Module
4
Signal Representation and Baseband Processing

Lesson
17
Noise

After reading this lesson, you will learn about

- ***Basic features of Short Noise;***
- ***Thermal (Johnson) Noise;***
- ***Various other forms of Noise;***
- ***Shannon's channel capacity equation and its interpretation;***

As noted earlier, when send some information-bearing signal through a physical channel, the signal undergoes changes in several ways. Some of the ways are the following:

- The signal is usually reduced or attenuated in strength (measured in terms of received power or energy per symbol)
- The signal propagates at a speed comparable to the speed of light, which is high but after all, finite. This means, the channel delays the transmission
- The physical channel may, occasionally introduce additive noise. A transmission cable, for example, may be a source of noise.
- The physical channel may also allow some interfering signals, which are undesired
- The channel itself may have a limitation in bandwidth which may lead to some kind of distortion of the transmitted signal.

Usually the strength of the signal at the receiver input is so low that it needs amplification before any significant signal processing can be carried out. However, the amplifier, while trying to boost the strength of the weak signal, also generates noise within. The power of this noise (and in some other modules down the line such as the frequency down converter in a heterodyne radio receiver) is not negligible. This internally generated noise is always present in a communication receiver. Various mathematical models exist to depict different noise processes that originate in the receiver and affect the transmitted signal. We will consider a simple additive noise model wherein an 'equivalent noise source' will be assumed ahead of a 'noise-less receiver' [n(t) in **Fig. 4.17.1**]. This noise, sometimes referred as 'channel noise', is additive in the sense that the instantaneous noise amplitude is added with the instantaneous amplitude of the received signal.

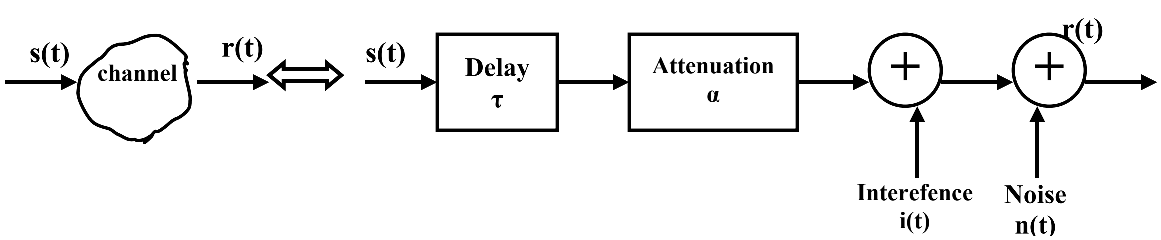

Fig. 4.17.1: *An equivalent model for the physical propagation channel, including the noise generated by the receiver front end*

If s(t) is the transmitted signal and 'α' is the attenuation introduced by the channel, the received signal r(t) can be expressed as,

$$r(t) = \alpha s(t-\tau) + I(t) + n(t) \quad 4.17.1$$

$I(t)$ represents the interfering signal, if any.

In this lesson, we briefly discuss about the physical features of noise and a short discussion on a baseband channel model for additive white Gaussian noise (AWGN) under certain assumptions.

It is a common knowledge that movable electrons within a passive or active electronic component are responsible for current when excited by external voltage. However, even when no voltage is applied externally, electrons are always in random motion, interacting with other electrons and the material's lattice sites and impurities. The average velocity in any direction remains zero. This statistically random electron motion creates a noise voltage.

Noise is a very important factor that degrades the transmitted signal in a receiver. It is necessary to know the noise level. Two important and relevant forms of noise are, a) *thermal noise* produced by random, thermally produced, motions of carriers in metals and semiconductors and b) *shot noise* produced by 'particle-like' behavior of electrons and photons when an external excitation is available to produce current. Shot noise is avoidable only if we reduce all current to zero.

Shot Noise

Let us consider a 'steady' or dc electric current *I* between two points A and B with each electron carrying a charge '*q*'. On an average, the number of charges moving from A to B during time '*t*' is

$$n_{av} = \frac{I.t}{q} \qquad 4.17.2$$

Now, at the microscopic level, the electrons do not move in a perfectly regular fashion. The rate of flow varies unpredictably within short spans of time. This means that the instantaneous current is usually different from 'I'. This fluctuation around a nominal average value of 'I' is modeled as a noise current (i_n). It has been established that the observed mean squared value of this fluctuating current is,

$$\mathrm{E}\left[i_n^2\right] = 2.q.I.B \qquad 4.17.3$$

where *B* is the bandwidth of the system used for measurement. Interestingly, the mean squared value of the noise current is proportional to the gross current 'I'. So, if the average (bias) current in a photo detector is high, there is a possibility of considerable shot noise. This is an important issue in the design of optical detectors in fiber optic communication. Shot noise in optical devices is widely called as 'quantum noise'. Low noise active electronic amplifiers for wireless receivers are intelligently designed to suppress the shot noise by electrostatic repulsion of charge carriers.

Shot noise is closely described and modeled as a Poisson process. The charge carriers responsible for the shot noise follow Poisson distribution [Lesson #7]. Analytically, the noise power may be obtained from the Fourier transform of the auto-correlation of this random process.

Thermal Noise (also known as Johnson-Nyquist noise and Johnson noise) :

Thermal noise is generated by the equilibrium fluctuations of the carriers in electronic components, even in absence of an applied voltage. It originates due to random thermal motion of the charge carriers. It was first measured by J. B. Johnson in 1928 and theoretically established by H. Nyquist through a fluctuation – dissipation relationship of statistical thermodynamics. Thermal noise is different from shot noise, which is due to current fluctuations that occur only when a macroscopic current exists.

The thermal noise power P, in watts, is given by $P = 4kT\Delta f$, where k is Boltzmann's Constant [$k = 1.380\,6505(24) \times 10^{-23}$ J/K], T is the component temperature in Kelvin and Δf is the bandwidth in Hz. Thermal noise power spectral density, Watt per Hz, is constant throughout the frequency spectrum of interest (typically upto 300 GHz). It depends only on k and T. That is why thermal noise is often said to be a white noise in the context of radio communication.
A quick and good estimate of thermal noise, in dBm [0 dBm = 1 mWatt], at room temperature (about 27^0C) is:

$$P = -174 + 10\log(\Delta f) \qquad 4.17.4$$

A quick calculation reveals that the total noise power in a receiver, with a bandwidth of 1 MHz and equivalent noise temperature of 27^0C, may be about -114 dBm.
The thermal noise voltage, v_n, that is expected across an 'R' Ohm resistor at an absolute temperature of 'T' K is given by:

$$v_n = \sqrt{4kT\Delta f} \qquad 4.17.5$$

So, thermal noise in a receiver can be made very low by cooling the receiver subsystems, which is a costly proposition.

Colour of noise

Several possible forms of noise with various frequency characteristics are some times named in terms of colors. It is assumed that such noise has components at all frequencies, with a spectral density proportional to $\frac{1}{f^{\alpha}}$.

White noise

It is a noise process with a flat spectrum vs. frequency, i.e. with same power spectral density, $\frac{N_o}{2}$ W/Hz. This means, a 1 KHz frequency range between 2 KHz and 3KHz contains the same amount of power as the range between 2 MHz and 2.001 MHz. Let us note here that the concept of an infinite-bandwidth white noise is only theoretical as the noise power is after all, finite in a physically realizable receiver. The additive Gaussian noise process is white.

Pink noise [flicker noise, 1/f noise]

The frequency spectrum of flicker noise is flat in logarithmic space, i.e., it has same power in frequency bands that are proportionally wide. For example, flicker noise in a system will manifest equal power in the range from 30 to 50 Hz and in the band from 3KHz to 5KHz. Interestingly, the human auditory system perceives approximately equal magnitude on all frequencies.

Brown noise

Similar to pink noise, but with a power density decrease of 6 dB per octave with increasing frequency (density proportional to $\frac{1}{f^2}$) over a frequency range which does not include DC. It can be generated by simulating Brownian motion and by integration*Blue noise:* Power spectral density of blue noise increases 3 dB per octave with increasing frequency (α = -1) over a finite frequency range. This kind of noise is sometimes useful for dithering.

Shannon's Channel Capacity Equation

The amount of noise present in the receiver can be represented in terms of its power $N = \frac{v_n^2}{R_{ch}}$, where R_{ch} is the characteristic impedance of the channel, as seen by the receiver and v_n is the rms noise voltage. Similarly, the message bearing signal can be represented by its power we can represent a typical message in terms of its average signal power $S = \frac{v_s^2}{R_{ch}}$, where v_s is the rms voltage of the signal. Now, it is reasonable to assume that the signal and noise are uncorrelated i.e., they are not related in any way and we cannot predict one from the other. If 'P_r' is the total power received due to the combination of signal and noise, which are uncorrelated random processes, we can write $v_r^2 = v_s^2 + v_n^2$, i.e.,

$$P_r = S + N \qquad 4.17.6$$

Now, let the received signal with rms voltage 'v_s' contain 'b' bits of information per unit time and noise with rms voltage 'v_n'. If, for the sake of simplicity, we decide to sample the received signal once per unit time, we can hope to recover the b bits of information correctly from the received signal sample by adopting the following strategy:

We quantize the sample in a manner such that the noise is not likely to make our decision about b-bits of information wrong. This is achievable if we adopt a b-bit quantizer(i.e. 2^b quantizer levels) and the noise sample voltage is less than half the step size. The idea then, is simply to read the quantizer output as the received b-bit information. So, the limiting condition may be stated as:

$2^b = \frac{V_{r\,max}}{V_{n\,max}}$, where $V_{r\,max}$ is the maximum allowable received signal amplitude and $V_{n\,max}$ is the maximum allowable noise amplitude. With this quantizer, our decision will be correct when

$2^b \geq \frac{V_r}{V_n}$ and our decision will be erroneous if $2^b \leq \frac{V_r}{V_n}$. So, the limiting condition for extracting b-bits of information from noise-corrupted received signal is,

$$2^b = \frac{V_r}{V_n} \qquad 4.17.7$$

Now, we can write,

$$2^b = \frac{V_r}{V_n} = \sqrt{\frac{v_r^2}{v_n^2}} = \sqrt{\frac{V_S^2 + V_n^2}{V_n^2}} = \sqrt{1 + \left(\frac{S}{N}\right)} \qquad 4.17.8$$

Or, equivalently,

$$\log_2\left(\sqrt{1 + \left(\frac{S}{N}\right)}\right) \qquad 4.17.9$$

Now, from Nyquist's sampling theorem, we know that, for a signal of bandwidth 'B', the maximum number of such samples that can be obtained per unit time is 2B and hence, the maximum amount of information (in bits) that can be obtained per unit time, is,

$$I_{\max} = 2Bb = 2B\log_2\left(\sqrt{1 + \left(\frac{S}{N}\right)}\right) = 2B\log_2\left(1 + \frac{S}{N}\right). \qquad 4.17.10$$

Eq. 4.17.10 is popularly expressed as,

$$C = B\log_2\left(1 + \frac{S}{N}\right) \qquad 4.17.11$$

'C' indicates the 'capacity of the waveform channel', i.e. the maximum amount of information that can be transmitted through a channel with bandwidth 'B' and enjoying signal-to-noise ratio of S/N. Eq. 4.17.11 is popularly known as *Shannon-Hartley Channel Capacity Equation* for additive white Gaussian noise waveform channel.

Interpretation of Shannon-Hartley Channel Capacity Equation

a) We observe that the capacity of a channel can be increased by either i) increasing the channel bandwidth or ii) increasing the signal power or iii) reducing the in-band noise power or iv) any judicious combination of the three. Each approach in practice has its own merits and demerits. It is indeed, interesting to note that, all practical digital communication systems, designed so far, operate far below the capacity promised by Shannon-Hartley equation and utilizes only a fraction of the capacity. There are multiple yet interesting reasons for this. One of the overriding requirements in a practical system is sustained and reliable performance within the regulations in force. However, advances in coding theory (especially turbo coding), signal processing techniques and VLSI techniques are now making it feasible to push the operating point closer to the Shannon limit.

b) If, $B \rightarrow \infty$, we apparently have infinite capacity but it is not true. As $B \rightarrow \infty$, the in-band noise power, N also tends to infinity [$N = N_o.B$, N_o : single-sided noise power spectral density, a constant for AWGN] and hence, $S/N \rightarrow 0$ for any finite signal power 'S' and $\log_2\left(1+\frac{S}{N}\right)$ also tends to zero. So, it needs some more careful interpretation and we can expect an asymptotic limit.

At capacity, the bit rate of transmission $R_b = C$ and the duration of a bit $= T_b = \frac{1}{R_b} = \frac{1}{C}$. If the energy received per information bit is E_b, the signal power S can be expressed as, S = energy received per unit time $= E_b.R_b = E_b.C$. So, the signal-to-noise ratio $\frac{S}{N}$ can be expressed as,

$$\frac{S}{N} = \frac{E_b C}{N_0 B} \qquad 4.17.12$$

Now, from Eq. 4.17.11, we can write,

$$\frac{C}{B} = \log_2\left(1+\frac{E_b C}{N_0 B}\right) \qquad 4.17.13$$

This implies,

$$\begin{aligned}\frac{E_b}{N_0} &= \frac{B}{C}\left(2^{\frac{C}{B}} - 1\right)\\ &\cong \frac{B}{C}\left[\left(1+\frac{C}{B}\ln 2\right) - 1\right], \quad \text{for } B >> C\\ &= \log_e 2, \quad \text{for } B >> C\\ &= -1.6 \text{ dB}\end{aligned} \qquad 4.17.14$$

So, the limiting $\frac{E_b}{N_0}$, in dB is -1.6 dB. So, ideally, a system designer can expect to achieve almost errorless transmission only when the $\frac{E_b}{N_0}$ is more than -1.6 dB and there is no constraint in bandwidth.

c) In the above observation, we set $R_b = C$ to appreciate the limit in $\frac{E_b}{N_0}$ and we also saw that if $R_b > C$, the noise v_n is capable of distorting the group of 'b' information bits. We say that the bit rate has exceeded the capacity of the channel and hence errors are not controllable by any means.

To reiterate, all practical systems obey the inequality $R_b < C$ and most of the civilian digital transmission systems utilize the available bandwidth efficiently, which means B (in Hz) and C (in bits per second) are comparable. For bandwidth efficient transmission, the strategy is to increase the bandwidth factor $\frac{R_b}{B}$ while $R_b < C$. This is achieved by adopting suitable modulation and reception strategies, some of which will be discussed in Module #5.

Problems

Q4. 17.1) Name two passive electronic components, which may produce noise.

Q4. 17.2) If a resistor generates 1 nano-Watt/Hz, determine the temperature of the resistor.

Q4. 17.3) Determine the capacity of a waveform channel whose bandwidth is 10 MHz and signal to noise rotation is 10dB.