① (a) The system is not memoryless because $y[n]$ depends on past values of $x[n]$

(b) The o/p of the system will be $y[n] = \delta(n)\,\delta[n-2] = 0$.

The system o/p is always zero for inputs of the form $\delta(n-k)$, $k \in \mathbb{Z}$. Therefore, the system is not Invertible.

② (a) The system is not causal because the o/p $y(t)$ at some time may depend on future values of $x(t)$.

For instance $y(-\pi) = x(0)$.

(b) Consider two arbitarary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \longrightarrow y_1(t) = x_1(\sin(t))$$

$$x_2(t) \longrightarrow y_2(t) = x_2(\sin(t))$$

Additivity:

$$x_1(t) + x_2(t) \longrightarrow x_1(\sin(t)) + x_2(\sin(t))$$

$$= y_1(t) + y_2(t)$$

Homoginity

$$\alpha\, x(t) \longrightarrow \alpha\, x(\sin(t)) = \alpha\, y(t)$$

Therefore, the system is Linear.

(3) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$

(a) $x_1[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$x_2[n] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$

Additive Property

$$x_1[n] + x_2[n] \longrightarrow \sum_{k=n-n_0}^{n+n_0} \left[x_1[k] + x_2[k]\right]$$

$$= \sum_{k=n-n_0}^{n+n_0} x_1[k] + \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= y_1[n] + y_2[n]$$

Homoginity

$$\alpha x[n] \longrightarrow \sum_{k=n-n_0}^{n+n_0} (\alpha x[k]) = \alpha \sum_{k=n-n_0}^{n+n_0} x[k] = \alpha y[n]$$

Therefore, the system is linear

(b) $y[n] = x[n-n_0] + x[n-n_0+1] + \cdots + x[n+n_0-1] + x(n+n_0]$

If the $x[n]$ is delayed by $n_1$ units then

$$x[n-n_1] \longrightarrow x[n-n_1-n_0] + x[n-n_1-n_0+1] + \cdots +$$
$$x[n-n_1+n_0-1] + x[n-n_1+n_0]$$
$$= y[n-n_1]$$

$\therefore$ Therefore, the system is ~~Invariant~~ time-Invariant.

3 (c) If $|x(n)| < B$, then

$$y(n) \le (2n_0+1)B$$

Therefore $C \le (2n_0+1)B$.

(4)

(a) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$

Additivity

$$x_1(t) \longrightarrow y_1(t) = t^2 x_1(t-1)$$

$$x_2(t) \longrightarrow y_2(t) = t^2 x_2(t-1)$$

$$x_1(t)+x_2(t) \longrightarrow t^2[x_1(t-1)+x_2(t-1)]$$

$$= t^2 x_1(t-1) + t^2 x_2(t-1)$$

$$= y_1(t) + y_2(t)$$

Homoginity

$$\alpha\, x(t) \longrightarrow t^2[\alpha\, x(t-1)] = \alpha\, t^2 x(t-1)$$

$$= \alpha\, y(t)$$

$\therefore$ The System is linear.

(b) ~~Consider two~~

If the I/p is delayed by $t_0$ units

$$x(t-t_0) \longrightarrow t^2 x(t-t_0) \neq y(t-t_0)$$

$\therefore$ The system is Time-Inverient

4 (b) $y[n] = x^2[n-2]$

$x_1[n] \longrightarrow y_1[n] = x_1^2[n-2]$

$x_2[n] \longrightarrow y_2[n] = x_2^2[n-2]$

$x_1[n] + x_2[n] \longrightarrow \left[x_1[n-2] + x_2[n-2]\right]^2$

$\neq y_1[n] + y_2[n]$

$\therefore$ The system is Non-Linear

$x[n-n_0] \longrightarrow x^2[n-n_0-2]$

$= y[n-n_0]$

$\therefore$ The system is time-Invariant

4 (c) $x_1[n] \longrightarrow y_1[n] = x_1[n+1] - x_1[n-1]$

$x_2[n] \longrightarrow y_2[n] = x_2[n+1] - x_2[n-1]$

Additive

$x_1[n] + x_2[n] \longrightarrow \left[x_1[n+1] + x_2[n+1]\right] - \left[x_1[n-1] + x_2(n-1)\right]$

$= \left[x_1[n+1] - x_1[n-1]\right] + \left[x_2[n+1] - x_2(n-1)\right]$

$= y_1[n] + y_2[n]$

Homoginity

$\alpha x(n) \longrightarrow \alpha x_1(n+1) - \alpha x(n-1)$

$= \alpha y[n]$

$\therefore$ The System is Linear

$$x[n-n_0] \longrightarrow x[n-n_0+1] - x[n-n_0-1]$$
$$= y[n-n_0]$$

$\therefore$ The System is time-Invariant

4 (d)

$$y(t) = odd(x(t))$$
$$= \frac{x(t) + x(-t)}{2}$$

$$x_1(t) \longrightarrow y_1(t) = \frac{1}{2}(x_1(t) + x_1(-t))$$
$$x_2(t) \longrightarrow y_2(t) = \frac{1}{2}(x_2(t) + x_2(-t))$$
$$x_1(t) + x_2(t) \longrightarrow \frac{1}{2}\left[(x_1(t) + x_2(t)) + (x_1(-t) + x_2(-t))\right]$$
$$= y_1(t) + y_2(t)$$

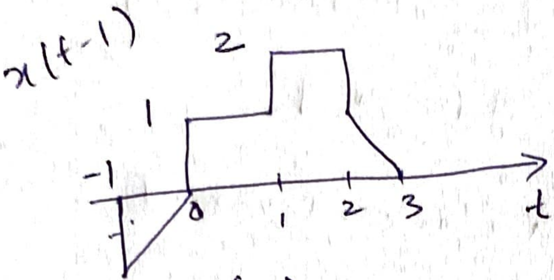
$$\alpha x(t) \longrightarrow \frac{1}{2}\left[\alpha x(t) + \alpha x(-t)\right]$$
$$= \alpha \frac{1}{2}\left[x(t) + x(-t)\right]$$
$$= \alpha y(t)$$

$\therefore$ The System is Linear

$$x(t-t_0) \longrightarrow \frac{1}{2}\left[x(t-t_0) + x(-t-t_0)\right]$$
$$\neq y(t-t_0)$$

$\therefore$ The System is Time-Variant

5

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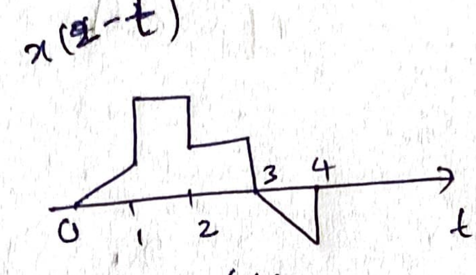


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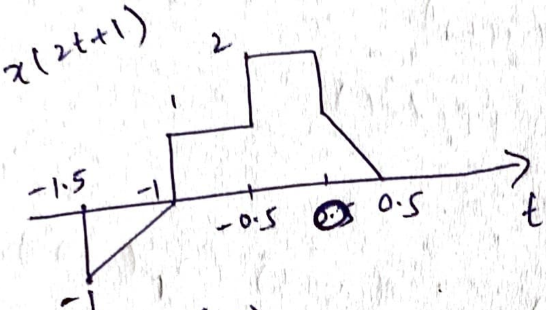


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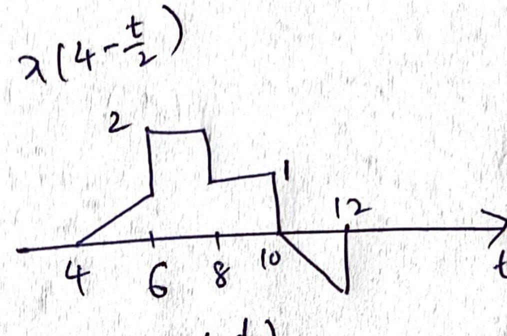


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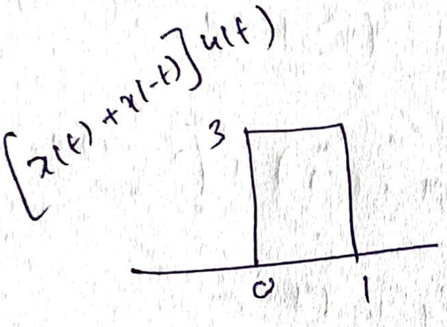


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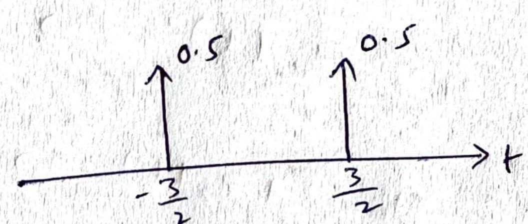
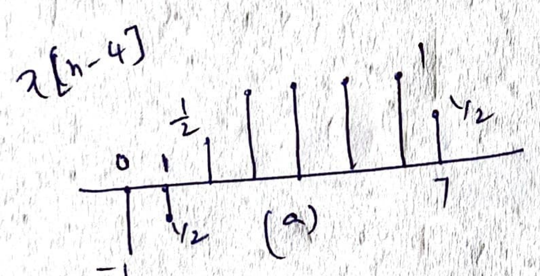



(f)

6

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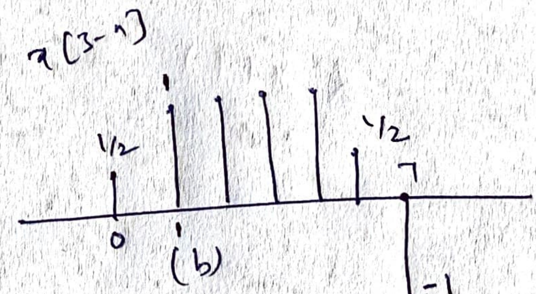


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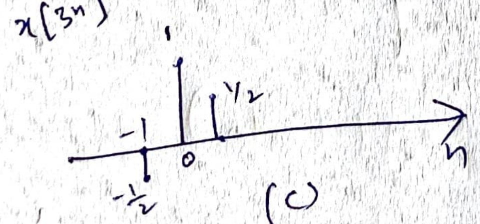


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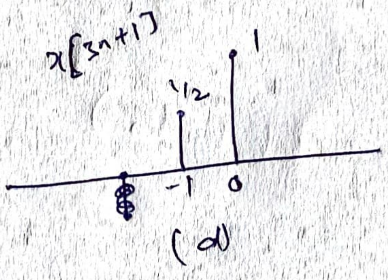
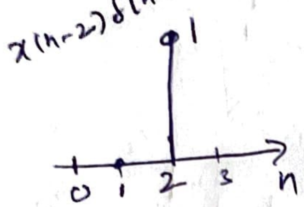



(d)

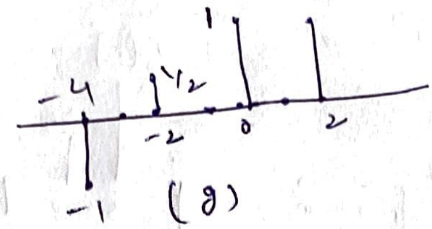
(e) $x[n]\, x[3-n] = x(n)$

$x(n-2)\delta(n-2)$



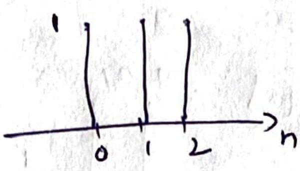
(f)

$\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$



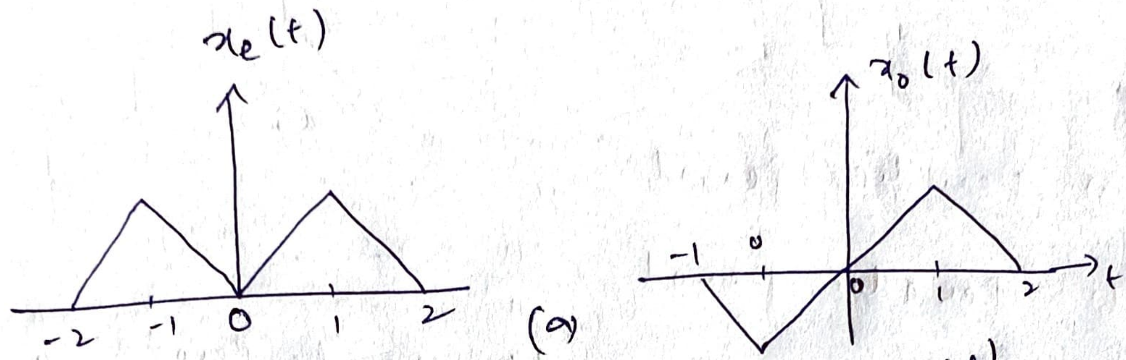
(g)

$x(n-1)^2$



(h)

7

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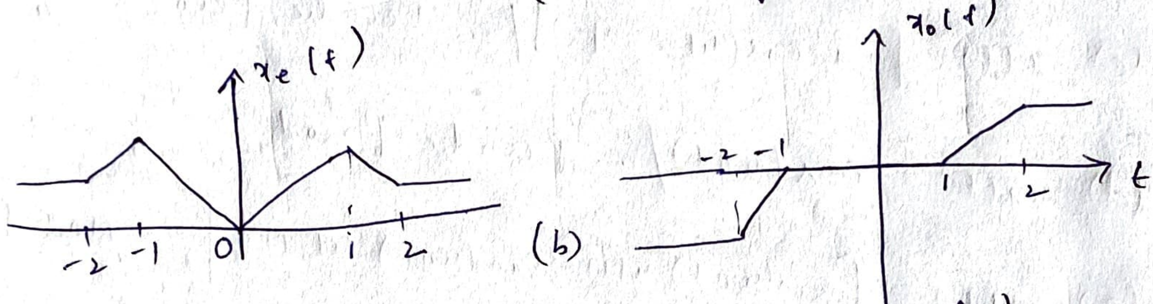


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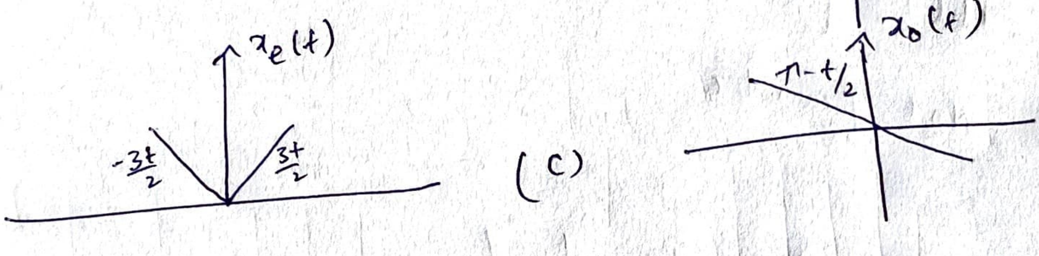
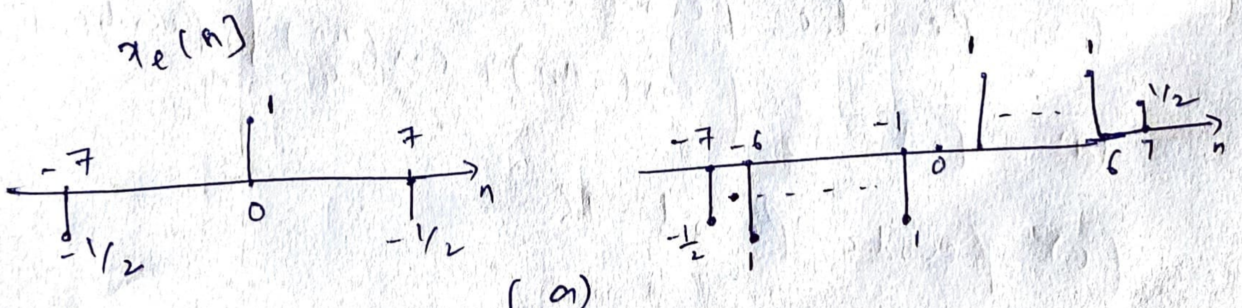
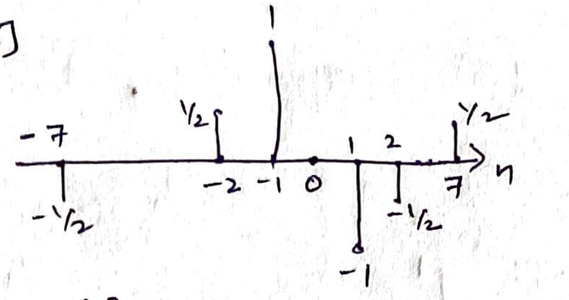


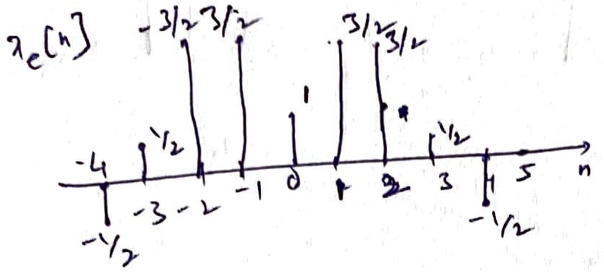

(c)

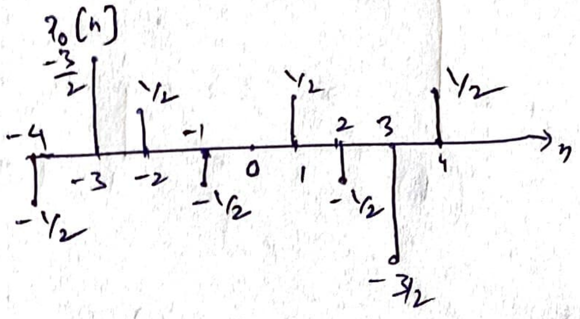
8




(a)


9

(a) Periodic, Period $= \frac{2\pi}{4} = \frac{\pi}{2}$.

(b) Periodic, Period $= \frac{2\pi}{\pi} = 2$.

(c) $x(t) = [1 + \cos(4t - 2\pi/3)]/2$. Periodic, Period $= \frac{2\pi}{4} = \frac{\pi}{2}$.

(d) $x(t) = \cos(4\pi t)/2$. Periodic, Period $= \frac{2\pi}{4\pi} = \frac{1}{2}$

(e) $x(t) = [\sin(4\pi t)\, u(t) - \sin(4\pi t)\, u(-t)]/2$. Not Periodic

(f) Not Periodic

10

(a) Periodic, Period = 7

(b) Not Periodic

(c) Periodic, Period = 8

(d) $x[n] = \frac{1}{2}\left[\cos\left(\frac{3\pi n}{4}\right) + \cos\left(\frac{\pi n}{4}\right)\right]$. Periodic, Period = 8.

(e) Periodic, Period = 16.

11) a) Linear, Stable

b) Memoryless, Linear, Causal, Stable

c) Linear

d) Linear, Causal, Stable

e) Time invariant, Linear, Causal, Stable

f) Linear, Stable

g) Time-Invariant, Linear, Causal.

12) a) Linear, Stable

b) Time-Invariant, Linear, Causal, Stable

c) Memoryless, Linear, Causal

d) Linear, Stable

e) Linear, Stable

f) Memoryless, Linear, Causal, Stable

g) Linear, Stable

13) a) Invertible, Inverse system; $y(t) = x(t+4)$

b) Non-Invertible. The signals $x(t)$, $x(t+2\pi)$ give the same o/p.

c) Non-Invertible. $\delta(n)$ and $2\,\delta[n]$ give the same o/p.

d) Invertible. Inverse system: $y(t) = \frac{d}{dt} x(t)$

e) Invertible. Inverse system: $y[n] = x[n+1]$ for $n \geq 0$ and $y[n] = x[n]$ for $n < 0$.

f) Non-Invertible $x[n]$ and $-x[n]$ give the same result.

g) Invertible. Inverse system: $y[n] = x[1-n]$

h) Invertible. Inverse system $y(t) = x(t) + \frac{d}{dt}x(t)$

(i) Invertible. Inverse system $y[n] = x[n] - \frac{1}{2}x[n-1]$

(j) Non Invertible. $\delta[n]$ and $2\delta[n]$ result in $y[n] = 0$

(k) Invertible. Inverse system: $y(t) = x(t/2)$.

(l) Non-Invertible. $x_1[n] = \delta[n] + \delta[n-1]$ and $x_2[n] = \delta[n]$ give $y[n] = \delta[n]$

(m) Invertible. Inverse system: $y[n] = x[2n]$.