MRM Quant Challenge – Week 1

- Problems marked with ★ are difficult problems
- Problem marked with ★★ are M.R.M originals. These problems were asked by fellow validators and originated in their line of work.

Numerical Problems

- Find the maximum number of queens that can be placed in a chessboard such that no two queens threaten each other. Also, give the position of those queens.

- Disappointed with the security threat posed by Ali Baba, the thieves from "Ali Baba and the 40 thieves" decided to move to a conventional methodology that uses simple locks and keys. The 40 thieves came to an agreement that the vault can be opened only if 30 of them or more is willing to open it. You are a lock smith and you can create as many locks and keys you can. Devise a strategy with minimum number of locks to solve their problem.

★ *(Coffin Problem)* All angle bisectors of a triangle is of length less than 1. What can be the maximum area of the triangle?

"Coffin problems" are extremely difficult math problems which, nevertheless, have elementary solutions. They were devised for and used by admission committees at Soviet universities in the 70's and 80's to keep Jewish candidates (and other "undesirables") out of the most prestigious universities, such as Moscow State. Only undesirable candidates were asked these questions in the oral entrance exams.

Algorithm/Programming Questions

- Given the stock price for the next N days : a_1, a_2, ….. , a_N. Devise an algorithm to generate maximum profit by buying and selling a stock exactly once (no short selling allowed). The algorithm must find the day in which the buy and sell order must be placed.

★ Given that integers are read from a data stream. Find median of elements read so for in efficient way. For simplicity assume there are no duplicates. For example, let us consider the stream 5, 15, 1, 3 …

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○ After reading 1st element of stream - 5 -> median - 5
○ After reading 2nd element of stream - 5, 15 -> median - 10
○ After reading 3rd element of stream - 5, 15, 1 -> median - 5
○ After reading 4th element of stream - 5, 15, 1, 3 -> median -
  4, so on...
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Note: For programming questions, the solution must contain the algorithm (pseudo-code) and the time complexity. The best solution will be judged on the complexity and ease of understanding.

Probability and Statistics

- Given a biased coin, how can we conduct a fair trial using it. Similarly, given a fair coin, how can we simulate from a biased coin with some probability p.

★★ A discrete time stochastic process is said to be markov iff $P(X_n \mid X_{n-1}, X_{n-2}, \dots, X_0) = P(X_n \mid X_{n-1})$ and the process is said to be time homogeneous if $P(X_n \mid X_{n-1}) = P(X_1 \mid X_0)$. Give an example of a process which is non-markov but time homogeneous.

★★ Given a random variable, the value u that minimizes the function $E(\rho(X,u))$ is the mean if the metric ρ is Eucledian and u is median if ρ is L1 metric. Given some quantile $k \in (0,1)$, find the metric ρ, such that the function $E(\rho(X,u))$ is minimized by the k^{th} quartile.

π is an infinite, non-repeating decimal — meaning that every possible number combination exists somewhere in pi.

- *Somewhere in that infinite string of digits is the name of every person you will ever love; the date, time and manner of your death; and the answers to all the great questions of the universe,*
- *Somewhere in that infinite string of digits is a pixel-perfect representation of the first thing you saw on this earth; the last thing you will see before your life leaves you; and all the moments, momentous and mundane, that will occur between these two points,*
- *Somewhere in that infinite string of digits is the ASCII representation of all the models that you validate,*
- *All the information that has ever existed or will ever exist, the DNA of every being in the universe.*

And all this contained in the ratio of a circumference and a diameter.

***Now, for the bummer**: Though this is a popular notion, mathematicians have not yet proved/disproved that π contains all finite sequences. This is still an open problem waiting to be conquered.*

Finance

- Prove that in discrete time the American call option has the same no-arbitrage price as the European call option