MRM Weekly Converge - Week 9

August 23, 2019

Problem (Pattern Recognition). Given that 2^{2004} is a 604 digit number starting with 1. How many of the set $\{2^1, 2^2, 2^3, ..., 2^{2003}\}$ starts with 4?

Problem (Number Theory). Find all ordered positive integer pairs (k, n) s.t $k! = (2^n - 2^{n-1})(2^n - 2^{n-2})...(2^n - 1)$

Problem (Combinatorics). There are 12 boys and 10 girls in a class. In how many ways can we make a group consisting of equal number of boys and girls? Give a neat simplified expression.

Problem (Number theory). What is the unit digit of the greatest integer less than or equal to $\frac{10^{2000}}{10^{100}+3}$?

Solution - Week 8

Solution (Number Theory). *(Solution - Paulina Wardzala)* $\frac{1}{a} + \frac{1}{b} = \frac{5}{2019} \implies \frac{a+b}{ab} = \frac{5}{2019} \implies ab$ is divisible by 2019. $2019 = 1 * 3 * 673 \implies$ WLOG, $a|2019$ or $a|673$ and $b|3$. Let $a = 2019k$, for some positive integer k, substituting and enforcing the positivity of a, we get $b \in [404, 504]$. k is an integer $\implies b = 404$. Similarly doing for the other case, we get no solution for b. Thus the only ordered pair is $(404, 815676)$ and $(815676, 404)$.

Solution (Number Theory). *(Solution - Richard Harris)* Taking $a = 0, b = k$, $a = 1, b = k - 1$ and equating them we get, $f(k) - f(k-1) = \frac{f(2)-f(0)}{2} = \frac{2f(1)-f(0)}{2}$. This implies that the function f is linear. Taking $f(x) = mx + c$, for $\forall c \in \mathbb{Z}$, we see that $f(x) = 2x + c$, for $\forall c \in \mathbb{Z}$ and $f(x) = 0$ satisfy the equation.

Solution (Statistics). We need that $\sum p \in \mathbb{Z}\frac{1}{p} = 2$. By trial and error, we see that the weakest player will wear a number of 24. The rest of the players wear 1, 5, 6, 8, 9, 10, 12, 15, 18 and 20.