Indian Institute of Technology Guwahati

KRITI '24

Gravitational wave Analysis

(Mid Prep - 400 Points)

Organizer

Equinox & CnA

Gravitational Waves

400 Points

Start : 29/01

End : 08/02

Introduction

Gravitational waves, those cosmic ripples in the fabric of spacetime, are like the heartbeats of the universe, revealing its most dramatic and mysterious events. Imagine massive objects, like black holes or neutron stars, engaged in a cosmic dance, and with each twirl and spin, they send out gravitational waves that traverse the cosmos at the speed of light.

In 2015, scientists achieved one of the greatest scientific feats when they de- tected these elusive waves directly using the Laser Interferometer Gravitational- Wave Observatory (LIGO). This discovery not only confirmed Einstein's mind- boggling predictions but also opened up a cosmic Pandora's box, letting us eavesdrop on the most extreme events in the universe.

Forget what you know about traditional astronomy – gravitational wave astronomy is the rebel of the cosmic sciences. These waves barely interact with matter, giving them the stealth to slip through the universe unnoticed, until now. Picture researchers as cosmic detectives, armed with advanced instruments and a thirst for discovery, chasing after the subtle whispers of the cosmos.

Since that groundbreaking moment, we've detected the gravitational waves from colossal collisions, like black holes crashing into each other and neutron stars doing their celestial tango. Each detection unveils a new chapter in the cosmic saga, offering insights into the nature of space, time, and the breathtaking phenomena playing out on the grand stage of the universe.

So buckle up! Gravitational waves have given us a front-row seat to the cosmic fireworks, and the show is just getting started. As we ride this wave of discovery, who knows what cosmic secrets we'll uncover next? It's a thrilling journey into the unknown, where the universe itself becomes a storyteller, and we're just beginning to grasp the first few chapters of its epic tale.

This being said, wouldn't it be exciting to analyse with our own brains the data collected by these state of the art detectors?
In this problem statement, we will be working out the details of gravitational waves and then use the publicly available data from LIGO to see gravitational waves in action.

Details

There are two parts of the PS.

1. The first part deals with the theoretical aspects of the gravitational waves.
2. The second part aims to use the publicly available data of LIGO detectors and specific libraries to understand a particular binary system(GW150914).

Theoretical Basis of Gravitational Waves

1. Consider a pair of in-spiraling objects(for example, a binary black hole system). Take the objects to be point-like. In this situation, the objects revolve a common centre of mass in circular orbits. Find an expression for the common angular frequency ω with which the objects revolve. Also calculate the total energy of the system. Express this energy in terms of the angular frequency ω.

2. Just as accelerating charges radiate EM waves, accelerating masses radiate gravitational waves. However, this fact is not accounted for in Newtonian gravity. Given that one can determine the gravitational radiation using the time-varying quadrupole moment of a given mass distribution(which is same as the moment of inertia for the system), find an expression for the radiated gravitational power using dimensional analysis. Give a detailed explanation for all the physical quantities on which dependence of radiated power is expected.
   The proportionality constant required in expression of radiated power is 32/5.
   Hint: You may need to make a correspondence with radiation due to time-varying electric dipole moment.
   * The frequency of radiation emitted is twice of the orbital angular frequency ω. Can you explain why?

3. Express the power radiated in terms of only angular frequency ω and the masses in the binary system.

4. Equating the rate of change of total energy to the power radiated, derive an expression relating a mass parameter to the angular frequency ω and its time derivative. This parameter is called the chirp mass.
5. Write chirp mass in terms of the frequency of emitted gravitational radiation and its temporal derivative.
   Cross-check: This is a famous equation!

Data Analysis and NR Techniques

1. Analysing GW Data

A lot of gravitational wave data is openly available on gwosc. For this part, you can either procure data for GW150914 from gwosc(remember, this data is noisy!) or use the github link GW150914 DATA.

1. Using the data, generate a time-series plot for gravitational strain.
2. Analyse the different parts of the plot. Explain the reason why strain has a temporal dependence. Does the frequency depend on time?
3. Assuming frequency is constant in at least one cycle, write an algorithm to estimate the frequency of each cycle. You have to be a little creative from hereon.
4. Using the results from previous exercises, write an algorithm to determine the chirp mass from the gravitational wave data. Hint: You may require to do a curve fit.

5. When do you expect the masses to coalesce with certainty? In other words, when will we observe a binary system to be a single mass irrespective of the distance of observation? From here, give an expression for the total mass of system in terms of parameters we can determine from data.
   * You can make a plot of orbital separation as a function of time to verify your argument.

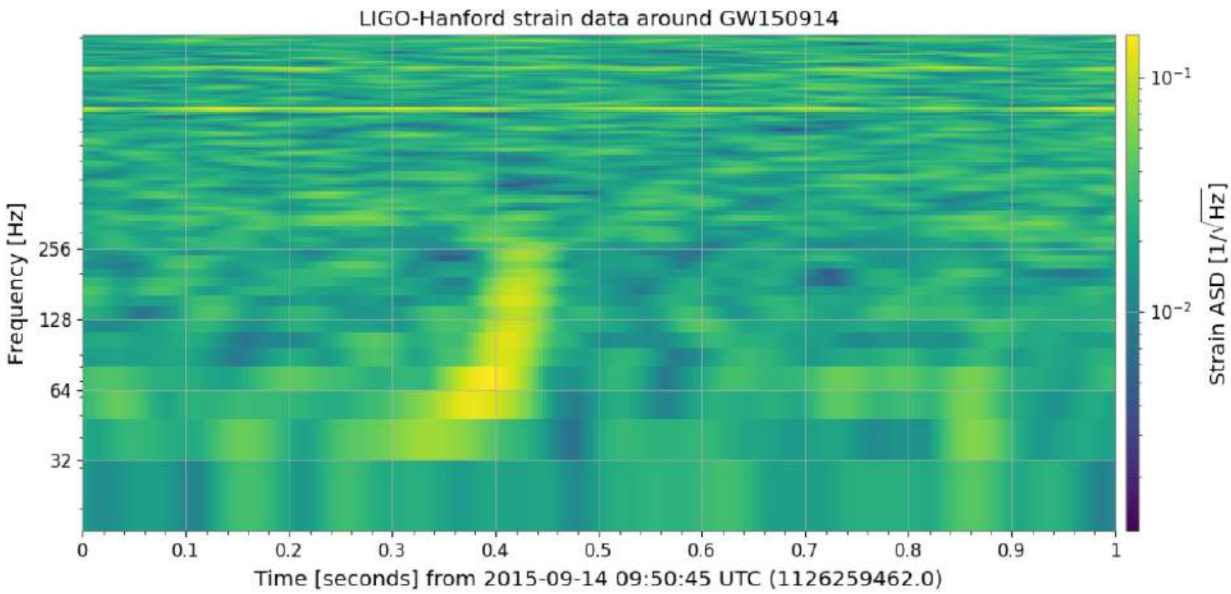

Figure 1: Spectrogram for GW 150914

6. Using Figure 1, and result from above, estimate the individual masses in the system. What sort of astrophysical objects can they possibly be?

2. Drawing Waveform From Parameters

This part requires you to familiarise yourself with the PyCBC library which is very popular in the gravitational wave physics community.

1. Generate a time-domain waveform using a suitable ”td approximant” us- ing the get td waveform function. Hint: Your choice should be such that you need the parameters from the previous exercises only. Remember, parameters can also be omitted. And some of the parameters are specific to the experiment.
2. Zoom in the plot near merger time.
3. How does the waveform change with change in individual masses? * Can you use your findings from this part to improve your estimate of the chirp mass?

3. Estimating the Distance

Estimating the distance of a source in conventional astronomy requires additional inputs. However, this is not the case with gravitational wave astronomy. In this part we will make a rough estimate to the distance of the source(GW150914) we have been studying.

1. As a gravitational wave propagates, it alternatively stretches and compresses the space. By the amplitude of gravitational wave we mean the strain in space of propagation. Use dimensional analysis, giving suitable arguments, to estimate an expression for the intensity of radiated gravitational waves. In the final expression, you can take the proportionality constant to be $\pi/2$.

2. Assume that as the radiation travels, the intensity fall off as square of distance from the source. Using results from previous exercises, derive an expression for the distance to the source.
3. Using the waveform you have drawn, and Figure 1 estimate the distance to our source. How well does it match with existing measurements?

Have fun!

We hope you enjoyed working on the PS and got yourself interested in the fascinating domain of gravitational wave physics. Of course you will be awarded for your hard work through points! But as a supplement, go through the audio signal equivalent of a black hole merger generated using PyCBC.
Sound of GW150914

Team formation

A team can consist of a maximum of 6 members, with the following rules:-

1. There should be at-least 1 first-year student.

Final Submission

Please ensure that you submit a PDF file containing the solution for the theoretical aspect, which comprises the first part of the Problem Statement.

Additionally, please upload the solution for the Data Analysis segment to the GitHub account of one of the team members and provide the link at the time of submission.

Judging Criteria

1. Clarity in the explanation of your solution.
2. Clarity in codes (with proper comments) and mathematically sound numerical techniques.
3. Part 1 : "Theoretical Basis of Gravitational Waves" accounts for 30% of the total points. Each part in the section has equal weight.
4. Part 2 "Data Analysis and NR Techniques " accounts for 70% of the total points.
5. Subsection 1 of Part 2, "Analysing GW Data" has 30% while subsection 2 "Drawing Waveform From Parameters" has 20% weightage. Subsection 3 "Estimating the Distance" accounts for 20% of the total points.
6. Each part in the subsections have equal weight.

General rules

1. **Who can participate**

- The Participating team can have maximum of 6 People out of which minimum 1 should be a fresher.
- The current Core Team Members of Equinox (Third Year and POR Holders) cannot participate in this PS.
- Only one team is allowed to participate from each hostel.

2. **Penalties**

- For any violations of rules, 50% of marks of this PS (i.e. 200 pts) will be subtracted from total points of the Hostel
- Penalty for late submission will be as follows

  i.) For every minute of late submission, 1% of points will be deducted for each minute for the first 15 minutes.

  ii.) 20% of the points will be deducted if submission is between 15 to 30 minutes.

  iii.) 50% of the marks will be deducted if submission is between 30 to 45 minutes.

  iv.)75% of the marks will be deducted if submission is between 45 to 50 minutes.

  v.)After 60 minutes there will be no submission.