MRM Weekly Converge - Week 4

May 3, 2019

"Just because something works,
doesn't mean it can't be improved"
- Shuri, Black-panther, 2018

Problem (Chess - Endgame). Black to play. Is there a winning strategy? If yes, what is the trick move?

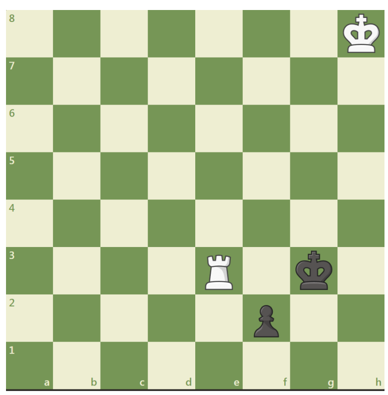

Figure 1: Black to move

Problem (Probability - Help Thanos!). (*Problem Submitted by Kousik Krishnan*) Thanos would like to wipe out **exactly** half the population. He want the selection to be completely random. He has a fair coin, using which he can make the selection. Find the minimum number of tosses he require to choose a set of n/2 people to kill.

Problem (Brain Teaser - Smart Avengers). All is lost, Thanos has got the infinity stone gauntlet and awaits to do the snap. The avengers are desperate and are willing to try anything to stop the chaos. Ant-man wants do end it "Back to the future" way - go back in time using pym-particles and kill baby Thanos. To do that they need to know the exact date in which Thanos was born(to kill him in birth - when he is the weakest).

The avengers know that Thanos was born in one of these days: May 15, May 16, May 19, June 17, June 18, July 14, July 16, August 14, August 15, August 17. Ant-Man(Lang) knows the month in which he was born and Iron-man(Tony) independently knows the day.

They cannot communicate to each other the month/day as Thanos will overhear it and instantaneously reverse time. So, they communicate to each other in the following secret way:

- Lang : I don't know the exact date and I am sure that Tony do not know the date either.
- Tony : I did not know the date earlier but now I know.
- Lang : Thanks. Now, I know too. Ant-man goes back in time, kills baby Thanos and saves the day.

When was Thanos born?

Problem (Combinatorics - The hero MCU deserves). (*Problem Submitted by Kousik Krishnan*) The unsung hero of End-game was a mouse - the mouse that accidentally brought Lang out of quantum realm. It makes us wonder, maybe in the other scenarios that Dr. Strange saw, the mouse did not press the correct sequence of buttons. How many such scenarios are possible?

Assume that the machine has 7 buttons, a combination consists of pressing one or more buttons simultaneously. A sequence consists of ordered 1 or more combinations. Example of a sequence is (1-2-3)(4-7)(5-6), where (1-2-3), (4-7), (5-6) are all combinations. A button can be pressed only once in a sequence, thus (1-2-3)(4-5)(2-7) is not a valid sequence as 2 appears in 2 combinations. Also, note that the combination (1-5-6) is same as (1-6-5). A valid sequence consists of pressing all the buttons exactly once. How many valid sequences are possible?

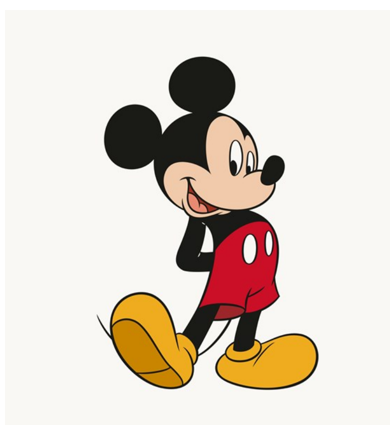

Figure 2: Disney wanted to show who the boss is by making a mouse hero

Solution - Week 3

Solution (Chess). *(Solution - Kousik Krishnan)*

- White - Qh1
- Black - Kxg5 or hxg5
- White - Ng2 or Qg2
- Black - hxg2 or hxg2
- White - h4++ or Nxg2++

Solution (Puzzle). *(Solution - Richard Harris)* Exactly 2 calls to the blackbox is required to get all the coefficients. Let $c = f(1) = \sum_{i=0}^{n} a_i x^i$. Then choose the smallest integer n s.t $10^n > c$. Finally, calculate $x = f(10^n)$ and read out the coefficients in block of n digits.

Solution (Geometry). *(Solution - Kousik Krishnan)* For the rays to produce triangles, there must be a ray from the point to all the vertices. Let the areas of the partitioned triangle be a,b and c. Any point can be uniquely denoted by the way it partitions the triangle - i.e a,b,c(Why?). Let the number of smaller triangles in each partition be n_a, n_b and n_c. Then $\frac{a}{n_a} = \frac{b}{n_b} = \frac{c}{n_c}$ and $a + b + c = area$. Also, $n_a + n_b + n_c = 27$. Putting it together, we have $a = \frac{n_a(area)}{27}$, $b = \frac{n_b(area)}{27}$ and $c = \frac{n_c(area)}{27}$. Therefore, the number of unique values of a, b and c is equal to the number of positive integer solutions of $n_a + n_b + n_c = 27$. Thus, the solution is $\binom{26}{2}$.

Solution (Game Theory). *(Solution - Sushant Vijayan)* Refer the book "Algorithmic Game Theory" by Noam Nisan, Tim Roughgarden, et al.