Module
5
Carrier Modulation

Lesson
31

Carrier Synchronization

After reading this lesson, you will learn about:

- ***Bit Error Rate (BER) calculation for BPSK;***
- ***Error Performance of coherent QPSK;***
- ***Approx BER for QPSK;***
- ***Performance Requirements;***

There are two major approaches in carrier synchronization:

a. To multiplex (FDM) a special signal, called pilot, which allows the receiver to extract the phase of the pilot and then use the information to synchronize its local carrier oscillator (L.O.) to the carrier frequency and phase of the received signal.

b. To employ a phase locked loop (PLL) or other strategy to acquire and track the carrier component. This approach has some advantages such as:
    a. No additional power is necessary at the transmitter for transmitting a pilot,
    b. It also saves overhead in the form of bandwidth or time. However, the complexity of the receiver increases on the whole. In this lesson, we will briefly discuss about a few basic approaches of this category.

A Basic Issue

Consider an amplitude-modulated signal with suppressed carrier

$$s(t) = A(t)\cos(\omega_c t + \overline{\varphi}) = A(t)\cos(2\pi f_c t + \overline{\varphi}) \qquad 5.31.1$$

Let the reference carrier generated in the demodulator be:

$$c(t) = \cos(2\pi f_c t + \phi) \qquad 5.31.2$$

Then, on carrier multiplication, we get,

$$s(t)c(t) = \frac{1}{2}.A(t).\cos\left(\phi - \overline{\phi}\right) + \frac{1}{2}.A(t).\cos\left(2\varpi_c t + \phi + \overline{\phi}\right) \qquad 5.31.3$$

The higher frequency term at twice the carrier frequency is removed by a lowpass filter in the demodulator and thus, we get the information-bearing signal as:

$$y(t) = \frac{1}{2}A(t)\cos(\varphi - \overline{\varphi}) \qquad 5.31.4$$

We see that the signal level is reduced by a factor $\cos(\varphi - \overline{\varphi})$ and its power is reduced by $\cos^2(\varphi - \overline{\varphi}) \rightarrow \varphi - \overline{\varphi} = 30^0$ means 1.25 dB decrease in signal power. So, if this phase offset is not removed carefully, the performance of the receiver is going to be poorer than expected. For simplicity, we did not consider any noise term in the above example and assumed that the phase offset ($\phi - \overline{\phi}$) is time-independent. Both these assumptions are not valid in practice.

Let us now consider QAM and M-ary PSK-type narrowband modulation schemes and let the modulated signal be:

$$s(t) = A(t)\cos(\omega_c t + \varphi) - B(t)\sin(\omega_c t + \varphi) \qquad 5.31.5$$

The signal is demodulated by the two quadrature carriers:

$$C_c(t) = (\cos\omega_c t + \overline{\varphi}) \text{ and } C_s(t) = -\sin(\omega_c t + \overline{\varphi}) \qquad 5.31.6$$

Now,

$$s(t)C_c(t) \xrightarrow[LPF]{after} y_I(t) = \frac{1}{2}A(t)\cos(\varphi - \overline{\varphi}) - \frac{1}{2}B(t)\sin(\varphi - \overline{\varphi}) \qquad 5.31.7$$

Similarly,

$$s(t)C_s(t) \xrightarrow[LPF]{after} y_Q(t) = \frac{1}{2}B(t)\cos(\varphi - \overline{\varphi}) + \frac{1}{2}A(t)\sin(\varphi - \overline{\varphi}) \qquad 5.31.8$$

'$y_I(t)$' and '$y_Q(t)$' are the outputs of the in-phase and Q-phase correlators. Note that the cross-talk interference from A(t) and B(t) in $y_I(t)$ and $y_Q(t)$. As the average power levels of A(t) and B(t) are similar, a small phase error ($\phi - \overline{\phi}$) results in considerable performance degradation. Hence, phase accuracy in a QAM or M-ary PSK receiver is very important.

Squaring Loop

Let us consider a modulated signal of the type:

$$s(t) = A(t)\cos(2\pi f_c t + \varphi)$$

The signal is dependent on the modulating signal A(t) and its mean, i.e. E[s(t)] = 0 when the signal levels are symmetric about zero. However, $s^2(t)$ contains a frequency component at $2f_c$. So, $s^2(t)$ can be used to drive a phase locked loop (PLL) tuned to $2f_c$ and the output of the voltage controlled oscillator (VCO) can be divided appropriately to get 'f_c' and 'φ' (refer **Fig.5.31.1**).

$$s^2(t) = A^2(t)\cos^2(2\pi f_c t + \varphi) = \frac{1}{2}A^2(t) + \frac{1}{2}A^2(t)\cos(4\pi f_c t + 2\varphi) \qquad 5.31.9$$

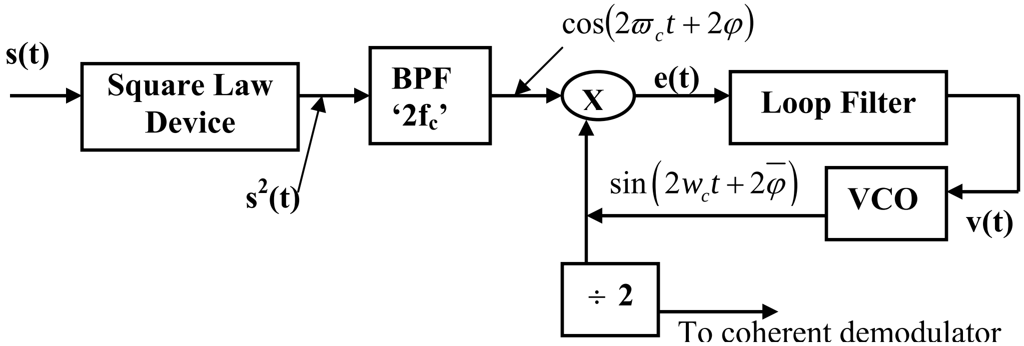

Fig. 5.31.1 *Block diagram illustrating the concept of a squaring loop*

Now, the expected value of $s^2(t)$ is:

$$E[s^2(t)] = \frac{1}{2}E[A^2(t)] + \frac{1}{2}E[A^2(t)]\cos(4\pi f_c t + 2\varphi) \qquad 5.31.10$$

Therefore, mean value of the output of the bandpass filter (BPF) at $2f_c$ is a sinusoid of frequency $2f_c$ and phase 2φ. So, the transmitted frequency can be recovered at the receiver from the modulated signal. An important point here is that the squaring operation has removed the sign information of the modulating signal A(t). As you may identify, the multiplier, along with the VCO and the loop filter constitute a PLL structure.

Principle of PLL

We will now briefly discuss about the principle of PLL. The peak amplitude of the input signal to the PLL is normalized to unit value. In practice, an amplitude limiter may be used to ensure this.

Let, $\bar{\varphi}$ represent the phase estimate at the output of VCO so that the 'error signal' at the output of the multiplier can be written as:

$$e(t) = \cos(4\pi f_c t + 2\varphi)\sin(4\pi f_c t + 2\bar{\varphi})$$

$$= \frac{1}{2}\sin 2(\bar{\varphi} - \varphi) + \frac{1}{2}\sin(8\pi f_c t + 2\varphi + 2\bar{\varphi}) \qquad 5.31.11$$

The loop filter is an LPF which responds to $\frac{1}{2}\sin 2(\bar{\varphi} - \varphi)$.

For a first order PLL (refer **Fig 5.31.2** for the block diagram of a first order PLL), the transfer function of the filter is of the type

$$G(s) = \frac{1+\tau_2 s}{1+\tau_1 s}\ ; \qquad \tau_1 >> \tau_2 \qquad 5.31.12$$

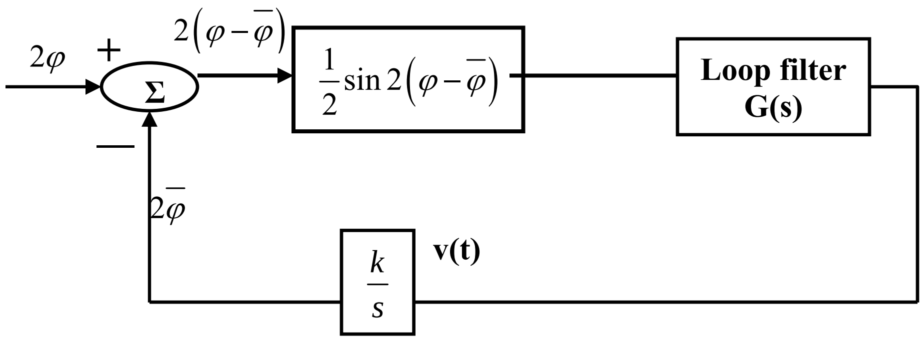

Fig. 5.31.2 *Equivalent block diagram of a first order PLL*

Now, the output of LPF, v(t) drives the VCO whose instant phase is given by,

$$4\pi f_c t + 2\bar{\varphi}(t) = 4\pi f_c t + k\int_{-\alpha}^{t} v(\tau)d\tau \qquad 5.31.13$$

$$\therefore 2\bar{\varphi} = k\int_{-\alpha}^{t} v(\tau)d\tau \qquad 5.31.14$$

The closed loop transfer function of this first order PLL is:

$$H(s) = \frac{kG(s)/s}{1+kG(s)/s} \qquad 5.31.15$$

Now, for small $(\varphi-\bar{\varphi})$, we may write $\frac{1}{2}\sin 2(\varphi-\bar{\varphi}) \cong (\varphi-\bar{\varphi})$

With this approximation, the PLL is linear and has a closed loop transfer function:

$$H(s) = \frac{k\ G(s)/s}{1+k\ G(s)/s} \qquad = \frac{1+\tau_2 s}{1+\left(\tau_2+\frac{1}{k}\right)s+\frac{\tau_1}{k}s^2} \qquad 5.31.16$$

The parameter 'τ_2' controls the position of 'zero' while 'k' and 'τ_1' together control the position of poles of the closed loop system.

It is customary to express the denominator of H(s) as:

$$D(s) = 1+\left(\tau_2+\frac{1}{k}\right)s+\frac{\tau_1}{k}s^2 = s^2+2\sigma\omega_n s+\omega_n{}^2 \qquad 5.31.17$$

Where, σ: Loop damping factor and $\left[=\frac{\tau_2+1/k}{2\omega n}\right]$;

ω_n : Natural frequency of the loop. $\left[\omega_n = \sqrt{k/\tau_1}\right]$

Now, the closed loop transfer function can be expressed as,

$$H(s) = \frac{\left(2\sigma\omega_n - \omega_n^2 / k\right)s + \omega_n^2}{s^2 + 2\sigma\omega_n s + \omega_n^2} \tag{5.31.18}$$

The one-sided noise equivalent BW of the loop is

$$B_{eq} = \frac{\tau_2^2\left(\frac{1}{\tau_2^2} + \frac{k}{\tau_1}\right)}{4\left(\tau_2 + \frac{1}{k}\right)} = \frac{1 + \left(\tau_2\omega_n\right)^2}{8\sigma\omega_n} \tag{5.31.19}$$

Effect of noise on phase estimation

Now, we assume that the phase of the carrier varies randomly but slowly with time and that the received signal is also corrupted by Gaussian noise.

Let, us express the narrowband signal and noise separately as:

$s(t) = A_c \cos[w_c t + \varphi(t)]$ and

$n(t) = x(t) \cos w_c t - y(t) \sin w_c t$, with power spectral density of $N_0 / 2$ (w/Hz)

$= n_c(t) \cos[w_c t + \varphi(t)] - n_s(t) \sin[w_c t + \varphi(t)]$

where

$n_c(t) = x(t) \cos \varphi(t) + y(t) \sin \varphi(t)$

$n_s(t) = - x(t) \cos \varphi(t) + y(t) \cos \varphi(t)$

Note that,

$n_c(t) + j\, n_s(t) = [\, x(t) + j\, y(t)]\, e^{-j\,\varphi(t)}$

$\therefore$ $n_c(t)$ and $n_s(t)$ have same stat. as $x(t)$ and $y(t)$.

So, the input to loop filter e(t) can be expressed as [see **Fig.5.31.3(a)**],

$e(t) = A_c \sin \Delta\varphi(t) + n_1(t)$

$$= A_c \sin \Delta\varphi(t) + n_c(t)\ \sin \Delta\varphi(t) - n_s(t) \cos \Delta\varphi(t) \tag{5.31.20}$$

Note that the model in **Fig.5.31.3(a)** is not linear. **Fig. 5.31.3(b)** shows the linearized but approximate model. In the model, $n_2(t)$ is a scaled version of $n_1(t)$.

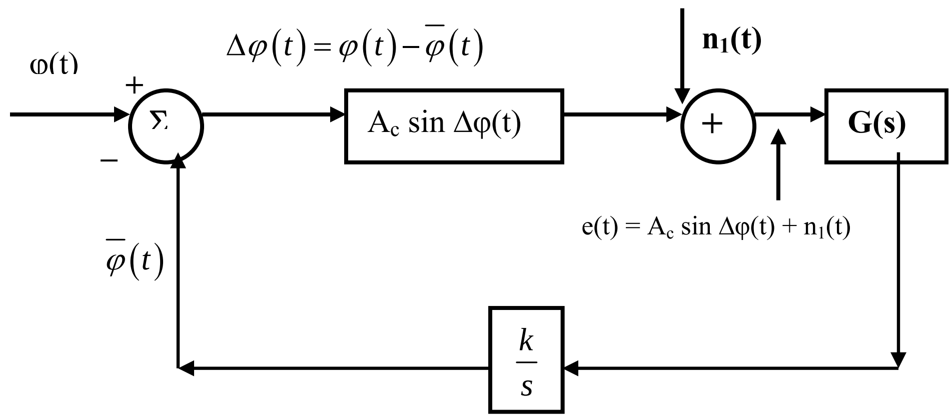

Fig.5.31.3(a) *The equivalent PLL model in presence of additive noise*

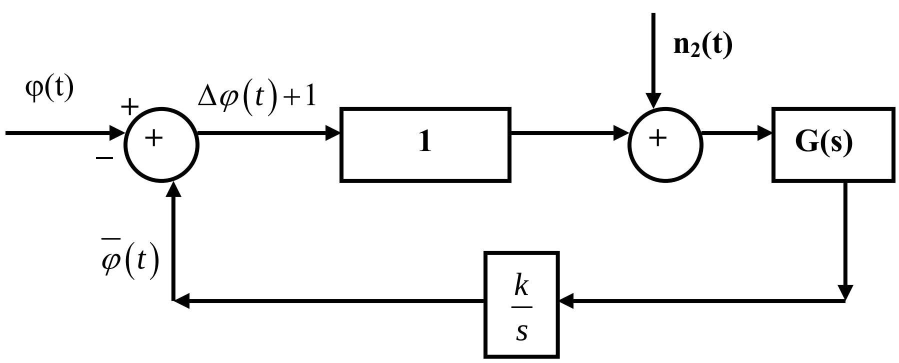

Fig. 5.31.3(b) *A linear equivalent PLL model in presence of additive noise*

To be specific,

$$n_2(t)=\frac{n_1(t)}{A_c}=\frac{n_c(t)}{A_c}\sin\Delta\varphi(t)-\frac{n_s(t)}{A_c}\cos\Delta\varphi(t) \qquad 5.31.21$$

Now the variance of phase error Δφ(t) is also the variance of the VCO output. After some analysis, this variance can be shown as:

$$\sigma_{\hat{\varphi}}^2=\frac{2N_0B_{eq}}{A_c^2}=\frac{1}{\gamma_L}, \text{ where } \gamma_L = loopSNR=\frac{A_c^2/2}{N_0B_{eq}} \qquad 5.31.22$$

An important message from the above summary on the principle of PLL is that, the variance of the VCO output, which should be small for the purpose of carrier synchronization, is inversely related to the loop SNR and hence much depends on how nicely the PLL is designed.

Costas Loop

2nd method for generating carrier phase ref. for DSB – SC type signal.

$$y_c(t) = \left[s(t)+n(t)\right]\cos\left(w_c t+\bar{\varphi}\right)$$
$$= \frac{1}{2}\left[A(t)+n_c(t)\right]\cos\Delta\varphi+\frac{1}{2}n_s(t)\sin\Delta\varphi+\text{double frequency term}$$
$$y_s(t) = \left[s(t)+n(t)\right]\sin\left(w_c t+\bar{\varphi}\right)$$
$$= \frac{1}{2}\left[A(t)+n_c(t)\right]\sin\Delta\varphi-\frac{1}{2}n_s(t)\cos\Delta\varphi+\text{double frequency term}$$

Refer **Fig 5.31.4** for the block diagram of a Costas Loop

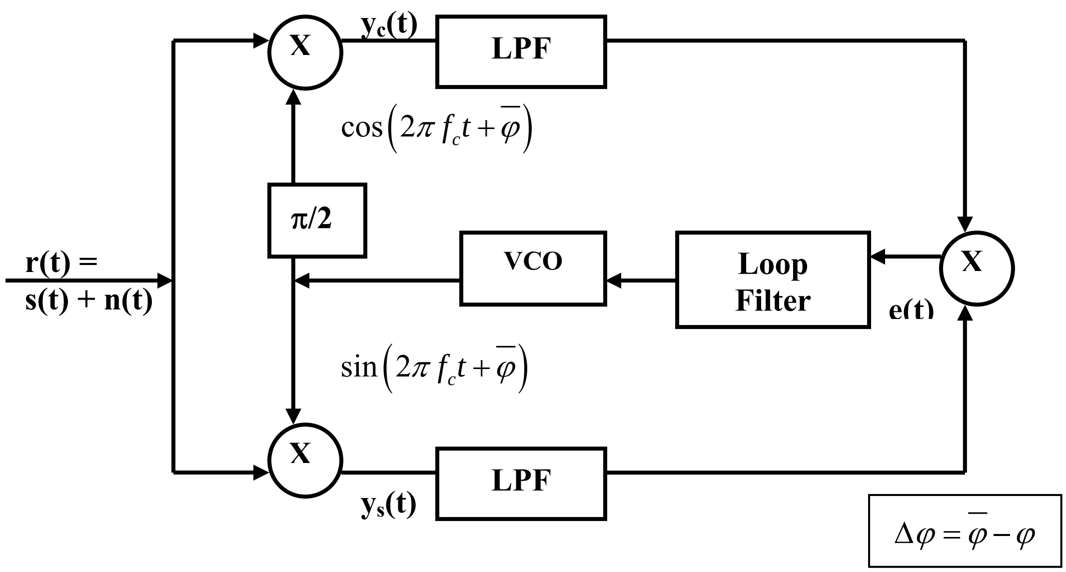

Fig. 5.31.4 *Block diagram of a Costas Loop*

Error signal

e(t) = multiplication of the low frequency components of $y_c(t)$ and $y_s(t)$

$$= \frac{1}{4}\left\{\left(\left[A(t)+n_c(t)\right]\cos\Delta\varphi+n_s(t)\sin\Delta\varphi\right)\times\left(\left[A(t)+n_c(t)\right]\sin\Delta\varphi-n_s(t)\cos\Delta\varphi\right)\right\}$$
$$= \left\{\frac{1}{8}\left(\left[A(t)+n_c(t)\right]^2\sin 2\Delta\varphi-n_s^2(t)\sin 2\Delta\varphi\right)+\frac{1}{4}n_s(t)\left[A(t)+n_c(t)\right]\cos 2\Delta\varphi\right\}$$
$$= \left\{\frac{1}{8}\left(\left[A(t)+n_c(t)\right]^2-n_s^2(t)\right)\sin 2\Delta\varphi+\frac{1}{4}n_s(t)\left[A(t)+n_c(t)\right]\cos 2\Delta\varphi\right\}$$

This composite error signal is filtered by the loop filter to generate the control voltage.

Note that the desired term is:

$$\frac{1}{8}A^2(t)\sin 2\left(\overline{\varphi}-\varphi\right)$$

The other terms are (signal × noise) and (noise × noise) type. For a good design of the loop filter, performance similar to a squaring loop may be obtained [without using a square circuit]. Also, the LPF in the I & Q path, are identical to the matched filters, matched to the signal pulse.

Decision Feedback Loop

[For DSB-SC type mod.]

$$y_c(t)=r(t)\cos\left(w_c t+\overline{\varphi}\right)=\left[s(t)+n(t)\right]\cos\left(w_c t+\overline{\varphi}\right)$$

$$=\frac{1}{2}\left[A(t)+n_c(t)\right]\cos\Delta\varphi+\frac{1}{2}n_s(t)\sin\Delta\varphi+\text{double frequency term}$$

This 'y_c (t)' is used to recover information A(t).

Now the error input e(t) to the loop filter, in absence of decision error is:

$$e(t)=\frac{1}{2}A(t)\left\{\left[A(t)+n_c(t)\right]\sin\Delta\varphi-n_s(t)\cos\Delta\varphi\right\}+\text{double frequency term}$$

$$=\frac{1}{2}A^2(t)\sin\Delta\varphi+\frac{1}{2}A(t)\left[n_c(t)\sin\Delta\varphi-n_s(t)\cos\Delta\varphi\right]+\text{double frequency term}$$

— Performs better than PLL or Costas Loop, if BER<10^{-2} [4 to 10 times better]

Refer **Fig 5.31.5** for the block diagram of a Decision Feedback Loop

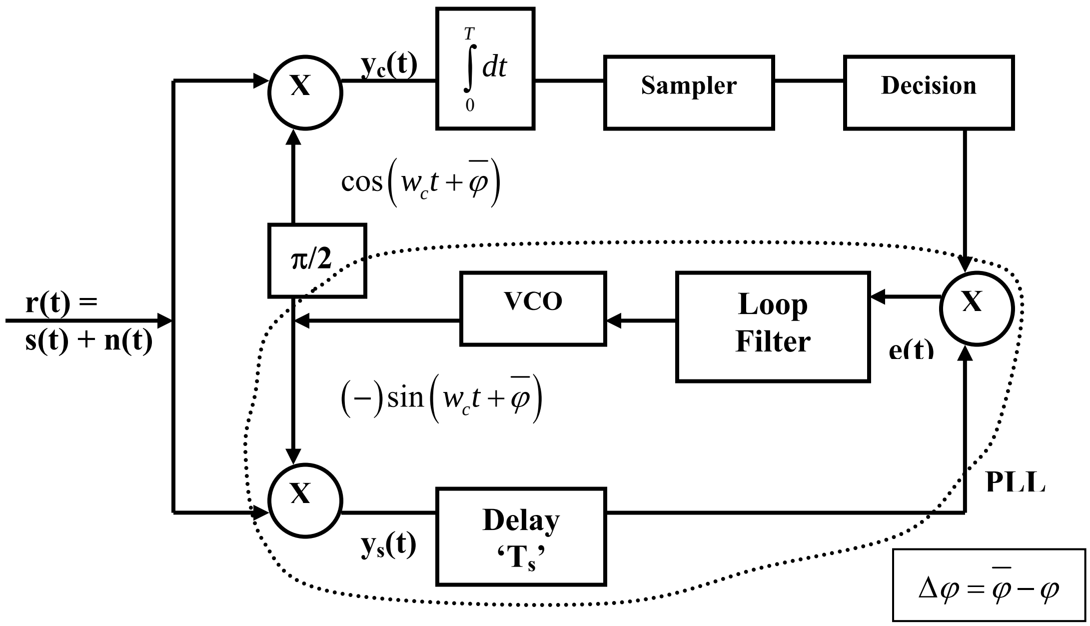

Fig. 5.31.5 *Block diagram of a decision feedback loop*

Decision Feedback PLL for M-ary PSK Modulation

This is a relatively simple scheme with good performance

$$s(t)=A_c\cos\left[w_c t+\varphi+\frac{2\pi}{M}(m-1)\right], \qquad m=1,2,\ldots,M.$$

$$=A_c\cos\left[w_c t+\varphi+\theta_m\right]$$

The received signal is of the form:

$$r(t) = s(t) + n(t)$$

The carrier recovery / tracking scheme removes the information-dependent phase component to obtain $\cos(w_c t + \varphi)$ as the received phase reference.

The received signal is demodulated (using coherent demodulator) to obtain a phase estimate

$$\overline{\theta_m}=\frac{2\pi}{M}(m-1)$$

In absence of decision error, $\overline{\theta_m}=\theta_m$, the transmitted signal phase.

Refer **Fig 5.31.6** for the block diagram of a Decision Feedback Loop for PSK Modulation.

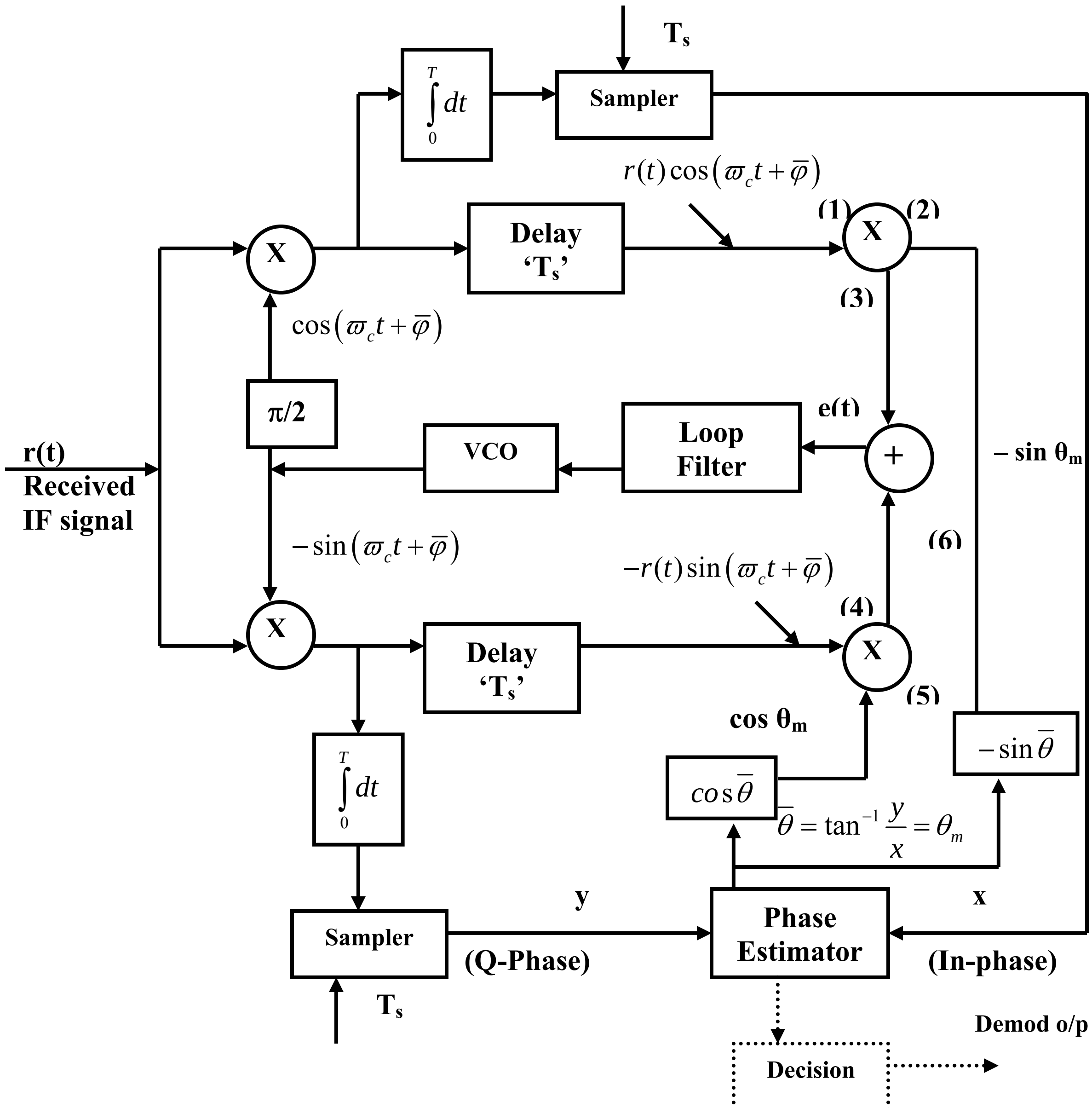

Fig. 5.31.6 *Block diagram of a decision feedback loop for PSK modulations*

Let us assume that there is no decision error and hence, $\overline{\theta_m} = \bar{\theta} = \theta_m$

Now the signal at (1) in the diagram may be written as $\gamma(t)\cos\left(w_c t + \bar{\varphi}\right)$.

Assuming no decision error, the signal at (2) is $-\sin\bar{\theta} = -\sin\overline{\theta_m} = -\sin\theta_m$

At (3), the signal is $-\gamma(t)\cos\left(w_c t + \bar{\varphi}\right).\sin\theta_m$

$= -\left[s(t) + n(t)\right]\cos\left(w_c t + \bar{\varphi}\right).\sin\theta_m$

$$= -\left\{A_c \cos\left[w_c t + \varphi + \theta_m\right] + n_c(t)\cos\left[w_c t + \varphi\right] - n_s(t)\sin\left[w_c t + \varphi\right]\right\}\cos\left[w_c t + \overline{\varphi}\right].\sin\theta_m$$

$$= -A_c \sin\theta_m \cos\left(w_c t + \varphi + \theta_m\right)\cos\left(w_c t + \overline{\varphi}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A)}$$

$$-n_c(t)\sin\theta_m \cos\left(w_c t + \varphi\right)\cos\left(w_c t + \overline{\varphi}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

$$+n_s(t)\sin\theta_m \sin\left(w_c t + \varphi\right)\cos\left(w_c t + \overline{\varphi}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(C)}$$

Now consider the terms at (A):

$$-A_c \sin\theta_m \cos\left(w_c t + \varphi + \theta_m\right)\cos\left(w_c t + \overline{\varphi}\right)$$

$$= -A_c \sin\theta_m \left(\frac{1}{2}\right)\left\{\cos\left(2w_c t + \varphi + \overline{\varphi} + \theta_m\right) + \cos\left(\varphi - \overline{\varphi} + \theta_m\right)\right\}$$

Of interest are the low frequency terms as we are using a loop filter later:
Low frequency terms in (A):

$$-\frac{1}{2}A_c \sin\theta_m \cos\left(\varphi - \overline{\varphi} + \theta_m\right)$$

$$= -\frac{1}{2}A_c \sin\theta_m \left[\cos\left(\varphi - \overline{\varphi}\right)\cos\theta_m - \sin\left(\varphi - \overline{\varphi}\right)\sin\theta_m\right]$$

Now consider terms (B):

$$-n_c(t)\sin\theta_m \cos\left(w_c t + \varphi\right)\cos\left(w_c t + \overline{\varphi}\right)$$

$$= -\frac{1}{2}n_c(t)\sin\theta_m \left[\cos\left(2w_c t + \varphi + \overline{\varphi}\right) + \cos\left(\varphi - \overline{\varphi}\right)\right]$$

Considering Low Frequency component:

$$-\frac{1}{2}n_c(t)\sin\theta_m \cos\left(\varphi - \overline{\varphi}\right)$$

Now consider terms (C):

$$n_s(t)\sin\theta_m \sin\left(w_c t + \varphi\right)\cos\left(w_c t + \overline{\varphi}\right)$$

$$= \frac{1}{2}n_s(t)\sin\theta_m \left[\sin\left(2w_c t + \varphi + \overline{\varphi}\right) + \sin\left(\varphi - \overline{\varphi}\right)\right]$$

Considering low frequency term:

$$\frac{1}{2}n_s(t)\sin\theta \sin\left(\varphi - \overline{\varphi}\right)$$

So, the overall low – frequency component at (3):
$(A)_{LF} + (B)_{LF} + (C)_{LF}$

$$= -\frac{1}{2}A_c \sin\theta_m \left[\cos\theta_m \cos\left(\varphi - \overline{\varphi}\right) - \sin\theta_m \sin\left(\varphi - \overline{\varphi}\right)\right]$$

$$-\frac{1}{2}n_c(t)\sin\theta_m\cos\left(\varphi-\overline{\varphi}\right)+\frac{1}{2}n_s(t)\sin\theta_m\sin\left(\varphi-\overline{\varphi}\right)$$

$$=-\frac{1}{2}\left[A_c\cos\theta_m+n_c(t)\right]\sin\theta_m\cos\left(\varphi-\overline{\varphi}\right)$$

$$+\frac{1}{2}\left[A_c\sin\theta_m+n_s(t)\right]\sin\theta_m\sin\left(\varphi-\overline{\varphi}\right)$$

By similar straightforward expansion, the overall low-frequency term at (6) may be shown as:

$$\frac{1}{2}\left[A_c\cos\theta_m+n_c(t)\right]cos\theta_m\sin\left(\varphi-\overline{\varphi}\right)+\frac{1}{2}\left[A_c\sin\theta_m+n_s(t)\right]\cos\theta_m\cos\left(\varphi-\overline{\varphi}\right)$$

These two signals [at (3) and (6)] are added to obtain the error signal e(t):

$$e(t)=\frac{1}{2}A_c\begin{Bmatrix}-\cos\theta_m\sin\theta_m\cos\left(\varphi-\overline{\varphi}\right)+\sin^2\theta_m\sin\left(\varphi-\overline{\varphi}\right)\\+cos^2\theta_m\sin\left(\varphi-\overline{\varphi}\right)+\sin\theta_m\cos\theta_m\cos\left(\varphi-\overline{\varphi}\right)\end{Bmatrix}$$

$$+\frac{1}{2}n_c(t)\sin\left(\varphi-\overline{\varphi}-\theta_m\right)+\frac{1}{2}n_s(t)cos\left(\varphi-\overline{\varphi}-\theta_m\right)$$

$$=\frac{1}{2}A_c\sin\left(\varphi-\overline{\varphi}\right)+\frac{1}{2}n_c(t)\sin\left(\varphi-\overline{\varphi}-\theta_m\right)+\frac{1}{2}n_s(t)cos\left(\varphi-\overline{\varphi}-\theta_m\right)$$