MRM Weekly Converge - Week 5

May 17, 2019

"Today, I do what others won't do
Tomorrow, I do what others can't do."

Problem (Chess - Endgame). White to play and mate in exactly 3 moves.

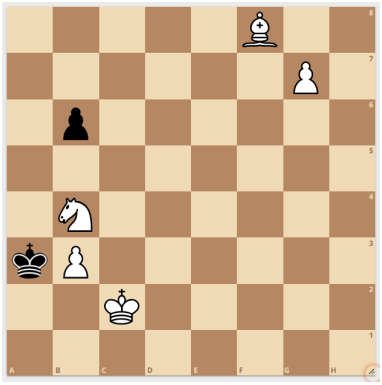

Figure 1: White to move

Problem (Brain Teaser). (*Problem Submitted by Saloni Sardana*) There are 512 bags of coins. One of these bags contain only counterfeit coin and others contain fair coins. Counterfeit coins weigh more than fair coins. Given a weight balance, what is the minimum number of times to use the balance to identify the counterfeit bag.

Problem (Combinatorics). A white solid cube is painted red in all its faces. This cube is cut into 27 identical smaller cubes and shuffled randomly. The smaller cubes are then assembled back randomly into a bigger cube. What is the probability that the re-assembled bigger cube will be red in all its faces?

Problem (Probability). Given a standard uniform(0,1) random number generator, devise an algorithm to simulate uniformly from a solid sphere. The number of calls to the random number generator must be deterministic.

Solution - Week 4

Solution (Chess). *(Solution - Sushant Vijayan/ Kashish Garg)*
1... Kg4 (the only move which avoids the draw. Kf4 fails to Re8.)
2.Re4+ Kg5
3.Re5+ Kg6
4.Re6+ Kf7
White's checks have seemingly ended and black can simply queen but...
5.Re5 (an amazing resource if black promotes hastily and queens then 6... Rf5 Qxf5 stalemate!!) g1=R!!
Surely a draw with two rooks facing down each other but... for the fact that black threatens mate with Rh1.White thwarts it with Rh5 but..
6.Rh5 Kg6!!
Now black threatens mate with Rf8!! and now finally white has no tricks left to play and meets his doom.
7.Kg8 Kxh5.

Solution (Probability). *(Solution - Ravi Sah)* If by *"Completely Random"*, we want each individual to have the same probability of death, then it can be done in 1 toss(divide the population equally and choose randomly). Else, if we want each selection of n/2 people to be equally likely, it can be approximately done in $log(\binom{n}{n/2})$ number of tosses. Note, there is no deterministic simulation mechanism to choose.

Solution (Brain Teaser). *(Solution - Paulina Wardaza/Chandan Kumar/Ajay Mehndiratta/Priyadarshi Abhishek/Nishant Agrawal)* **July 16**

- Lang doesnt know the exact date and is sure that Tony doesnt know either. Lang does know the month. If the month was either May or June, he wouldnt be sure that Tony doesnt know the date the 18 and 19 appear only once (in May and June respectively), so Tony could know already whats the date. To sum up, Lang says its neither May nor June.

- Tony says that now he knows the date. So based on the knowledge that the month is July or August, he knows the exact date. It means, that the day can be only 15, 16 or 17 (the days which appear only either in July or in August, but not in both months).

- Now Lang also knows. If the month was August, he still wouldnt be sure if its 15th or 17th. But in July there is only one option left July 16th. And thats the answer.

Solution (Combinatorics). *(Solution - Mikolaj Fido)* The number of ways to divide a set of n elements into k parts is given by the sterling number of second kind($S(n,k)$). A sequence is an ordered combination, thus the total number of sequences is given by $\sum_{k=1}^{7}(k!)S(7,k)$. The answer is **47,293**.