MRM Converge - Problem Set 13

February 3, 2020

Problem (Puzzle). There are 12 coins, in which we know at-most 1 is faulty (either weighs higher or lower). Note that it can also be the case that none is faulty. Given a weight balance, can we check if any coin is faulty, if yes, output if heavier/lighter and which coin in just 3 uses of the balance.

Problem (Puzzle). Express:

- absolute function in terms of max
- max in terms of absolute

Problem (Finance). Assuming our PnL distribution follows a standard normal, the expected shortfall at percentile p is equal to VaR at what percentile? The answer must be a closed form function of p.

Problem (Arithmetic). When a function $f(x)$ is differentiated n times, the function we get is denoted by $f^n(x)$, let $f(x) = \frac{e^x}{x}$, find the value of $\lim_{n\to\infty} \frac{f^n(1)}{n!}$.

Solution - Problem Set 12

Solution (Probability). *(Kousik Krishnan)* The project allocated the minimum fund can either have 1 or 2 units of money (Since if all projects have at least 3 units, 18 units will be required).

The number of configurations which have the minimum number of funds to be 2 can be found by binomial expansion, i.e. it is the coefficient of x^{15} in $(x^2 + x^3 + ...)^6$. We get it to be $\binom{8}{3}$. Thus, the answer is $\frac{1*(\binom{14}{5}-\binom{8}{3})+2*(\binom{8}{3})}{\binom{14}{5}}$

Similarly, to find the maximum, we compute all the configurations that have the maximum to be less than or equal to k as the coefficient of x^{15} in $(x^2 + x^3 + ...x^k)^6$. We can find the number of configurations that have the maximum to be exactly k as the number of configurations in less than or equal to k subtracted by configurations in less than or equal to $k+1$. The final answer is 5.314685.

Solution (Finance). Hint : The expectation of product of independent random variables is the product of expectation of the random variables.

Solution (Graph Theory - Emerging risk). No solution is received. A quick literature survey revealed that certain parts of the question are open problems.

Solution (Arithmetic). (Protyaya Halder) Taking the log and doing few algebraic reductions leaves us with $-log(a)-\sum_{p=1}^{\infty}((d/a)^(2p)/((2p)*(2p+1)))$, which can be simplified to an ugly logarithm.

Hint : An elegant solution can be produced with the use of right hand approximation method.