Calculate norms, distances and scalar products
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EXERCISE 1

Question 1 Calculate the Euclidean, Manhattan and Maximum norm of the following vectors:

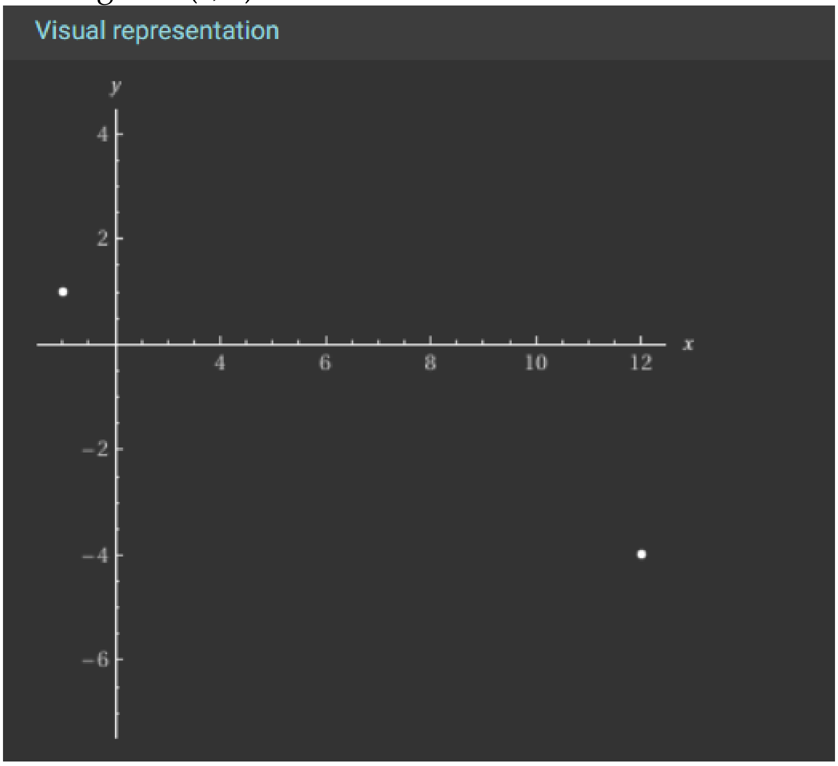
	Euclidean	Manhattan	Maximum
a = (1, 1)	$\sqrt{1^2+1^2}=\sqrt{2}$	1 + 1 = 2	Max(1+1) = 2
b = (1, 2, 3, 4, 5)	$\sqrt{1^2+2^2+3^2+4^2+5^2}=\sqrt{55}$	1+2+3+4+5 = 15	Max(1,2,3,4,5)=5
c = (0, 0, 0, 0)	$\sqrt{0^2+0^2+0^2+0^2}=0$	0+0+0+0 = 0	0
d= (0)	0	0	0
d1 = (2)	$\sqrt{2^2}=2$	2	2
e = (12, -4)	$\sqrt{12^2+(-4)^2}=\sqrt{160}$	12+4=16	12
f = (-2, 2, 1000, 1)	$\sqrt{(-2)^2+2^2+1000^2+1^2} = \sqrt{1000009}$	2+2+1000+1=1005	1000
g = (1, -1, 100 ,1.1, -1000)	$\sqrt{1+1+100^2+1.1^2+1000^2} = 1010003.21$	1+1+100+1.1+1000 =1103.1	1000
h = (1, 1, 100, 1.1, 0)	$\sqrt{1+1+100^2+1.1^2+0} = 10003.21$	1+1+100+1.1+0= 103.1	100

Question 2 Calculate the Euclidean, Manhattan and Maximum distances between the vectors:

	Euclidean	Manhattan	Maximum
a and e	$\sqrt{(12-1)^2+(-4-1)^2}=\sqrt{146}$	$\|12-1\|+\|4-1\|=16$	Max(|12-1|,|-4-1|)=11
c and f	Since c=(0,0,0,0) -> f-c=$\sqrt{1000009}$	1005	1000
g and h	$\sqrt{0+2^2+0+0+1000^2} = \sqrt{1000004}$	0+2+0+0+1000=1002	1000
d and d1	$\sqrt{0+2^2}=2$	2	2

Question 3 Plot points of Question 1, whenever possible.
Plotting 1-D: (d, d1)

Plotting 2-D: (a, e)



No 3-D vector

Question 4 What is the angle between

a) (1,1,1) and (-1,-1,-1) in degrees?
b) (1, 1) and (12, -4)?
c) (2,3) and (4,5)?
d) (3,4) and (4,3) in degrees by using math.acos in Python
e) (1,2,2) and (2,2,1) in degrees by using math.acos in Python

We use the formula

$$a^T \cdot b = a1 \times b1 + a2 \times b2 + \cdots = ||a|| \times ||b|| \times \cos(\theta) \Rightarrow \cos(\theta) = \frac{a^T \cdot b}{||a|| \times ||b||}$$

In python:

```
import numpy as np
from math import acos, degrees

# Define the vectors
v1 = np.array(<vector_coords>)
v2 = np.array(<vector_coords>)
# Compute the dot product and magnitudes
dot_product = np.dot(v1, v2)
magnitude_v1 = np.linalg.norm(v1)
magnitude_v2 = np.linalg.norm(v2)
# Calculate the cosine of the angle
cos_theta = dot_product / (magnitude_v1 * magnitude_v2)
# Compute the angle in degrees
angle_degrees = degrees(acos(max(min(cos_theta,1),-1)))
print(f'{angle_degrees:.2f} degrees')
```

a) 180°
b) 63.43°
c) 4.97°
d) 16.26°
e) 27.27°

Question 5

a) Let u and v be two vectors in the n-space. If $u^T v = 0$, what is your conclusion?

U and V are orthogonial to eachother

b) Let u=(1,-1,-1) and v=(0,0,2). What is the result of $u + \frac{1}{2}(v - u)$

$$(1, -1, -1) + \frac{1}{2}(0 - 1, 0 + 1, 2 + 1) = (1, -1, -1) + \left(-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right) =$$

$$(1/2, -1/2, 1/2)$$

c) What is the scalar product of (-1,-1,1) and (1,2,1)? Are these separated by more than 90^0, less than 90^0, or exactly 90^0?

$$(-1, -1, 1)^T \cdot (1,2,1) = -1 \times 1 - 1 \times 2 + 1 \times 1 = -1 - 2 + 1 = -2$$

Since the scalar product is < 0 the $cos(\theta) < 0 \Rightarrow \boldsymbol{\theta > 90°}$