3. Estimating the distance

Q.) As a gravitational wave propagates, it alternatively stretches and compresses the space. By the amplitude of gravitational wave we mean the strain in space of propagation. Use dimensional analysis, giving suitable arguments, to estimate an expression for the intensity of radiated gravitational waves. In the final expression, you can take the proportionality constant to be $\pi/2$.

We know that the Poynting vector in the case of electromagnetic waves, which is proportional to $E^2 \propto \left(\frac{\partial A}{\partial t}\right)^2$ (where A is the vector potential). One then expects the energy flux in gravitational waves to also scale as $\left(\frac{\partial h}{\partial t}\right)^2$. One can think of $\frac{\partial}{\partial t}$ as f, the frequency of radiation. Note that because of the quadrupolar nature of gravitational waves, the frequency of the waves is *twice* the orbital frequency of such a binary system. (In the electromagnetic case, the frequency of the emitted radiation from a rotating dipole is the same as the frequency of rotation.) Therefore the flux is $\propto h^2 f^2$. This should be multiplied by combinations of G and c in order to give us the traditional units of energy flux, and one gets,

$$F \approx \frac{c^3}{G} h^2 f^2$$

Here energy flux 'F' is basically $\frac{E}{At}$ which is intensity
so

$$I \propto \frac{c^3}{G} h^2 f^2$$

Q.) Assume that as the radiation travels, the intensity fall off as square of distance from the source. Using results from previous exercises, derive an expression for the distance to the source.

$$I \propto P$$

$$I \propto \frac{1}{r^2}$$

$$I = \frac{P}{4\pi r^2}$$

Q.) Using the waveform you have drawn, and Figure.1 estimate the distance to our source. How well does it match with existing measurements?

$$P = \frac{32}{5} \frac{G^4 \mu^2 M^3}{c^5 d^5}$$

Equating both intensity expression

$$\frac{P}{4\pi r^2} = \frac{\pi}{2}\frac{c^3}{G}h^2 f^2$$

$$\frac{32}{5}\frac{G^4\mu^2 M^3}{c^5 d^5 \cdot 4\pi r^2} = \frac{\pi}{2}\frac{c^3}{G}h^2 f^2$$

$$r = \sqrt{\frac{64G^5\mu^2 M^3}{5c^8 h^2 d^5 \times 4\pi^2 f^2}}$$