eg: C.E: $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$. Determine stability.

s^5	1	3	2
s^4	1	3	2
s^3	$\cancel{0}$ 4	$\cancel{0}$ 6	
s^2	$\frac{3}{2}$	2	
s^1	$2/3$		
s^0	2		

A.E: $s^4 + 3s^2 + 2 \Rightarrow \psi$

$$\frac{\partial \psi}{\partial s} = 4s^3 + 6s$$

Replace zeros with these coefficients

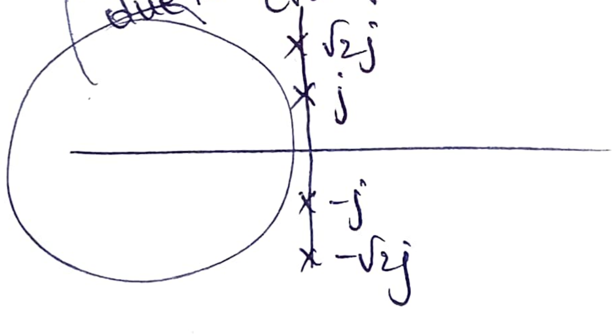
→ Whenever sequence of zero appears in a row, write auxiliary equation ~~[illegible]~~. (A.E)

$$A.E(\psi) \Rightarrow s^4 + 3s^2 + 2 = 0$$

$$(s^2+2)(s^2+1) = 0$$

$$s = \pm\sqrt{2}j \; ; \; s = \pm j$$

Another pole in LHS ~~due~~ to no sign change.



eg: C.E: $s^6+3s^5+4s^4+6s^3+5s^2+3s+2=0$

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	$\cancel{0}$ 8	$\cancel{0}$ 8	0	
s^2	2	2		
s^1	$\cancel{0}$ 4	0		
s^0	2			

A.E: $2s^4+4s^2+2=0$ (φ_1)

$$\frac{\partial \varphi_1}{\partial s} = 8s^3+8s$$

A.E(φ_2): $2s^2+2=0$

$$\frac{\partial \varphi}{\partial s} = 4s$$

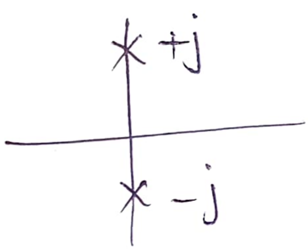
$\varphi_1 = 2s^4+4s^2+2=0$

$(s^2+1)^2=0$

$s = \pm j ; \pm j$

$\varphi_2 = 2s^2+2 = 0$

$\underline{s = \pm j}$



Whenever poles are repeating on Imaginary axis, the system is <u>unstable</u>.

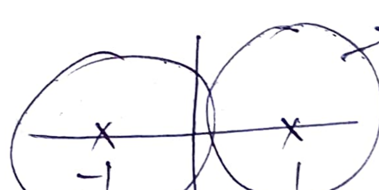
eg: C.E: $s^4+s^3-s-1=0$. Find no. of poles in LH s-plane.

s^4	1	0	-1
s^3	1	-1	0
s^2	1	-1	
s^1	$\cancel{0}$ 2	0	
s^0	-1		

A.E: $\varphi \Rightarrow s^2-1=0$

$$\frac{\partial \varphi}{\partial s} = 2s$$

$$\boxed{s = \pm 1}$$



2 poles in RHS plane

2 poles in LHS s-plane

eg: find the range of k for which system is stable,

$C.E: s^3+9s^2+4s+k=0$

$$
\begin{array}{c|cc}
s^3 & 1 & 4 \\
s^2 & 9 & k \\
s^1 & \frac{36-k}{9} & 0 \\
s^0 & k &
\end{array}
$$

$\frac{36-k}{9} > 0$

$\boxed{36 > k \text{ and } k > 0}$

if $\boxed{0 < K < 36} \rightarrow$ stable

if $\boxed{K = 36} \rightarrow$ marginally stable

$\Downarrow$

$$
\begin{array}{c|cc}
s^3 & 1 & 4 \\
s^2 & 9 & 36 \\
s^1 & \cancel{0}\ 18 & 0 \\
s^0 & 36 &
\end{array}
$$

$\rightarrow A.E: 9s^2+36 \ (\psi)$

$\frac{\partial \psi}{\partial s} = 18s$

$\psi \Rightarrow 9s^2+36=0$

$s = \pm 2j$

$s = \pm j\omega_n$

$\boxed{\omega_n = 2 \text{ rad/s}}$

$2\pi f_n = 2$

$\boxed{f_n = \frac{1}{\pi} \text{ Hz}} \rightarrow$ frequency of oscillation.

eg: Find value of 'k' for which system is stable,

$$GH = \frac{k}{s(s+2)(s+4)(s+6)}$$

CE: $1+GH=0$

$$1+\frac{k}{s(s+2)(s+4)(s+6)} = 0$$

$(s^2+2s)(s^2+10s+24)+k=0$

$s^4+12s^3+44s^2+48s+k=0$

s^4	1	44	K
s^3	12	48	0
s^2	40	K	0
s^1	$48-\frac{3k}{10}$	0	
s^0	K		

$$\frac{12(40)}{12}$$

$$\frac{40(48)-12k}{40}$$

$$48-\frac{3k}{10}>0 \quad , \boxed{K>0}$$

$$\frac{\cancel{3}k}{10} < \cancel{48}^{16}$$

$$\boxed{K<160}$$

if $\boxed{0<K<160}$ → stable

if $\boxed{K=160}$ → marginally stable

$\Downarrow$

~~s^4~~			
~~s^3~~	~~1~~	~~44~~	~~K~~
~~s^2~~	~~12~~	~~48~~	~~0~~
~~s^1~~	~~40~~	~~160~~	
~~s^0~~			

s^4	1	44	k
s^3	12	48	0
s^2	40	160	
s^1	$\cancel{0}$ → 80	0	
s^0	160		

A·E

$$(\psi)=40s^2+160=0 \rightarrow \frac{\partial\psi}{\partial s}=80s$$

$$s=\pm 2j$$

$$\omega_n = 2\,rad/s$$

$$f_n=\frac{1}{\pi}\,Hz$$

eg: Determine value of K & P so that oscillates at 2 rad/s.

$$G(s) = \frac{K(s+1)}{s^3 + ps^2 + 3s + 1}$$

Sol:

if H(s) is not given, Assume unity feedback.

$\cancel{\frac{G(s)}{1+}}$

$\omega_n = 2\,rad/s$

C.E: $1 + GH = 0$

$$1 + \frac{K(s+1)}{s^3 + ps^2 + 3s + 1} = 0$$

$p = 1$

$k = 3$

$$s^3 + ps^2 + s(k+3) + (k+1) = 0$$

$$\begin{array}{c|cc} s^3 & 1 & k+3 \\ s^2 & p & k+1 \\ s^1 & k+3 - \frac{k+1}{p} & \\ s^0 & k+1 & \end{array}$$

$$k+3 - \left(\frac{k+1}{p}\right) = 0$$

$$p = \frac{k+1}{k+3}$$

A.E: $(\varphi) \Rightarrow ps^2 + (k+1) = 0$

$$\frac{\cancel{(k+1)}}{(k+3)} s^2 + \cancel{(k+1)} = 0$$

$$s^2 + (k+3) = 0$$

$s = \pm j\omega_n \longrightarrow s = \pm\sqrt{k+3}\,j$

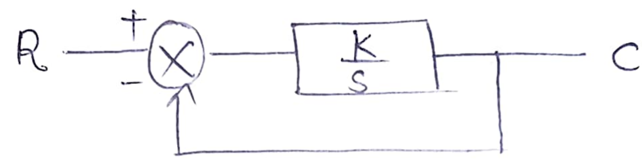
$\boxed{k = 1}$

$\boxed{p = 1/2}$

Root Locus :

→ Root locus is nothing but variation of closed loop system poles as the system gain(K) varies.

eg:

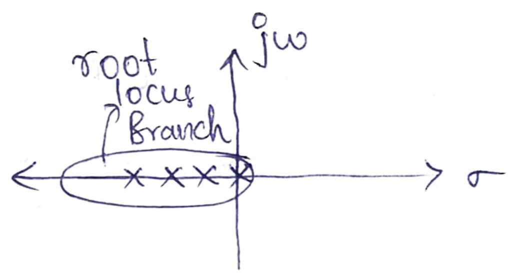


characteristic equation → $1+GH=0$

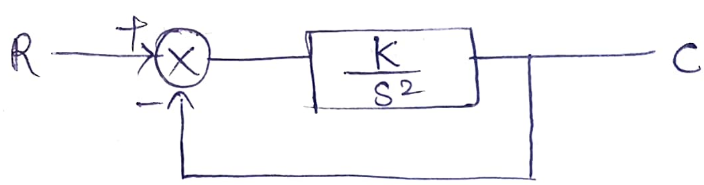
$$1+\frac{K}{S}(1)=0$$

$$\boxed{S=-K}$$

K	S
0	0
1	-1
2	-2
⋮	⋮
∞	-∞



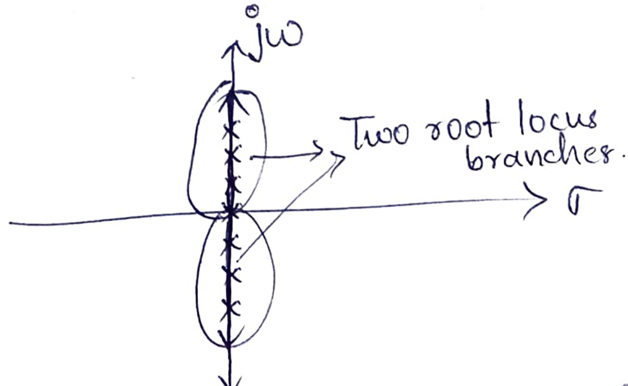
eg:



$$1+GH=0$$

$$1+\frac{K}{S^2}(1)=0 \rightarrow \boxed{S=\pm j\sqrt{K}}$$

K	S
0	0
1	$\pm j$
2	$\pm\sqrt{2}j$
⋮	⋮



$$\boxed{\text{No. of root locus branches} = \text{order of system}}$$

$GH \rightarrow$ open loop gain,

$1+GH=0 \rightarrow$ characteristic equation,

Let, $GH = \dfrac{K\,N(s)}{D(s)}$ —— ①

$$1+\frac{K\,N(s)}{D(s)}=0$$

from ① & ②,

$$D(s)+K\,N(s)=0 \quad \text{—— ②}$$

Case-①: If $D(s)=0$ and $K=0$

Then poles of open loop gain = poles of closed loop system

Case-②: if $K=\infty$ and $N(s)=0$

then zeroes of open loop gain = poles of closed loop system

→ The root locus diagram starts from $K=0$ (poles of GH) and ends at $K=\infty$ (zeroes of GH)

eg: find start and end points of the root locus diagram

$$GH=\frac{K(s+5)}{s(s+10)(s+20)}$$

Sol:-

No. of branches = 3 (order of GH)

Start points : $s=0,-10,-20$

End points : $s=-5,\infty,\infty$

→ Angle condition,

To check whether $s = s_0$ is present in root locus or not,

$$1 + G(s_0)H(s_0) = 0$$

$$\boxed{GH = -1}$$

$$\angle GH = \angle -1$$

$$\boxed{\angle GH = \pm(2q+1)180^\circ \quad q = 0, 1, 2 \ldots}$$

eg: $GH = \dfrac{K}{s(s+2)(s+4)}$

check whether the following pts are in root locus or not

(a) $s = -0.75$ (b) $s = -1 + 4j$

Sol:- (a)

$$\angle GH\Big|_{s=-0.75} = \frac{\angle K}{\angle(-0.75)(1.25)(3.25)}$$

$$= \frac{0^\circ}{\pm 180^\circ \cdot 0^\circ \cdot 0^\circ}$$

$$= \pm 180^\circ$$

So, $s = -0.75$ is in the RL.

(b)

$$\angle GH\Big|_{s=-1+4j} = \frac{\angle K}{\angle -1+4j \; \angle 1+4j \; \angle 3+4j}$$

$$= \frac{0^\circ}{104^\circ \cdot 76^\circ \cdot 53^\circ}$$

$$= -233^\circ$$

Since it is not the odd multiples of 180°, it is not in Root Locus.

→ Magnitude condition,

To find the system gain(K) at a point on RL

$$1 + GH = 0$$

$$GH = -1$$

$$\therefore \boxed{|GH| = 1}$$

eg: $GH = \dfrac{K}{S(S+4)}$ find system gain at a point, $S = -2 + 5j$

Sol:

$$|GH|_{S=-2+5j} = \left|\frac{K}{(-2+5j)(2+5j)}\right| = 1$$

$$= \left|\frac{K}{-25-4}\right| = 1$$

$$\therefore |K| = 29$$

$$\angle GH|_{S=-2+5j} = \frac{\angle K}{\angle -2+5j \ \angle 2+5j}$$

$$= \frac{0^\circ}{112^\circ \cdot 68^\circ}$$

$$= \underline{\pm 180^\circ}$$

$180 - 68 = 112$

eg: $GH = \dfrac{(s+1)}{s(s+2)((s+1)^2+1)}$

$s = 0, -2, -1+j, -1-j$

→ If s is real then it is on the root locus if and only iff there are odd no. of real open poles and zeroes to the right side of 's'.

Rules for construction of Root Locus:

Rule-1: Locate the open loop poles and zeroes in S-plane

Rule-2: Find no. of root locus branches.
↳ equal to open-loop poles / order of characteristic eq.

Rule-3: Identify and draw the real axis root locus branches

Rule-4: Find the centroid and the angle of asymptotes

Rule-5: Find the intersection pt of root locus branches with an imaginary axis.

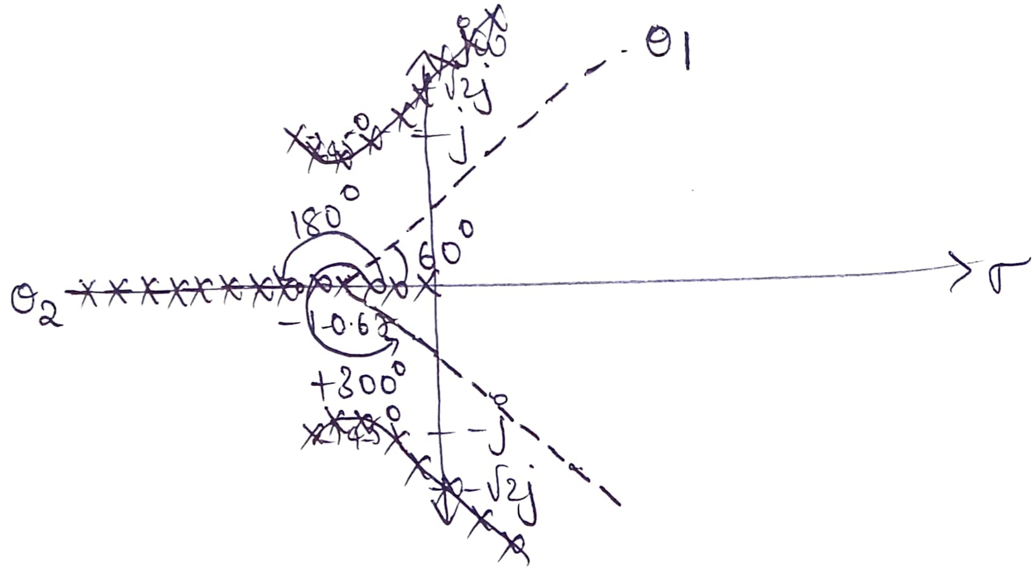
Rule-6: Find the Break-away & Break-in pts.

Rule-7: Find the angle of departure and the angle of arrival.

eg: Draw the root locus for the system,

$$GH = \frac{k}{s(s^2+2s+2)}$$

Sol: poles of GH : $s = 0$, $s = -1 \pm j$



No. of roots locus branches = 3

no. of poles (n) = 3

no. of zeroes (m) = 0

$$\text{Centroid} = \frac{0+(-1+j)+(-1-j)}{3} = \frac{-2}{3} = \underline{-0.67}$$

angle of asymptotes, $\frac{2q+1}{n-m} \times 180^\circ \quad (0,1,2)$

$\hookrightarrow 60^\circ, 180^\circ, 300^\circ$

ch. eq $\rightarrow 1+GH=0$

$$1+\frac{K}{s(s^2+2s+2)} = 0$$

$$s^3+2s^2+2s+k=0$$

s^3	1	2
s^2	2	K
s^1	$\frac{4-k}{2}$	0
s^0	K	

at $K=4 \rightarrow$ marginally stable. (or) $j\omega$ crossing pt

$$2s^2+K=0$$
$$2s^2+4=0$$
$$s^2=-2 \rightarrow \boxed{s=\pm\sqrt{2}j}$$

intersection pts

$$K=-s^3-2s^2-2s$$

$$\frac{dk}{ds}=0 \rightarrow +3s^2+4s+2=0$$

$$\underline{s=-0.67\pm0.47j}$$

$\phi_d = 180-\phi$

where $\phi = \Sigma\theta_{poles} - \Sigma\theta_{zeroes}$

$$=(135^\circ+90^\circ)-0^\circ$$
$$=225^\circ$$

$\phi_{d_1} = -45^\circ$

$\phi_{d_2} = 45^\circ$

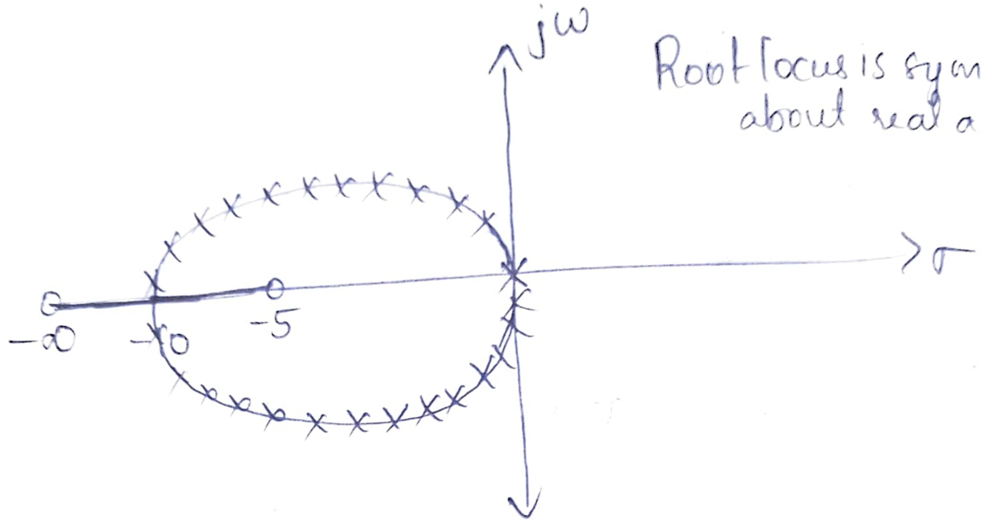
- eg: $GH = \frac{K(s+5)}{s^2}$

Poles of GH : $s = 0, 0$

No. of branches $= 2$

no. of poles $(n) = 2$

no. of zeroes $(m) = 1$



Root locus is symmetric about real axis

$$\text{Centroid} = \frac{0-(-5)}{1} = 5$$

$$\text{angle of asymptotes} = \left(\frac{2q+1}{1}\right)180^\circ \Rightarrow q = 0 \text{ only}$$

$$\underline{\theta = 180^\circ}$$

$1 + GH = 0$

$$1 + \frac{K(s+5)}{s^2} = 0$$

$$s^2 + ks + 5k = 0$$

$$K = \frac{-s^2}{(s+5)}$$

s^2	1	5k
s^1	k	0
s^0	5k	

$$\frac{dk}{ds} = 0$$

↳ $s = 0, -10$

↳ Break in pts