Reconstruction of the Seabottom Reflection Coefficient

Evgeny O. Kovalenko, Igor V. Prokhorov, Andrey A. Sushchenko
Institute for Applied Mathematics, FEB RAS, Vladivostok, Russia,
Far Eastern Federal University, Vladivostok, Russia;
e-mail: kovalenko.eo@dvfu.ru, sushchenko.aa@dvfu.ru

The kinetic model, describing sound propagation in the ocean with diffuse reflection by Lambert's cosine law on the bottom surface, is considered. Based on it the inverse problem of bottom scattering reconstruction is formulated.
Inverse problem is reduced to finding of solution of integral equation of the first kind. Iteration algorithm for finding of solution inverse problem is proposed and numerical experiments by recovering of coefficient of bottom scattering at different width of directivity pattern are carried out.

1 Introduction

There is mainstream problem in the World ocean research which relates to conservation and rational use of its resources. Nowadays, ocean can barely afford to many times increased the anthropogenic influence. Its variations is required a permanent monitoring. One of the most effective method of registration for local changes is survey of the seabottom and the water column using by underwater unmanned vehicle which equipped by the side-scan sonar. The principle of sonar is based on the emitting and the detection of the reflected echo signal. There are several approaches for the description of this process. The wave models, taking into account an amplitude and a phase of the propagated signal, is most common use. However, if wavelength is comparable to the sampling scale then the beam signal propagation theory is also valid [Quanjo, Turner]. It is based on the kinetic model of the transfer acoustic radiation in the randomly inhomogeneous media [SMJ]. The main advantage of this model is taking into account the middle scattering in the media, which is constitute about 10% of the total attenuation in the demersal layer, according to the practical research I.B. Andreeva [Andreeva]. The influence of the volume scattering on the propagating signal is widely studied in the previous works. For instance, in paper [Acoustical Physics] the problem of the determination the seabottom scattering coefficient based on the received signal by sonar is considered. That problem was solved using by following approximations. There are the single scattering in the media, narrow directivity pattern of the receiving antenna and a pointwise source. Authors deduced an explicit formula for the determination of the seabottom scattering coefficient which is taking into account the additive correction to the volume scattering in the total signal. Further evolution of this problem is continued in paper [SPIE - Kov], in which authors used a source, close to the real experiments. It emits parcels during finite time interval. Thus, authors reduced the reconstruction problem to the Fredholm integral equation of the first kind, in the left side of which is a total measured signal by SSS, on the other side is unknown function, describing inhomogeneous of the seabottom surface. There are so many papers about solving the Fredholm integral equation of the first kind. The most ones were based on the researches by A.N. Tikhonov and A.B. Bakushinskii [Tikhonov (спроси у Всилича), Bakushinskii]. Further, authors considered papers which is the most suitable for the specifics of the problem studying. It is worth to note authors research the problem with a receiver equipped by a wide directivity pattern. In [PRUAC] authors solved one using by the narrow directivity pattern approximation which leads to the object defocusing of the sebottom reconstruction. However, the solution of the directivity diagram has a small aperture. Highly successful convolution algorithms for the reconstruction of the Earth surface using by a high-resolution satellite images are developed in the works of Shcherbinina N. V. [Щербинина]Method is based on using the frequency for the improving image sharpness. This approach could be applicable for the time spectrum also. In paper [Goncharskii] the focusing problem of laser radiation is solved in the approximation of the geometric optic. Authors were obtained the condition that if the incident radiation have a central symmetry and small domain of irradiation then problem is solvable. In paper [Yagola] authors interpreted the blur and defocusing as additive space-invariant distortion. The problem is solved using by the Fourier transform decomposition combined with the regularization. In this paper authors develop a generalized algorithm of the image focusing for the reconstruction of seabed scattering coefficient based on the received echosignal by SSS equipped by the widely directivity pattern. Thus, a solution of the Fredholm integral equation of the first kind is reduced to the solving of SLE by an iterative method combined with the regularization.

2 Formulation of the problem

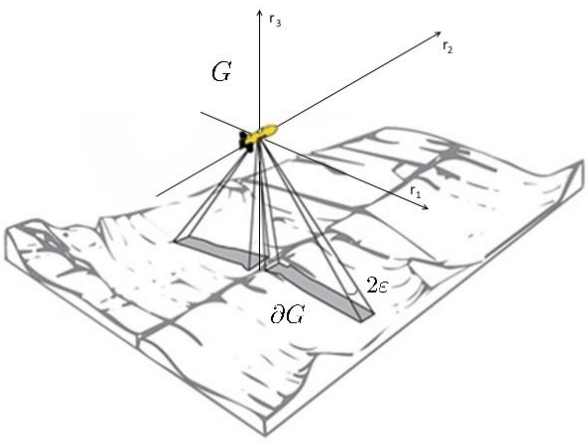

Рис. 1: АНПА с ГБО на борту

Propagation of acoustic waves on the tens order kHz frequencies in the fluctuating ocean is described by radiation transfer equation[?],[?]:

A long formula:

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{k}\cdot\nabla I(\mathbf{r},\mathbf{k},t) + \mu I(\mathbf{r},\mathbf{k},t) = J(\mathbf{r},\mathbf{k},t), \quad (1)$$

here $\mathbf{r}\in\mathbb{R}^3, t\in[0,T]$ and wave vector $\mathbf{k}$ belongs to the unit sphere $\Omega := \{\mathbf{k}\in\mathbb{R}^3 : |\mathbf{k}| = 1\}$. The function $I(\mathbf{r},\mathbf{k},t)$ denotes the wave energy of flux density at time t at the point $\mathbf{r}$ which propagates in the direction $\mathbf{k}$ with velocity c. μ and σ denote the attenuation and the scattering coefficients, respectively, and the function J describes the sources of wave field.

Sonar signal is propagated in the medium $G := \{\mathbf{r}\in\mathbb{R}^3 : r_3 > -l\}$, which is top half-space bounded by the surface $\partial G = \gamma := \{\mathbf{y}\in\mathbb{R}^3 : y_3 = -l\}$, interpreted as the bottom of the ocean. и пусть $\mathbf{n}(\mathbf{y})$ — внешняя нормаль к границе области G.

Further, authors consider the case of an isotropic pointwise source which moves with constant velocity V along r_2 axis and emits a pulse parcels in times t_i, $\overline{1,m}$ with intensity J_i, respectively:

$$J(\mathbf{r},\mathbf{k},t) = \delta(\mathbf{r}-\mathbf{V}t)\sum_{i=0}^{N} J_i\delta(t-t_i). \quad (2)$$

Here, δ denotes Dirac delta function, $\chi_{[a,b]}$ is characteristic function of interval $[a,b]$.

Assume, sources in medium are missing before initial time

$$I|_{t=0} = 0. \quad (3)$$

Let, $\Omega^{\pm} := \{\mathbf{k}\in\Omega : \pm k_3 < 0\}$ then the reflective properties of ∂G are determined by diffuse reflection by the Lambert's cosine law:

$$I(\mathbf{y},\mathbf{k}) = \frac{\sigma_d}{\pi}\int\limits_{\Omega^+} |\mathbf{n}(\mathbf{y})\cdot\mathbf{k}'| I(\mathbf{y},\mathbf{k}')d\mathbf{k}', \quad (4)$$

where $\mathbf{y}\in\partial G, \mathbf{k}\in\Omega^-$.

Let, $\Gamma := \{(\mathbf{V}t,\mathbf{k},t) : \mathbf{k}\in\Omega, t\in(0,T)\}$ then measuring of the receiving signal on Γ is a sum of the signal, caused by the reflection from the bottom surface, and a signal scattered by the inhomogeneities of the medium G:

$$I^{\pm}(t) = \int_{\Omega} S^{\pm}(\mathbf{k}) I|_{\Gamma}(\mathbf{V}t,\mathbf{k},t)d\mathbf{k}. \quad (5)$$

Here, $S^+(\mathbf{k})$ and $S^-(\mathbf{k})$ denote the directivity pattern of the receiving antenna on the starboard and portside, respectively. Further, authors consider the case of a narrow directivity pattern of the receiving antenna, focused on orthogonal plane to vehicle path:

$$S^{\pm}(\mathbf{k}) = \chi_{[0,\mp 1]}(k_1)\exp(-k_2^2/\varepsilon^2), \quad (6)$$

where, ε - angle of the width of directivity pattern.

3 Determination of the bottom scattering coefficient

The authors consider the case when the receiving antenna detects signals from one sensing interval only. The decision was presented as (1) - (5) as:

$$I_i^{\pm}(t) = \frac{1}{\pi}\int\limits_{-1}^{1} \sigma_d(y_1,y_2)S^{\pm}(\mathbf{k})\times$$
$$\times\frac{cl^2 J_i\exp(-\mu c(t-t_i))dk_2}{|\mathbf{V}t_i-\mathbf{y}|^2|\mathbf{y}-\mathbf{V}t|y_1(t-t_i)|k_2\mathbf{V}-c|}, \quad (7)$$

here, $|\mathbf{V}t - \mathbf{y}| = |\frac{c(t-t_i)}{2}(1+\frac{V^2}{c^2})|$, $|\mathbf{x} - \mathbf{y}| = |\frac{c(t-t_i)}{2}(1-\frac{V^2}{c^2})|$, $y_1 = \sqrt{|\mathbf{V}t - \mathbf{y}|^2 - l^2}$. The equation 7 describes receiving signal from SSS in the time t from starboard and port side in the i-th sensing interval. Integral in right part corresponds to set of points on the bottom from which the signal was reflected. Directivity pattern $S(\mathbf{k})$ determines reflection area. $|Vt-y|$ - slant range for the receiver, $|Vt_i-y|$ - slant range for the source. y_1 - downrange.

Уравнение (7) описывает принимаемый сигнал гидролокатором бокового обзора в момент времени t слева и справа по борту в i-ый интервал зондирования. Интеграл в правой части соответствует множеству точек на дне, от которых отразился сигнал. Диаграмма направленности S(k) определяет область отражения. $|Vt-y|$ - slant range for the receiver, $|Vt_i-y|$ - slant range for the source. y_1 - downrange.

4 The Inverse Problem

The decision of inverse problem represents big interest from the application point of view that is finding the coefficient of bottom scattering based on receiving signal. Thus, the problem reduced to solving fredholm integral equation of the first kind (7) with respect to the function σ_d. It is well known that numerical solution this kind of equation is unsustainably and strongly depends on kind of core, and in our case by kind of directivity pattern $S(\mathbf{k})$. Next, the authors consider two approaches to solving this equation.

Большой интерес с прикладной точки зрения представляет решение обратной задачи, заключающейся в определении коэффициента донного расеяния на основе принятого сигнала. Таким образом, задача сводится to solving fredholm integral equation of the first kind (7) относительно функции σ_d. Хорошо известно, что численное решение данного типа уравнения неустойчиво и сильно зависит от вида ядра, а в нашем случае от вида диаграммы направленности S(k). Далее расмотрим 2 подхода к решению данного уравнения.

4.1 Narrow DP Approximation

In previous paper **[WHAT]** the authors obtained a solution of equation (7) in the approximation of narrow directivity pattern of the receiving antenna. In this case the solution of the equation reduces to the explicit formula for finding of bottom reflection σ_d. Thus, each moment at the bottom corresponds to a single time of receiving of the signal.

В предыдущей работе **[]** авторы получили решение уравнения (7) в приближении узкой диаграммы направленности приемной антенны. В таком случае решение уравнения сводится к явной формуле для определения донного рассеяния σ_d. Таким образом, каждой точке на дне ставится в соответствие единственный момент времени приема сигнала.

$$S^{\pm}(\mathbf{k}) = \chi_{0,\mp 1}(k_1)\delta(k_2) \tag{8}$$

Thus, the solution of (7) is

$$\sigma_d\,(y_1, y_2) = \frac{2\pi}{J_i c l^2} l_i^4 y_1 \exp(2\mu l_i) I^{\pm}(t). \tag{9}$$

Here, a slant range $l_i = c(t-t_i)/2$, and $y_1 = \sqrt{l_i^2 - l^2}, y_2 = Vt_i$. The decision (7) in the form of (9) is valid when angle of directivity pattern is small enough ($\varepsilon < 0.1$). However, this assumption strongly narrows the applicability of formula (9) and leads to defocusing of objects on the bottom, i.e. the diameter of the objects increases several times, and this effect is enhanced with increasing the sounding range.

Решение (7) в виде (9) справедливо при достаточно малых растворах диаграммы направленности ($\varepsilon < 0.1$). Однако данное предположение сильно сужает применимость формулы (9) и приводит к расфокусировке объектов на дне, т.е. диаметр объектов увеличивается в несколько раз, причем этот эффект усиливается с увеличением дальности зондирования.

4.2 Discrete Method

From a mathematical point of view, the equation (7) is integral equation of I kind relatively of function σ_d. The authors introduce the sampling method for it solving. In other words authors define the areas in which σ_d is constant. Inasmuch as σ_d is defined on the set Γ authors introduce a decomposition by axis y_1, y_2.

С математической точки зрения уравнение (7) относится к интегральному уравнению I рода относительно функции σ_d. Для его решения введём метод дискретизации. Тоесть определим области, в которых σ_d константа. Так как σ_d задана на множестве Γ, введём разбиение по осям

y_1, y_2:

$$\{(y_1^p, y_2^q) \in \gamma : y_1^p = ph, y_2^q = qh\},$$
$$p \in \overline{1, H}, q \in \overline{1, M}, \qquad (10)$$

where, h is the grid spacing. In this way, the area limited of points (y_1^p, y_2^q), (y_1^{p+1}, y_2^q), (y_1^p, y_2^{q+1}), (y_1^{p+1}, y_2^{q+1}) σ_d is constant and the equation (7) reduces to:

где h - шаг сетки. Таким образом, в области, ограниченной точками (y_1^p, y_2^q), (y_1^{p+1}, y_2^q), (y_1^p, y_2^{q+1}), (y_1^{p+1}, y_2^{q+1}) σ_d считается постоянной, и уравнение (7) сводится к:

$$I(t_{ij}) = \sum_{p=1}^{N}\sum_{q=1}^{M} a_{ijpq}\sigma_d^{pq}, \qquad (11)$$

where,

$$t_{ij} = t_i + j\tau, \tau = \frac{\Delta_t}{M}, a_{ijpq} =$$
$$= \int_{-1}^{1} \frac{cl^2 J_i \exp(-\mu c(t - t_i))dk_2}{|\mathbf{V}t_i - \mathbf{y}|^2 |\mathbf{y} - \mathbf{V}t|y_1(t - t_i)| k_2 \mathbf{V} - c|},$$
$$\sigma_d^{pq} = \sigma_d(\mathbf{y}^{pq}), \mathbf{y}^{pq} = (y_1^p, y_2^q).$$

The equation (11) is S-diagonal SLE. Parameter s depends on width directivity pattern receiving and transmitting antennas and duration of emission. For solving SLE (11) authors use Seidel's method.

Уравнение (11) представляет собой s-диагональную СЛАУ. Параметр s зависит от ширины диаграммы направленности приёмной и передающей антенн и длительности испускания сигнала. For solving SLE (11) authors use Seidel's method.

5 Numerical experiments

In the experiments, authors consider a cases of the focusing at different width of directivity pattern of receiving antenna. The authors took a decision got by formula (9) as a zero iteration in Seidel's method pictured on figures (3) - (8) over character a). Experiments were conducted with directivity pattern is equal to 1, 2, 4, 8, 14, 40 degrees. For the evaluation of **SOMETHING**, it was introduced **SOMETHING**. Parameters applicable for construction of picture of sea bottom based on real data got from SSS are in the table 5. For solving of task of numerical integration authors applied Monte-Carlo's method.

В эксперименте рассматривались случаи фокусировки при разной ширине диаграммы направленности приемной антенны. В качестве нулевой итерации в методе Зейделя было взято решение, полученное по формуле (9), изображенное на рисунках (3 - 8) над буквой а). В данном эксперименте рассмотрены случаи при диаграмме направленности равной 1, 2, 4, 8, 14, 40 градусов. Для оценки ЧЕГОТОТАМ было введено ЧТОТОТАМ(отклонение). Параметры, применяемые для построения изображения морского дна на основе реальных данных, получаемых с гидролокатора бокового обзора, представлены в таблице 1. Для решения задачи численного интегрирования применялся метод Монте-Карло.

Таблица 1: Значения параметров для эксперимента

μ,м$^{-1}$	Δt,с	c,м/с	J	l,м	y_1,м	y_2,м
0.018	0.4	1500	1	12	[0,40]	[0, 40]

Поверхность дна описывается функцией

$$\sigma_d(y_1, y_2) = \begin{cases} 0.3, & ((y_1 - (12-b)a) - (y_2 - 6a))^2 + \\ & +\frac{3}{10}((y_2 - (12-b)a) + (y_1 - 6a))^2 < 9a^2, \\ 0.25, & \sqrt{(y_1 - (8-b)a)^2 + (y_2 - 9a)^2} < \frac{a^2}{4}, \\ 0.2, & \sqrt{(y_1 - (17-b)a)^2 + (y_2 - 3a)^2} < \frac{a^2}{4}, \\ 0.1, & \text{иначе.} \end{cases} \qquad (12)$$

и изображена на рисунке 2.

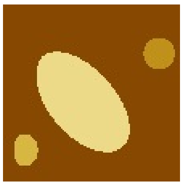

Рис. 2: Точное решение

Таблица 2: Нормы для 1°×

	$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$		$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$
a	0.0426	0.0749	e	0.0426	0.0749
b	0.0426	0.0749	f	0.0426	0.0749
c	0.0426	0.0749	g	0.0426	0.0749
d	0.0426	0.0749	h	0.0426	0.0749

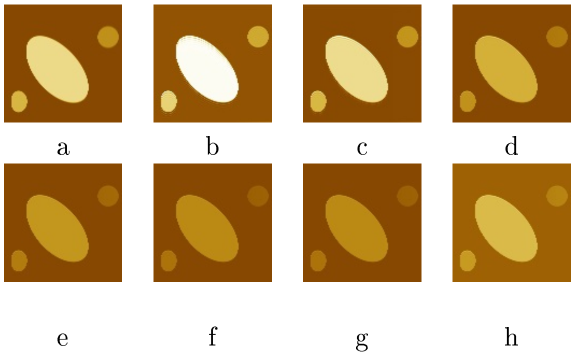

Рис. 3: 1°

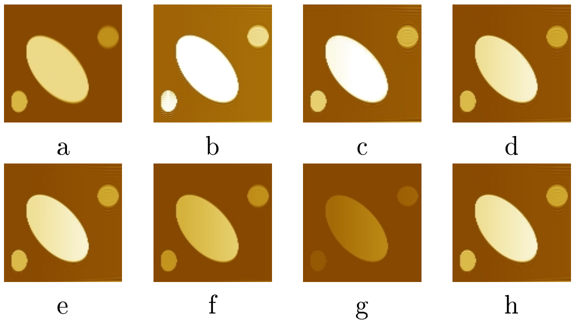

Рис. 4: 2°

Таблица 3: Нормы для 2°×

	$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$		$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$
a	0.0426	0.0749	e	0.0426	0.0749
b	0.0426	0.0749	f	0.0426	0.0749
c	0.0426	0.0749	g	0.0426	0.0749
d	0.0426	0.0749	h	0.0426	0.0749

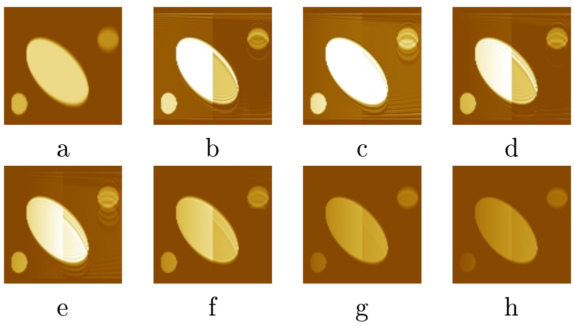

Рис. 5: 4°

5.1 Subsection

Body of subsection 2ε

Numbered formula should be written as

$$a + b = 2. \tag{13}$$

Таблица 4: Нормы для 4°

	$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$		$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$
a	0.3333	0.1035	e	0.1653	0.0198
b	0.236	0.0627	f	0.1636	0.0266
c	0.2	0.0247	g	0.1819	0.0406
d	0.1634	0.0234	h	0.1983	0.0526

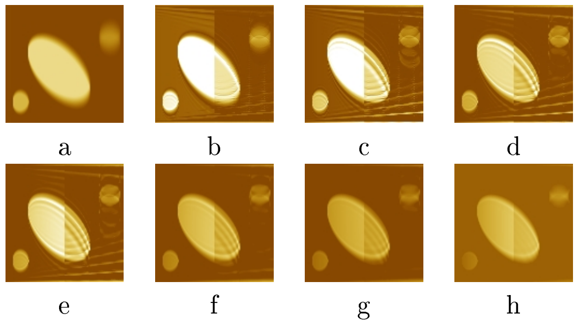

Рис. 6: 8°

Таблица 5: Нормы для 8°

	$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$		$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$
a	0.1959	0.0521	e	0.2	0.0299
b	0.2766	0.0424	f	0.1933	0.0324
c	0.2239	0.0319	g	0.1755	0.0399
d	0.1984	0.0291	h	0.1817	0.0477

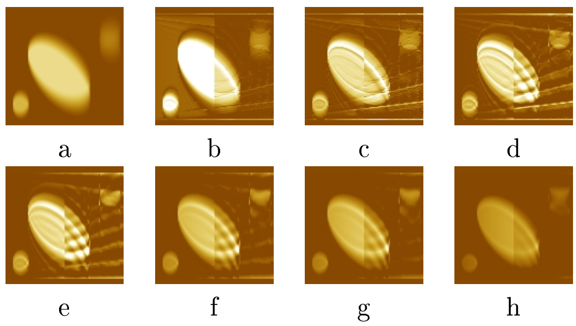

Рис. 7: 14°

Таблица 6: Нормы для 14°

	$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$		$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$
a	0.1926	0.0536	e	0.2225	0.0382
b	0.3106	0.0524	f	0.2	0.0355
c	0.3066	0.0428	g	0.2	0.0369
d	0.2505	0.0440	h	0.2	0.0423

Acknowledgements

Put acknowledgements in the last section, please do not use footnotes for that.

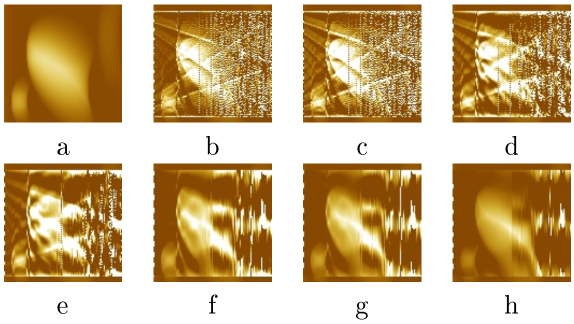

Рис. 8: 40°

Таблица 7: Нормы для 40°

	$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$		$\|\|\Delta\sigma\|\|_2$	$\|\|\Delta\sigma\|\|_\infty$
a	0.0886	0.4	e	0.1332	0.4
b	0.1438	0.4	f	0.1263	0.4
c	0.1444	0.4	g	0.1204	0.4
d	0.1384	0.4	h	0.1165	0.4

Список литературы

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