Straight Lines

11^{th} Maths - Chapter 10

This is Problem-12 from Exercise 10.4

1. Find the equation of the line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

   Solution:

   Given lines can be written in the form of

   $$\mathbf{n}^{\top}\mathbf{x} = c \tag{1}$$

   Therefore,

   $$\begin{pmatrix} 4 & 7 \end{pmatrix}\mathbf{x} = 3 \tag{2}$$

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -1 \tag{3}$$

Now, line equation that has equal intercepts on the axes is

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = c \tag{4}$$

Solving equations (2) and (3)

augumented matrix is

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \tag{5}$$

$$\xleftrightarrow{R_1 \leftarrow 4R_1} \begin{pmatrix} 1 & \frac{7}{4} & \frac{3}{4} \\ 2 & -3 & -1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & \frac{7}{4} & \frac{3}{4} \\ 0 & \frac{-13}{2} & \frac{-5}{2} \end{pmatrix} \tag{6}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{-2}{13} R_2} \begin{pmatrix} 1 & \frac{7}{4} & \frac{3}{4} \\ 0 & 1 & \frac{5}{13} \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - \frac{7}{4} R_2} \begin{pmatrix} 1 & 0 & \frac{1}{13} \\ 0 & 1 & \frac{5}{13} \end{pmatrix} \tag{7}$$

Therfore,

$$\mathbf{x} = \begin{pmatrix} \frac{1}{13} \\ \frac{5}{13} \end{pmatrix} \tag{8}$$

Also this point lies on the equation (4)

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{13} \\ \\ \frac{5}{13} \end{pmatrix} = c \tag{9}$$

$$\frac{1}{13} + \frac{5}{13} = c \tag{10}$$

Therefore, the equation is

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = \frac{6}{13} \tag{11}$$

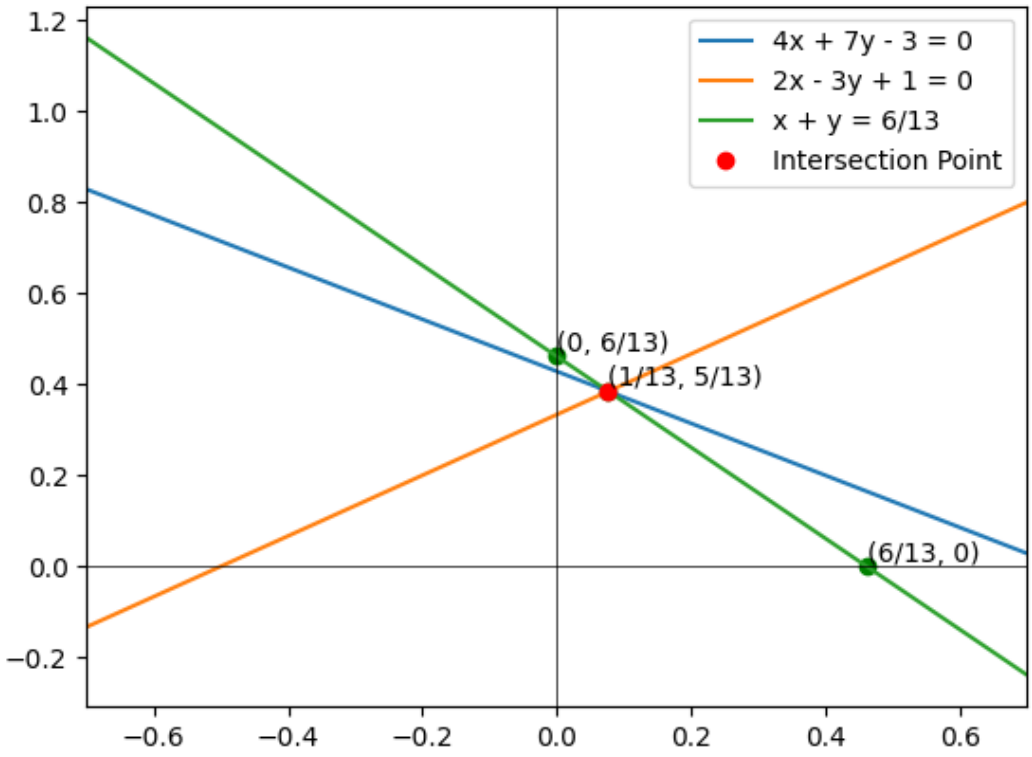

Figure 1: Straight Lines