MRM Weekly Converge - Week 8

August 2, 2019

"The art of proposing a question must be held of higher value than solving it"
- Georg Cantor

Problem (Puzzle). A football coach is assembling a team of 11 players to play. The coach may pick any player he wants from an infinite pool of players. Its hard to remember that many names, so each player wears a jersey with a unique positive integer on the back (so that there is a player for every integer). That number also represents the average number of games needed for him/her to score a goal. The coach would like for his team to average exactly three goals per game but would also like for his weakest player to be as strong as possible. What number does the ideal weakest player wear? What are the numbers of the other 10 players the coach should select?

Problem (Number Theory). Find all ordered positive integer pairs (a, b) $s.t$ $\frac{1}{a} + \frac{1}{b} = \frac{5}{2019}$

Problem (Programming). Let $c = 1723$ and a be an integer. b is a huge number whose digits are inputted in the form of a list (array), e.g b = [1,2,4,3,5,6] denotes the number 124356. Find an efficient algorithm to compute $a^b \bmod c$

Problem (Number theory). Let $\mathbb{Z}$ be the set of integers. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that for all integers a and b, $f(2a) + 2f(b) = f(f(a + b))$.

Solution - Week 7

Solution (Statistics). Refer Solution - German tank problem

Solution (Probability). Refer Solution - 100 prisoners problem