GCLC Prover Output for conjecture "thm-0110-Parallelogram2"

Groebner bases method used

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1 Construction and prover internal state

Construction commands:

- Point A
- Point B
- Point C
- Line ab: A B
- Line bc: B C
- Parallel, p: A bc
- Parallel, q: C ab
- Intersection of lines, D: p q
- Line ac: A C
- Line bd: B D
- Intersection of lines, E: ac bd

Coordinates assigned to the points:

- $A = (0, 0)$
- $B = (u_1, 0)$
- $C = (u_2, u_3)$
- $D = (x_2, u_3)$
- $E = (x_4, x_3)$

Conjecture(s):

1. Given conjecture

    - **GCLC code:**

      ```
      same_length A E E C
      ```

    - **Expression:**
      $AE \cong EC$

2 Resolving constructed lines

- $ab \ni A, B$; line is horizontal (i.e., $y(A) = y(B)$)
- $bc \ni B, C$
- $p \ni A, D$
- $q \ni C, D$; line is horizontal (i.e., $y(C) = y(D)$)
- $ac \ni A, C, E$
- $bd \ni B, D, E$

3 Creating polynomials from hypotheses

- Point A
  no condition
- Point B
  no condition
- Point C
  no condition
- Line ab: A B
  - point A is on the line (A, B)
    no condition
  - point B is on the line (A, B)
    no condition
- Line bc: B C
  - point B is on the line (B, C)
    no condition
  - point C is on the line (B, C)
    no condition

- Parallel, p: $A\ bc$
  - Line (A, D) parallel with line (B, C)

$$p_1 = u_3x_2 + (-u_3u_2 + u_3u_1)$$

- Parallel, q: $C\ ab$
  - Line (C, D) parallel with line (A, B)
    — true by the construction
- Intersection of lines, D: $p\ q$
  - point D is on the line (A, D)
    no condition
  - point D is on the line (C, D)
    no condition
- Line ac: $A\ C$
  - point A is on the line (A, C)
    no condition
  - point C is on the line (A, C)
    no condition
- Line bd: $B\ D$
  - point B is on the line (B, D)
    no condition
  - point D is on the line (B, D)
    no condition
- Intersection of lines, E: $ac\ bd$
  - point E is on the line (A, C)

$$p_2 = -u_3x_4 + u_2x_3$$

  - point E is on the line (B, D)

$$p_3 = -u_3x_4 + x_3x_2 - u_1x_3 + u_3u_1$$

4 Creating polynomial from the conjecture

- Processing given conjecture(s).
- Segment $[A, E]$ equal size as segment $[E, C]$

$$p_4 = 2u_2x_4 + 2u_3x_3 + (-u_3^2 - u_2^2)$$

Conjecture 1:

$$p_5 = 2u_2x_4 + 2u_3x_3 + (-u_3^2 - u_2^2)$$

5 Invoking the theorem prover

The used proving method is Buchberger's method.

Input polynomial system is:

$$\begin{array}{rcl} p_0 & = & u_3x_2 + (-u_3u_2 + u_3u_1) \\ p_1 & = & -u_3x_4 + u_2x_3 \\ p_2 & = & -u_3x_4 + x_3x_2 - u_1x_3 + u_3u_1 \end{array}$$

5.1 Iteration 1

Current set is $S_1 =$

$$\begin{array}{rcl} p_0 & = & u_3x_2 + (-u_3u_2 + u_3u_1) \\ p_1 & = & -u_3x_4 + u_2x_3 \\ p_2 & = & -u_3x_4 + x_3x_2 - u_1x_3 + u_3u_1 \end{array}$$

1. Creating S-polynomial from the pair (p_0, p_1).
   Skipping pair p_0 and p_1 because gcd of their leading monoms is zero.
2. Creating S-polynomial from the pair (p_0, p_2).
   Skipping pair p_0 and p_2 because gcd of their leading monoms is zero.
3. Creating S-polynomial from the pair (p_1, p_2).
   Forming S-pol of p_1 and p_2:
   $$p_{12} = u_3x_3x_2 + (-u_3u_2 - u_3u_1)x_3 + u_3^2u_1$$
   S-pol added.

5.2 Iteration 2

Current set is $S_2 =$

$$\begin{array}{rcl} p_0 & = & u_3x_2 + (-u_3u_2 + u_3u_1) \\ p_1 & = & -u_3x_4 + u_2x_3 \\ p_2 & = & -u_3x_4 + x_3x_2 - u_1x_3 + u_3u_1 \\ p_3 & = & -2u_3^2u_1x_3 + u_3^3u_1 \end{array}$$

1. Creating S-polynomial from the pair (p_0, p_3).
   Skipping pair p_0 and p_3 because gcd of their leading monoms is zero.
2. Creating S-polynomial from the pair (p_1, p_3).
   Skipping pair p_1 and p_3 because gcd of their leading monoms is zero.
3. Creating S-polynomial from the pair (p_2, p_3).
   Skipping pair p_2 and p_3 because gcd of their leading monoms is zero.

5.3 Groebner Basis

Groebner basis has 4 polynomials:

$$\begin{aligned}
p_0 &= u_3x_2 + (-u_3u_2 + u_3u_1) \\
p_1 &= -u_3x_4 + u_2x_3 \\
p_2 &= -u_3x_4 + x_3x_2 - u_1x_3 + u_3u_1 \\
p_3 &= -2u_3^2u_1x_3 + u_3^3u_1
\end{aligned}$$

Groebner basis succesfully computed.

6 Reducing Polynomial Conjecture

Reducing with polynomial p_1, the result is:

$$p_{21} = (-2u_3^2 - 2u_2^2)x_3 + (u_3^3 + u_3u_2^2)$$

Reducing with polynomial p_3, the result is:

$$p_{22} = 0$$

Conclusion is reduced to zero.

7 Prover report

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 4 terms.

Time Complexity: Time spent by the prover is 0.001 seconds.There are no ndg conditions.