ΕΘΝΙΚΟ ΚΑΙ ΚΑΠΟΔΙΣΤΡΙΑΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ΤΜΗΜΑ ΟΙΚΟΝΟΜΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΠΡΟΓΡΑΜΜΑ ΜΕΤΑΠΤΥΧΙΑΚΩΝ ΣΠΟΥΔΩΝ «ΕΦΑΡΜΟΣΜΕΝΗΣ ΟΙΚΟΝΟΜΙΚΗΣ ΚΑΙ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗΣ»

ΚΑΤΕΥΘΥΝΣΗ
«ΔΙΟΙΚΗΣΗ, ΑΝΑΛΥΤΙΚΗ ΚΑΙ ΠΛΗΡΟΦΟΡΙΑΚΑ ΣΥΣΤΗΜΑΤΑ ΕΠΙΧΕΙΡΗΣΕΩΝ»

Master of Science in
Business Administration, Analytics and Information Systems

Data Analysis (Business Statistics) - Optimization

Assignment 3 :

Probabilities Exercises

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Exercise 1

Table 1 Time that accident happened

		8 am - 4 pm	4 pm - 12 mid.	12 mid. - 8 am
Alcohol influence	Not at all	41	28	16
	High	10	20	33
	Very high	8	29	45

A1. Calculate the probabilities that a car accident:

(i) *happened between 8 am and 4 pm (A) :*

$$P(A) = \frac{41+10+8}{41+10+8+28+20+29+16+33+45} = \frac{59}{230} \approx 0.26 = 26\%$$

(ii) *happened subject to high alcohol influence (B) :*

$$P(B) = \frac{10+20+33}{230} = \frac{63}{230} \approx 0.27 = 27\%$$

(iii) *happened subject to very high alcohol influence (C) given that it happened between 4 pm - 12 mid. (D):*

$$P(C|D) = \frac{P(C\cap D)}{P(D)} = \frac{\frac{29}{230}}{\frac{28+20+29}{230}} = \frac{29}{77} \approx 0.38 = 38\%$$

(iv) *happened between 12 mid. - 8 am (E) given that the driver did not operate subject to alcohol influence (F) :*

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{16}{230}}{\frac{41+28+16}{230}} = \frac{16}{84} \approx 0.19 = 19\%$$

(iv) *is due to high alcohol influence (G) or happened between 12 mid. - 8 am (H):*

$$P(G\cup H) = P(G) + P(H) - P(G\cap H) = \frac{10+20+33+16+45}{230} = \frac{124}{230} \approx 0.54 = 54\%$$

A2. Are the very high alcohol influence (A) and the chance of a car accident between 12 mid. - 8 am (B) independent events?

In order for A and B to be independent events the following must be true:

$$\mathrm{P(A\ \cap B) = P(A)P(B)}$$

So lets calculate:

$$\mathrm{P(A)} = \frac{8+29+45}{230} = \frac{82}{230} \approx 0.36 = 36\%$$

$$\mathrm{P(B)} = \frac{16+33+45}{230} = \frac{94}{230} \approx 0.41 = 41\%$$

$$\mathrm{P(A\cap B)} = \frac{45}{230} = \frac{82}{230} \approx 0.20 = 20\%$$

$$P(A)P(B) = 36\% \times 41\% = 14.76\%$$

So $P(A)P(B) \neq P(A \cap B)$ => ***A and B are not independent***

B. TOYS Company produces toys. After a quality control on produced toys, the company has found that the probability of a defective product is 3%. The quality control department installed a diagnostic machine in the production line that

- **identifies defective toys with probability 98%**
- **decides wrongly that a toy is defective with probability 0.1%**

According to the statement we define the events:
A: toy is defective
A': toy is not defective (compliment of A),
B: diagnostic machine decides defectiveness
B': diagnostic machine decides non defectiveness.

The data now translates to :

- P(A) = 3% (P(A') = 97%)
- P(B|A) = 98%
- P(B|A') = 0.1%

(i) If a toy is selected randomly, what is the probability that the diagnostic machine decides defectiveness?

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = 0.98 \times 0.03 + 0.001 \times 0.97 \approx 0.03 = 3\%$$

(ii) What is the probability that the toy is defective while the diagnostic machine decides that it is not?

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(B'|A)P(A)}{P(B')} = \frac{\big(1\text{-}P(B|A)\big)P(A)}{1 - P(B)} = \frac{0.02 \times 0.03}{0.97} \approx 0.0006 \ = \ 0.06\%$$

EXERCISE 2

1. What is the probability of being born on Wednesday; 1/7
2. What is the probability that the Athens Stock Exchange will be open for business on Christmas day this year? 0 , experimental probability
3. What is the probability that the price of gasoline will be higher next year than this year? How did you arrive at your answer? $p \in [0,1]$ subjective probability
4. What is the probability of throwing exactly 7 with two dice? 6/36 (from dice rolling table)
5. What is the probability that the difference between the numbers showing when two dice are rolled is 2? 8/36 (from dice rolling table)

EXERCISE 3

With respect to the Testing for Covid-19,

1. **Calculate P(T) when P(T|B) = 0.99 and P(T|B') = 0.05**

!Note we assume that P(B) = 0.1% = 0.001

$$P(T) = P(T|B)P(B) + P(T|B')P(B') = 0.99 \times 0.001 + 0.05 \times 0.999 \approx 0.05 = 5\%$$

2. **Calculate P(B|T) and P(B|T') if P(T|B) = 0.99 and P(T|B') = 0.05**

$$P(B|T) = \frac{P(B \cap T)}{P(T)} = \frac{P(T|B)P(B)}{P(T)} = \frac{0.99 \times 0.001}{0.05} \approx 0.0198 = 1.98\%$$

$$P(B|T') = \frac{P(B \cap T')}{P(T')} = \frac{P(T'|B)P(B)}{P(T')} = \frac{0.01 \times 0.001}{0.95} \approx 0.00001 = 0.001\%$$