CSE411

Simulation and Modeling : Assignment 3

Testing Random Number Generators

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Problem Definition:

In this assignment, we will create a *linear congruential generator* and perform four empirical tests (Uniformity test, Serial test, Runs test and Correlation test) on the generated random numbers. And check whether the hypothesis is rejected or not for particular tests.
We will calculate the values for n = 20, 500, 4000 and 10000. Here n is total random number count.
For each n, we need to perform 10 tests in total:
(1) Uniformity test at k = 10 and 20
(2) Serial test using d = 2, 3 and k = 4, 8
(3) Run length test
(4) Correlation test with j =1, 3 and 5.
α = 0.1 for all tests

Tests:

Seed = 1505102

1. **Uniformity test**
   Uniformity test is a means of measuring the extent to which a sample of random numbers comply with a uniform distribution.
   Here we checked the null hypothesis. For seed = 1505102, no set of random numbers are rejected.

n	k	χ^2	$\chi^2_{k-1,\,1-\alpha}$	Rejected?
20	10	2.0	14.684	No
	20	18.0	27.204	No
500	10	9.0	14.684	No
	20	26.7	27.204	No
4000	10	6.98	14.684	No
	20	18.096	27.204	No
10000	10	5.094	14.684	No
	20	22.548	27.204	No

2. **Serial test**

The serial test is the chi-square test in higher dimensions. This checks the assumption that every individual U_i's of a set are independent.
Here with a smaller dimension, *d* and interval, *k*, the null hypothesis is not rejected mostly. But if we increase the dimension and intervals, that hypothesis may get rejected more frequently.

n	d	k	χ^2	$\chi^2_{k^d-1,\,1-\alpha}$	Rejected?
20	2	4	9.2	22.307	No
	3	4	58.0	77.745	No
	2	8	66.8	77.745	No
	3	8	506.0	552.374	No
500	2	4	12.016	22.307	No
	3	4	57.614	77.745	No
	2	8	66.928	77.745	No
	3	8	512.554	552.374	No
4000	2	4	8.32	22.307	No
	3	4	76.776	77.745	No
	2	8	47.744	77.745	No
	3	8	512.581	552.374	No
10000	2	4	15.846	22.307	No
	3	4	81.505	77.745	Yes
	2	8	64.2432	77.745	No
	3	8	585.267	552.374	Yes

3. **Runs test**

Runs test is a direct test of the independence assumption. Uniformity is not tested here.

We examine the U_i sequence for run-ups. A run-up is the unbroken subsequences of maximal length within which the U_i's increase monotonically.

n	R	$\chi^2_{6,\,1-\alpha}$	Rejected?
20	1.469	10.644	No
500	4.241		No
4000	1.813		No
10000	3.848		No

4. Correlation test

Correlation tests directly assess whether the generated U_i's exhibit discernible correlation at j lag.

n	j	A_j	$z_{1-\alpha/2}$	Rejected?
20	1	0.007659	1.645	No
	3	0.508		No
	5	1.009		No
500	1	0.391		No
	3	1.845		Yes
	5	1.696		Yes
4000	1	1.027		No
	3	0.193		No
	5	0.431		No
10000	1	0.212		No
	3	0.189		No
	5	0.823		No

Distribution of generated Random numbers: