Vector Algebra

11^{th} Maths - Chapter 10

This is Problem-12 from Exercise 10.4

1. Find the equation of the line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.
   Solution:
   Given lines can be written in the form of $\mathbf{n}^T\mathbf{x} = \text{c}$
   Therefore,

$$\begin{pmatrix} 4 & 7 \end{pmatrix} \mathbf{x} = 3 \tag{1}$$

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -1 \tag{2}$$

   Now, line equation that has equal intercepts on the axes is

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = c \tag{3}$$

   Solving equations (1) and (2)
   augumented matrix is

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \tag{4}$$

   $R_1 \leftarrow 4R_1$

$$\begin{pmatrix} 1 & \frac{7}{4} & \frac{3}{4} \\ 2 & -3 & -1 \end{pmatrix} \tag{5}$$

$R_2 \leftarrow R_2 - 2R_1$

$$\begin{pmatrix} 1 & \frac{7}{4} & \frac{3}{4} \\ 0 & \frac{-13}{2} & \frac{-5}{2} \end{pmatrix} \tag{6}$$

$R_2 \leftarrow \frac{-2}{13} R_2$

$$\begin{pmatrix} 1 & \frac{7}{4} & \frac{3}{4} \\ 0 & 1 & \frac{5}{13} \end{pmatrix} \tag{7}$$

$R_1 \leftarrow R_1 - \frac{7}{4} R_2$

$$\begin{pmatrix} 1 & 0 & \frac{1}{13} \\ 0 & 1 & \frac{5}{13} \end{pmatrix} \tag{8}$$

Therfore, $\mathbf{x} = \begin{pmatrix} \frac{1}{13} \\ \frac{5}{13} \end{pmatrix}$ Also this point lies on the equation (3)

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{13} \\ \frac{5}{13} \end{pmatrix} = c$$

$$\frac{1}{13} + \frac{5}{13} = c$$

Therefore, the equation is $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = \frac{6}{13}$

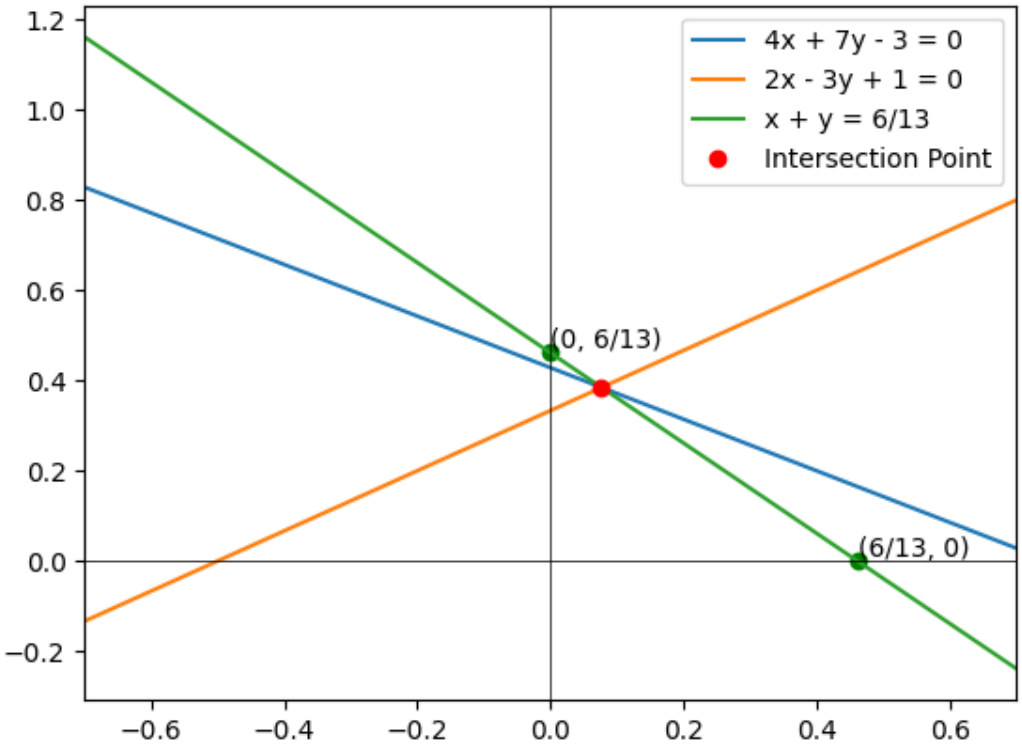

Figure 1: Straight Lines