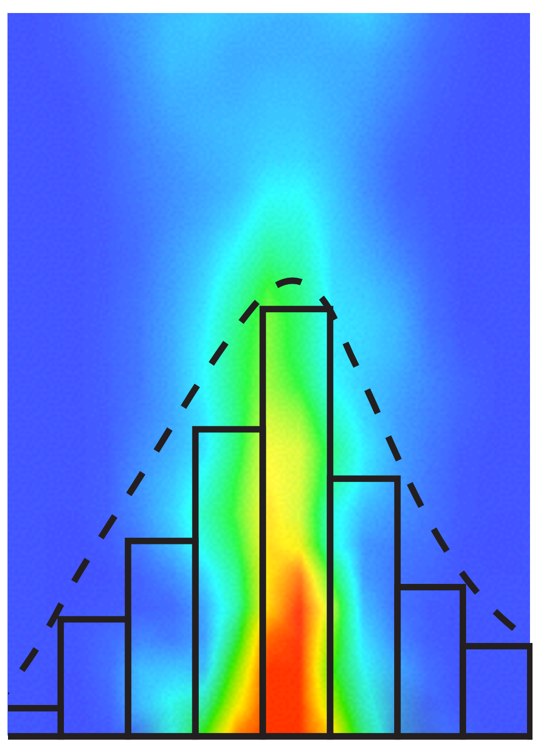
Quantitative Testing of Fire Scenario Hypotheses: A Bayesian Inference Approach



Kristopher J. Overholt

November 25, 2013

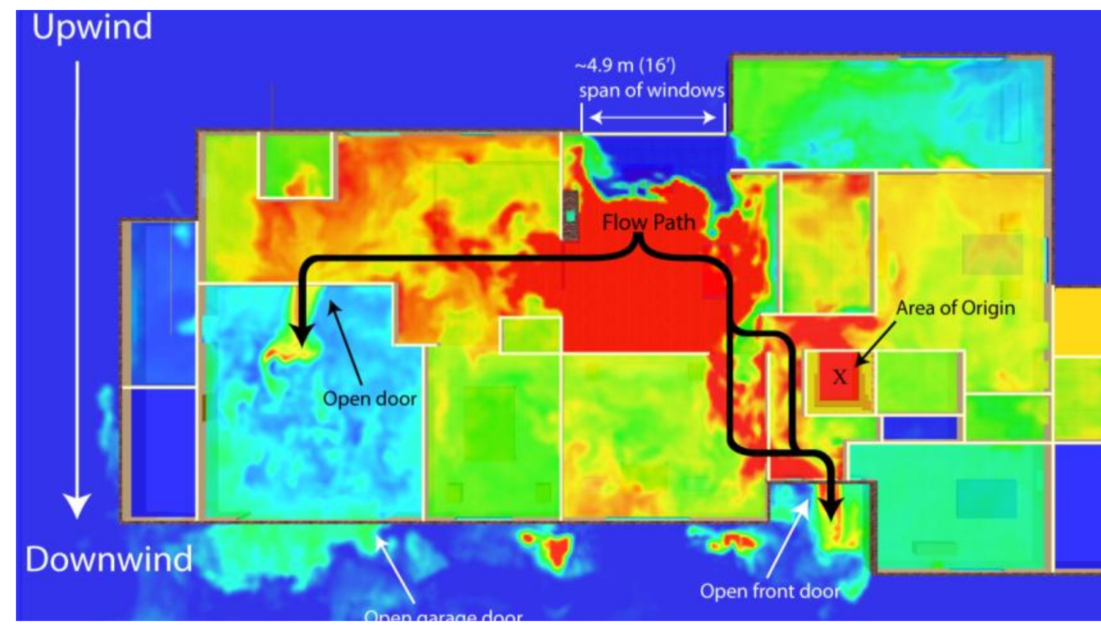
Outline

- Fire Modeling Objectives
- Probability Distributions
- Bayes' Theorem and Bayesian Inference Framework
- Examples Using Bayesian Inference in Fire Scenarios:
  - 1) Estimating Fire Size using HGL Temperature Data
  - 2) Estimating Fire Size using Heat Flux Data
  - 3) Estimating Fire Location using Heat Flux Data
  - 4) Estimating Material Properties
  - 5) Estimating Transient Fire Size
- Conclusions

Fire Models in Fire Investigations

Fire modeling tools can be applied to model validation exercises, fire and arson investigations, and reconstructions of firefighter line-of-duty deaths (LODDs) and injuries.

Barowy and Madrzykowski, 2012

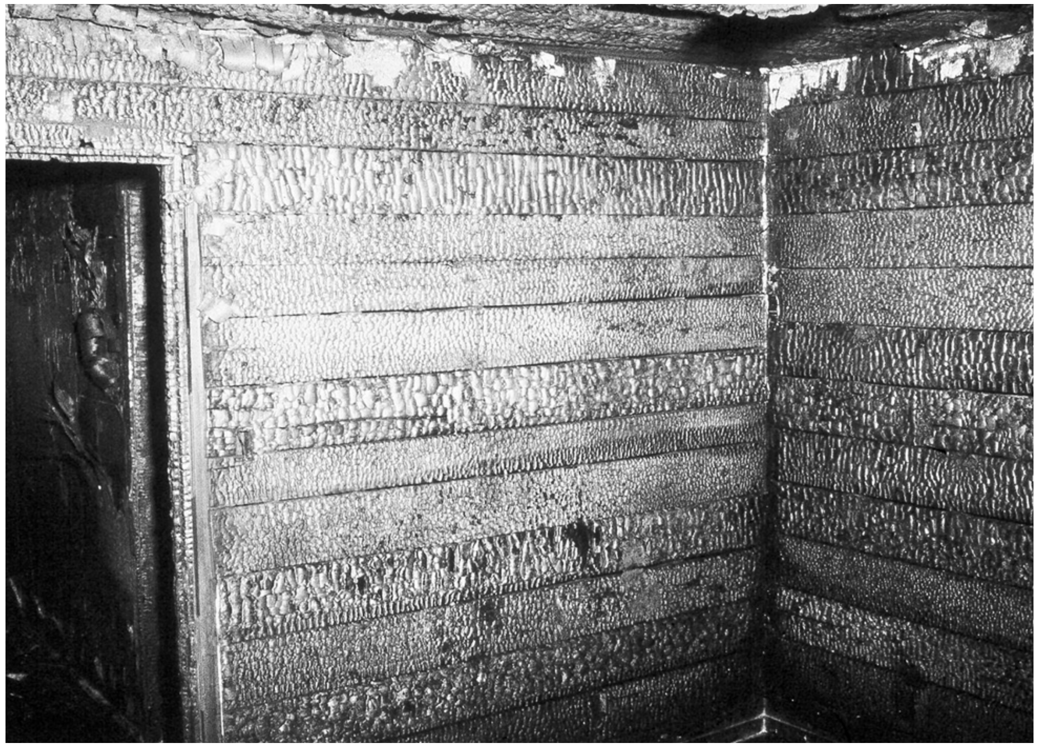


Barowy and Madrzykowski, 2012

The location and intensity (i.e., heat release rate) of a fire in a compartment are important model inputs that govern the evolution of thermal conditions in the fire compartment.

Fire Models in Fire Investigations

Compartment fires leave behind a record of fire activity and history (i.e., fire signatures).



NFPA 921, 2011 ed.

NFPA 921, 2011 ed.

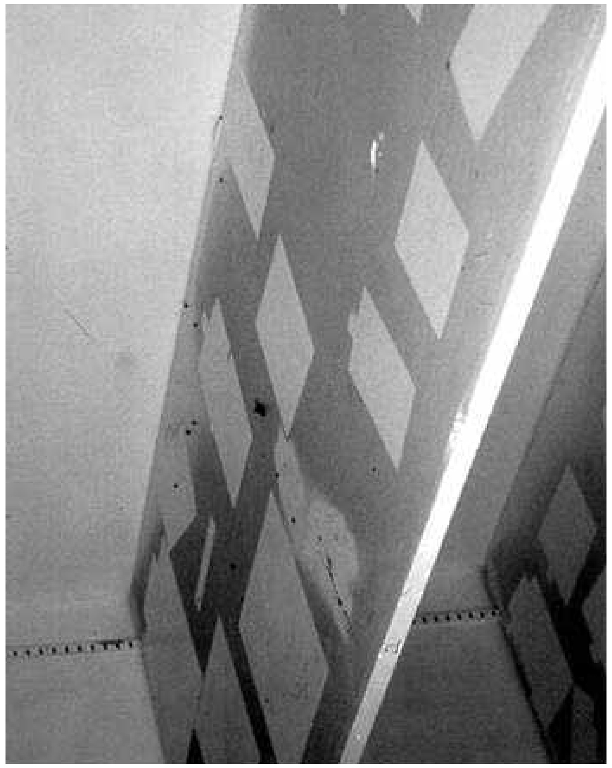
- Gypsum wallboard undergoes chemical reactions/calcination,
- Metal and plastic components melt and deform,
- Soot deposition to surfaces occurs.

Fire Models in Fire Investigations

In fire scenarios, observations of fire damage can be a surrogate for measured data.

We can use various fire models and inversion techniques to estimate the size and location of a fire in a compartment.

We want to **calculate unknown model inputs and outputs** and **quantify our degree of belief** in them.



Ezekoye, Implementation of Aerosol Smoke Physics into Fire Dynamics Modeling for Fire Reconstruction

Riahi, 2011

Institutional Decision Analysis

In institutional settings (businesses, governments, or research organizations), **decisions need to be justified**, and **formal decision analysis** has a role to play […]

[…] forensic scientists use a particular quantitative approach to evaluating forensic laboratory results, the Bayesian approach, as a means of **quantifying uncertainty and communicating it accurately to judges, prosecutors, and defense lawyers** […]

[…] using the Bayesian approach also brings about a particular type of intersubjectivity; […] **quantifications must be consistent** across forensic specializations, which brings about a **transparency based on shared understandings and practices**.

Gelman, Institutional decision analysis, http://andrewgelman.com/2004/11/16/institutional_d/, 2004; Kruse, The Bayesian approach to forensic evidence: Evaluating, communicating and distributing responsibility, Social Studies of Science, 2013.

Institutional Decision Analysis

Numerical scale	Verbal scale	Likelihood ratio interval
+4	The results of the examination extremely strongly support that …	lr ≥ 1,000,000
+3	The results of the examination strongly support that …	6000 ≤ lr < 1,000,000
+2	The results of the examination support that …	100 ≤ lr < 6000
+1	The results of the examination support to some extent that …	6 ≤ lr < 100
0	The results of the examination support neither … nor …	1/6 < lr < 6
−1	The results of the examination support to some extent that … *was not* …	1/6 ≥ lr > 1/100
−2	The results of the examination support that … *was not* …	1/100 ≥ lr > 1/6000
−3	The results of the examination strongly support that … *was not* …	1/6000 ≥ lr > 1/1,000,000
−4	The results of the examination extremely strongly support that … *was not* …	lr ≤ 1/1,000,000

Kruse, The Bayesian approach to forensic evidence: Evaluating, communicating and distributing responsibility, Social Studies of Science, 2013.

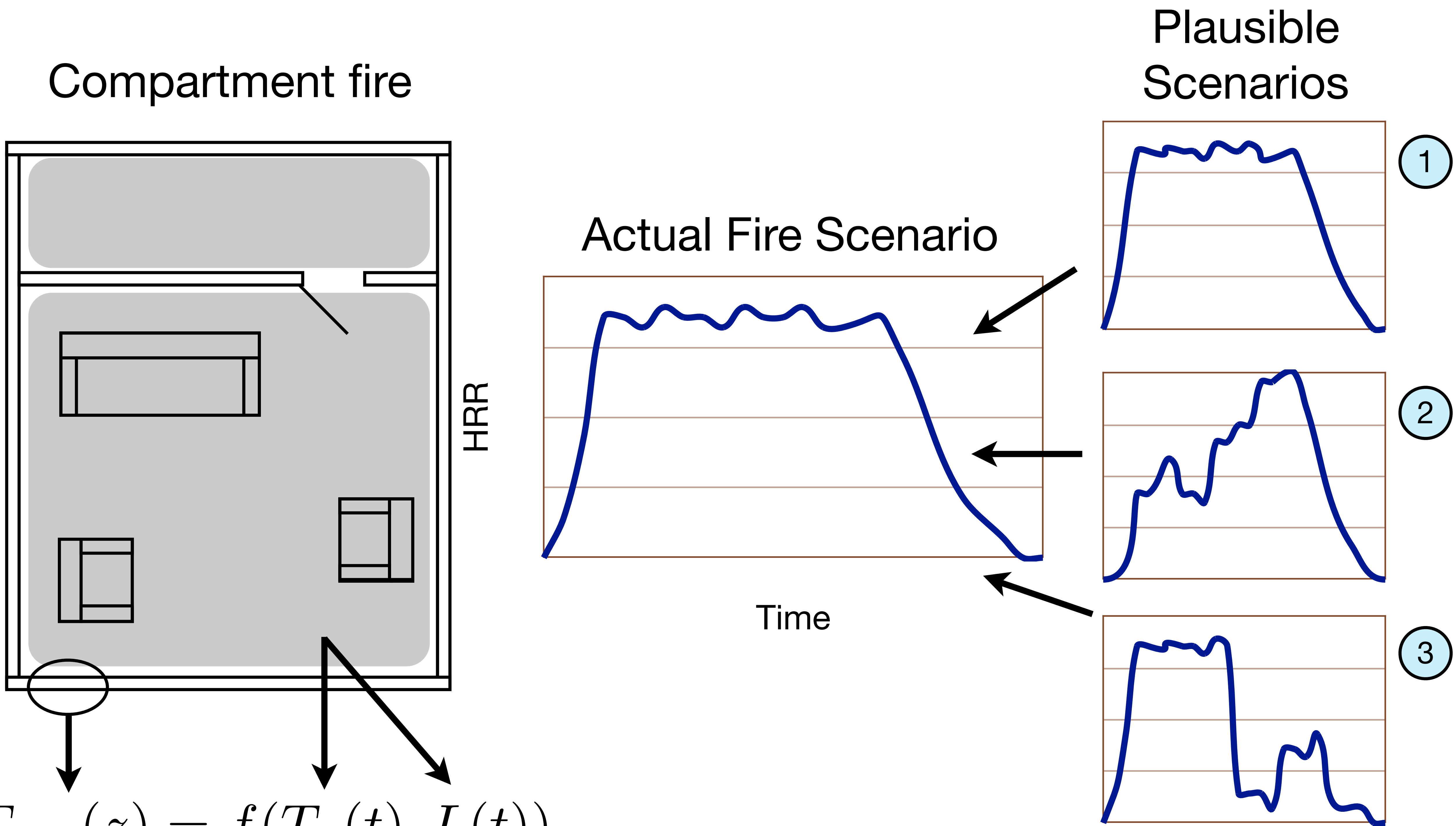
Objectives

1) To describe the **amount of uncertainty in model inputs and outputs**.

2) To quantify our **degree of certainty (or degree of belief)** in unknown input parameters for fire scenarios.

3) To apply a **statistical parameter inversion framework** to various fire scenarios to determine: 1) fire size, 2) fire location, and 3) material property parameters.

Quantifying Plausible Scenarios



$$T_{gyp}(z) = f(T_u(t), L(t))$$

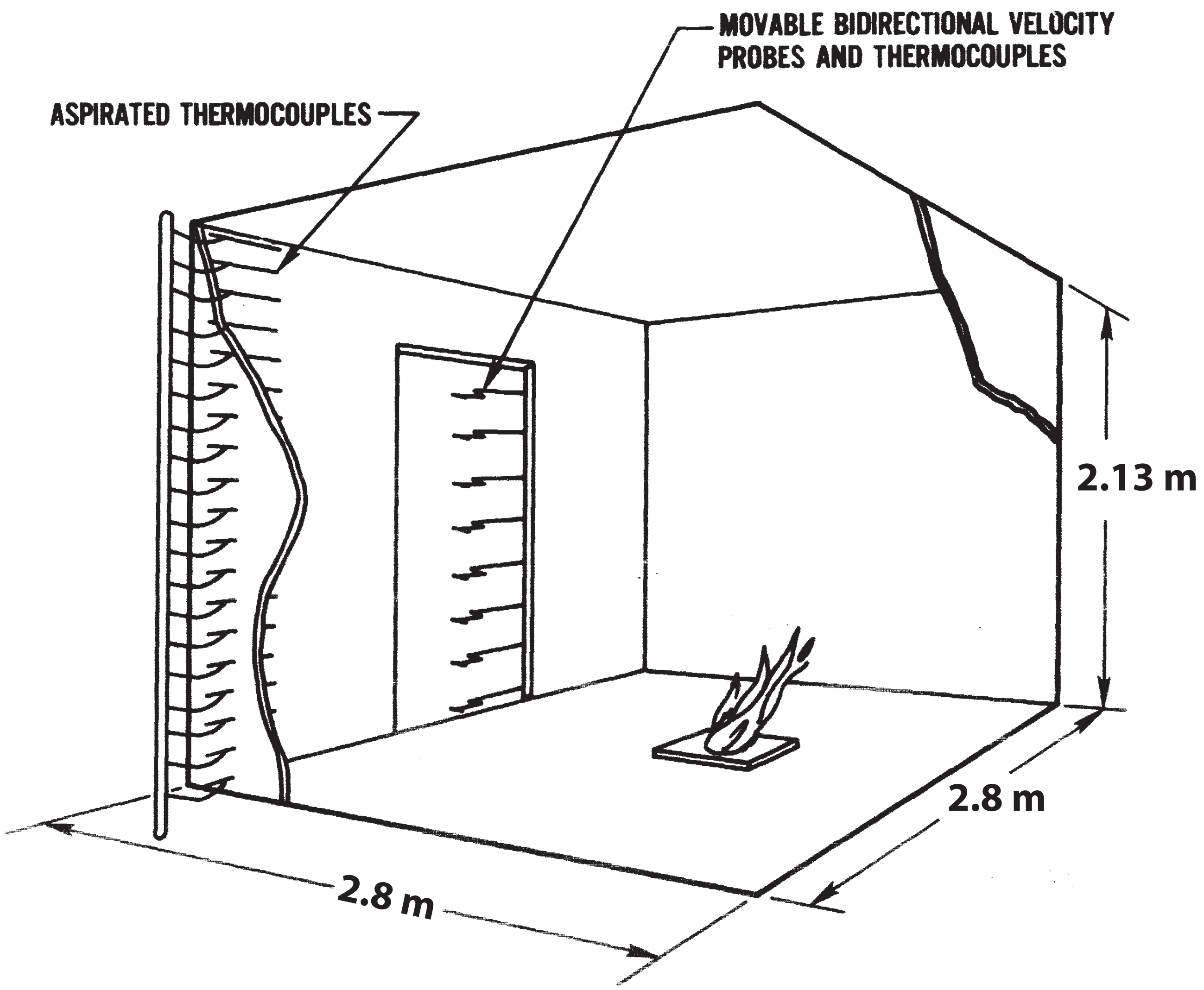
Example Case - Estimating Fire Size

Fire in a compartment with an estimate of the hot gas layer (HGL) temperature:

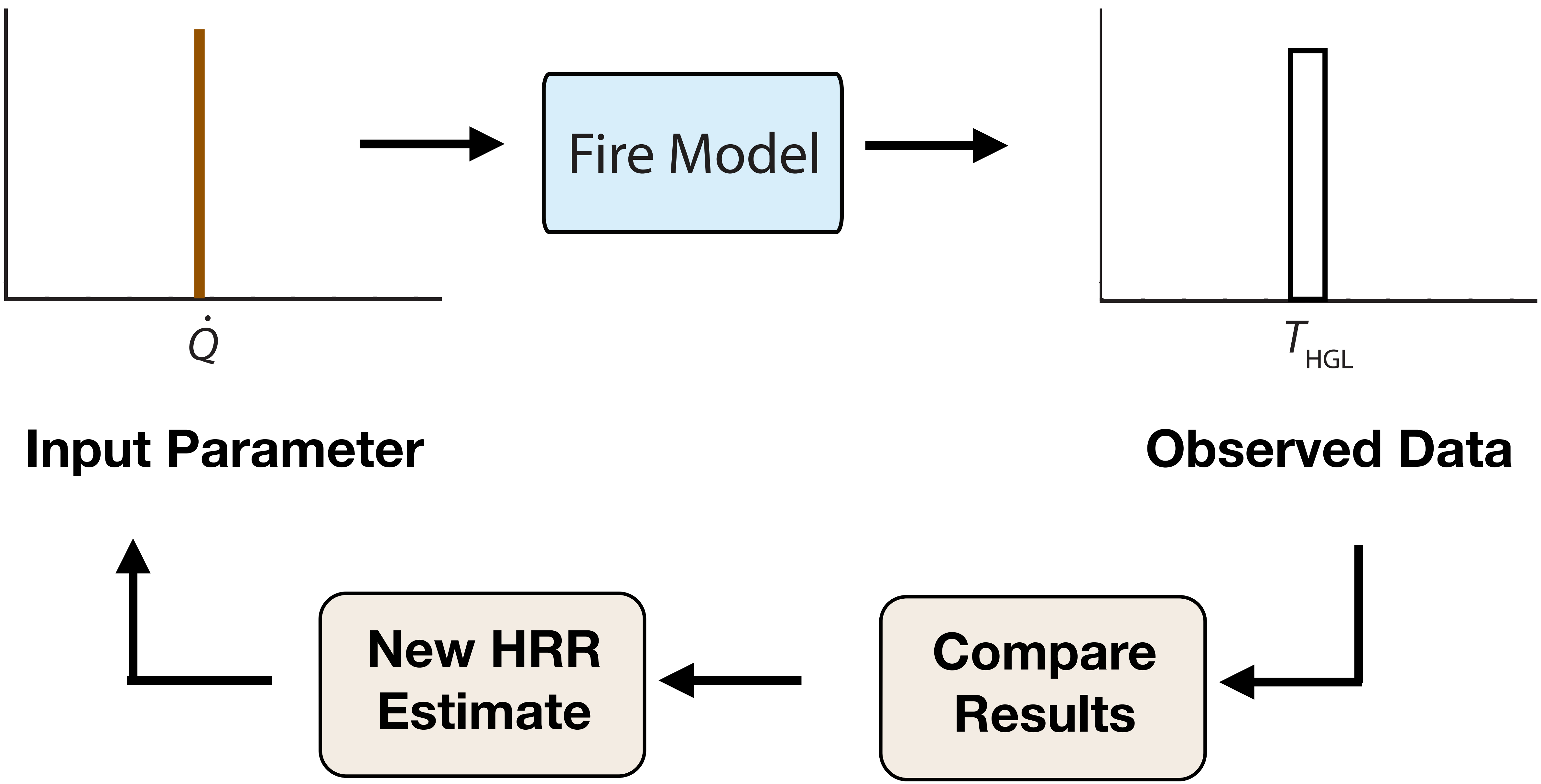
200 °C

Unknown parameter:

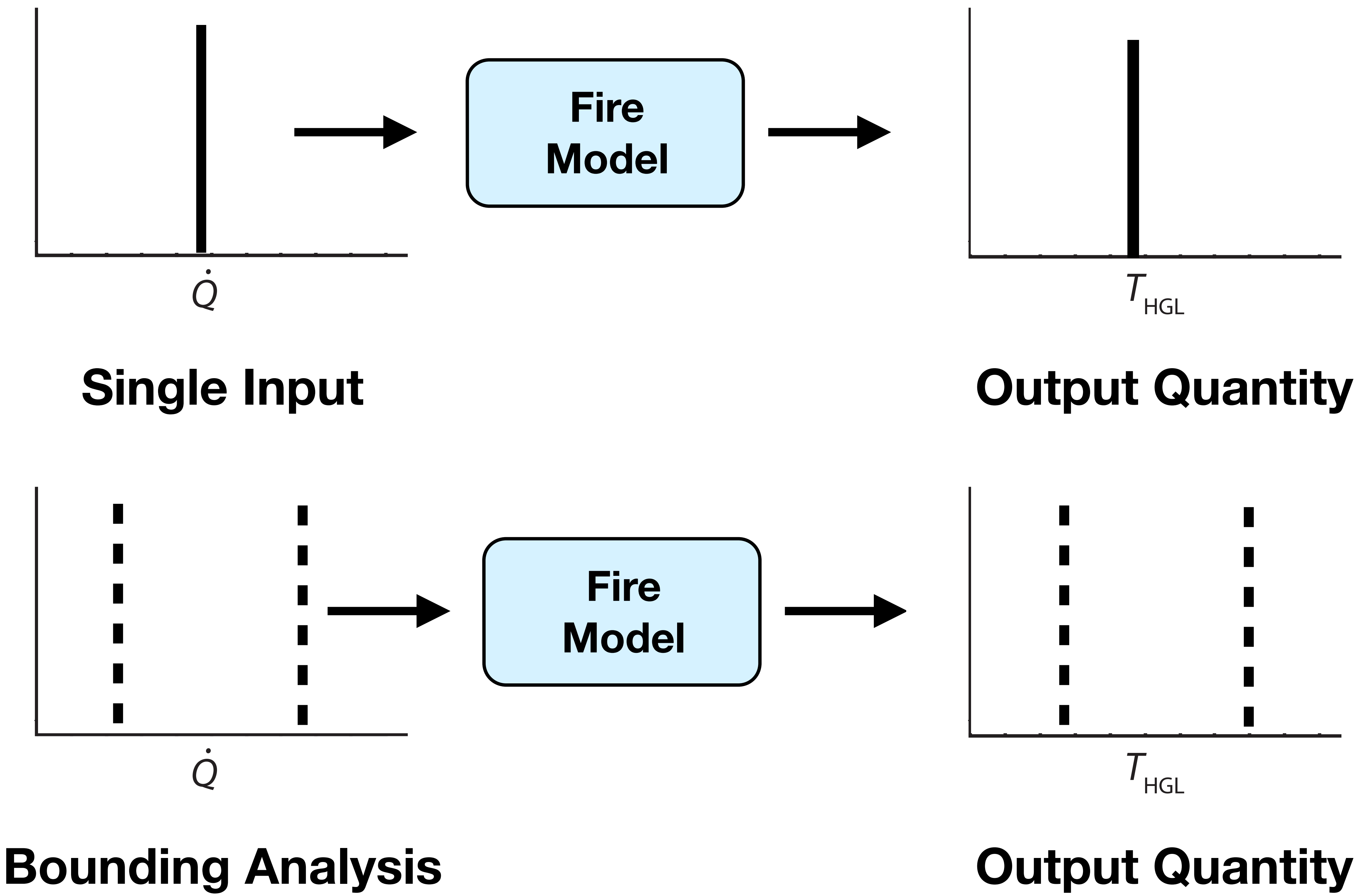
HRR $\dot{Q}$



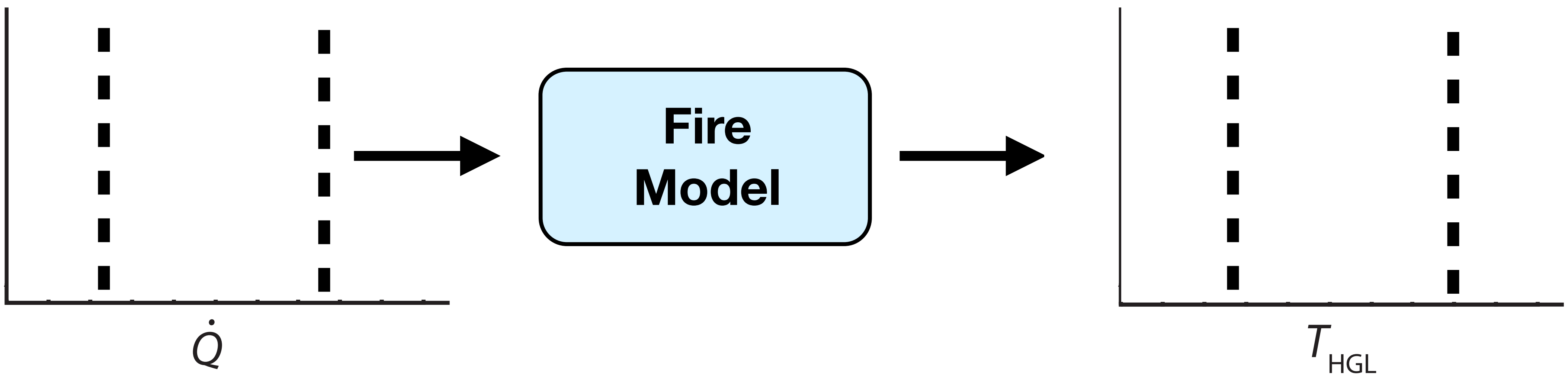
Inverse - Calculating a Single Input HRR



Forward - Single Input vs. Bounding Analysis

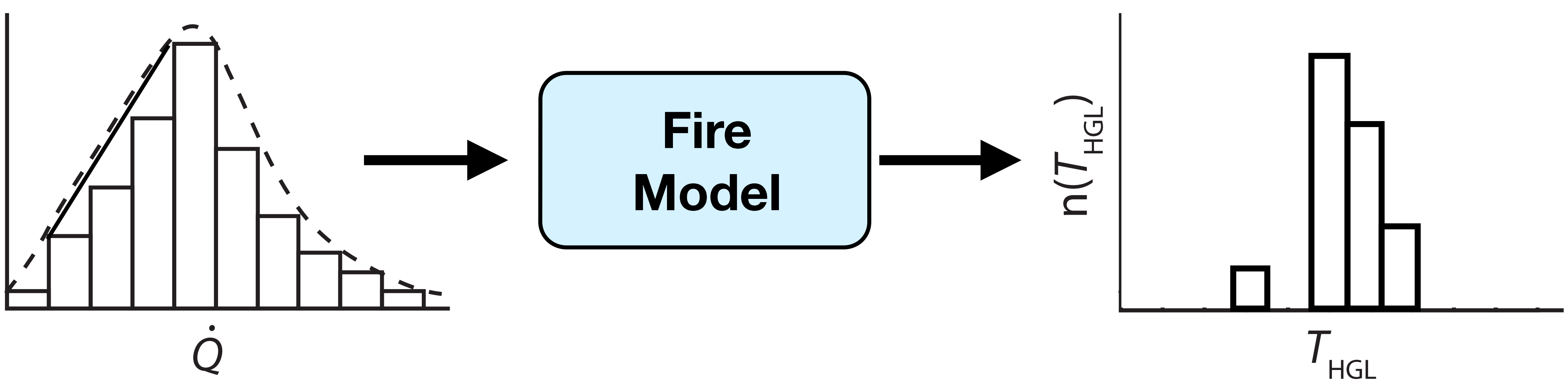


Forward - Bounding Analysis vs. Probability Distribution



Bounding Analysis

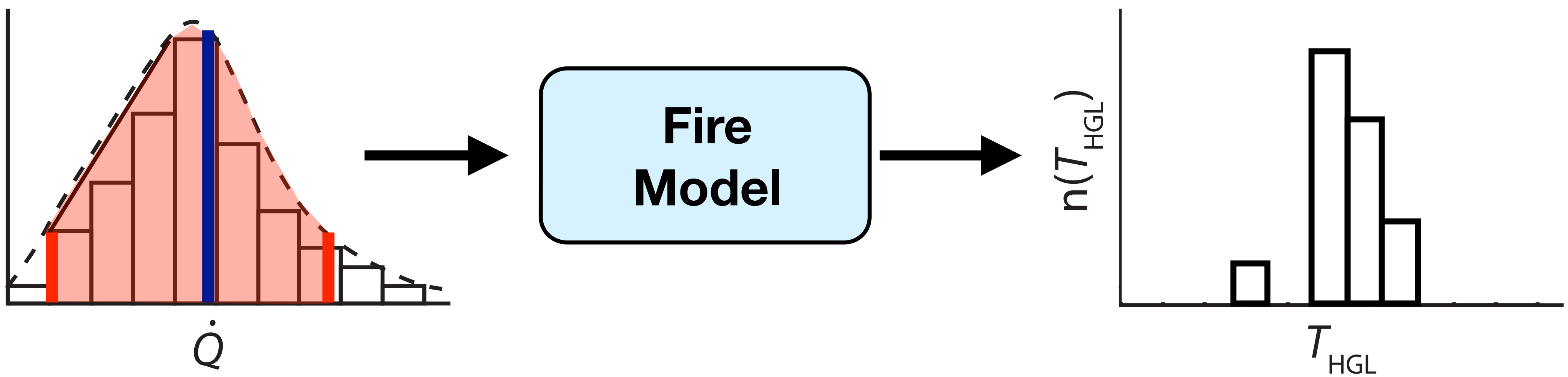
Output Quantity



Probability Distribution

Output Quantity

Benefits of a Probability Distribution



Input Parameter **Output Quantity**

We can:

1) propagate a distribution to the output quantity

2) approximate using two parameters: average value and width

3) describe uncertainty directly using **credible intervals**

Benefits of a Probability Distribution

More data observations give us more certainty in the true value.

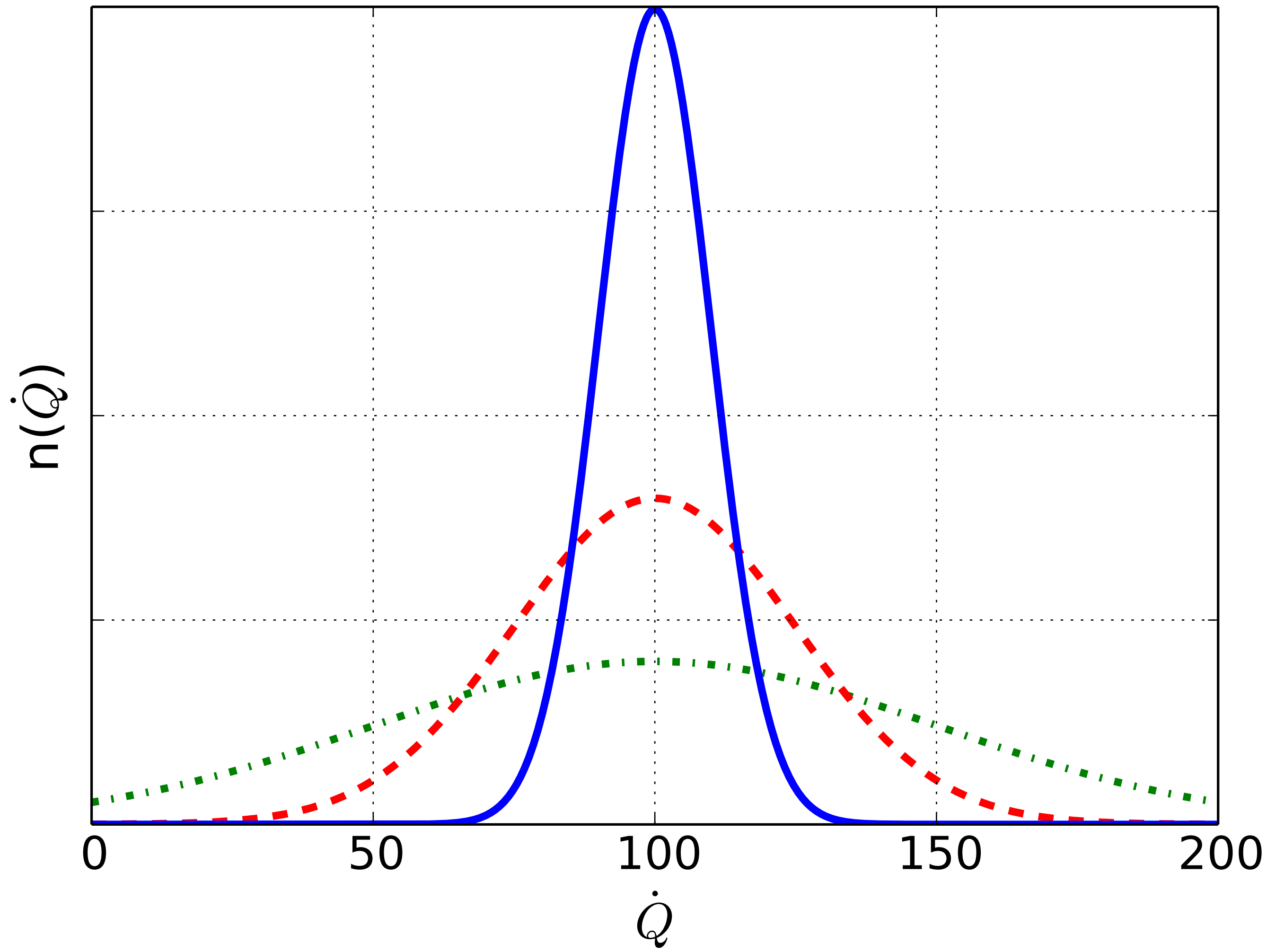
95 % credible

intervals of:

0 to 200 kW

50 to 150 kW

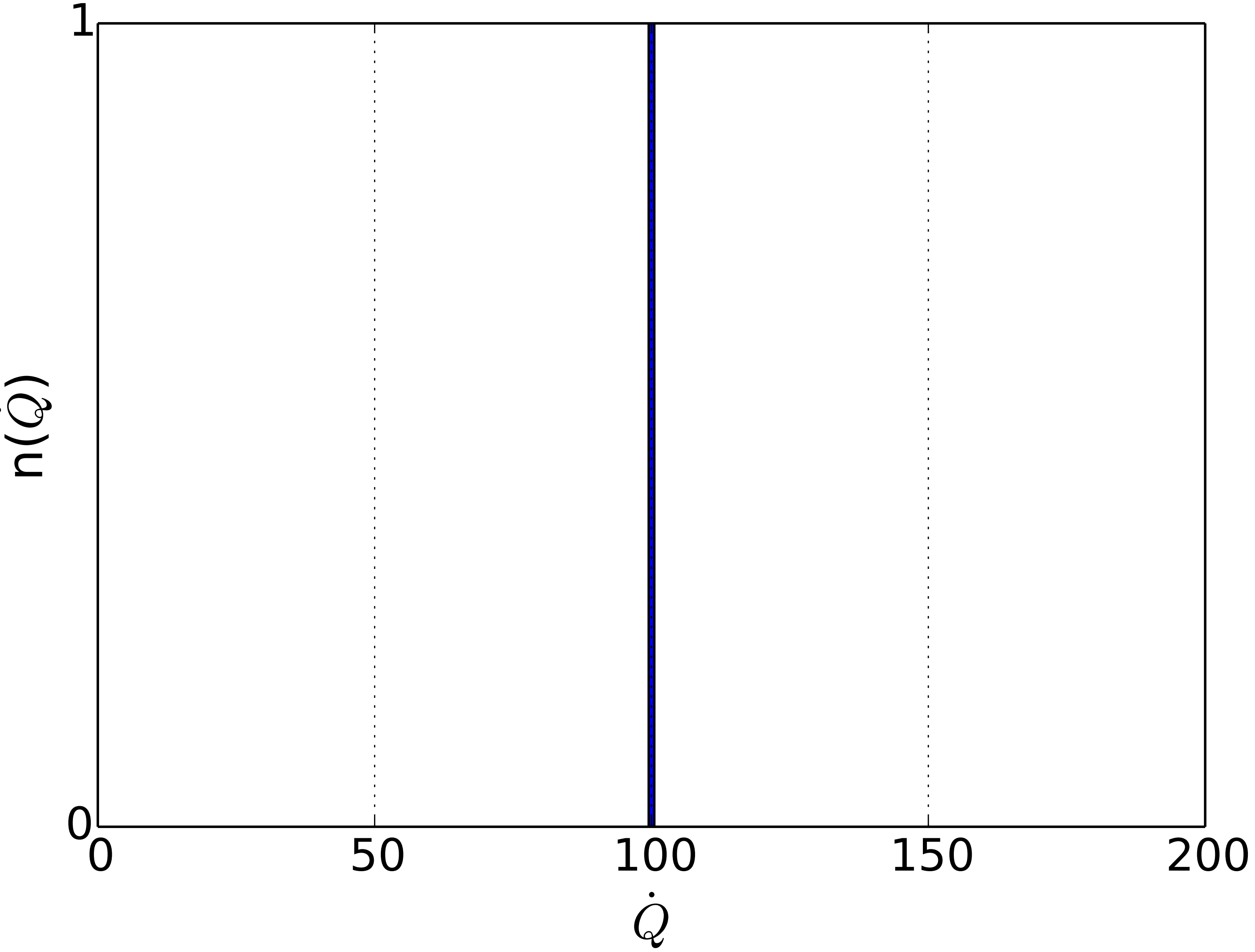
80 to 120 kW



Which estimate is more certain?

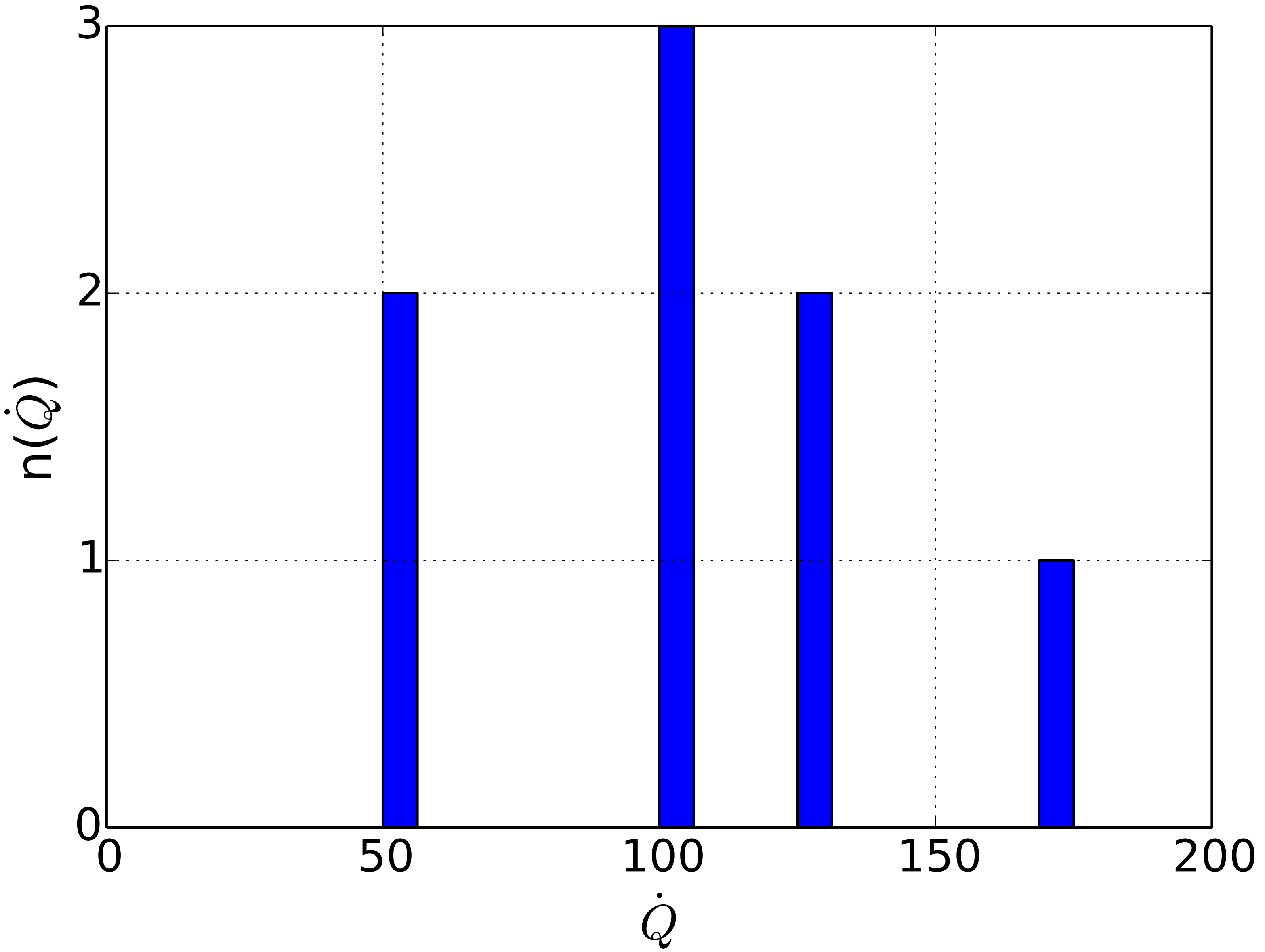
Calculating a Probability Distribution

How can we calculate a probability distribution?



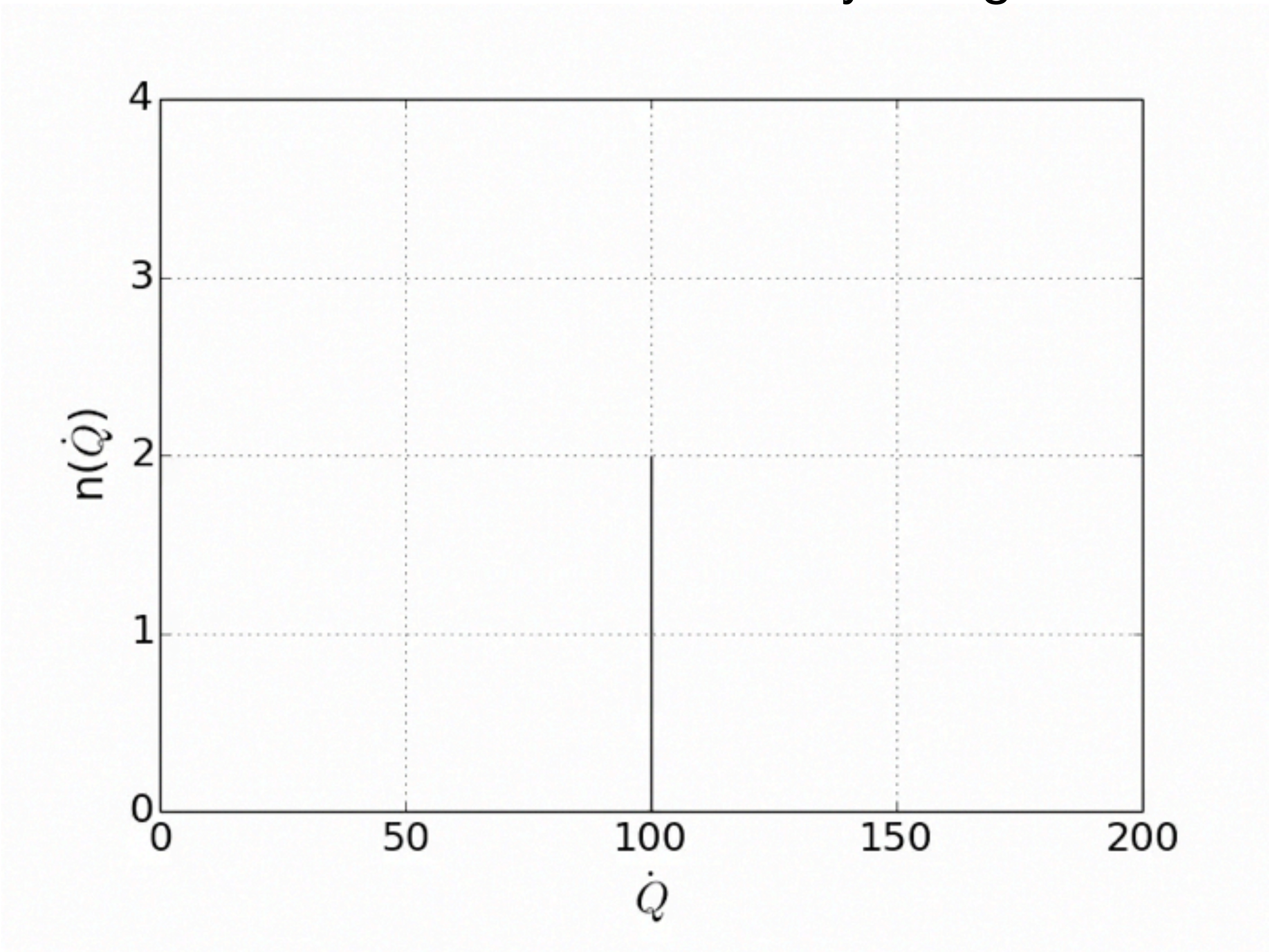
Calculating a Probability Distribution

Obtain multiple estimates from experts.



Calculating a Probability Distribution

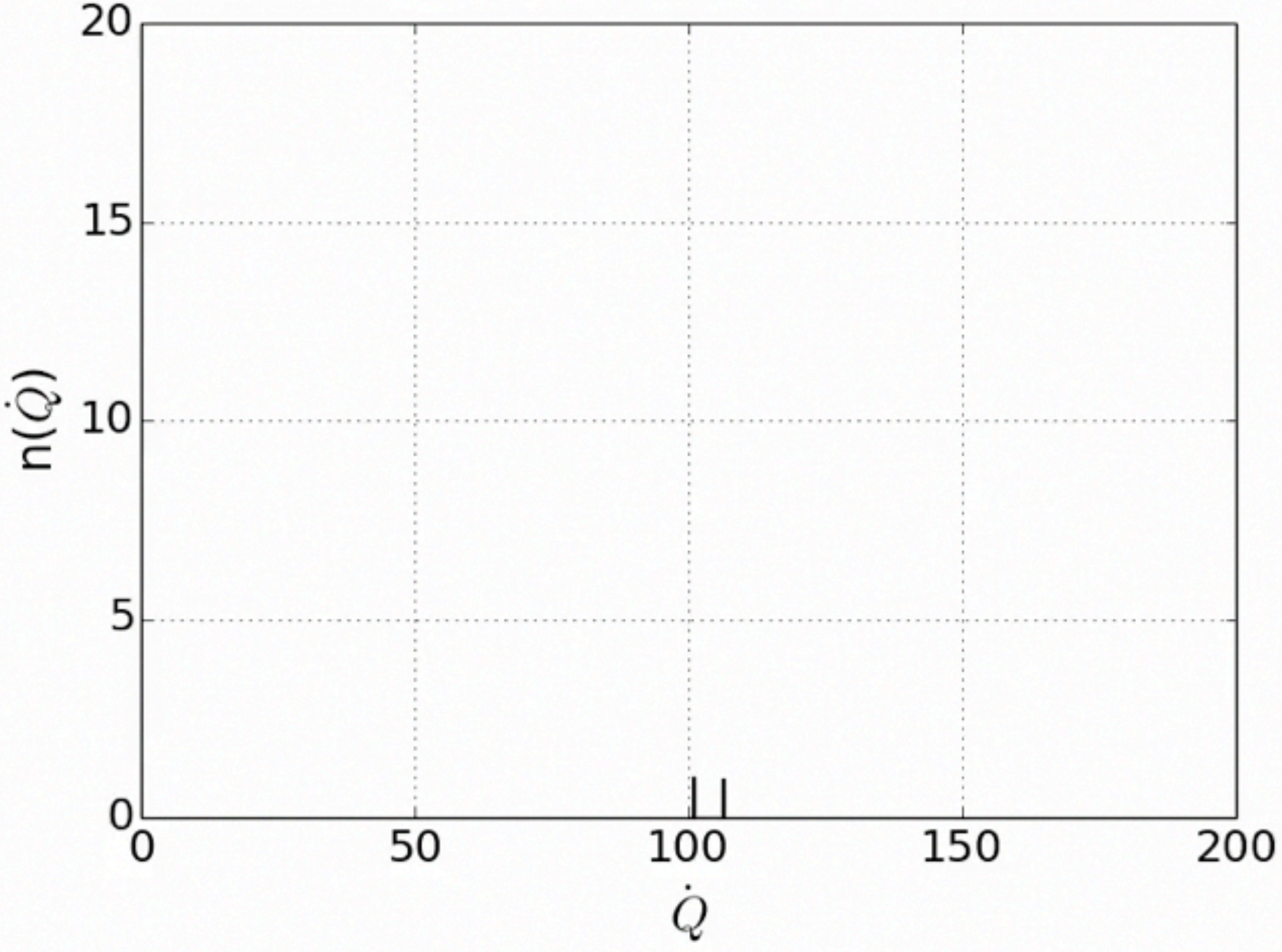
Calculate the distribution manually using a model.



Calculating a Probability Distribution

Calculate a distribution automatically with a model

and a statistical inference method.

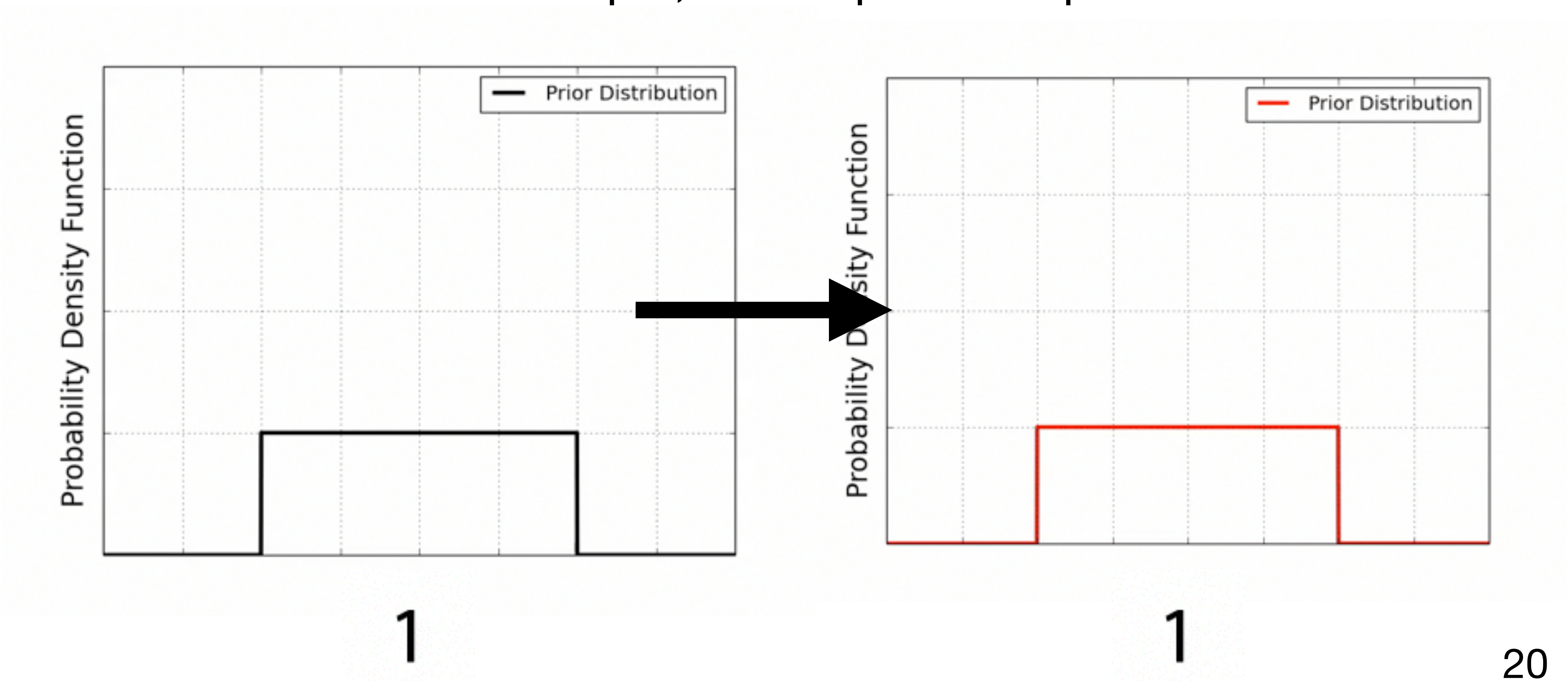


Calculating a Probability Distribution

How do we update the probability distribution at each step?

Bayes' Theorem

For example, three update steps:



Bayes' Theorem

Definition of Bayes' Theorem

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(B|A) \cdot \mathrm{P}(A)}{\mathrm{P}(B)}$$

"An Essay Toward Solving a Problem in the Doctrine of Chances"

(Reverend Thomas Bayes, 1764)

Bayes' Theorem

Definition of Bayes' Theorem

$$\overbrace{\mathrm{P}(A|B)}^{\text{Posterior}} = \frac{\overbrace{\mathrm{P}(B|A)}^{\text{Likelihood}} \cdot \overbrace{\mathrm{P}(A)}^{\text{Prior}}}{\underbrace{\mathrm{P}(B)}_{\text{Normalizing constant}}}$$

Bayes' Theorem

Definition of Bayes' Theorem

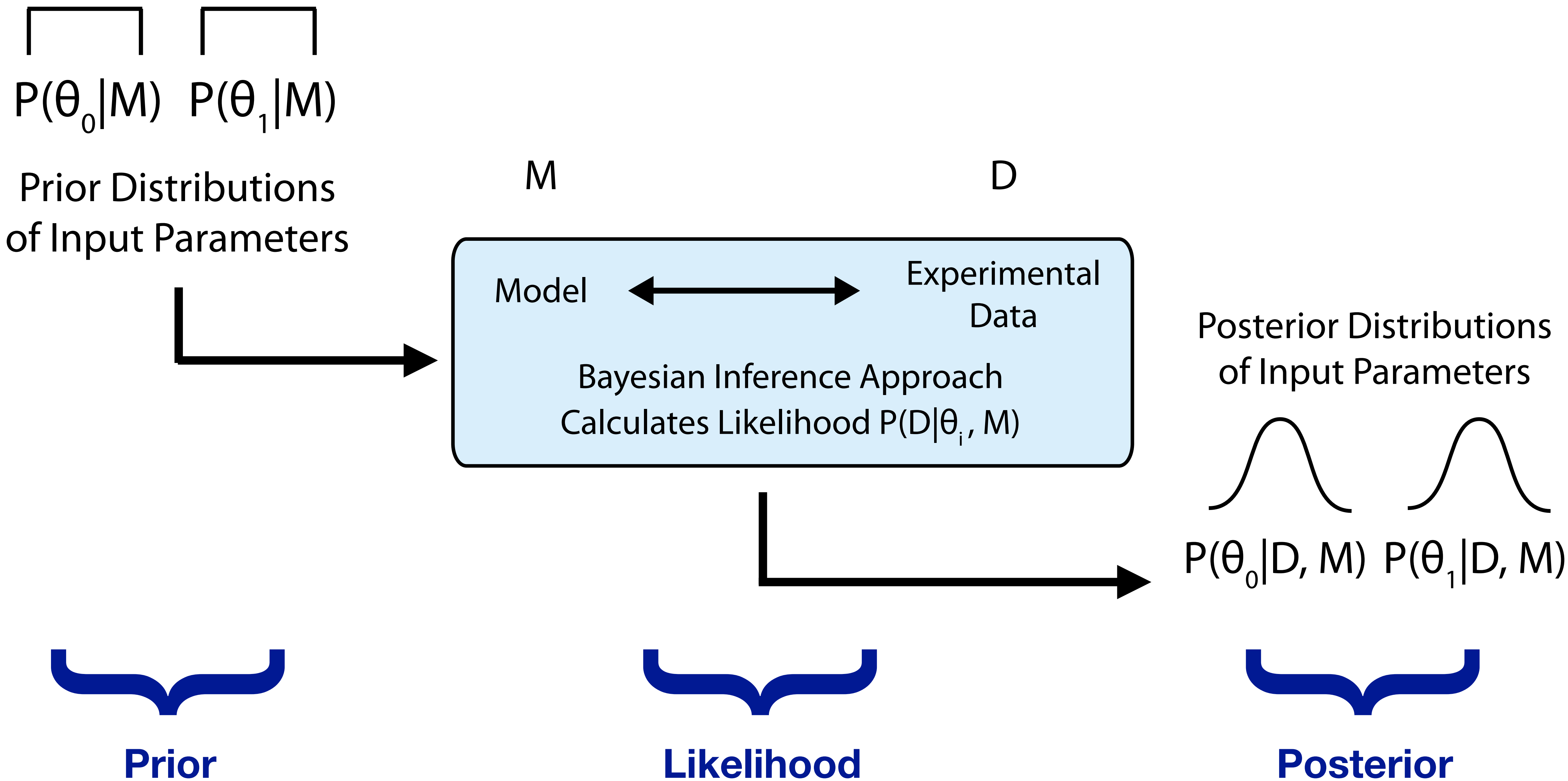
$$\overbrace{\mathrm{P}(A|B)}^{\text{Posterior}} = \frac{\overbrace{\mathrm{P}(B|A)}^{\text{Likelihood}} \cdot \overbrace{\mathrm{P}(A)}^{\text{Prior}}}{\underbrace{\mathrm{P}(B)}_{\text{Normalizing constant}}}$$

Bayes' Theorem for Model Applications

$$\underbrace{\mathrm{P}(\overbrace{\theta}^{\text{Unknown parameter}}|\underbrace{D}_{\text{Data}}, \overbrace{\mathcal{M}}^{\text{Model}})}_{\textbf{Posterior}} \propto \underbrace{\mathrm{P}(D|\theta, \mathcal{M})}_{\textbf{Likelihood}} \cdot \underbrace{\mathrm{P}(\theta|\mathcal{M})}_{\textbf{Prior}}$$

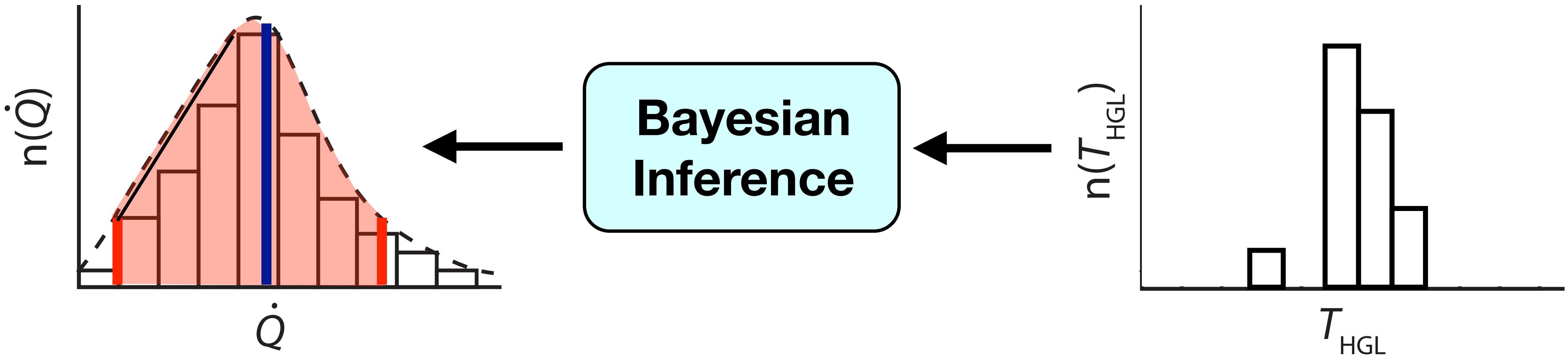
Bayesian Inference Framework

The Bayesian inference process



Bayesian Inference Framework

Bayesian Approach



Distribution of Input Parameter Values

Observed Data

We can describe a probability distribution using:

Average Value

Credible Interval

Applications of Bayesian Inference

Examples using Bayesian inference in fire scenarios

1. Estimate **fire size** using measured HGL temperature data and the CFAST model.

2. Estimate **fire size** using measured heat flux data and a radiation correlation.

3. Estimate **fire location** using heat flux data and a correlation.

4. Use the FDS model to estimate **material properties** using measured mass loss data.

5. Estimate **transient fire size** using HGL temperature data and the CFAST model.

Computational Framework

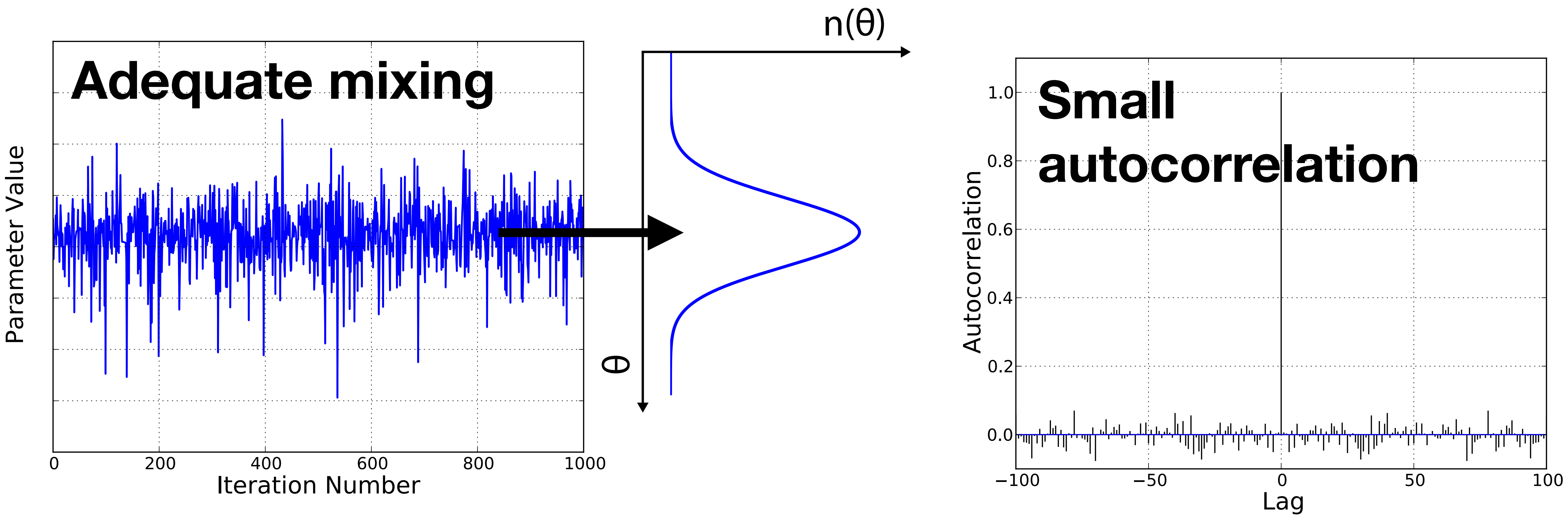
Fire modeling and visualization tools:

- Empirical correlations
- Consolidated Model of Fire and Smoke Transport (CFAST)
- Fire Dynamics Simulator (FDS)
- Smokeview (SMV)

Computational, analysis, and visualization tools:

- Python
- Numeric Python (NumPy)
- Scientific Python (SciPy)
- Math plotting library (matplotlib)
- Bayesian Inference in Python (PyMC)

Results of Bayesian Inference



Example 1 - Estimating Fire Size

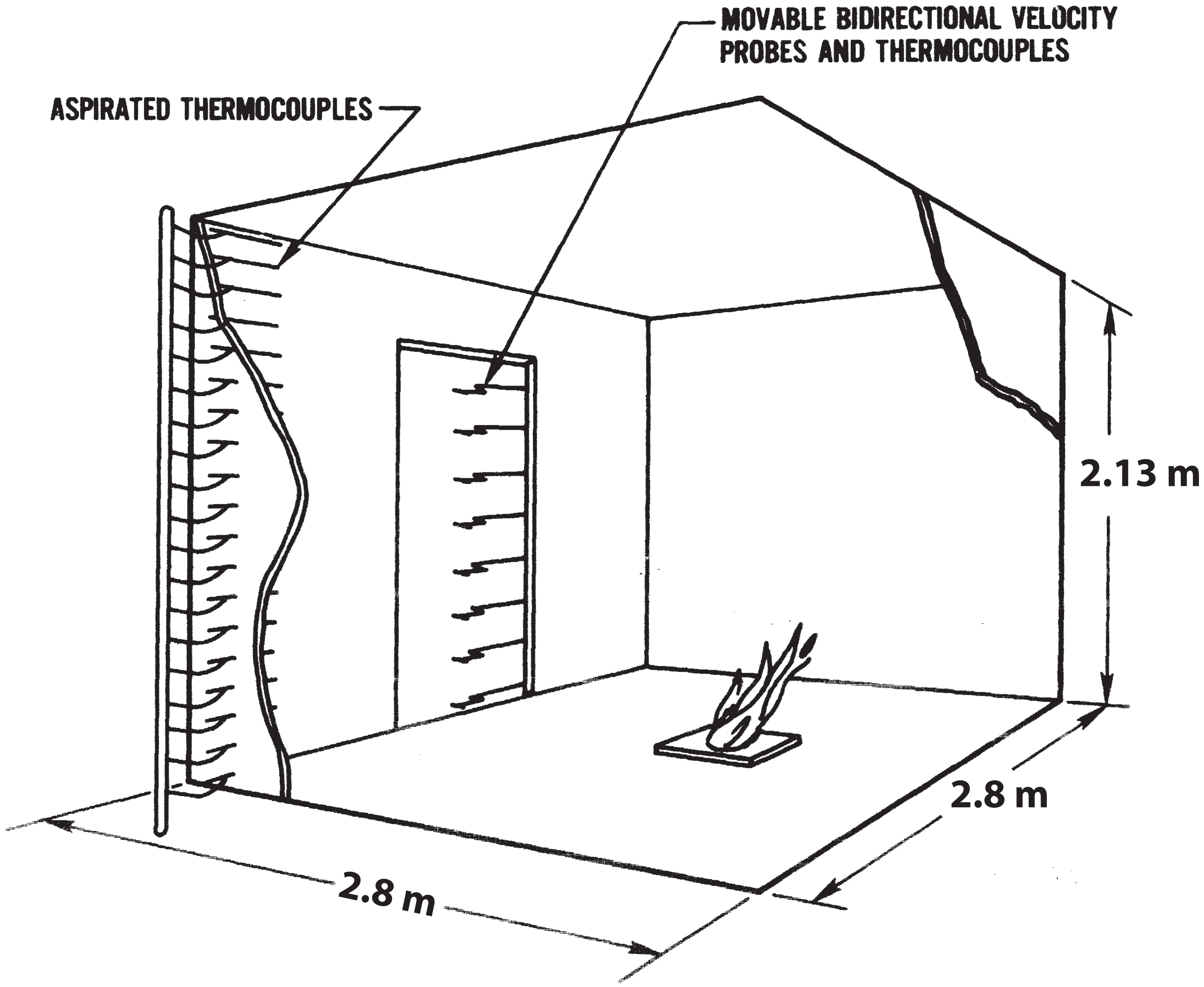
Methane fire in a compartment

Model: CFAST

Unknown parameter: $\dot{Q}$

Prior: Uniform distribution

$$1 \text{ kW} > \dot{Q} > 100 \text{ kW}$$
$$\dot{Q}(i=0) = 50 \text{ kW}$$



K. Steckler, J. Quintiere, and W. Rinkinen, “Flow Induced By Fire in A Compartment,” NBSIR 82-2520, National Bureau of Standards, Gaithersburg, Maryland, September 1982.

Example 1 - Estimating Fire Size

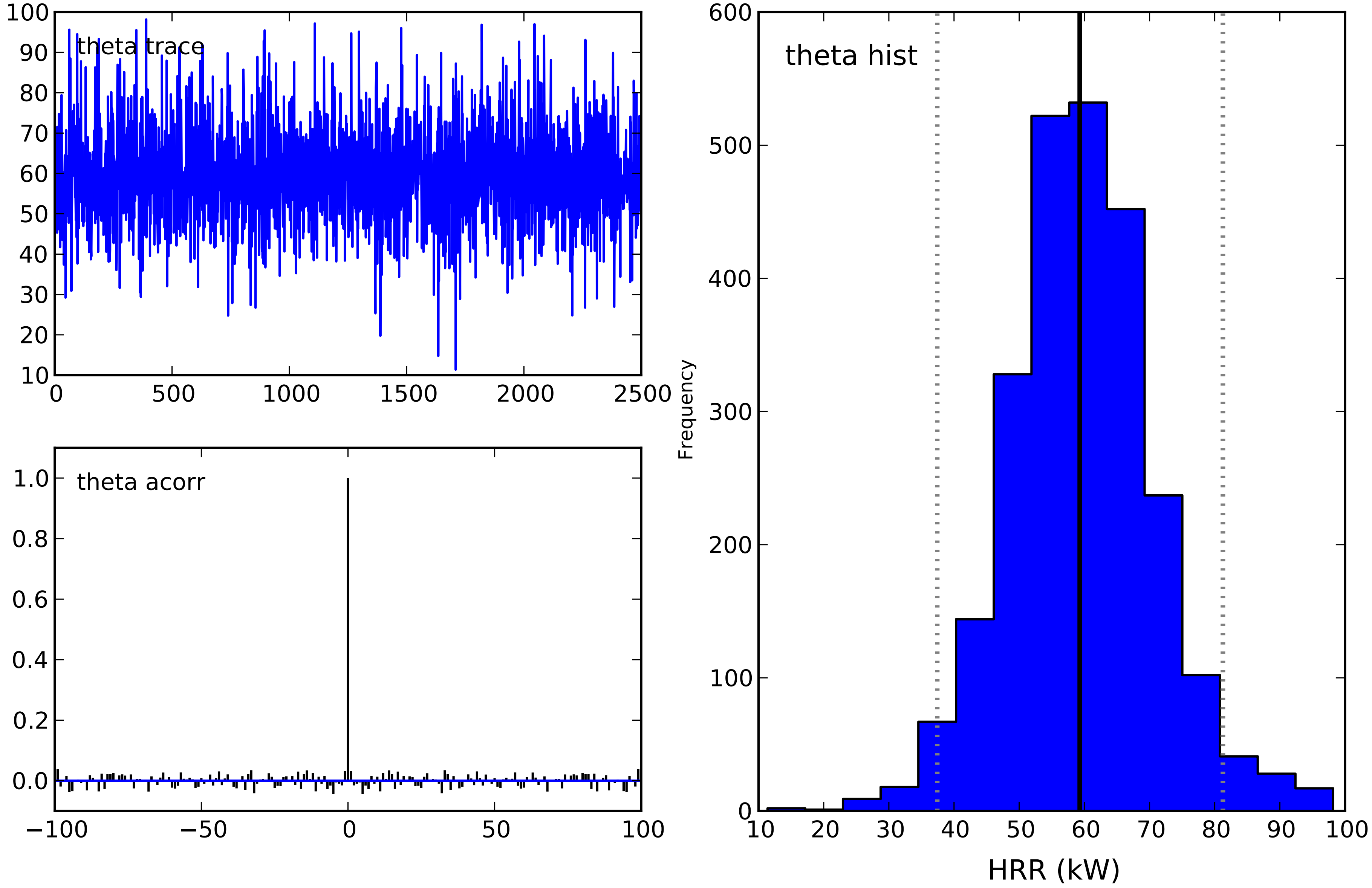
Summary of experimental parameters and measured HGL temperature data for compartment fire tests

Opening Configuration	Door Width (m)	Door Height (m)	$\dot{Q}_{\mathrm{exp}}$ (kW)	T_{HGL} (°C)	T_{∞} (°C)
2/6 Door	0.24	1.83	62.9	190	26
3/6 Door	0.36	1.83	62.9	164	28
4/6 Door	0.49	1.83	62.9	141	22
4/6 Door	0.49	1.83	62.9	135	13
5/6 Door	0.62	1.83	62.9	129	23
6/6 Door	0.74	1.83	62.9	129	29
6/6 Door	0.74	1.83	62.9	130	31
6/6 Door	0.74	1.83	62.9	109	12
6/6 Door	0.74	1.83	62.9	116	13
7/6 Door	0.86	1.83	62.9	120	26
8/6 Door	0.99	1.83	62.9	109	22

Observed data

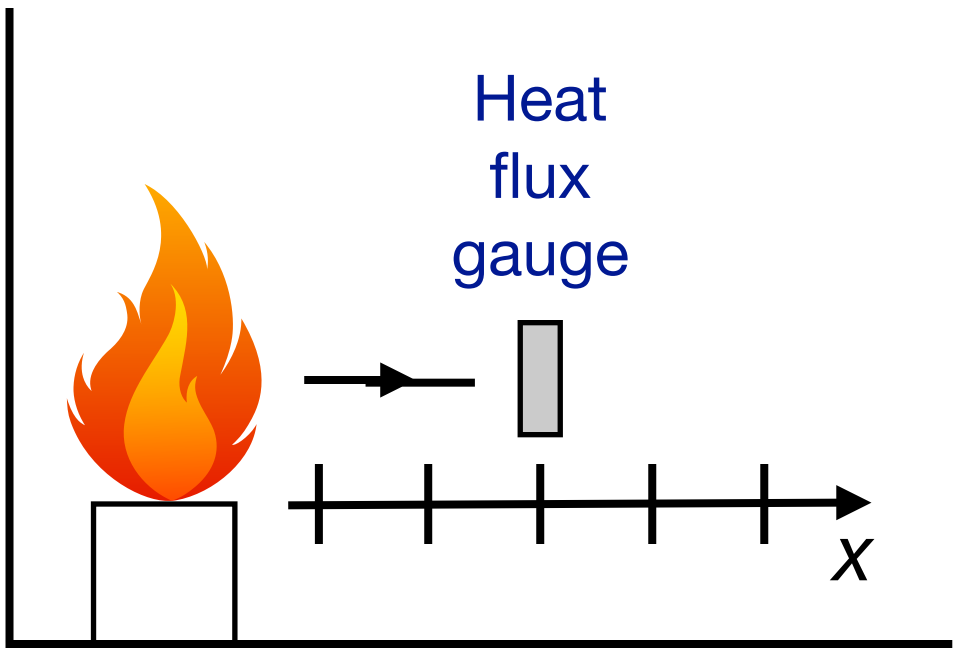
Example 1 - Estimating Fire Size

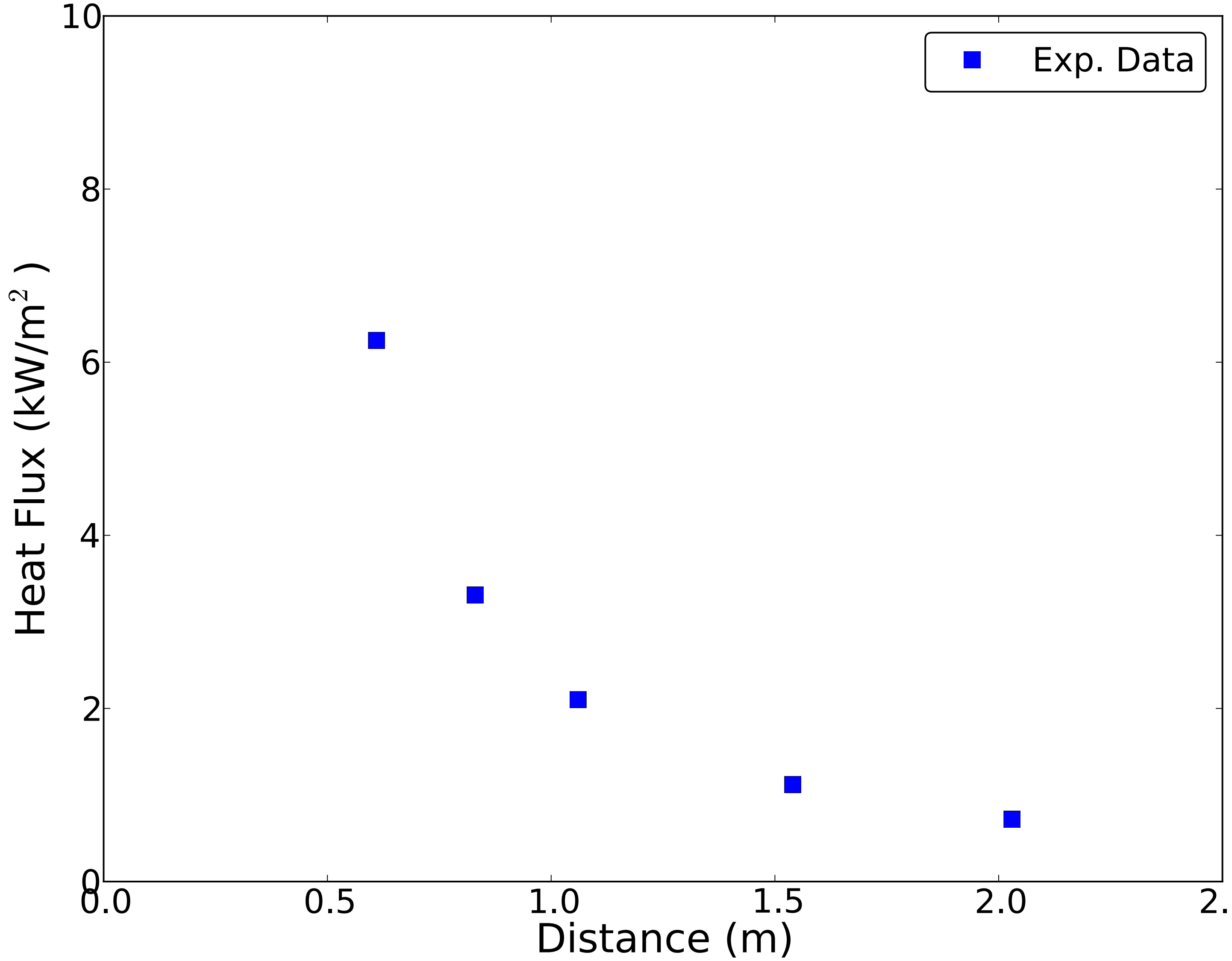
PyMC results for HRR parameter $\dot{Q}$:
Mean: 59.7 kW (reported 63 kW); Standard Deviation: 11.1 kW;
95 % credible interval = [37 kW, 81 kW]



Example 2 - Estimating Fire Size

Measured heat flux vs. distance data; propane burner





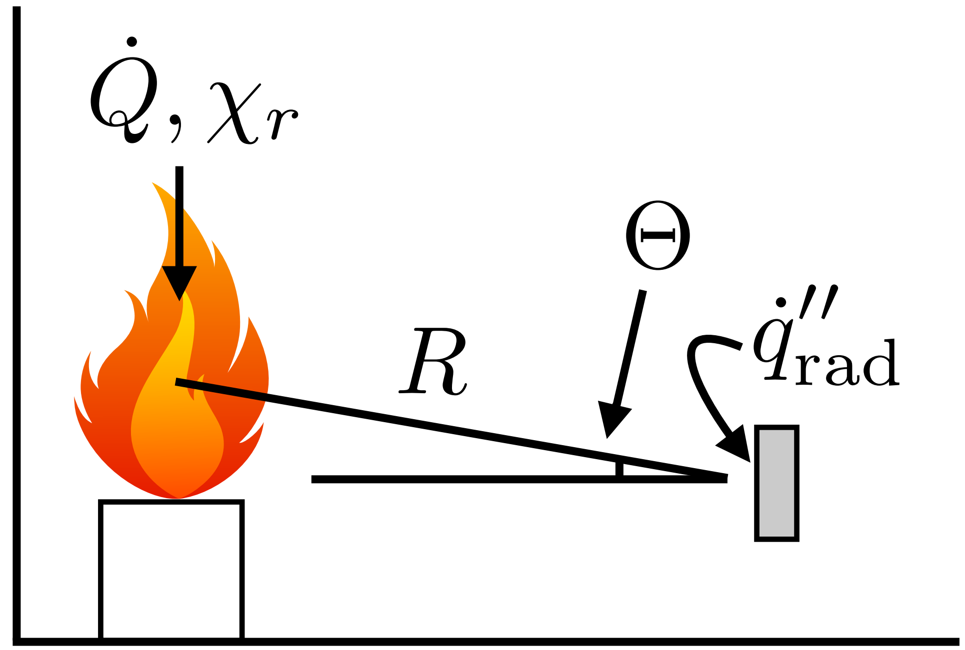
R. Fleury, "Evaluation of Thermal Radiation Models for Fire Spread Between Objects," Master's thesis, University of Canterbury, Christchurch, New Zealand, 2010.

Example 2 - Estimating Fire Size

Model: Point source radiation equation

Known radiative fraction

$$\underbrace{\dot{q}''_{\mathrm{rad}}}_{\text{Measured heat flux data}} = \cos(\Theta)\,\frac{\chi_{\mathrm{r}}\overbrace{\dot{Q}}^{\textbf{Unknown fire size parameter}}}{4\pi \underbrace{R^2}_{\text{Known distance}}}$$



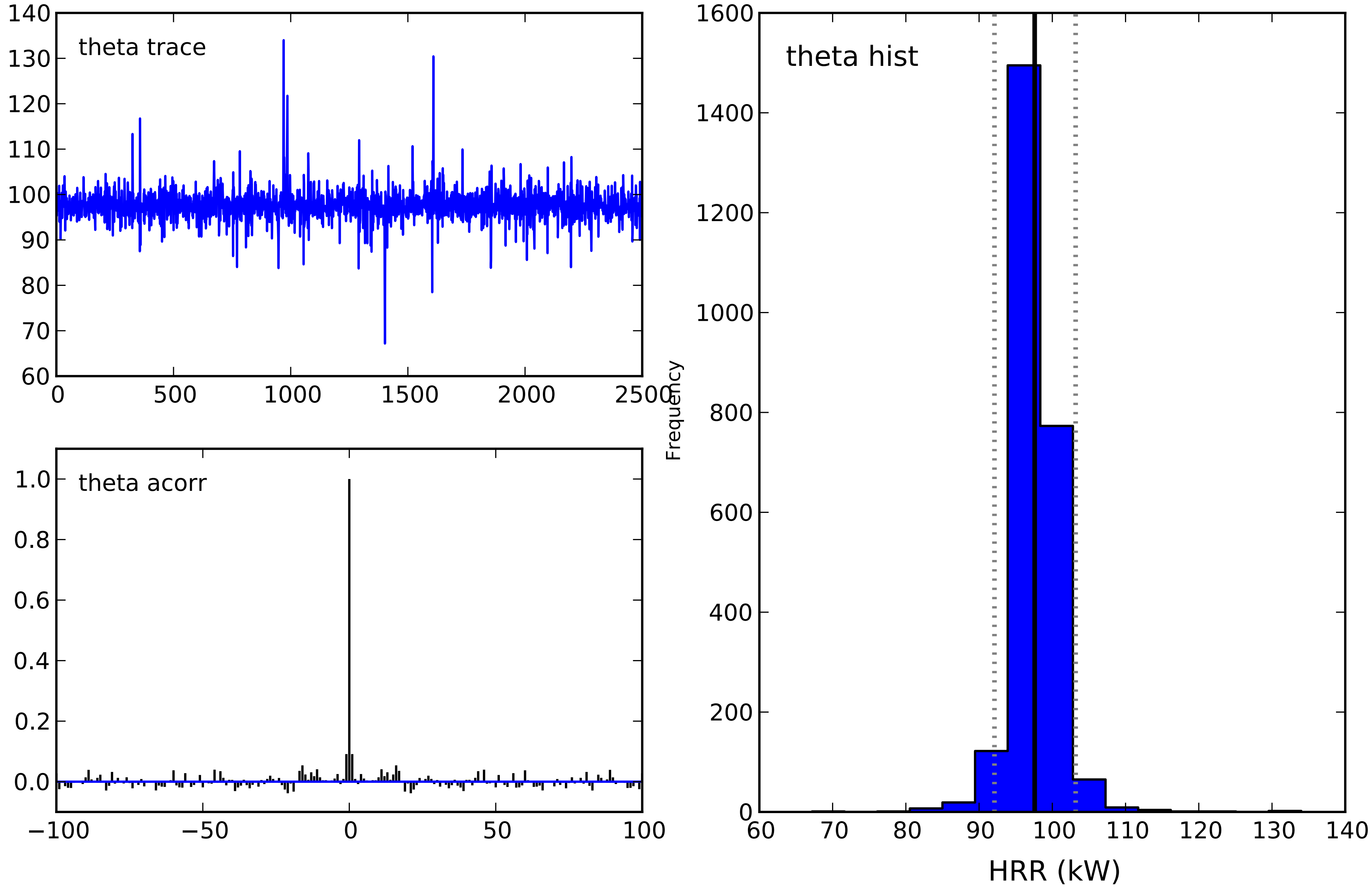
Unknown parameter: $\dot{Q}$

Prior: Uniform distribution: $50\ \mathrm{kW} < \dot{Q} < 300\ \mathrm{kW}$

$$\dot{Q}(i=0) = 200\ \mathrm{kW}$$

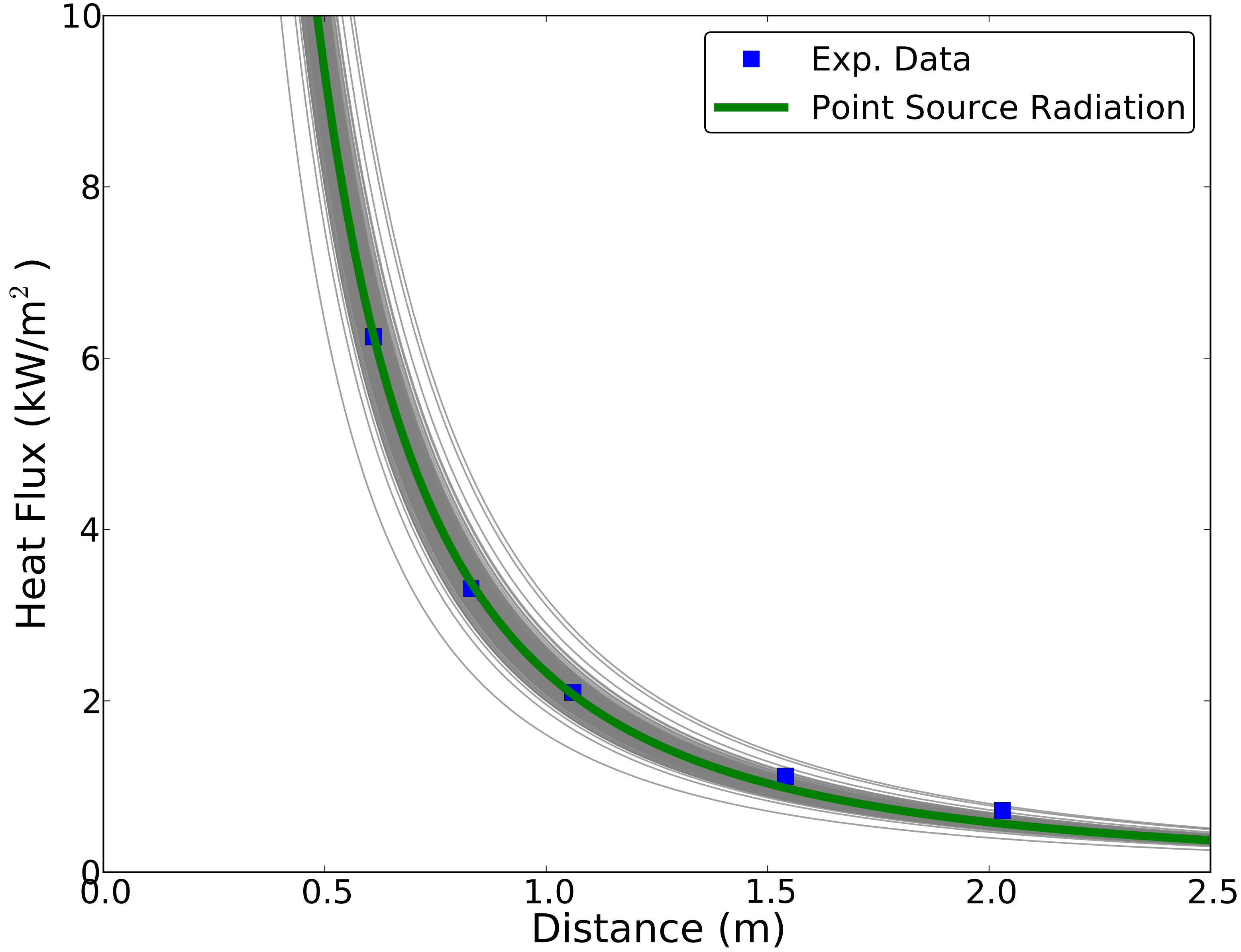
Example 2 - Estimating Fire Size

PyMC results for HRR parameter $\dot{Q}$:
Mean: 97.6 kW (report 100 kW); Standard Deviation: 3.1 kW;
95 % credible interval = [92 kW, 103 kW]



Example 2 - Estimating Fire Size

Results of point source radiation model at posterior mean value (solid green line) and for all values from the posterior distribution (shaded gray area)



Example 3 - Inverse Fire Localization

A fire is located in a compartment measuring 10 m × 10 m × 2.4 m with an open door measuring 0.9 m x 2.4 m. A 300 kW propane fire at (*x*, *y*) = (2, 2) was used in FDS to generate synthetic heat flux data. Six heat flux gauges are located near the fire at a height of 1.2 m.

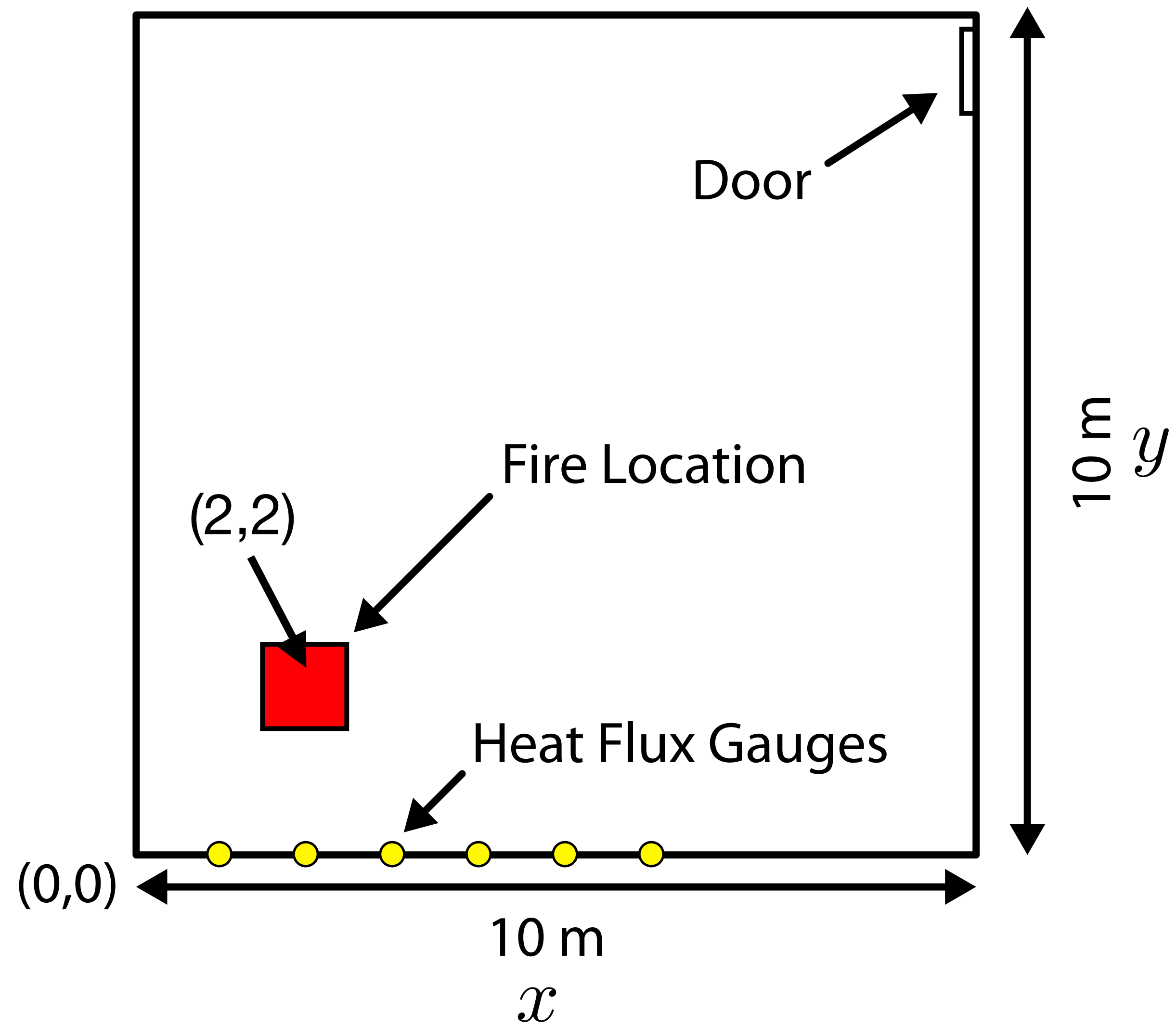
Model: Point source radiation equation

Unknown parameters: x, y

Priors: Uniform distributions

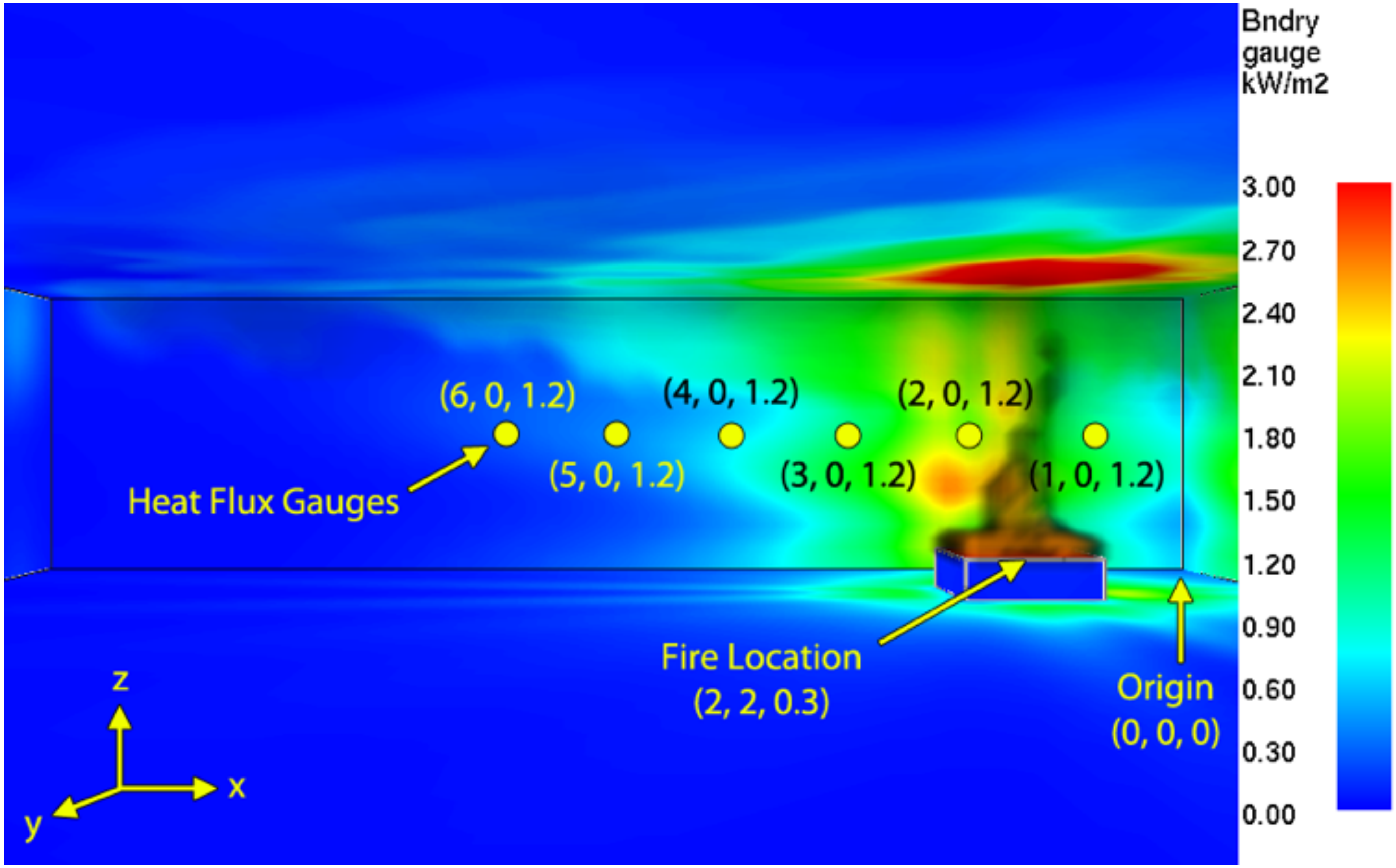
$$0 \text{ m} > x, y > 10 \text{ m}$$

$$x(i = 0) = y(i = 0) = 5 \text{ m}$$



Example 3 - Inverse Fire Localization

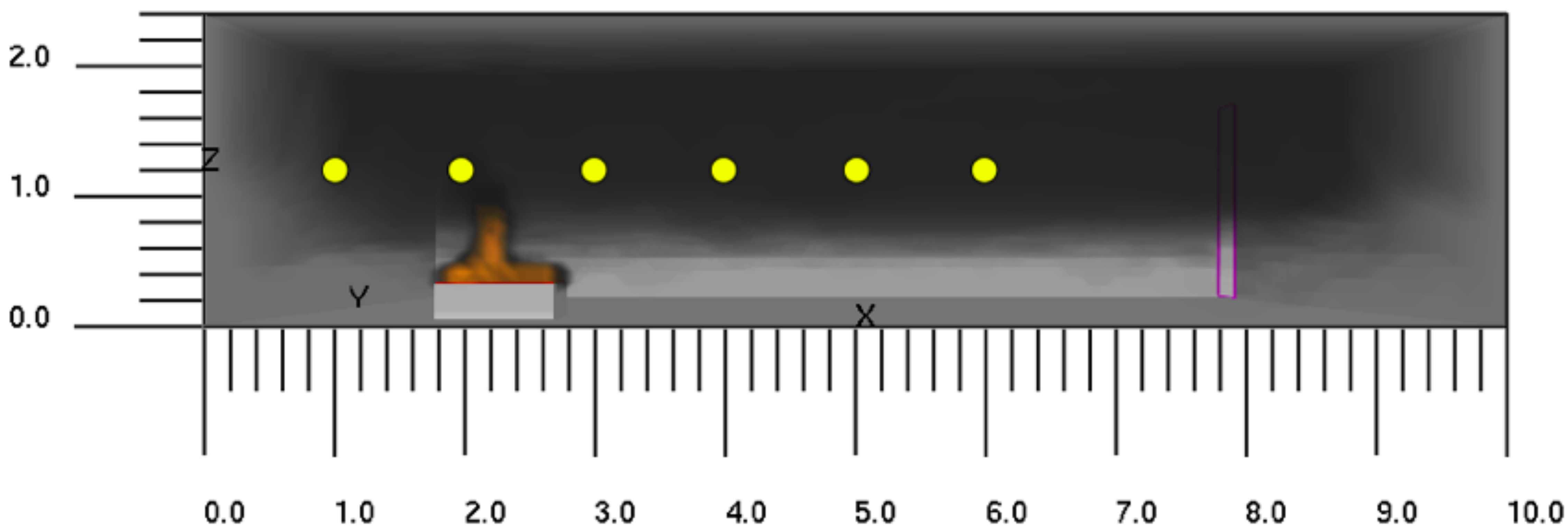
Visualization of gauge heat flux at walls (T_w = 20 °C) for 300 kW case



$$\dot{q}''_{\mathrm{gauge}} = \dot{q}''_{\mathrm{r}}/\varepsilon + \dot{q}''_{\mathrm{c}} + h(T_{\mathrm{w}} - T_{\mathrm{G}}) + \sigma(T_{\mathrm{w}}^4 - T_{\mathrm{G}}^4)$$

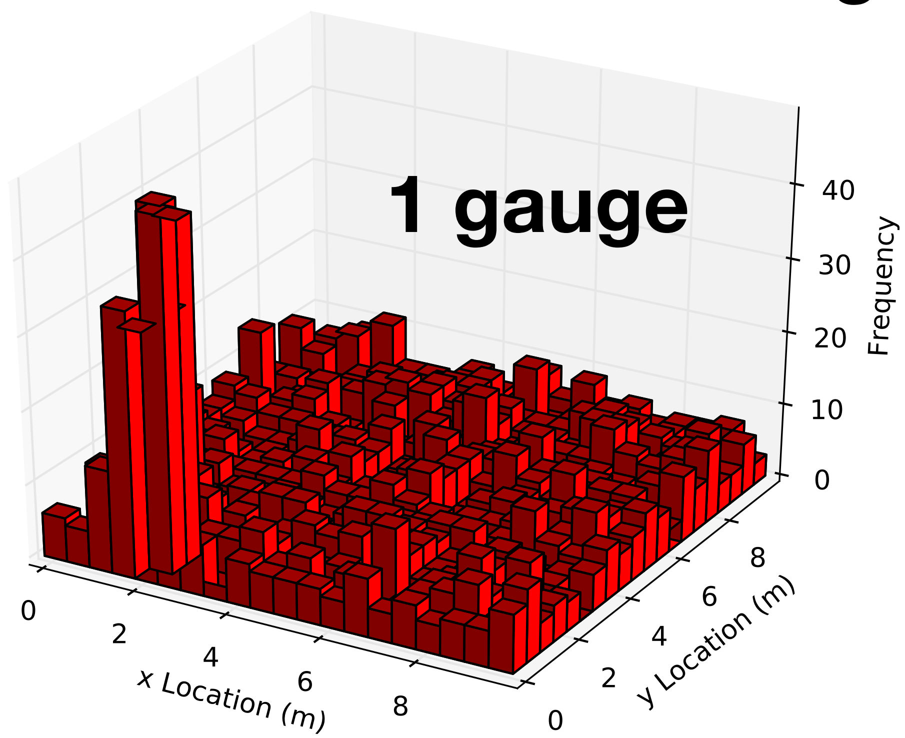
Example 3 - Inverse Fire Localization

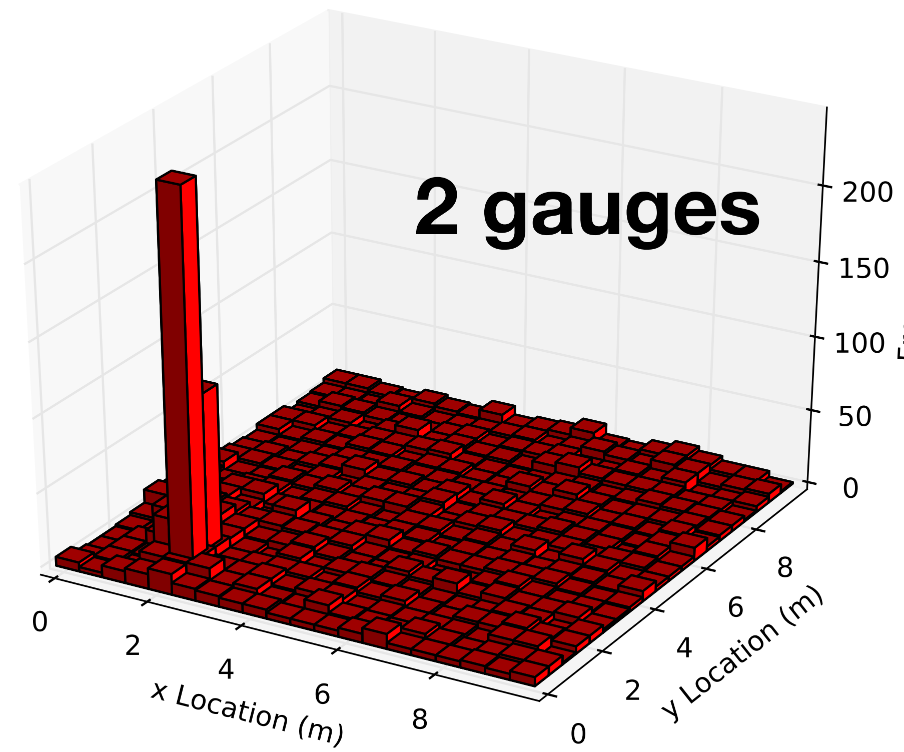
Visualization of fire and smoke layer for 300 kW case at 300 s

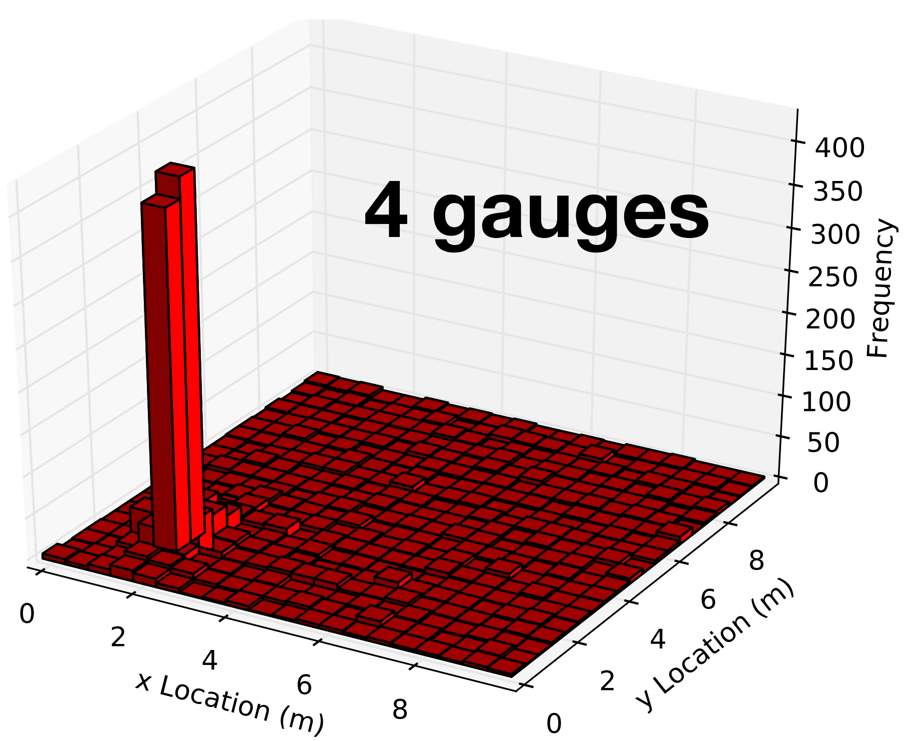


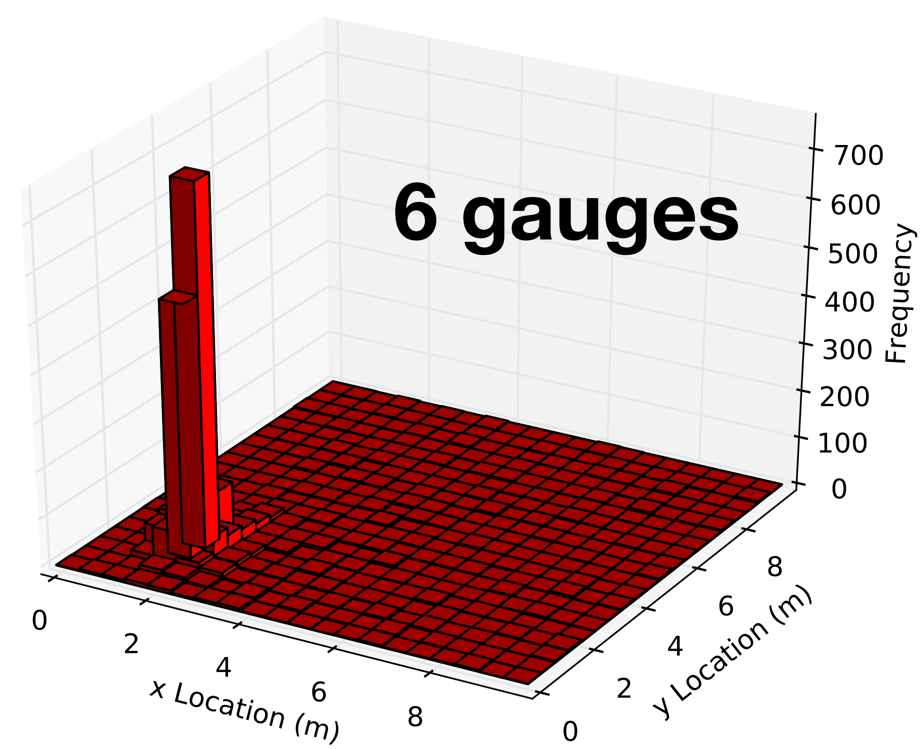
Example 3 - Inverse Fire Localization

300 kW case using gauge heat flux value at 300 s



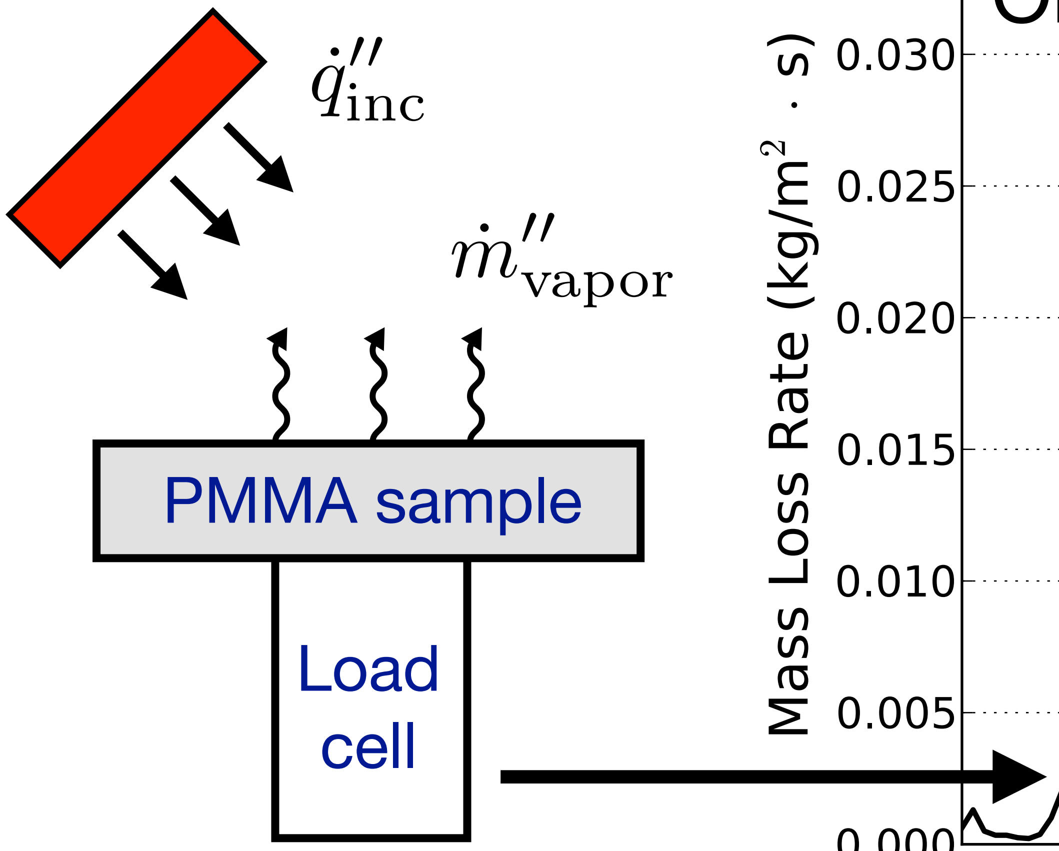


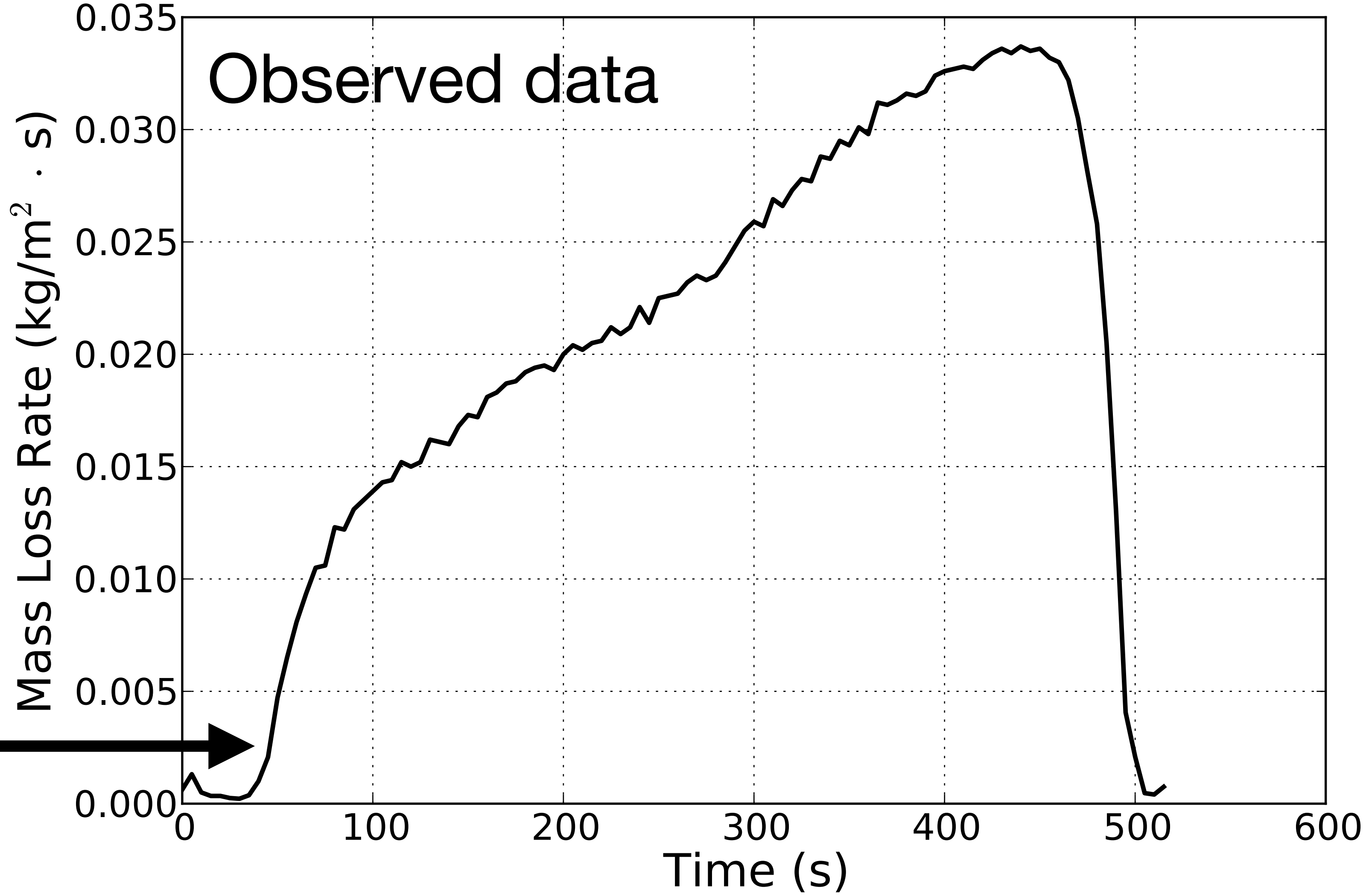




Example 4 - Material Property Estimation

Transient mass loss rate of sample in the NIST gasification apparatus: 8.5 mm thick PMMA sample; incident heat flux was 52 kW/m^2; 1 cm layer of insulation under the PMMA sample.





Example 4 - Material Property Estimation

To predict the mass loss rate of the PMMA sample exposed to an external heat flux, FDS was run in “solid-phase only” mode in which no gas-phase combustion occurs.

Specific Heat

Thermal Conductivity

Heat of reaction

$$\rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} k_s \frac{\partial T}{\partial x} + \dot{q}_s''' \; ; \quad \dot{q}_s''' = \frac{\partial \rho_s}{\partial t} \Delta H_r - \dot{q}_r'''$$

Density

Function of pre-exponential factor, and activation energy

Function of absorption coefficient, emissivity

The pyrolysis model, external heat flux, and 1D heat conduction solver determine the reaction rate of the solid material.

Example 4 - Material Property Estimation

Material properties for PMMA (from literature) and their associated uncertainty values.

Parameter	Literature Value	Uncertainty (%)	Measurement Technique	Source
Absorption Coefficient	2700 1/m	50	FTIR	[119]
Pre-Exponential Factor	8.5×10^{12} 1/s	50	TGA	[120]
Activation Energy	188 000 kJ/kmol	3	TGA	[120]
Emissivity	0.85	20	IS	[121]
Heat of Reaction	870 kJ/kg	15	DSC	[122]
Thermal Conductivity	0.20 $W/m \cdot K$	15	TLC	[120]
Density	1100 kg/m^3	5	Direct	[120]
Specific Heat	2.2 $kJ/kg \cdot K$	15	DSC	[122]

Model: FDS
Unknown parameters: Eight material properties
Priors: Uniform distribution around literature values

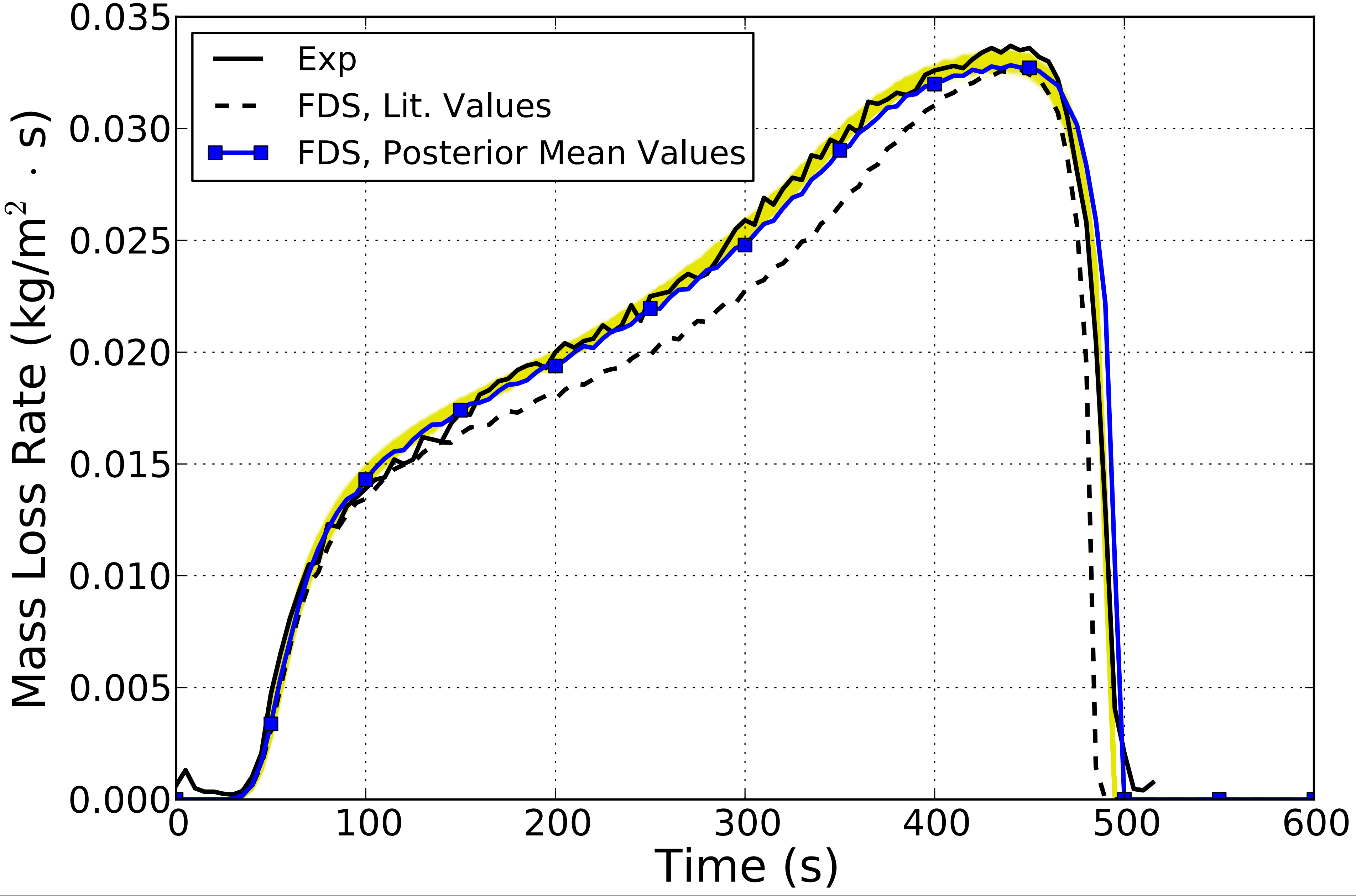
Example 4 - Material Property Estimation

Resulting posterior mean values for the input parameters compared to literature values

Parameter	Units	Posterior Mean Value	Literature Value	Relative Difference
Heat of Reaction	kJ/kg	958	870	10%
Thermal Conductivity	$W/m \cdot K$	0.22	0.20	10%
Density	kg/m^3	1208	1100	9.8%
Specific Heat	$kJ/kg \cdot K$	2.0	2.2	9.1%

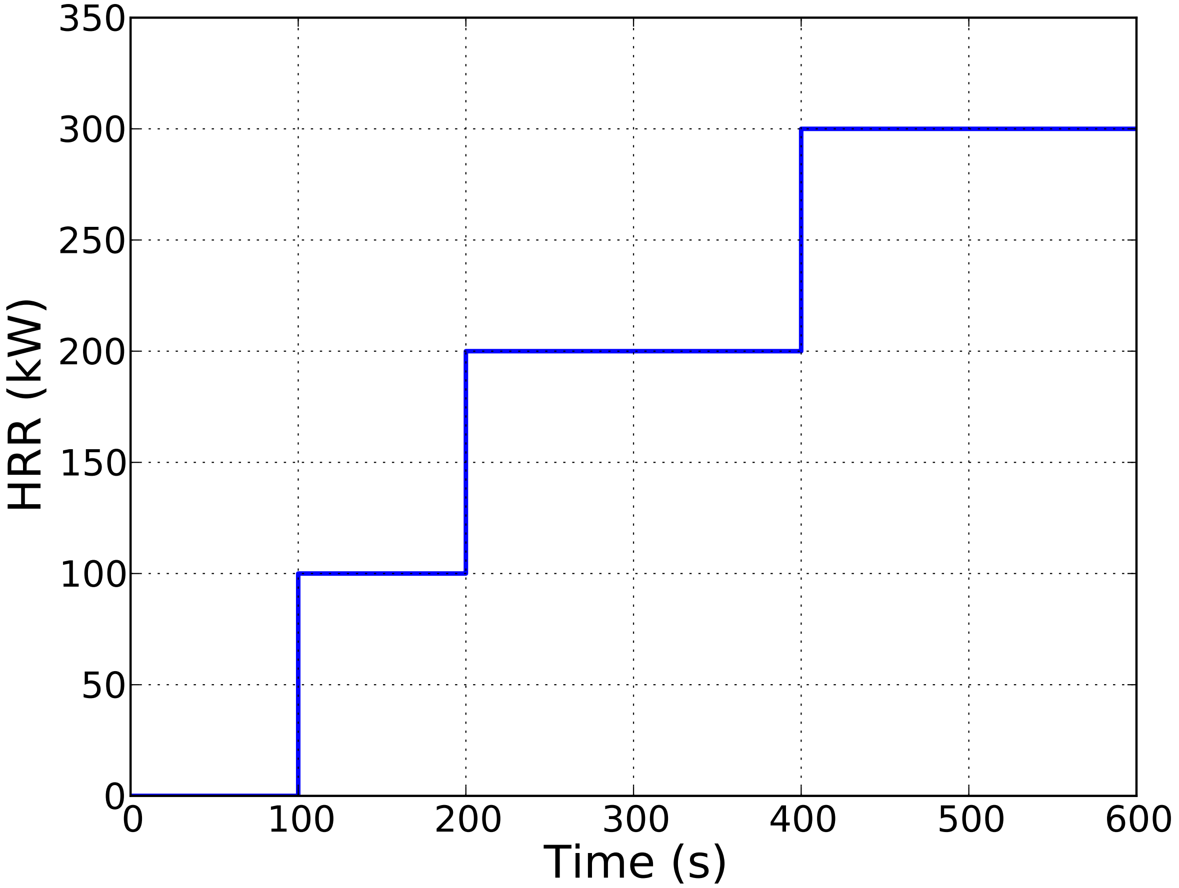
Example 4 - Material Property Estimation

Results from experiment (black line), literature values (dashed line), posterior mean values (blue line with boxes), and individual realizations over all posterior values (yellow region)

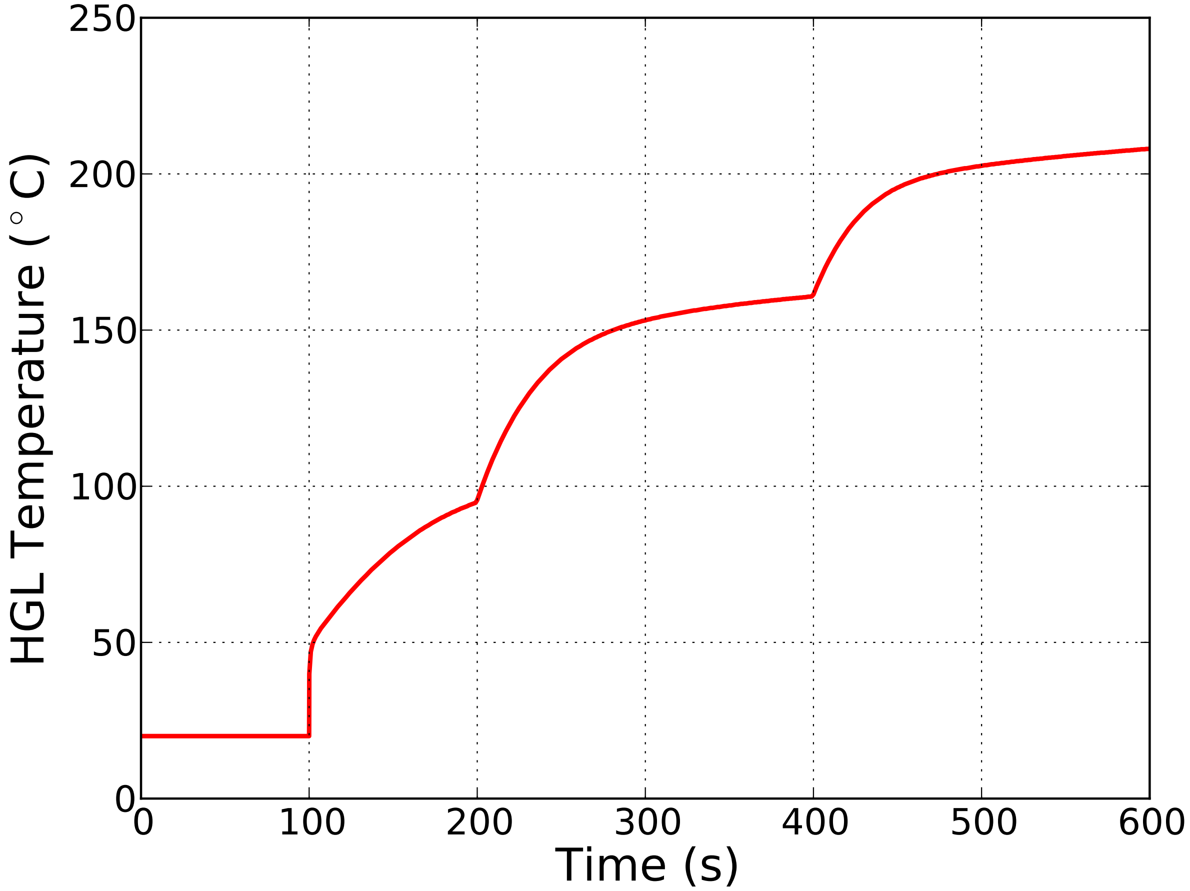


Example 5 - Estimating Transient HRR

Multiple step function HRR curve



**Input HRR
(Actual HRR)**



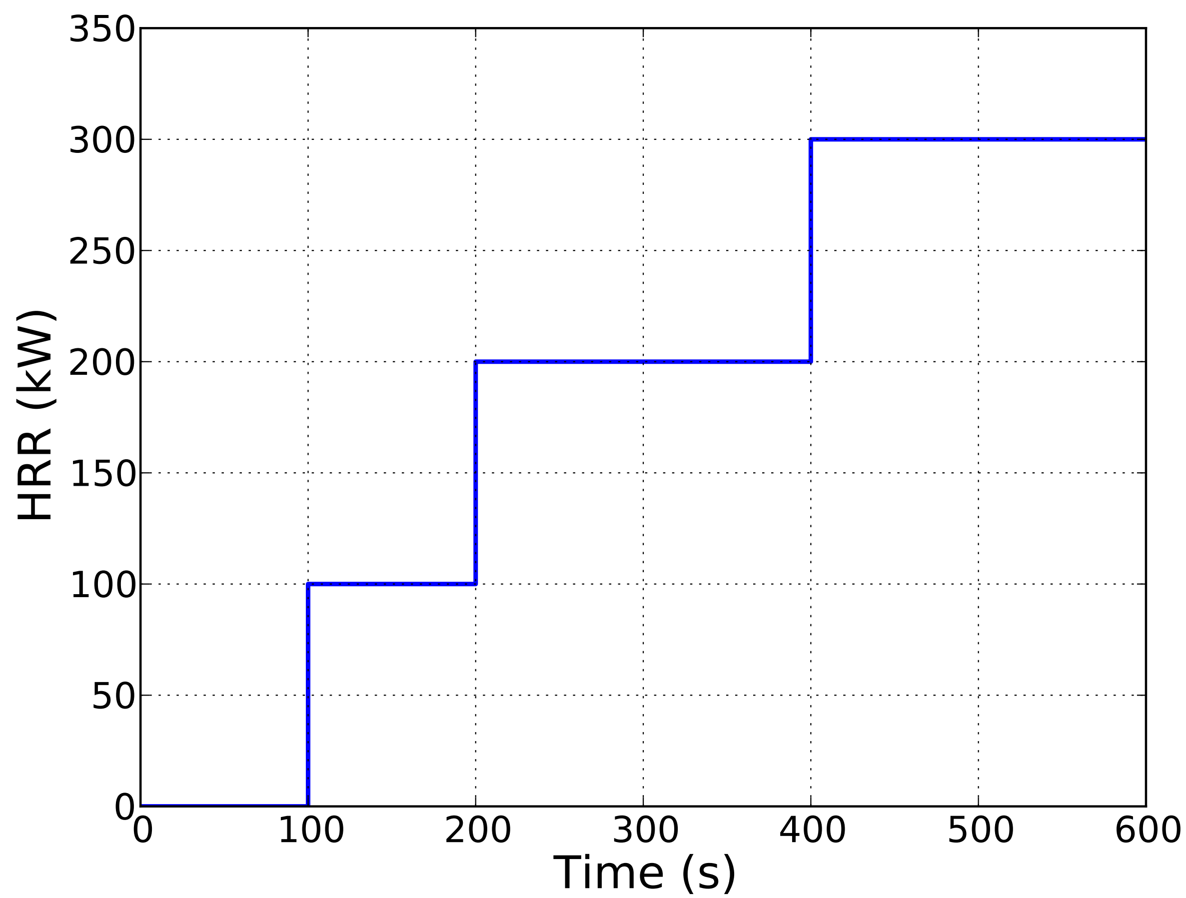
**HGL Temperature
(Observed Data)**

Example 5 - Estimating Transient HRR

The transient HRR is parameterized as a piecewise linear function

$$\dot{Q}(t) = \dot{Q}_i \left(\frac{t - t_{i+1}}{t_i - t_{i+1}} \right) + \dot{Q}_{i+1} \left(\frac{t - t_i}{t_{i+1} - t_i} \right)$$

Unknown fire size parameters



Model: CFAST

61 unknown parameters: $\dot{Q}_i$

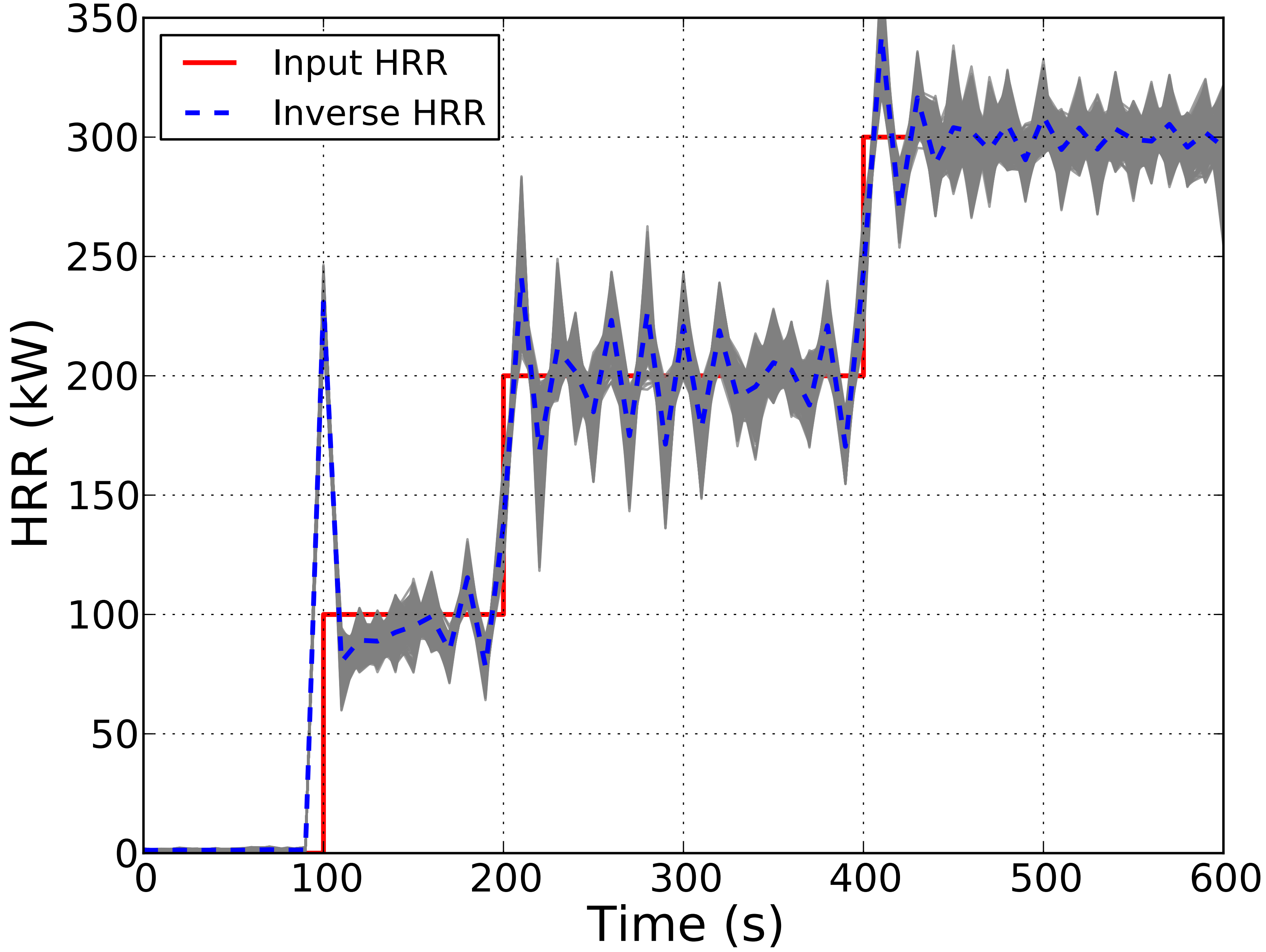
Priors: Uniform distribution

$$0 \text{ kW} > \dot{Q}_i > 500 \text{ kW}$$

$$\dot{Q}_i(i = 0) = 50 \text{ kW}$$

Example 5 - Estimating Transient HRR

Results of CFAST model at posterior mean value (dashed blue line) and for all values from the posterior distribution (shaded gray area)



Conclusions

A Bayesian inference framework was applied to various fire scenarios (in both univariate and multivariate cases) to determine fire size, fire location, and material properties. We can also combine these problems.

The Bayesian inference approach can be used to:

1) Quantify our degree of certainty (or degree of belief) in input parameters or plausible scenarios.

2) Justify decisions based on plausible scenarios.

3) Quantify and accurately communicate uncertainty.

Conclusions

These inversion techniques have applications towards:

1) Fire and arson investigations, and reconstructions of firefighter line-of-duty deaths (LODDs) and injuries (quantifying our uncertainty in input parameters and plausible scenarios).

2) Model validation exercises (validation-driven model development).

3) Risk analyses, probabilistic risk assessments (calculating probability distributions).

Acknowledgements

Funding for this research was provided by the
NIST Department of Commerce
Grant No. 60NANB7D6122,
Los Alamos National Laboratory,
and research internships at NIST and SwRI.

Questions?

- Questions and discussion

koverholt.com

code.google.com/p/bayes-fire