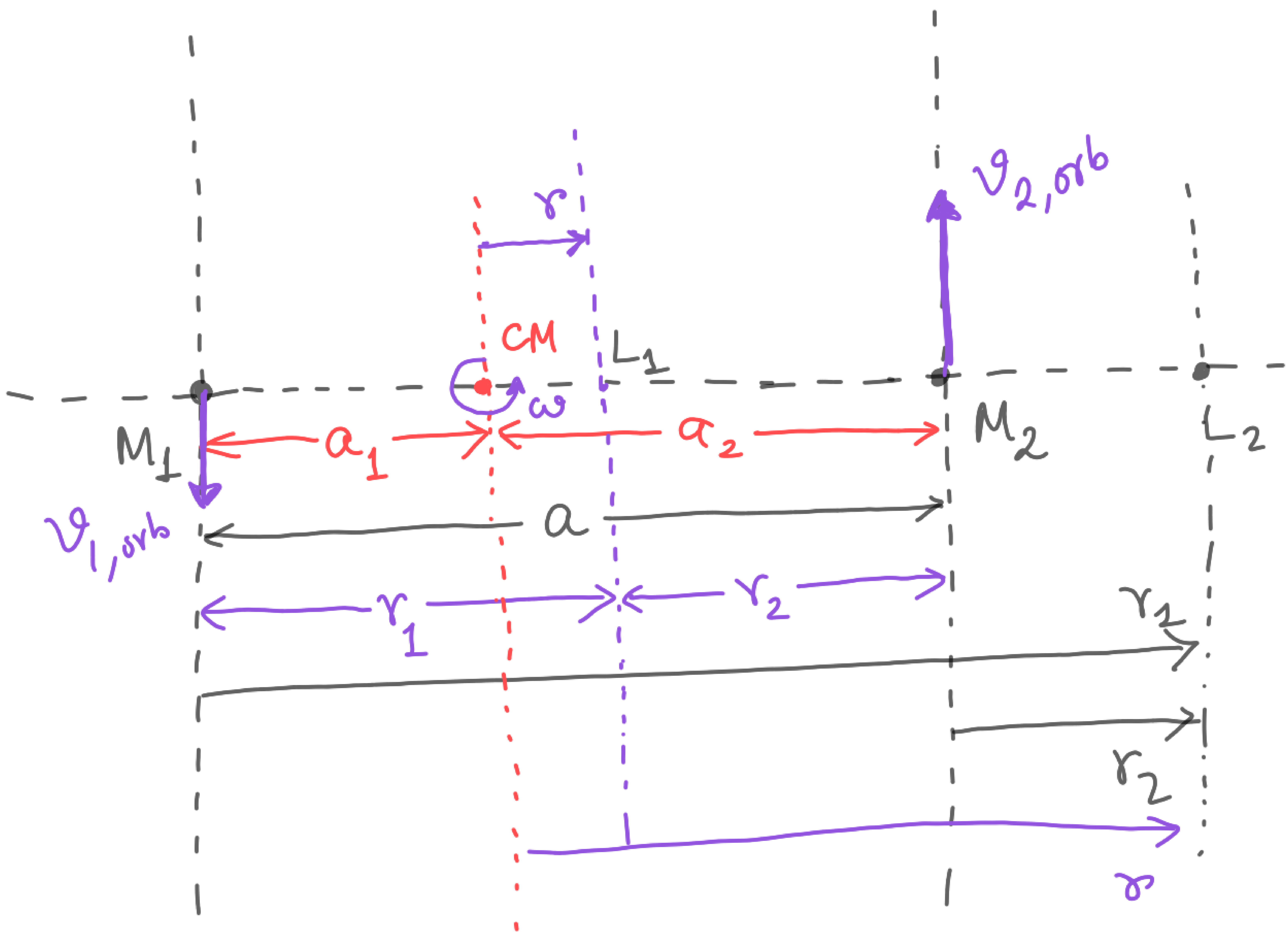
Mass transfer in binary systems

Assumptions

1) Circular orbit
2) Point masses
3) $M_1 > M_2 \ggg$ mass of test particle



Center of mass

$$M_1 a_1 = M_2 a_2$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{a_2}{a_1} \Rightarrow \frac{M_1}{M_2} + 1 = \frac{a_2}{a_1} + 1$$

$$\Rightarrow \frac{M_1+M_2}{M_2} = \frac{a_2+a_1}{a_1}$$

$$\Rightarrow \frac{M_T}{M_2} = \frac{a}{a_1} \Rightarrow a_1 = \frac{M_2}{M_T} \cdot a$$

By induction, $a_2 = \frac{M_1}{M_T} \cdot a$.

Question: If a test particle is placed at the C.M. of the binary, will it stay stationary?

Gravitational force balance

$$\frac{G M_1 m}{a_1^2} \qquad \frac{G M_2 m}{a_2^2}$$

$$\frac{\cancel{G} M_1 \cancel{m} \cancel{M_T^2}}{M_2^2 \cancel{a^2}} \qquad \frac{\cancel{G} M_2 \cancel{m} \cancel{M_T^2}}{M_1^2 \cancel{a^2}}$$

$$\frac{M_1}{M_2^2} > \frac{M_2}{M_1^2}$$

because $M_1 > M_2$

* So, the point along the axis where a co-rotating test particle will be stationary is not the center of mass.

* At any other point in the co-rotating system, there will be a centrifugal force.

Centrifugal potential

$$\phi_{cf} = -\frac{1}{m}\int_0^r F_{cf} \cdot dr$$

$$= -\frac{1}{m}\int_0^r m\omega^2 r \, dr = -\frac{1}{2}\omega^2 r^2.$$

Total potential

$$\phi_T = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{1}{2}\omega^2 r^2$$

Question: where will the force be zero?

$$F = -m\nabla\phi_T$$

How does this look in 3D? Let's plot!

How do we find these stationary pts?

For L_1

$$r_1 = a - r_2 \qquad r = a_2 - r_2$$

$$\phi_{roche} = -\frac{GM_1}{a-r_2} - \frac{GM_2}{r_2} - \frac{1}{2}\omega^2(a_2-r_2)^2$$

What is ω? Kepler's 3rd law.

$$v_1 = \omega a_1$$

$$\frac{GM_1 M_2}{a^2} = \frac{M_1 v_1^2}{a_1}.$$

$$\frac{GM_2}{a^2} = \omega^2 a_1.$$

$$\frac{G\cancel{M_2}}{a^2} = \omega^2 \frac{\cancel{M_2}}{M_T} a \Rightarrow \omega^2 = \frac{GM_T}{a^3}.$$

$$\phi_{roche} = -\frac{GM_1}{a-r_2} - \frac{GM_2}{r_2} - \frac{1}{2}\frac{GM_T}{a^3}(a_2-r_2)^2$$

$$-\nabla\phi_{roche} = 0$$

$$\Rightarrow \frac{M_1}{(a-r_2)^2} - \frac{GM_2}{r_2^2} - \frac{M_T}{a^3}(a_2-r_2) = 0$$

:(Can't solve analytically. But for the curious, try Newton-Rahpson to get r_2.

for L_2

$$r = a_2 + r_2 \qquad r_1 = a + r_2 .$$

Scale and shape of the Roche potential

$$\phi_{roche} = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{1}{2}\frac{GM_T}{a^3} r^2 .$$

$$= -\frac{GM_T}{a}\left[\frac{M_1/M_T}{r_1/a} - \frac{M_2/M_T}{r_2/a} - \frac{1}{2}\left(\frac{r}{a}\right)^2\right]$$

Question: What is the physical significance of the L_1 point?

"Roche-lobe overflow"