Module 5

Carrier Modulation

Lesson 30

Orthogonal Frequency Division Multiplexing (OFDM)

After reading this lesson, you will learn about:

- ***Basic concept of OFDM;***
- ***Peak to Average Power Ratio (PAPR);***
- ***Effect of the transmission channel on OFDM;***
- ***Effect of ISI on OFDM;***
- ***OFDM applications;***

Orthogonal Frequency Division Multiplexing (OFDM)

OFDM is a relatively new spectrally efficient digital modulation scheme which employs multiple carriers that are mutually orthogonal to one another over a given time interval. Each carrier, consisting of a pair of sine wave and a cosine wave, is referred as a 'sub-carrier'.

Let us consider an OFDM scheme with N sub-carriers. The available transmission bandwidth is equally divided amongst the N sub-carriers. If 'W' Hz is the single-sided transmission bandwidth available, the bandwidth allocated to each sub-carrier is $\frac{W}{N}$ Hz. The difference between two adjacent sub carriers is called the sub carrier spacing, which is also $\frac{W}{N}$ Hz in our example. Each sub carrier, upon data modulation may often be categorized as a narrowband modulated signal but the overall OFDM signal is a wideband signal for moderate or large value of 'N'. Usually, the modulation operation is carried out at the baseband level and the baseband-modulated signal is translated in the frequency domain by frequency up-conversion to the required radio frequency band.

A data symbol consists of several bits and the symbol is used to modulate all the carriers simultaneously. The symbol rate and the sub carrier spacing are so chosen that all the carriers are orthogonal over one OFDM symbol duration. **Fig. 5.30.1** shows three sub carriers, which are orthogonal over one symbol duration. If the duration of an OFDM symbol is 'T' second, we see from the figure that the sub carrier frequencies are $f_0 = \frac{1}{T}$ Hz, $f_1 = \frac{2}{T}$ Hz and $f_2 = \frac{3}{T}$ Hz. It is interesting to note that, many other combinations of three frequencies are possible which are orthogonal over T second, such as $(\frac{1}{T}, \frac{2}{T}, \frac{4}{T})$, $(\frac{1}{T}, \frac{3}{T}, \frac{4}{T})$, $(\frac{1}{T}, \frac{5}{T}, \frac{9}{T})$ and so on. However, all combinations are not optimally bandwidth efficient.

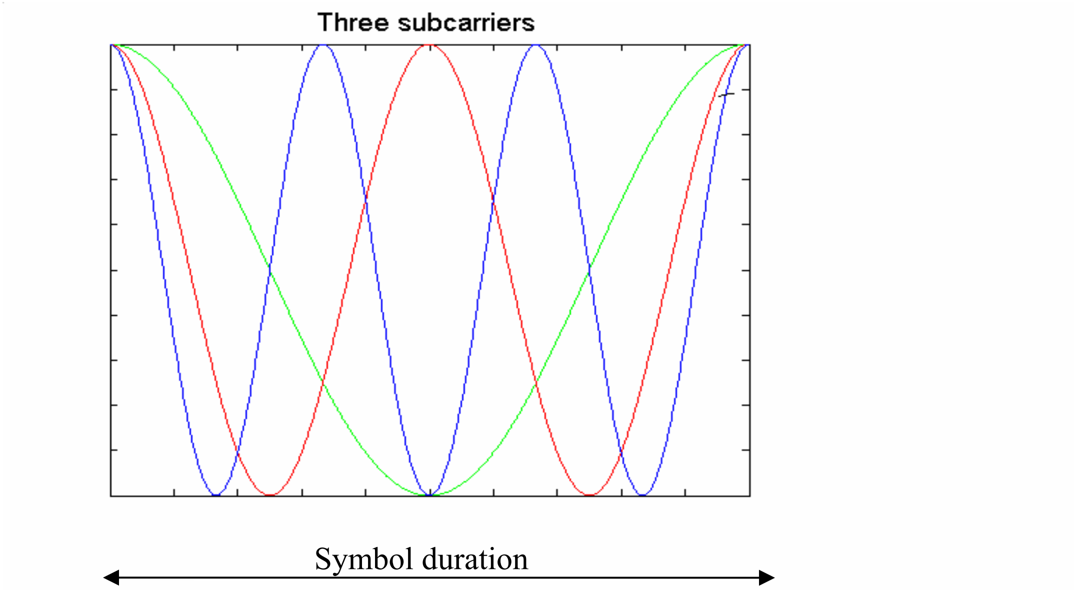


Fig. 5.30.1 *Three sub carriers, which are orthogonal over one symbol duration*

A particularly compact arrangement may be like this: We accept a group of 2N bits every T second and form N dibits (one dibit is made of two bits) from these bits. All these N dibits are then fed in parallel to the N sub carriers with the first dibit modulating the first sub carrier, second dibit modulating the second sub carrier and so forth. As noted earlier, each sub carrier consists of a pair of orthogonal sinusoids at the same frequency. So, each sub carrier receives one dibit every T second and hence its own symbol rate of arrival is 1 symbol per T second. We set the sub carrier frequencies as

$$f_0 = \frac{1}{T}\ \text{Hz},\ f_1 = \frac{2}{T}\,\text{Hz},\ \ldots\ldots,\ f_{N-1} = \frac{N}{T}\ \text{Hz}.$$

Fig.5.30.2 shows a conceptual diagram highlighting the orthogonal multiple carrier modulation scheme. The a_i-s in the diagram indicates the modulating signal in the I-path and the b_i-s are the modulating signals in the Q-path. The ‘encoder’ in a practical system performs several operations but is of no special significance at the moment. Let us refer a ‘cosine’ carrier as an in-phase carrier and a ‘sine’ carrier as a quadrature-phase carrier. All the in-phase carrier modulated signals are added algebraically and similarly are the quadrature-phase modulated signals. The overall I-phase and Q-phase signals together form the complex baseband OFDM signal. At this point, one may interpret the scheme of **Fig.5.30.2** as consisting of a bank of N parallel QPSK modulators driven by N orthogonal sub carriers.

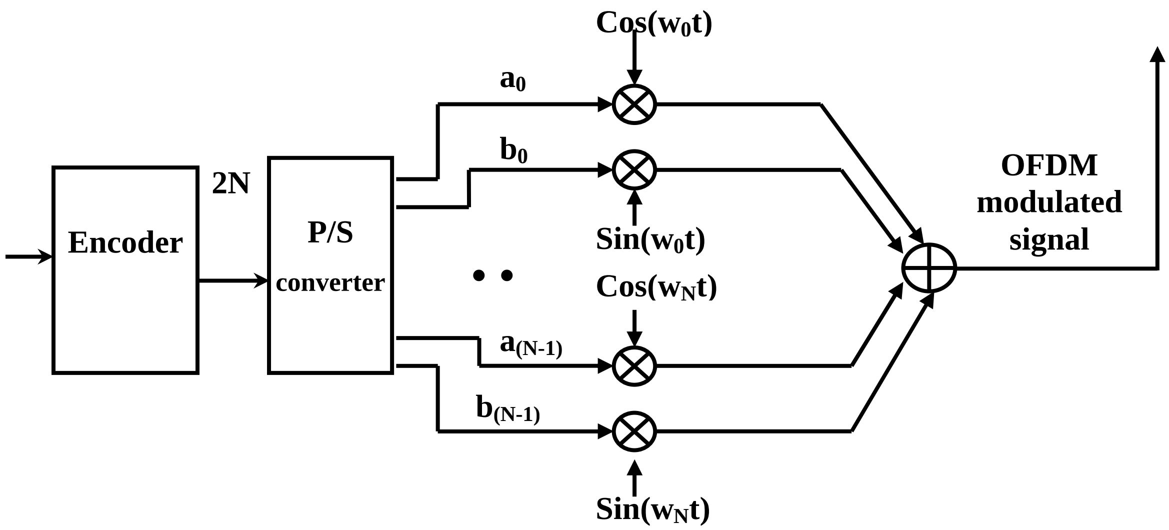


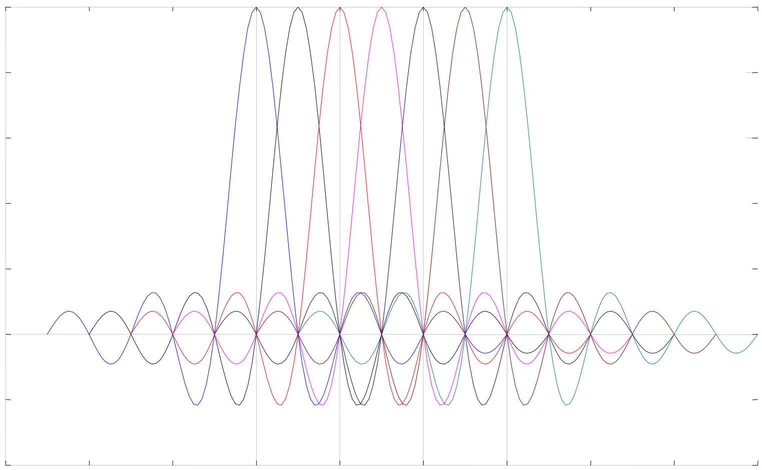
Fig.5.30.2 *A conceptual diagram highlighting orthogonal multiple carrier modulation ($\omega_o = 2\pi f_o = 2\pi/T$)*

Mathematically, if $\{\tilde{X}(k)\}$ represents a sequence of N complex modulating symbols, the complex baseband modulated OFDM symbol is represented as

$$\tilde{x}(t) = \frac{1}{N}.\sum_{k=0}^{N-1} \tilde{X}(k).e^{j2\pi k f_0 t}, \quad 0 \le t \le T \text{ and } k = 0, 1, \ldots, (N-1) \qquad 5.30.1$$

$\tilde{X}(k)$ is the k-th symbol modulating the k-th sub carrier and f_o is the inter-sub carrier spacing.

An interesting and practically useful feature of the OFDM modulation scheme is that pulse shaping is not necessary for the modulating signals because a bunch of orthogonal carriers, when modulated by random pulse sequences, have a very orderly spectrum as sketched in **Fig. 5.30.3**. As indicated, the orthogonal sub-carriers occupy the spectral zero crossing positions of other sub-carriers. This ensures that a sub carrier modulated signal with seemingly infinite spectrum does not interfere with the signals modulated by other sub carriers.



→ Frequency

Figure 5.30.3: *OFDM and the orthogonality principle. The sub-carriers occupy the spectral zero crossing positions of other sub-carriers. Sub-carriers are orthogonal*

Further, it also implies that the OFDM modulated signal of **Eq. 5.30.1** can be generated by simple Inverse Discrete Fourier Transform (IDFT) which can be implemented efficiently as an N-point Inverse Fast Fourier Transform (IFFT). If the modulated signal in time domain (**Eq. 5.30.1**) is uniformly sampled with an interval $\frac{T}{N}$, the n-th sample of the OFDM signal is:

$$\tilde{x}\left(n\frac{T}{N}\right) \equiv \tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k)e^{j2\pi k f_0 nT/N}, \quad 0 \le n \le (N-1) \qquad 5.30.2$$

Similarly, at the receiving end, an N-point FFT operation does the equivalent job of demodulation of OFDM signal. This makes digital design and implementation of OFDM modulator and demodulator very convenient. However, there are several practical issues which demand proper attention.

Fig.5.30.4 shows a magnitude plot of complex time domain samples $\tilde{x}\left(n\frac{T}{N}\right)$ for 10 OFDM symbols. Sixty four sub-carriers have been used to obtain the OFDM signal. Note that the baseband modulated signal looks random with great variation in amplitude. This amplitude fluctuation is expressed by a parameter called '*Peak to Average Power Ratio (PAPR)*'. Higher is this ratio, more fluctuating is the envelope of the modulated signal.

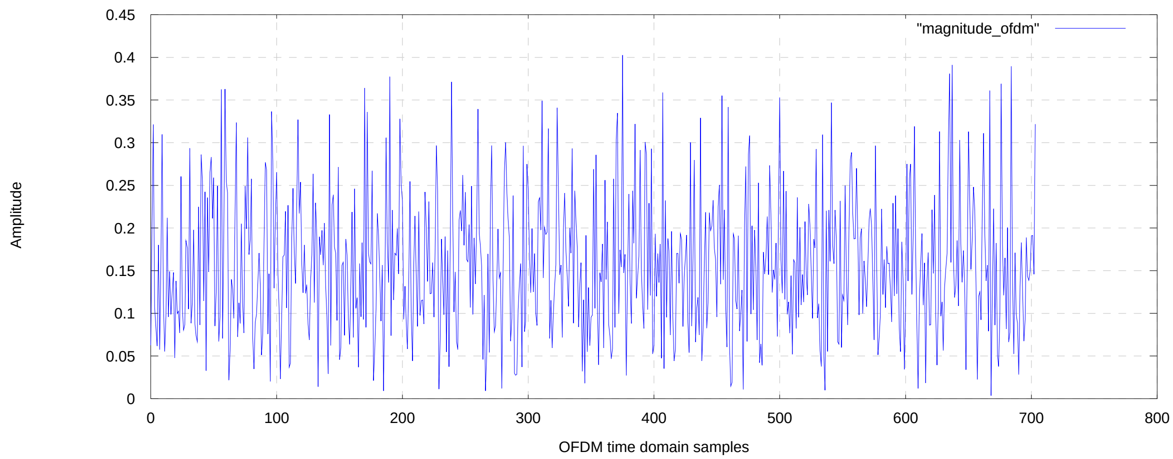


Fig.5.30.4 *Magnitude of complex time domain samples* $\tilde{x}\left(n\frac{T}{N}\right)$ *for ten OFDM symbols*

For a constant envelope modulation scheme, the PAPR is 1.0. An OFDM signal with high PAPR needs a power amplifier with a large linear range of operation in the transmitter. Otherwise, the OFDM signal gets distorted and produces harmonic distortions while being amplified by the power amplifier and the orthogonality among the sub carriers is affected in the process. This results in considerable Inter Symbol Interference (ISI) at the receiver and the quality of demodulated data degrades. Fortunately, several schemes such as clipping, filtering and special forms of coding are available to contain the PAPR of an OFDM signal before it is actually transmitted.

Another important issue is the effect of the transmission channel. Due to its high spectral efficiency (**Fig. 5.30.5** shows a representative amplitude spectrum for OFDM signal), OFDM is favoured for data transfer at high rates (typically beyond 1Mbps) in wireless networks. The transmission channels in practical environments often manifest multiple signal paths and signal fading at a receiver. These channel-induced phenomena can potentially degrade the quality of the received signal by a) affecting the sub carrier orthogonality and b) introducing ambiguities in carrier phase and symbol timing. As a result, the quality of received data degrades. For example, **Fig.5.30.6** shows the error performance of a simulated OFDM scheme using 64 sub carriers in presence of thermal noise only.

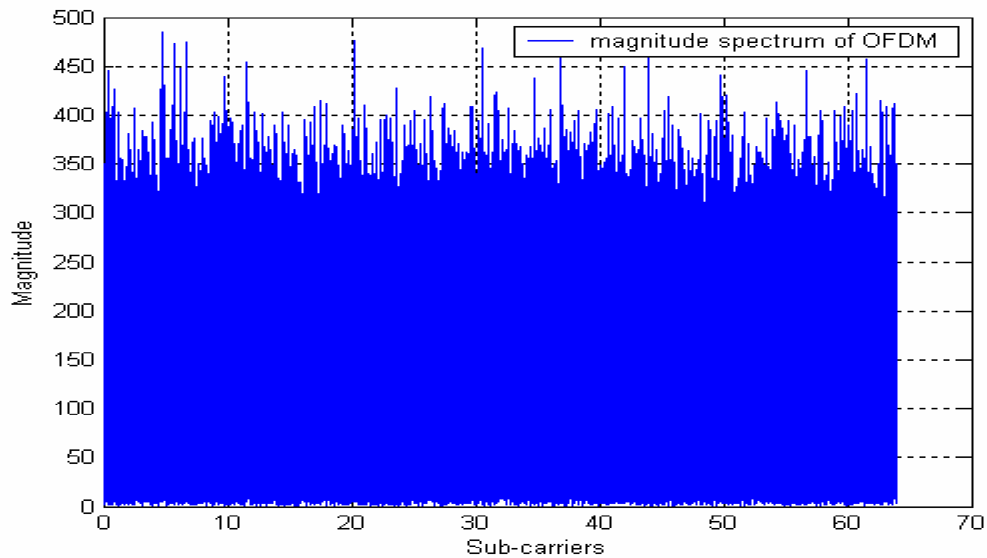


Fig.5.30.5 *Magnitude spectrum of OFDM signal with 64 subcarriers. The normalized subcarriers are spaced at 1 Hz apart. The subcarrier values are shown along the horizontal axis. The random fluctuation at the top of the spectrum is largely due to the finite number of OFDM symbols that was considered in simulation*

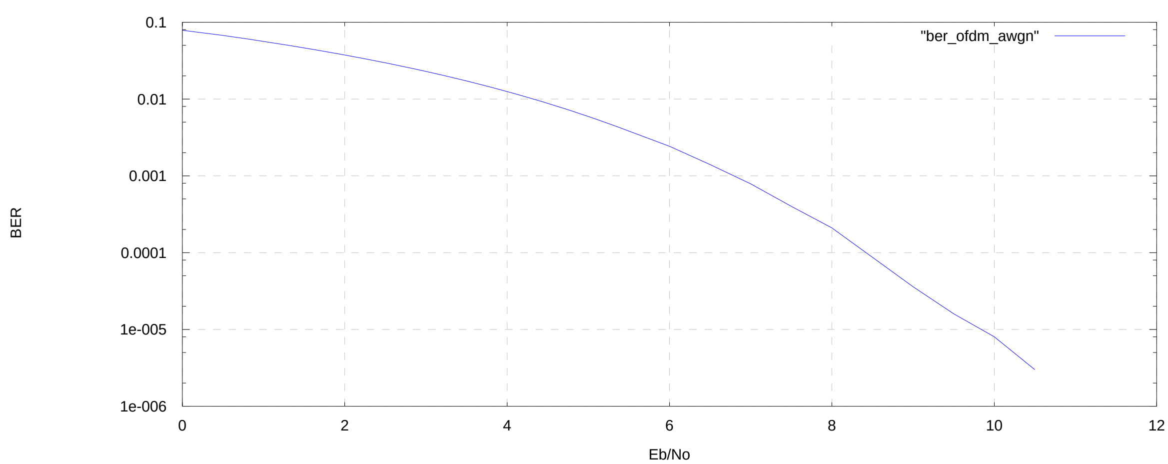


Fig.5.30.6 *BER vs Eb/No (in dB) for a 64 carrier OFDM system in AWGN channel*

One may note that the OFDM modulation scheme performs very close to the BER performance of QPSK modulation scheme. However, lack of orthogonality and lack of synchronization can make the situation very poor. **Fig.5.30.6** shows that less than 10 dB in E_b/N_o is sufficient to achieve an average error rate of 10^{-5} in presence of additive Gaussian noise and with perfect synchronization at the receiver; but in an indoor wireless local area network environment, it may need more than 20 dB of E_b/N_o to ensure same error rate.

In a practical system, the problems of phase and timing synchronization are addressed by inserting appropriate training sequence in a frame of symbols. The receiver knows the training sequence and hence it can estimate the quality of transmission channel from a sizeable number of samples of the distorted training sequence, sent as preamble to the information-carrying symbols. The frequency or phase synchronization process is vital for restoring the orthogonality among sub carriers before data demodulation takes place in the receiver. Often this is achieved by following a two-step scheme: a coarse synchronization followed by a fine synchronization process.

Inter symbol interference (ISI) may manifest in an OFDM demodulator either a) due to channel multipath or b) due to lack of orthogonality or c) combination of both. As a result, delayed portion of the previous symbol superimposes on the initial portion of the present symbol at the receiver. Thus initial portion of the present symbol gets affected more due to multi-path fading. To reduce this effect, usually the duration of an OFDM symbol (expressed by N samples) is stretched a little in time and a few samples of the latter part of an OFDM symbol are appended at the beginning of the symbol. These samples are called 'cyclic prefix'. If N_g is the number of time domain samples of OFDM symbol used as cyclic prefix, the total number of samples in a cyclic-prefixed OFDM symbol becomes $N_s = N + N_g$.

Applications of OFDM

Some applications of OFDM in modern wireless digital transmission systems are mentioned below:

a) Asynchronous Digital Subscriber Line (ADSL), High speed DSL, Very high speed DSL use OFDM for transmission of high rate data transfer
b) Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB).
c) IEEE 802.11a, IEEE 802.11g and HYPERLAN2 wireless Local Area Network (WLAN) standards include OFDM for supporting higher bit rates
d) IEEE 802.16 Wireless Metropolitan Area Network (MAN) standard also includes OFDM.

Problems

Q5.30.1) Sketch two waveforms other than sinusoid which are orthogonal to each other over some time interval 'T'.

Q5.30.2) Mention two disadvantages of signaling using OFDM.

Q5.30.3) Explain how and where the concept of FFT is useful in an OFDM transceiver.