MRM Converge - Problem Set 12

December 13, 2019

Problem (Probability). There are 6 projects and 15 units of money (a unit could be thought of as million dollars). The 15 units of money is distributed to these 6 projects such that all projects are funded. What is the expected number of units of money in the project which is allocated the least amount? Also, what is the expected number of units of money in the project that is allocated the maximum amount of money?

Problem (Finance). A financial instrument returns a stochastic annual return which follows the probability distribution $f(x)$, i.e. the return at the any year t is given by r_t which comes from the probability distribution $f(x)$. Suppose the probability distribution is taken to be uniform between -5% and 15%, answer the following questions:

- What is the expected CAGR (compound annual growth rate) of the instrument if it is sold after 10 years?
- What is the expected CAGR (compound annual growth rate) of the instrument over infinite time?

Problem (Graph theory - Emerging risk). A bank has funded the water distribution network of Manhattan which is made up of 10 nodes and 25 edges. The nodes could be thought of as reservoirs or sinks (industries, apartments) while the edges are the pipes connecting them. The distribution network is a connected graph (i.e. traversing the edges one could get from one node to any other). An information has come that a mafia group is randomly blowing up 5 pipes (edges). If the water distribution network could not service a node (i.e. there exists no edge connecting the node to the rest of the graph), the bank losses 1 million \$. What is the expected number of dollars that the bank will lose after the mafia group's activities?

Problem (Arithmetic). Compute $\lim_{n\to\infty}(\prod_{k=1}^{n}(a^2 - (\frac{kd}{n})^2))^{\frac{1}{2n}}$

Solution - Problem Set 11

Solution (Probability). *(Protyaya Halder, Abhishek Priyadarshi)* Let there exist a minimum k such that $\forall s \geq k$, $x_s \geq y_s$, such that $\sum_{i=1}^{n} |x_i - y_i| = \sum_{i=1}^{k-1}(y_i - x_i) + \sum_{i=k}^{n}(x_i - y_i)$. There cannot exist greater than k number of numbers less than n in the subset X. Suppose not, then we will have less than $n-k$ elements less than n in subset Y $\implies y_k \leq x_k \implies$ contradiction. Therefore, all the elements less than or equal to n will be subtracted while all the elements greater than n will be added. Thus, the final answer is $\sum_{i=n+1}^{2n} i - \sum_{j=1}^{n} j = n^2$

Solution (Probability/Programming). *(Richard Harris, Sanjeev Awasare)* The Manhattan distance is a linear equation in the absolute differences on the x and y axes and so $E[|dx| + |dy|] = E[|dx|] + E[|dy|]$.

$E[d] = 2E[|x-y|] = 2\int_0^1(\int_0^y (y-x)dx + \int_y^1 (x-y)dx)dy$. Solving the integral, we get the final answer to be $\frac{2}{3}$.

Solution (Probability). *(Kousik K)* The trick is to count the number of cards which has the number i on it. Note, there can exist only one card with the number 1 on it. The number of cards with the number p, where p is a prime is $p-1$. The only divisors of p are 1 and $p \implies f(1) + f(p) = p$, where $f(x)$ denotes the number of cards with the number x on it. The number of cards with the number p^n, where p is a prime is given by $p^{n-1}(p-1)$ (can be proved by induction). Similarly, we can prove that the number of cards with the number $p_1^{n_1}p_2^{n_2}...p_k^{n_k}$ is given by $p_1^{n_1-1}(p_1-1)p_2^{n_2-1}(p_2-1)...p_k^{n_k-1}(p_k-1)$. Thus, all the naturals numbers are present in the sequence.

Solution (Number Theory). *(Richard Harris, Protyaya Halder, Abhishek Priyadarshi)* -1 can be written as $e^{(2n+1)\pi} \implies (-1)^{\sqrt{2}} = e^{(2n+1)\pi\sqrt{2}}$ which has infinite solutions. Setting n as 0, we get -0.27 - 0.96i. It can also be proven that no solution is real (left to the reader as an exercise).