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Physics
ASSIGNMENT - 3

4.5

1) Work function, $\phi = 2.4 eV$
voltage at which no electron reach cathode $= 4.8V$

a) Number of electron, $q = 1$

i) Maximum kinetic energy, $K.E = V_s . q$
$= 4.8 . 1$
$K.E_m = 4.8 eV$

ii) Mass of an electron, $m = 9.1 \times 10^{-31} kg$
we know that, $KE = mv^2/2$
$4.8e = 9.1 \times 10^{-31} \times v^2/2$
so, $\frac{4.8 \times 2 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = v^2$
$v^2 = \frac{15.36 \times 10^{-19}}{9.1 \times 10^{-31}}$
$v^2 = 1.69 \times 10^{+12}$
$v = 1.3 \times 10^6 m/s$
speed at maximum, $v = 1.3 \times 10^6 m/s$

b) Initial Energy, $E_{in} = K.E + \phi$
$= 4.8 + 2.4$
$E_{in} = 7.2 eV$
$h = 4.1357 \times 10^{-15} eV$
$c = 3 \times 10^8 m/s$ (speed of light)
$E = hc/\lambda$
wavelength, $\lambda = \frac{hc}{E} = \frac{4.1357 \times 10^{-15} \times 3 \times 10^8}{7.2}$
$= 1.72 \times 10^{-7}$
$\lambda = 172 nm$

c) Minimum Energy to emit electron, $E_{min} = \phi = hf_0$

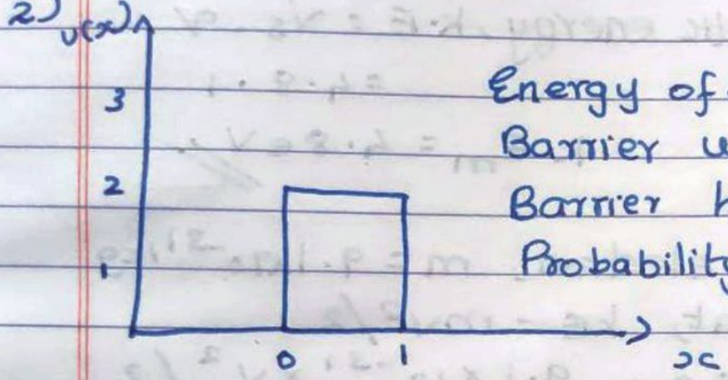
minimum frequency, $f_0 = \phi / h$

$= 2.4 / 4.1357 \times 10^{-15}$

$f_0 = 0.58 \times 10^{15}$

$\therefore f_0 = 5.8 \times 10^{14}\ Hz$

2)



Energy of electron $= 1.2\ ev$

Barrier width $= 1 \times 10^{-10}\ m$

Barrier height $= 2\ ev$

Probability $\Rightarrow T = e^{-2KL}$

$$K = \frac{\sqrt{2m(V-E)}}{h}$$

(i) Mass of a electron, $m_e = 9.1 \times 10^{-31}\ kg$

$h = 1.055 \times 10^{-34}\ Js$

$$k_e = \frac{\sqrt{2m_e(V-E)}}{h}$$

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31}(2-1.2) \times 1.6 \times 10^{-19}}}{1.055 \times 10^{-34}}$$

$$= \frac{\sqrt{23.312 \times 10^{-50}}}{1.055 \times 10^{-34}}$$

$$= \frac{4.826 \times 10^{-25}}{1.055 \times 10^{-34}}$$

$k_e = 4.574 \times 10^{9}\ m^{-1}$

$2k_eL : 2 \times 4.574 \times 10^{9} \times 10^{-10}$

$2k_eL = 9.148 \times 10^{-1}$

$T = e^{-2k_eL} = e^{-9.148 \times 10^{-1}}$

$\therefore$ Probability, $T_e = 0.40060$; i.e., 40%

Ans:- 0.633

ii) Mass of a proton $m_p = 1.6 \times 10^{-27}$ kg

$$k_p = \frac{\sqrt{2m_p(U-E)}}{\hbar}$$

$$= \frac{\sqrt{2 \times 1.6 \times 10^{-27} (2-1.2) \times 1.6 \times 10^{-19}}}{1.055 \times 10^{-34}}$$

$$= \frac{\sqrt{4.096 \times 10^{-46}}}{1.055 \times 10^{-34}} = \frac{2.023 \times 10^{-23}}{1.055 \times 10^{-34}}$$

$$k_p = 1.917 \times 10^{11} m^{-1}$$

$$2k_pL = 3.834 \times 10 = 38.34$$

$$T_p = e^{-2k_pL}$$

$$= e^{-38.34} = 1/e^{38.34}$$

Tunneling propability of a proton $T_p = 2.23 \times 10^{-17}$ //

Ans:- 3×10^{-9}

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3) Pontential, $V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise} \end{cases}$

wavefunction, $\psi_n = A \sin\left(\frac{n\pi x}{L}\right)$

a) calculate A,

normalisation, $\int_0^L \psi_{(x)} \cdot (\psi)'_x \, dx = 1$

$$= \int_0^L A \sin\left(\frac{n\pi x}{L}\right) \cdot A \sin\left(\frac{n\pi x}{L}\right) \cdot dx = 1$$

$$= A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) \cdot dx = 1$$

$$A^2 \int_0^L \frac{1}{2} - \frac{\cos 2(n\pi x/L)}{2} \cdot dx = 1 \quad \left(\int \sin^2 dx = \frac{1}{2} \cdot \frac{\cos 2x}{2} dx\right)$$

$$A^2 \left[\frac{x}{2} - \frac{\sin 2(n\pi x/L)}{2} \cdot \frac{L}{n\pi}\right]_0^L = 1$$

$\sin(n\pi) = 0$

$\therefore$

$$A^2\left[\frac{L}{2} - 0\right] = 1$$

$$A^2 = 2/L$$

$$\boxed{A = \sqrt{2/L}}$$

$$\boxed{\psi_n = \sqrt{2/L}\,\sin\left(\frac{n\pi x}{L}\right)}$$

b) calculate $\langle x \rangle$ and $\langle p \rangle$

$$\langle x \rangle = \int_0^L \psi^*(x) \cdot x \cdot \psi(x) \cdot dx$$

$$= \int_0^L \sqrt{2/L} \cdot \sin\left(\frac{n\pi x}{L}\right) \cdot x \cdot \sqrt{2/L} \cdot \sin\left(\frac{n\pi x}{L}\right) \cdot dx$$

$$= \frac{2}{L}\int_0^L x \cdot \sin^2\left(\frac{n\pi x}{L}\right) \cdot dx$$

substitute $t = \frac{x}{L}$

$$\frac{dt}{dx} = \frac{1}{L}, \quad dx = L \cdot dt$$

$$\langle x \rangle = \frac{2}{L} \cdot L^2 \int t \cdot \sin^2(n\pi t) \cdot dt$$

$$= \frac{\cancel{2}L}{\cancel{2}} \int (t - t\cos(2\pi n t) \cdot dt)$$

$$\langle x \rangle = L \int t \cdot dx - \int t\cos(2n\pi t) \cdot dt$$

Solving this we get,

$$\langle x \rangle = L\left[\frac{t^2}{2} - \frac{t\sin(2\pi x t)}{2n\pi} - \frac{\cos(2\pi n t)}{4n^2\pi^2}\right]$$

$t = x/L$ and limit is 0 to L, so

$$\langle x\rangle = L\left[\frac{L^2}{L^2\cdot 2} - \frac{x\sin(2\pi L n/L)}{2n\pi L} - \frac{\cos(2n\pi L/L)}{4n^2\pi^2} - 0 + \frac{x\sin(0)}{2n\pi L} + \frac{\cos(0)}{4n^2\pi^2}\right]$$

$$\langle x\rangle = L\left[\frac{1}{2} - 0 - \frac{1}{4n^2\pi^2} + 0 + \frac{1}{4n^2\pi^2}\right]$$

$$\langle x\rangle = L\left[\frac{1}{2}\right]$$

$$\therefore \boxed{\langle x\rangle = \frac{L}{2}}$$

$$\langle p\rangle = \int_0^L \psi^*(x)\cdot\hat{p}\cdot\psi(x)\cdot dx \quad \left(\hat{p} = -ih\frac{\partial}{\partial x}\right)$$

$$= \int_0^L \sqrt{2/L}\,\sin\left(\frac{n\pi x}{L}\right)\cdot -ih\frac{\partial}{\partial x}\cdot\sqrt{2/L}\,\sin\left(\frac{n\pi x}{L}\right)\cdot dx$$

$$= \int_0^L \sqrt{2/L}\cdot\sin\left(\frac{n\pi x}{L}\right)\cdot -ih\sqrt{2/L}\cdot\frac{n\pi}{L}\cos\left(\frac{n\pi x}{L}\right)\cdot dx$$

$$= \int_0^L -\frac{ih2}{L}\cdot\sin\left(\frac{n\pi x}{L}\right)\cdot\frac{n\pi}{L}\cos\left(\frac{n\pi x}{L}\right)\cdot dx$$

$$= -\frac{ih2n\pi}{L^2}\int_0^L \sin\left(\frac{n\pi x}{L}\right)\cdot\cos\left(\frac{n\pi x}{L}\right)\cdot dx$$

Subsitute $\frac{n\pi x}{L} = t$, $dx = \frac{dt\cdot L}{n\pi\cos\left(\frac{n\pi x}{L}\right)}$

$$\langle p\rangle = -\frac{ih2\pi n}{L^2}\cdot\frac{L}{n\pi}\int t\cdot dt$$

$$\langle p\rangle = -\frac{ih2}{L}\left[t^2/2\right]$$

$t = \sin\left(\frac{n\pi x}{L}\right)$

$$\langle p \rangle = \frac{-ih2}{L}\left[\frac{\sin^2(n\pi x/L)}{2}\right]_0^L$$

$$= \frac{-ih2}{L}\left[\frac{\sin^2(n\pi)}{2} - \frac{\sin^2 0}{2}\right]$$

$\sin \pi n = 0$

so,

$$\langle p \rangle = -\frac{ih2}{L}\left[0 - 0\right] = 0$$

$$\boxed{\therefore \langle p \rangle = 0}$$

$$\boxed{\langle p \rangle = 0 \quad \text{and} \quad \langle x \rangle = \frac{L}{2}}$$

c) Calculate $\langle x^2 \rangle$ and $\langle p^2 \rangle$

limit 0 to L

$$\langle x^2 \rangle = \int_0^L \psi(x) \cdot x^2 \cdot \psi(x) \cdot dx$$

$$\psi(x) = \sqrt{2/L} \cdot \sin\left(\frac{n\pi x}{L}\right)$$

so, $\langle x^2 \rangle = \int_0^L 2/L \cdot x^2 \cdot \sin^2\left(\frac{n\pi x}{L}\right) \cdot dx$

$t = x/L$, then $dx = L \cdot dt$, $x^2 = L^2 t$

$$\langle x^2 \rangle = \int \frac{2}{L} \cdot L^3 t^2 \sin^2(n\pi x) \cdot dt$$

$$= \frac{2L^3}{L}\int t^2 \sin^2(n\pi x) \cdot dt$$

$$= 2L^2 \int t^2 \left(\frac{1}{2} - \frac{\cos(2\pi nt)}{2} \cdot dt\right)$$

$$= 2L^2 \int \frac{t^2}{2} - \frac{t^2 \cdot \cos(2\pi n t)}{2} \cdot dt$$

$$\frac{2L^2}{2} \int t^2 \cdot dt - \int t^2 \cdot \cos(2n\pi t) \cdot dt$$

when we solve $\int t^2 \cdot dt = t^3/3$

and $\int t^2 \cdot \cos(2n\pi) \cdot dt = + \frac{t^2 \cdot \sin(2n\pi t)}{2\pi n} - \frac{\sin(2n\pi t)}{4\pi^3 n^3}$

$$+ \frac{t \cos(2\pi n t)}{2\pi^2 n^2}$$

so, $\langle x^2 \rangle = \frac{2L^2}{2} \left(\frac{t^3}{3} - \frac{t^2 \sin(2\pi n t)}{2\pi n} + \frac{\sin(2\pi n t)}{4\pi^3 n^3} - \frac{t \cos(2\pi n t)}{2\pi^2 n^2} \right)$

Substitute $t = x/L$

$$\langle x^2 \rangle = L^2 \left[\frac{L^3}{L^3 \cdot 3} - L^2 \sin(2\pi n) \right.$$

$$\langle x^2 \rangle = L^2 \left[\frac{x^3}{L^3 \cdot 3} - \frac{x^2 \sin(2n\pi x/L)}{L^2 \, 2\pi n} + \frac{\sin\left(\frac{2\pi n x}{L}\right)}{4\pi^3 n^3} - \frac{x \cos(2n\pi x/L)}{L \, 2\pi^2 n^2} \right]_0^L$$

$$= L^2 \left[\frac{L^3}{L^3 \cdot 3} - \frac{L^2 \sin(2n\pi)}{L^2 \, 2n\pi} + \frac{\sin(2n\pi)}{4\pi^3 n^3} - \frac{L \cos(2n\pi)}{L \, 2\pi^2 n^2} \right.$$

$$\left. - 0 - 0 + \frac{\sin(0)}{4\pi^3 n^3} - 0 \right]$$

$$= L^2 \left[\frac{1}{3} - \frac{\sin(2n\pi)}{2n\pi} + \frac{\sin(2\pi n)}{4\pi^3 n^3} - \frac{\cos(2n\pi)}{2\pi^2 n^2} \right.$$

$$\left. + \frac{\sin(0)}{4\pi^3 n^3} \right] = L^2 \left[\frac{1}{3} - \frac{1}{2\pi^2 n^2} \right]$$

$$\boxed{\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2\pi^2 n^2}}$$

$$\langle p^2 \rangle = \int_0^L \psi^*(x)\,(\hat{p})^2 \cdot \psi x \cdot dx$$

$$= \int_0^L \frac{2}{L}\sin\left(\frac{n\pi x}{L}\right) - h^2 \frac{d^2}{dx^2} \cdot \sin\left(\frac{n\pi x}{L}\right) \cdot dx$$

$$= \int_0^L \frac{-2h^2}{L} \cdot \sin\left(\frac{\pi x n}{L}\right) \cdot -\left(\frac{n\pi}{L}\right)^2 \sin\left(\frac{n\pi x}{L}\right) dx$$

$$\langle p^2 \rangle = \frac{2h^2n^2\pi^2}{L^3} \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) \cdot dx$$

$$= \frac{2h^2n^2\pi^2}{L^3} \int_0^L \frac{1}{2} - \frac{\cos(2\pi n x/L)}{2} \cdot dx$$

$$\frac{h^2n^2\pi^2}{L^3}\left[\int_0^L 1\,dx - \int_0^L \frac{\cos(2\pi n x/L)}{2} \cdot dx\right]$$

$$\frac{h^2n^2\pi^2}{L^3}\left[[x]_0^L - \left[\frac{\sin(2n\pi x/L)}{2\pi n/L}\right]_0^L\right]$$

$$\frac{h^2n^2\pi^2}{L^3}\left[L - 0 + 0\right]$$

$$\boxed{\langle p^2 \rangle = \frac{h^2n^2\pi^2}{L^2} = \left(\frac{hn\pi}{L}\right)^2}$$

$$\therefore \langle p^2 \rangle = \left(\frac{hn\pi}{L}\right)^2 \text{ and } \langle x^2 \rangle = L^2\left[\frac{1}{3} - \frac{1}{2\pi^2 n^2}\right]$$

d) $\Delta x \, \Delta p$

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

$$= L^2\left(\frac{1}{3} - \frac{1}{2(n\pi)^2} - \frac{1}{4}\right) = \frac{L^2}{4}\left(\frac{1}{3} - \frac{2}{(\pi n)^2}\right)$$

$$\Delta x = \frac{L}{2}\sqrt{\frac{1}{3} - \frac{2}{(n\pi)^2}} \quad \text{or} \quad L\sqrt{\frac{1}{12} - \frac{1}{2(n\pi)^2}}$$

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

$$\Delta p = \sqrt{\left(\frac{\hbar n\pi}{L}\right)^2 - 0^2} = \frac{\hbar n\pi}{L}$$

$$\Delta x . \Delta p = \frac{\cancel{L}}{2}\sqrt{\frac{1}{3} - \frac{2}{(n\pi)^2}} \cdot \frac{\hbar n\pi}{\cancel{L}}$$

$$= \boxed{\frac{\hbar n\pi}{2}\sqrt{\frac{1}{3} - \frac{2}{(n\pi)^2}} \quad \text{or} \quad \hbar n\pi\sqrt{\frac{1}{12} - \frac{1}{2(n\pi)^2}}}$$

Based on uncertainity principle,

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

i.e, $\frac{\hbar n\pi}{2}\sqrt{\frac{1}{3} - \frac{2}{(n\pi)^2}} \geq \frac{\hbar}{2}$

4) f, Eighen function = $\cos nx$

General equation:-

$$Df = \lambda f$$

where λ is eighen value and D is operator

$$D = \frac{d^2}{dx^2} \text{ (Given)}$$

$$\frac{d^2}{dx^2}(\cos nx) = \lambda \cos nx$$

$$\lambda \cos nx = \frac{d}{dx}\left(\frac{d(\cos nx)}{dx}\right)$$

$$\lambda \cos nx = \frac{d}{dx}(-n \sin nx)$$

$$\lambda \cos nx = -n^2 \cos nx$$

$$\lambda \cancel{\cos nx} = -n^2 (\cancel{\cos nx})$$

$$\therefore \lambda = -n^2$$

$\therefore$ corrasponding Eighen values of $\cos nx$ is $\lambda = -1, -4, -9 \ldots$

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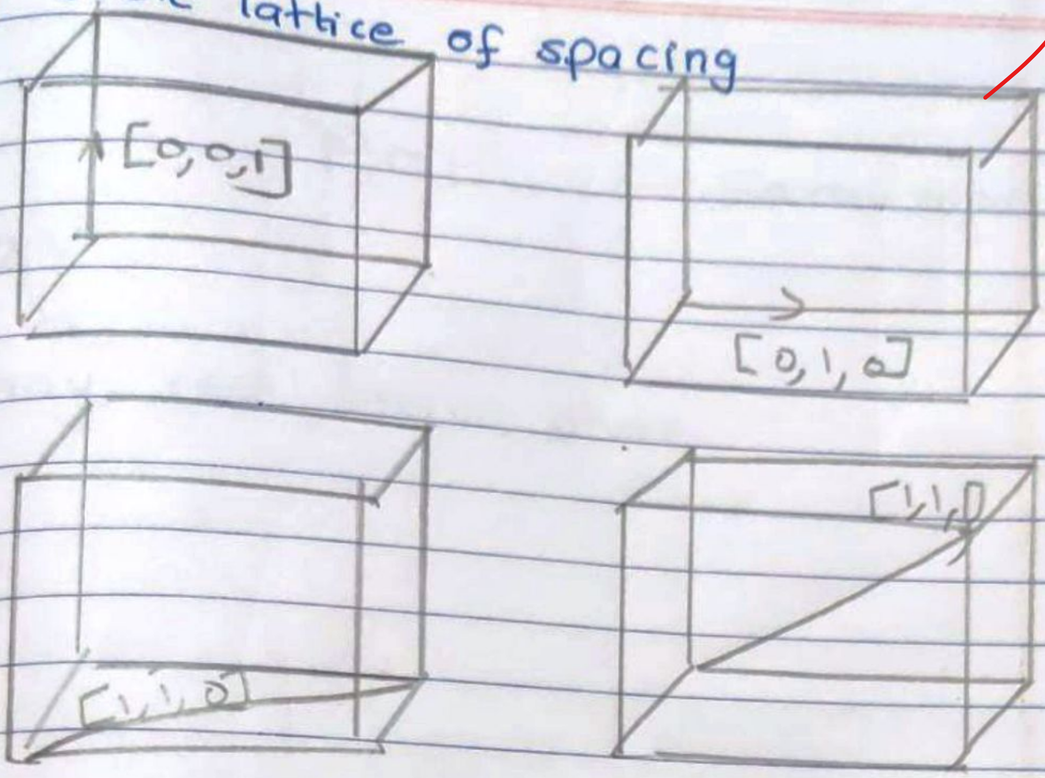
5) a) Cubic lattice of spacing



have to show the co-ordinates points

b) Intercepts : 2a, -4b and 3c

co-efficient of interepts : 2, -4, 3   no commas

Reciprocal of co-efficient : $\frac{1}{2}$, $\frac{-1}{4}$, $\frac{1}{3}$

multiply 12 with recipocal : $12 \times \frac{1}{2}$, $12 \times \frac{-1}{4}$, $12 \times 1/3$
to clear fraction

∴ : 6, -3, 4

∴ miller indices for the given matrix is 6, -3, 4

parentheses brackets