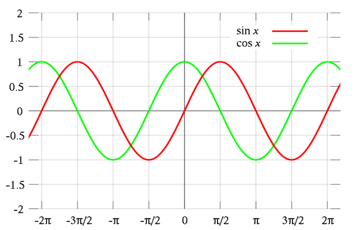
First mini-lecture: Functions, signals, and transforms

Functions

From secondary school mathematics, you are familiar with the concept of a continuous function such as $f : \mathbf{R} \rightarrow \mathbf{R}$, $y = f(x)$

- $y = x + 1$
- $y = 2x$
- $y = x^2$
- $y = \sin x$
- $y = \cos x$



Signals

Moving from mathematics towards the physical sciences and engineering, physical processes in nature often have behaviours that can be described mathematically.

When we record the behaviours of these physical processes we often do so by creating a ***signal*** that has the same mathematical description as the physical process.

- E.g. a microphone picking up a person's voice is an example of transforming pressure changes in the air into an electrical ***signal***, and
- an old-fashioned film camera transforms the field of view (whatever the camera is pointing at) into a two-dimensional ***signal*** recorded in photographic film.

Signals

The next step often is to store this ***signal*** in computer memory.

Often, these ***signals*** are more complicated than the simple functions mentioned previously, and so do not have a concise description.

Also, often we cannot even measure the physical behaviour continuously, we must measure it at discrete intervals in time (or space). Then, a concise and simple mathematical description such as a sin() or x^2 is unlikely to be found and we must explicitly store the measurements in an array to represent our signal.

For example, $f(x) = [4, 7, 2, 8, 1, 9, 12, 4, 7, 1, 6]$.

Let's next look at the formal description of such signals...

Signals

These signals, sampled at discrete intervals, are called discrete signals:

$f : \mathbf{N} \rightarrow \mathbf{R}$, $y = f(x)$

If discrete signals are stored in a computer memory they will be limited to a finite number of bits to represent each sampled value. Irrespective of whether these values are encoded as fixed point or floating point data types, they are no more expressive than natural numbers, and so digital signals have the form:

$f : \mathbf{N} \rightarrow \mathbf{N}$, $y = f(x)$

although we sometimes informally consider them for convenience as $f : \mathbf{N} \rightarrow \mathbf{R}$

Let's consider the example functions (we may have looked at earlier in class) as discrete signals in both time and value…

Digital signal processing concerns the mathematical manipulation of digital signals to change them from one type to another (such as noise cancellation and speech enhancement in mobile phone telecommunications, or image compression to store photographs more efficiently in one's camera memory) or to extract some basic information from the signals (such as determining the average brightness in an image).

Transforms

A transform rearranges the signal so that it is easier to process, or allows processing that would not be possible otherwise (from undergraduate data structures, consider sorting a list as a way to make it easier to find the largest element in the list)

An example of a simple reversable transformation:

$T : \mathbf{R} \rightarrow \mathbf{R}$, $T(f(x)) = f(-x)$

Examples:

- Volume control (reversible)
- Image compression (can be reversible or not)
- Fourier transformation (reversible, except for floating point rounding errors)

Signals and images

- The term signal is often reserved for one-dimensional signals that are stored on computer in a single array
- A two-dimensional discrete signal
  $f(x, y), f : \mathbf{N} \times \mathbf{N} \to \mathbf{R}$ or $f(x, y), f : \mathbf{N} \times \mathbf{N} \to \mathbf{C}$
  or its digital equivalent
  $f(x, y), f : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ or $f(x, y), f : \mathbf{N} \times \mathbf{N} \to \mathbf{N} \times \mathbf{N}$
  is often called an image, and can be stored on computer as an array of arrays (a 2D array).

Processing of signals

- Digital signals are usually processed on a computer using computer software
- However, there are other possibilities, such as electrical or analog electronic processing (hearing aid) and optical processing (advanced computer networking switches).