Module
5

Carrier Modulation

Lesson 23

Amplitude Shift Keying (ASK) and Frequency Shift Keying (FSK) Modulations

After reading this lesson, you will learn about

- ➢ *Amplitude Shift Keying (ASK) Modulation;*
- ➢ *On-off keying;*
- ➢ *Frequency Shift Keying (FSK) Modulation;*
- ➢ *Power spectra of BFSK;*

Amplitude Shift Keying (ASK) Modulation:

Amplitude shift keying (ASK) is a simple and elementary form of digital modulation in which the amplitude of a carrier sinusoid is modified in a discrete manner depending on the value of a modulating symbol. Let a group of 'm' bits make one symbol. Hence one can design $M = 2^m$ different baseband signals, $d_m(t)$, $0 \le m \le M$ and $0 \le t \le T$. When one of these symbols modulates the carrier, say, $c(t) = \cos\omega_c t$, the modulated waveform is:

$$s_m(t) = d_m(t).\cos\omega_c t \qquad\qquad 5.23.1$$

This is a narrowband modulation scheme and we assume that a large number of carrier cycles are sent within a symbol interval, i.e. $\dfrac{T}{\left(2\pi\big/\varpi_c\right)}$ is a large integer. It is obvious that the information is embedded only in the peak amplitude of the modulated signal. So, this is a kind of digital amplitude modulation technique. From another angle, one can describe this scheme of modulation as a one-dimensional modulation scheme where one basis function $\varphi_1(t) = \sqrt{\dfrac{2}{T}}.\cos\varpi_c t$, defined over $0 \le t \le T$ and having unit energy is used and all the baseband signals are linearly dependent.

Ex. #5.23.1 Let m = 2 and $d_0 = 0$, $d_1 = 1$, $d_2 = 2$ and $d_3 = 3$. It is simple to generate such distinct and fixed levels in practice. Further, let us arbitrarily assume the following information to signal mapping: $d_0 \equiv (1,1)$, $d_1 \equiv (1,0)$, $d_2 \equiv (0,1)$ and $d_3 \equiv (0,0)$. So, we have four symbols and the modulated waveforms are:

$$s_0(t) = d_0(t).\sqrt{\frac{2}{T}}.\cos\varpi_c t = 0, \quad s_1(t) = d_1(t).\sqrt{\frac{2}{T}}.\cos\varpi_c t = \sqrt{\frac{2}{T}}.\cos\varpi_c t , \quad s_2(t) =$$

$$d_2(t).\sqrt{\frac{2}{T}}.\cos\varpi_c t = 2.\sqrt{\frac{2}{T}}.\cos\varpi_c t \quad \text{and } s_3(t) = d_3(t).\sqrt{\frac{2}{T}}.\cos\varpi_c t = 3.\sqrt{\frac{2}{T}}.\cos\varpi_c t$$

The signal constellation consists of four points on a straight line. The distances of the points from the origin (signifying zero energy) are 0, 1, 2 and 3 respectively. Note that in this example, no-transmission indicates that 'd_0', i.e. the symbol (1,1) is 'transmitted'. This is not surprising and it also should not give an impression that we are able to transmit 'information' without spending any energy. In fact, it is a bad practice to assign zero energy to a symbol for any good quality carrier modulation scheme because,

in this case, it becomes difficult to recover the basis carrier accurately for coherent demodulation at the receiving end and that ultimately leads to poor SER and BER. Another interesting feature to note is that the modulated symbols have different energy levels, viz. 0, 1, 4 and 9 units. This feature does not make the highest energy symbol d_3 more immune to thermal noise.

On the contrary, the large range of energy level, namely, from '0' to '9' implies that the power amplifier in the transmitter has to have a large linear range of operation – sometime a costly proposition. If the power amplifier goes into its non-linear range while amplifying $s_3(t)$, harmonics of the carrier sinusoid will be generated which will rob some power from $s_3(t)$ away and may interfere with other wireless transmissions in frequency bands adjacent to $\pm 2\omega_c$, $\pm 3\omega_c$, etc. The point to note is that, the Euclidean distance of $s_3(t)$ from the nearest point $s_2(t)$ in the receiver signal space decreases because of amplifier nonlinearity and it means that the receiver will confuse more between $s_3(t)$ and $s_2(t)$ while trying to detect the symbols in presence of noise.

Assuming that all the symbols are equally likely to appear at the input of the modulator, we see that the average energy per symbol ($\overline{E_s}$) is 14/4 = 3.5 unit. This is an important parameter for transmission of digital signals because it is ultimately proportional to the average transmission power. A system designer would always try to ensure low transmission power to save cost and to enhance reliability of the system. So, we see the simple example of ASK modulation of four symbols could be cited in such a way that the signal points were better placed in the constellation diagram such that $\overline{E_s}$ is minimum. ♦

Now, ASK being a form of amplitude modulation, we can say that the bandwidth of the modulated signal will be the same as the bandwidth of the baseband signal. The baseband signal is a long and random sequence of pulses with discrete values. Hence, ASK modulation is not bandwidth efficient. It is implemented in practice when simplicity and low cost are principal requirements.

On-off keying

On-Off Keying (OOK) is a particularly simple form of ASK that represents binary data as the presence or absence of a sinusoid carrier. For example, the presence of a carrier over a bit duration T_b may represent a binary '1' while its absence over a bit duration T_b may represent a binary '0'. This form of digital transmission (OOK) has been commonly used to transmit Morse Codes over a designated radio frequency for telegraph services. As mentioned earlier, OOK is not a spectrally efficient form of digital carrier modulation scheme as the amplitude of the carrier changes abruptly when the data bit changes. So, this mode of transmission is suitable for low or moderate data rate. When the information rate is high, other bandwidth efficient phase modulation schemes are preferable.

Frequency Shift Keying Modulation

Frequency Shift Keying (FSK) modulation is a popular form of digital modulation used in low-cost applications for transmitting data at moderate or low rate over wired as well as wireless channels. In general, an M-ary FSK modulation scheme is a power efficient modulation scheme and several forms of M-ary FSK modulation are becoming popular for spread spectrum communications and other wireless applications. In this lesson, our discussion will be limited to binary frequency shift keying (BFSK).

Two carrier frequencies are used for binary frequency shift keying modulation. One frequency is called the 'mark' frequency (f_2) and the other as the space frequency (f_1). By convention, the 'mark' frequency indicates the higher of the two carriers used. If T_b indicates the duration of one information bit, the two time-limited signals can be expressed as :

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos 2\pi f_i t, & 0 \le t \le T_b, i = 1,2 \\ 0, & \text{elsewhere.} \end{cases} \qquad 5.23.2$$

The binary scheme uses two carriers and for special relationship between the two frequencies one can also define two orthonormal basis functions as shown below.

$$\varphi_l(t) = \sqrt{\dfrac{2}{T_b}} \cos 2\pi f_i t \quad ; \quad 0 \le t \le T_b \text{ and } j = 1,2 \qquad 5.23.3$$

If $T_1 = 1/f_1$ and $T_2 = 1/f_2$ denote the time periods of the carriers and if we choose $m.T_1 = n.T_2 = T_b$, where 'm' and 'n' are positive integers, the two carriers are orthogonal over the bit duration T_b. If $R_b = 1/T_b$ denotes the data rate in bits/second, the orthogonal condition implies, $f_1 = m.R_b$ and $f_2 = n.R_b$. Let us assume that $n > m$, i.e. f_2 is the 'mark' frequency. Let the separation between the two carriers be, $\Delta f = f_2 - f_1 = (n-m).R_b$.
Now, the scalar coefficients corresponding to **Eq. (5.23.1)** and **(5.23.3)** are:

$$s_{ij} = \int_0^{T_b} s_i(t)\varphi_j(t)dt = \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \ne j \end{cases} \quad ; \quad i = 1,2 \text{ and } j = 1,2 \qquad 5.23.4$$

i.e. $\left. \begin{aligned} s_{11} = s_{22} = \sqrt{E_b} \\ s_{12} = s_{21} = 0 \end{aligned} \right\} \begin{aligned} i = 1,2 \\ j = 1,2 \end{aligned}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 5.23.5

So, the two signal victors can be expressed as:

$$\overline{s_1} = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad \text{and} \quad \overline{s_2} = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \qquad 5.23.6$$

Please note that one can generate an FSK signal without following the above concept of orthogonal carriers and that is often easy in practice. **Fig. 5.23.1** shows a

possible FSK modulated waveform. Notice the waveform carefully and verify if the two carriers are orthogonal. An obvious feature of an FSK modulated signal, analogous to analog FM signal is that envelop of the modulated signal is constant. All modulation schemes which exhibit constant envelope, are preferable for high power digital transmission because, operation of the power amplifier in a non-linear region may not produce considerable harmonics.
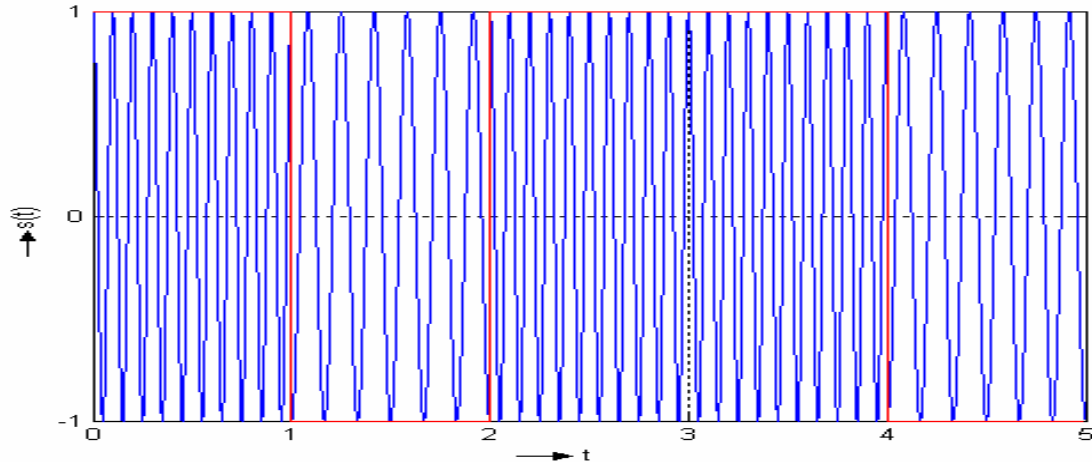

Fig. 5.23.1 *Binary FSK waveform*

Fig. 5.23.2 shows the constellation diagram for binary FSK. **Fig. 5.23.3** shows a conceptual diagram for generating binary FSK modulated signal. Note that the input random binary sequence is represented by '1' and '0' where '0' represents no voltage at the input of the multipliers. A '0' input to the inverter results in a '1' at its output. That is, the inverter, along with the two multipliers and the summing unit, may be thought to behave as a 'switch' which selects output of one of the two oscillators.
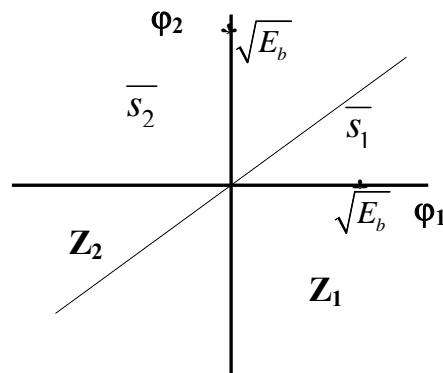

Fig. 5.23.2 *Signal constellation for binary FSK. The diagram also shows the two decision zones, Z_1 and Z_2.*
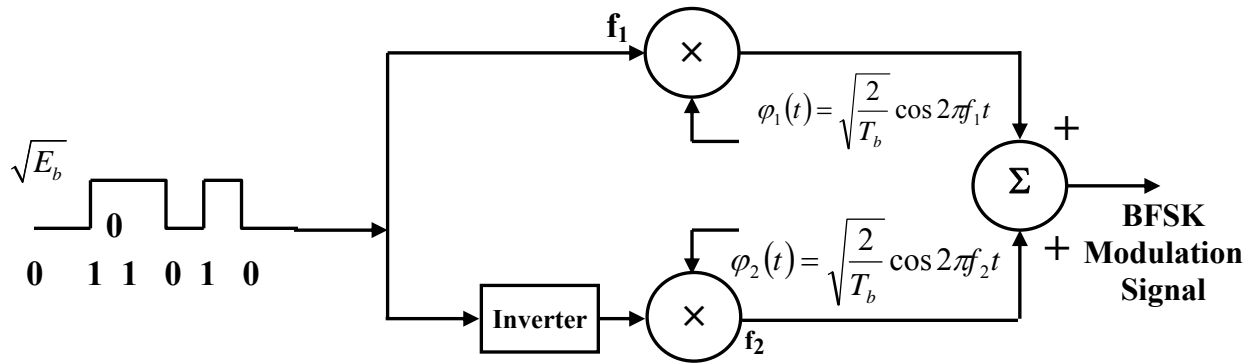
Fig. 5.23.3 *A schematic diagram for BFSK modulation*

In practice, however, this scheme will not work reliably because, the two oscillators being independent, it will be difficult to maintain the orthogonal relationship between the two carrier frequencies. Any relative phase shift among the two oscillators, which may even occur due to thermal drift, will result in deviation from the orthogonality condition. Another disadvantage of the possible relative phase shift is random discontinuity in the phase of the modulated signal during transition of information bits. A better proposition for physical implementation is to use a voltage controlled oscillator (VCO) instead of two independent oscillators and drive the VCO with an appropriate baseband modulating signal, derived from the serial bit stream. The VCO free-running frequency (f_{free}) should be chosen as:

$$f_{free} = \frac{f_1 + f_2}{2} = \frac{m+n}{2}.R_b \qquad 5.23.6$$

Fig. 5.23.4 (a) shows the form of a coherent FSK demodulator, based on the concepts of correlation receiver as outlined in Module #4. The portion on the LHS of the dotted line shows the correlation detector while the RHS shows that the vector receiver reduces to a subtraction unit. Output of the subtraction unit is compared against a threshold of zero to decide about the corresponding transmitted bit.
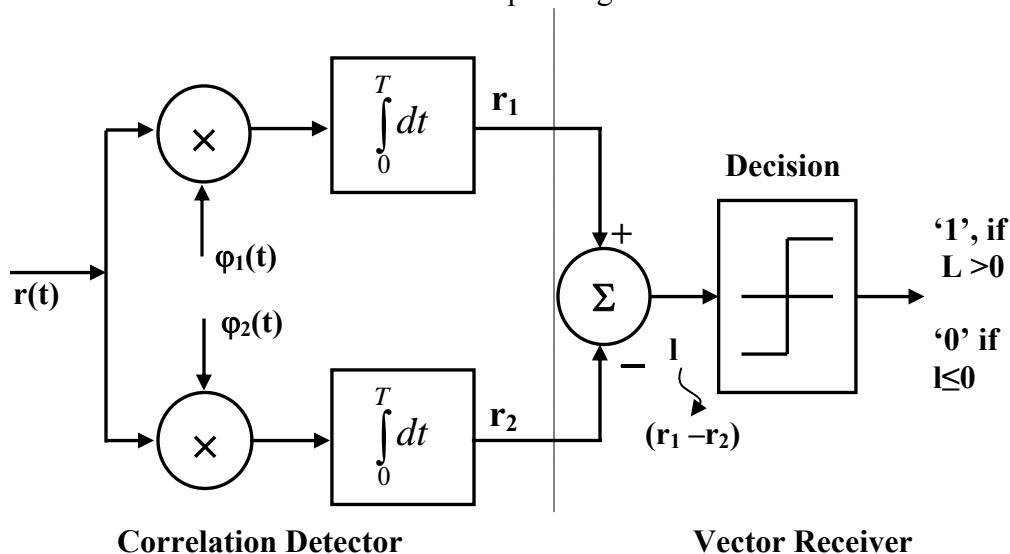

Correlation Detector   **Vector Receiver**

Fig. 5.23.4(a) *Schematic diagram of a coherent BFSK demodulator*

Fig. 5.23.4(b) gives a scheme for non-coherent demodulation of BFSK signal using matched filters. It is often easier to follow this approach than the coherent demodulation scheme without sacrificing error performance.
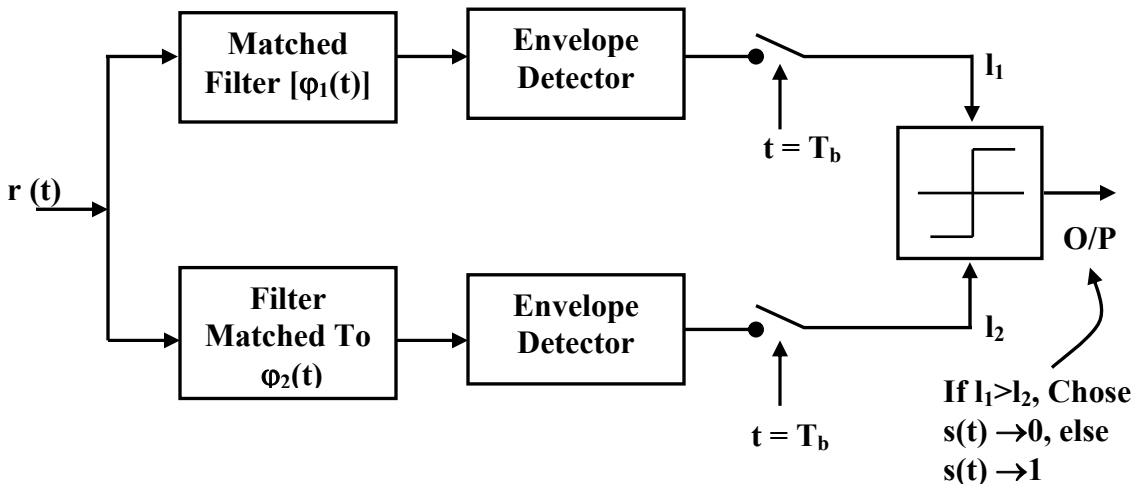

Fig. 5.23.4(b) *General scheme for non-coherent demodulation of BFSK signal using matched filters*

When the issues of performance and bandwidth are not critical and the operating frequencies are low or moderate, a low complexity realization of the demodulator is also possible. Two bandpass filters, one centered at f_1 and the other centered at f_2 may replace the matched filters in **Fig. 5.23.4(b).**

Power Spectra of BFSK

Power spectrum of an FSK modulated signal depends on the choice of f_1 and f_2, i.e. on 'm' and 'n'. When (n-m) is large, we may visualize BFSK as the sum of two OOK signals (see **Fig.5.23.3**) with carriers f_1 and f_2. However, such choice of (n-m) does not a result in bandwidth efficiency.

In the following, we consider n = (m+1), i.e. $f_1 = m.R_b = m/T_b$, $f_2 = f_1 + \dfrac{1}{T_b} =$

$n.R_b = (m+1).R_b = (m+1)/T_b$ and $f_c = \dfrac{f_2 + f_1}{2} = f_1 + \dfrac{1}{2T_b} = \varpi_c \Big/ 2\pi = f_{free}.$

5.23.7

Considering the spectrum of a random binary sequence, as we have presented in Module #4, it is easy to see that, ISI can be avoided in detecting the signals for the above choice of f_1 and f_2.

Now, the BFSK modulated signal can be expressed as:

$$s(t) = \sqrt{\frac{2E_b}{T_b}}.\cos\left[w_c t \pm \frac{\pi t}{T_b}\right] = \underbrace{\sqrt{\frac{2E_b}{T_b}}.\cos\left(\frac{\pi t}{T_b}\right)}_{u_I(t)}.\cos w_c t \mp \sqrt{\frac{2E_b}{T_b}}.\sin\left(\frac{\pi t}{T_b}\right)\sin w_c t$$

5.23.8

The '±' sign in the above expression contains information about the information sequence, d(t). It is interesting to note that $u_I(t)$, the real part of the lowpass complex equivalent of the modulated signal s(t) is independent of the information sequence d(t) This portion of s(t) gives rise to a set of two delta functions, each of strength $\frac{E_b}{2T_b}$ and

located at $f = +\frac{1}{2T_b} = \frac{f_b}{2}$ and $f = -\frac{1}{2T_b} = -\frac{f_b}{2}$, where $f_b = R_b$.

'$u_Q(t)$', the imaginary part of the lowpass complex equivalent of the modulated signal s(t) can be expressed in terms of a shaping function $g_Q(t)$ as,

$$u_Q(t) = \mp\sqrt{\frac{2E_b}{T_b}}.\sin\left(\frac{\pi t}{T_b}\right) = \mp g_Q(t),$$

5.23.9

where $g_Q(t) = \sqrt{\frac{2E_b}{T_b}}.\sin\left(\frac{\pi t}{T_b}\right)$, $0 \le t \le T_b$

5.23.10

Now, the energy spectral density (esd) of the shaping function $g_Q(t)$ is,

$$\Psi g_Q(f) = \frac{8\,E_b T_b.\cos^2\left(\pi T_b f\right)}{\pi^2\left(4T_b^2 f^2 - 1\right)^2}$$

5.23.11

From the above expression, we define the psd of $u_Q(t)$ as:

$$= \frac{esd\ of\ g(t)}{T_b} = \frac{8\,E_b \cos^2\left(\pi T_b f\right)}{\pi^2\left(4T_b^2 f^2 - 1\right)^2}$$

5.23.12

As $u_I(t)$ and $u_Q(t)$ are statistically independent of each other, we can now construct the baseband spectrum $U_B(f)$ of the BFSK modulated signal s(t) as:

$$U_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b \cos^2\left(\pi T_b f\right)}{\pi^2\left(4T_b^2 f^2 - 1\right)^2}$$

Fig. 5.23.5 shows a sketch (approximate) of the power spectrum of binary FSK signal. In a subsequent lesson (**Lesson #29**), we will discuss about another form of FSK, known as Minimum Shift Keying (MSK), which is operates with minimum possible separation between two frequencies f_1 and f_2.
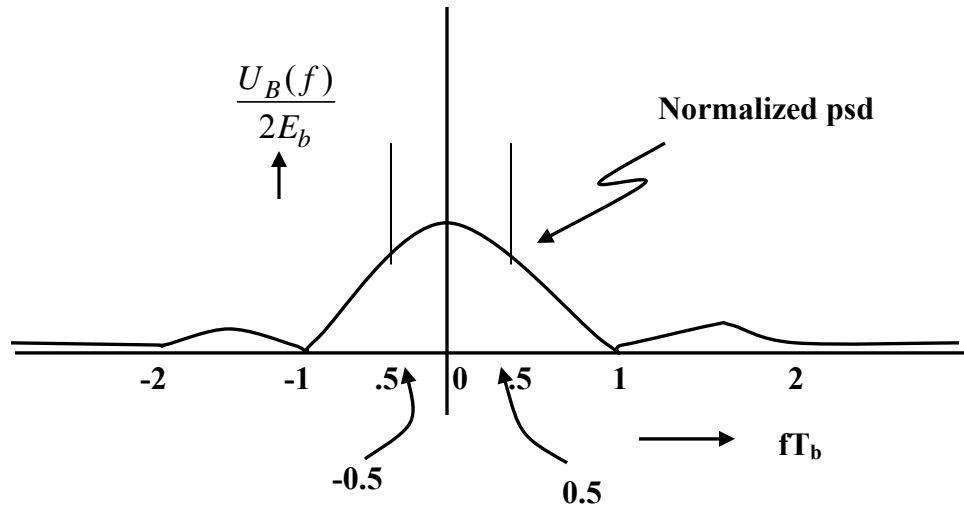

Fig. 5.23.5 *Sketch of the power spectrum of binary FSK signal vs. 'fT$_b$' when $(f_2-f_1) = 1/T_b$.*

Problems

Q5.23.1) Draw the signal constellation of an ASK modulation scheme.

Q5.23.2) How is binary FSK modulation scheme different from binary PSK?

Q5.23.3) Comment if it is a good practice to generate a binary FSK signal by switching an oscillation?

Q5.23.4) Explain in Fig. 5.23.5 why two spikes appear in the spectrum of binary FSK signal for fT$_b$ = ±0.5?