Nine-point circle

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- based on definintions of Wikipedia -

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The nine points

How to define Euler's circle?

Also known as Feuerbach's circle or Euler's circle (not to be confused with Euler's circle in graph theory) the nine-point circle (not to be confused with Pont Neuf ;))

In geometry, the nine-point circle is a circle that can be constructed for any given triangle.

It is so named because it passes through nine significant concyclic points defined from the triangle.

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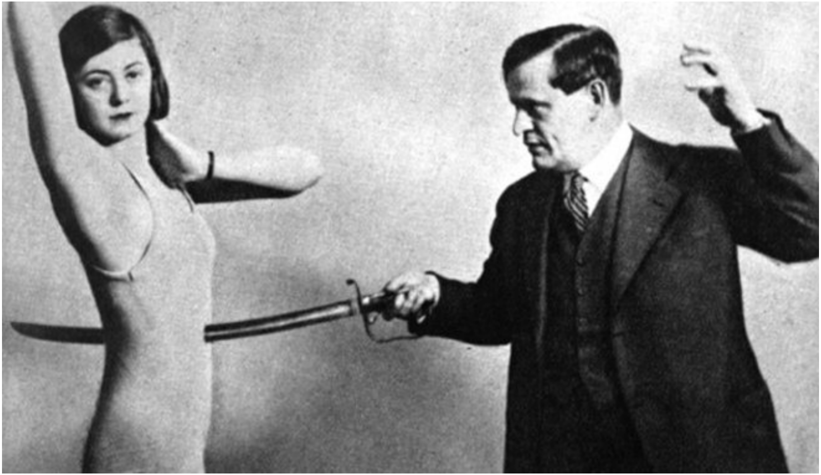
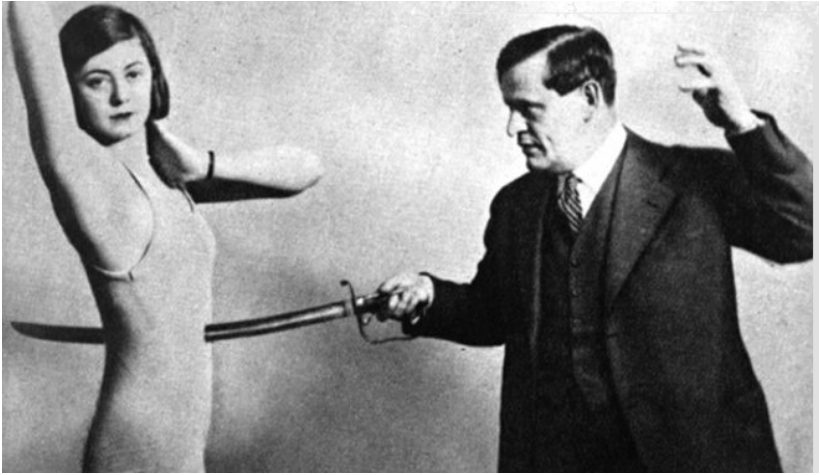
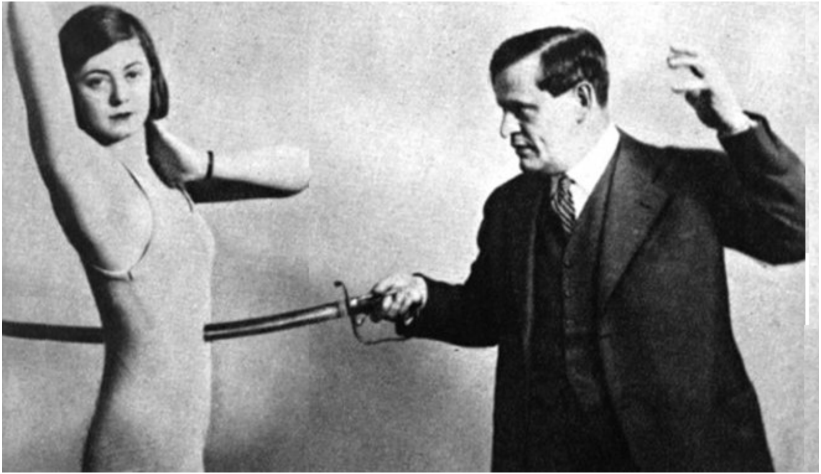
Nine significant points

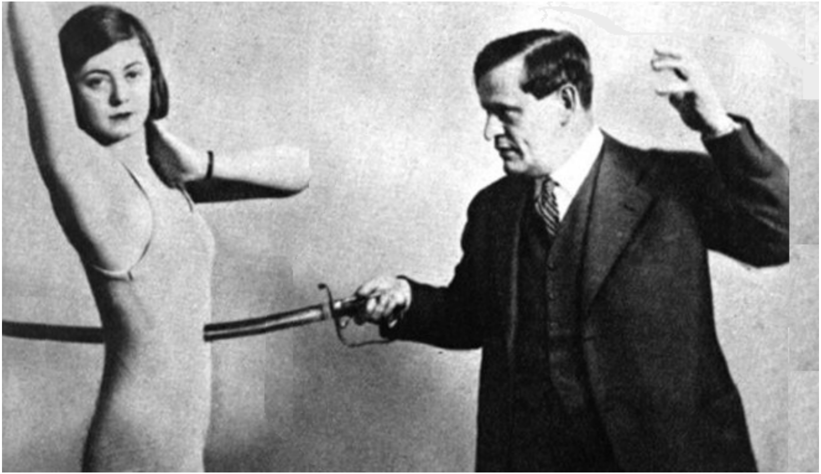
The diagram above shows the nine significant points of the nine-point circle.

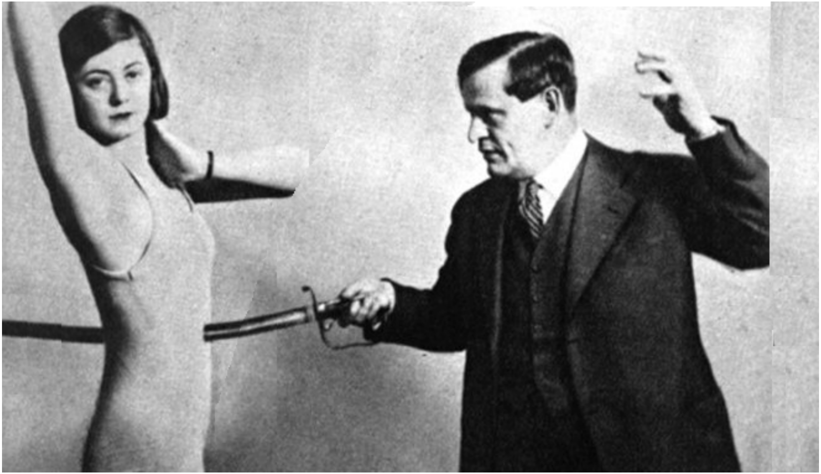
Points D, E, and F are the midpoints of the three sides of the triangle. Points G, H, and I are the feet of the altitudes of the triangle. Points J, K, and L are the midpoints of the line segments between each altitude's vertex intersection (points A, B, and C) and the triangle's orthocenter (point S). For an acute triangle, six of the points (the midpoints and altitude feet) lie on the triangle itself; for an obtuse triangle two of the altitudes have feet outside the triangle, but these feet still belong to the nine-point circle.











Discovery

Although he is credited for its discovery, Karl Wilhelm Feuerbach did not entirely discover the nine-point circle, but rather the six-point circle,

recognizing the significance of the midpoints of the three sides of the triangle and the feet of the altitudes of that triangle.

(At a slightly earlier date, Charles Brianchon and Jean-Victor Poncelet had stated and proven the same theorem.)

Soon after Feuerbach, mathematician Olry Terquem himself proved the existence of the circle.

He was the first to recognize the added significance of the three midpoints

between the triangle's vertices and the orthocenter.

Thus, Terquem was the first to use the name nine-point circle.

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The nine-point center N is one-fourth of the way along the Euler line from the centroid G to the orthocenter HN = 3NG.

Források

- geogebra
- wikipedia