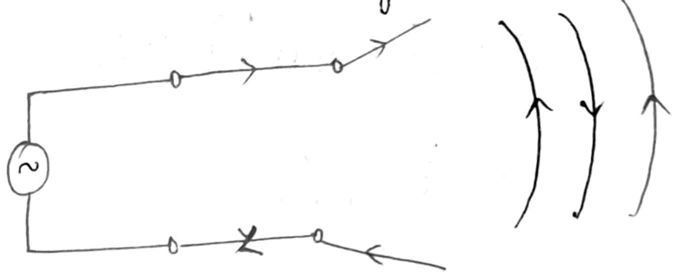
23/08/22

# EC305

- **Antenna**: It is a transducer which converts electric signals into electromagnetic waves.

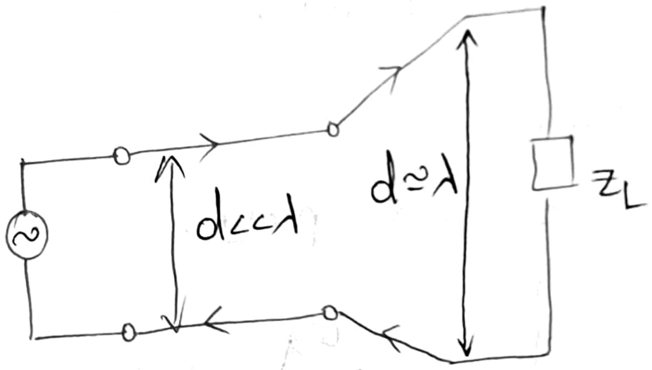


→ Radiation is evolved by two processes.

(i) Time varying charge.

(ii) Spacial Imbalance ↓

(It is nothing but distance b/w the lines is equal to wavelength of radiation $d \simeq \lambda$.)




- **Transverse Electromagnetic wave**: (travels in the same direction as the wave)

$$\vec{E} = E_0 e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{j\omega t}\, \hat{x} \ \ V/m$$

$$\vec{H} = \frac{E_0}{\eta_0} e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\, \hat{y} \ \ A/m$$

$$\boxed{\vec{E} \times \vec{H} = \text{direction}}$$

$$\boxed{\eta_0 = \frac{|\vec{E}|}{|\vec{H}|}}$$

$$\vec{P}(t) = \vec{E} \times \vec{H} \quad W/m^2$$

$$\text{Power} = \int_S \vec{P}_{avg} \cdot \vec{ds}$$

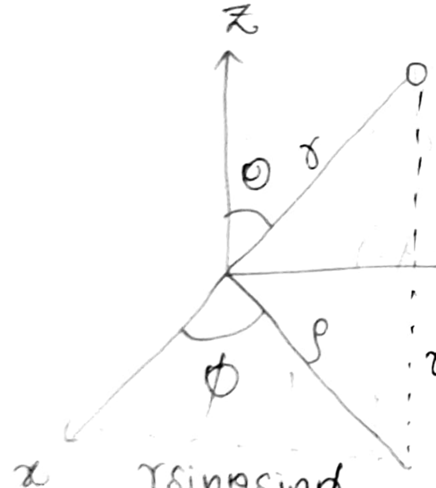
$$\vec{P}_{avg} = \frac{1}{T}\int_0^T \vec{P}(t)\, dt$$

power density

$$= \frac{1}{2} \text{Re}\left[\vec{E} \times \vec{H}^*\right] \quad W/m^2$$

$$= \frac{1}{2} \frac{|\vec{E}|^2}{\eta_0} \cdot \hat{a}_z$$

- Spherical co-ordinate system:




$(r, \theta, \phi)$

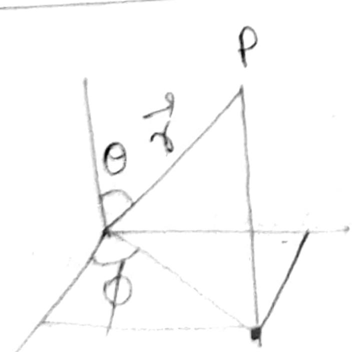
$r$ – radius

$\theta$ – 0 to $\pi$

$\phi$ – 0 to $2\pi$

$\rho = r\sin\theta$

- Projection of $r$ in XY-plane:





Differential Area

$ds_1 = r\sin\theta\, d\phi\, dr$

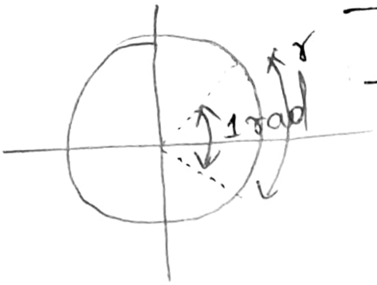
$ds_2 = (dr)\, r\, d\theta$

$ds_3 = r\sin\theta\, (d\phi)\, r\, d\theta$


$\boxed{r\,d\theta \; r\sin\theta\, d\phi}$

$$\boxed{\vec{ds} = \hat{r} \cdot r\,d\theta \cdot r\sin\theta\, d\phi}$$

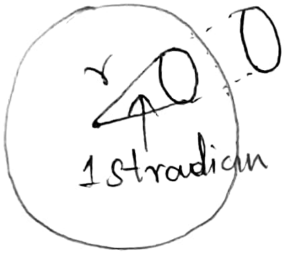
## Radian & Steradian:




→ Circle has a total angle of $2\pi$ rad

→ 1 radian is angle created by an arc of length 'r'.

1 radian = Arc length $\hat{r}$.

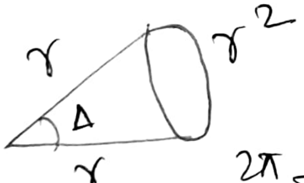



$A = r^2$

→ Sphere has a total solid angle of $4\pi$ rad

→ 1 steradian is square of angle (solid angle) created by cone of surface area "$r^2$"

Unit of solid angle is Steradian




Surface area of sphere $= 4\pi r^2$

$$ds = \int_0^{2\pi}\int_0^{\pi} (r\,d\theta)(r\sin\theta)\,d\phi$$

$$\int ds = 4\pi r^2$$

$$2\pi r^2 \int_0^{\pi} \sin\theta\,d\theta = \underline{4\pi r^2}$$

$4\pi r^2$ = Total surface area of sphere.

$4\pi$ = Total solid angle of sphere

$$d\Omega = \frac{ds}{r^2} = \frac{(\cancel{r}\sin\theta)(\cancel{r}d\theta)\,d\phi}{\cancel{r^2}}$$

$$d\Omega = \frac{ds}{r^2} = \sin\theta\, d\theta\, d\phi$$

$$\int d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta\, d\theta\, d\phi$$

$$= 2\pi \int_0^{\pi} \sin\theta\, d\theta$$

$$\underline{\underline{d\Omega = 4\pi}}$$

$$\vec{P}_{avg} = \frac{|E|^2}{2\eta_0}\,\hat{a}_r$$

$$\boxed{Power = \frac{|E|^2}{2\eta_0}\cdot 4\pi r^2}$$

$$Power = \int \vec{P}_{avg}\cdot\vec{ds}$$

$$\boxed{Power = P_{avg}\cdot(area)}$$

$$\boxed{Power = |P_{avg}|\cdot(4\pi r^2)}$$

$$P_{avg} = \frac{Power}{4\pi r^2}$$

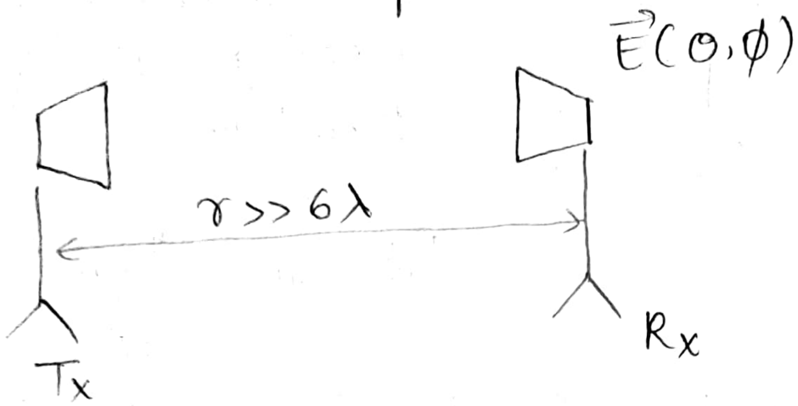
$$\boxed{\text{Radiation Intensity }(U) = P_{avg}(r^2) = \frac{Power}{4\pi}}$$

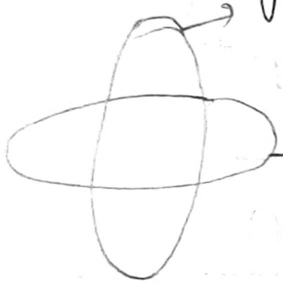
↓

Power per solid angle

$$\boxed{\vec{E} = \frac{E_0}{4\pi r}\cdot e^{-j(\omega t - \beta r)}\,\hat{\theta}}$$

- **Radiation pattern**: Plot of electromagnetic field intensity in a space.



→ Vertical Plane (Elevation plane) (YZ-plane) → $\phi = 90^\circ$, $\theta$ varies

→ Horizontal plane (Azimuthal plane) (XY-plane) → $\theta = 90^\circ$, $\phi$ varies

→ We can understand the radiation pattern of an antenna by finding electromagnetic field along Azimuthal and Elevation planes.

→ Isotropic radiation: Radiation is same along the whole surface.

- Directional pattern
- Omnidirectional pattern
- Radiation pattern

$U_{rad} = \frac{Power}{4\pi}$

→ The following antenna is radiating uniformly with Prad Watts.

$$P_{density} = \frac{P_{rad}}{4\pi r^2}$$

→ If antenna is not radiating uniformly then,

$$P_{density} = \frac{\iint_S P_{density} \cdot \vec{ds}}{4\pi r^2}$$

$$\rightarrow \text{Directivity}(\theta, \phi) = \frac{U(\theta, \phi)}{[\text{Avg radiation Intensity}]}$$

$$\boxed{\text{Directivity}(\theta, \phi) = \frac{U(\theta, \phi)}{U_{avg}}}$$

$$\rightarrow P_{density} = \frac{1}{2} \frac{|\vec{E}(\theta, \phi)|^2}{\eta_0}$$

$$U(\theta, \phi) = \frac{1}{2} \frac{|\vec{E}(\theta, \phi)|^2 \cdot r^2}{\eta}$$

$$\text{Space avg } P_{density} = \frac{\iint_S P_{density} \cdot \vec{ds}}{4\pi r^2}$$

$$U_{avg} = \frac{\iint P_{density} \cdot ds}{4\pi}$$

$$\boxed{D(\theta, \phi) = \frac{\frac{1}{2\eta} |\vec{E}(\theta, \phi)|^2 \cdot r^2}{\iint P_{den} \cdot \vec{ds}} \cdot 4\pi}$$

$$D(\theta,\phi)\Big| = \frac{\frac{1}{2\pi}|E(\theta,\phi)|^2 \cdot r^2 \cdot 4\pi}{\iint_S \frac{|E(\theta,\phi)|^2}{2\pi} \cdot r^2 \cdot \sin\theta\, d\theta\, d\phi}$$

$$= \frac{4\pi}{\iint_S \left(\frac{|E(\theta,\phi)|}{|E(\theta,\phi)|_{max}}\right)^2 \cdot \sin\theta\, d\theta\, d\phi}$$

$$D(\theta,\phi) = \frac{4\pi}{\iint_S |E(\theta,\phi)|_n^2 \sin\theta\, d\theta\, d\phi}$$

Examples:

1) $\vec{E} = \frac{E_0}{4\pi\varepsilon r} e^{j(\omega t - \beta r)} \hat{a}_\theta$

(i) isotropic antenna:

wave propagation = +ve r-direction

$$U(\theta,\phi) = \frac{P_{rad}}{d\Omega} = \left(\frac{P_{rad}}{dA}\right) \cdot r^2$$

$$= \text{Power density} \cdot r^2$$

$$\text{Power density} = \vec{P}_{avg} = \frac{|E|^2}{2\eta_0} \cdot \hat{a}_r$$

$$\vec{P}_{avg} = \frac{E_0^2}{(4\pi\varepsilon r)^2} \cdot \frac{1}{2\eta_0} \hat{a}_r$$

$$\boxed{U(\theta,\phi) = \frac{E_0^2}{(4\pi\varepsilon)^2} \cdot \frac{1}{2\eta_0}}$$

$$U_{avg} = \frac{\text{Power}}{4\pi} \quad \text{where power} = \iint_S \vec{P}_{avg} \cdot \vec{ds}$$

$$\vec{ds} = (r\,d\theta)\cdot(r\sin\theta\, d\phi)\cdot \hat{r}$$

$$\text{Power} = \iint_S \frac{E_0^2}{(4\pi\varepsilon r)^2} \cdot \frac{1}{2\eta_0}\, \hat{a}_r\, \vec{ds}$$

$$= \frac{E_0^2}{(4\pi\varepsilon)^2 2\eta_0} \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi$$

$$= \frac{E_0^2}{(4\pi\varepsilon)^2 \cdot 2\eta_0} \cdot (4\pi)$$

$$\text{Power} = \frac{E_0^2}{4\pi\varepsilon^2 \cdot (2\eta_0)}$$

$$\boxed{U_{avg} = \frac{\text{Power}}{4\pi} = \frac{E_0^2}{(4\pi\varepsilon)^2} \cdot \frac{1}{2\eta_0}}$$

$$U(\theta,\phi) = U_{avg}$$

for all values of $\theta, \phi$.

(ii) <u>directional antenna</u>:

$$\vec{E} = j\, \frac{\text{const} = E_0}{4\pi\omega\varepsilon_r r}\, \beta^2 \sin\theta\, e^{j(\omega t - \beta x)}\, \hat{a}_\theta$$

wave = +ve $r$-direction

$$\vec{E} = j\, \frac{E_0 \sin\theta}{r}\, e^{j(\omega t - \beta r)}\, \hat{a}_\theta$$

$\left( \text{Power density } \vec{P}_{avg} = \frac{|E_0|^2}{2\eta_0} \right)$

$$\vec{P}_{avg} = \frac{E_0^2}{r^2} \cdot \frac{1}{2\eta_0} \sin^2\theta \, \hat{a}_r$$

$$U(\theta,\phi) = \frac{E_0^2}{2\eta_0} \sin^2\theta \quad \left( U(\theta,\phi) = \vec{P}_{avg} \cdot r^2 \right)$$

$$U_{avg} = \frac{\text{Power}}{4\pi}$$

$$\text{Power} = \iint_S \vec{P}_{avg} \cdot \vec{ds}$$

$$\text{Power} = \iint_S \frac{E_0^2}{2\eta_0} \sin^2\theta \, \hat{a}_r \cdot \vec{ds} \qquad \vec{ds} = (r\,d\theta)(r\sin\theta\,d\phi)\,\hat{r}$$

$$= \frac{E_0^2}{2\eta_0} \iint \sin^2\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$= \frac{E_0^2}{2\eta_0} \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$= \frac{E_0^2}{2\eta_0} \int_0^{2\pi} \int_0^{\pi} (1-\cos^2\theta) \sin\theta \, d\theta \, d\phi$$

$$t = \cos\theta$$

$$= \frac{E_0^2}{2\eta_0} \int_0^{2\pi} d\phi \int_1^{-1} (1-t^2)\, dt$$

$$= \frac{E_0^2}{2\eta_0} \cdot (2\pi) \cdot \frac{4}{3}$$

$$= \frac{4E_0^2\pi}{3\eta_0} \qquad (\eta_0 = 120\pi)$$

$$\boxed{\text{Power} = \frac{E_0^2}{90}}$$

$$U_{avg} = \frac{Power}{4\pi} = \frac{E_0^2}{90 \times 4\pi}$$

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{avg}} = \frac{E_0^2 \sin^2\theta}{2 \times 120\pi} \times \frac{90 \times 4\pi}{E_0^2}$$

$$\boxed{D(\theta, \phi) = \frac{3}{2} \sin^2\theta}$$

→ if $D(\theta = 90°) \Rightarrow D(\theta, \phi) = 3/2$

↓

performs better compared to isotropic.

- Half Power Beam Width :

$$D(\theta, \phi) = \frac{3}{2} \sin^2\theta \qquad 0 \le \theta \le \pi$$

$$U(\theta, \phi) = \frac{E_0^2}{2\eta_0} \sin^2\theta$$

$$\left( \frac{U(\theta, \phi)_{max}}{2} = \frac{E_0^2}{2\eta_0} \times \frac{1}{2} \right)$$

if $U(\theta, \phi) = \frac{U(\theta, \phi)_{max}}{2}$

then $\Rightarrow \frac{\cancel{E_0^2}}{\cancel{2\eta_0}} \sin^2\theta_{HP} = \frac{\cancel{E_0^2}}{\cancel{2\eta_0}} \cdot \frac{1}{2}$

$$\sin\theta_{H.P} = \pm \frac{1}{\sqrt{2}}$$

$$\boxed{\theta_{H.P} = 45°, 135°}$$

$$\therefore \boxed{\theta_{H.P.B.W} = 90°}$$

2) $U(\theta, \phi) = \cos^2(2\theta) \quad 0 \le \theta \le 90^\circ$

Determine HPBW.

Sol: $U = \cos^2(2\theta) \longrightarrow \frac{U_{max}}{2} = \frac{1}{2}$

$$\frac{1}{2} = \cos^2(2\theta) \Rightarrow \cos(2\theta) = \pm \frac{1}{\sqrt{2}}$$

$$2\theta = 45^\circ \text{ (or) } 135^\circ$$

$$\theta = 22.5^\circ \text{ (or) } 67.5^\circ$$

$$\therefore \boxed{\theta_{HPBW} = 45^\circ}$$

3) $P_{rad} = 10\,W$ ; max power density at 1000 m from antenna

Sol: $U(\theta, \phi) = B_0 \cos^2\theta$

$$P_{density} = \frac{U(\theta, \phi)}{r^2} \qquad \begin{array}{l} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array}$$

$$= \frac{\max(U(\theta, \phi))}{r^2}$$

$$P_{density} = \frac{B_0}{r^2}$$

$$P_{rad} = \iint U \cdot d\Omega$$

$$= \int_0^{2\pi} \int_0^{\pi} B_0 \cos^2\theta \, \sin\theta \, d\theta \, d\phi$$

$$10\,W = \frac{4\pi}{3} B_0 \longrightarrow \boxed{B_0 = \frac{30}{4\pi}}$$ ✓

max power density,

$$\frac{B_0}{r^2} = \frac{30}{4\pi} \times \frac{1}{(1000)^2}$$

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}}$$

$$\left( U_{avg} = \frac{P_{rad}}{4\pi} \right)$$

$$D(\theta,\phi) = \frac{B_0 \cos^2\theta}{10/4\pi}$$

$$= \frac{\frac{30}{\cancel{4\pi}} \cos^2\theta}{10/\cancel{4\pi}}$$

$$\boxed{D(\theta,\phi) = 3\cos^2\theta}$$

→ In the direction of antenna, max directivity is $\underline{\underline{3}}$

$$\text{HPBW} \Rightarrow \frac{\max(U(\theta,\phi))}{2} = U(\theta,\phi)$$

$$\frac{\cancel{B_0}}{2} = \cancel{B_0} \cos^2\theta$$

$$\cos\theta = \pm \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ \text{ (or) } 135^\circ$$

$$\boxed{\theta_{HPBW} = 90^\circ}$$

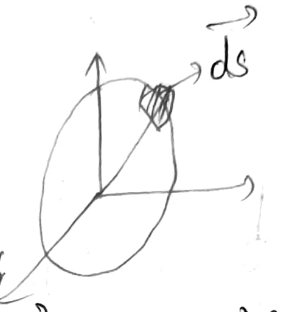
## Radiation Resistance: $(R_r)$

Given, $\vec{E} = \frac{j I_0 dl \beta^2 \sin\theta}{4\pi \varepsilon \omega r} e^{j(\omega t - \beta r)} \hat{a}_\theta$

$$\vec{P}_{avg} = \frac{1}{2\eta_0}|E|^2 \hat{a}_r$$

$$\vec{P}_{avg} = \frac{1}{2\eta_0}\frac{(I_0 dl)^2\beta^4}{(4\pi\omega\varepsilon r)^2}\sin^2\theta \, \hat{a}_r \ \text{W/m}^2$$

$$P_{rad} = \iint \vec{P}_{avg}\cdot\vec{ds}$$




$$P_{rad} = \frac{1}{2\eta_0}\frac{I_0^2 dl^2 \beta^4}{(4\pi\omega\varepsilon r)^2}\int_0^{2\pi}\int_0^{\pi} r^2 \sin^3\theta \, d\theta \, d\phi$$

$$\vec{ds} = (r\sin\theta \, d\phi)(r \, d\theta)\,\hat{a}_r$$

$$P_{rad} = \frac{1}{2\eta_0}\frac{I_0^2 dl^2 \beta^4}{(4\pi\omega\varepsilon)^2}\left[\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\phi\right]$$

$\beta = \omega\sqrt{\mu\varepsilon}$

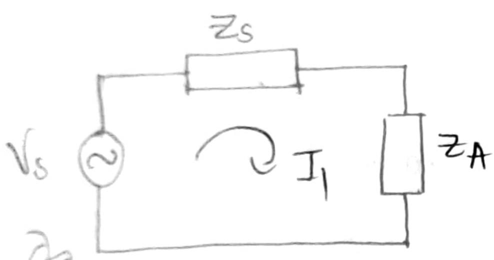
$\beta = 2\pi/\lambda$

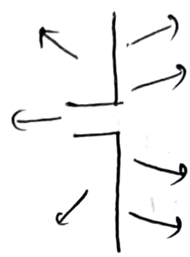
$= (I_0 dl)^2 .$

$$\star \quad \boxed{R_r = \frac{2P_{rad}}{I_0^2} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2}$$

$dl$ - length of antenna.

$Z_A = R_r + jX_A$

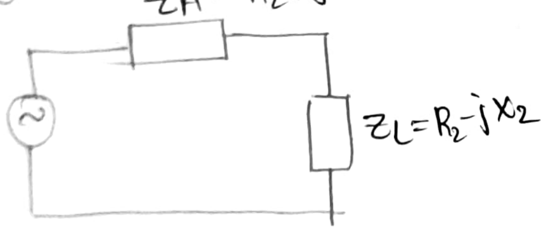



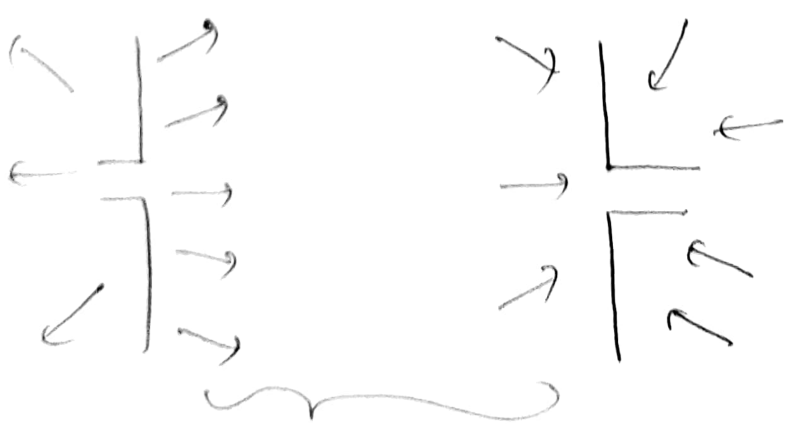
Equivalent circuit of transmitter antenna.

Z-mutual

$Z_A = R_2 + jX_2$




Equivalent circuit of Receiver antenna.



Zmut

↓

for coupling b/w antenna's

→ for transmitter - directivity.

for Receiver - Effective appearance (Effective apperature)

(*) Effective Apperature: Power capturing ability of the antenna.

$(\theta, \phi)$

→ Pden of the incident wave.

→ It is directional dependent.

$$\text{Power density}_{(\theta,\phi)} = \frac{P_r}{4\pi r^2} \cdot D(\theta,\phi)$$

$$\text{Power radiated } (P_{rad}) = \frac{1}{2} I_2^2 R_2$$

$$I_2 = \frac{V_{oc}}{2R_2} \; ; \; V_{oc} = I_1 (Z_{mut})$$

$$\boxed{P_{load} = \frac{1}{2} \frac{V_{oc}^2}{(4R_2)} = \frac{1}{8R_2} |Z_{mut}|^2 |I_1|^2}$$

$$\text{Power transmitted} : \frac{1}{2} I_1^2 R_1$$

$$\text{Pdensity of radiating signal} = \frac{P_t}{4\pi r^2} \cdot D_r(\theta,\phi)$$

Power received by antenna at a distance 'r' from tx is $P_{den} * A_{E_2}(\theta,\phi)$

$$\boxed{P_{received} \; \frac{\frac{1}{2} I_1^2 R_1}{4\pi r^2} D_1 \times A_{e_2}}$$

$A_e$ - effective aperature

$$\frac{1}{\cancel{8}R_2} |Z_{mut}|^2 |\cancel{I_1}|^2 = \frac{1}{\cancel{2}} \frac{\cancel{I_1^2} R_1}{\cancel{4}\pi r^2} D_1 A_{e_2}$$

$$|Z_{mut}|^2 = \frac{R_1 R_2}{\lambda r^2} D_1 A_{e2}$$

$$\boxed{|Z_{mut}|^2_{1\to 2} = \frac{R_1 R_2 D_1 A_{e2}}{\lambda r^2}}$$

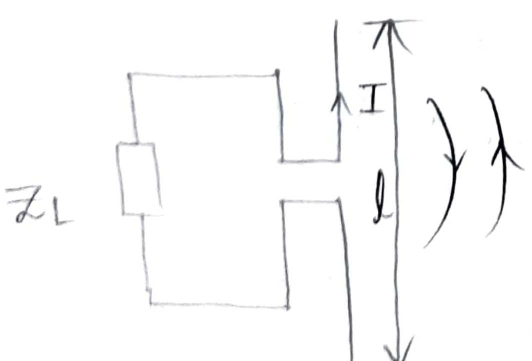
$$\boxed{|Z_{mut}|^2_{2\to 1} = \frac{R_2 R_1 D_2 A_{e1}}{\lambda r^2}}$$

$$\boxed{\frac{D_1}{A_{e1}} = \frac{D_2}{A_{e2}}} \longrightarrow \boxed{D_1 \propto A_{e1}}$$

$$\boxed{A_e = \frac{\text{Power captured } (\theta,\phi)}{\text{incident EM wave power density}}}$$

→ The ratio of directivity to the effective aperture is always constant.

$$\boxed{\frac{D}{A_e} = \text{const}}$$

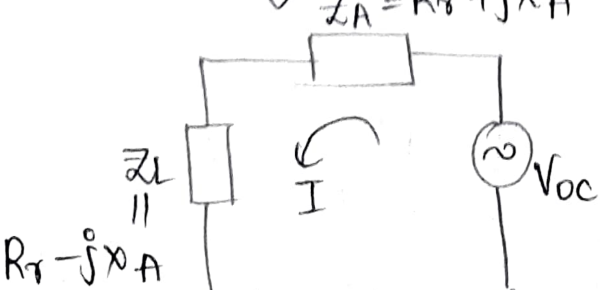



$$A_e = \frac{\text{Power captured}}{\text{Power density}}$$

$$P_{load} = \frac{1}{2}|I|^2 \cdot R_r$$

$\Downarrow$ $Z_A = R_r + jX_A$

$$= \frac{1}{2}\frac{|V_{oc}|^2}{(2R_r)^2} \cdot R_r$$




$$\boxed{P_{load} = \frac{1}{8}\frac{|V_{oc}|^2}{R_r}}$$

$$\left(|V_{oc}| = |E| \times |l|\right)$$

So, $P_{load} = \dfrac{1}{8R_r} |E|^2 \cdot |l|^2$

$$\left(\text{But, } R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2\right)$$

$$A_e = \frac{1}{8} \cdot \frac{\cancel{|E|^2} \cdot \cancel{|l|^2}}{\cancel{|E|^2} \cdot 80\pi^2 \left(\frac{\cancel{l}}{\lambda}\right)^2} \cdot 2\eta_0$$

$$= \frac{1}{8} \times \frac{\cancel{2} \times \cancel{120}^{3} \cancel{\pi}}{\cancel{80}\, \pi^{\cancel{2}} / \lambda^2}$$

$$\boxed{A_e = \frac{3\lambda^2}{8\pi}} \quad (*)$$

$$\rightarrow \frac{D_0}{A_e} = \frac{\cancel{3/2}}{\frac{\cancel{3}\lambda^2}{\cancel{8}_4\pi}} = \frac{4\pi}{\lambda^2} \text{ is always constant.}$$

→ Power density (isotropic antenna) $= \dfrac{P_r}{4\pi r^2}$

→ Power density (directional antenna) $= \dfrac{P_r}{4\pi r^2} \cdot D(\theta, \phi)$

→ Effectively we have 3 major losses in an antenna:

(1) Conductor loss } can't be controlled.
(2) Dielectric loss }
(3) Reflection loss → can be controlled.

• Antenna efficiency:

$$\boxed{\text{Antenna efficiency } (e_0) = e_{cd} \cdot e_r}$$

$e_{cd}$ → conductor + dielectric losses.

$e_r$ → reflection loss

$$\boxed{e_0 \le 1}$$

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

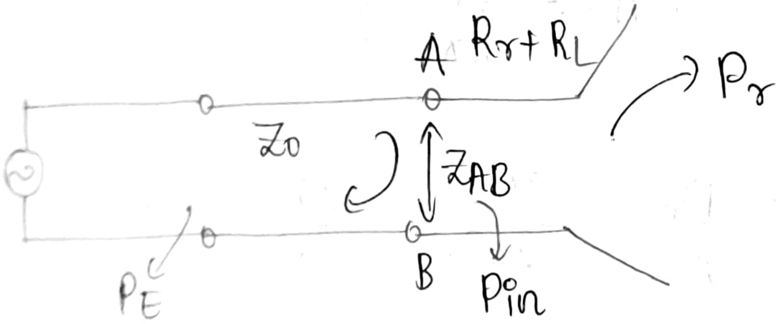
$R_r$ → radiation resistance

$R_L$ → loss resistance

$$|\Gamma_L| = \frac{Z_{AB} - Z_0}{Z_{AB} + Z_0}$$

$$\boxed{e_r \le 1}$$

$$\boxed{e_r = 1 - |\Gamma_L|^2}$$ ✱




$$\boxed{P_r = P_{in} \cdot e_{cd}}$$

$$\boxed{P_r = P_E \cdot e_{cd} \cdot e_r}$$

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{U(\theta,\phi)}{P_r/4\pi}$$

$$\text{Gain}(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}\big|_{(P_{in})}} = \frac{U(\theta,\phi)}{P_{in}/4\pi}$$

$$\text{Gain}(\theta,\phi) = \frac{U(\theta,\phi)}{\frac{P_r}{e_{cd}} \times \frac{1}{4\pi}} = e_{cd} \times \frac{U(\theta,\phi)}{(P_r/4\pi)}$$

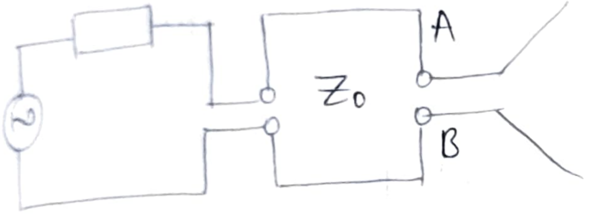
$$\boxed{\text{Gain}(\theta,\phi) = e_{cd} \times D(\theta,\phi)}$$ ✱

$$\boxed{\text{Absolute Gain} = e_r \cdot e_{cd} \times D(\theta,\phi)}$$ ⊛

$$\boxed{\text{Gain}\Big|_{\text{decibels}} = 10\log(\text{Gain})\ \text{dB}}$$

$$\boxed{\text{Gain} = \frac{4\pi}{\theta_{HPBW}\cdot\phi_{HPBW}}} \longrightarrow \text{directional beam.}$$

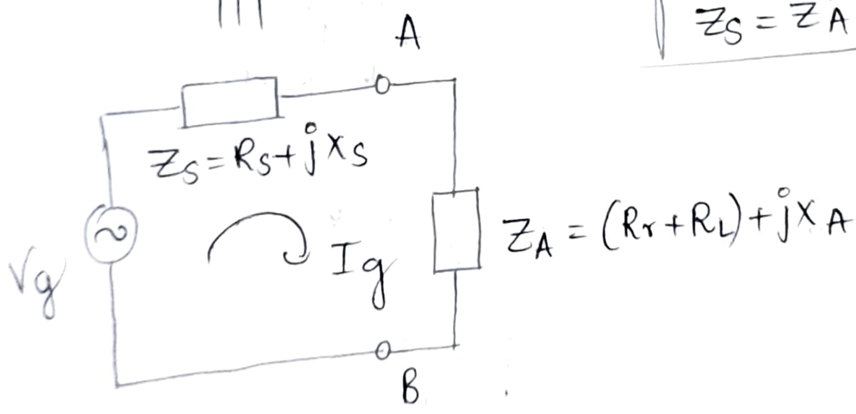
---




Transmitter Antenna.

|||

$$\boxed{Z_S = Z_A^*}$$




$$I_g = \frac{V_g}{Z_S + Z_A}$$

$$|I_g| = \frac{|V_g|}{\left[(R_S + R_r + R_L)^2 + (X_S + X_A)^2\right]^{1/2}}$$

$$\boxed{Z_S = Z_A^*} \longrightarrow R_S = R_r + R_L \checkmark$$

$$X_S = -X_A \checkmark$$

$$\text{Power lost in } Z_s = \frac{1}{2}|I_g|^2 \cdot R_s$$

$$= \frac{1}{2} \frac{|V_g|^2}{(2R_s)^2} \cdot R_s$$

$$= \frac{1}{8} \cdot \frac{|V_g|^2}{R_s}$$

$$\text{Power radiated by antenna} = \frac{1}{2}|I_g|^2 \cdot R_r$$

$$= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)^2} \cdot R_r$$

$$\text{Power lost by antenna} = \frac{1}{2}|I_g|^2 \cdot R_L$$

$$= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)^2} \cdot R_L$$

$$\text{Power supplied } P_s = \frac{1}{2}|V_g||I_g|$$

$$= \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)}$$

$$\text{Power delivered} = \frac{1}{2} \frac{|V_g|^2}{(R_s+R_r+R_L)^2} \times (R_r+R_L)$$

Radiation pattern → Variation of $\vec{E}$ wrt direction

- Polarization: Locus of $\vec{E}$ in given place as a function of time.

→ state of Antenna for which state of polarization responds maximaly.

$$\vec{E} = E_{xo}\cos(\omega t - \beta z)\hat{a}_x + E_{yo}\cos(\omega t - \beta z + \phi)\hat{a}_y$$

↳ generalised form

+z-direction

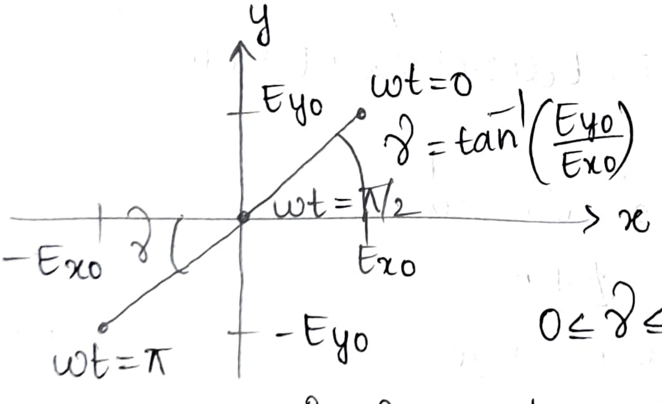
↳ propagation of wave.

- Linear Polarized

At $z=0$,

$$\vec{E} = E_{xo}\cos\omega t\,\hat{a}_x + E_{yo}\cos(\omega t + \phi)\,\hat{a}_y$$

At $\phi = 0^\circ$ → $\vec{E} = E_{xo}\cos\omega t\,\hat{a}_x + E_{yo}\cos(\omega t)\,\hat{a}_y$




$\gamma = \tan^{-1}\left(\frac{E_{yo}}{E_{xo}}\right)$

$0 \le \gamma \le \pi/2$

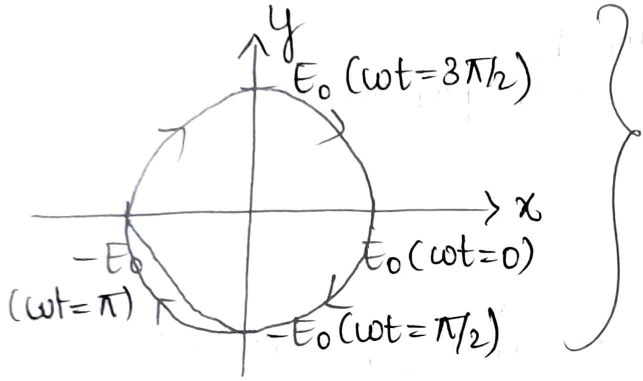
if $\gamma = 0$ → horizontal polarization

if $\gamma = \pi/2$ → Vertical polarization

- Circular polarized

$$\phi = \pm \pi/2$$
$$E_{x0} = E_{y0} = E_0$$
$$E = E_0 \cos\omega t \, \hat{a}_x + E_0 \cos(\omega t + \phi) \, \hat{a}_y$$

at $\phi = \pi/2$,
$$E = E_0 \cos\omega t \, \hat{a}_x - E_0 \sin\omega t \, \hat{a}_y$$




Left Hand Circular Polarization

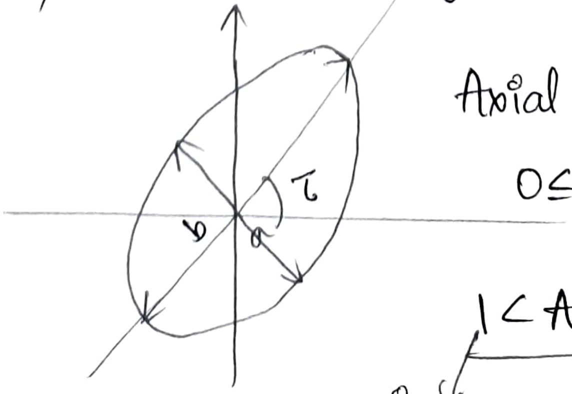
at $\phi = -\pi/2$, $\longrightarrow$ Right Hand Circular Polarization.
$$E = E_0 \cos\omega t \, \hat{a}_x + E_0 \sin\omega t \, \hat{a}_y$$

- Elliptical Polarized

$$\phi \neq 0$$
$$E_{x0} \neq E_{y0}$$

$+AR \rightarrow LH$
$-AR \rightarrow RH$

if $\phi = +ve \rightarrow$ LHEP
if $\phi = -ve \rightarrow$ RHEP
(+z-direction)




Axial Ratio $= \dfrac{a}{b}$

$0 \leq \tau \leq 180^\circ$

$1 < AR < \infty$,

circular polarization

linear polarization

Point Care Sphere:

- $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2} \rightarrow$ Electric field parameters.
- $0 \leq \tau \leq 180^\circ$

$1 \leq \left(\frac{a}{b}\right) \leq \infty$

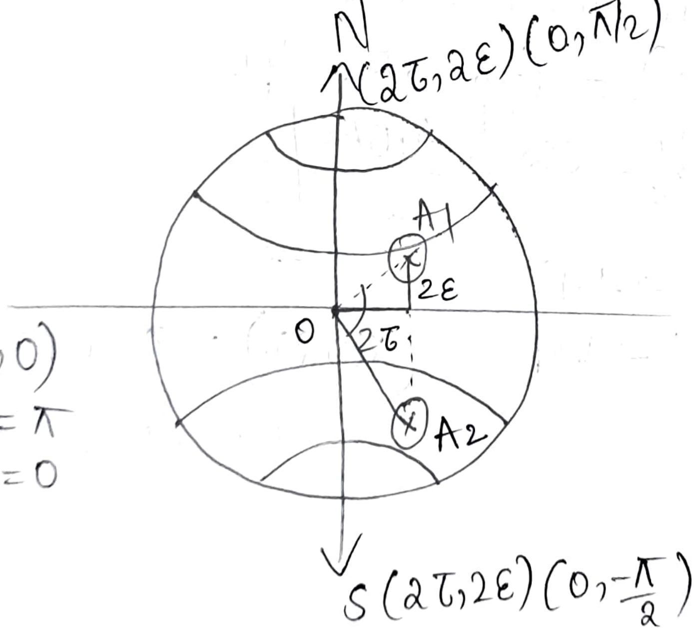
AR

$\varepsilon = \cot^{-1}(\pm AR)$

- $-\frac{\pi}{4} < \varepsilon < \pi/4$

$\gamma = \tan^{-1}\left(\frac{E_{yo}}{E_{xo}}\right)$

- $0 \leq \gamma \leq \pi/2$




$(2\tau, 0)$
$2\tau = \pi$
$2\varepsilon = 0$

Northern Hemisphere (NH) $\rightarrow 2\varepsilon$ positive
Southern Hemisphere (SH) $\rightarrow 2\varepsilon$ negative

if $\varepsilon > 0 \rightarrow AR > 0 \rightarrow$ LH
if $\varepsilon < 0 \rightarrow AR < 0 \rightarrow$ RH

Equator, $\varepsilon = 0$
$AR = \infty \rightarrow$ linear polarisation

North pole,

$$\tau = 0$$
$$\epsilon = \pi/4 \rightarrow LHCP$$

$(0,0) \rightarrow$ Horizontal polarisation
$(\pi,0) \rightarrow$ Vertical polarisation

South pole,

$$\tau = 0$$
$$\epsilon = -\pi/4 \rightarrow RHCP$$

- Orthogonal state of Polarization:

The state for which response of Antenna is minimum. is called Orthogonal Polarization.

$(2\epsilon, 2\tau) = (-2\epsilon, 2\tau + 180^\circ)$

$$\boxed{\eta = \cos^2 \left| \frac{\angle A_1 O A_2}{2} \right|} \rightarrow$$ Efficiency in which two states share information.

$A_1, A_2 \rightarrow$ States

$(0,0) \rightarrow$ H.P
$(0,\pi) \rightarrow$ V.P

$(2\epsilon, 2\tau) = \left(\frac{\pi}{2}, 0\right) \rightarrow LHCP$
$\hookrightarrow \left(\frac{-\pi}{2}, 0\right) \rightarrow RHCP$

→ The efficiency is max b/w same state of polarization
→ The efficiency is zero, b/w orthogonal state of polarisation

eg: $(0,0) \rightarrow$ H.P $(\pi,0) \rightarrow$ V.P

$$\eta = \cos^2\left|\frac{\pi}{2}\right| = 0$$

$\left(\frac{\pi}{2}, 0\right) \rightarrow LHCP$ $\left(\frac{-\pi}{2}, 0\right) \rightarrow RHCP$

$$\eta =$$

→ In satellite communication we use,
Circular polarisation (not HP or VP)

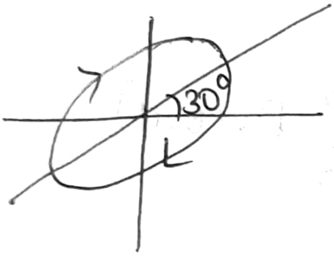
- Since we can increase rate by '2' times
  information sharing
  (but need to design 2 antennas for two states)

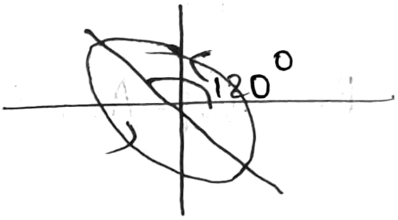
eg: Orthogonal state of (120, 60) ?

$$(2\varepsilon, 2\tau) = (120, 60) \rightarrow \text{LHEP}$$

orthogonal states

$$(-2\varepsilon, 2\tau + 180) = (-120, 240)$$




→ $\eta = 1$ (no loss)

$\eta = 0$ → (Severe loss)

→ Polarization state of Antenna :

$$\hat{\rho}_a = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$$

$$E_{x0} = E_{y0} \; ; \; \phi = \pi/2$$

LHCP

→ Incident EM wave :

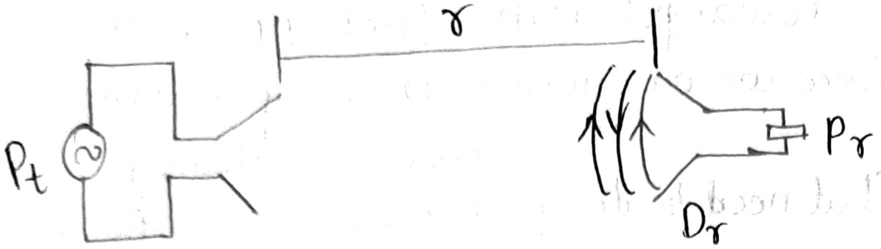
$$\hat{\rho}_w = \hat{a}_x$$

~~Energy exchange rate~~ : $\eta = |\hat{\rho}_a \cdot \hat{\rho}_w|^2$

Polarization loss factor

$$\eta = |\cos\psi|^2$$

# Friis Transmission Relation:




$$A_e = \frac{\lambda^2}{4\pi} D(\theta,\phi)$$

$$W_i = \frac{P_t}{4\pi r^2} \cdot D_t(\theta,\phi)$$

$$P_r = W_i \cdot A_{er} = W_i \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)$$

$$P_r = \frac{P_t}{4\pi r^2} D_t(\theta,\phi) \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)$$

↓ Power received by antenna (lossless)

→ Power received by antenna (including losses)

$$W_i = \frac{P_t}{4\pi r^2} \cdot D_t(\theta,\phi) \cdot e_{cdt} \cdot (1-|\Gamma|^2)_t$$

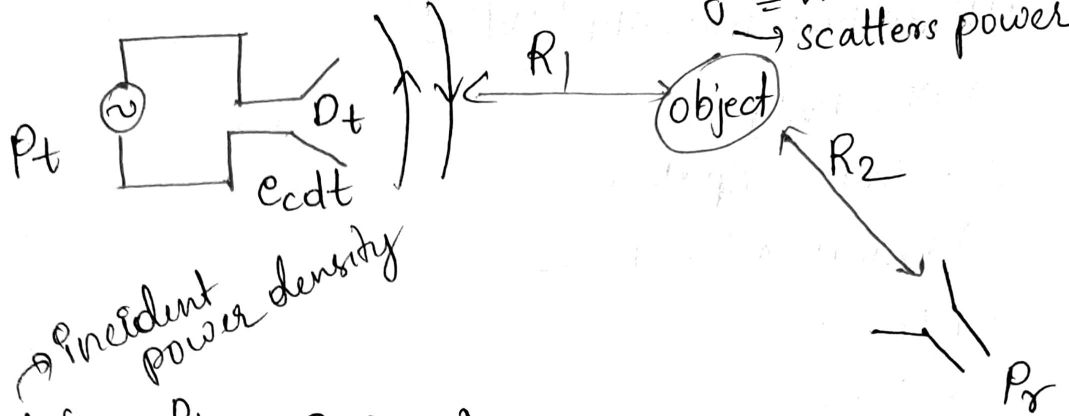
$$P_r = W_i \cdot A_{er} = W_i \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)\, e_{cdr} (1-|\Gamma|^2)_r$$

$$P_r = \left[\frac{P_t}{4\pi r^2} D_t(\theta,\phi) \cdot \frac{\lambda^2}{4\pi} D_r(\theta,\phi)\right] \cdot e_{cdt} \cdot e_{cdr} \cdot (1-|\Gamma_L|^2)_t \cdot (1-|\Gamma_L|^2)_r \, |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

- **Radar Range Equation:**

Radar cross section.

$\sigma = m^2$ → scatters power




→ incident power density

$$W_i = \frac{P_t}{4\pi R_1^2} \cdot D_t(\theta,\phi) \cdot e_{cdt}$$

Power captured by the object located at a distance $R_1$ from transmitter $= W_i \cdot \sigma$

At location $R_2$,

Scattered power density

$$W_s = \lim_{R_2 \to \infty} \frac{P_{cap}}{4\pi R_2^2}$$

$$W_s = \lim_{R_2 \to \infty} \frac{W_i \sigma}{4\pi R_2^2}$$

$$\sigma = \lim_{R_2 \to \infty} 4\pi R_2^2 \cdot \frac{W_s}{W_i}$$

→ $\sigma = 0$ if $W_s = 0$

(In this case we can't track object)

→ It tells about ability of object to scatter power.

Power received ($P_r$) $= W_s \cdot A_{er}$

$$\boxed{P_r = \frac{P_t \cdot D_t(\theta,\phi)}{4\pi R_1^2} \cdot e_{cdt} \cdot \frac{\sigma}{4\pi R_2^2} \cdot \frac{\lambda^2}{4\pi} \cdot D_r(\theta,\phi)\, e_{cdr}}$$