Dart Madness

Goal: Our goal is to study four factors that could potentially impact our group's accuracy with respect to throwing darts. If all goes according to plan, we will find an optimal technique to wow all our friends.

Response: Score (defined as the sum of 3 tosses, using the scoring system below)

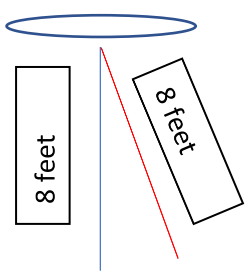
Factor	Levels
A = distance from dart board	3 (8 ft, 10 ft, 12 ft)
B = horizontal position	2 (straight on, to the right)
C = vertical position	2 (standing, on one knee)
D = specific hand throwing	2 (dominant, non dominant)

Narrative:
While traditional darts has a very specific way of scoring, for the purposes of this experiment we have created our own scoring system. Our scoring system, which can be seen drawn on the above image of a dartboard, gives 10 points for a bullseye (red and green), 7 points for inside the first circle, 5 for inside the second circle, and 3 for landing on the board but outside the second circle. If a player does not hit the dartboard, they will get 0 points. If a player hits the boundary between the rings, the higher value is rewarded. The motivation for this scoring system is that while typically a player wants to aim for the bullseye, if it is just outside the bullseye and in the "1" slice of the dartboard, the player's accuracy is unfairly rewarded. Under our scoring system, accuracy is rewarded.

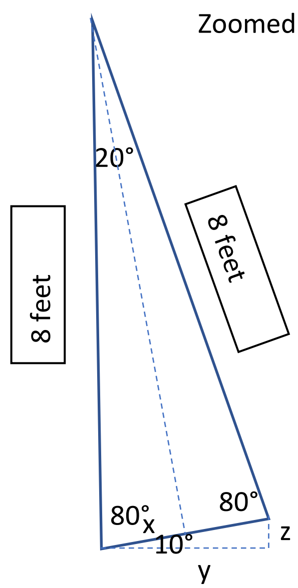
Thinking about our data a little more critically, it is evident that our data will be discrete. This played a role in how we determined what will constitute an observation in our experiment. The sum of three tosses will be recorded as one observation. This step will hopefully lead to our data behaving in a slight bit more continuous fashion, as our response will be able to take on values 0 – 30, as opposed to 0 – 10 if we just used a single toss as an observation.

We will have 4 factors in our experiment, 3 of which have 2 levels and 1 of which has 3 levels. Factor A is the distance from the dartboard. This factor has 3 levels: 8 feet, 10 feet, and 12 feet. Factor B is horizontal position and has two levels, straight on and to the right. Straight on is self-explanatory, where the person throwing the dart is facing the dartboard straight on. The "to the right" position requires a bit more explaining, as it is critical that we keep the distance from the person to the dartboard fixed. We will demonstrate this for our first distance level of 8 feet using the diagrams below.

Overhead



Zoomed In



$\cos(80°) = x/8$
$x = 1.389185421$ ft
$2*x=2.778370843$ ft

$\cos(10°) = y/2.778370843$
$y = 2.736161147$ ft

$(2.778370843)^2 = (2.736161147)^2 + z^2$
$z = 0.4824590331$ ft

The rudimentary illustration above shows an overhead view of what we mean by off to the right. The red line represents a “to the right” horizontal position, whereas the blue line represents a “straight on” horizontal position. It is important to keep the distance from dartboard fixed, so the “off to the right” horizontal position is a slight bit closer to the wall that the dartboard is hanging on. In order to ensure consistency, it is important that we fix the angle from which we will be throwing. We chose 20 degrees as our fixed angle. The calculations above show how we will determine where we will throw our darts from the “off to the right” position. For our distance of 8 feet, we will measure 2.736161147 feet (32.83”) to the right of our straight on position, and then move 0.4824590331 feet (5.79”) forward. This resulting point will be the one in which we throw our darts from for the “to the right” horizontal position. Similar calculations can be done for our distance of 10 feet and 12 feet. For our distance of 10 feet, we will measure 3.420201433 feet (41.04”) to the right of our straight on position, and then move 0.6030737916 feet (7.24”) forward. For our distance of 12 feet, we will measure 4.10424172 feet (49.25”) to the right of our straight on position, and then move 0.72368855 feet (8.68”) forward. Factor C is vertical position and has two levels, standing and kneeling. Our last factor, Factor D, represents throwing hand and has two levels (dominant vs non-dominant).

We will have three team members throwing darts in this experiment, with each of the three being able to allot roughly an hour for data collection. Because we have 3 factors of 2 levels, and 1 factor of 3 levels, this will equate to 24 treatments. Additionally, to help control person to person variation in score, we are planning to block by person.

The next natural question that needs to be answered is with respect to the number of blocks we will need to achieve a power greater than 0.80. There are a couple of values that need to be explained before we push ahead with our calculations. For starters, we need to determine a value for our desired detectable difference for each of our factors. We will denote these differences as D_A, D_B, D_C, and D_D. We expect Factor A, distance from the dartboard, to have a strong impact on score, so we have assigned $D_A = 4$. Our gut feeling as a group is that Factor B and C will not have quite as large of an impact on score, so we set these two difference values to $D_B = 3$ and $D_C = 3$. The factor that we expect to have the largest impact on score is Factor D, whether the darts are being thrown with the dominant vs non-dominant hand. Because we suspect this factor will have the largest impact on score, we have set this difference value the highest with $D_D = 5$. The final value needed for us to determine the number of blocks we will need is an estimate for σ^2. Our estimate of σ^2 was obtained by heading out and obtaining a pilot sample. Because we are assuming equal variance among the 24 treatments, it was okay for us to obtain our pilot sample using the treatment of our choosing. Our pilot sample was collected from the treatment where Factor A = 12 feet, Factor B = to the right, Factor C = kneeling, and Factor D = non-dominant hand. Using this pilot sample, we were able to calculate our estimated σ^2 value of 27.46667 (see R code below for calculation of sample variance of our pilot sample). Using these difference values and our σ^2 estimate, we found that to have a power greater than 0.8 we must use 5 blocks.

To clarify, 1 block will consist of 1 of our team members going through each of the 24 treatments. The order in which the blocks will be completed has been randomized, as well as the order in which the treatments are completed within each block. One important thing to note: the order in which each block is run will be randomized, but each person will complete his or her share of observations (the entire block) before we move on to the next block. See our data table below that shows the order in which the data will be collected.

R Code used to estimate σ²:

```
pilot_sample2 = c(10, 14, 12, 19, 12, 3)
var(pilot_sample2)
```

R Code used to determine number of blocks:

```
# number of blocks of factor A
alpha=.05
a=3  #levels of factor A
b=2  #levels of factor B
c=2  #levels of factor C
d=2  #levels of factor D
Da=4  #desired diff in means to detect with prob 1-beta
sigsq= 27.46667 #our estimate of sigma^2 (mse, here, in practice, more difficult)
biggestn=15  #largest blocks to try
n=2:biggestn  #the common blocks for each treatment (try several)
Fcrit=qf(1-alpha,a-1,(a*b*c*d-1)*(n-1))  #value at which we reject H0
lama=n*b*c*d*(Da^2)/(2*sigsq)  #non-centrality paramter (ncp)
beta=pf(Fcrit,a-1,(a*b*c*d-1)*(n-1),ncp=lama)
power=1-beta
cbind(n,Fcrit,beta,power)  #display results by combining in columns

      n    Fcrit         beta     power
 [1,]  2 3.422132 0.5764546259 0.4235454
 [2,]  3 3.199582 0.3757784401 0.6242216
 [3,]  4 3.129644 0.2331641042 0.7668359
 [4,]  5 3.095433 0.1383759948 0.8616240
```

```
# number of blocks of factor B
alpha=.05
a=3  #levels of factor A
b=2  #levels of factor B
c=2  #levels of factor C
d=2  #levels of factor D
Db=3  #desired diff in means to detect with prob 1-beta
sigsq= 27.46667 #our estimate of sigma^2 (mse, here, in practice, more difficult)
biggestn=15  #largest blocks to try
n=2:biggestn  #the common blocks for each treatment (try several)
Fcrit=qf(1-alpha,b-1,(a*b*c*d-1)*(n-1))  #value at which we reject H0
lamb=n*a*c*d*(Db^2)/(2*sigsq)  #non-centrality paramter (ncp)
beta=pf(Fcrit,b-1,(a*b*c*d-1)*(n-1),ncp=lamb)
power=1-beta
cbind(n,Fcrit,beta,power)  #display results by combining in columns

      n    Fcrit         beta     power
 [1,]  2 4.279344 0.5238747884 0.4761252
 [2,]  3 4.051749 0.3380819845 0.6619180
 [3,]  4 3.979807 0.2103786071 0.7896214
 [4,]  5 3.944539 0.1266348800 0.8733651
```

```
# number of blocks for factor C
alpha=.05
a=3   #levels of factor A
b=2   #levels of factor B
c=2   #levels of factor C
d=2   #levels of factor D
Dc=3   #desired diff in means to detect with prob 1-beta
sigsq= 27.46667 #our estimate of sigma^2 (mse, here, in practice, more difficult)
biggestn=15   #largest blocks to try
n=2:biggestn   #the common blocks for each treatment (try several)
Fcrit=qf(1-alpha,c-1,(a*b*c*d-1)*(n-1))   #value at which we reject H0
lamc=n*a*b*d*(Dc^2)/(2*sigsq)   #non-centrality paramter (ncp)
beta=pf(Fcrit,c-1,(a*b*c*d-1)*(n-1),ncp=lamc)
power=1-beta
cbind(n,Fcrit,beta,power)   #display results by combining in columns
```

```
      n    Fcrit         beta     power
 [1,]  2 4.279344 0.5238747884 0.4761252
 [2,]  3 4.051749 0.3380819845 0.6619180
 [3,]  4 3.979807 0.2103786071 0.7896214
 [4,]  5 3.944539 0.1266348800 0.8733651
```

```
# number of blocks of factor D
alpha=.05
a=3   #levels of factor A
b=2   #levels of factor B
c=2   #levels of factor C
d=2   #levels of factor D
Dd=5   #desired diff in means to detect with prob 1-beta
sigsq= 27.46667 #our estimate of sigma^2 (mse, here, in practice, more difficult)
biggestn=15   #largest blocks to try
n=2:biggestn   #the common blocks for each treatment (try several)
Fcrit=qf(1-alpha,d-1,(a*b*c*d-1)*(n-1))   #value at which we reject H0
lamc=n*a*b*c*(Dd^2)/(2*sigsq)   #non-centrality paramter (ncp)
beta=pf(Fcrit,d-1,(a*b*c*d-1)*(n-1),ncp=lamc)
power=1-beta
cbind(n,Fcrit,beta,power)   #display results by combining in columns
      n    Fcrit         beta     power
 [1,]  2 4.279344 1.142689e-01 0.8857311
```

R Code used to randomize experiment:

```
randomized_name=c('Austin', 'Kowshik', 'Xin','Austin', 'Kowshik', 'Xin')
sample(randomized_name, 5)

Treatment=c(1:24)
Distance=c(8,10,12,8,10,12,8,10,12,8,10,12,8,10,12,8,10,12,8,10,12,8,10,12)
Posture=c('S','S','S','S','S','S','K','K','K','K','K','K','S','S','S','S','S','S','K','K','K','K','K','K')
Horizontal=c('H','H','H','H','H','H','H','H','H','H','H','H','R','R','R','R','R','R','R','R','R','R','R','R')
Hand=c('D','D','D','ND','ND','ND','D','D','D','ND','ND','ND','D','D','D','ND','ND','ND','D','D','D','ND','ND','ND')
treatment_mat=cbind(Treatment,Distance,Posture,Horizontal,Hand)
treatment_mat

run_order=treatment_mat[sample(1:24),]
run_order=treatment_mat[sample(1:24),]
run_order=treatment_mat[sample(1:24),]
run_order=treatment_mat[sample(1:24),]
run_order=treatment_mat[sample(1:24),]
```

Data Table:

Reference the table below for determining what levels of each factor go into each treatment.

		Dominant Hand			**Non Dominant Hand**		
	Distance	8 feet	10 feet	12 feet	8 feet	10 feet	12 feet
Posture	**Horizontal**						
Standing	Straight	1	2	3	4	5	6
Kneeling	Straight	7	8	9	10	11	12
Standing	Right	13	14	15	16	17	18
Kneeling	Right	19	20	21	22	23	24

The data table that we will fill out when conducting the experiment is below.

Person Throwing	Treatment	Distance	Posture	Horizontal	Hand	Score
Austin	11	10	K	H	ND	
Austin	16	8	S	R	ND	
Austin	7	8	K	H	D	
Austin	19	8	K	R	D	
Austin	15	12	S	R	D	
Austin	6	12	S	H	ND	
Austin	4	8	S	H	ND	
Austin	21	12	K	R	D	
Austin	3	12	S	H	D	
Austin	17	10	S	R	ND	
Austin	13	8	S	R	D	
Austin	10	8	K	H	ND	
Austin	18	12	S	R	ND	
Austin	20	10	K	R	D	
Austin	23	10	K	R	ND	
Austin	1	8	S	H	D	
Austin	22	8	K	R	ND	
Austin	8	10	K	H	D	
Austin	9	12	K	H	D	
Austin	5	10	S	H	ND	
Austin	2	10	S	H	D	
Austin	24	12	K	R	ND	
Austin	12	12	K	H	ND	
Austin	14	10	S	R	D	
Xin	21	12	K	R	D	
Xin	3	12	S	H	D	
Xin	2	10	S	H	D	
Xin	14	10	S	R	D	
Xin	19	8	K	R	D	
Xin	7	8	K	H	D	
Xin	5	10	S	H	ND	
Xin	22	8	K	R	ND	
Xin	13	8	S	R	D	
Xin	23	10	K	R	ND	
Xin	15	12	S	R	D	
Xin	1	8	S	H	D	
Xin	24	12	K	R	ND	
Xin	10	8	K	H	ND	
Xin	9	12	K	H	D	
Xin	6	12	S	H	ND	

Xin	4	8	S	H	ND	
Xin	8	10	K	H	D	
Xin	16	8	S	R	ND	
Xin	17	10	S	R	ND	
Xin	20	10	K	R	D	
Xin	18	12	S	R	ND	
Xin	12	12	K	H	ND	
Xin	11	10	K	H	ND	
Austin	13	8	S	R	D	
Austin	14	10	S	R	D	
Austin	11	10	K	H	ND	
Austin	6	12	S	H	ND	
Austin	2	10	S	H	D	
Austin	8	10	K	H	D	
Austin	19	8	K	R	D	
Austin	9	12	K	H	D	
Austin	12	12	K	H	ND	
Austin	24	12	K	R	ND	
Austin	1	8	S	H	D	
Austin	21	12	K	R	D	
Austin	15	12	S	R	D	
Austin	5	10	S	H	ND	
Austin	16	8	S	R	ND	
Austin	7	8	K	H	D	
Austin	4	8	S	H	ND	
Austin	3	12	S	H	D	
Austin	22	8	K	R	ND	
Austin	23	10	K	R	ND	
Austin	18	12	S	R	ND	
Austin	17	10	S	R	ND	
Austin	20	10	K	R	D	
Austin	10	8	K	H	ND	
Kowshik	6	12	S	H	ND	
Kowshik	16	8	S	R	ND	
Kowshik	11	10	K	H	ND	
Kowshik	3	12	S	H	D	
Kowshik	1	8	S	H	D	
Kowshik	10	8	K	H	ND	
Kowshik	23	10	K	R	ND	
Kowshik	19	8	K	R	D	
Kowshik	20	10	K	R	D	
Kowshik	22	8	K	R	ND	
Kowshik	21	12	K	R	D	

Kowshik	8	10	K	H	D	
Kowshik	9	12	K	H	D	
Kowshik	2	10	S	H	D	
Kowshik	12	12	K	H	ND	
Kowshik	17	10	S	R	ND	
Kowshik	24	12	K	R	ND	
Kowshik	5	10	S	H	ND	
Kowshik	4	8	S	H	ND	
Kowshik	18	12	S	R	ND	
Kowshik	15	12	S	R	D	
Kowshik	14	10	S	R	D	
Kowshik	7	8	K	H	D	
Kowshik	13	8	S	R	D	
Xin	2	10	S	H	D	
Xin	18	12	S	R	ND	
Xin	4	8	S	H	ND	
Xin	12	12	K	H	ND	
Xin	3	12	S	H	D	
Xin	15	12	S	R	D	
Xin	23	10	K	R	ND	
Xin	10	8	K	H	ND	
Xin	16	8	S	R	ND	
Xin	6	12	S	H	ND	
Xin	9	12	K	H	D	
Xin	24	12	K	R	ND	
Xin	11	10	K	H	ND	
Xin	17	10	S	R	ND	
Xin	7	8	K	H	D	
Xin	22	8	K	R	ND	
Xin	13	8	S	R	D	
Xin	19	8	K	R	D	
Xin	5	10	S	H	ND	
Xin	14	10	S	R	D	
Xin	1	8	S	H	D	
Xin	8	10	K	H	D	
Xin	21	12	K	R	D	
Xin	20	10	K	R	D	

Data Collection

The data was collected at a local bar, Campus Quarters, on Monday April 29th, 2019. To ensure that we would have an adequate amount of time to perform our experiment, we arrived promptly when Campus Quarters opened at 4:00 PM. The first thing we did upon arrival was mark the various positions in which we would be throwing the darts. Despite that fact we had 24 different treatments, there were only 6 unique physical locations that necessitated being measured. The 6 locations were 8 feet (straight on), 8 feet (to the right), 10 feet (straight on), 10 feet (to the right), 12 feet (straight on), and 12 feet (to the right). We marked these positions with masking tape, so we would not have to remeasure for each treatment. It is worth noting that our other factors, throwing hand (dominant vs nondominant) and posture (standing vs kneeling), had no impact on the physical location from which we threw the darts.

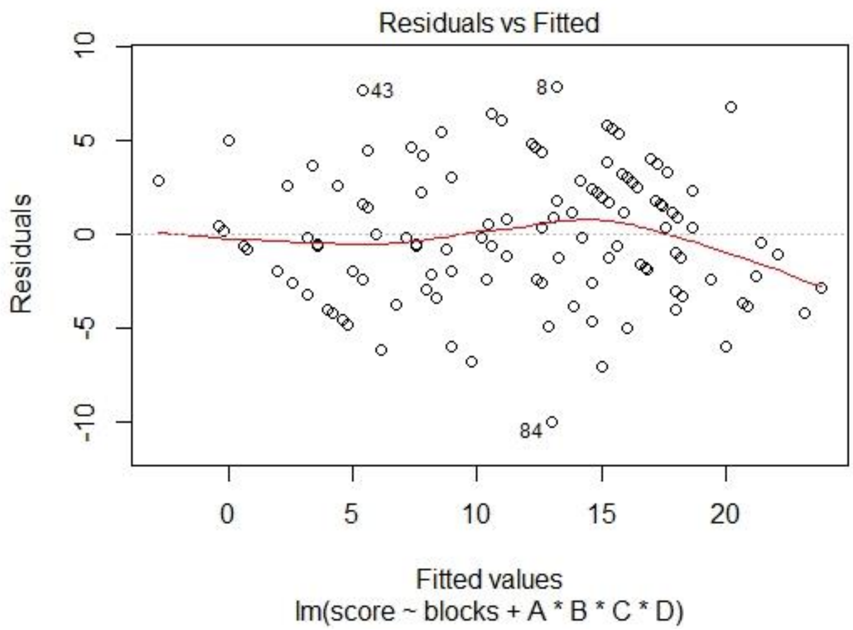
While running the experiment, we encountered next to no difficulties. In the days prior to the experiment, we performed extensive reconnaissance to ensure that there would not be any conflicts with our treatments (i.e. a structure in the bar that might impede us from performing a given treatment). The order in which we performed the treatments mirrors the data table shown above. For more information on how the experiment was run, please reference the final approved proposal above, specifically the narrative section.

Discussion of Analysis

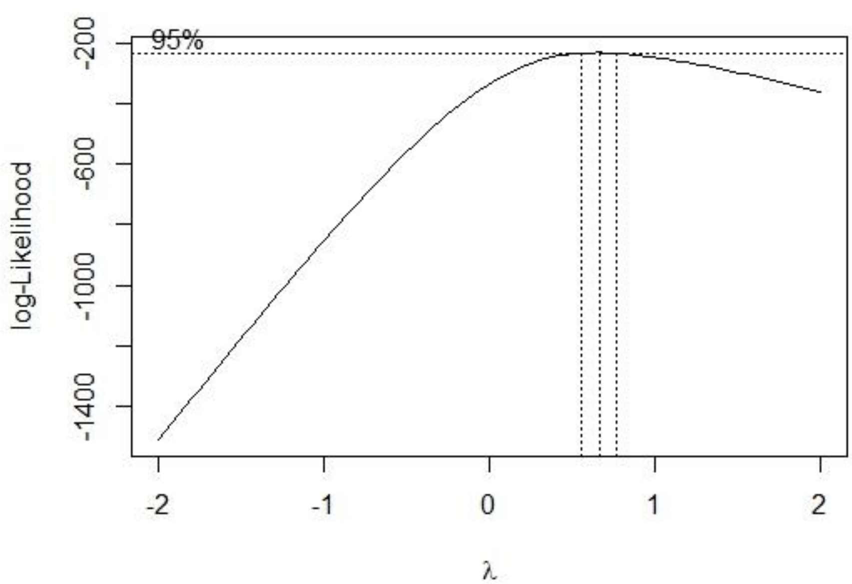
All the analysis on our data was performed in R, and the exact R code that was used is included as an appendix. After manually entering our data into R, the first output that we obtained was the ANOVA table and can be seen below.

```
            Df Sum Sq Mean Sq F value   Pr(>F)    
blocks       4   3355   838.7  53.978  < 2e-16 ***
A            2    653   326.3  20.998 3.08e-08 ***
B            1     75    75.2   4.840   0.0303 *  
C            1     15    15.4   0.992   0.3220    
D            1      3     3.0   0.194   0.6610    
A:B          2    121    60.4   3.885   0.0240 *  
A:C          2     22    11.1   0.715   0.4919    
B:C          1      1     0.7   0.043   0.8354    
A:D          2     99    49.5   3.186   0.0459 *  
B:D          1      1     1.4   0.091   0.7640    
C:D          1      2     1.9   0.121   0.7291    
A:B:C        2     75    37.4   2.409   0.0956 .  
A:B:D        2      9     4.3   0.274   0.7609    
A:C:D        2     10     5.0   0.320   0.7268    
B:C:D        1      8     8.0   0.515   0.4746    
A:B:C:D      2     17     8.6   0.551   0.5784    
Residuals   92   1430    15.5                     
```

Before diving into what looks like several significant variables, it was imperative that we perform residual analysis to ensure that the nonconstant variance and normality assumptions are not violated. The first plot that was analyzed was the residuals vs fitted plot.



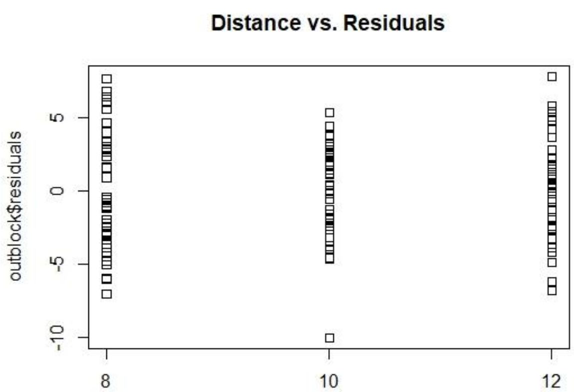
Upon initial visual inspection, this plot looked slightly spurious but not glaringly problematic. Due to the inconclusive nature of our visual inspection, we took additional measures to assess the validity of our constant variance assumption. Performing the Box-Cox test, we received the following graph.

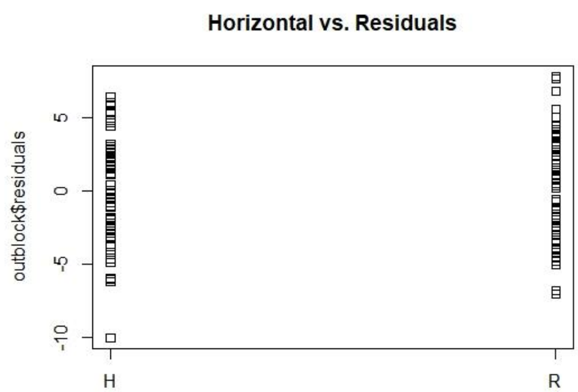


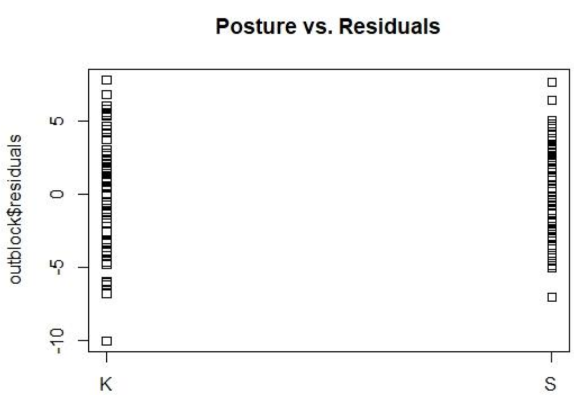
The resulting graph from our Box-Cox test would indicate that a variable transformation is recommended, specifically one with a lambda value close to 0.67. The combination of having skepticism of our residuals vs fitted plot and now a Box-Cox procedure recommending a variable transformation, had us believing that are constant variance assumption might be violated; however, after further analysis, this view began to change, starting with the results from running a Breusch-Pagan Test in R.

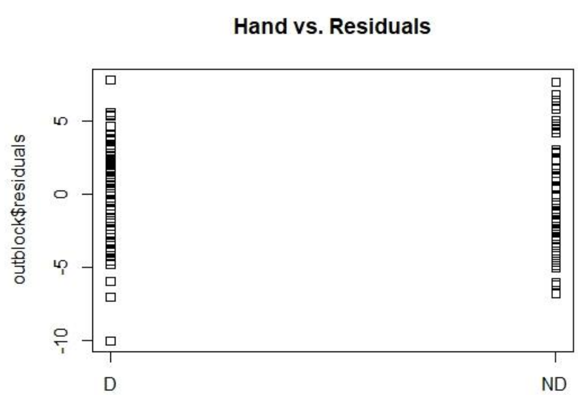
```
Non-constant Variance Score Test
Variance formula: ~ fitted.values
Chisquare = 0.02546167, Df = 1, p = 0.87322
```

The output above was yielded from running the Breusch-Pagan Test using the ncvTest function in R. The null hypothesis of this test is that variance is constant, so the p-value of 0.87322 indicates that we have insufficient evidence to reject this null hypothesis. This piece of evidence would lend to the notion that our constant variance assumption has not be violated. The final thing that we did in our quest to determine the validity of our constant variance assumption was plot each of the factors against the residuals. The resultant graphs can be seen below.

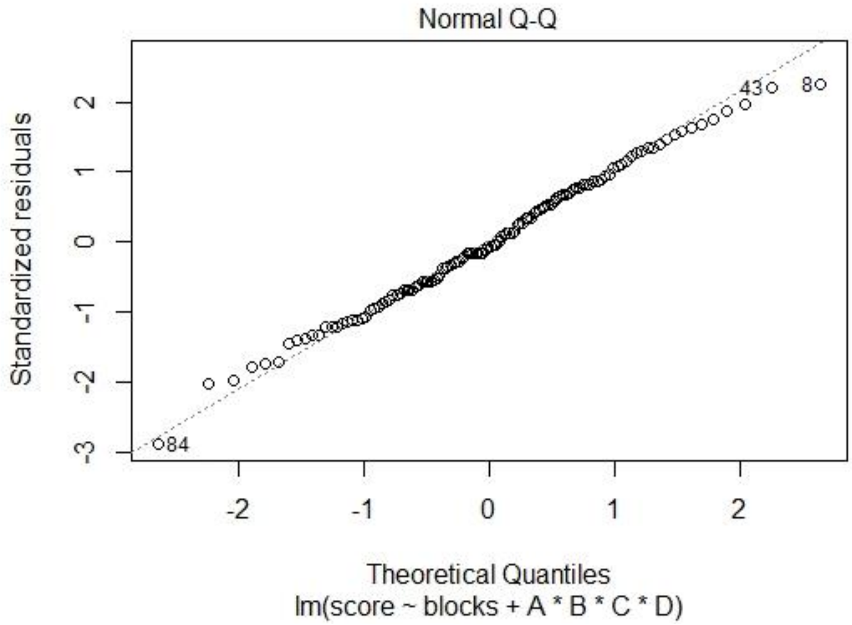




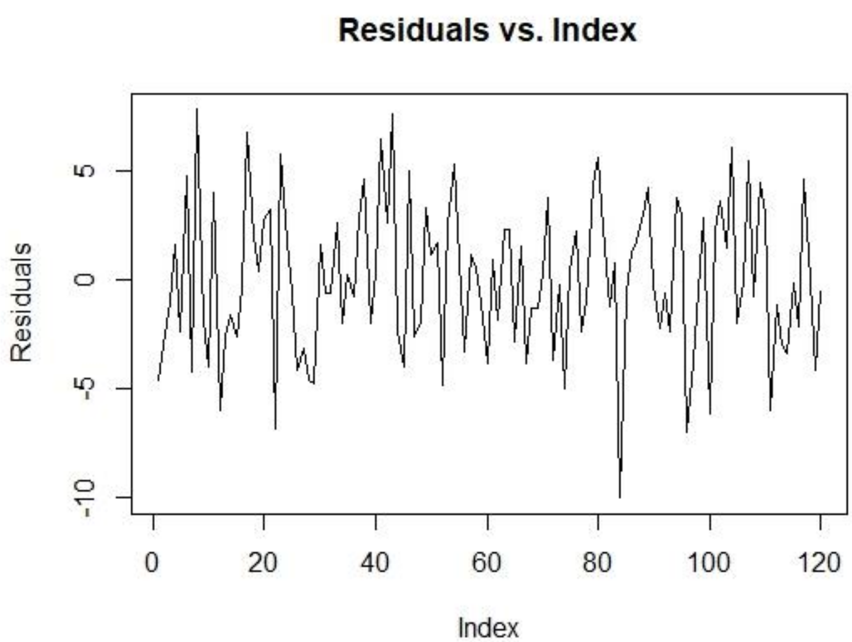




None of the above graphs are indicative of nonconstant variance. The combination of the results of our ncvTest and the individual analysis of each factor plotted against the residuals outweighed our prior concerns of nonconstant variance. After consulting with Dr McGrath, it seems that our residuals vs fitted plot took on a slightly spurious look because our response variable (score) is strictly integer. This could also explain why the Box-Cox test suggests a variable transformation. After ample analysis, it is time to check our normality assumption. We do this by looking at the Normal QQ plot, which can be seen below.



The Normal QQ plot gives us no reason to believe that our normality assumption has been violated. The last assumption that we need to check before we move on to our data analysis is independence. Initially there was a little concern that independence might be an issue, as it intuitively makes sense that there may be some natural improvement in score as we throw more darts; however, looking at the residuals vs index plot, there is nothing to suggest that our independence assumption has been violated. This plot can be seen below.

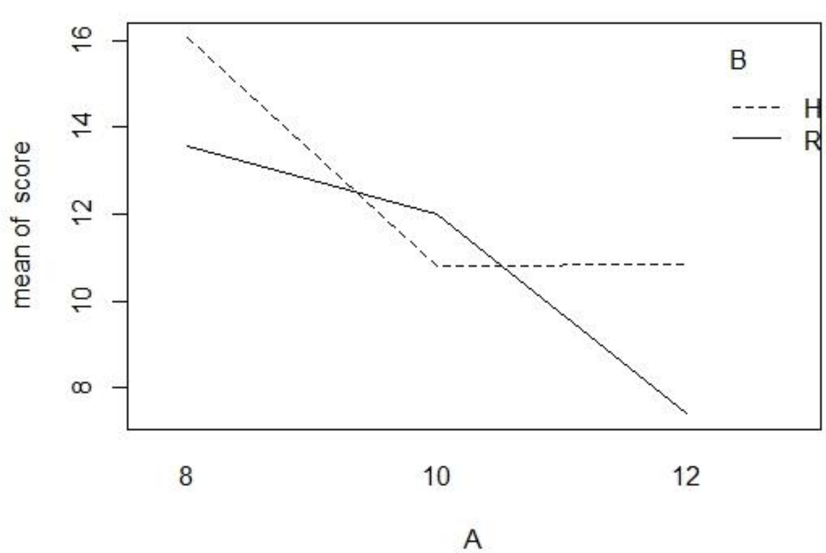


In conclusion, our three assumptions (constant variance, normality, and independence) are not violated. Thus, we do not need to perform a variable transformation, and we can begin our data analysis.

As we dive in, we thought it might be helpful to place another copy of our ANOVA table below. Additionally, just a friendly reminder that Factor A is distance (8 feet, 10 feet, and 12 feet), Factor B is horizontal position (straight on vs to the right), Factor C is posture (standing vs kneeling), and Factor D is hand (dominant vs nondominant).

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
blocks	4	3355	838.7	53.978	< 2e-16	***
A	2	653	326.3	20.998	3.08e-08	***
B	1	75	75.2	4.840	0.0303	*
C	1	15	15.4	0.992	0.3220	
D	1	3	3.0	0.194	0.6610	
A:B	2	121	60.4	3.885	0.0240	*
A:C	2	22	11.1	0.715	0.4919	
B:C	1	1	0.7	0.043	0.8354	
A:D	2	99	49.5	3.186	0.0459	*
B:D	1	1	1.4	0.091	0.7640	
C:D	1	2	1.9	0.121	0.7291	
A:B:C	2	75	37.4	2.409	0.0956	.
A:B:D	2	9	4.3	0.274	0.7609	
A:C:D	2	10	5.0	0.320	0.7268	
B:C:D	1	8	8.0	0.515	0.4746	
A:B:C:D	2	17	8.6	0.551	0.5784	
Residuals	92	1430	15.5			

Using a level of significance of 0.05, it is clear that the following variables are significant: blocks, A (distance), B (horizontal position), the A:B (distance and horizontal position) interaction, and the A:D (distance and posture) interaction. Despite A and B both being significant, because they are each involved in a significant interaction term, it does not behoove us to analyze either Factor A or Factor B independently. Let's first take a look at our A:B interaction term, which is significant with a p-value of 0.0240. An interaction plot can be seen below.



There are a couple of interesting things to note with the A:B interaction plot. When throwing from a horizontal position of straight on, there is a clear decrease in score as we increase distance from 8 feet to 10 feet; however, we see very little change (a slight increase) when we move from 10 feet to 12 feet, keeping our straight on orientation. When throwing from the horizontal position to the right, there is a slight change (minimal decrease) when we increase distance from 8 feet to 10 feet; however, there is a clear decrease in score as we increase distance from 10 feet to 12 feet, keeping the to the right orientation. The patterns described above are indicative that Factor A and Factor B interact with one another, and this notion is supported by our p-value of 0.0240.

Continuing our analysis of the A:B interaction variable, we first need to note a couple of things. Factor A has 3 levels (8 feet, 10 feet, and 12 feet), and Factor B has 2 levels (H and R). Additionally, we must also note that Factor A is quantitative, and Factor B is qualitative. Because Factor A is quantitative and has 3 levels, we can test for both a linear and quadratic effect. We need to keep in mind though that Factor B is qualitative, so we will test for linear and quadratic effects while holding Factor B fixed at straight on, and then we will test for linear and quadratic effects while holding Factor B fixed at to the right. The calculations to test for the significance of these terms can be seen below.

	Horizontal Position	
Distance	H (Straight On)	R (To the Right)
8	16.05	13.55
10	10.80	12.00
12	10.85	7.40

Factor B Fixed at H (Straight On)

B_{lin} = -1(16.05) + 1(10.85) = -5.20
B_{quad} = 1(16.05) – 2(10.80) + 1(10.85) = 5.30

F stat for B_{lin} = c^2*(n/sum(c_i^2)/MSE = $(-5.20)^2$*(20/4)/15.5 = 8.722580645
F stat for B_{quad} = c^2*(n/sum(c_i^2)/MSE = $(5.30)^2$*(20/12)/15.5 = 3.020430108

P-value for B_{lin} = 0.003989327
P-value for B_{quad} = 0.08556764

Factor B Fixed at R (To The Right)

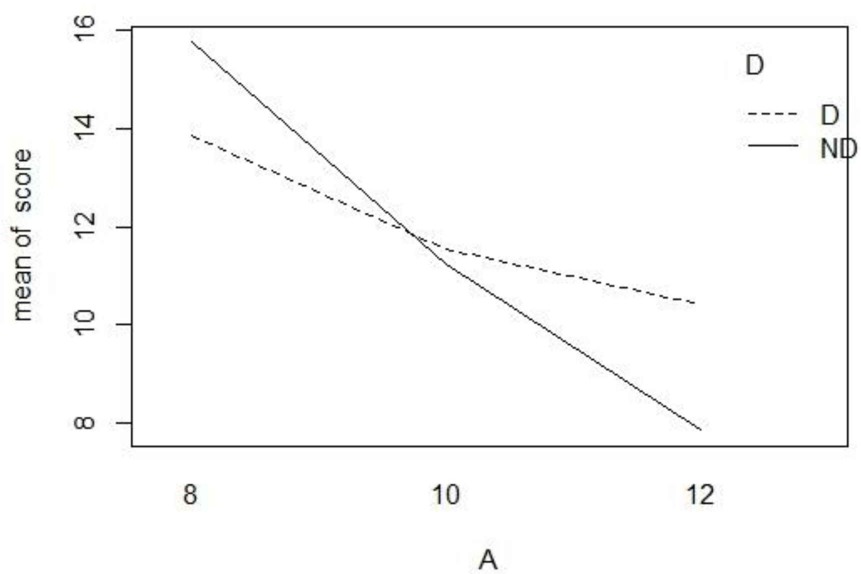
B_{lin} = -1(13.55) + 1(7.4) = -6.15
B_{quad} = 1(13.55) – 2(12) + 1(7.4) = -3.45

F stat for B_{lin} = c^2*(n/sum(c_i^2)/MSE = $(-6.15)^2$*(20/4)/15.5 = 12.20080645
F stat for B_{quad} = c^2*(n/sum(c_i^2)/MSE = $(-3.45)^2$*(20/12)/15.5 = 1.279983871

P-value for B_{lin} = 0.0007362503
P-value for B_{quad} = 0.2608425

These hand calculations (though R was used for the p-values) match the conclusions seen in our R code as well. Fixing Factor B at straight on (H), we find that there is a linear effect between score and distance. Looking at our means table, it is apparent that this linear effect is also negative—as distance increases, score decreases. Fixing Factor B at to the right (R), we find that there is a linear effect between score and distance. Similar to when Factor B was fixed at straight on, it is apparent that this linear effect is negative too. In both cases we had insufficient evidence to prove that there was a significant quadratic effect. Simply looking at the A:B interaction plot, none of these findings are particularly surprising.

Next, let's look at our A:D interaction term, which is significant with a p-value of 0.0459. An interaction plot can be seen below.



There are a couple of interesting things to note with the A:D interaction plot. For starts, the first thing that jumps off this plot is that at 8 feet, the nondominant mean is higher than the dominant mean. This is quite surprising, and maybe speaks to our groups superior dart throwing skill (or lack thereof). When throwing with our dominant hand, there is a clear decrease in score as we increase distance from 8 feet to 10 feet to 12 feet; however, the change is gradual and somewhat minimal. Comparing the 8-foot distance and 12-foot distance, the means only differed by 3.45. When throwing with our nondominant hand, there is a steeper decrease in score as we increase distance from 8 feet to 10 feet to 12 feet. Comparing the 8-foot and 12-foot distance, the means differed by 7.9. This hints that distance has a larger impact on the nondominant hand as compared to dominant hand. These patterns described are indicative that Factor A and Factor D interact with one another, and this notion is supported by our p-value of 0.0459.

Continuing our analysis of the A:D interaction variable, we again need to note a couple of things. Factor A has 3 levels (8 feet, 10 feet, and 12 feet), and Factor D has 2 levels (D and ND). Additionally, we must also note that Factor A is quantitative, and Factor D is qualitative. Because Factor A is quantitative and has 3 levels, we can test for both a linear and quadratic effect. We need to keep in mind though that

Factor D is qualitative, so we will test for linear and quadratic effects while holding Factor D fixed at dominant, and then we will test for linear and quadratic effects while holding Factor D fixed at nondominant. The calculations to test for the significance of these terms can be seen below.

	Hand	
Distance	D (Dominant)	ND (Nondominant)
8	13.85	15.75
10	11.55	11.25
12	10.40	7.85

Factor D Fixed at D (Dominant)

B_{lin} = -1(13.85) + 1(10.40) = -3.45
B_{quad} = 1(13.85) – 2(11.55) + 1(10.40) = 1.15

F stat for B_{lin} = c^2*(n/sum(c_i^2)/MSE = $(-3.45)^2$*(20/4)/15.5 = 3.839516129
F stat for B_{quad} = c^2*(n/sum(c_i^2)/MSE = $(1.15)^2$*(20/12)/15.5 = 0.1422043011

P-value for B_{lin} = 0.053085
P-value for B_{quad} = 0.7069678

Factor B Fixed at ND (Nondominant)

B_{lin} = -1(15.75) + 1(7.75) = -8
B_{quad} = 1(15.75) – 2(11.25) + 1(7.75) = 1

F stat for B_{lin} = c^2*(n/sum(c_i^2)/MSE = $(-8)^2$*(20/4)/15.5 = 20.64516129
F stat for B_{quad} = c^2*(n/sum(c_i^2)/MSE = $(1)^2$*(20/12)/15.5 = 0.1075268817

P-value for B_{lin} = 1.675551e-05
P-value for B_{quad} = 0.7437226

These hand calculations (though R was used for the p-values) do not perfectly match the conclusions seen in our R code. Discussing the findings from these hand calculations first, when we fix Factor D at dominant (D), we have insufficient evidence to prove that there is a linear effect or quadratic effect. When we fix Factor D at nondominant (ND), we find that there is a linear effect between score and distance. Looking at our means table, it is apparent that this linear effect is also negative—as distance increases, score decreases. Additionally, we have insufficient evidence to prove that there is a quadratic effect. The surprising result here is that we have insufficient evidence to prove there is a linear effect between distance and score when Factor D is fixed at dominant. This relationship appears linear on the interaction plot, but after some additional thought, it is plausible that because the rate of change is so gradual, there is not a statistically significant linear effect.

It is good practice in statistical analysis to check for the significance of interaction terms before checking the significance of the individual variables. We have already analyzed the two significant interaction terms, A:B and A:D, so now it is time check for single variable significance. The only two individual variables that are significant are A and B. Due to their involvement in the interaction terms, it would be statistically irresponsible for us to analyze these two variables by themselves. The last variable to analyze is our most significant variable, blocking.

The blocks variable was the most significant variable in our experiment, having a p-value of 2e-16. One way to gauge the effectiveness of blocking is by calculating the relative efficiency. The calculation for relative efficiency can be seen below.

$$RE = \frac{(dfe_{ercbd} + 1)(df_{ecrd} + 3)}{(dfe_{ercbd} + 3)(df_{ecrd} + 1)} * \frac{\sigma^2_{crd}}{\sigma^2_{rcbd}}$$
$$RE = \frac{(93)(99)}{(95)(97)} * \frac{49.8}{15.5}$$
$$RE = 3.210113945$$

We would need 3.21 times the amount of data in a completely randomized design to achieve the same variance of treatment means we did with the completely randomized block design. Because our relative efficiency is over 3, I would make the argument that blocking is worthwhile in this instance.