# Design, Analysis and Optimization of Phase Changing Fluid Balloons for Planetary Exploration

A Project Report submitted in partial fulfillment for the award of the Degree of

#### **BACHELOR OF TECHNOLOGY**

in

#### **AEROSPACE**

by

### MRIDUL SONGARA

pursued in

#### DEPARTMENT OF AEROSPACE

to



# INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY Thiruvananthapuram

**April 22, 2016** 

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**April 22, 2016** 

CERTIFICATE

This is to certify that the project report entitled "Design, Analysis and Optimization

of Phase Changing Fluid Balloons for Planetary Exploration" submitted by Mridul

Songara, to the Indian Institute of Space Science and Technology, Thiruvananthapu-

ram, in partial fulfillment for the award of the degree B.Tech. in Aerospace Engineer-

ing, is a bonafide record of the project work carried out by him under my supervision.

The contents of this report, in full or in parts, have not been submitted to any other

Institute or University for the award of any degree or diploma.

Head of the department

Department of Aerospace Engineering

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Technology

Shri Pankaj Priyadarshi

Head ADSD

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Vikram Sarabhai Space Center

Place: Thiruvananthapuram

April 2016

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**DECLARATION** 

I declare that this report titled "Design, Analysis and Optimization of Phase Chang-

ing Fluid Balloons for Planetary Exploration" submitted in partial fulfillment of the

Degree of "Bachelor of Technology in Aerospace Engineering" is a record of origi-

nal project work carried out by me under the supervision of Shri Pankaj Priyadarshi,

and has not formed the basis for the award of any degree, diploma, fellowship or other

titles in this or any other Institution or University of higher learning. In keeping with

the ethical practice in reporting scientific information, due acknowledgments have been

made wherever the findings of others have been cited.

Place: Thiruvananthapuram

April 2016

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### **ABSTRACT**

The present work focus on the balloon systems for planetary exploration. The particular types of balloons used are phase changing fluid balloons[2]. The system consist of two balloons - one containing lifting gas and other containing phase changing fluid. Balloon theory - the buoyancy concept, balloon shape and design is discussed in the present work. A one dimensional trajectory equation is used along with heat transfer equations of the five components - primary balloon film, lifting gas, secondary balloon film, phase changing fluid in liquid phase and vapour phase. The phase change thermodynamics is used (during which temperature derivatives of liquid and vapour are zero). The one DoF trajectory of balloon is calculated. The code is validated with ALICE balloons of Jet Propulsion Laboratory which used the same concept to flew balloons in Earth atmosphere. Several differences were noted and the reasons were discussed. Integration time step,  $\Delta t$ , independence study is performed and it is found that for small  $\Delta t$ 's of 0.00625 seconds and 0.0003125 seconds the variation of about 100 metres exist after 5 hours of flight. A sensitivity analysis is performed and the trajectory is found to be very sensitive to variations in mass of lifting gas. Finally, Helium balloon is analysed by studying the effects of drag and heat transfer on the balloon.

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# **CHAPTER 1**

# Introduction

India is moving towards Planetary Exploration missions. Landing on the Moon surface and orbiting Mars is just a beginning in space exploration. Venus is the next potential stop of Indian space program after successfully reaching Moon and Mars. Fast forwarding few missions, we will be planning missions to explore Venus atmosphere and perhaps also its surface.

There is a lot[10] of scientific curiosity regarding Venus - the super rotation of atmosphere (winds on Venus flow faster than the planet rotate, also, it rotates in the same direction), Venus tectonics, chemistry of Sulphur in the atmosphere etc. The environment there is completely different from the Earth despite of similarities in planet size and density. A little is known about the planet through a few previous missions. It is established that Venus is covered with thick carbon dioxide atmosphere, which reaches high pressure and temperature of 9.2 MPa and 740 K respectively near the surface. Oceans can not exist there. It is suggested from D/H ratio that once abundant water existed there but how was it lost is unknown. It is speculated, however, that it might have happened due to weak intrinsic magnetic field of Venus. Plate tectonics are considered to be absent since no direct evidence of volcanic activity has been acquired.

All the Venus missions attempted in the past - Venera, Pioneer, Magellan, Vega - are summarized by NASA's National Space Science Data Centre(NSSDC) on their website. Most of the these missions were flyby missions, few lander and orbiter missions. Two notable missions most relevant to this project are Vega 1 and Vega 2 launched in December 1984 by the Soviet Union. Both were helium filled spherical, super pressure balloons and floated at approximately 50 km altitude in middle layer of Venus' three-tiered cloud system, for about 47 hrs.

# 1.1 Aim of Present Study

The Present study aims to design phase changing fluid balloons for planetary exploration mission specially Venus, since it is nearest to Earth and similar in shape and size,

yet, has entirely different kind of environment. The balloon system is composed of a lifting gas along with the phase changing fluid. Next, a design optimization study is proposed. Also, 3 dof inertial frame trajectory equations have been derived for the purpose of detailed analysis of balloon trajectory.

Balloons provide an excellent medium for excursion through atmosphere.

- They can effectively[6] investigate the atmospheric rotation to execute in situ measurements of atmospheric dynamics and composition.
- They can also provide the necessary long duration (potentially many days) measurements under the thick clouds.
- Balloons can provide global coverage circumnavigate in several days with super rotation and meridional circulation
- They can be used to carry deep atmosphere/ surface dropsondes.
- They are less expensive, since they are not designed for landing.

It is a challenging[6] task to float a balloon in high-temperature, high-pressure and acid atmosphere.

- Deploying and inflating is a delicate phase.
  - The balloon has to be packed into a storage container for long term.
  - The balloon system has to stabilize with the main parachute during descent in the atmosphere, then deploy and inflate the balloon.
  - The parachute, inflation system and the balloon has to, then, separate.
- No validation facilities are available for full scale balloon testing in the temperature.
- It is difficult to achieve a 'balloon in earth' like qualification due to differences in atmospheric composition, atmospheric profile, atmospheric turbulence, wind gradients and gravity.

Different options for a planetary balloon mission have been explored, as shared by Jeffery L. Hall[7] of JPL in his presentation on 'Venus Balloons for High Altitude' -

- zero pressure balloons
- superpressure balloons
- altitude cycling balloon
  - hot air (Montgolfiere)
  - phase change fluid

The first two types have been perfected for Earth atmosphere and can be readily adapted to other planets. Zero pressure balloons implies very little pressure difference across the balloon envelope. It is achieved by venting out the pressurized gas. They can be used for vertical excursions but since they require ballast and venting of balloon gas, mission durations are very small. Superpressure balloons are constant volume, closed balloons and provide longer mission duration but lesser altitude variations. The altitude cycling concept is relatively less mature and a detailed analysis has not yet been performed but they can provide both longer flights and vertical excursions. Montgolfiere balloons require accurate radiative properties of planet's atmosphere and its performance model is yet to be quantified. Phase change fluid balloons' performance has been modelled and validated by actual flight in Earth's atmosphere. Building on the groundwork, it is possible to make models for bobbing balloon system on other planets.

The Phase change balloons' performance prediction model[1] was prepared using altitude control experiment(ALICE)[3] balloon system which consist of a helium lifting balloon and buoyancy controlling freon R114 balloon. The analytical model is correlated well with flight trajectories of four different day and night balloons.

The excursion about a stabilization altitude in atmosphere can be achieved using different phase changing fluids - water, methanol or fluid combinations - water + ammonium or even using reversible chemisorption. The options are summarized by Jack A. Jones[2] in his paper on 'Reversible fluid balloon altitude control concept'.

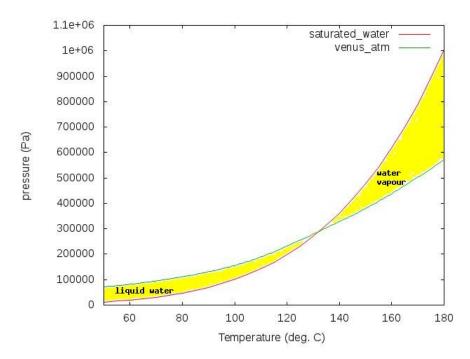


Figure 1.1: Pressure-Temperature profile of Saturated Water and Venus atmosphere

The above plot shows that water is a potential candidate as a reversible fluid in Venus atmosphere since its saturation point lies at about 42 km altitude, below which it exist as vapour and above as liquid.

Use of steam as a balloon fluid has been explored by Richard M. Dunlap[9] and a model has been made by N. Izutsu et. al. [10]. The later has also studied various options for envelope material.

# 1.2 Outline of Present Study

The balloon system consists of two balloons - primary balloon containing a lifting gas and secondary balloon containing the phase changing fluid. A 1 dof trajectory simulation is performed using a heat transfer model[1]. A thermodynamic analysis of phase change is performed in the second balloon for use in the trajectory simulation. The balloon performance is analysed assuming there is no drag and no heat transfer. Then the drag term is introduced followed by introduction of heat transfer.

The balloon deployment strategy and options in the envelope material are not studied in present work.

# **CHAPTER 2**

# Balloon Theory, Governing Equations and Solution Procedure

# 2.1 Balloon Theory

The equations and theory in this section are taken from reference[5]

# 2.1.1 Buoyancy

The buoyant force on a body in a fluid is

$$B = p_2 S - p_1 S$$

where,  $p_2$  is force applied by the fluid on the lower surface of the body and  $p_1$  is applied on the upper surface and S is the surface area on which the pressure is applied. Using fluid density the buoyant force becomes

$$B = \rho_{fluid} \, g \, Volume_{body}$$

For a balloon containing volume V of a buoyant gas of density  $\rho_g$  in external air of density  $\rho_a$ , the net upward force (i.e. buoyant force - total weight of the balloon system) will become

$$F = (\rho_a V - mass_{system})g$$

Thus, the balloon will float at an altitude where buoyant force equals the total weight of the balloon system. At a given altitude,  $\rho_a$  is known from standard atmosphere profile data. Thus, for a specified design altitude and mass of payload to be carried, the volume of balloon can be determined. Also, with higher altitude,  $rho_a$  decreases resulting in lower mass for a specified volume, i.e. reaching higher altitudes require light weight systems.

# 2.1.2 Balloon Shape

The most appropriate shape of a balloon can be designed keeping in mind uniform distribution of suspended load and maximizing the balloon strength while minimizing the tensile force which should be uniformly distributed on the film subjected to pressure. Also, the above points should be maintained during ascent when the balloon shape changes.

#### 2.1.2.1 Natural Shaped Balloons

A perfect design geometry of sphere, cylinder or tetrahedral can be assumed by a balloon only when it fully inflate after ascent. Above figure shows partial inflation of the



Figure 2.1: A partially inflated balloon[5]

balloon on the ground, which has resulted in a large number of folds parallel to the longitudinal axis, indicating at the excess film in the circumferential direction. Since the length of the film remains constant irrespective of inflation of balloon, tension is not generated in circumferential direction. R.H.Upson did a significant work regarding formulation of balloon shape assuming only longitudinal tension.

As a solution to this problem natural-shape balloon has been proposed which maintains same shape throughout the process of its inflation. It assumes an infinitesimal

amount of excess film in the circumferential direction at full inflation. Balloons a and b

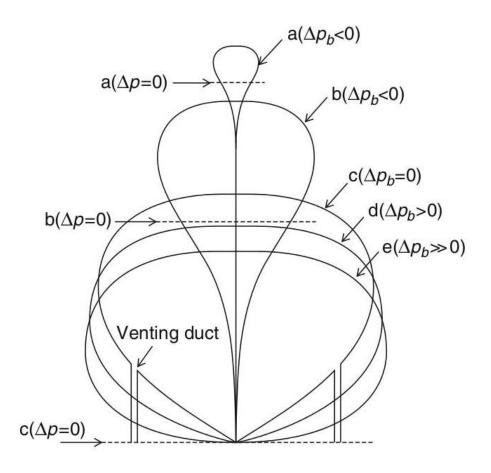


Figure 2.2: Natural shaped balloon at different amount of inflations. $\Delta p_b$  is pressure difference between interior and exterior of the balloon at the base of the balloon[5]

are at partially inflated state, c fully inflated zero-pressure balloon and d and e at a state with pressurised interior of balloon.

# 2.1.3 Balloon Design with Load Tapes

Load tapes are vertical reinforcement fibers needed to transmit the concentrated load of a payload as a distributed load on the balloon film. Their extensibility is low compared to the film. All load tapes concentrate at the point where payload is suspended on the balloon, thus it distributes the load all over the film.

# 2.2 Governing Equations

The equation in this section as taken from reference[1]



Figure 2.3: A balloon reinforced with load tapes[11]

## 2.2.1 Equations of Motion

The Balloon system consist of a helium balloon and a water balloon with a rope and gondola attached to it. To study the dynamics of the balloon, it is considered as a point mass. The forces acting on it are buoyant force, gravity and drag. Assuming 'z' coordinate axis in vertical direction, with positive upwards, the dynamic equations becomes:

$$(m_{sys} + c_m \rho_\infty V) \frac{d^2 z}{dt^2} = g(\rho_\infty V - m_{sys}) - \frac{1}{2} \rho_\infty C_D \left( \frac{dz}{dt} + W_z \right) \left| \frac{dz}{dt} + W_z \right| A \quad (2.1)$$

where,

$$m_{sys} = m_{tot} - \int_0^t (\dot{\mathcal{L}}_l + \dot{\mathcal{L}}_g) \,\mathrm{d}t$$

 $\dot{\mathcal{L}}_l = \text{liquid water leak rate}$ 

 $\dot{\pounds}_g = \text{Water vapour leak rate}$ 

 $C_m = Virtual mass coefficient$ 

$$V = V_g + V_l$$

 $W_z =$ wind velocity in z direction

A =balloon cross section area

 $V_g$ , the volume occupied by the gas is calculated using pressure balance considering the balloon to be infinitely inflatable. Using the ideal gas equation

$$V_g = \frac{m_g R T_g}{M_g P_\infty}$$

also, since density of liquid is constant

$$V_l = \frac{m_l}{\rho_l}$$

The kinematic equation is

$$\frac{dx}{dt} = -w\sin(\theta) \tag{2.2}$$

$$\frac{dy}{dt} = -w\cos(\theta) \tag{2.3}$$

$$\frac{dz}{dt} = U_z \tag{2.4}$$

w is wind velocity and  $\theta$  wind direction measured from north.

# 2.2.2 Heat Balance

The temperature change of balloon gas is due to heat transfer and adiabatic expansion/compression of gas. If the leakage of liquid and gas is also considered, following are equations to calculate temperature change in lifting gas, liquid and balloon film:

$$\frac{dQ_{pg}}{dt} = \dot{q}_{pg} - \rho_{\infty} V_p g \frac{dz}{dt} - \dot{\mathcal{L}}_p \frac{Q_p}{m_p}$$
(2.5)

$$\frac{dQ_{pf}}{dt} = \dot{q}_{pf} \tag{2.6}$$

$$\frac{dt}{dQ_g} = \dot{q}_g - \rho_\infty V_g g \frac{dz}{dt} - \dot{\mathcal{L}}_g \frac{Q_g}{m_g}$$
(2.7)

$$\frac{dQ_l}{dt} = \dot{q}_l - \dot{\mathcal{L}}_l \frac{Q_l}{m_l} \tag{2.8}$$

$$\frac{dQ_f}{dt} = \dot{q}_f \tag{2.9}$$

For simplicity, initially no leakage is considered, thus the above equations reduce to

$$m_{pg} c_{pg} \frac{dT_{pg}}{dt} = \dot{q}_{pg} - \rho_{\infty} V_{pg} g \frac{dz}{dt}$$

$$m_{pf} c_{pf} \frac{dT_{pf}}{dt} = \dot{q}_{pf}$$

$$m_{g} c_{g} \frac{dT_{g}}{dt} = \dot{q}_{g} - \rho_{\infty} V_{g} g \frac{dz}{dt}$$

$$m_{l} c_{l} \frac{dT_{l}}{dt} = \dot{q}_{l}$$

$$m_{f} c_{f} \frac{dT_{f}}{dt} = \dot{q}_{f}$$

#### **2.2.2.1** Heat Flux

The thermal model consist of absorption of solar radiation, infra-red as well as solar radiation reflected from ground and convection from air to film as well as from film to liquid.

$$\dot{q}_{pg} = \left[G\alpha_{pgeff}(1+r_v) + \epsilon_{pint}\sigma(T_{gp}^4 - T_{fp}^4) - \epsilon_{pgeff}\sigma T_{gp}^4 + \epsilon_{pgeff}\sigma T_{BB}^4\right]S_p$$

$$- CH_{pgf}(T_{gp} - T_{fp})S_p \quad (2.10)$$

$$\dot{q}_{pf} = \left[G\alpha_{pweff}\left(\frac{1}{4} + \frac{1}{2}r_v\right) + \epsilon_{pint}\sigma(T_{gp}^4 - T_{fp}^4) - \epsilon_{pweff}\sigma T_{fp}^4 + \epsilon_{pweff}\sigma T_{BB}^4\right]S_p + \left[CH_{pgf}(T_{gp} - T_{fp}) + CH_{pfa}(T_{\infty} - T_{fp})\right]S_p \quad (2.11)$$

$$\dot{q}_g = \left[G\alpha_{geff}(1+r_v) + \epsilon_{int}\sigma(T_g^4 - T_f^4) - \epsilon_{geff}\sigma T_g^4 + \epsilon_{geff}\sigma T_{BB}^4\right]S_g$$
$$-CH_{gf}(T_g - T_f)S_g \quad (2.12)$$

$$\dot{q}_{l} = \left[G\alpha_{leff}\left(\frac{1}{2} + r_{v}\right) + \epsilon\sigma(T_{BB}^{4} - T_{l}^{4}) + CH_{lfa}(T_{\infty} - T_{l})\right]S_{l}$$
(2.13)

$$\dot{q}_f = \left[G\alpha_{weff}\left(\frac{1}{4} + \frac{1}{2}r_v\right) + \epsilon_{int}\sigma(T_g^4 - T_f^4) - \epsilon_{weff}\sigma T_f^4 + \epsilon_{weff}\sigma T_{BB}^4\right]S + \left[CH_{qf}(T_q - T_f) + CH_{fa}(T_\infty - T_f)\right]S \quad (2.14)$$

G is solar constant and  $r_v$  is reflectivity (albedo) of planet the planet.

where, the effective radiative heat transfer coefficients are defined in terms of the actual gas and wall radiative properties

$$\alpha_{geff} = \frac{\alpha_g \tau_{wsol}}{1 - r_{wsol}(1 - \alpha_q)} \tag{2.15}$$

$$\epsilon_{int} = \frac{\epsilon_g \epsilon_w}{1 - r_w (1 - \epsilon_g)} \tag{2.16}$$

$$\epsilon_{geff} = \frac{\epsilon_g \tau_w}{1 - r_w (1 - \epsilon_g)} \tag{2.17}$$

$$\alpha_{weff} = \alpha_w \left( 1 + \frac{\tau_{wsol}(1 - \alpha_g)}{1 - r_{wsol}(1 - \alpha_g)} \right)$$
 (2.18)

$$\epsilon_{weff} = \epsilon_w \left( 1 + \frac{\tau_w (1 - \epsilon_g)}{1 - r_w (1 - \epsilon_g)} \right) \tag{2.19}$$

and convective heat transfer coefficient  $CH = Nu \times \frac{k}{D}$ 

$$k =$$
thermal conductivity (2.20)

$$D = \text{characteristic length}$$
 (2.21)

$$Nu = \text{Nusselt number}$$
 (2.22)

Heat transfer is evaluated using the non dimensional number - Nusselt Number. For different heat transfer processes, it is calculated[1] as follows

 Primary balloon film to air - assuming sperical configuration Nusselt number for forced convection

$$Nu_1 = 0.37Re^{0.6} Vp < 53800 (2.23)$$

$$Nu_1 = 0.74Re^{0.6} Vp \ge 53800 (2.24)$$

Nusselt number for free convection

$$Nu_2 = 2 + 0.6(Gr\,Pr_a)^{1/4} \tag{2.25}$$

Nusselt number for primary balloon film to air convection: Nu=Max $Nu_1,Nu_2$ 

2. Primary balloon lifting gas to film assuming spherical configuration

$$Nu_1 = 2.5(2 + 0.6Rn^{1/4})$$
  $Rn < 1.34681 \times 10^8$  (2.26)

$$Nu_1 = 0.325 \ Rn^{1/3}$$
  $Rn \ge 1.34681 \times 10^8$  (2.27)

3. Secondary Balloon film to air - assuming vertical flat plate configuration Reynolds number:

$$Re = \frac{\rho_{\infty}L|U_z + W_z|}{\mu_{\infty}} \tag{2.28}$$

Nusselt number for forced convection in air  $(Pr_{\infty} = 0.71)$ :

$$Nu_1 = 0.5924Re^{1/2}$$
  $Re < 487508.3$  (2.29a)

$$Nu_1 = 0.033Re^{0.8} - 758.3$$
  $Re \ge 487508.3$  (2.29b)

The Nusselt number for free convection:

$$Nu^T = 0.515Rn^{1/4} (2.30a)$$

$$Nu_a = \frac{2.8}{\ln\left(1 + \frac{2.8}{Nu^T}\right)}$$
 (2.30b)

$$Nu_b = 0.103Rn^{1/3} (2.30c)$$

$$Nu_2 = \left[ (Nu_a)^6 + (Nu_b)^6 \right]^{1/6} \qquad 1 < Rn < 10^{12}$$
 (2.30d)

Nusselt number for balloon film to air convection:

$$Nu = Max\{Nu_1, Nu_2\}$$
 (2.31)

4. Secondary Balloon gas to film - assuming vertical flat plate configuration

$$Nu^T = 0.515Rn^{1/4} (2.32a)$$

$$Nu_a = \frac{2.8}{\ln\left(1 + \frac{2.8}{Nu^T}\right)}$$
 (2.32b)

$$Nu_b = 0.103Rn^{1/3} (2.32c)$$

$$Nu = \left[ (Nu_a)^6 + (Nu_b)^6 \right]^{1/6} \qquad 1 < Rn < 10^{12}$$
 (2.32d)

Secondary Balloon film in liquid region to air - assuming horizontal cylinder
 Nusselt number for forced convection in air

$$Nu_1 = 0.38 + 0.44Re^{1/2}$$
  $Re < 1297.742$  (2.33a)

$$Nu_1 = 0.22Re^{0.6}$$
  $Re > 1297.742$  (2.33b)

Nusselt number for free convection in air

$$Nu_2 = \left[0.6 + 0.322(Rn)^{1/6}\right]^2 \tag{2.34}$$

Nusselt number for balloon film to air convection

$$Nu = Max\{Nu_1, Nu_2\} \tag{2.35}$$

Equations 1-7 describe the balloon motion.

## 2.2.3 Phase Change

The phase change thermodynamics is used to obtain volume of the balloon during the process. When the saturation temperature is reached the heat is absorbed by liquid or released by vapour as latent heat  $(h_{avg})$ . Thus, the quality,  $\mathbf{x} \left( = \frac{m_{vapour}}{m_{total}} \right)$  can be found as

$$x = \frac{h_{avg} - h_f}{h_{fg}}$$

where,  $h_f$  is specific enthalpy of saturated liquid,  $h_g$  is specific enthalpy of saturated vapour and  $h_{fg} = h_g - h_f$ . When the total heat supplied becomes equal to or greater that  $h_{fg}$ , complete phase transfer occurs and additional heat is used to increase the temperature of fluid. The standard saturated water tables are used for the above properties.

# 2.2.4 3 DoF Trajectory Equations in Inertial Coordinate

After, simulating the balloon in vertical direction and validating the code, the present work is intended to simulate the balloon motion using 3 dof trajectory code in inertial coordinate frame. For this purpose, 3 dof trajectory code is written in C++ using already written planet fixed reference frame code. The latter code is inherited in the present

code.

The generalised inertial coordinate frame equations are derived as follows. The forces included in the equations are lift, drag, thrust, gravity and buoyancy.

From Newton's second law of motion we have.

$$m\left(\frac{d\vec{V}}{dt}\right)_{I} = \vec{L} + \vec{D} + \vec{G} + \vec{T}$$
 (2.36)

 $\vec{L}, \vec{D}, \vec{T}$  are in velocity aligned where as  $\vec{G}$  is in radius aligned. So first transformation is from velocity to radius aligned.

$$\vec{F}_V = T_{V \leftarrow R} \vec{F}_R \tag{2.37}$$

$$(T_{V \leftarrow R})^{-1} \vec{F}_V = (T_{V \leftarrow R})^{-1} T_{V \leftarrow R} \vec{F}_R$$
 (2.38)

$$\vec{F}_R = T_{R \leftarrow V} \vec{F}_V \tag{2.39}$$

Where  $\overrightarrow{F}$  is a force vector.

$$T_{V \leftarrow R} = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & \sin(\psi) \\ 0 & -\sin(\psi) & \cos(\psi) \end{bmatrix}$$

$$T_{R \leftarrow V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & -\sin(\psi) \\ 0 & \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(2.40)

$$T_{R \leftarrow V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & -\sin(\psi) \\ 0 & \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(2.41)

From above transformation we can say that inverse of a transformation matrix is equal

to transpose of the matrix.

$$T_{R \leftarrow V} = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\cos(\psi)\sin(\gamma) & \cos(\psi)\cos(\gamma) & -\sin(\psi) \\ -\sin(\gamma)\sin(\psi) & \cos(\gamma)\sin(\psi) & \cos(\psi) \end{bmatrix}$$
(2.42)

$$\vec{L}_V = L \begin{bmatrix} \cos(\sigma) \\ 0 \\ \sin(\sigma) \end{bmatrix}$$
 (2.43)

$$\vec{D}_V = D \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \tag{2.44}$$

$$\vec{T}_{V} = T \begin{bmatrix} \sin(\theta_{T})\cos(\sigma_{T}) \\ \cos(\sigma_{T}) \\ \sin(\theta_{T})\sin(\sigma_{T}) \end{bmatrix}$$

$$\vec{L}_{R} = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\cos(\psi)\sin(\gamma) & \cos(\psi)\cos(\gamma) & -\sin(\psi) \\ -\sin(\gamma)\sin(\psi) & \cos(\gamma)\sin(\psi) & \cos(\psi) \end{bmatrix} \vec{L}_{V}$$

$$\vec{L}_{R} = L \begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\psi)\sin(\sigma) & \cos(\psi)\cos(\sigma)\sin(\gamma) \\ \cos(\psi)\sin(\sigma) & \cos(\psi)\sin(\gamma)\sin(\psi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\psi)\sin(\sigma) & \cos(\psi)\cos(\sigma)\sin(\gamma) \\ \cos(\psi)\sin(\sigma) & \cos(\phi)\sin(\gamma)\sin(\psi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\gamma) \\ \cos(\psi)\sin(\sigma) & \cos(\phi)\sin(\gamma)\sin(\psi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\gamma)\sin(\sigma) & \cos(\phi)\cos(\sigma)\sin(\gamma) \\ \cos(\psi)\sin(\sigma) & \cos(\phi)\sin(\gamma)\sin(\psi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\gamma)\sin(\sigma) & \cos(\phi)\cos(\sigma)\sin(\gamma) \\ \cos(\psi)\sin(\sigma) & \cos(\phi)\sin(\gamma)\sin(\psi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\gamma)\sin(\sigma) & \cos(\phi)\cos(\sigma)\sin(\gamma) \\ \cos(\phi)\sin(\gamma)\sin(\phi) & \cos(\phi)\sin(\gamma) \\ \cos(\phi)\sin(\gamma)\sin(\phi) & \cos(\phi)\sin(\gamma) \\ \cos(\phi)\sin(\gamma)\sin(\phi) & \cos(\phi)\sin(\gamma) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\phi)\sin(\sigma) & \cos(\phi)\cos(\sigma)\sin(\gamma) \\ \cos(\phi)\sin(\gamma)\sin(\phi) & \cos(\phi)\sin(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\phi)\sin(\sigma) & \cos(\phi)\cos(\sigma) \\ \cos(\phi)\sin(\gamma)\sin(\phi) & \cos(\phi)\sin(\gamma) \\ \cos(\phi)\sin(\phi) & \cos(\phi)\sin(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\phi) \\ -\sin(\phi)\sin(\phi) & \cos(\phi)\sin(\gamma) \\ \cos(\phi)\sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma)\cos(\phi) \\ -\sin(\phi)\sin(\phi) & \cos(\phi)\sin(\phi) \\ \cos(\phi)\sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ -\sin(\phi)\sin(\phi) & \cos(\phi)\sin(\phi) \\ \cos(\phi)\sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ -\sin(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ -\sin(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ -\sin(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ -\sin(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \\ \cos(\phi)\cos(\phi) & \cos(\phi) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\phi)\cos(\phi)\cos(\phi) \\ \cos(\phi)\cos(\phi) \\$$

$$\vec{L}_R = \begin{bmatrix}
\cos(\gamma) & \sin(\gamma) & 0 \\
-\cos(\psi)\sin(\gamma) & \cos(\psi)\cos(\gamma) & -\sin(\psi) \\
-\sin(\gamma)\sin(\psi) & \cos(\gamma)\sin(\psi) & \cos(\psi)
\end{bmatrix} \vec{L}_V$$
(2.46)

$$\vec{L}_R = L \begin{bmatrix} \cos(\gamma)\cos(\sigma) \\ -\sin(\psi)\sin(\sigma) - \cos(\psi)\cos(\sigma)\sin(\gamma) \\ \cos(\psi)\sin(\sigma) - \cos(\sigma)\sin(\gamma)\sin(\psi) \end{bmatrix}$$
(2.47)

$$\vec{D}_R = D \begin{bmatrix} -\sin(\gamma) \\ -\cos(\gamma)\cos(\psi) \\ -\cos(\gamma)\sin(\psi) \end{bmatrix}$$
(2.48)

$$\vec{D}_{R} = D \begin{bmatrix} -\sin(\gamma) \\ -\cos(\gamma)\cos(\psi) \\ -\cos(\gamma)\sin(\psi) \end{bmatrix}$$

$$\vec{T}_{R} = T \begin{bmatrix} \cos(\gamma)\sin(\gamma) + \cos(\gamma)\cos(\sigma_{T})\sin(\theta_{T}) \\ \cos(\gamma)\cos(\psi)\cos(\theta_{T}) - \sin(\psi)\sin(\sigma_{T})\sin(\theta_{T}) - \cos(\psi)\cos(\sigma_{T})\sin(\gamma)\sin(\theta_{T}) \\ \cos(\psi)\sin(\sigma_{T})\sin(\theta_{T}) + \cos(\gamma)\cos(\theta_{T})\sin(\psi) - \cos(\sigma_{T})\sin(\gamma)\sin(\psi)\sin(\theta_{T}) \end{bmatrix}$$

$$(2.48)$$

Now all forces are in radius aligned now we need to transform from radius to earth fixed.

$$T_{R \leftarrow E} = \begin{bmatrix} \cos(\Phi) & 0 & \sin(\Phi) \\ 0 & 1 & 0 \\ -\sin(\Phi) & 0 & \cos(\Phi) \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{E \leftarrow R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\Phi) & 0 & -\sin(\Phi) \\ 0 & 1 & 0 \\ \sin(\Phi) & 0 & \cos(\Phi) \end{bmatrix}$$

$$T_{E \leftarrow R} = \begin{bmatrix} \cos(\Phi)\cos(\theta) & -\sin(\theta) & -\sin(\Phi)\cos(\theta) \\ \cos(\Phi)\sin(\theta) & \cos(\theta) & -\sin(\Phi)\sin(\theta) \\ \sin(\Phi) & 0 & \cos(\Phi) \end{bmatrix}$$

$$(2.50)$$

$$T_{E \leftarrow R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\Phi) & 0 & -\sin(\Phi) \\ 0 & 1 & 0 \\ \sin(\Phi) & 0 & \cos(\Phi) \end{bmatrix}$$
(2.51)

$$T_{E \leftarrow R} = \begin{bmatrix} \cos(\Phi)\cos(\theta) & -\sin(\theta) & -\sin(\Phi)\cos(\theta) \\ \cos(\Phi)\sin(\theta) & \cos(\theta) & -\sin(\Phi)\sin(\theta) \\ \sin(\Phi) & 0 & \cos(\Phi) \end{bmatrix}$$
(2.52)

 $L_{E1} = \sin(\theta)(L\sin(\psi)\sin(\sigma) + L\cos(\psi)\cos(\sigma)\sin(\gamma)) - \cos(\theta)\sin(\phi)(L\cos(\psi)\sin(\sigma) - \cos(\phi)\sin(\phi)) = 0$  $L\cos(\sigma)\sin(\gamma)\sin(\psi) + L\cos(\gamma)\cos(\phi)\cos(\sigma)\cos(\theta)$ 

 $\cos(\theta)(L\sin(\psi)\sin(\sigma) + L\cos(\psi)\cos(\sigma)\sin(\gamma))$ 

 $L_{E3} = \cos(\phi)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) + L\cos(\gamma)\cos(\sigma)\sin(\phi)$ 

$$ec{L}_E = egin{bmatrix} ec{L}_{E1} \ ec{L}_{E2} \ ec{L}_{E3} \end{bmatrix}$$

 $\vec{D}_{E1} = D\cos(\gamma)\cos(\psi)\sin(\theta) - D\cos(\phi)\cos(\theta)\sin(\gamma) + D\cos(\gamma)\cos(\theta)\sin(\phi)\sin(\psi)$ 

 $\overrightarrow{D}_{E2} = D\cos(\gamma)\sin(\phi)\sin(\psi)\sin(\theta) - D\cos(\phi)\sin(\gamma)\sin(\theta) - D\cos(\gamma)\cos(\psi)\cos(\theta)$ 

 $\vec{D}_{E3} = -D\sin(\gamma)\sin(\phi) - D\cos(\gamma)\cos(\phi)\sin(\psi)$ 

$$ec{D}_E = egin{bmatrix} ec{D}_{E1} \ ec{D}_{E2} \ ec{D}_{E3} \end{bmatrix}$$

 $\overrightarrow{T}_{E1} = \sin(\theta)(T\sin(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\gamma)\cos(\psi)\cos(\theta_T) + T\cos(\psi)\cos(\sigma_T)\sin(\gamma)\sin(\theta_T)) + T\cos(\psi)\cos(\sigma_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi)\cos(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\cos(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T)\sin(\phi_T) + T\cos(\phi_T)\sin(\phi_T$  $\cos(\phi)\cos(\theta)(T\cos(\theta_T)\sin(\gamma)+T\cos(\gamma)\cos(\sigma_T)\sin(theta_T))-\cos(\theta)\sin(\phi)(T\cos(\gamma)\cos(\theta_T)\sin(\psi)+T\cos(\gamma)\cos(\theta_T)\sin(\phi))$  $T\cos(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T)$ 

 $\overrightarrow{T}_{E2} = \cos(\phi)\sin(\theta)(T\cos(\theta_T)\sin(\gamma) + T\cos(\gamma)\cos(\sigma_T)\sin(\theta_T)) - \cos(\theta)(T\sin(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\gamma)\cos(\psi)\cos(\theta_T) + T\cos(\psi)\cos(\sigma_T)\sin(\gamma)\sin(\theta_T)) - \sin(\phi)\sin(\theta)(T\cos(\gamma)\cos(\theta_T)\sin(\psi) + T\cos(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\phi)\sin(\phi)\sin(\theta_T)) - T\cos(\phi)\sin(\phi)\sin(\phi)\sin(\theta_T)$ 

 $\overrightarrow{T}_{E3} = \sin(\phi)(T\cos(\theta_T)\sin(\gamma) + T\cos(\gamma)\cos(\sigma_T)\sin(\theta_T)) + \cos(\phi)(T\cos(\gamma)\cos(\theta_T)\sin(\psi) + T\cos(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T))$ 

$$ec{T}_E = egin{bmatrix} ec{T}_{E1} \ ec{T}_{E2} \ ec{T}_{E3} \ \end{bmatrix}$$

Gravitational force

$$\vec{G}_E = \begin{bmatrix} -G\cos(\phi)\cos(\theta) \\ -G\cos(\phi)\sin(\theta) \\ -G\sin(\phi) \end{bmatrix}$$

Where G =  $mg\left(\frac{r_o}{r}\right)^2$  Transformation matrix from earth fixed to inertial frame .

$$T_{E \leftarrow I} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (2.53)

$$\vec{F}_I = T_{I \leftarrow E} \vec{F}_E \tag{2.54}$$

$$T_{I \leftarrow E} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) & 0\\ \sin(\omega t) & \cos(\omega t) & 0\\ 0 & 0 & 1 \end{bmatrix}$$
 (2.55)

$$\vec{L}_I = T_{I \leftarrow E} \vec{L}_E \tag{2.56}$$

$$\vec{D}_I = T_{I \leftarrow E} \vec{D}_E \tag{2.57}$$

$$\vec{T}_I = T_{I \leftarrow E} \vec{T}_E \tag{2.58}$$

$$\vec{G}_I = T_{I \leftarrow E} \vec{G}_E \tag{2.59}$$

$$\vec{L}_{ECI} = \begin{bmatrix} \vec{L}_{ECI1} \\ \vec{L}_{ECI2} \\ \vec{L}_{ECI3} \end{bmatrix}$$
 (2.60)

 $\overrightarrow{L}_{ECI1} = \cos(\omega t)\sin(\theta)(L\sin(\psi)\sin(\sigma) + L\cos(\psi)\cos(\sigma)\sin(\gamma)) - \cos(\theta)\sin(\phi)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) + L\cos(\gamma)\cos(\phi)\cos(\sigma)\cos(\sigma)\cos(\theta)) + \sin(\omega t)(\cos(\theta)(L\sin(\psi)\sin(\sigma) + L\cos(\psi)\cos(\phi)\sin(\gamma)) + \sin(\phi)\sin(\theta)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) - L\cos(\gamma)\cos(\phi)\cos(\sigma)\sin(\theta))$ 

 $\overrightarrow{L}_{ECI2} = \sin(\omega t)(\sin(\theta)(L\sin(\psi)\sin(\sigma) + L\cos(\psi)\cos(\sigma)\sin(\gamma)) - \cos(\theta)\sin(\phi)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) + L\cos(\gamma)\cos(\phi)\cos(\sigma)\cos(\sigma)\cos(\theta)) - \cos(\omega t)(\cos(\theta)(L\sin(\psi)\sin(\sigma) + L\cos(\psi)\cos(\sigma)\sin(\gamma)) + \sin(\phi)\sin(\theta)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) - L\cos(\gamma)\cos(\sigma)\sin(\gamma)) + \sin(\phi)\sin(\theta)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) - L\cos(\gamma)\cos(\phi)\cos(\sigma)\sin(\theta))$ 

$$\overrightarrow{L}_{ECI3} = \cos(\phi)(L\cos(\psi)\sin(\sigma) - L\cos(\sigma)\sin(\gamma)\sin(\psi)) + L\cos(\gamma)\cos(\sigma)\sin(\phi)$$

Drag in inertial frame

$$ec{D}_{ECI} = egin{bmatrix} ec{D}_{ECI1} \ ec{D}_{ECI2} \ ec{D}_{ECI3} \end{bmatrix}$$

 $\overrightarrow{D}_{ECI1} = \sin(\omega t)(D\cos(\gamma)\cos(\psi)\cos(\theta) + D\cos(\phi)\sin(\gamma)\sin(\theta) - D\cos(\gamma)\sin(\phi)\sin(\psi)\sin(\theta)) + \cos(\phi)\sin(\phi)\sin(\phi)\sin(\phi)\sin(\phi)\sin(\phi)$ 

$$\overrightarrow{D}_{ECI2} = \sin(\omega t)(D\cos(\gamma)\cos(\psi)\sin(\theta) - D\cos(\phi)\cos(\theta)\sin(\gamma) + D\cos(\gamma)\cos(\theta)\sin(\phi)\sin(\psi)) - \cos(\phi)\cos(\theta)\sin(\phi)\sin(\phi)\sin(\phi) + D\cos(\phi)\cos(\phi)\sin(\phi)\cos(\phi)\sin(\phi)$$

$$\vec{D}_{ECI3} = -D\sin(\gamma)\sin(\phi) - D\cos(\gamma)\cos(\phi)\sin(\psi)$$

Thrust in inertial frame

$$ec{T}_{ECI} = egin{bmatrix} ec{T}_{ECI1} \ ec{T}_{ECI2} \ ec{T}_{ECI3} \ \end{bmatrix}$$

 $T\cos(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\psi)\sin(\theta_T)) + \sin(\omega t)(\cos(\theta)(T\sin(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\gamma)\cos(\psi)\cos(\theta_T) + T\cos(\psi)\cos(\sigma_T)\sin(\gamma)\sin(\theta_T)) - \cos(\phi)\sin(\theta)(T\cos(\theta_T)\sin(\gamma) + T\cos(\gamma)\cos(\sigma_T)\sin(\theta_T)) + \sin(\phi)\sin(\theta)(T\cos(\gamma)\cos(\theta_T)\sin(\psi) + T\cos(\psi)\sin(\sigma_T)\sin(\theta_T)) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T)))$ 

 $\overrightarrow{T}_{ECI2} = \sin(\omega t)(\sin(\theta)(T\sin(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\gamma)\cos(\psi)\cos(\theta_T) + T\cos(\psi)\cos(\sigma_T)\sin(\gamma)\cos(\phi)\cos(\phi)\cos(\theta_T) + T\cos(\psi)\sin(\gamma)\cos(\sigma_T)\sin(\gamma)\cos(\phi)\cos(\theta_T)\sin(\gamma) + T\cos(\gamma)\cos(\sigma_T)\sin(\theta_T)) - \cos(\theta)\sin(\phi)(T\cos(\gamma)\cos(\theta_T)\sin(\psi) + T\cos(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T)) - \cos(\omega t)(\cos(\theta)(T\sin(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\gamma)\cos(\psi)\cos(\theta_T) + T\cos(\psi)\cos(\sigma_T)\sin(\gamma)\sin(\theta_T)) - \cos(\phi)\sin(\theta)(T\cos(\theta_T)\sin(\gamma) + T\cos(\gamma)\cos(\phi)\cos(\theta_T)\sin(\phi)) + T\cos(\gamma)\cos(\sigma_T)\sin(\theta_T)) + \sin(\phi)\sin(\theta)(T\cos(\gamma)\cos(\theta_T)\sin(\psi) + T\cos(\psi)\sin(\sigma_T)\sin(\theta_T)) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T)) + T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T)) + T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T))$ 

 $\overrightarrow{T}_{ECI3} = \sin(\phi)(T\cos(\theta_T)\sin(\gamma) + T\cos(\gamma)\cos(\sigma_T)\sin(\theta_T)) + \cos(\phi)(T\cos(\gamma)\cos(\theta_T)\sin(\psi) + T\cos(\psi)\sin(\sigma_T)\sin(\theta_T) - T\cos(\sigma_T)\sin(\gamma)\sin(\psi)\sin(\theta_T))$ 

Gravitational force in inertial frame

$$ec{G}_{ECI} = egin{bmatrix} ec{G}_{ECI1} \ ec{G}_{ECI2} \ ec{G}_{ECI3} \end{bmatrix}$$

$$\vec{G}_{ECI1} = G\cos(\phi)\sin(\omega t)\sin(\theta) - G\cos(\omega t)\cos(\phi)\cos(\theta)$$

$$\vec{G}_{ECI2} = -G\cos(\omega t)\cos(\phi)\sin(\theta) - G\cos(\phi)\cos(\theta)\sin(\omega t)$$

$$\vec{G}_{ECI3} = -G\sin(\phi)$$

$$\begin{split} m\,\overrightarrow{u}_{dot} &= \overrightarrow{L}_{ECI1} + \overrightarrow{D}_{ECI1} + \overrightarrow{T}_{ECI1} + \overrightarrow{G}_{ECI1} \\ m\,\overrightarrow{v}_{dot} &= \overrightarrow{L}_{ECI2} + \overrightarrow{D}_{ECI2} + \overrightarrow{T}_{ECI2} + \overrightarrow{G}_{ECI2} \\ \\ m\,\overrightarrow{w}_{dot} &= \overrightarrow{L}_{ECI3} + \overrightarrow{D}_{ECI3} + \overrightarrow{T}_{ECI3} + \overrightarrow{G}_{ECI3} \end{split}$$

Buoyancy force -

$$ec{B}_{ECI} = egin{bmatrix} ec{B}_{ECI1} \ ec{B}_{ECI2} \ ec{B}_{ECI3} \ \end{bmatrix}$$

$$\vec{B}_{ECI1} = B\cos(\phi)\sin(\omega t)\sin(\theta) - B\cos(\omega t)\cos(\phi)\cos(\theta)$$

$$\vec{B}_{ECI2} = -B\cos(\omega t)\cos(\phi)\sin(\theta) - B\cos(\phi)\cos(\theta)\sin(\omega t)$$

$$\vec{B}_{ECI3} = -B\sin(\phi)$$

### 2.3 Procedure

The trajectory equation is solved using the Venus atmospheric data. The forces acting on the system are gravity, buoyancy and drag. The pressure inside the balloon is assumed to be equal to the outside atmospheric pressure until the volume of balloon is equal to design volume. The radiative and convective heat transfer properties of helium, water vapour, liquid water and balloon film are used for calculating the heat transfer rates and the temperature and pressure inside the balloon which is used to find the net buoyant force acting on the balloon. Finally, RK4 integration is used along with initial balloon conditions.

At every point in balloon trajectory, it is checked if the water temperature and pressure have reached their saturation value, to initiate the phase change process. Since a continuous temperature data can not be produced, the water temperature becoming exactly equal to saturation temperature at some point may not be possible. Thus, it is checked if difference between the two temperatures are within 0.001 K. Also, the temperatures may satisfy this condition just after a phase transfer, thus a phase transfer will begin only when the water vapour releases heat or liquid gains heat. Volume of balloon at any point during phase transition is found by comparing the amount of heat transferred to/from the water with its latent heat. The convective heat transfer coefficients - Nusselt number calculated using Grashoff number, Prandlt number and Reynold's number are calculated using standard correlations for spherical configuration. The effective thermal radiative heat transfer coefficients, considering a semi transparent balloon film are calculated using the values and correlations given in reference [2]. The temperature

is calculated using amount of heat transferred, mass and specific heat capacity. The volume occupied by vapour and Helium is then calculated using this temperature in ideal gas law and that occupied by liquid is calculated by its mass and density which is assumed constant. The surface area used in heat transfer equations and cross section area used in calculating drag are calculated using this volume. When a single phase is present, it is assumed to occupy spherical volume. If both phases are present, it is assumed that each one forms a hemisphere and surface areas of those hemispheres are then calculated.

The computer code was written in C++ and saturated water data and Venus atmosphere data were written as class in header files. The output is generated in a text file and plotted using gnuplot. The code is first used to simulate a flight in earth atmosphere for the purpose of validation with JPL's ALICE data.

## **CHAPTER 3**

# **Design Optimization Problem and Procedure**

The balloon design problem can be proposed as a design optimization problem as follows

# 3.1 Objectives

- 1. minimize the total mass of the balloon
- 2. maximize the vertical traverse of balloon

### 3.2 Constraints

1. minimum altitude of balloon > 100 m (this is the minimum altitude just after dropping the balloon)

3. 
$$\sigma_{film} < \frac{31.7 MPa(\sigma_{yield,film})}{1.25}$$

- 4. traverse range:  $altitude_{max} altitude_{min} = 1m$
- 5.  $mass_{total} \ll 100kg$
- 6.  $Temperature_{film} < 127^{o}C$  (polyethylene melting point)

## 3.3 Variables and their bounds

- 1.  $D_{balloon}$   $0.5m < D_{balloon} < 100m$
- 2.  $mass_{fluid}$   $0.5kg < mass_{fluid} < 80kg$
- 3. film thickness  $t_{film} > .1 \mu m$
- 4.  $mass_{air}$   $mass_{air} > 0$

#### 3.4 Procedure

The present code is modified and a C++ class is made, which takes design variables as inputs and objective functions and constraints are given as outputs. The objective function is fed to an optimizer while satisfying the constraints. In the class,the four variables which are taken as inputs are used as follows

- The mass of phase changing fluid and lifting gas are assigned to their respective variables in the code for use in trajectory, heat transfer and phase change equations.
- The diameter of balloon denotes the maximum diameter before the envelope starts to stretch. If the outside pressure decrease after this point, the inside pressure will no longer be equal to it. The inside of the balloon will start to pressurize and the envelop will stretch until the stress becomes equal to yield stress of the envelop material.
- The film thickness is used along with the balloon diameter to calculate the mass of the film as follows -

$$Volume_{balloon} = \frac{4}{3}\pi \left(\frac{D_{balloon}}{2}\right)^{3}$$

$$Volume_{film} = \frac{4}{3}\pi \left(\left(\frac{D_{balloon}}{2} + t\right)^{3} - \left(\frac{D_{balloon}}{2}\right)^{3}\right)$$

$$\implies mass_{film} = \rho_{film} \frac{4}{3}\pi \left(\left(\frac{D_{balloon}}{2} + t\right)^{3} - \left(\frac{D_{balloon}}{2}\right)^{3}\right)$$

Next, it is checked if the design constraints are satisfied

- using the constraint of payload mass, the variables phase change fluid mass and lifting gas mass and the film mass calculated above, the total mass is calculated.
- The pressure in the stretched balloon is used to calculate the stress on the envelope
- The constraint of minimum altitude and traverse range is satisfied by comparing
  it against the altitudes at which phase change occurs, since phase change is the
  reason the direction of traverse changes.

the film temp		ylene is chec	
the illm temp	perature.		

#### **CHAPTER 4**

#### **Results**

The purpose of analysis and before using the code for other planetary systems, Code Validation is an important step. Section 4.1 in the present work describes the results from the reference[1] problem and compare them with the results from the code developed using present study. The section also provide  $\Delta t$  independence study to determine a suitable  $\Delta t$  time step for integration. Section 5.2 provides sensitivity studies, effect of variation in lifting gas mass is seen on the altitude achieved after five hours. Sensitivity study will tell about robustness of the system and margin of error allowed when filling the system with the lifting gas. The third section provides analysis studies on a helium balloon - effect on a balloon system if drag and heat transfer are switched off and if one of them if applied.

#### 4.1 Code Validation

Taking [1] and [3] as reference problem, a evaporating-condensing system is added to Helium balloon to provide repeated vertical excursions. R114 is used as phase changing fluid in a balloon along with the helium balloon. The mass distribution of this system is changed in order to compare with JPL's ALICE 0/D[1] balloon.

Component	mass(g))
Helium Gas	409
Primary balloon film	815
Freon R114	1000
Secondary balloon film	157
Total payload	624.4

Table 4.1: Mass distribution for code validation

The mass distribution of ALICE 0/D balloon system is provided in Table 4.1 Before beginning the code validation process,  $\Delta t$  independence of results are found out in following subsection-

# 4.1.1 $\Delta t$ independence study

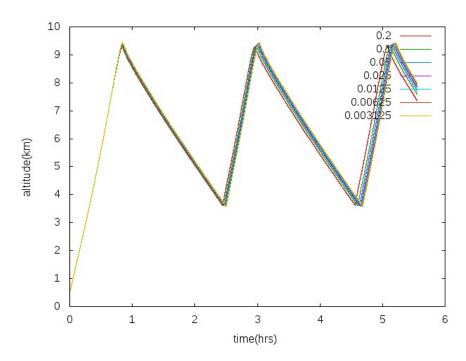


Figure 4.1: Altitude-time profile of phase change fluid balloon calculated using different  $\Delta$  t

The figure shows time-altitude profile for various  $\Delta t$  time steps. If integration time step is more than 0.2 seconds, the results vary by large amounts. Near accurate results can be obtained if  $\Delta t$  of 0.2 s is used.

$\Delta t$ (s)	Altitude	after	5	hrs
	(km)			
0.4	35.4708			
0.2	8.70643			
0.1	8.17611			
0.05	7.84855			
0.025	7.58859			
0.0125	7.41822			
0.00625	7.29113			
0.003125	7.19138			

Table 4.2: Variation in altitude due to difference in  $\Delta t$ 

The altitude after 5 hrs is tabulated against the  $\Delta t$  time step in the above table.

### 4.1.2 Comparison with JPL Data

Differences in the two plots are clearly visible - the edges are sharp, the vertical traverse altitude range is small. The reasons are numerous assumptions involved

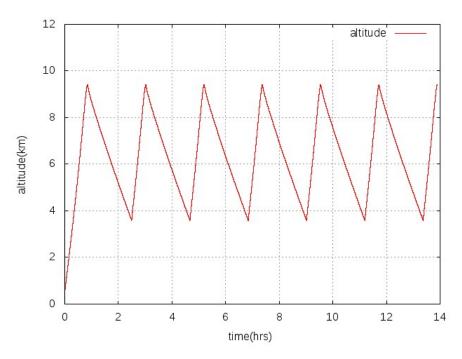


Figure 4.2: Altitude-time profile of phase change fluid balloon using present code

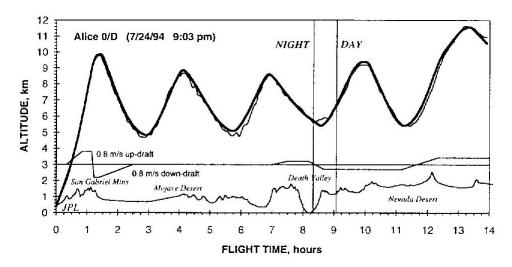


Figure 4.3: Altitude-time profile of JPL's ALICE

- The porosity factor for Freon R114 liquid is accounted in ALICE 0/D simulation.
- Leak rates of fluids are accounted. Leak rate depends on temperature, surface
  area of balloon, fluid pressure, permeation of balloon film. The overall leak rate
  was calculated as sum of partial contributions of permeation and temperature
  independent factors. The former was calculated using the actual flight data of
  ALICE 0/D balloon.
- The JPL plots have accounted for the weather conditions and cloud cover of that place that time. It changes the heat transfer rates due to convection and radiation.

- They have also incorporated the changes due to different ground contours, e.g.
  when the balloon passes over a mountain the differences in wind velocity in the
  windward and leeward side are accounted.
- Variations in albedo and wind speeds is considered.

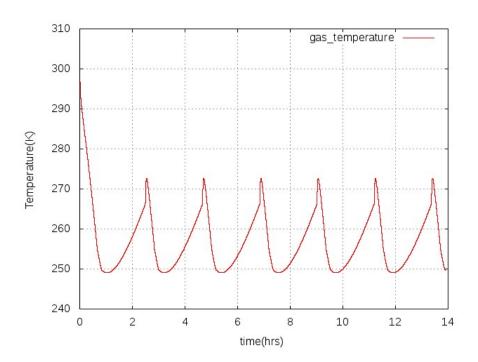


Figure 4.4: Gas temperature-time profile of phase change fluid balloon

The above figure shows the variation of lifting gas temperature with time. The lifting gas temperature never goes beyond 271 K. In the above plot, there is a difference in the slopes during ascent and descent of the balloon. The reason is using drag coefficient of 0.4 during ascent and 0.8 during descent to provide a better aerodynamic model for natural shaped balloons. Maintaining a perfect geometry during ascent and descent is not possible.

# 4.2 Sensitivity Studies

Figure 4.5 is a altitude-time plot for different masses of lifting gas. The plot show significant variations that can occur due to small variations in lifting gas mass. It implies that a small mistake during filling of the system with lifting mass can result in some significant change in the altitude. The maximum altitude and time taken to complete a cycle are both changed due to variation in lifting gas mass.

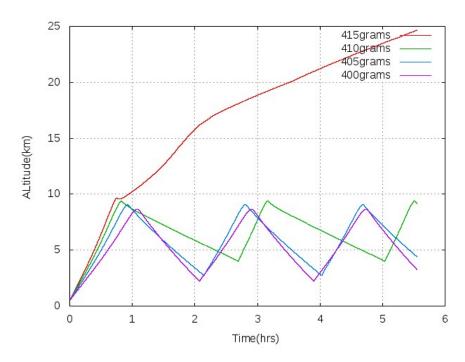


Figure 4.5: Altitude-time profile for different lifting gas mass

# 4.3 Helium Balloon Analysis

Component	Mass(kg)
Helium	0.400
Balloon film	0.800
Payload	0.400

Table 4.3: Mass distribution for first simulation

The mass of lifting gas, balloon film and payload for the Helium balloon analysis is listed in above Table.

The balloon maximum diameter is taken as 3m. The balloon volume change is assumed to be negligible beyond this point. The inside of the balloon, though, will be pressurised. We can find the stress on the film using the pressure value inside the balloon and pose it as a design criteria  $\sigma_{film} < \frac{\sigma_{yield}}{factor of safety}$ .

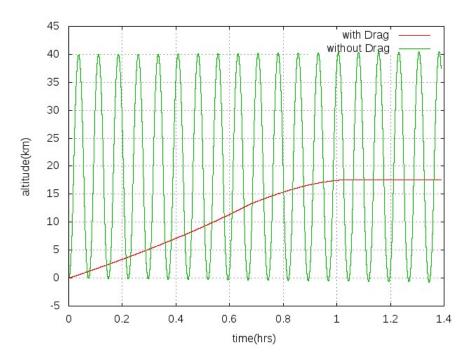


Figure 4.6: Altitude-time profile for Helium balloon with and without drag

The plot shows comparison of altitude-time profile for the balloon system, when drag is considered and when it is not considered. The case when drag is not there, the balloon clearly keep on oscillating while once the drag is considered the altitude damps out and stabilize at an altitude.

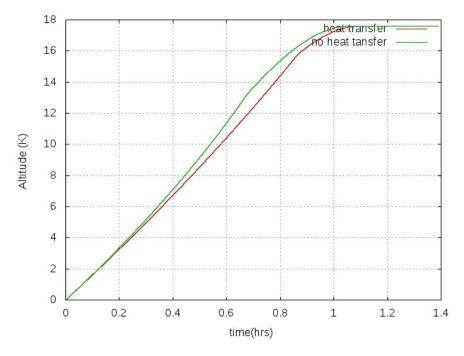


Figure 4.7: Altitude-time profile for Helium balloon with and without heat transfer

A comparison has been made in figure 4.7 between the cases when heat transfer is considered and when it is not present, to analyse the effect heat transfer has on a

helium balloon. The atmospheric pressure variation is considered in both the cases. When heat transfer takes place, the temperature of balloon reduces as shown in figure 4.8, and cause the volume of balloon to reduce. For a smaller balloon to stabilize will need higher value of atmospheric density. Thus, after heat transfer the balloon should stabilize at lower temperature. In figure 4.7 both the balloons end up stabilizing at the same altitude because of design constraint of balloon diameter, which fixes its volume.

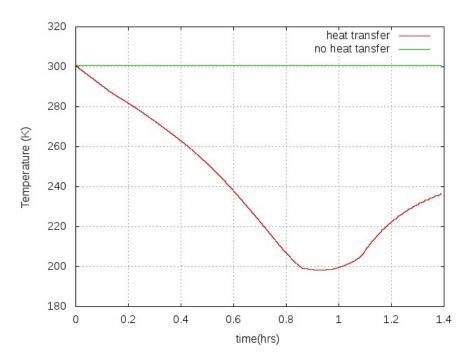


Figure 4.8: Gas Temperature-time profile for Helium balloon with and without heat transfer

The above figure shows the variation of lifting gas temperature with time. In the case heat transfer is taking place, the temperature starts to increase after 1 hr, around the time it reaches the stabilizing altitude, because the black ball[12] temperature is higher than the lifting gas temperature. The black ball temperature is used to calculate heating due to atmospheric infrared radiations.

### **CHAPTER 5**

### **Conclusion**

The thermodynamic model used with the trajectory model for one dimensional motion has been used to simulate the trajectory of a balloon with lifting gas. The stabilization altitude of such a balloon depends on the design limit of balloon diameter. The various types of balloons have been studied in theory[7] and it is established that phase changing fluid balloons are most suitable for planetary exploration. They provide vertical excursions for long period of time without any external control. A simplified trajectory code for phase changing fluid balloons is written. In addition to the phase changing fluid, the balloon system consist of a lifting gas balloon. The code is validated with JPL's ALICE[1] balloon and is found to vary significantly in some aspects. The probable reasons and assumptions in the analysis are stated. The integration time step  $\Delta t$ independence study is performed, which can help in selecting a suitable  $\Delta t$ , depending on the time upto which simulation needed to be performed. It is found out through sensitivity studies that the balloon trajectory is very sensitive to variation in lifting gas mass. The analysis of Helium balloon study revealed the significance of heat transfer in balloon performance. The design optimization problem is currently running and about to give some results which will soon be updated in the Project report. The next immediate step of the present work is to study the feasibility of a phase changing fluid balloon mission in planetary atmospheres. Venus is the first candidate for such study. A similar simulation code has been written for Venus atmosphere also and it use water as phase changing fluid along with helium as lifting gas. Such studies have been performed in the past indicating the feasibility of Venus balloon missions. The contribution through the future work of present study will be design optimization of balloon systems for Venus atmosphere. The knowledge gained in the present work will be utilized to reach that step and possibly become a part of future ISRO mission to Venus.

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