OpenGeoProver Output for conjecture "Chou 346 (Desargues' Theorem)"

Wu's method used

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1 Validation of Construction Protocol

Construction steps:

- Free point S
- Free point A
- Free point B
- Free point C
- Line SA through two points S and A
- Line SB through two points S and B
- Line SC through two points S and C
- Random point A1 from line SA
- Random point B1 from line SB
- Random point C1 from line SC
- Line BC through two points B and C
- Line B1C1 through two points B1 and C1
- Intersection point P of point sets BC and B1C1
- Line CA through two points C and A
- Line C1A1 through two points C1 and A1
- Intersection point Q of point sets CA and C1A1
- Line AB through two points A and B
- Line A1B1 through two points A1 and B1
- Intersection point R of point sets AB and A1B1

Theorem statement:

• Points P, Q, R are collinear

Validation result: Construction protocol is valid.

2 Transformation of Construction Protocol to algebraic form

Transformation of Construction steps

- 2.1 Transformation of point S:
 - Point S has been assigned following coordinates: (0, 0)
- 2.2 Transformation of point A:
 - Point A has been assigned following coordinates: $(0, u_1)$
- 2.3 Transformation of point B:
 - Point B has been assigned following coordinates: (u_2, u_3)
- 2.4 Transformation of point C:
 - Point C has been assigned following coordinates: (u_4, u_5)
- 2.5 Transformation of point A1:
 - Point A1 has been assigned following coordinates: (u_6, x_1)
 - Polynomial that point A1 has to satisfy is:

$$p = x_1$$

• Processing of polynomial

$$p = x_1$$

Info: Will try to rename X coordinate of point A1

Info: X coordinate of point A1 renamed by zero

• Point A1 has been renamed. Point A1 has been assigned following coordinates: $(0, u_6)$

2.6 Transformation of point B1:

- Point B1 has been assigned following coordinates: (u_7, x_1)
- Polynomial that point B1 has to satisfy is:

$$p = u_2x_1 - u_7u_3$$

• Processing of polynomial

$$p = u_2x_1 - u_7u_3$$

Info: Polynomial

$$p = u_2x_1 - u_7u_3$$

added to system of polynomials that represents the constructions

• New polynomial added to system of hypotheses

2.7 Transformation of point C1:

- Point C1 has been assigned following coordinates: (u_8, x_2)
- Polynomial that point C1 has to satisfy is:

$$p = u_4x_2 - u_8u_5$$

• Processing of polynomial

$$p = u_4x_2 - u_8u_5$$

Info: Polynomial

$$p = u_4x_2 - u_8u_5$$

added to system of polynomials that represents the constructions

• New polynomial added to system of hypotheses

2.8 Transformation of point P:

- Point P has been assigned following coordinates: (x_3, x_4)
- Polynomial that point P has to satisfy is:

$$p = (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3)$$

• Processing of polynomial

$$p = (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3)$$

Info: Polynomial

$$p = (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3)$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point P has to satisfy is:

$$p = (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1$$

• Processing of polynomial

$$p = (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1$$

Info: Polynomial

$$p = (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1$$

added to system of polynomials that represents the constructions

• New polynomial added to system of hypotheses

2.9 Transformation of point Q:

- Point Q has been assigned following coordinates: (x_5, x_6)
- Polynomial that point Q has to satisfy is:

$$p = u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1$$

• Processing of polynomial

$$p = u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1$$

Info: Polynomial

$$p = u_4 x_6 + (-u_5 + u_1) x_5 - u_4 u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point Q has to satisfy is:

$$p = u_8 x_6 - x_5 x_2 + u_6 x_5 - u_8 u_6$$

• Processing of polynomial

$$p = u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6$$

Info: Polynomial

$$p = u_8 x_6 - x_5 x_2 + u_6 x_5 - u_8 u_6$$

added to system of polynomials that represents the constructions

• New polynomial added to system of hypotheses

2.10 Transformation of point R:

- Point R has been assigned following coordinates: (x_7, x_8)
- Polynomial that point R has to satisfy is:

$$p = u_2 x_8 + (-u_3 + u_1)x_7 - u_2 u_1$$

• Processing of polynomial

$$p = u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1$$

Info: Polynomial

$$p = u_2 x_8 + (-u_3 + u_1)x_7 - u_2 u_1$$

added to system of polynomials that represents the constructions

- New polynomial added to system of hypotheses
- Polynomial that point R has to satisfy is:

$$p = u_7x_8 - x_7x_1 + u_6x_7 - u_7u_6$$

• Processing of polynomial

$$p = u_7 x_8 - x_7 x_1 + u_6 x_7 - u_7 u_6$$

Info: Polynomial

$$p = u_7 x_8 - x_7 x_1 + u_6 x_7 - u_7 u_6$$

added to system of polynomials that represents the constructions

• New polynomial added to system of hypotheses

Transformation of Theorem statement

• Polynomial for theorem statement:

$$p = x_8x_5 - x_8x_3 - x_7x_6 + x_7x_4 + x_6x_3 - x_5x_4$$

Time spent for transformation of Construction Protocol to algebraic form

 \bullet 0.059 seconds

3 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

```
\begin{array}{rcl} p_1 & = & u_2x_1 - u_7u_3 \\ p_2 & = & u_4x_2 - u_8u_5 \\ p_3 & = & (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\ p_4 & = & (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1 \\ p_5 & = & u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\ p_6 & = & u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6 \\ p_7 & = & u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \\ p_8 & = & u_7x_8 - x_7x_1 + u_6x_7 - u_7u_6 \end{array}
```

3.1 Triangulation, step 1

Choosing variable: Trying the variable with index 8.

Variable x_8 selected: The number of polynomials with this variable, with indexes from 1 to 8, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_8 from all other polynomials by reducing them with polynomial p_7 from previous step.

Finished a triangulation step, the current system is:

```
\begin{array}{rcl} p_1 & = & u_2x_1 - u_7u_3 \\ p_2 & = & u_4x_2 - u_8u_5 \\ p_3 & = & (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\ p_4 & = & (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1 \\ p_5 & = & u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\ p_6 & = & u_8x_6 - x_5x_2 + u_6x_5 - u_8u_6 \\ p_7 & = & -u_2x_7x_1 + (u_7u_3 - u_7u_1 + u_6u_2)x_7 + (-u_7u_6u_2 + u_7u_2u_1) \\ p_8 & = & u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \end{array}
```

3.2 Triangulation, step 2

Choosing variable: Trying the variable with index 7.

Variable x_7 selected: The number of polynomials with this variable, with indexes from 1 to 7, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_7 . No reduction needed.

The triangular system has not been changed.

3.3 Triangulation, step 3

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable, with indexes from 1 to 6, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_6 from all other polynomials by reducing them with polynomial p_5 from previous step.

Finished a triangulation step, the current system is:

```
\begin{array}{rcl} p_1 & = & u_2x_1 - u_7u_3 \\ p_2 & = & u_4x_2 - u_8u_5 \\ p_3 & = & (u_4 - u_2)x_4 + (-u_5 + u_3)x_3 + (u_5u_2 - u_4u_3) \\ p_4 & = & (u_8 - u_7)x_4 - x_3x_2 + x_3x_1 + u_7x_2 - u_8x_1 \\ p_5 & = & -u_4x_5x_2 + (u_8u_5 - u_8u_1 + u_6u_4)x_5 + (-u_8u_6u_4 + u_8u_4u_1) \\ p_6 & = & u_4x_6 + (-u_5 + u_1)x_5 - u_4u_1 \\ p_7 & = & -u_2x_7x_1 + (u_7u_3 - u_7u_1 + u_6u_2)x_7 + (-u_7u_6u_2 + u_7u_2u_1) \\ p_8 & = & u_2x_8 + (-u_3 + u_1)x_7 - u_2u_1 \end{array}
```

3.4 Triangulation, step 4

Choosing variable: Trying the variable with index 5.

Variable x_5 selected: The number of polynomials with this variable, with indexes from 1 to 5, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_5 . No reduction needed.

The triangular system has not been changed.

3.5 Triangulation, step 5

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable, with indexes from 1 to 4, is 2.

Minimal degrees: 2 polynomial(s) with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3 from previous step.

Finished a triangulation step, the current system is:

```
\begin{array}{lll} p_1 &=& u_2x_1-u_7u_3\\ p_2 &=& u_4x_2-u_8u_5\\ p_3 &=& (-u_4+u_2)x_3x_2+(u_4-u_2)x_3x_1+\\ && (u_8u_5-u_8u_3-u_7u_5+u_7u_3)x_3+(u_7u_4-u_7u_2)x_2+\\ && (-u_8u_4+u_8u_2)x_1+\\ && (-u_8u_5u_2+u_8u_4u_3+u_7u_5u_2-u_7u_4u_3)\\ p_4 &=& (u_4-u_2)x_4+(-u_5+u_3)x_3+(u_5u_2-u_4u_3)\\ p_5 &=& -u_4x_5x_2+(u_8u_5-u_8u_1+u_6u_4)x_5+(-u_8u_6u_4+u_8u_4u_1)\\ p_6 &=& u_4x_6+(-u_5+u_1)x_5-u_4u_1\\ p_7 &=& -u_2x_7x_1+(u_7u_3-u_7u_1+u_6u_2)x_7+(-u_7u_6u_2+u_7u_2u_1)\\ p_8 &=& u_2x_8+(-u_3+u_1)x_7-u_2u_1 \end{array}
```

3.6 Triangulation, step 6

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable, with indexes from 1 to 3, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_3 . No reduction needed.

The triangular system has not been changed.

3.7 Triangulation, step 7

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable, with indexes from 1 to 2, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_2 . No reduction needed.

The triangular system has not been changed.

3.8 Triangulation, step 8

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable, with indexes from 1 to 1, is 1.

Single polynomial with chosen variable: Chosen polynomial is p_1 . No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{array}{lll} p_1 &=& u_2x_1-u_7u_3\\ p_2 &=& u_4x_2-u_8u_5\\ p_3 &=& (-u_4+u_2)x_3x_2+(u_4-u_2)x_3x_1+\\ && (u_8u_5-u_8u_3-u_7u_5+u_7u_3)x_3+(u_7u_4-u_7u_2)x_2+\\ && (-u_8u_4+u_8u_2)x_1+\\ && (-u_8u_5u_2+u_8u_4u_3+u_7u_5u_2-u_7u_4u_3)\\ p_4 &=& (u_4-u_2)x_4+(-u_5+u_3)x_3+(u_5u_2-u_4u_3)\\ p_5 &=& -u_4x_5x_2+(u_8u_5-u_8u_1+u_6u_4)x_5+(-u_8u_6u_4+u_8u_4u_1)\\ p_6 &=& u_4x_6+(-u_5+u_1)x_5-u_4u_1\\ p_7 &=& -u_2x_7x_1+(u_7u_3-u_7u_1+u_6u_2)x_7+(-u_7u_6u_2+u_7u_2u_1)\\ p_8 &=& u_2x_8+(-u_3+u_1)x_7-u_2u_1 \end{array}$$

4 Final Remainder

4.1 Final remainder for conjecture Chou 346 (Desargues' Theorem)

Calculating final remainder of the conclusion:

$$g = x_8x_5 - x_8x_3 - x_7x_6 + x_7x_4 + x_6x_3 - x_5x_4$$

with respect to the triangular system.

1. Pseudo remainder with p_8 over variable x_8 :

$$g = -u_2x_7x_6 + (u_3 - u_1)x_7x_5 + u_2x_7x_4 + (-u_3 + u_1)x_7x_3 + u_2x_6x_3 - u_2x_5x_4 + u_2u_1x_5 - u_2u_1x_3$$

2. Pseudo remainder with p_7 over variable x_7 :

$$g = -u_2^2 x_6 x_3 x_1 + (u_7 u_3 u_2 - u_7 u_2 u_1 + u_6 u_2^2) x_6 x_3 + (-u_7 u_6 u_2^2 + u_7 u_2^2 u_1) x_6 + u_2^2 x_5 x_4 x_1 + (-u_7 u_3 u_2 + u_7 u_2 u_1 - u_6 u_2^2) x_5 x_4 - u_2^2 u_1 x_5 x_1 + (u_7 u_6 u_3 u_2 - u_7 u_6 u_2 u_1 + u_6 u_2^2 u_1) x_5 + (u_7 u_6 u_2^2 - u_7 u_2^2 u_1) x_4 + u_2^2 u_1 x_3 x_1 + (-u_7 u_6 u_3 u_2 + u_7 u_6 u_2 u_1 - u_6 u_2^2 u_1) x_3$$

3. Pseudo remainder with p_6 over variable x_6 :

$$\begin{array}{lll} g & = & u_4u_2^2x_5x_4x_1 + \\ & & (-u_7u_4u_3u_2 + u_7u_4u_2u_1 - u_6u_4u_2^2)x_5x_4 + \\ & & (-u_5u_2^2 + u_2^2u_1)x_5x_3x_1 + \\ & & (u_7u_5u_3u_2 - u_7u_5u_2u_1 - u_7u_3u_2u_1 + u_7u_2u_1^2 + \\ & & u_6u_5u_2^2 - u_6u_2^2u_1) \\ & & x_5x_3 \\ & & -u_4u_2^2u_1x_5x_1 + \\ & & (-u_7u_6u_5u_2^2 + u_7u_6u_4u_3u_2 - u_7u_6u_4u_2u_1 + \\ & & u_7u_6u_2^2u_1 + u_7u_5u_2^2u_1 - u_7u_2^2u_1^2 + \\ & & u_6u_4u_2^2u_1) \\ & & x_5 \\ & & + (u_7u_6u_4u_2^2 - u_7u_4u_2^2u_1)x_4 + \\ & & (-u_7u_6u_4u_3u_2 + u_7u_6u_4u_2u_1 + u_7u_4u_3u_2u_1 - \\ & & -u_7u_4u_2u_1^2) \\ & & x_3 \\ & & + (-u_7u_6u_4u_2^2u_1 + u_7u_4u_2^2u_1^2) \end{array}$$

4. Pseudo remainder with p_5 over variable x_5 :

$$\begin{array}{lll} g&=&(-u_7u_6u_4^2u_2^2+u_7u_4^2u_2^2u_1)x_4x_2+\\ &&(u_8u_6u_4^2u_2^2-u_8u_4^2u_2^2u_1)x_4x_1+\\ &&(u_8u_7u_6u_5u_4u_2^2-u_8u_7u_6u_4^2u_3u_2+\\ &&u_8u_7u_6u_4^2u_2u_1-u_8u_7u_6u_4u_2^2u_1\\ &&-u_8u_7u_5u_4u_2^2u_1+u_8u_7u_4^2u_3u_2u_1\\ &&-u_8u_7u_4^2u_2u_1^2+u_8u_7u_4u_2^2u_1^2\\ &&-u_8u_6^2u_4^2u_2^2+u_8u_6u_4^2u_2^2u_1+\\ &&u_7u_6^2u_4^2u_2^2-u_7u_6u_4^2u_2^2u_1)\\ &&x_4\\ &+\\ &&(u_7u_6u_4^2u_3u_2-u_7u_6u_4^2u_2u_1-u_7u_4^2u_3u_2u_1+\\ &&u_7u_4^2u_2u_1^2)\\ &&x_3x_2\\ &&+\\ &&(-u_8u_6u_5u_4u_2^2+u_8u_6u_4u_2^2u_1+u_8u_5u_4u_2^2u_1\\ &&-u_8u_4u_2^2u_1^2)\\ &&x_3x_1\\ &&+\\ &&(u_8u_6^2u_5u_4u_2^2-u_8u_6^2u_4u_2^2u_1\\ &&-u_8u_6u_5u_4u_2^2u_1+u_8u_6u_4u_2^2u_1^2\\ &&-u_8u_6u_5u_4u_2^2u_1+u_8u_6u_4u_2^2u_1^2\\ &&-u_8u_6u_5u_4u_2^2u_1+u_8u_6u_4u_2^2u_1^2\\ \end{array}$$

$$\begin{aligned} &-u_7u_6^2u_4^2u_3u_2+u_7u_6^2u_4^2u_2u_1+\\ &u_7u_6u_4^2u_3u_2u_1-u_7u_6u_4^2u_2u_1^2)\\ &x_3\\ &+(u_7u_6u_4^2u_2^2u_1-u_7u_4^2u_2^2u_1^2)x_2+\\ &(-u_8u_6u_4^2u_2^2u_1+u_8u_4^2u_2^2u_1^2)x_1+\\ &(-u_8u_7u_6^2u_5u_4u_2^2+u_8u_7u_6^2u_4^2u_3u_2\\ &-u_8u_7u_6^2u_4^2u_2u_1+u_8u_7u_6^2u_4u_2^2u_1+\\ &u_8u_7u_6u_5u_4u_2^2u_1-u_8u_7u_6u_4^2u_3u_2u_1+\\ &u_8u_7u_6u_4^2u_2u_1^2-u_8u_7u_6u_4u_2^2u_1^2+\\ &u_8u_6u_4^2u_2^2u_1-u_8u_6u_4u_2^2u_1^2\\ &-u_8u_6^2u_4^2u_2^2u_1-u_8u_6u_4^2u_2^2u_1^2\\ &-u_7u_6^2u_4^2u_2^2u_1+u_7u_6u_4^2u_2^2u_1^2)\end{aligned}$$

5. Pseudo remainder with p_4 over variable x_4 :

$$\begin{array}{lll} g&=&(-u_7u_6u_5u_4^2u_2^2+u_7u_6u_4^3u_3u_2\\ &-u_7u_6u_4^3u_2u_1+u_7u_6u_4^2u_2^2u_1+\\ &u_7u_5u_4^2u_2^2u_1-u_7u_4^3u_3u_2u_1+\\ &u_7u_4^3u_2u_1^2-u_7u_4^2u_2^2u_1^2)\\ &x_3x_2\\ &+\\ &(u_8u_6u_5u_4u_2^3-u_8u_6u_4^2u_3u_2^2+\\ &u_8u_6u_4^2u_2^2u_1-u_8u_6u_4u_2^3u_1\\ &-u_8u_5u_4u_2^3u_1+u_8u_4^2u_3u_2^2u_1\\ &-u_8u_4^2u_2^2u_1^2+u_8u_4u_2^3u_1^2)\\ &x_3x_1\\ &+\\ &(u_8u_7u_6u_5^2u_4u_2^2-u_8u_7u_6u_5u_4^2u_3u_2+\\ &u_8u_7u_6u_5u_4^2u_2u_1-u_8u_7u_6u_5u_4u_3u_2^2\\ &-u_8u_7u_6u_5u_4u_2^2u_1+u_8u_7u_6u_4u_3u_2^2u_1\\ &-u_8u_7u_6u_4^2u_3u_2u_1+u_8u_7u_6u_4u_3u_2^2u_1\\ &-u_8u_7u_5^2u_4u_2^2u_1+u_8u_7u_5u_4^2u_3u_2u_1+\\ &-u_8u_7u_5u_4^2u_2u_1^2-u_8u_7u_4u_3u_2^2u_1+\\ &u_8u_7u_5u_4u_2^2u_1^2-u_8u_7u_4u_3u_2^2u_1+\\ &u_8u_7u_4^2u_3u_2u_1^2-u_8u_7u_4u_3u_2^2u_1+\\ &u_8u_6u_5u_4u_2^3u_1-u_8u_6u_4^2u_3u_2+\\ &-u_8u_6^2u_5u_4u_2^3+u_8u_6^2u_4^2u_3u_2+\\ &-u_8u_6^2u_4^2u_2^2u_1+u_8u_6u_4^2u_3u_2^2+\\ &-u_8u_6^2u_4^2u_2^2u_1+u_8u_6u_4^2u_3u_2+\\ &u_8u_6u_5u_4u_2^3u_1-u_8u_6u_4^2u_3u_2+\\ &u_8u_6u_5u_4u_2^3u_1-u_8u_6u_4^2u_3u_2+\\ &u_8u_6u_4^2u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_5u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_4u_2^2u_1^2-u_8u_6u_4u_2^3u_1+\\ &u_8u_6u_4u_$$

$$\begin{array}{l} u_7u_6^2u_4^3u_2u_1 - u_7u_6^2u_4^2u_2^2u_1 \\ -u_7u_6u_5u_4^2u_2^2u_1 + u_7u_6u_4^3u_3u_2u_1 \\ -u_7u_6u_5u_4^2u_2^2u_1 + u_7u_6u_4^2u_2^2u_1^2) \\ x_3 \\ + \\ (u_7u_6u_5u_4^2u_2^3 - u_7u_6u_4^3u_3u_2^2 + \\ u_7u_6u_4^3u_2^2u_1 - u_7u_6u_4^2u_2^3u_1 \\ -u_7u_5u_4^2u_2^3u_1 + u_7u_4^3u_3u_2^2u_1 \\ -u_7u_4^3u_2^2u_1^2 + u_7u_4^2u_2^3u_1^2) \\ x_2 \\ + \\ (-u_8u_6u_5u_4^2u_2^3 + u_8u_6u_4^3u_3u_2^2 \\ -u_8u_6u_4^3u_2^2u_1 - u_8u_4^3u_3u_2^2u_1 + \\ u_8u_5u_4^2u_2^3u_1 - u_8u_4^3u_3u_2^2u_1 + \\ u_8u_5u_4^2u_2^3u_1 - u_8u_4^3u_3u_2^2u_1 + \\ u_8u_7u_6^2u_5u_4^2u_2^2 + u_8u_7u_6^2u_5u_4u_2^3 + \\ u_8u_7u_6^2u_4^3u_3u_2 - u_8u_7u_6^2u_4^3u_2u_1 - \\ -u_8u_7u_6^2u_4^2u_3u_1^2 - u_8u_7u_6u_4^2u_2^2u_1 + \\ u_8u_7u_6u_4u_2^3u_1 - u_8u_7u_6u_4^2u_2^2u_1^2 + \\ u_8u_7u_6u_4u_2^3u_1^2 + u_8u_7u_6u_4^2u_2^2u_1^2 + \\ u_8u_7u_6u_4u_2^3u_1^2 + u_8u_7u_5u_4u_2^2u_1^2 + \\ u_8u_7u_5u_4u_2^3u_1^2 + u_8u_7u_5u_4^2u_2^2u_1^2 + \\ u_8u_7u_5u_4u_2^3u_1^2 + u_8u_7u_4^2u_3^2u_2u_1 - \\ -u_8u_7u_5u_4u_2^3u_1^2 + u_8u_7u_4^2u_3^2u_2u_1 + \\ -u_8u_6u_5u_4^2u_3^2u_1 - u_8u_6^2u_4^2u_3^2u_1 - \\ -u_8u_6u_4u_2^2u_1^2 - u_8u_6u_4^2u_2^3u_1 + \\ -u_8u_6u_5u_4^2u_3^2u_1 + u_8u_6u_4^3u_3u_2^2u_1 + \\ -u_8u_6u_4u_2^2u_1^2 + u_8u_6u_4^2u_3^2u_1 + \\ -u_8u_6u_5u_4^2u_3^2u_1 + u_8u_6u_4^3u_3u_2^2u_1 + \\ -u_8u_6u_4^2u_2^2u_1^2 + u_8u_6u_4^2u_3^2u_1 + \\ -u_8u_6u_4^2u_2^2u_1^2 + u_7u_6^2u_4^2u_3^2u_1 + \\ u_7u_6u_5u_4^2u_3^2u_1 - u_7u_6u_4^2u_3^2u_1 + \\ u_7u_6u_4^2u_2^2u_1^2 - u_7u_6u_4^2u_3^2u_1 +$$

6. Pseudo remainder with p_3 over variable x_3 :

$$g = (u_7^2 u_6 u_5 u_4^3 u_2^2 - u_7^2 u_6 u_5 u_4^2 u_2^3)$$

```
-u_7^2u_6u_4^4u_3u_2+u_7^2u_6u_4^4u_2u_1+
u_7^2 u_6 u_4^3 u_3 u_2^2 - 2 u_7^2 u_6 u_4^3 u_2^2 u_1 +
u_7^2u_6u_4^2u_2^3u_1 - u_7^2u_5u_4^3u_2^2u_1 +
u_7^2 u_5 u_4^2 u_2^3 u_1 + u_7^2 u_4^4 u_3 u_2 u_1
-u_7^2u_4^4u_2u_1^2-u_7^2u_4^3u_3u_2^2u_1+
2u_7^2u_4^3u_2^2u_1^2 - u_7^2u_4^2u_2^3u_1^2
-u_7u_6u_5u_4^3u_2^3 + u_7u_6u_5u_4^2u_2^4 +
u_7u_6u_4^4u_3u_2^2 - u_7u_6u_4^4u_2^2u_1
-u_7u_6u_4^3u_3u_2^3+2u_7u_6u_4^3u_2^3u_1
-u_7u_6u_4^2u_2^4u_1+u_7u_5u_4^3u_2^3u_1
-u_7u_5u_4^2u_2^4u_1-u_7u_4^4u_3u_2^2u_1+
u_7u_4^4u_2^2u_1^2 + u_7u_4^3u_3u_2^3u_1
-2u_7u_4^3u_2^3u_1^2+u_7u_4^2u_2^4u_1^2
x_2^2
+
(-u_8u_7u_6u_5u_4^3u_2^2 + u_8u_7u_6u_5u_4u_2^4 +
u_8u_7u_6u_4^4u_3u_2 - u_8u_7u_6u_4^4u_2u_1 +
u_8u_7u_6u_4^3u_2^2u_1 - u_8u_7u_6u_4^2u_3u_2^3 +
u_8u_7u_6u_4^2u_2^3u_1 - u_8u_7u_6u_4u_2^4u_1 +
u_8u_7u_5u_4^3u_2^2u_1 - u_8u_7u_5u_4u_2^4u_1
-u_8u_7u_4^4u_3u_2u_1 + u_8u_7u_4^4u_2u_1^2
-u_8u_7u_4^3u_2^2u_1^2 + u_8u_7u_4^2u_3u_2^3u_1
-u_8u_7u_4^2u_2^3u_1^2 + u_8u_7u_4u_2^4u_1^2 +
u_8u_6u_5u_4^3u_2^3 - u_8u_6u_5u_4^2u_2^4
-u_8u_6u_4^4u_3u_2^2 + u_8u_6u_4^4u_2^2u_1 +
u_8u_6u_4^3u_3u_2^3 - 2u_8u_6u_4^3u_2^3u_1 +
u_8u_6u_4^2u_2^4u_1 - u_8u_5u_4^3u_2^3u_1 +
u_8u_5u_4^2u_2^4u_1 + u_8u_4^4u_3u_2^2u_1
-u_8u_4^4u_2^2u_1^2-u_8u_4^3u_3u_2^3u_1+\\
2u_8u_4^3u_2^3u_1^2 - u_8u_4^2u_2^4u_1^2 +
u_7u_6u_5u_4^3u_2^3 - u_7u_6u_5u_4^2u_2^4
-u_7u_6u_4^4u_3u_2^2 + u_7u_6u_4^4u_2^2u_1 +
u_7u_6u_4^3u_3u_2^3 - 2u_7u_6u_4^3u_2^3u_1 +
u_7u_6u_4^2u_2^4u_1 - u_7u_5u_4^3u_2^3u_1 +
u_7u_5u_4^2u_2^4u_1 + u_7u_4^4u_3u_2^2u_1
-u_7u_4^4u_2^2u_1^2 - u_7u_4^3u_3u_2^3u_1 +
2u_7u_4^3u_2^3u_1^2 - u_7u_4^2u_2^4u_1^2
x_2x_1
(-u_8u_7^2u_6u_5^2u_4^2u_2^2 + u_8u_7^2u_6u_5^2u_4u_2^3 +
```

```
u_8u_7^2u_6u_5u_4^3u_3u_2 - u_8u_7^2u_6u_5u_4^3u_2u_1 +
2u_8u_7^2u_6u_5u_4^2u_2^2u_1
-u_8u_7^2u_6u_5u_4u_3u_2^3-u_8u_7^2u_6u_5u_4u_2^3u_1
-u_8u_7^2u_6u_4^3u_3^2u_2 + u_8u_7^2u_6u_4^3u_3u_2u_1 +
u_8u_7^2u_6u_4^2u_3^2u_2^2
-2u_8u_7^2u_6u_4^2u_3u_2^2u_1+
u_8u_7^2u_6u_4u_3u_2^3u_1 + u_8u_7^2u_5^2u_4^2u_2^2u_1
-u_8u_7^2u_5^2u_4u_2^3u_1-u_8u_7^2u_5u_4^3u_3u_2u_1+
u_8u_7^2u_5u_4^3u_2u_1^2 - 2u_8u_7^2u_5u_4^2u_2^2u_1^2 +
u_8u_7^2u_5u_4u_3u_2^3u_1 + u_8u_7^2u_5u_4u_2^3u_1^2 +
u_8u_7^2u_4^3u_3^2u_2u_1 - u_8u_7^2u_4^3u_3u_2u_1^2
-u_8u_7^2u_4^2u_3^2u_2^2u_1 +
2u_8u_7^2u_4^2u_3u_2^2u_1^2 - u_8u_7^2u_4u_3u_2^3u_1^2 +
u_8u_7u_6^2u_5u_4^3u_2^2 - u_8u_7u_6^2u_5u_4^2u_2^3
-u_8u_7u_6^2u_4^4u_3u_2 + u_8u_7u_6^2u_4^4u_2u_1 +
u_8u_7u_6^2u_4^3u_3u_2^2 - 2u_8u_7u_6^2u_4^3u_2^2u_1 +
u_8u_7u_6^2u_4^2u_2^3u_1 + u_8u_7u_6u_5^2u_4^2u_2^3
-u_8u_7u_6u_5^2u_4u_2^4 - u_8u_7u_6u_5u_4^3u_3u_2^2 +
u_8u_7u_6u_5u_4^2u_3u_2^3 - u_8u_7u_6u_5u_4^2u_2^3u_1 +
u_8u_7u_6u_5u_4u_2^4u_1 + u_8u_7u_6u_4^4u_3u_2u_1
-u_8u_7u_6u_4^4u_2u_1^2 - u_8u_7u_6u_4^3u_3u_2^2u_1 +
2u_8u_7u_6u_4^3u_2^2u_1^2 - u_8u_7u_6u_4^2u_2^3u_1^2
-u_8u_7u_5^2u_4^2u_2^3u_1+u_8u_7u_5^2u_4u_2^4u_1+
u_8u_7u_5u_4^3u_3u_2^2u_1 - u_8u_7u_5u_4^3u_2^2u_1^2
-u_8u_7u_5u_4^2u_3u_2^3u_1 + 2u_8u_7u_5u_4^2u_2^3u_1^2
-u_8u_7u_5u_4u_2^4u_1^2-u_8u_6^2u_5u_4^3u_2^3+
u_8u_6^2u_5u_4^2u_2^4 + u_8u_6^2u_4^4u_3u_2^2
-u_8u_6^2u_4^4u_2^2u_1-u_8u_6^2u_4^3u_3u_2^3+
2u_8u_6^2u_4^3u_2^3u_1 - u_8u_6^2u_4^2u_2^4u_1 +
u_8u_6u_5u_4^3u_2^3u_1 - u_8u_6u_5u_4^2u_2^4u_1
-u_8u_6u_4^4u_3u_2^2u_1+u_8u_6u_4^4u_2^2u_1^2+
u_8u_6u_4^3u_3u_2^3u_1 - 2u_8u_6u_4^3u_2^3u_1^2 +
u_8u_6u_4^2u_2^4u_1^2 - u_7^2u_6^2u_5u_4^3u_2^2 +
u_7^2 u_6^2 u_5 u_4^2 u_2^3 + u_7^2 u_6^2 u_4^4 u_3 u_2
-u_7^2u_6^2u_4^4u_2u_1-u_7^2u_6^2u_4^3u_3u_2^2+
2u_7^2u_6^2u_4^3u_2^2u_1 - u_7^2u_6^2u_4^2u_2^3u_1
-u_7^2 u_6 u_5 u_4^3 u_3 u_2^2 + u_7^2 u_6 u_5 u_4^3 u_2^2 u_1 + \\
u_7^2 u_6 u_5 u_4^2 u_3 u_2^3 - u_7^2 u_6 u_5 u_4^2 u_2^3 u_1 +
u_7^2 u_6 u_4^4 u_3^2 u_2 - 2 u_7^2 u_6 u_4^4 u_3 u_2 u_1 + \\
u_7^2u_6u_4^4u_2u_1^2 - u_7^2u_6u_4^3u_2^2u_2^2 +
```

```
3u_7^2u_6u_4^3u_3u_2^2u_1 - 2u_7^2u_6u_4^3u_2^2u_1^2
-u_7^2u_6u_4^2u_3u_2^3u_1+u_7^2u_6u_4^2u_2^3u_1^2+
u_7^2 u_5 u_4^3 u_3 u_2^2 u_1 - u_7^2 u_5 u_4^2 u_3 u_2^3 u_1
-u_7^2u_4^4u_3^2u_2u_1+u_7^2u_4^4u_3u_2u_1^2+
u_7^2 u_4^3 u_3^2 u_2^2 u_1 - 2 u_7^2 u_4^3 u_3 u_2^2 u_1^2 +
u_7^2 u_4^2 u_3 u_2^3 u_1^2 + u_7 u_6^2 u_5 u_4^3 u_2^3
-u_7u_6^2u_5u_4^2u_2^4 - u_7u_6^2u_4^4u_3u_2^2 +
u_7 u_6^2 u_4^4 u_2^2 u_1 + u_7 u_6^2 u_4^3 u_3 u_2^3
-2u_7u_6^2u_4^3u_2^3u_1+u_7u_6^2u_4^2u_2^4u_1
-u_7u_6u_5u_4^3u_2^3u_1+u_7u_6u_5u_4^2u_2^4u_1+
u_7u_6u_4^4u_3u_2^2u_1 - u_7u_6u_4^4u_2^2u_1^2
-u_7u_6u_4^3u_3u_2^3u_1 + 2u_7u_6u_4^3u_2^3u_1^2
-u_7u_6u_4^2u_2^4u_1^2)
x_2
(u_8^2u_6u_5u_4^2u_2^3-u_8^2u_6u_5u_4u_2^4)
-u_8^2 u_6 u_4^3 u_3 u_2^2 + u_8^2 u_6 u_4^3 u_2^2 u_1 +
u_8^2u_6u_4^2u_3u_2^3 - 2u_8^2u_6u_4^2u_2^3u_1 +
u_8^2u_6u_4u_2^4u_1-u_8^2u_5u_4^2u_2^3u_1+
u_8^2 u_5 u_4 u_2^4 u_1 + u_8^2 u_4^3 u_3 u_2^2 u_1
-u_8^2u_4^3u_2^2u_1^2 - u_8^2u_4^2u_3u_2^3u_1 +
2u_8^2u_4^2u_2^3u_1^2 - u_8^2u_4u_2^4u_1^2
-u_8u_6u_5u_4^3u_2^3 + u_8u_6u_5u_4^2u_2^4 +
u_8u_6u_4^4u_3u_2^2 - u_8u_6u_4^4u_2^2u_1
-u_8u_6u_4^3u_3u_2^3 + 2u_8u_6u_4^3u_2^3u_1
-u_8u_6u_4^2u_2^4u_1+u_8u_5u_4^3u_2^3u_1
-u_8u_5u_4^2u_2^4u_1-u_8u_4^4u_3u_2^2u_1+
u_8u_4^4u_2^2u_1^2 + u_8u_4^3u_3u_2^3u_1
-2u_8u_4^3u_2^3u_1^2 + u_8u_4^2u_2^4u_1^2
x_1^2
(u_8^2u_7u_6u_5^2u_4^2u_2^2 - u_8^2u_7u_6u_5^2u_4u_2^3
-u_8^2u_7u_6u_5u_4^3u_3u_2 + u_8^2u_7u_6u_5u_4^3u_2u_1
-2u_8^2u_7u_6u_5u_4^2u_2^2u_1+
u_8^2u_7u_6u_5u_4u_3u_2^3 + u_8^2u_7u_6u_5u_4u_2^3u_1 +
u_8^2 u_7 u_6 u_4^3 u_3^2 u_2 - u_8^2 u_7 u_6 u_4^3 u_3 u_2 u_1
-u_8^2u_7u_6u_4^2u_3^2u_2^2 +
2u_8^2u_7u_6u_4^2u_3u_2^2u_1
-u_8^2u_7u_6u_4u_3u_2^3u_1
-u_8^2u_7u_5^2u_4^2u_2^2u_1+u_8^2u_7u_5^2u_4u_2^3u_1+\\
```

```
u_8^2 u_7 u_5 u_4^3 u_3 u_2 u_1 - u_8^2 u_7 u_5 u_4^3 u_2 u_1^2 +
2u_8^2u_7u_5u_4^2u_2^2u_1^2
-u_8^2u_7u_5u_4u_3u_2^3u_1-u_8^2u_7u_5u_4u_2^3u_1^2
-u_8^2u_7u_4^3u_3^2u_2u_1+u_8^2u_7u_4^3u_3u_2u_1^2+
u_8^2u_7u_4^2u_3^2u_2^2u_1
-2u_8^2u_7u_4^2u_3u_2^2u_1^2+u_8^2u_7u_4u_3u_2^3u_1^2
-u_8^2u_6^2u_5u_4^2u_2^3+u_8^2u_6^2u_5u_4u_2^4+
u_8^2 u_6^2 u_4^3 u_3 u_2^2 - u_8^2 u_6^2 u_4^3 u_2^2 u_1
-u_8^2u_6^2u_4^2u_3u_2^3+2u_8^2u_6^2u_4^2u_2^3u_1
-u_8^2u_6^2u_4u_2^4u_1-u_8^2u_6u_5^2u_4^2u_2^3+
u_8^2u_6u_5^2u_4u_2^4 + u_8^2u_6u_5u_4^3u_3u_2^2
-u_8^2u_6u_5u_4^3u_2^2u_1-u_8^2u_6u_5u_4^2u_3u_2^3+
3u_8^2u_6u_5u_4^2u_2^3u_1 - 2u_8^2u_6u_5u_4u_2^4u_1
-u_8^2u_6u_4^3u_3u_2^2u_1+u_8^2u_6u_4^3u_2^2u_1^2+
u_8^2u_6u_4^2u_3u_2^3u_1 - 2u_8^2u_6u_4^2u_2^3u_1^2 +
u_8^2u_6u_4u_2^4u_1^2 + u_8^2u_5^2u_4^2u_2^3u_1
-u_8^2u_5^2u_4u_2^4u_1-u_8^2u_5u_4^3u_3u_2^2u_1+
u_8^2 u_5 u_4^3 u_2^2 u_1^2 + u_8^2 u_5 u_4^2 u_3 u_2^3 u_1
-2u_8^2u_5u_4^2u_2^3u_1^2+u_8^2u_5u_4u_2^4u_1^2+
u_8u_7u_6^2u_5u_4^2u_2^3 - u_8u_7u_6^2u_5u_4u_2^4
-u_8u_7u_6^2u_4^3u_3u_2^2+u_8u_7u_6^2u_4^3u_2^2u_1+
u_8u_7u_6^2u_4^2u_3u_2^3 - 2u_8u_7u_6^2u_4^2u_2^3u_1 +
u_8u_7u_6^2u_4u_2^4u_1 + u_8u_7u_6u_5u_4^3u_3u_2^2
-u_8u_7u_6u_5u_4^2u_3u_2^3 - u_8u_7u_6u_5u_4^2u_2^3u_1 +
u_8u_7u_6u_5u_4u_2^4u_1 - u_8u_7u_6u_4^4u_3^2u_2 +
u_8u_7u_6u_4^4u_3u_2u_1 + u_8u_7u_6u_4^3u_3^2u_2^2
-u_8u_7u_6u_4^3u_3u_2^2u_1-u_8u_7u_6u_4^3u_2^2u_1^2+
2u_8u_7u_6u_4^2u_2^3u_1^2 - u_8u_7u_6u_4u_2^4u_1^2
-u_8u_7u_5u_4^3u_3u_2^2u_1+u_8u_7u_5u_4^2u_3u_2^3u_1+
u_8u_7u_4^4u_3^2u_2u_1 - u_8u_7u_4^4u_3u_2u_1^2
-u_8u_7u_4^3u_3^2u_2^2u_1 + 2u_8u_7u_4^3u_3u_2^2u_1^2
-u_8u_7u_4^2u_3u_2^3u_1^2 + u_8u_6^2u_5u_4^3u_2^3
-u_8u_6^2u_5u_4^2u_2^4 - u_8u_6^2u_4^4u_3u_2^2 +
u_8u_6^2u_4^4u_2^2u_1 + u_8u_6^2u_4^3u_3u_2^3
-2u_8u_6^2u_4^3u_2^3u_1+u_8u_6^2u_4^2u_2^4u_1
-u_8u_6u_5u_4^3u_2^3u_1+u_8u_6u_5u_4^2u_2^4u_1+
u_8u_6u_4^4u_3u_2^2u_1 - u_8u_6u_4^4u_2^2u_1^2
-u_8u_6u_4^3u_3u_2^3u_1+2u_8u_6u_4^3u_2^3u_1^2
-u_8u_6u_4^2u_2^4u_1^2-u_7u_6^2u_5u_4^3u_2^3+\\
u_7u_6^2u_5u_4^2u_2^4 + u_7u_6^2u_4^4u_3u_2^2
```

```
-u_7u_6^2u_4^4u_2^2u_1-u_7u_6^2u_4^3u_3u_2^3+
2u_7u_6^2u_4^3u_2^3u_1 - u_7u_6^2u_4^2u_2^4u_1 +
u_7u_6u_5u_4^3u_2^3u_1 - u_7u_6u_5u_4^2u_2^4u_1
-u_7u_6u_4^4u_3u_2^2u_1+u_7u_6u_4^4u_2^2u_1^2+\\
u_7u_6u_4^3u_3u_2^3u_1 - 2u_7u_6u_4^3u_2^3u_1^2 +
u_7u_6u_4^2u_2^4u_1^2)
x_1
(-u_8^2u_7u_6^2u_5^2u_4^2u_2^2 +
u_8^2u_7u_6^2u_5^2u_4u_2^3 +
u_8^2 u_7 u_6^2 u_5 u_4^3 u_3 u_2
-u_8^2u_7u_6^2u_5u_4^3u_2u_1 +
2u_8^2u_7u_6^2u_5u_4^2u_2^2u_1
-u_8^2u_7u_6^2u_5u_4u_3u_2^3
-u_8^2u_7u_6^2u_5u_4u_2^3u_1
-u_8^2u_7u_6^2u_4^3u_3^2u_2 +
u_8^2u_7u_6^2u_4^3u_3u_2u_1 +
u_8^2 u_7 u_6^2 u_4^2 u_3^2 u_2^2
-2u_8^2u_7u_6^2u_4^2u_3u_2^2u_1+
u_8^2u_7u_6^2u_4u_3u_2^3u_1 +
u_8^2 u_7 u_6 u_5^2 u_4^2 u_2^2 u_1
-u_8^2u_7u_6u_5^2u_4u_2^3u_1
-u_8^2u_7u_6u_5u_4^3u_3u_2u_1 +
u_8^2u_7u_6u_5u_4^3u_2u_1^2
-2u_8^2u_7u_6u_5u_4^2u_2^2u_1^2 +
u_8^2u_7u_6u_5u_4u_3u_2^3u_1 +
u_8^2u_7u_6u_5u_4u_2^3u_1^2 +
u_8^2 u_7 u_6 u_4^3 u_3^2 u_2 u_1
-u_8^2u_7u_6u_4^3u_3u_2u_1^2
-u_8^2u_7u_6u_4^2u_3^2u_2^2u_1 +
2u_8^2u_7u_6u_4^2u_3u_2^2u_1^2
-u_8^2u_7u_6u_4u_3u_2^3u_1^2 +
u_8^2 u_6^2 u_5^2 u_4^2 u_2^3 - u_8^2 u_6^2 u_5^2 u_4 u_2^4
-u_8^2u_6^2u_5u_4^3u_3u_2^2 +
u_8^2u_6^2u_5u_4^3u_2^2u_1 +
u_8^2 u_6^2 u_5 u_4^2 u_3 u_2^3
-2u_8^2u_6^2u_5u_4^2u_2^3u_1+u_8^2u_6^2u_5u_4u_2^4u_1
-u_8^2u_6u_5^2u_4^2u_2^3u_1+u_8^2u_6u_5^2u_4u_2^4u_1+
u_8^2u_6u_5u_4^3u_3u_2^2u_1
-u_8^2u_6u_5u_4^3u_2^2u_1^2
```

```
-u_8^2u_6u_5u_4^2u_3u_2^3u_1+\\
2u_8^2u_6u_5u_4^2u_2^3u_1^2-u_8^2u_6u_5u_4u_2^4u_1^2+\\
u_8u_7^2u_6^2u_5^2u_4^2u_2^2
-u_8u_7^2u_6^2u_5^2u_4u_2^3
-u_8u_7^2u_6^2u_5u_4^3u_3u_2 +
u_8u_7^2u_6^2u_5u_4^3u_2u_1
-2u_8u_7^2u_6^2u_5u_4^2u_2^2u_1+
u_8u_7^2u_6^2u_5u_4u_3u_2^3 +
u_8u_7^2u_6^2u_5u_4u_2^3u_1 +
u_8 u_7^2 u_6^2 u_4^3 u_3^2 u_2^2
-u_8u_7^2u_6^2u_4^3u_3u_2u_1
-u_8u_7^2u_6^2u_4^2u_3^2u_2^2 +
2u_8u_7^2u_6^2u_4^2u_3u_2^2u_1
-u_8u_7^2u_6^2u_4u_3u_2^3u_1
-u_8u_7^2u_6u_5^2u_4^2u_2^2u_1+
u_8u_7^2u_6u_5^2u_4u_2^3u_1 +
u_8u_7^2u_6u_5u_4^3u_3u_2u_1
-u_8u_7^2u_6u_5u_4^3u_2u_1^2 +
2u_8u_7^2u_6u_5u_4^2u_2^2u_1^2
-u_8u_7^2u_6u_5u_4u_3u_2^3u_1
-u_8u_7^2u_6u_5u_4u_2^3u_1^2
-u_8u_7^2u_6u_4^3u_3^2u_2u_1 +
u_8u_7^2u_6u_4^3u_3u_2u_1^2 +
u_8u_7^2u_6u_4^2u_3^2u_2^2u_1
-2u_8u_7^2u_6u_4^2u_3u_2^2u_1^2+
u_8u_7^2u_6u_4u_3u_2^3u_1^2
-u_8u_7u_6^2u_5^2u_4^2u_2^3 + u_8u_7u_6^2u_5^2u_4u_2^4
-u_8u_7u_6^2u_5u_4^3u_2^2u_1+
2u_8u_7u_6^2u_5u_4^2u_2^3u_1
-u_8u_7u_6^2u_5u_4u_2^4u_1+u_8u_7u_6^2u_4^4u_3^2u_2
-u_8u_7u_6^2u_4^4u_3u_2u_1
-u_8u_7u_6^2u_4^3u_3^2u_2^2 +
2u_8u_7u_6^2u_4^3u_3u_2^2u_1
-u_8u_7u_6^2u_4^2u_3u_2^3u_1+\\
u_8u_7u_6u_5^2u_4^2u_2^3u_1
-u_8u_7u_6u_5^2u_4u_2^4u_1+
u_8u_7u_6u_5u_4^3u_2^2u_1^2
-2u_8u_7u_6u_5u_4^2u_2^3u_1^2 +
u_8u_7u_6u_5u_4u_2^4u_1^2 - u_8u_7u_6u_4^4u_3^2u_2u_1 + \\
u_8u_7u_6u_4^4u_3u_2u_1^2 + u_8u_7u_6u_4^3u_3^2u_2^2u_1
```

$$\begin{array}{l} -2u_8u_7u_6u_4^3u_3u_2^2u_1^2 + \\ u_8u_7u_6u_4^2u_3u_3^2u_1^2 + \\ u_7u_6^2u_5u_4^3u_3u_2^2 \\ -u_7^2u_6^2u_5u_4^2u_3u_2^3 - u_7^2u_6^2u_4^4u_3^2u_2 + \\ u_7^2u_6^2u_4^4u_3u_2u_1 + u_7^2u_6^2u_4^3u_3^2u_2^2 \\ -2u_7^2u_6^2u_4^3u_3u_2^2u_1 + \\ u_7^2u_6^2u_4^2u_3u_2^3u_1 \\ -u_7^2u_6u_5u_4^3u_3u_2^2u_1 + \\ u_7^2u_6u_5u_4^2u_3u_3^2u_1 + u_7^2u_6u_4^4u_3^2u_2u_1 \\ -u_7^2u_6u_4^4u_3u_2u_1^2 - u_7^2u_6u_4^3u_3^2u_2^2u_1 + \\ 2u_7^2u_6u_4^4u_3u_2u_1^2 - u_7^2u_6u_4^3u_3^2u_2^2u_1 + \\ 2u_7^2u_6u_4^3u_3u_2^2u_1^2 \\ -u_7^2u_6u_4^2u_3u_3^2u_1^2) \end{array}$$

7. Pseudo remainder with p_2 over variable x_2 :

$$\begin{array}{lll} g&=&(u_8^2u_6u_5u_4^4u_2^3-u_8^2u_6u_5u_4^3u_2^4\\ &-u_8^2u_6u_4^4u_3u_2^2+u_8^2u_6u_4^4u_2^3u_1+\\ &u_8^2u_6u_4^4u_3u_2^3-2u_8^2u_6u_4^4u_2^3u_1+\\ &u_8^2u_6u_4^3u_2^4u_1-u_8^2u_5u_4^4u_2^3u_1+\\ &u_8^2u_5u_4^3u_2^4u_1-u_8^2u_5^4u_3u_2^2u_1\\ &-u_8^2u_5^4u_2^2u_1^2-u_8^2u_4^4u_3u_2^3u_1+\\ &2u_8^2u_4^4u_2^3u_1^2-u_8^2u_4^3u_2^4u_1^2\\ &-u_8u_6u_5u_4^5u_2^3+u_8u_6u_5u_4^4u_2^4+\\ &u_8u_6u_4^6u_3u_2^2-u_8u_6u_4^6u_2^2u_1\\ &-u_8u_6u_4^5u_3u_2^3+2u_8u_6u_2^5u_2^3u_1\\ &-u_8u_6u_4^4u_2^4u_1+u_8u_5u_2^5u_2^3u_1\\ &-u_8u_6u_4^4u_2^4u_1-u_8u_4^6u_3u_2^2u_1+\\ &u_8u_6^6u_2^2u_1^2+u_8u_4^5u_3u_2^3u_1\\ &-2u_8u_4^5u_2^3u_1^2+u_8u_4^4u_2^4u_1^2)\\ &x_1^2\\ &+\\ &(-u_8^2u_7u_6u_5^2u_4^2u_2^2\\ &-u_8^2u_7u_6u_5u_4^2u_2^2u_1+\\ &2u_8^2u_7u_6u_5u_4^2u_2^2u_1+u_8^2u_7u_6u_5^5u_4^2u_2^2\\ &-u_8^2u_7u_6u_5u_4^2u_2^2u_1+u_8^2u_7u_6u_5^4u_3^2u_2\\ &-u_8^2u_7u_6u_5u_4^2u_2^2u_1+u_8^2u_7u_6u_5^4u_3^2u_2\\ &-u_8^2u_7u_6u_5u_4^2u_2^2u_1+u_8^2u_7u_6u_5^4u_3^2u_2\\ &-u_8^2u_7u_6u_4^5u_3u_2u_1\\ &-u_8^2u_7u_6u_4^5u_3u_2u_1\\ &-u_8^2u_7u_6u_4^4u_3^2u_2^2+\\ &2u_8^2u_7u_6u_4^4u_3u_2^2u_1\end{array}$$

```
-u_8^2u_7u_6u_4^3u_3u_2^3u_1 +
u_8^2 u_7 u_5^2 u_4^3 u_2^3 u_1
-u_8^2u_7u_5^2u_4^2u_2^4u_1 +
u_8^2 u_7 u_5 u_4^4 u_2^2 u_1^2
-2u_8^2u_7u_5u_4^3u_2^3u_1^2+
u_8^2 u_7 u_5 u_4^2 u_2^4 u_1^2 - u_8^2 u_7 u_4^5 u_3^2 u_2 u_1 +
u_8^2 u_7 u_4^5 u_3 u_2 u_1^2 + u_8^2 u_7 u_4^4 u_3^2 u_2^2 u_1
-2u_8^2u_7u_4^4u_3u_2^2u_1^2+
u_8^2u_7u_4^3u_3u_2^3u_1^2 - u_8^2u_6^2u_5u_4^4u_2^3 +
u_8^2 u_6^2 u_5 u_4^3 u_2^4 + u_8^2 u_6^2 u_4^5 u_3 u_2^2
-u_8^2u_6^2u_4^5u_2^2u_1-u_8^2u_6^2u_4^4u_3u_2^3+
2u_8^2u_6^2u_4^4u_2^3u_1 - u_8^2u_6^2u_4^3u_2^4u_1 +
u_8^2 u_6 u_5 u_4^4 u_2^3 u_1 - u_8^2 u_6 u_5 u_4^3 u_2^4 u_1
-u_8^2u_6u_4^5u_3u_2^2u_1+u_8^2u_6u_4^5u_2^2u_1^2+
u_8^2 u_6 u_4^4 u_3 u_2^3 u_1 - 2 u_8^2 u_6 u_4^4 u_2^3 u_1^2 +
u_8^2 u_6 u_4^3 u_2^4 u_1^2 + u_8 u_7 u_6^2 u_5 u_4^4 u_2^3
-u_8u_7u_6^2u_5u_4^3u_2^4 - u_8u_7u_6^2u_4^5u_3u_2^2 +
u_8u_7u_6^2u_4^5u_2^2u_1 + u_8u_7u_6^2u_4^4u_3u_2^3
-2u_8u_7u_6^2u_4^4u_2^3u_1+u_8u_7u_6^2u_4^3u_2^4u_1+
u_8u_7u_6u_5^2u_4^4u_2^3 - u_8u_7u_6u_5^2u_4^3u_2^4 +
u_8u_7u_6u_5u_4^5u_2^2u_1 - 3u_8u_7u_6u_5u_4^4u_2^3u_1 +
2u_8u_7u_6u_5u_4^3u_2^4u_1 - u_8u_7u_6u_4^6u_3^2u_2 +
u_8u_7u_6u_4^6u_3u_2u_1 + u_8u_7u_6u_4^5u_3^2u_2^2
-u_8u_7u_6u_4^5u_3u_2^2u_1-u_8u_7u_6u_4^5u_2^2u_1^2+
2u_8u_7u_6u_4^4u_2^3u_1^2 - u_8u_7u_6u_4^3u_2^4u_1^2
-u_8u_7u_5^2u_4^4u_2^3u_1 + u_8u_7u_5^2u_4^3u_2^4u_1
-u_8u_7u_5u_4^5u_2^2u_1^2 + 2u_8u_7u_5u_4^4u_2^3u_1^2
-u_8u_7u_5u_4^3u_2^4u_1^2 + u_8u_7u_4^6u_3^2u_2u_1
-u_8u_7u_4^6u_3u_2u_1^2-u_8u_7u_4^5u_3^2u_2^2u_1+
2u_8u_7u_4^5u_3u_2^2u_1^2 - u_8u_7u_4^4u_3u_2^3u_1^2 +
u_8u_6^2u_5u_4^5u_2^3 - u_8u_6^2u_5u_4^4u_2^4
-u_8u_6^2u_4^6u_3u_2^2+u_8u_6^2u_4^6u_2^2u_1+
u_8u_6^2u_4^5u_3u_2^3 - 2u_8u_6^2u_4^5u_2^3u_1 +
u_8u_6^2u_4^4u_2^4u_1 - u_8u_6u_5u_4^5u_2^3u_1 +
u_8u_6u_5u_4^4u_2^4u_1 + u_8u_6u_4^6u_3u_2^2u_1
-u_8u_6u_4^6u_2^2u_1^2 - u_8u_6u_4^5u_3u_2^3u_1 +
2u_8u_6u_4^5u_2^3u_1^2 - u_8u_6u_4^4u_2^4u_1^2
-u_7u_6^2u_5u_4^5u_2^3+u_7u_6^2u_5u_4^4u_2^4+
u_7u_6^2u_4^6u_3u_2^2 - u_7u_6^2u_4^6u_2^2u_1
-u_7u_6^2u_4^5u_3u_2^3 + 2u_7u_6^2u_4^5u_2^3u_1
```

```
-u_7u_6^2u_4^4u_2^4u_1+u_7u_6u_5u_4^5u_2^3u_1
-u_7u_6u_5u_4^4u_2^4u_1-u_7u_6u_4^6u_3u_2^2u_1+
u_7u_6u_4^6u_2^2u_1^2 + u_7u_6u_4^5u_3u_2^3u_1
-2u_7u_6u_4^5u_2^3u_1^2+u_7u_6u_4^4u_2^4u_1^2
(u_8^2u_7^2u_6u_5^2u_4^3u_3u_2^2)
-u_8^2u_7^2u_6u_5^2u_4^2u_3u_2^3
-u_8^2u_7^2u_6u_5u_4^4u_3^2u_2 +
u_8^2 u_7^2 u_6 u_5 u_4^4 u_3 u_2 u_1 + \\
u_8^2 u_7^2 u_6 u_5 u_4^3 u_3^2 u_2^2
-2u_8^2u_7^2u_6u_5u_4^3u_3u_2^2u_1+
u_8^2 u_7^2 u_6 u_5 u_4^2 u_3 u_2^3 u_1
-u_8^2u_7^2u_5^2u_4^3u_3u_2^2u_1 +
u_8^2 u_7^2 u_5^2 u_4^2 u_3 u_2^3 u_1 +
u_8^2 u_7^2 u_5 u_4^4 u_3^2 u_2 u_1
-u_8^2 u_7^2 u_5 u_4^4 u_3 u_2 u_1^2
-u_8^2u_7^2u_5u_4^3u_3^2u_2^2u_1 +
2u_8^2u_7^2u_5u_4^3u_3u_2^2u_1^2
-u_8^2u_7^2u_5u_4^2u_3u_2^3u_1^2 +
u_8^2 u_7 u_6^2 u_5 u_4^4 u_3 u_2^2
-u_8^2u_7u_6^2u_5u_4^3u_3u_2^3
-u_8^2u_7u_6^2u_4^5u_3^2u_2 +
u_8^2u_7u_6^2u_4^5u_3u_2u_1 +
u_8^2 u_7 u_6^2 u_4^4 u_3^2 u_2^2
-2u_8^2u_7u_6^2u_4^4u_3u_2^2u_1+\\
u_8^2 u_7 u_6^2 u_4^3 u_3 u_2^3 u_1
-u_8^2u_7u_6u_5u_4^4u_3u_2^2u_1+
u_8^2u_7u_6u_5u_4^3u_3u_2^3u_1 +
u_8^2 u_7 u_6 u_4^5 u_3^2 u_2 u_1
-u_8^2u_7u_6u_4^5u_3u_2u_1^2
-u_8^2u_7u_6u_4^4u_3^2u_2^2u_1+
2u_8^2u_7u_6u_4^4u_3u_2^2u_1^2
-u_8^2u_7u_6u_4^3u_3u_2^3u_1^2
-u_8u_7^2u_6^2u_5u_4^4u_3u_2^2 +
u_8u_7^2u_6^2u_5u_4^3u_3u_2^3 +
u_8u_7^2u_6^2u_4^5u_3^2u_2
-u_8u_7^2u_6^2u_4^5u_3u_2u_1
-u_8u_7^2u_6^2u_4^4u_3^2u_2^2 +
2u_8u_7^2u_6^2u_4^4u_3u_2^2u_1
```

```
-u_8u_7^2u_6^2u_4^3u_3u_2^3u_1
-u_8u_7^2u_6u_5^2u_4^4u_3u_2^2+
u_8u_7^2u_6u_5^2u_4^3u_3u_2^3 +
u_8u_7^2u_6u_5u_4^5u_3^2u_2
-u_8u_7^2u_6u_5u_4^5u_3u_2u_1
-u_8u_7^2u_6u_5u_4^4u_3^2u_2^2 +
3u_8u_7^2u_6u_5u_4^4u_3u_2^2u_1
-2u_8u_7^2u_6u_5u_4^3u_3u_2^3u_1
-u_8u_7^2u_6u_4^5u_3^2u_2u_1+
u_8u_7^2u_6u_4^5u_3u_2u_1^2 +
u_8u_7^2u_6u_4^4u_3^2u_2^2u_1
-2u_8u_7^2u_6u_4^4u_3u_2^2u_1^2+
u_8u_7^2u_6u_4^3u_3u_2^3u_1^2 +
u_8u_7^2u_5^2u_4^4u_3u_2^2u_1
-u_8u_7^2u_5^2u_4^3u_3u_2^3u_1
-u_8u_7^2u_5u_4^5u_3^2u_2u_1 +
u_8u_7^2u_5u_4^5u_3u_2u_1^2 +
u_8u_7^2u_5u_4^4u_3^2u_2^2u_1
-2u_8u_7^2u_5u_4^4u_3u_2^2u_1^2+
u_8u_7^2u_5u_4^3u_3u_2^3u_1^2
-u_8u_7u_6^2u_5u_4^5u_3u_2^2 +
u_8u_7u_6^2u_5u_4^4u_3u_2^3 + u_8u_7u_6^2u_4^6u_3^2u_2
-u_8u_7u_6^2u_4^6u_3u_2u_1
-u_8u_7u_6^2u_4^5u_3^2u_2^2+
2u_8u_7u_6^2u_4^5u_3u_2^2u_1
-u_8u_7u_6^2u_4^4u_3u_2^3u_1+
u_8u_7u_6u_5u_4^5u_3u_2^2u_1
-u_8u_7u_6u_5u_4^4u_3u_2^3u_1
-u_8u_7u_6u_4^6u_3^2u_2u_1+u_8u_7u_6u_4^6u_3u_2u_1^2+
u_8u_7u_6u_4^5u_3^2u_2^2u_1
-2u_8u_7u_6u_4^5u_3u_2^2u_1^2+
u_8u_7u_6u_4^4u_3u_2^3u_1^2 +
u_7^2 u_6^2 u_5 u_4^5 u_3 u_2^2
-u_7^2u_6^2u_5u_4^4u_3u_2^3 - u_7^2u_6^2u_4^6u_3^2u_2 +
u_7^2 u_6^2 u_4^6 u_3 u_2 u_1 + u_7^2 u_6^2 u_4^5 u_3^2 u_2^2
-2u_7^2u_6^2u_4^5u_3u_2^2u_1+
u_7^2 u_6^2 u_4^4 u_3 u_2^3 u_1
-u_7^2u_6u_5u_4^5u_3u_2^2u_1+
u_7^2 u_6 u_5 u_4^4 u_3 u_2^3 u_1 + u_7^2 u_6 u_4^6 u_3^2 u_2 u_1
-u_7^2 u_6 u_4^6 u_3 u_2 u_1^2 - u_7^2 u_6 u_4^5 u_3^2 u_2^2 u_1 + \\
```

$$2u_7^2 u_6 u_4^5 u_3 u_2^2 u_1^2 -u_7^2 u_6 u_4^4 u_3 u_2^3 u_1^2)$$

8. Pseudo remainder with p_1 over variable x_1 :

$$g = 0$$

5 Prover results

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 10 terms.

Time Complexity: Time spent by the prover is 0.198 seconds.

6 NDG Conditions

NDG Conditions in readable form

- Points A and B are not identical
- Points S and C are not identical
- \bullet Line through points B and C is not parallel with line through points C1 and B1
- Points A, S, B and C are not collinear
- \bullet Line through points A1 and C1 is not parallel with line through points A and C
- Line through points A1 and B1 is not parallel with line through points A and B

Time spent for processing NDG Conditions

 \bullet 0.658 seconds