Лабораторная работа №3. Вариант 3.

In [1]:

```
import numpy as np
from matplotlib import pyplot as plt
```

In [3]:

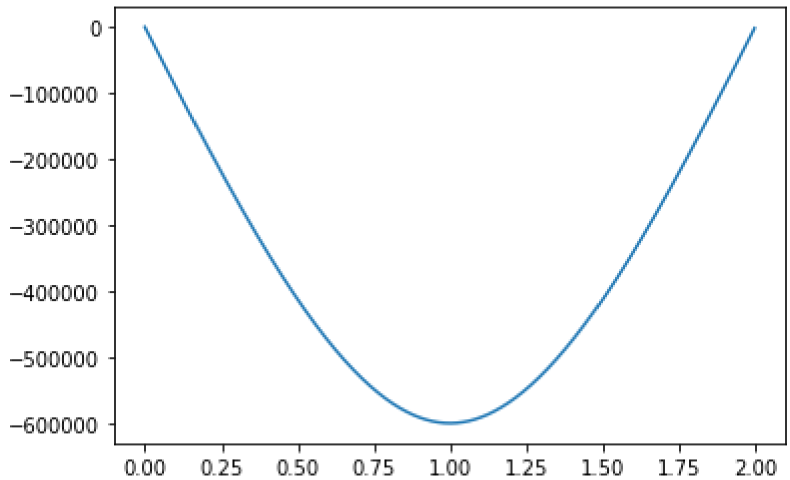
```
#Дано:
C = 1
L = 2
E = 200
P = 6000
b = 0.1
h = 0.1
```

In [4]:

```
#Точное решение:
def uex(x):
    L = 2
    E = 200
    P = 6000
    b = 0.1
    h = 0.1
    p1 = 6 * P / (E * b * h * h * h)
    c2 = - p1 * L * L * 3 / 8
    c1 = c2 + (p1 * L * L / 4)
    c22 = - (p1 * L * L * L / 3) - (c2 * L)
    if ((0 <= x) & (x < L * 0.5)):
        a = (p1 * x * x * x / 6) + c1 * x
        return a
    #elif (((L * 0.5) <= x) & (x <= L)):
    else:
        a = p1 * ((L * x * x * 0.5) - (x * x * x / 6)) + (c2 * x) + c22
        return a
#График:
Q = 1000
X = [i * 0.001 * L for i in range(Q)]
# График точного решения
y_1 = [uex(x) for x in X]
plt.plot(X, y_1, label='y')
```

Out[4]:

```
[<matplotlib.lines.Line2D at 0x1a81ce5d9d0>]
```



In [5]:

```
# функция f
def f(x):
    L = 2
    E = 200
    P = 6000
    b = 0.1
    h = 0.1
    p1 = 6 * P / (E * b * h * h * h)
    if ((0 <= x) & (x < L * 0.5)):
        a = p1 * x
        return a
    elif (((L * 0.5) <= x) & (x <= L)):
        a = p1 * (L - x)
        return a
```

In [6]:

```
# функция формы
def fi(x, i, N):
    delta = L / N
    if (i == 0):
        if ((0 <= x) & (x <= delta)):
            a = 1 - (x / delta)
            return a
        else:
            return 0
    elif (i == N):
        if ((((N - 1) * delta) <= x) & (x <= L)):
            a = (x / delta) - N + 1
            return a
        else:
            return 0
    else:
        if (((i - 1) * delta <= x) & (x <= i * delta)):
            a = (x / delta) - i + 1
            return a
        elif ((i * delta <= x) & (x <= (i + 1) * delta)):
            a = i + 1 - (x / delta)
            return a
        else:
            return 0
```

In [7]:

```
# локальная матрица для левой части
def Ke(N, i):
    c = np.zeros((N + 1, N + 1))
    c[i][i] = 1
    c[i][i+1] = -1
    c[i+1][i] = -1
    c[i+1][i+1] = 1
    return c
```

In [8]:

```
# матрица жесткости
def K(N):
    #коэффициент
    ck = N / L
    #матрица
    c = np.zeros((N + 1, N + 1))
    #складываем локальные матрицы
    for i in range(N):
        g = Ke(N, i)
        c = c + g
    #зануляем первую и последнюю строки, ставим 1
    for i in range(N + 1):
        c[0][i] = 0
        c[N][i] = 0
    c[0][0] = 1
    c[N][N] = 1
    #домножаем на коэффициент
    c = ck * c
    return c
```

In [9]:

```
# локальная матрица правой части
def Pe(N, i):
    c = np.zeros((N + 1, N + 1))
    c[i][i] = 2
    c[i][i+1] = 1
    c[i+1][i] = 1
    c[i+1][i+1] = 2
    return c
```

In [10]:

```
# вектор нагрузок
def P(N):
    L = 2
    #коэффициент
    cp = - L / (6 * N)
    #матрица
    c1 = np.zeros((N + 1, N + 1))
    #складываем локальные матрицы
    for i in range(N):
        g = Pe(N, i)
        c1 = c1 + g
    #зануляем первую и последние строки
    for i in range(N + 1):
        c1[0][i] = 0
        c1[N][i] = 0
    #домножаем на коэффициент
    c1 = cp * c1
    #домножаем на вектор
    c2 = np.full(N + 1, 0) #пока заполним нулями, позже заполним f(x)
    for i in range(N + 1):
        x = i * L / N
        c2[i] = f(x)
    c3 = np.transpose(c2)
    c = c1.dot(c3)
    return c
```

In [11]:

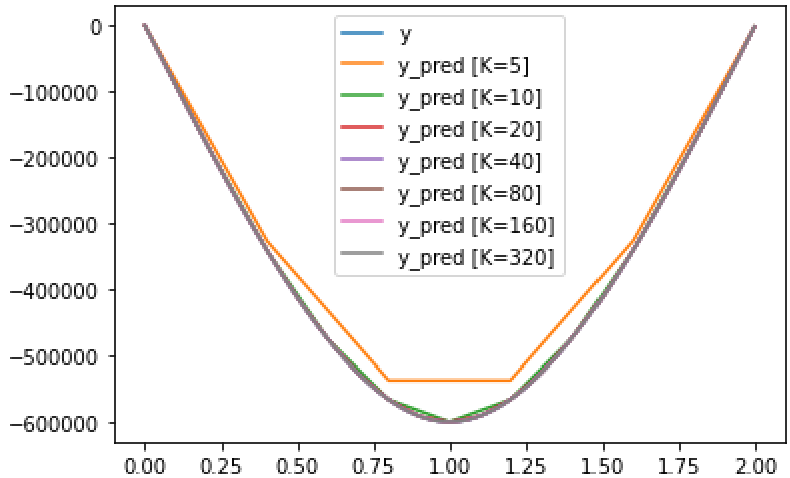
```
# решаем слау
def mke(N):
    u = np.zeros(N + 1)
    Q, R = np.linalg.qr(K(N))
    u = np.linalg.solve(R, np.dot(Q.transpose(), P(N)))
    return u
# приближенное решение
def uh(x, N):
    u = mke(N)
    s = 0
    for i in range(N + 1):
        k = fi(x, i, N)
        a = k * u[i]
        s = s + a
    return s
```

In [12]:

```
import scipy.integrate as integrate
import scipy.special as special
from scipy.integrate import quad
from numpy import sqrt
# относительная погрешность на бесконечной норме
def e_inf(N):
    Q = 1000
    X = [i * 0.001 * L for i in range(Q)]
    a1 = max([np.abs(uex(x) - uh(x, N)) for x in X])
    a2 = max([np.abs(uex(x)) for x in X])
    a = a1 / a2
    return a

def u1(x, N):
    return ((uex(x) - uh(x, N)) ** 2)

def u2(x):
    return (uex(x) ** 2)
# относительная погрешность на L2
def e_l2(N):
    Q = 1000
    X = [i * 0.001 * L for i in range(Q)]
    a1 = sum(u1(x, N) * 0.001 * L for x in X)
    a2 = sum(u2(x) * 0.001 * L for x in X)
    a = sqrt (a1 / a2)
    return a
# число обусловленности
def mu(A):
    return np.linalg.cond(A)
```

In [14]:

```
Q = 1000
X = [i * 0.001 * L for i in range(Q)]
# График точного решения
y_1 = [uex(x) for x in X]
plt.plot(X, y_1, label='y')
logs = [] # Таблица результатов
b1 = e_inf(2)
for N in [5, 10, 20, 40, 80, 160, 320]:
    y = [uh(x, N) for x in X]

    a1 = e_inf(N)
    a2 = e_l2(N)
    plt.plot(X, y, label=f'y_pred [K={str(N)}]')
    logs.append(f"{N}\t"
                f"{a1}\t"
                f"{np.log2(b1/a1)}\t"
                f"{a2}\t"
                f"{mu(K(N))}\n")
    b1 = a1
plt.legend()
plt.show()
for line in ['N\tEr_inf\tПорядок_сходимости\tEr_L2\tmu(K)\n'] + logs:
    print(line)
```




```
N       Er_inf  Порядок_сходимости      Er_L2   mu(K)

5       0.10400075555555557     0.8878899998765459      0.0811945406009728
6       10.99460031976541

10      0.013505401888890574    2.9449855326478294      0.0090323027502599
76      40.689754983885145

20      0.0035633184722160985   1.9222432362159894      0.0022667834319651
517     161.87377437504045

40      0.0009137233055550908   1.9633921719351295      0.0005677711681325
716     648.0040189479324

80      0.00023192838459780l2   1.9780779580233652      0.0001425497249466
7052    2593.263437826876

160     5.895267540162117e-05   1.9760502028194913      3.621426229253557e
-05     10374.676473872563
```

```
320     1.5116177630455538e-05  1.9634639104101417      9.631695489410261e
-06     41500.517125946324
```

In [ ]: