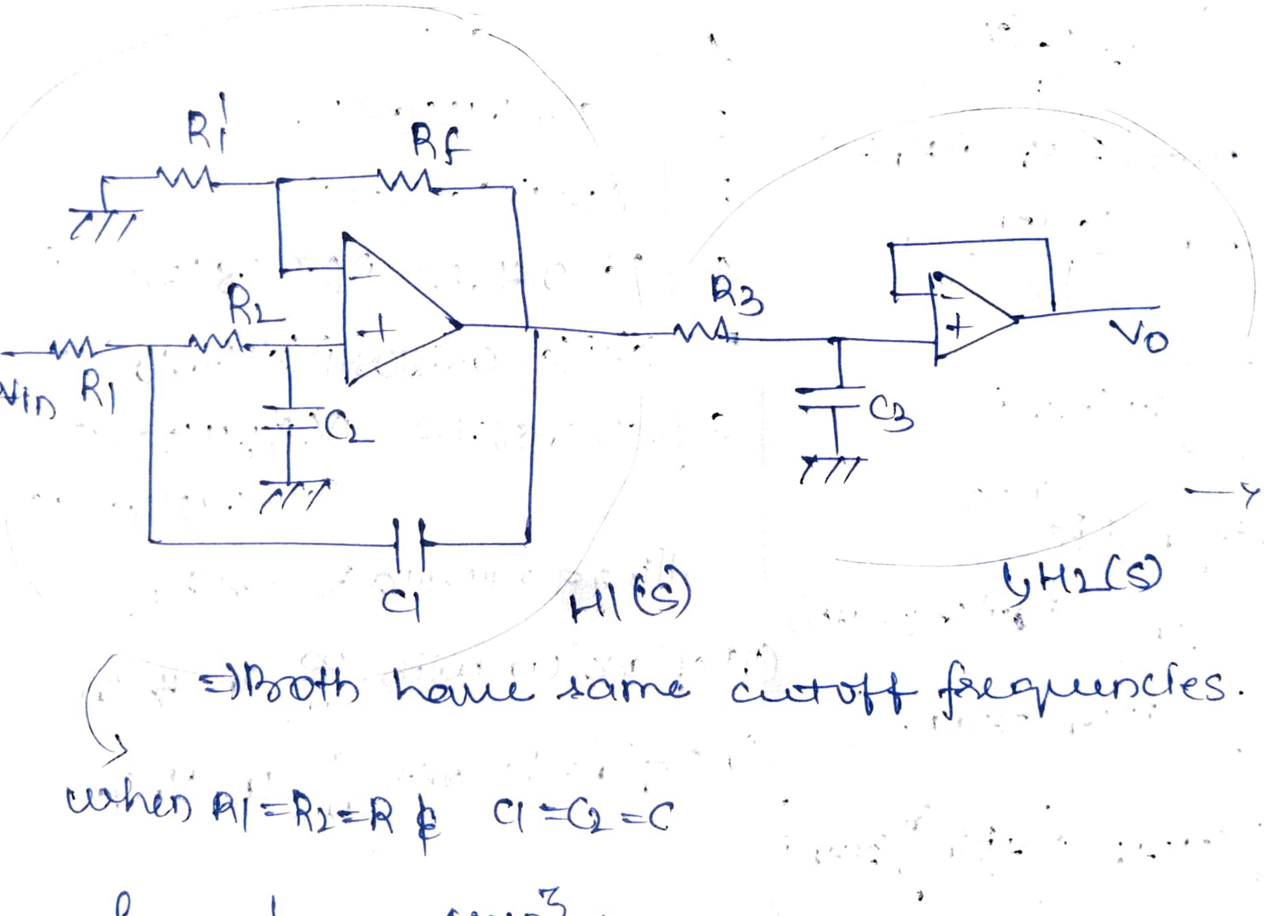
Q] Design a 3rd order butterworth LPF with cutoff freq 1 KHz.

$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$H_n(s) = \left(\frac{1}{s^2+s+1}\right)\left(\frac{1}{s+1}\right)$$

let cuttoff angular frequency $= \omega_0$

$$H(s) = \frac{1}{\left[\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right) + 1\right]} \times \frac{1}{\left(\frac{s}{\omega_0}\right)+1} = \left(\frac{\omega_0^2}{s^2+\omega_0 s+\omega_0^2}\right)\left(\frac{\omega_0}{s+\omega_0}\right)$$



$\rightarrow f_0 = \frac{1}{2\pi R_3 C_3}$

⇒ Both have same cutoff frequencies.

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$f_0 = \frac{1}{2\pi RC} = 1 \times 10^3 \text{ Hz}.$$

$C = 0.01\mu F$

$$\Rightarrow R = \frac{1}{2\pi f c} \Rightarrow \boxed{R \simeq 16K\Omega}$$

$$H_1(S) = \frac{A_0/R_1C_1R_2C_2}{S^2 + S\left[\frac{R_1C_2 + R_2C_2 + R_1C_1(1-A_0)}{R_1R_2C_1C_2}\right] + \frac{1}{R_1R_2C_1C_2}}$$

when $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$\Downarrow$$

$$H_1(S) = \frac{A_0/R^2C^2}{S^2 + \frac{S(3-A_0)}{RC} + \frac{1}{R^2C^2}}$$

By comparing

$$\Rightarrow \boxed{\omega_0 = 1/RC \text{ and } 3 - A_0 = 1 \Rightarrow A_0 = 2}$$

$$H_1(S) = \frac{A_0\omega_0^2}{S^2 + \omega_0 S + \omega_0^2}$$

$$A_0 = 1 + \frac{R_F}{R_1'} \Rightarrow \boxed{R_F = R_1'} = \underline{10K\Omega}$$

$$\left[1 - \frac{1}{\sqrt{1+\epsilon^2}} = A_{max}\right]$$

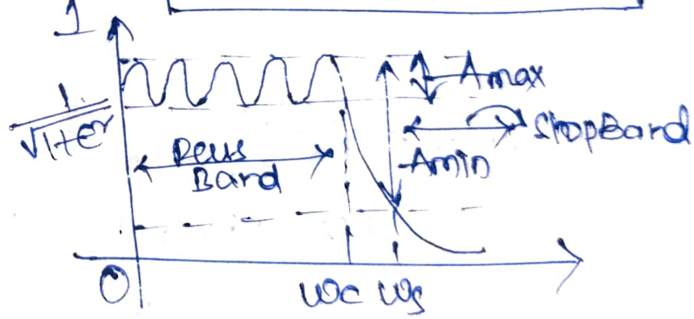
chebyshev filter (LPF)

$$H(j\omega) = \frac{1}{\left(1 + \epsilon^2 C_n^2\left(\frac{\omega}{\omega_p}\right)\right)^{1/2}}$$

$\omega_p \to \omega_c$

where $\boxed{\epsilon = \left(10^{\frac{A_{max}}{10}} - 1\right)^{1/2}}$

$$\boxed{H_n(j\omega) = \frac{1}{\left(1 + \epsilon^2 C_n^2(\omega)\right)^{1/2}}}$$



Amax & Amin are in dB

$\omega_c = \omega_p$ = pass band edge

ω_s = stop band edge

$$20\log 1 - 20\log\frac{1}{\sqrt{1+\epsilon^2}} = A_{max}$$

$$\downarrow$$

$$\boxed{10\log(1+\epsilon^2) = A_{max}} \Rightarrow \log(1+\epsilon^2) = \frac{A_{max}}{10} = \log 10^{\frac{A_{max}}{10}}$$

$$1 + \epsilon^2 = 10^{\frac{A_{max}}{10}} \Rightarrow \epsilon = \left(10^{\frac{A_{max}}{10}} - 1\right)^{1/2}$$

$F(\omega^2) = \epsilon^2 C_n^2(\omega)$

= Chebyshev polynomial

$$\boxed{\begin{aligned} C_n(\omega) &= \cos(n\cos^{-1}\omega) \quad \text{for } \omega \le 1 \\ &= \cosh(n\cosh^{-1}\omega) \quad \text{for } \omega > 1 \end{aligned}}$$

$n = 0 \; ; \; C_0(\omega) = 1$

$n = 1 \; ; \; C_1(\omega) = \omega$

$C_{n+1}(\omega) = 2\omega C_n(\omega) - C_{n-1}(\omega)$

n	$C_n(\omega)$
0	1
1	ω
2	$2\omega^2 - 1$
3	$4\omega^3 - 3\omega$
4	$8\omega^4 - 8\omega^2 + 1$

ripple width $= 1 - \frac{1}{(1+\epsilon^2)^{1/2}}$

order of chebyshev filter (LPF)

Given $A_{min}, A_{max}, \omega_s, \omega_p$;

find 'n'

$-A_{min} = 10\log(1 + \epsilon^2 C_n^2(\omega))$

for $\omega = \frac{\omega_s}{\omega_c}$ normaliced.

$\epsilon^2 C_n^2(\omega) = \epsilon^2 (\cosh(n\cosh^{-1}(\omega)))^2$

$= 10^{-A_{min}/10} - 1$

$|A_{min}| = 10\log_{10}(1 + \epsilon^2(\cosh(n\cosh^{-1}\omega))^2)$

$\epsilon^2 = 10^{\frac{-A_{max}}{10}} - 1$

$$\cosh(n\cosh^{-1}\omega) = \left[\frac{10^{\frac{-A_{min}}{10}} - 1}{10^{\frac{-A_{max}}{10}} - 1}\right]^{1/2}$$

$$n \geq \frac{\cosh^{-1}\left[\left(10^{\frac{-A_{min}}{10}} - 1\right) \Big/ \left(10^{\frac{-A_{max}}{10}} - 1\right)\right]^{1/2}}{\cosh^{-1}\left(\frac{\omega_s}{\omega_c}\right)} \quad \text{— (1)}$$

Poles:

$1 + \epsilon^2 C_n^2(\omega) = 0$

$S_K = \sigma_K \pm j\omega_K$

$K = 0, 1, 2, \ldots 2n-1$

$\sigma_K = \sinh a \sin(2K+1)\pi/2n$

$\omega_K = \cosh a \cos(2K+1)\pi/2n$

$\theta_K = (2K+1)\pi/2n$

for $K = 0, 1, 2, \ldots 2n-1$

& $a = \frac{1}{n}\sinh^{-1}(1/\epsilon)$

$\frac{\sigma_K}{\sinh a} = \sin(2K+1)\frac{\pi}{2n}$

$\frac{\omega_K}{\cosh a} = \cos(2K+1)\frac{\pi}{2n}$

$\left(\frac{\sigma_K}{\sinh a}\right)^2 + \left(\frac{\omega_K}{\cosh a}\right)^2 = 1$

Q The specification for chebyshev filter is given by $A_{min} = 20dB$ $A_{max} = 3dB$, $\left(\frac{\omega_s}{\omega_c}\right) = 1.5$; determine order of filter & confirm stop band requirement is fullfilled.

$$n = \frac{\cosh^{-1}\left[(10^{2}-1)/(10^{0.3}-1)\right]^{1/2}}{\cosh^{-1}(1.5)} \Rightarrow n = 3.11$$

choose $n=4$, $\epsilon = (10^{0.3}-1)^{1/2} = 1$

$$A_{min} = 10\log_{10}\left[1 + 1^2\{\cosh(4\cosh^{-1}(1.5))\}^2\right] = 27.4\,dB > 20\,dB$$

$\therefore$ satisfied.

Q] A chebyshev filter is required to provide the following specification:-

→ passband maximum ripple width 0.5dB upto 3KHz

→ stopband maximum to be 60dB at 30KHz

$A_{max} = 0.5dB$ | $f_c = f_p = 3KHz$

$A_{min} = 60dB$ | $f_s = 30KHz$.

$$n = \frac{\cosh^{-1}\left[(10^{6}-1)/(10^{0.05}-1)\right]^{1/2}}{\cosh^{-1}\left(\frac{30}{3}\right)} = 2.88 \Rightarrow \boxed{n=3}.$$

$$\epsilon = (10^{A_{max}/10}-1)^{1/2} = 0.35$$

$$a = \frac{1}{3}\sinh^{-1}\left(\frac{1}{0.35}\right) = 0.59$$

$$\theta_K = (2K+1)\frac{\pi}{2n} \Rightarrow \theta_0 = \frac{\pi}{6} = 30^\circ$$

$$\theta_1 = 90^\circ$$

$$s_1 = -\sinh a = -\sinh(0.59) = -0.625$$

$$s_2 = -0.625\sin 30^\circ \pm j1.18\cos 30^\circ \Rightarrow s_2 = -0.313 \pm j1.022$$

$$s_3 = -0.625\sin 90^\circ \pm j1.18\cos 90^\circ = -0.625 \text{ (repeated root)}$$

$$H(s) = \frac{K}{(s+0.625)(s+0.313+j1.022)(s+0.313-j1.022)}$$

$$\therefore H(s) = \frac{K}{(s+0.625)(s^2+0.626s+1.142)}$$

where 'K' value occurs when $H(0)=1$ as n is odd.

$\Rightarrow K = (0.625)(1.142)$.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_N^2(\omega)}}$$

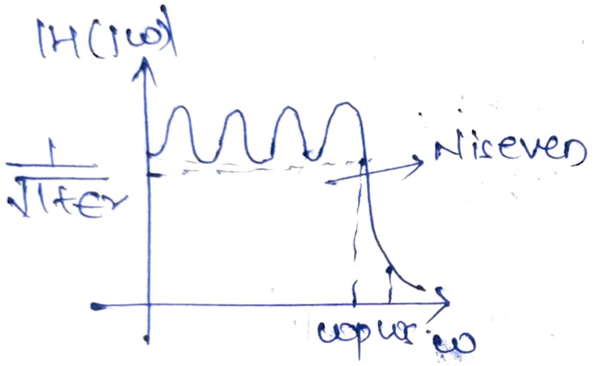
when $\omega = 0$

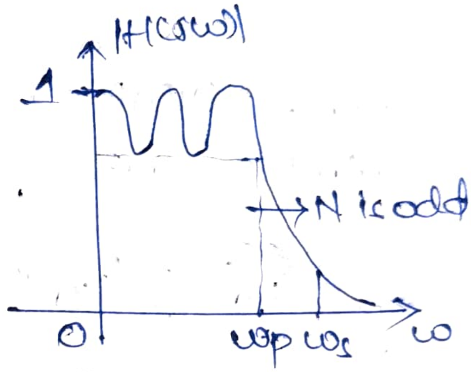
$$C_N(\omega=0) = \cos(N\cos^{-1}(0))$$

$$C_N(\omega=0) = \cos(N \times \pi/2)$$

when $N \to$ even $\Rightarrow C_N(\omega=0) = \pm 1 \to |H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2}}$

when $N \to$ odd $\Rightarrow C_N(\omega=0) = 0 \to |H(j\omega)| = 1$





Q] Design a LPF with maximum gain of 5dB with passband ripple of 0.5dB & cutoff frequency = 2500 rad/sec. stopband frequency of filter is 12,500 rad/sec with stop band attenuation of 30dB or more.

$A_{max} = 0.5dB$, $\omega_p = 2500$ rad/se, $\omega_s = 12500$ rad/sec

$A_{min} = 30dB$

$\epsilon = 0.349$

$n = 3$

$P_1 = -0.312 + j1.029$

$P_2 =$

$P_3 =$

[K × gain]

$$H(s) = \frac{0.715 \times gain}{(s^2 + 0.6264 s + 1.1424)(s + 0.6264)} \quad (1)$$

Since, N is odd, $H(s=0) = 1$

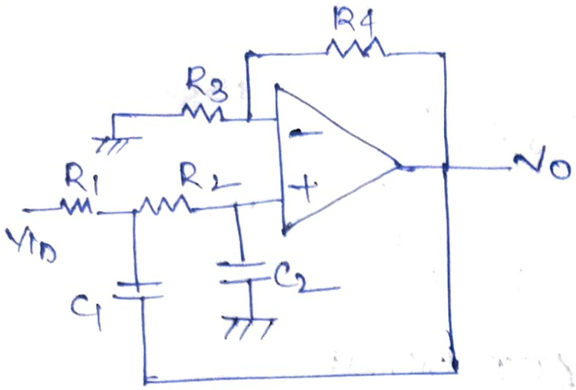
$$1 = \frac{K}{1.1424 \times 0.6264}$$

$$\Rightarrow \boxed{K = 0.715}$$

$20\log(gain) = 5$

$\Rightarrow gain = 10^{0.25} = 1.7783$

second order filter



$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_o/R_1C_1R_2C_2}{s^2 + s\left[\frac{R_1C_2 + R_2C_2 + R_1C_1(1-A_o)}{R_1R_2C_1C_2}\right] + \frac{1}{R_1R_2C_1C_2}} \quad \text{— ②}$$

when $R_1 = R_2 = R$, $C_1 = C_2 = C$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_o/R^2C^2}{s^2 + \frac{1}{RC}s(3-A_o) + \left(\frac{1}{RC}\right)^2}$$

on comparing ① & ② eqn's.

$$\left(\frac{1}{RC}\right)^2 = 1.1424 \Rightarrow RC = \sqrt{\frac{1}{1.424}}$$

$$\frac{3-A_o}{RC} = 0.6264 \Rightarrow (3-A_o)\sqrt{1.424} = 0.6264$$

$$\Rightarrow A_o = 2.41$$

Set $A_o = 1$, (unity gain of filter)

Then,

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/R_1R_2C_1C_2}{s^2 + \left[\frac{R_1C_2 + R_2C_2}{R_1R_2C_1C_2}\right] + \frac{1}{R_1R_2C_1C_2}}$$

since $\omega_p = 2500$ rad/sec

$$s \to s/\omega_p$$

$$\Rightarrow H(s) = \frac{0.715 \times 1.7783}{\left(\left(\frac{s}{\omega_p}\right)^2 + 0.6264\left(\frac{s}{\omega_p}\right) + 1.1424\right)\left(\frac{s}{\omega_p} + 0.6264\right)}$$

$$H(s) = \frac{0.715 \times 1.7783 \times \omega_p^3}{(s^2 + 0.6264 s\,\omega_p + 1.1424\,\omega_p^2)(s + 0.6264\,\omega_p)} \quad \text{— ③}$$

Compare ③ and ②

$$\frac{1}{R_1R_2C_1C_2} = 1.1414\times(2500)^2$$

$$\frac{R_1C_2 + R_2C_2}{R_1R_2C_1C_2} = 0.6264\times 2500$$

$$\frac{1}{R^2C_1C_2} = 71,40,000$$

$$\frac{2RC_2}{R^2C_1C_2} = 1566$$

$$\frac{2}{RC_1} = 1566$$

Let, $R_1 = R_2 = R = 10K\,\Omega$ $R_1 = R_2$

$\frac{1}{C_1}$ assume, $C_1 = C_2 = 0.01\mu F$

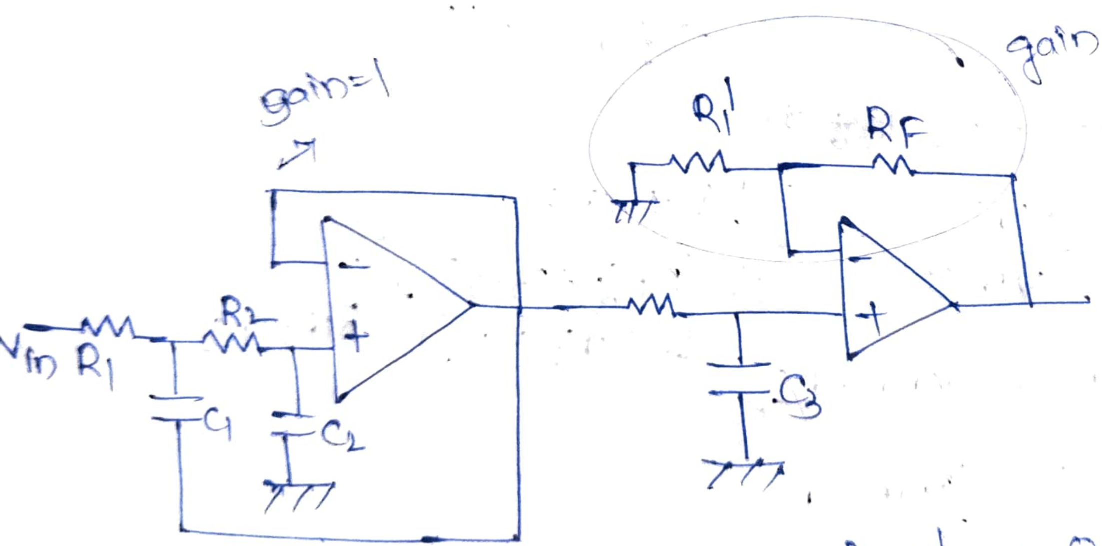
→ R_1 & R_2

$$\frac{1}{R_1R_2} = 1.1414\times(2500)^2(1\times10^{-6})^2$$

$$\frac{R_1+R_2}{R_1R_2} = 1.1414\times(2500)^2(1\times10^{-6})^{4}\times 0.6264\times 2500$$

$\Rightarrow R_1 =$

$R_2 =$



$$1 + \frac{R_F}{R_1'} = 1.77$$

$R_1' = 10K\,\Omega$

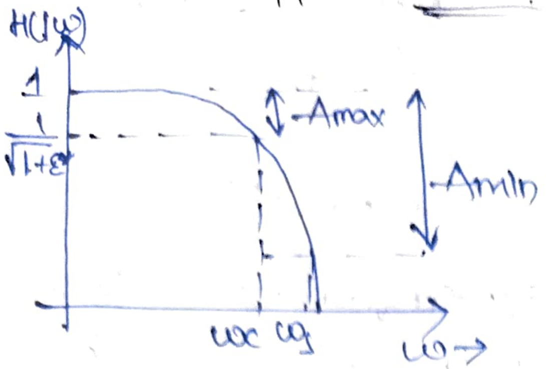
$\Rightarrow R_F = 0.77\times 10k\Omega$

$$\Rightarrow \frac{1}{R_3C_3} = 0.6264\times 2500$$

$C_3 = 1\mu F$

$R_3 = ?$

Butterworth approximation:



ω_c = pass band edge

ω_s = stop band edge

Amax = Maximum pass band attenation

Amin = Minimum stopband attenation

in dB

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\omega^{2n}}}$$

$$|-A_{max}| = 10\log_{10}(1+\epsilon^2)$$

$$\epsilon = \left(10^{-A_{max}/10} - 1\right)^{1/2} \quad -①$$

At $\omega = \frac{\omega_s}{\omega_c}$

$$|-A_{min}| = 10\log_{10}(1+\epsilon^2\omega^{2n}) \quad -②$$

Sub. Eq① in②

$$-A_{min} = 10\log_{10}\left(1+\left\{10^{\frac{-A_{max}}{10}} - 1\right\}(\omega^{2n})\right)$$

$$\omega^{2n} = \left\{\frac{10^{-A_{min}} - 1}{10^{-A_{max}} - 1}\right\}$$

$$\omega = \frac{\omega_s}{\omega_c}$$

Taking log on both sides

⇒

$$n = \frac{1}{2\log_{10}\left(\frac{\omega_s}{\omega_c}\right)} \times \log_{10}\left\{\frac{10^{-A_{min}/10} - 1}{10^{-A_{max}/10} - 1}\right\}$$

Q) A butterworth LPF is required to provide the following specification passband upto 10KHz with 3 dB of loss, stop band to be atleast 100 dB down at 100K Hz calculate the order of the filter & roots of butterworth polynomials.

$-A_{max} = 3\text{dB}$

$-A_{min} = 100\text{dB}$

$f_s = 100\text{K Hz}$

$f_c = 10\text{K Hz}$

$\Rightarrow \boxed{n=5}$

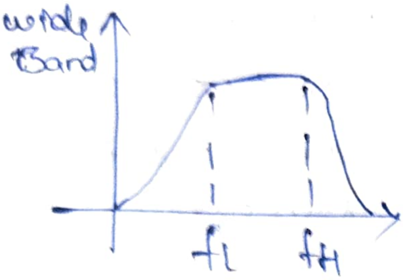
$$H(S) = \frac{1}{(S+1)(S^2+1.618S+1)(S^2+0.618S+1)}$$

Bandpass filter:

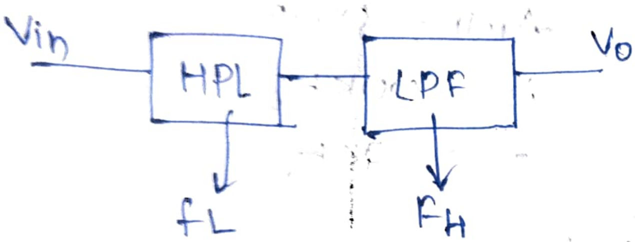
wide bandpass ($Q<10$)

Narrow bandpass ($Q>10$)

$$Q = \frac{f_c}{BW}$$



wide band pass filter

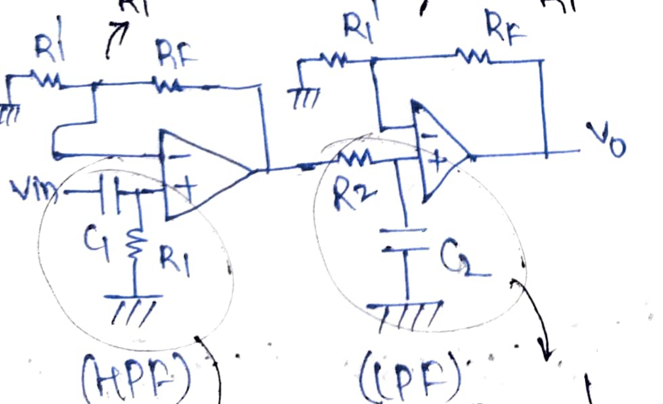


$f_H > f_L$

Q) Design a wide bandpass filter so that $f_L = 200Hz$, $f_H = 1KHz$, & a pass band gain = 4.

$1+\frac{R_F}{R_1'} = 2$

$1+\frac{R_F}{R_1'} = 2$



$$f_L = \frac{1}{2\pi R_1 C_1} = 200$$

$C_1 = 0.1\mu F$

$R_1 = ?$

$$f_H = \frac{1}{2\pi R_2 C_2} = 1\times10^3$$

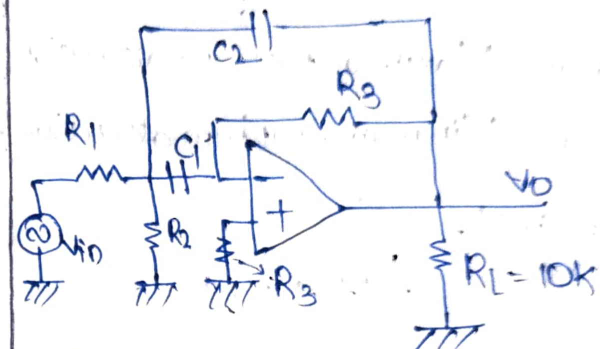
$C_2 = 0.1\mu F$

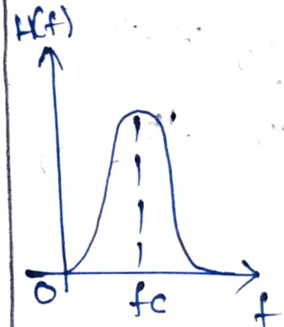
$R_2 = ?$

$$Q = \frac{f_c}{BW} = \frac{\sqrt{f_L f_H}}{(f_H - f_L)} = 0.56$$

- Narrow band pass filter

→ High frequencies and low frequencies are rejected by the filter and only narrow frequencies are allowed.





at low freq.

$$\omega = 0$$

$$\frac{1}{j\omega C} = \infty$$

impedence is high, at C, open ckt

Q..

given fc, Q, BW, Design

Equations are,

Assume,

$$C_1 = C_2 = C$$

$$R_1 = \frac{Q}{2\pi f_c C A_F}$$

$$R_2 = \frac{Q}{2\pi f_c C(2Q^2 - A_F)}$$

$$R_3 = \frac{Q}{\pi f_c C}$$

Pass band gain $= A_F = \dfrac{R_3}{2R_1}$

$\boxed{A_F < 2Q^2}$ → condition must be satisfied.

⇒ centre frequency (f_c) can be changed to new centre frequency (f_c'), keep gain & BW Constant by changing R_2 to R_2'. $R_2' = R_2\left(\dfrac{f_c}{f_c'}\right)^2$

Q]. a] Design a Narrow bandpass filter with $f_c = 1\,KHz$, $Q = 3$, $A_F = 10$.

b] Change the Center freq. to 1.5 KHz, keeping AF and BW constant.

$C_1 = C_2 = C = 0.01\,\mu F$

$R_1 = 4.72\,k\Omega$

$R_2 = 5.97\,k\Omega$

$R_3 = 95.5\,k\Omega$

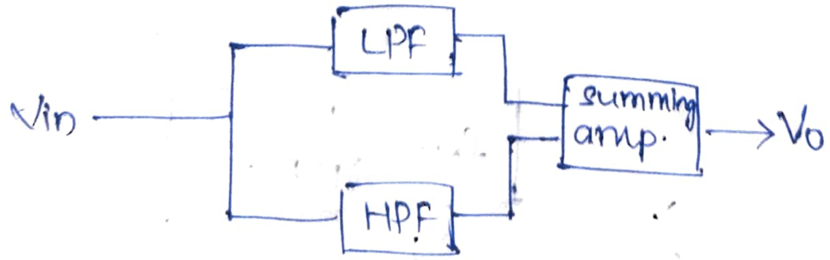
$R_2' = 5.97 \times \left(\dfrac{1K}{1.5k}\right)^2 = 2.65\,k\Omega$

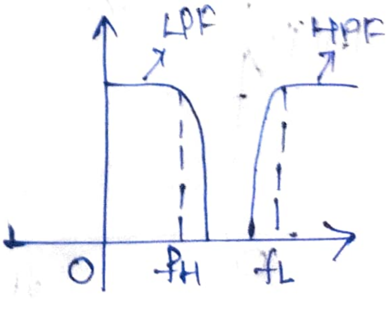
Band reject filters (Band stop)

↓ wide Band — ↓ Narrow band

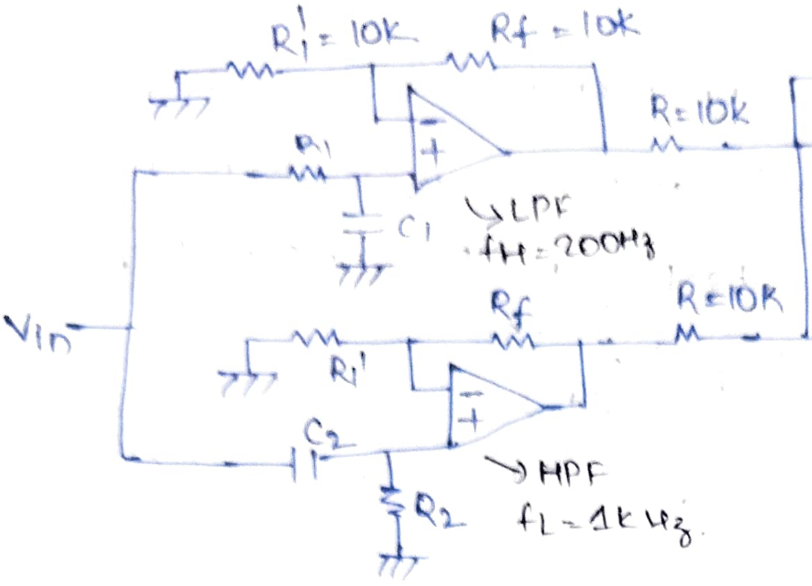
$Q = \dfrac{f_c}{BW}$ | $Q < 10$ — $Q > 10$

wideband reject filter:





Q) Design a wide band reject filter with $f_H = 200Hz$ & $f_L = 1kHz$



Assume $C_1 = C_2 = 0.01\mu F$

LPF

$$\frac{1}{2\pi R_1 C_1} = 200$$

$$R_1 = \frac{1}{2\pi \times 200 \times 0.01 \times 10^{-6}}$$

$\therefore R_1 = \underline{79.5\,k\Omega}$

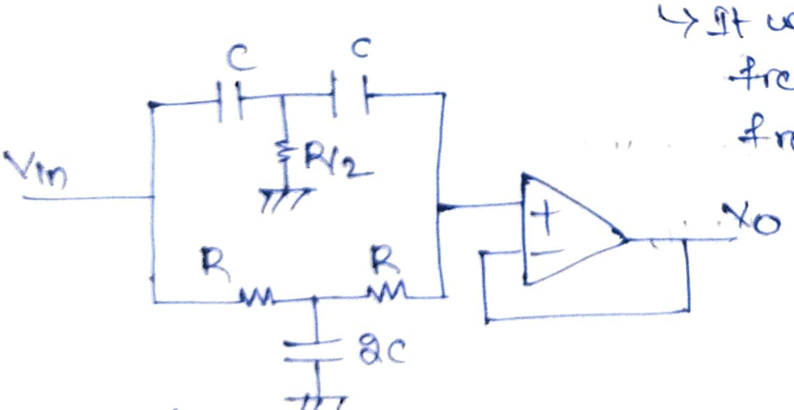
HPF

$$\frac{1}{2\pi R_2 C_2} = 1000$$

$R_2 = \underline{15.9k\Omega}$

Narrow band reject filter (notch filter) :- → Twin-T network

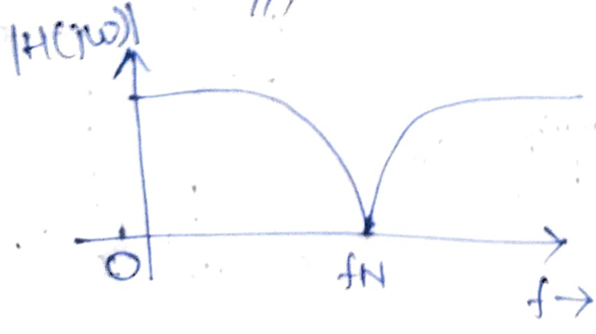
↳ It will pass both high and low frequencies and reject medium freq.



$X_c = \frac{1}{j\omega C}$

$f_{low} \to X_c \simeq \infty$ O.C

$f_{High} \to X_c \simeq 0$ S.C



$$\boxed{f_N = \frac{1}{2\pi RC}}$$

↳ notch frequency

Q. Design a 60Hz active notch filter.

$f_N = 60Hz$

let $C = 0.068\mu F$

$R = \frac{1}{2\pi f_N C} = 39.01 k\Omega$

Frequency Transformations

① Normalised LPF to LPF;

$$s \rightarrow \frac{s}{\omega_c}$$

$H(s) \rightarrow$ Normalised LPF

$$\text{LPF} \leftarrow H'(s) = H\left(\frac{s}{\omega_c}\right)$$

② Normalised LPF to HPF

~~$s \rightarrow \frac{s}{\omega_c}$~~ $s \rightarrow \frac{\omega_c}{s}$; $H(s) \rightarrow$ normalised LPF

$$H'(s) = H\left(\frac{\omega_c}{s}\right) \rightarrow \text{HPF}$$

③ Normalised LPF to BPF

$$s \longrightarrow \frac{s^2 + \omega_l \omega_h}{s(\omega_h - \omega_l)}$$

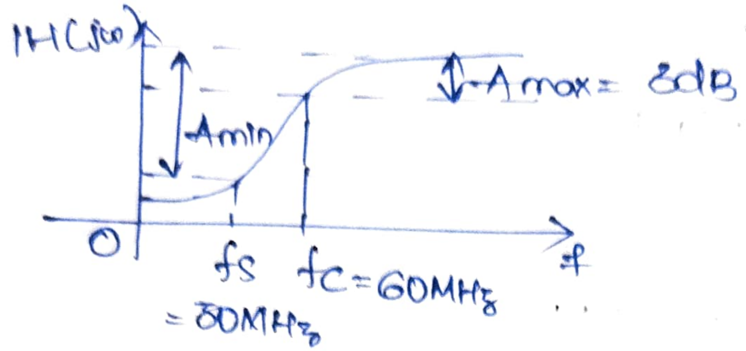
$$H'(s) = H\left[\frac{s^2 + \omega_l \omega_h}{s(\omega_h - \omega_l)}\right] \rightarrow \text{Band Pass Filter}$$

④ Normalised LPF to BSF

$$s \rightarrow \frac{s(\omega_h - \omega_l)}{s^2 + \omega_l + \omega_h}$$

$$H'(s) = H\left[\frac{s(\omega_h - \omega_l)}{s^2 + \omega_h \omega_l}\right] \rightarrow \text{Band stop filter.}$$

Q] A butterworth HPF has $f_c = 60\,MHz$, $A_{min} = 40\,dB$
$f_s = 30\,MHz$, $A_{max} = 3\,dB$ find order of filter.



$$n = \frac{1}{2\log\left(\frac{\omega_s}{\omega_c}\right)} \times \log_{10}\left(\frac{10^{\frac{A_{min}}{10}} - 1}{10^{A_{max}/10} - 1}\right) \rightarrow \text{normalised LPF}$$

normalised HPF $\rightarrow$ $n = \dfrac{1}{2\log\left(\frac{\omega_c}{\omega_s}\right)} \log_{10}\left(\dfrac{10^{\frac{A_{min}}{10}} - 1}{10^{\frac{A_{max}}{10}} - 1}\right)$

$\underline{\underline{n = 6.64}} \Rightarrow n = 7$

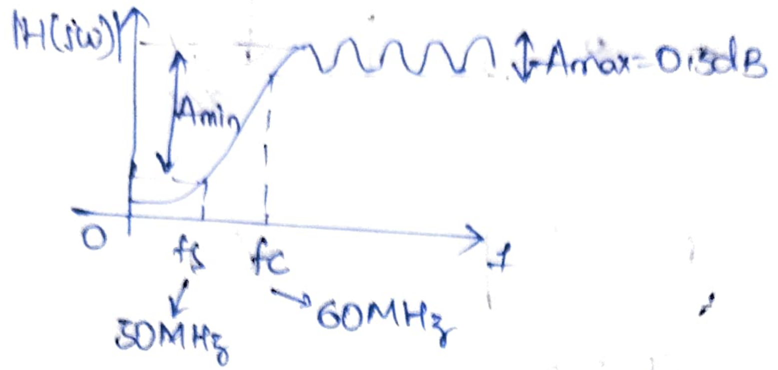
poles: $\rightarrow$?

Transfer fxn $\rightarrow$?

Q. A chebyshev HPF has,

$f_c = 60MHz$, $A_{min} = 40dB$, $f_s = 30MHz$, $A_{max} = 0.5dB$

find order of filter.



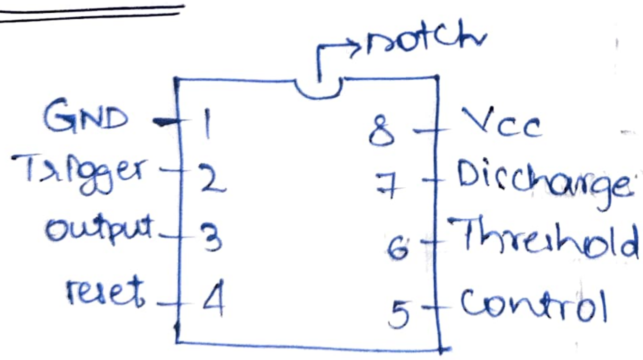
norm HPF ⇒ $n = \dfrac{\cosh^{-1}\left(\left(10^{\frac{A_{min}}{10}} - 1\right) / \left(10^{\frac{A_{max}}{10}} - 1\right) \right)^{1/2}}{\cosh^{-1}\left(\frac{\omega_s}{\omega_c}\right)}$

n → ?

poles → ?

T.F → ?

IC555:



pin diagram