

# Module 3

## Quantization and Coding

# Lesson 13

## Differential Pulse Code Modulation (DPCM)

After reading this lesson, you will learn about

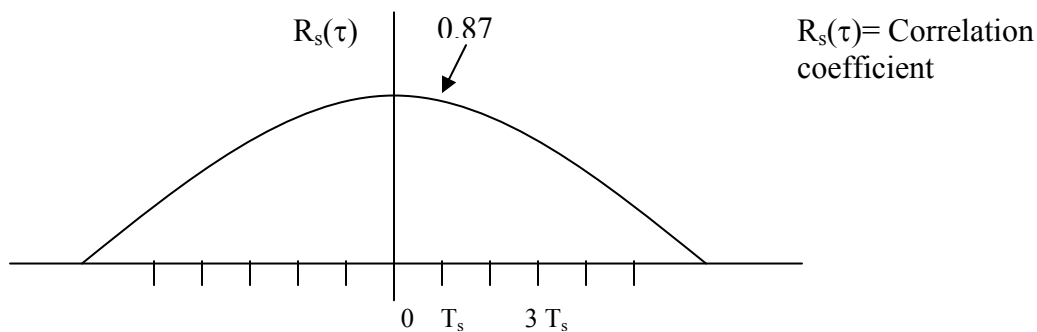
- *Principles of DPCM;*
- *DPCM modulation and de-modulation;*
- *Calculation of SQNR;*
- *One tap predictor;*

The standard sampling rate for pulse code modulation (PCM) of telephone grade speech signal is  $f_s = 8$  Kilo samples per sec with a sampling interval of  $125 \mu \text{ sec}$ . Samples of this band limited speech signal are usually correlated as amplitude of speech signal does not change much within  $125 \mu \text{ sec}$ . A typical auto correlation function  $R(\tau)$  for speech samples at the rate 8 Kilo samples per sec is shown in **Fig 3.13.1**.  $R(\tau = 125 \mu \text{ sec})$  is usually between 0.79 and 0.87. This aspect of speech signal is exploited in differential pulse code modulation (DPCM) technique. A schematic diagram for the basic DPCM modulator is shown in **Fig 3.13.2**. Note that a predictor block, a summing unit and a subtraction unit have been strategically added to the chain of blocks of PCM coder instead of feeding the sampler output  $x(kT_s)$  directly to a linear quantizer. An error sample  $e_p(kT_s)$  is fed.

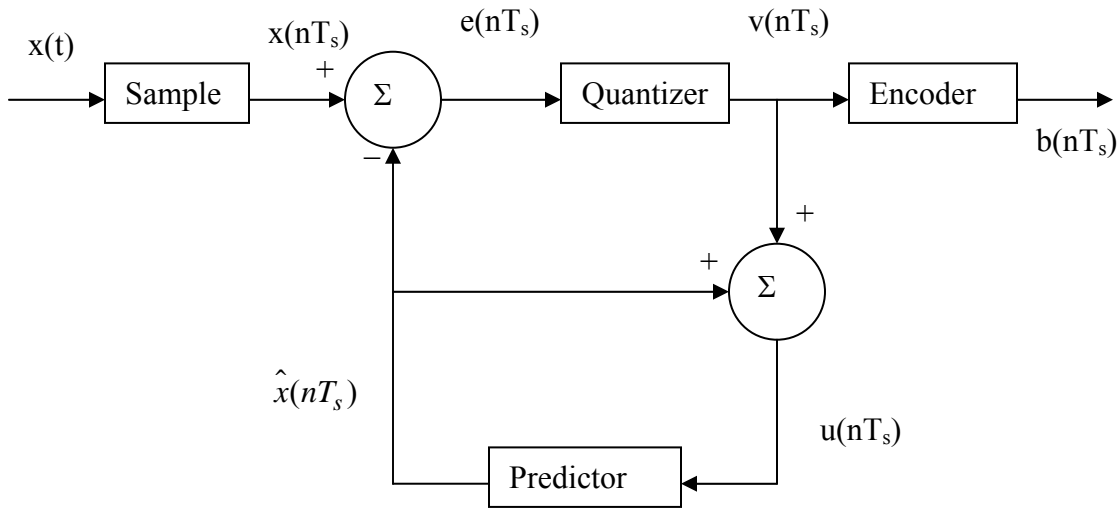
The error sample is given by the following expression:

$$e_p(kT_s) = x(kT_s) - \hat{x}(kT_s) \quad 3.13.1$$

$\hat{x}(kT_s)$  is a predicted value for  $x(kT_s)$  and is supposed to be close to  $x(kT_s)$  such that  $e_p(kT_s)$  is very small in magnitude.  $e_p(kT_s)$  is called as the ‘prediction error for the n-th sample’.



**Fig. 3.13.1** Typical normalized auto-correlation coefficient for speech signal



**Fig. 3.13.2** Schematic diagram of a DPCM modulator

If we assume a final enclosed bit rate of 64kbps as of a PCM coder, we envisage smaller step size for the linear quantizer compared to the step size of an equivalent PCM quantizer. As a result, it should be possible to achieve higher SQNR for DPCM codec delivering bits at the same rate as that of a PCM codec.

There is another possibility of decreasing the coded bit rate compared to a PCM system if an SQNR as achievable by a PCM codec with linear equalizer is sufficient. If the predictor output  $\hat{x}(kT_s)$  can be ensured sufficiently close to  $x(kT_s)$  then we can simply encode the quantizer output sample  $v(kT_s)$  in less than 8 bits. For example, if we choose to encode each of  $v(kT_s)$  by 6 bits, we achieve a serial bit rate of 48 kbps, which is considerably less than 64 Kbps. This is an important feature of DPCM, especially when the coded speech signal will be transmitted through wireless propagation channels.

We will now develop a simple analytical structure for a DPCM encoding scheme to bring out the role that may be played by the prediction unit.

As noted earlier,

$$e_p(kT_s) = \text{k-th input to quantizer} = x(kT_s) - \hat{x}(kT_s)$$

$$\hat{x}(kT_s) = \text{prediction of the k-th input sample } x(kT_s).$$

$$e_q(kT_s) = \text{quantizer output for k-th prediction error.}$$

$$= c[e_p(kT_s)], \text{ where } c[] \text{ indicates the transfer characteristic of the quantizer}$$

If  $q(kT_s)$  indicates the quantization error for the k-th sample, it is easy to see that

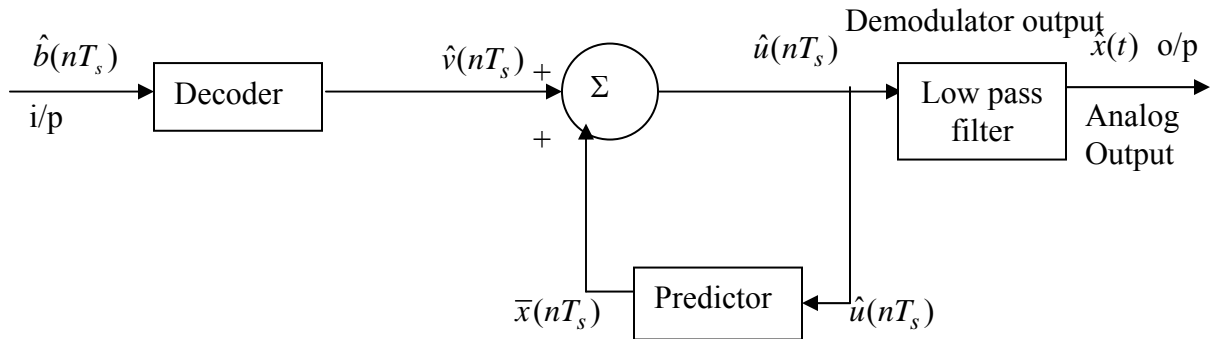
$$e_q(kT_s) = e_p(kT_s) + q(kT_s) \quad 3.13.2$$

Further the input  $u(kT_s)$  to the predictor is,

$$\begin{aligned} u(kT_s) &= \hat{x}(kT_s) + e_q(kT_s) = \hat{x}(kT_s) + e_p(kT_s) + q(kT_s) \\ &= x(kT_s) + q(kT_s) \end{aligned} \quad 3.13.3$$

This equation shows that  $u(kT_s)$  is indeed a quantized version of  $x(kT_s)$ . For a good prediction  $e_p(kT_s)$  will usually be small compared to  $x(kT_s)$  and  $q(kT_s)$  in turn will be very small compared to. Hence, the predictor unit should be so designed that variance of  $q(kT_s) < \text{variance of } e_p(kT_s) \ll \text{variance of } x(kT_s)$ .

A block schematic diagram of a DPCM demodulator is shown in **Fig 3.13.3**. The scheme is straightforward and it tries to estimate  $u(kT_s)$  using a predictor unit identical to the one used in the modulator. We have already observed that  $u(kT_s)$  is very close to  $x(kT_s)$  within a small quantization error of  $q(kT_s)$ . The analog speech signal is obtained by passing the samples  $\hat{u}(kT_s)$  through an appropriate low pass filter. This low pass filter should have a 3 dB cut off frequency at 3.4kHz.



**Fig. 3.13.3** Schematic diagram of a DPCM demodulator; note that the demodulator is very similar to a portion of the modulator

## Calculation of SQNR for DPCM

The expression for signal to quantization noise power ratio for DPCM coding is:

$$\begin{aligned} SQNR &= \frac{\text{Variance of } x(kT_s)}{\text{Variance of } q(kT_s)} \\ &= \left[ \frac{\text{Variance of } x(kT_s)}{\text{Variance of } e_p(kT_s)} \right] \cdot \left[ \frac{\text{Variance of } e_p(kT_s)}{\text{Variance of } q(kT_s)} \right] \end{aligned}$$

As in PCM coding, we are assuming instantaneous sampling and ideal low pass filtering. The first term in the above expression is the ‘predictor gain ( $G_p$ )’. This gain visibly increases for better prediction, i.e., smaller variance of  $e_p(kT_s)$ . The second term,  $SNR_p$  is a property of the quantizer. Usually, a linear or uniform quantizer is used for simplicity in a DPCM codec. Good and efficient design of the predictor plays a crucial role in enhancing quality of signal or effectively reducing the necessary bit rate for transmission. In the following we shall briefly take up the example of a single-tap predictor.

### Single-Tap Prediction

A single-tap predictor predicts the next input example  $x(kT_s)$  from the immediate previous input sample  $x([k-1]T_s)$ .

$$\begin{aligned} \text{Let, } \hat{x}(kT_s) &= \hat{x}(k|k-1) = \text{the } k\text{-th predicted sample, given the } (k-1)\text{th input sample} \\ &= a.u(k-1|k-1) \end{aligned}$$

Here, ‘a’ is known as the prediction co-efficient and  $u(k-1|k-1)$  is the (k-1)-th input to the predictor given the (k-1)-th input speech sample, i.e.,  $x(k-1)$ .

Now the k-th prediction error sample at the input of the quantizer may be expressed as

$$\begin{aligned} e_p(kT_s) &\equiv e_p(k) \\ &= x(k) - \hat{x}(k|k-1) \\ &= x(k) - a.u(k-1|k-1) \end{aligned} \tag{3.13.4}$$

The mean square error or variance of this prediction error samples is the statistical expectation of  $e_p^2(k)$ .

Now,

$$\begin{aligned} E[e_p^2(k)] &= E[\{x(k) - a.u(k-1|k-1)\}^2] \\ &= E[x(k).x(k) - 2.a.x(k).u(k-1|k-1) + a^2.u(k-1|k-1).u(k-1|k-1)] \\ &= E[x(k).x(k) - 2aE[x(k).u(k-1|k-1)] + a^2.E[u(k-1|k-1).u(k-1|k-1)]] \end{aligned} \tag{3.13.5}$$

Let us note that  $E[x(k).x(k)]=R(0)$ . Where ( $R(\tau)$ ) indicates the autocorrelation coefficient. For the second term, let us assume that  $u(k-1|k-1)$  is an unbiased estimate of  $x(k-1)$  and that  $u(k-1|k-1)$  is a satisfactorily close estimate of  $x(k-1)$ , so that we can use the following approximation:

$$\begin{aligned} E[x(k).u(k-1|k-1)] &\approx E[x(k).x(k-1)] \\ &= R(\tau = 1.T_s) \equiv R(1), \text{ say} \end{aligned}$$

The third term in the expanded form of  $E[e_p^2(k)]$  can easily be identified as:

$$a^2.E[u(k-1|k-1).u(k-1|k-1)] = a^2.R(\tau = 0) = a^2.R(0)$$

$$\therefore E[e_p^2(k)] = R(0) - 2.a.R(1) + a^2.R(0)$$

$$= R(0)[1 - 2a.\frac{R(1)}{R(0)} + a^2] \quad 3.13.6$$

The above expression shows that the mean square error or variance of the prediction error can be minimized if  $a = R(1)/R(0)$ .

## Problems

- Q3.13.1) Is there any justification for DPCM, if the samples of a signal are known to be uncorrelated with each other?
- Q3.13.2) Determine the value of prediction co-efficient for one tap prediction unit if  $R(0) = 1.0$  and  $R(1) = 0.9$ .