Small length DVB-S2 type LDPC Codes

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Abstract— **LDPC codes achieve a capacity very close to Shannon limit. The DVB Standard was one of the earliest standards to adopt LDPC codes for FEC. The DVB-S2 standard has specified LDPC codes of length 64800 and 16200. The codes in the DVB-S2 standard almost touch the capacity curve. But the codes are too long to be used in most of the wireless applications. In this paper we propose LDPC codes of rate – 1/2, 2/3, 3/4 and 5/6 having the same structure as DVB-S2 code but of length 480. The codes proposed in this paper are of irregular weight and they have good bit error rate performance.**

Keywords-DVB-S2; irregular; LDPC; small length

I. INTRODUCTION

LDPC codes proposed by Gallager [1] have become the favorite of the new generation coding theorists due to their superior performance as compared to other error correcting codes. It must be mentioned that though the codes have been introduced in the 60's decade, it did not find any use due to the complex hardware required for its encoding and decoding. Because of their capacity achieving performance, LDPC codes have been adopted in the physical layer of many recent communication standards such as DVB-S2, DVB-T, 10 G Base - T, 802.16e and 802.11n.

The Low- Density Parity Check Codes derive its name from the characteristic of its parity check matrix H – i.e., very few 1's in its H matrix. LDPC code can be described by its H matix or alternatively by Tanner graph [2]. The codes that are represented by a sparse parity-check matrix with a constant number of ones (weight) in each column and in each row are called regular LDPC codes. Gallager described the construction of regular LDPC codes. Later it was shown that the performance of LDPC codes can be improved by using irregular LDPC codes, i.e., codes having non uniform number of 1's per column and also non uniform number of 1's per row [3-6]. In standard literature LDPC codes conforming to a specified block length for a specified rate are described. In IEEE 802.16-2009 [7] standard, a single LDPC code for rate (R) half and that of rate 5/6 and 2 different types of LDPC codes for rates 2/3 and 3/4 are presented. All six code classes have the same general matrix structure that allows for a linear encoding scheme which simplifies the decoding process significantly. It consists of 24 columns and (1-*R*)*24 rows, with each entry describing a *m*-by-*m* sub-matrix which is either a permuted identity matrix or a zero matrix. In the DVB-S2 [8] standards high rate LDPC codes are presented. But the block length of the codes are very high i.e., length = 16,200 and 64,800. In practical applications, such a long length of data may not be present.

In this paper we present 4 new LDPC codes of block length 480 based on the construction technique of LDPC codes presented in DVB-S2 standard. In the first section of the paper we give a brief introduction to LDPC codes. In the second section, we describe the existing encoding and decoding techniques of LDPC codes. The existing methods of LDPC code construction are presented in third section. The new H matrices along with their simulated BER results are presented in the fourth section. Section five summarizes the paper.

II. LDPC CODES

LDPC codes are block codes with parity-check matrices that contain a very few non-zero entries. They differ from classical block codes in the decoding technique. Classical block codes are generally decoded with ML like decoding algorithms and so are usually short and designed algebraically [9]. LDPC codes are decoded iteratively. The LDPC code-words satisfy a set of linear parity-check constraints [1]. These constraints are typically defined by an m-by-n parity-check matrix H, whose m rows specify each of the m constraints (the number of parity check equations), and n represents the length of the codeword. The alternate way of representing the LDPC code can be by the corresponding Tanner graph. Fig. 1 shows the H matrix representation of a (2, 3) regular LDPC code and Fig. 2 shows its matching Tanner graph representation.

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Figure 1 H matrix of a (2,3) LDPC code

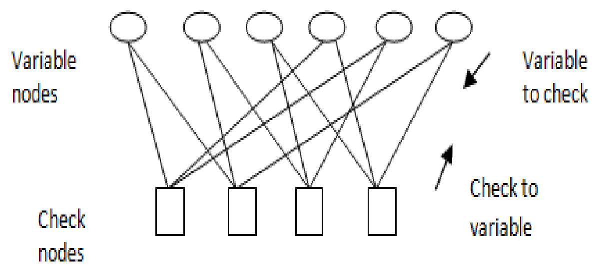

Figure 2 Tanner graph of the above H matrix

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Let us define few terms which will be used in this paper:

Cycle: In representing a H matrix by Tanner graph [10], [11], if a set of t check nodes and t variable nodes can be connected by line or edge so that a path exists that connects every node in the set and connects each node with itself in a fashion that it does not cover an edge two times, is called a length 2t cycle [3]. Short cycles (length 4 and 6) degrade the performance of LDPC codes [10].

Regular LDPC codes: A LDPC code is called (w_c,w_r)-regular if each column has fixed number w_c 1's and each row a fixed number, w_r of 1's [10].

Irregular LDPC codes: A LDPC code is irregular if the number of ones is different in each row and column. In irregular LDPC codes, the degrees of the nodes are represented by polynomial equations [10].

A. Encoding of LDPC

There are various methods for encoding LDPC codes. LDPC codes can be encoded using the same method as of other linear codes i.e., bringing the H matrix in systematic format using Gauss-Jordan elimination [12] and then obtaining the corresponding **G** matrix. But by this elimination process, the complexity of encoding will be huge i.e., O(n^2) for n length code. So to reduce the complexity of encoding, H matrix is designed to have some particular structure [10].

LDPC being linear codes, the H matrix and the codeword vector x satisfy the following condition:

$$Hx^T = 0 \quad (1)$$

This property of H matrix can be exploited to reduce the encoding complexity. The authors of [13] have employed this property and proposed an encoding technique which significantly reduces the encoding complexity. In the Richardson - Urbanke technique, in short RU technique, the H matrix is divided into 6 sub matrices **A**, **B**, **C**, **D**, **E** and **T**, where matrices **A**, **B**, **C**, and **E** are sparse, matrix **D** is dense and matrix **T** is sparse and also lower triangular. The codeword vector **x** is written as (**s**, $\mathbf{p_1}$, $\mathbf{p_2}$), **s** stands for the systematic part, $\mathbf{p_1}$ and $\mathbf{p_2}$ stand for the parity parts. The length of $\mathbf{p_1}$ is termed as the 'gap'. In WiMAX standard, the codes are so constructed that they can be encoded using the technique in [13].

B. Decoding of LDPC

The decoding algorithm of LDPC codes was discovered numerous times independently and is known by various names [10]. The most popular names are the message passing algorithm (MPA), the sum-product algorithm(SPA) and the belief propagation (BP) algorithm. In BP, in each iteration the computed probability or 'belief' values are propagated between the two set of nodes or processing units [14], [15] the Variable Node Unit (VNU) and the Check Node Unit (CNU). Figure 3, shows the exchange of the 'belief' values called check to variable node (CTV) values or messages from CNU and variable to check node (VTC) values or messages from VNU to CNU.

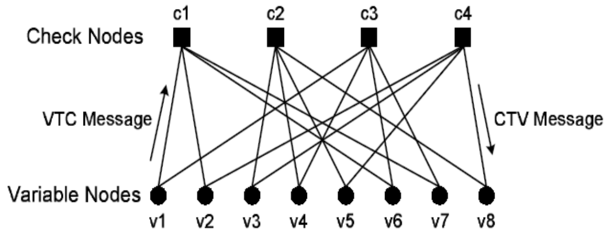

Figure 3 Decoding in Tanner graph

The complex CNU operations can be simplified using the computationally less stringent Min-Sum algorithm (MSA). Since the estimation of AWGN channel noise is not required, MSA is suitable for hardware implementation. In the normalized MSA, the CTV messages L_{cv} is computed by CNU as

$$L_{cv} = \alpha \times \prod_{v' \in N(c) \setminus v} sign(L_{v'c}) \times \min_{v' \in N(c) \setminus v} \left| L_{v'c} \right| \quad (4)$$

where, α is a normalizing factor, L_{cv} are the VTC messages, *N(c)* represents the set of VNU for the check node *c*. The VNU computes the VTC messages L_{vc} as

$$L_{vc} = \sum_{c' \in M(v) \setminus c} L_{c'v} + I_v \quad (5)$$

where, *M(v)* stand for the set of CNU's connected to the VNU's *v* except for the variable node *c*. I_v is the intrinsic message of VNU *v*. Though, MSA helps in simplifying the implementation of decoding process it comes at the cost of decoding performance [10].

III. LDPC CONSTRUCTION

The main points to be addressed during the construction of the H matrix of LDPC codes are that it should be 4 cycle free and it should also be full rank. Gallager has given construction techniques for the H matrix of column weight w_c i.e., each column having w_c number of 1's and row weight w_r i.e., each row having w_r number of 1's in his Ph. D thesis [1], [10] and the technique is as follows:

- A sub - matrix H_1 having only 1 entry of value 1 in every column and w_r 1s in every row is constructed.
- The i-th row is filled with 1s in the column numbers (i - 1) w_r +1 to w_r.
- Remaining wc – 1 sub - matrices are random permutations of H_1.
- All the w_c sub–matrices are concatenated to give the final H matrix.

Though this construction technique is easy and gives good performance, there is no guarantee that small cycles (4 and 6) are not present in the constructed H matrix [10].

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Several construction methods for LDPC codes were given by Mackay [16] and these can be modified to be length 4 cycle free, by verifying for every pair of columns of H for any overlap of 1 in two entries. [12], [17] and [18] give methods of constructing H matrix using combinatorial logic. In WiMAX standard, the final H matrix is obtained by suitably scaling up of a base matrix H_b. The base matrix H_b can be written as, $H_b = [H_{b1}\ H_{b2}]$ where H_{b1} is for encoding the information bits and H_{b2} is for the parity-check bits. Sub matrix H_{b2} can be written as,

$$H_{b2} = [h_b H'_{b2}]$$

where vector h_b has odd number of 1's, and H'_{b2} is a dual-diagonal matrix with 1 at i = j and i = j+1, and 0 at other values of i and j, where i is the row index and j the column index [10].

IV. NEW H MATRICES

In DVB-S2 Standard, codes of length 16200 and 64800 are presented and the value of M is chosen as 360. For length 64800 codes of rates: 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 8/9 and 9/10 are specified and for length 16200 codes of all the above rates except for 9/10 are specified. All the codes are irregular LDPC codes. In this paper we present four new DVB-S2 type LDPC codes of rate 1/2, 2/3, 3/4 and 5/6. For the codes presented in this paper, the block length of the code is chosen as 480. The codes are presented in tables 2, 3, 4 and 5 for rate 1/2, 2/3, 3/4 and 5/6 respectively. In DVB-S2 codes, certain structure is imposed on parity check matrices H, to facilitate the description of the codes and for easy encoding [19].

If the H matrix is converted to the generator matrix G in systematic format, then encoding can be simplified. But the resultant matrix will no longer be sparse and will lead to storage and encoding complexity problems. So, in DVB-S2 the parity check matrix is restricted to be of the form:

$$H = [A \quad B]$$

where *B* is staircase lower triangular as shown in fig 4 [19].

$$B = \begin{bmatrix} 1 & & & & & \\ 1 & 1 & & & \mathbf{0} & \\ & 1 & 1 & & & \\ & & 1 & \ddots & & \\ & \mathbf{0} & & \ddots & 1 & \\ & & & & 1 & 1 \end{bmatrix}$$

Figure 4 Submatrix of Parity Check Matrix

Then any information block $\boldsymbol{i = (i_0, i_1, \ldots, i_{k-1})}$ is encoded to a codeword $\boldsymbol{c = (i_0, i_1, \ldots, i_{k-1}, p_0, p_1, \ldots, p_{n-k-1})}$ using $\boldsymbol{Hc^T = 0}$, and recursively solving for parity bits [19].

$$\begin{aligned} &a_{00}i_0 + a_{01}i_1 + \ldots + a_{0,k-1}i_{k-1} + p_0 = 0 \Rightarrow Solve\ p_0 \\ &a_{10}i_0 + a_{11}i_1 + \ldots + a_{1,k-1}i_{k-1} + p_0 + p_1 = 0 \Rightarrow Solve\ p_1 \\ &\qquad \vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad\qquad \vdots \\ &a_{N-K-1,0}i_0 + a_{N-K-1,1}i_1 + \ldots + a_{N-K-1,k-1}i_{k-1} + p_{N-K-2} + p_{N-K-1} = 0 \Rightarrow Solve\ p_{N-K-1} \end{aligned}$$

It must be noted that the matrix A is sparse and hence encoding has linear complexity with respect to the block length.

Few restrictions are also applied on the A sub - matrix of parity check matrix such that the storage requirement of the H matrix is decreased by a factor of M. For the codes presented in this paper, the value of M is chosen as 40.

For a group of M bit nodes, if the check nodes connected to the first bit node of degree, say d_v , are numbered as c_1, c_2, …, c_{dv} then the check nodes connected to the i^{th} bit node ($i \le M$) are numbered as$\{c_1 + (i-1)q\}$ mod(N-K), $\{c_2 + (i-1)q\}$ mod(N-K), …, $\{c_{dv} + (i-1)q\}$ mod(N-K), where N-K = total number of check nodes and

$$q = \frac{N-K}{M}.$$

The q values for the 4 different types of codes presented in this paper are given in table 1.

TABLE I. Q VALUES

Code Rate	q
1/2	6
2/3	4
3/4	3
5/6	2

For the following group of M bit nodes, the check nodes connected to the first bit node is given in the second row of the tables 2, 3, 4 and 5 as the case may be. For the next group of M bit nodes entries from the third row are considered and so on and so forth. For example in constructing a rate ½ code, the check nodes 5, 75, 53 and 14, given by the entries of the 1^{st} row of table 2 are connected to the first bit node. So, the check nodes connected to the second bit node can be calculated to be 5 + 6 = 11, 75 + 6 = 81, 53 + 6 = 59 and 14 + 6 = 20. The entries of the first row of table 2 are added by 6 as the q value of the proposed rate ½ code is 6. For codes of other rates the check nodes connected to the second bit node are obtained by adding proportionate value of q to the check nodes connected to bit node 1. The check nodes connected to the bit nodes upto 40 are obtained similarly. The check nodes connected to the 41^{st} bit node are given by the entries in the 2^{nd} row of table 2 and they are 6, 230, 34 and 83. The check nodes connected to the next 39 bit nodes are obtained in a similar fashion. The check nodes connected to the 81^{st} bit node is obtained from the third row of the table 2. Proceeding in the above fashion, the entire H matrix can be computed.

The check nodes connected to the first bit node of the group are in general randomly chosen so that the resulting LDPC code is cycle-4 free and occurrence of cycle-6 is minimized to the extent a solution can be found within a reasonable search time. Too many short cycles (such as cycle-4 and cycle-6), where one can find cycles in the bipartite graph containing 4 or 6 nodes, are detrimental to code performance since they lead to

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non-extrinsic information being fed back after a small number of iterations [19].

TABLE II. RATE 1/2

5	75	53	14
6	230	34	83
1	92	177	67
2	135	29	
3	209	43	
4	56	102	

TABLE III. RATE 2/3

1	75	53	14
2	20	34	83
3	92	77	
4	135	29	
1	9	43	
2	56	102	
3	39	65	
4	118	141	

TABLE IV. RATE 3/4

1	75	53	14
2	22	67	93
3	13	82	111
1	39	29	71
2	61	43	98
3	54	7	
1	105	57	
2	97	116	
3	11	31	

TABLE V. RATE 5/6

1	75	53	14
2	20	34	10
1	12	77	
2	38	29	
1	9	43	
2	56	7	
1	3	65	
2	18	41	
1	11	24	
2	49	69	

The presented H matrices are irregular in weight. The last m X m block of the presented codes is a staircase matrix and so except for the last column; it has 2 1's in each column in the m X m block. The rest of the code matrix has a column weight of 3 or 4. As the constructed H matrices are irregular, they give better bit error rate performance, compared to regular matrices. The list of bit node degrees of a code word of a particular rate and the total number of nodes with those degrees is given in table 6.

TABLE VI. H MATRIX PROPERTIES

Rate	Bit Nodes of Degree 4	Bit Nodes of Degree 3	Bit Nodes of Degree 2	Bit Nodes of Degree 1
1/2	120	120	239	1
2/3	80	240	159	1
3/4	200	160	119	1
5/6	80	320	79	1

Simulation results show that the constructed irregular matrices give good performance. Fig. 5 gives the bit error rate simulation of the 4 LDPC code matrices.

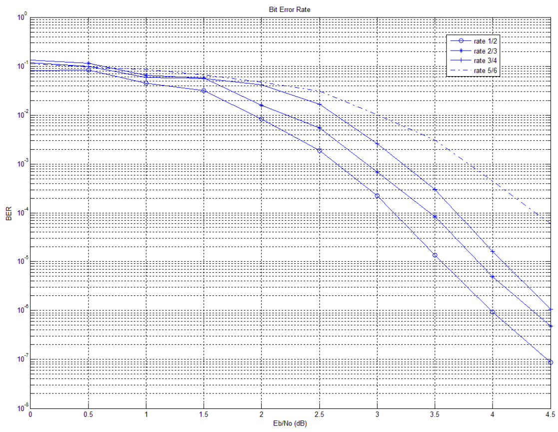

Figure 5 Bit error rate performance of the constructed codes

The matrices are decoded with min sum decoding algorithm and the maximum number of iteration was chosen to be 10. The performance can be improved by increasing the number of iterations and also by using belief propagation algorithm for decoding.

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