MRM Weekly Converge - Week 3

April 22, 2019

"There are three kinds of lies:
lies, damned lies and statistics."
- Benjamin Disraeli

Problem (Chess). White to play in Fig 1 and mate in three moves.

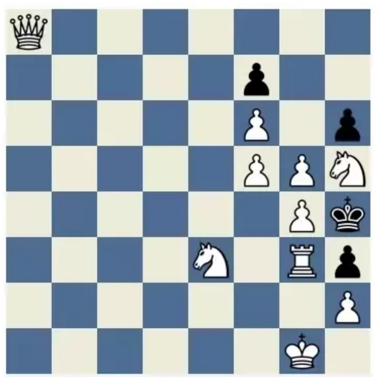

Figure 1: White to play and mate in three moves.

Problem (Puzzle). We are told a polynomial has positive integer coefficients. We have access to a blackbox which evaluates the polynomial for any real input and outputs the value. Find the minimum number of calls required to the blackbox to find all the coefficients of the polynomial. (*Problem submitted by Kashish Garg.*)

Problem (Geometry). Consider an interior point within a triangle. Assume that it is possible to draw 27 rays from this point such that it divides the triangle into 27 other smaller triangles and each of the smaller triangle has the same area as the others. How many such points are there for a given triangle? (*Problem submitted by Kashish Garg.*)

Problem (Game Theory). Consider a two player zero sum game where both players have a finite set of pure strategies. As it is a zero sum game the payoff matrix A can be written purely in terms of the row players payoff.

- Assuming a mixed strategy with probability vector p for the row player set up a linear optimisation problem such that he optimises his expected pay. (Hint : The expected payoffs for different strategies of the column player will be pA. Once p is known the column player will choose the minimum of the entires of pA. This minimises his cost. So the row player will then choose his p such that he maximises this minimum cost)

- Do the same for column player with strategy q. Show that the linear optimisation problems are dual of each other.

- Use the fact that the two linear optimisations are dual of each other to show that a nash equilibrium always exists for zero sum two player games with finite strategies.

(*Problem submitted by Sushant Vijayan.*)

Solution - Week 2

Solution (Chess). *(Solution - Richard Harris)* dxe6! (en passant) is the solution. No other move mates in a single move. The previous move of black was to block against the bishop check with e5.

Solution (Numerical Puzzle). *(Solution - Richard Harris)* The condition we get is $4 \times 10^{d-1} + b = 4.(10b + 4)$. d is the number of digits and b is the suffix of the number after the first digit 4. Substituting d=6 and solving one gets the number to be 410256.

Solution (Probability & Stats). *(Solution - Sushant Vijayan)* Generally $P(X_1 + X_2 + X_3 \cdots + X_n \leq c) = \frac{c^n}{n!10^n}$ when the X_i are iid unifrom and $0 < c < 10$. A straightforward proof is to show it by induction.

Fun Fact (*By Anirban Dutta*) - The sum of uniform distribution is called Irwin-Hall distribution. Joseph Irwin and Philipp Hall published two papers in Biometrics in 1927 in the same volume (consecutive pages actually) to discuss this distribution. They had different perspectives for looking at it. Irwin was a biostatistician and Hall was a mathematician, a group theorist to be precise. Interestingly enough, R C Bose, a group theorist whose work had great application in statistics was a dan of Irwin for helping him understand Fisher's work..

Solution (Math - Coffin). *(Solution - Kousik Krishnan)* Consider the following two lemmas:

Lemma - 1 Let $m = 2^{n_1} + 2^{n_2} + ... + 2^{n_k}$, where $n_1 > n_2 > ...n_k$ and let s_m denote the mth place of the series. Let $m^{'} = 2^{n_2} + ... + 2^{n_k}$. Then $s_m = s^c_{m^{'}}$.

Lemma - 2 Let $k = 2^m$ and $k^{'} = 2^{m+1}$. Then $s_k = s^c_{k^{'}}$

Main Proof : (By Contradiction) Let the decimal be rational. Then the decimal can be written as $0.a\bar{b}$ where a and b are 0-1 sequence with s and t length respectively. WLOG let s be 0. Let the binary representation of t be given by: $t = 2^{n_1} + 2^{n_2} + ... + 2^{n_k}$, where $n_1 > n_2 > ... > n_k$. Then, this means that $s_{t+i} = s_i \forall i \leq t$.

From lemma-1 we know, $s_t = s_{2^{n_k}}$ if k is odd and $s_t = s^c_{2^{n_k}}$ if k is even.

And $s_{2t} = 2^{n_1+1} + 2^{n_2+1} + ... + 2^{n_k+1}$ and from lemma-1, $s_{2t} = s_{2^{n_k+1}}$ if k is odd and $s_{2t} = s^c_{2^{n_k+1}}$ if k is even. From, lemma-2 we know that $s_{2^{n_k+1}} = s^c_{2^{n_k}}$. This implies that s_t is not equal to s_{2t}, which produces the necessary contradiction.