

Effect of magnetic field on flow behaviour of blood through a modeled Atherosclerotic artery

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Abstract— In this theoretical study a mathematical model is developed to study the effect of stenosis on flow behavior of the streaming blood through an atherosclerotic artery in the presence of magnetic field. The non-Newtonian character of blood is taken into account using Casson fluid model of the blood. The coupled differential equations governing the MHD flow of blood are solved by a combined use of analytical and numerical techniques in the appropriate boundary conditions. The geometry of the asymmetric shape of the stenosis assumed to be manifested in the arterial segment is given due consideration in the analysis. An extensive quantitative analysis is performed through large scale numerical computations of the measurable flow variables having more physiological significance by developing computer codes. Some important observations having medical interest in the flow of the blood in the stenosed arteries are obtained.

Keywords- Stenosis, Casson fluid, Magnetic field, shear stress, slip velocity.

I. INTRODUCTION

Stenosis, (borrowed from Ancient Greek $\sigma\tau\acute{\epsilon}\nu\omega\sigma\iota\varsigma$) is defined as the abnormal narrowing of body passage tube or orifice that cause serious circulatory disorders leading to cardiac failure. While the exact mechanism of the formation of stenosis in a conclusive manner is unclear from the standpoint of anatomy, physiology and pathology, the abnormal deposition of various substances like cholesterol, fat on the endothelium of the arterial wall and proliferation of connected tissues accelerate the growth of the disease. Once the constriction in the artery develops cholesterol, fat and other substances build up in the inner lining of the artery and this natural process is called *Carotid Circulatory Disorders*. It generally disturbs the normal blood flow leading to malfunctioning of the hemodynamic system and the cardiovascular system. Carotid Artery Stenosis is a major risk factor for Ischemic Stroke (most common form of stroke usually caused by blood clot plugging in artery.)

The fact that the hemodynamic factors play a commendable role in the genesis and growth of the disease has attracted many researchers to explore modern approach and sophisticated mathematical models for investigation on flow

through stenosed arteries. To illuminate the effects of stenosis present in the arterial lumen intensive experimental and theoretical researches have been carried out worldwide for both normal and stenotic arteries. In most of the investigations relevant to the domain under discussion, the Newtonian model of blood (single phase homogeneous viscous fluid) was accepted. This model of blood is acceptable for high shear rate in case of flow through narrow arteries of diameter ≤ 1000 μm . Experimental observations reveal that blood being predominantly a suspension of erythrocytes in plasma exhibits remarkable non-Newtonian behaviour when it flows through narrow arteries, at low shear rate particularly, in diseased state when clotting effects in small arteries are present. A more comprehensive study on the diagnosis, prevention and treatment of stenosis related diseases suggest that an accurate description of blood flow requires consideration of erythrocytes (red cells) as discrete particles in small arteries. In view of their observations it is preferable to represent the flow of blood in narrow arteries by a non-Newtonian fluid. H-B fluid model and Casson fluid models are generally used in the theoretical investigation of blood through narrow arteries. Due to the presence of the substances like proteins, fibrinogens and globulin in an aqueous base plasma human red blood cells can form a chain like structure, known as aggregates or rouleaux. As the rouleaux behaves like a plastic solid then there exists a yield stress that can be identified with the constant yield stress in Casson's fluid. So, many investigations have reported that blood may be better described by Casson fluid model at low shear rates as blood possesses a finite non-zero yield stress.

Also blood may be considered as a suspension of magnetic particles (red cells) in non-magnetic plasma due to the presence of hemoglobin (an iron compound) in red cells. Many researchers have investigated the effect of magnetic field on blood flow treating blood as an electrically conductive fluid. The conductive flow in the presence of a magnetic field induces voltage and current leading to a decrease in flow. The importance of heat transfer on artery diseases and blood flow was mentioned in several investigations. Ugulu and Abby [5] established that the heat transfer and magnetic field have a

considerable effect on blood flow through narrow constricted artery. Also it has been found that with the help of magnets, the flow of blood in the arteries may be regulated. In the proposed theoretical investigation a mathematical model is developed to study the effect of stenosis on flow behaviour of the streaming blood through an atherosclerotic artery in the presence of magnetic field using Casson fluid model. It gives us an opportunity to present quantitative analysis of the measurable flow variables based on numerical computations by taking different accepted values of the material constants and rheological parameters. Their graphical representations are presented at the end of the paper with appropriate scientific discussions. Finally comparisons are made with the other existing results to justify the applicability of the present model.

II. MODEL DESCRIPTION

Let us consider the axially-symmetric flow of blood through an artery of circular cross section with a mild stenosis specified at the position shown in Fig.1. The blood is modeled as Casson fluid. It is assumed that the flow is fully developed ($v_r=v_z=0$ in z -direction only). The velocity does not change in the direction of the flow except the entry and the exit regions. Since the red blood cell is a major bio-magnetic substance, blood flow may be influenced by magnetic field. Considering transverse magnetic field, the action of magnetization will introduce a rotational motion to orient the magnetic fluid particle with the magnetic field.

The geometry of the stenosis is assumed to be manifested in the arterial segment given by (Kapoor, et al [8])

$$R(z) = 1 - \frac{\delta}{R_0} \exp\left[-\frac{m^2 z^2 \epsilon^2}{R_0^2}\right] \quad (1)$$

Where δ is the maximum height of the Stenosis, m determines slope of Stenosis ($m \geq 2$), ϵ denotes the relative length of the constriction ($\epsilon = \frac{R_0}{L_0}$), R is the radius of obstructed tube. L_0 is the length of the stenosis. The schematic diagram is shown in Figure 1.

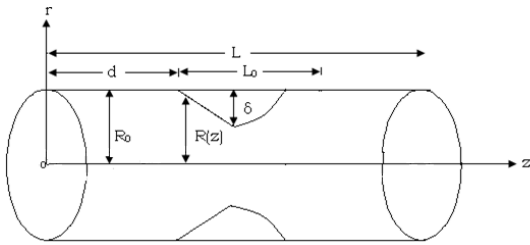


Fig1.Geometry of the stenosed artery

The governing equations are given below, where H is the magnetic intensity, P is the pressure, τ_c is the shear stress, Q is the volumetric flow, u is the velocity of blood.

$$-\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial(r\tau_c)}{\partial r} + F_1 \frac{\partial H}{\partial z} = 0 \quad (2)$$

$$\tau_c^{1/2} = \tau_0^{1/2} + F_2 \left(-\frac{\partial u}{\partial r}\right)^{1/2}, \text{ if } \tau_c \geq \tau_0 \quad (3)$$

$$\frac{\partial u}{\partial r} = 0 \text{ if } \tau_c \leq \tau_0 \quad (4)$$

$$\text{Where } F_1 = \frac{\mu_w M H_0}{\rho u_0^2} \text{ and } F_2 = \sqrt{\left(\frac{\mu}{R_0 \rho u_0}\right)}$$

The boundary conditions are :

$$u = u_s \text{ at } r = R_z$$

$$\tau_c \text{ is finite at } r = 0.$$

The velocity u , is given by solving the following equations:

$$-\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial(r\tau_c)}{\partial r} + F_1 \frac{\partial H}{\partial z} = 0$$

$$\Rightarrow \frac{1}{r} \frac{\partial(r\tau_c)}{\partial r} = \frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}$$

$$\text{Let, } \frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z} = c$$

$$\frac{1}{r} \frac{\partial(r\tau_c)}{\partial r} = c$$

$$\Rightarrow \frac{\partial(r\tau_c)}{\partial r} = cr$$

$$\Rightarrow \int_0^r \partial(r\tau_c) = \int_0^r cr$$

$$\Rightarrow [r\tau_c]_0^r = \left[\frac{cr^2}{2}\right]_0^r$$

$$\Rightarrow r\tau_c = \frac{cr^2}{2}$$

$$\Rightarrow \tau_c = \frac{cr}{2}$$

$$\Rightarrow \tau_0^{1/2} + F_2 \left(-\frac{\partial u}{\partial r}\right)^{1/2} = \left(\frac{cr}{2}\right)^{1/2}$$

$$\Rightarrow F_2 \left(-\frac{\partial u}{\partial r}\right)^{1/2} = \left(\frac{cr}{2}\right)^{1/2} - \tau_0^{1/2}$$

$$\Rightarrow F_2^2 \left(-\frac{\partial u}{\partial r}\right) = \frac{cr}{2} + \tau_0 - 2 \frac{(cr\tau_0)^{1/2}}{\sqrt{2}}$$

$$\Rightarrow \int \frac{\partial u}{\partial r} = \int \left[\frac{1}{F_2^2} \left(-\frac{cr}{2} - \tau_0\right) + \sqrt{2} \frac{(cr\tau_0)^{1/2}}{F_2^2} \right]$$

$$\Rightarrow u = -\frac{cr^2}{4F_2^2} - \frac{\tau_0 r}{F_2^2} + 2\sqrt{2}r^{3/2} \frac{(c\tau_0)^{1/2}}{3F_2^2} + k \quad (5)$$

At, $u = u_s$ at $r = R_z$

$$u_s = -\frac{cR^2(z)}{4F_2^2} - \frac{\tau_0 R(z)}{F_2^2} + 2\sqrt{2}R^{\frac{3}{2}}(z) \frac{(c\tau_0)^{1/2}}{3F_2^2} + k \quad (6)$$

$$\therefore u = -\left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}\right) \frac{r^2}{4F_2^2} - \frac{\tau_0 r}{F_2^2} + 2\sqrt{2}r^{3/2} \frac{(\tau_0)^{1/2}}{3F_2^2} \left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}\right)^{1/2} + u_s + \left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}\right) \frac{R^2(z)}{4F_2^2} + \frac{\tau_0 R(z)}{F_2^2} - 2\sqrt{2}R^{\frac{3}{2}}(z) \frac{(\tau_0)^{1/2}}{3F_2^2} \left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}\right)^{1/2} \quad (7)$$

The volumetric flow rate Q, is given by:

$$\begin{aligned} Q &= \int_0^{R(z)} ru \cdot dr \\ &= \left[-\frac{cr^4}{16F_2^2} - \frac{\tau_0 r^3}{3F_2^2} + 4\sqrt{2}r^{7/2} \frac{(\tau_0)^{1/2}}{7F_2^2} c^{1/2} + u_s r + c \frac{R^2(z)r^2}{8F_2^2} + \frac{\tau_0 R(z)r^2}{2F_2^2} - 2\sqrt{2}R^{\frac{3}{2}}(z) \frac{(\tau_0)^{1/2}}{6F_2^2} r^2 c^{1/2} \right]_0^{R(z)} \\ &= -\frac{cR^4(z)}{16F_2^2} - \frac{\tau_0 R^3(z)}{3F_2^2} + 4\sqrt{2}R^{\frac{7}{2}}(z) \frac{(\tau_0)^{1/2}}{7F_2^2} c^{1/2} + u_s R(z) + c \frac{R^4(z)}{8F_2^2} + \frac{\tau_0 R^3(z)}{2F_2^2} - 2\sqrt{2}R^{\frac{7}{2}}(z) \frac{(\tau_0)^{1/2}}{6F_2^2} c^{1/2} \quad (8) \end{aligned}$$

$$\therefore Q = -\left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}\right) \frac{R^4(z)}{8F_2^2} - \frac{\tau_0 R^3(z)}{6F_2^2} + \sqrt{2}R^{\frac{7}{2}}(z) \frac{(\tau_0)^{\frac{1}{2}}}{7F_2^2} \left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z}\right)^{1/2} + u_s R(z) \quad (9)$$

The shear stress τ_c , is given by, $\tau_w = \mu \left[\frac{du}{dr} \right]$ (10)

$$\begin{aligned} \tau_c &= \mu \left[\frac{\sqrt{2\tau_0 cr}}{F_2^2} - c \frac{r}{2F_2^2} - \frac{\tau_0}{F_2^2} \right]_{R(z)} \\ &= \frac{\mu}{F_2^2} \left[\sqrt{2\tau_0 cR(z)} - c \frac{R(z)}{2} - \tau_0 \right] \quad (11) \end{aligned}$$

$$\tau_c = \frac{\mu}{F_2^2} \left[\sqrt{2\tau_0 \left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z} \right) R(z)} - \left(\frac{\partial P}{\partial z} - F_1 \frac{\partial H}{\partial z} \right) \frac{R(z)}{2} - \tau_0 \right] \quad (12)$$

III. NUMERICAL RESULTS & DISCUSSION

A specific numerical illustration is presented in the analysis considering some particular values of the different physical and rheological parameters. The purpose of this numerical

computation is to examine the validity of the model under consideration. To discuss the results of the study quantitatively, the values of the different constants and rheological parameters from standard literatures are taken.

It is to be noted that the parameter value $u_s = 0$ corresponds to the case of no-slip in the artery.

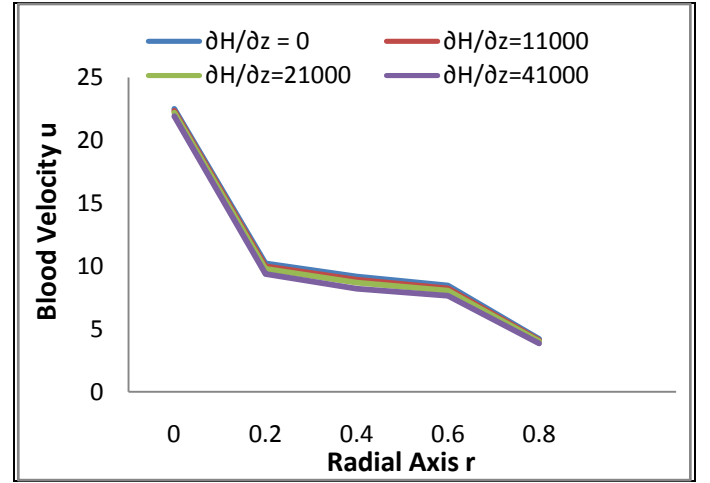


Fig.2 Variation of Blood velocity (u) vs Radial axis (r) for different values of $\partial H/\partial z$ with $u_s=0$

In Fig.2 the velocity, u shows a steep drop when the radius of stenosis increases from $r = 0$ to $r = 0.2$ and then there is a parabolic decrease to 0 at $r = 0.9$ which is $R(z)$ i.e. radius of the artery with varying magnetic field gradient dH/dz . Very little change in magnitude of u is observed.

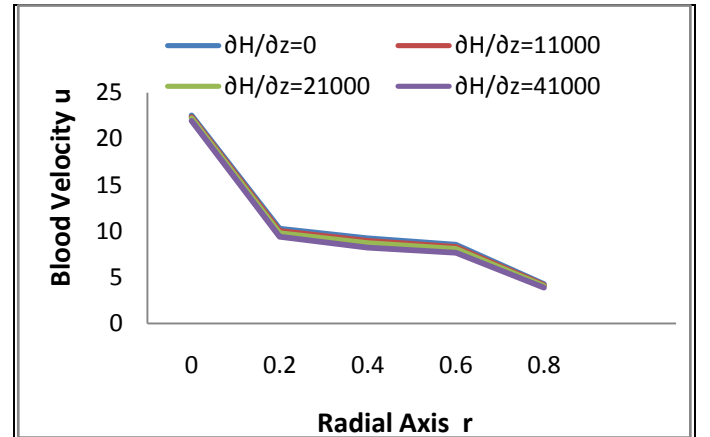


Fig.3 Variation of Blood velocity (u) vs Radial axis (r) for different values of $\partial H/\partial z$ with $u_s=0.05$

In Fig.3 the above case is considered with slip velocity and there is slight increase in the magnitude of u.

Fig.4 illustrates the variation of blood flow velocity with radial axis r with varying the pressure gradient $\frac{\partial p}{\partial z}$. The

velocity, u decreases when the radius of stenosis increases from $r=0$ to $r=0.9$ and drops to 0 at $r=0.9$ which is $R(z)$ i.e. radius of the artery. Fig.5 depicts the case with slip velocity.

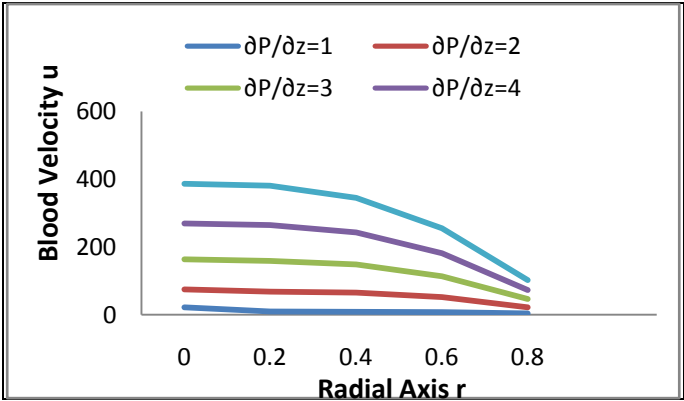


Fig.4 Variation of Blood velocity (u) vs radial axis (r) for different values of $\partial P/\partial z$ with $u_s=0$

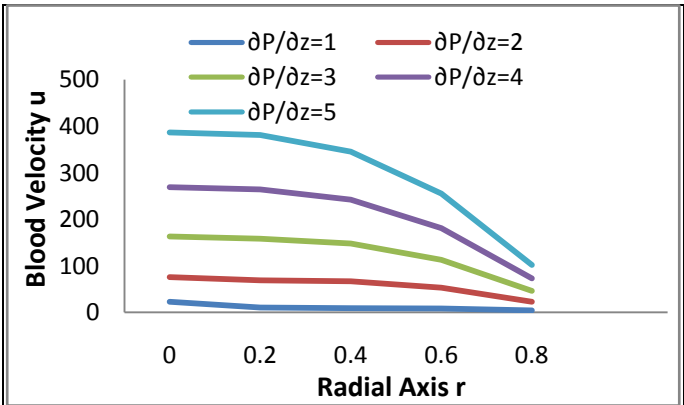


Fig.5 Variation of Blood velocity (u) vs radial axis (r) for different values of $\partial P/\partial z$ with $u_s=0.05$

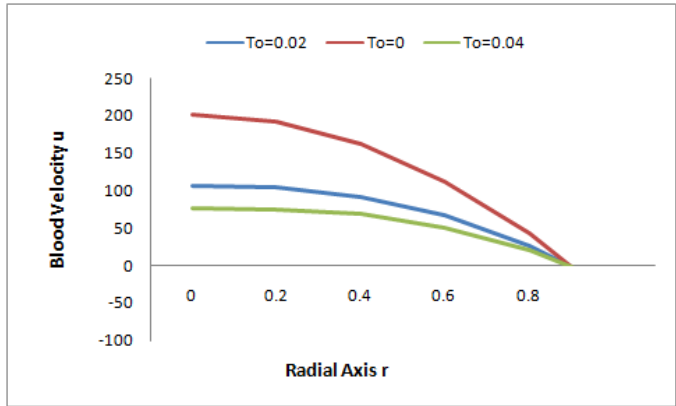


Fig.6 Variation of Blood velocity (u) vs radial axis (r) for different values of τ_0 (T_0) with $u_s=0$
Fig.6 exhibits the result of variation of flow velocity with radial axis r with different values of shear stress τ_0 . Significant change in magnitude of u is observed. It decreases with τ_0 and the same is depicted in Fig.7 by considering the slip velocity

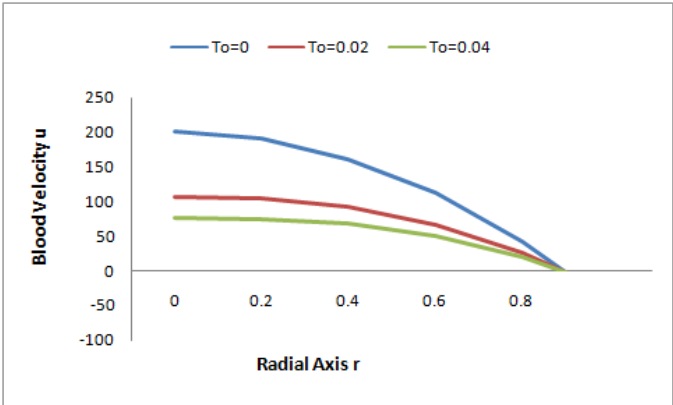


Fig.7 Variation of Blood velocity (u) vs radial axis (r) for different values of τ_0 (T_0) with $u_s=0.05$
It is observed that in Fig.8 the velocity, u experiences a steep drop when the radius of stenosis increases from $r=0$ to $r=0.2$ and then there is a parabolic decrease when the magnetic permeability parameter μ_0 is varied. A very small change in magnitude of u is observed but there is greater change from $r=0.4$ to $r=0.8$. We find that the magnitude of u increases with magnetic permeability μ_0

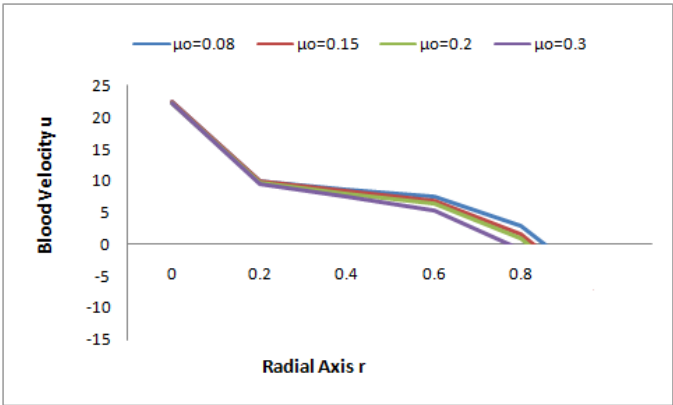


Fig.8 Variation of Blood velocity (u) vs Radial axis (r) for different values of μ_s with $u_s=0$
In Fig.9 the case with slip velocity we observe that there is an increase in the magnitude of the flow velocity.

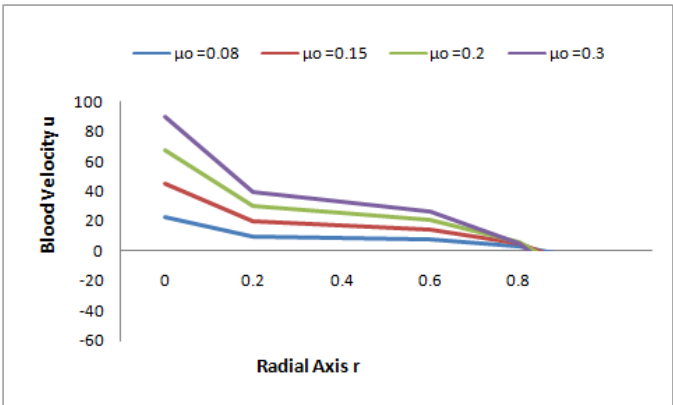


Fig.9 Variation of Blood velocity (u) vs Radial axis (r) for different values of μ_s with $u_s=0.05$

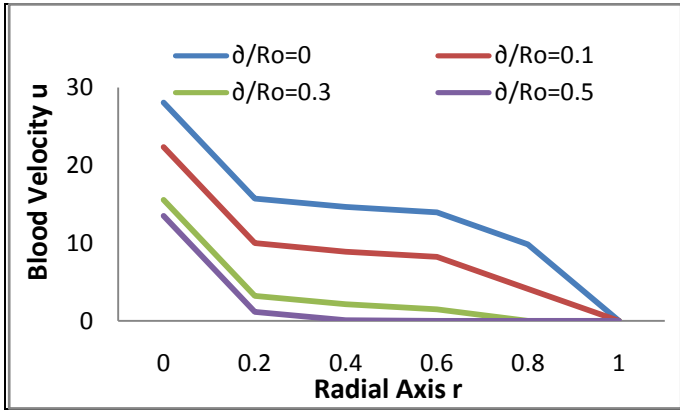


Fig.10 Variation of Blood velocity (u) vs Radial axis (r) for different values of ∂/R_o with $u_s=0$
From Fig.10 it is established that the flow velocity drops when the radius of the stenosis increases from $r = 0$ to $r = 0.2$ and then a parabolic decreasing trend is obtained. Also there is a significant change in the flow velocity with varying the stenosis height $\frac{\partial}{R_o}$. Fig.11 gives the case with slip velocity.

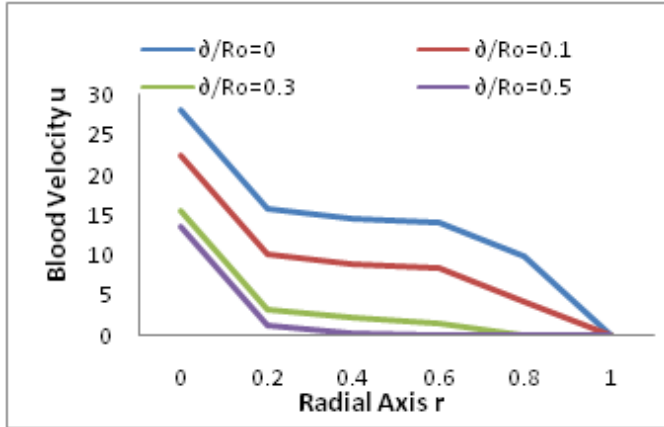


Fig.11 Variation of Blood velocity (u) vs Radial axis (r) for different values of ∂/R_o with $u_s=0.05$.

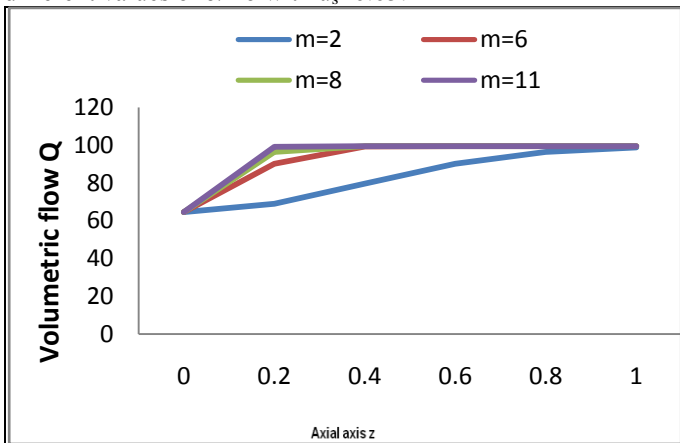


Fig.12 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of m with $u_s=0$
Fig.12 shows the change of volumetric flow rate with axial distance z for different values of stenosis shape parameter m . It is found that the volumetric flow rate, Q increases when the

axial distance z increases from $z=0$ to $z=1$. Q decreases with increase in m . Fig.13 illustrates the case with slip velocity.

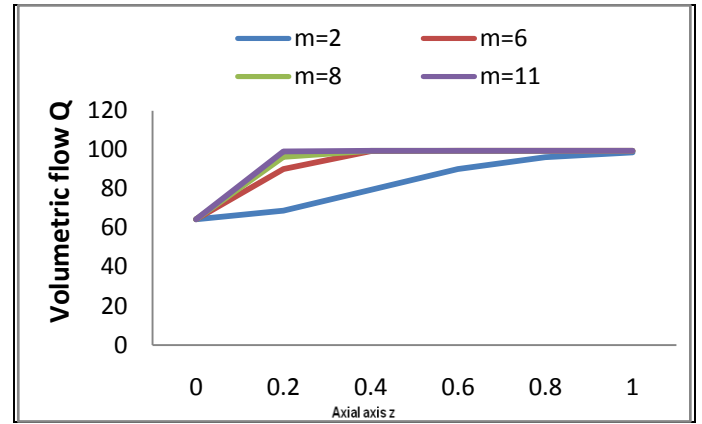


Fig.13 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of m with $u_s=0.05$.

Fig.14 shows the variation of flow rate with axial distance z for different values of $\frac{\partial H}{\partial z}$ and we find that Q increases with axial distance z but decreases with increase of $\frac{\partial H}{\partial z}$. In Fig. 15 by considering slip velocity only a slight increase in the magnitude of Q is observed keeping the trend same.

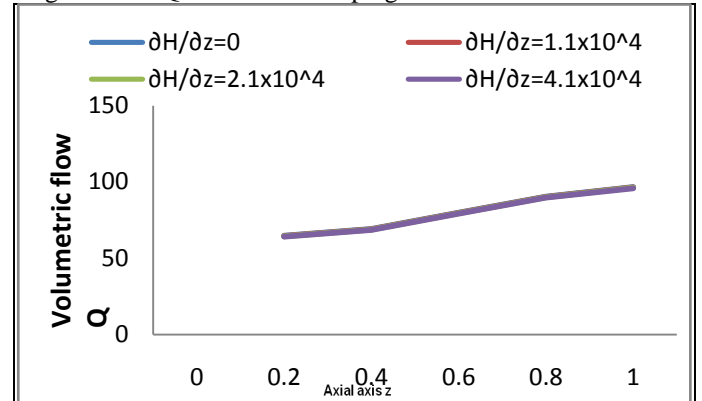


Fig.14 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of $\partial H/\partial z$ with $u_s=0$.

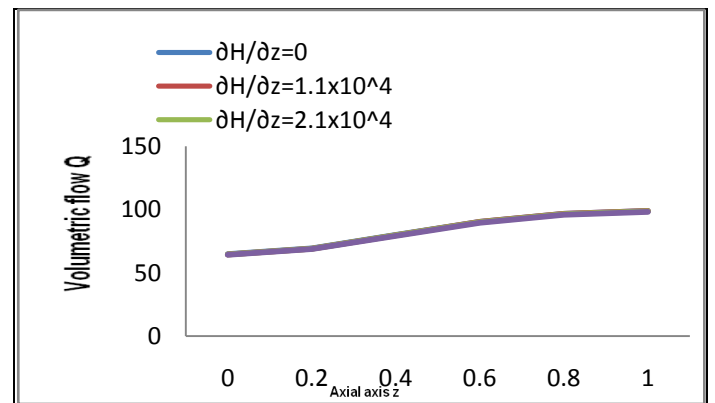


Fig.15 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of $\partial H/\partial z$ with $u_s=0.05$

Variation of flow rate Q with axial distance z for different values of stenosis height $\frac{\partial}{R_0}$ are given due consideration in Fig.16. It is observed that the flow rate increases from $z=0$ to $z=1$ and then remains constant. The consideration of slip velocity in Fig.17 enhances the magnitude of Q by a very small amount.

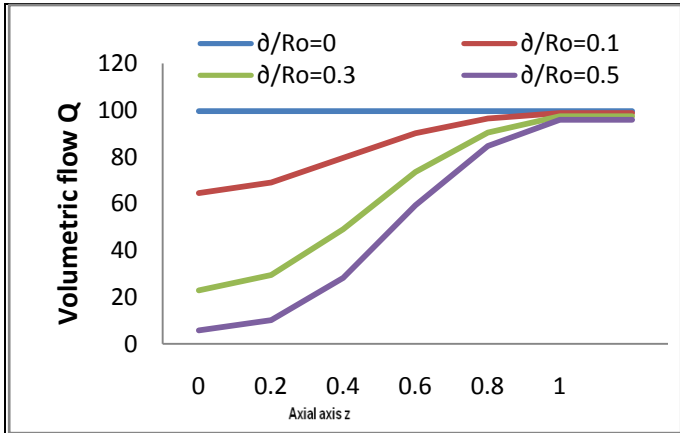


Fig.16 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of ∂/R_0 with $u_s=0$

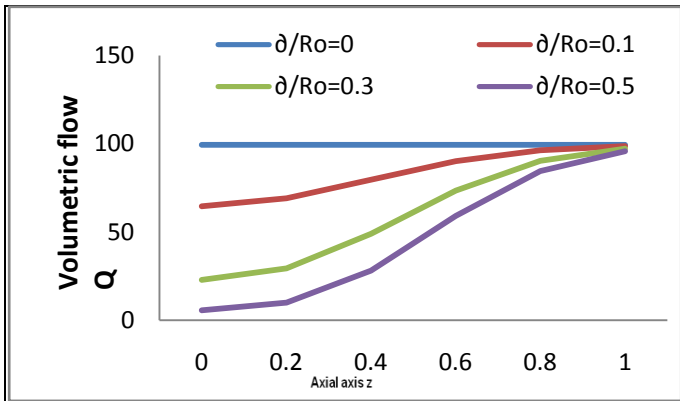


Fig.17 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of ∂/R_0 with $u_s=0.05$

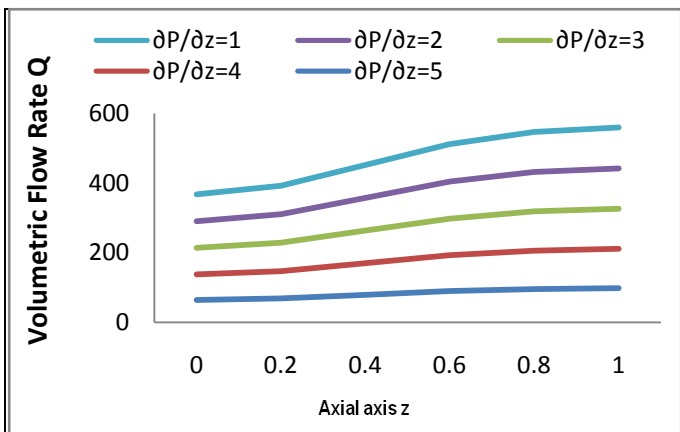


Fig.18 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of $\partial P/\partial z$ with $u_s=0$

Fig.18 exhibits the variation of Q with axial distance z with varying pressure gradient $\frac{\partial P}{\partial z}$. It is easy to see that flow rate shows an increasing trend but as pressure gradient increases its magnitude decreases.

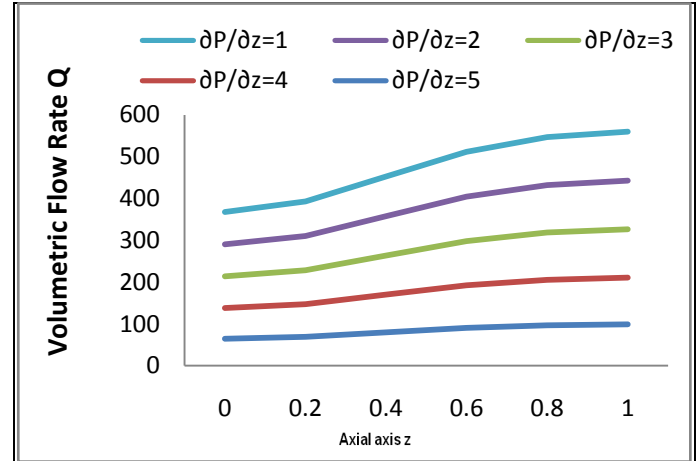


Fig.19 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of $\partial P/\partial z$ with $u_s=0.05$

In Fig.19 the above case is considered by simply considering slip velocity at the wall of the stenosis.

Fig.20, 21, 22 & 23 illustrates the variation of volumetric flow rate Q with axial distance z for different values of shear stress (τ_0) and magnetic permeability (μ_s) both with and without slip condition. We find that in all these cases the flow rate shows an increasing trend with increase in axial distance z .

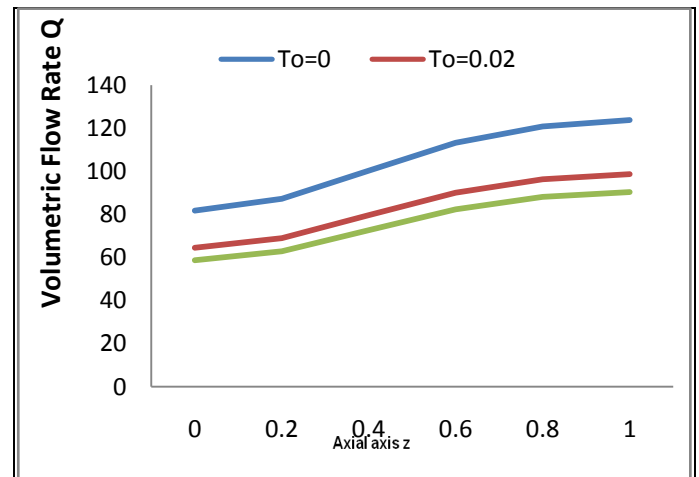


Fig.20 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of τ_0 (T_0) with $u_s=0$

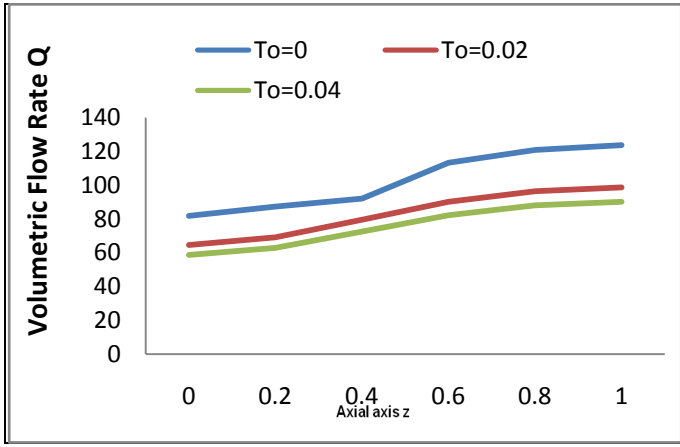


Fig.21 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of τ_0 (T_0) with $u_s=0.05$

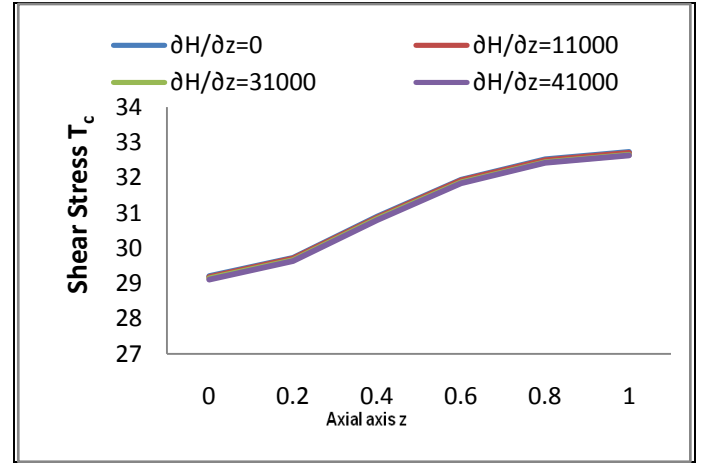


Fig.24 Variation of Shear stress (τ_c) vs Axial axis (z) for different values of $\partial H/\partial z$

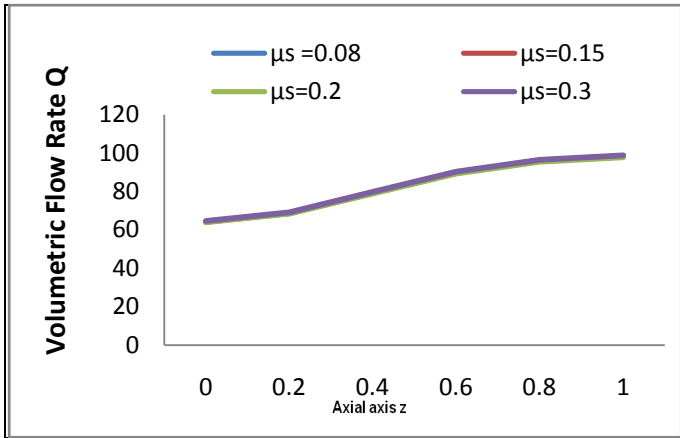


Fig.22 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of μ_s with $u_s=0$

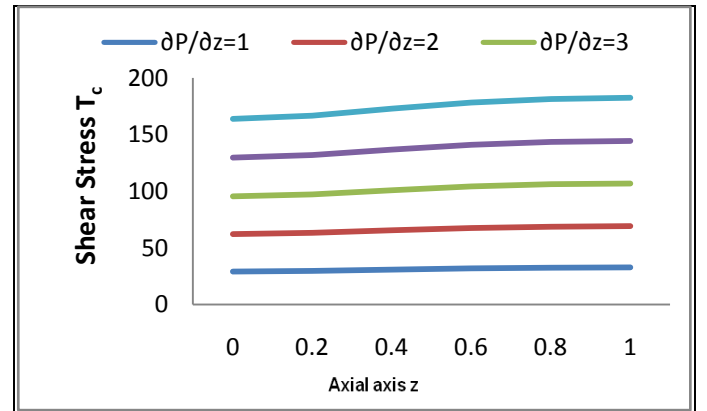


Fig.25 Variation of Shear stress (τ_c) vs Axial axis (z) for different values of $\partial H/\partial z$

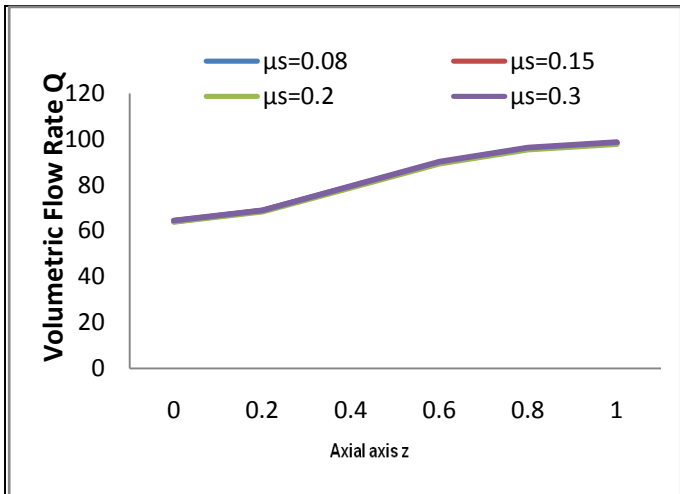


Fig.23 Variation of Volumetric flow (Q) vs Axial axis (z) for different values of μ_s with $u_s=0.05$

Fig.24 shows the change of shear stress (τ_c) with the axial distance z for different values of the magnetic field gradient $\frac{\partial H}{\partial z}$. It is observed that shear stress increases with z but decreases with decrease in $\frac{\partial H}{\partial z}$.

It is observed in Fig.26 that shear stress increases with axial distance z for different values of viscosity μ . We also find that as viscosity increases the shear stress at the wall of stenosis decreases.

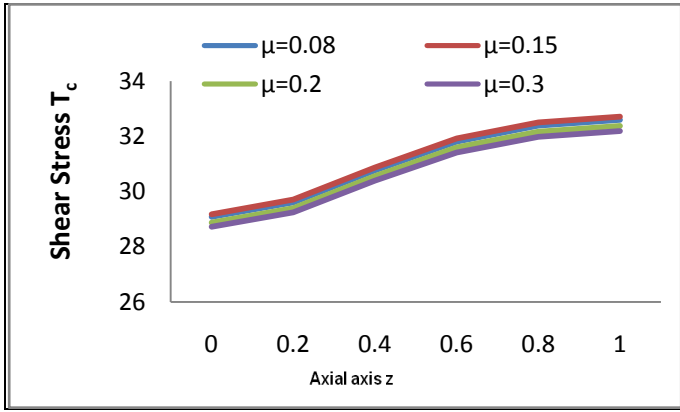


Fig.26 Variation of Shear stress (τ_c) vs Axial axis (z) for different values of μ .

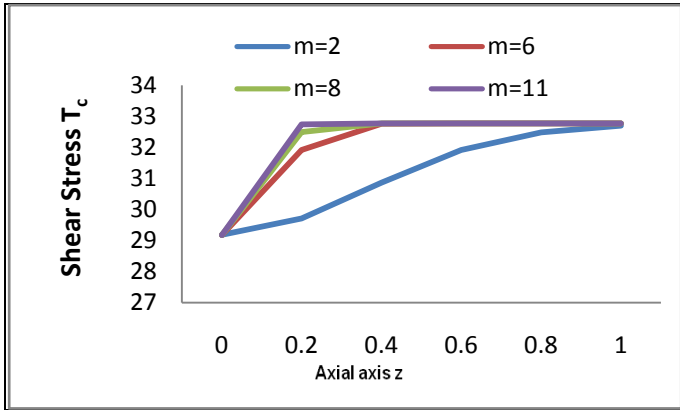


Fig.27 Variation of Shear stress (τ_c) vs Axial axis (z) for different values of m .

Fig.27 and 28 considers the variation of shear stress with axial distance z with varying stenosis shape parameter (m) and stenosis height $\frac{\partial}{R_0}$ respectively. It is interesting to note that with the increase of m there is a significant increase of shear stress upto a certain level and then it remains constant. We also find that as we increase the stenosis height $\frac{\partial}{R_0}$ shear stress decreases for a fixed z but from $z = 0.6$ it becomes almost constant.

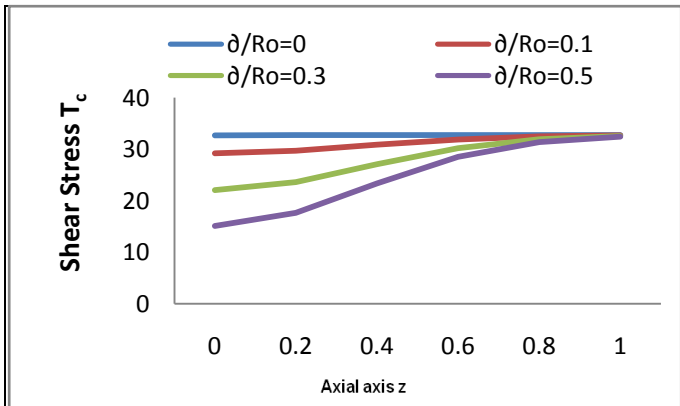


Fig.28 Variation of Shear stress (τ_c) vs Axial axis (z) for different values of ∂/R_0 .

IV. CONCLUSION

This theoretical analysis provides a scope in bringing out many interesting results on rheological properties of blood flow through narrow stenosed arteries considering Casson fluid model of blood under the influence of magnetic intensity. It is interesting to note that the rheological and length parameters influence the flow characteristics quantitatively. The effect of inclusion of slip velocity is given due consideration in the analysis which was neglected by the previous researchers. Also the slope of the stenosis and magnetic intensity are seen to have a dominating role in ascertaining the flow characteristics of blood through stenosed arteries. The investigation will provide a scope for ascertaining the role of various parameters in different conditions of arteriosclerosis. As blood is regarded as a suspension of magnetic particles (red cells) in non-magnetic plasma so the inclusion of magnetic field in the flow characteristics of blood will help the physicians and open an era in biomathematics. It is to conclude that the MHD principles may be used to decelerate the flow of blood in a human arterial system and the results may help the cardiologists in the treatment of certain cardiovascular disorders. It may also help the general physicians in the treatment of diseases that accelerate blood circulation like hypertension, hemorrhages.

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