Module
3

Quantization and Coding

Lesson
11

Pulse Code Modulation

After reading this lesson, you will learn about

> ➢ *Principle of Pulse Code Modulation;*
> ➢ *Signal to Quantization Noise Ratio for uniform quantizaton;*

A schematic diagram for Pulse Code Modulation is shown in **Fig 3.11.1**. The analog voice input is assumed to have zero mean and suitable variance such that the signal samples at the input of A/D converter lie satisfactorily within the permitted single range. As discussed earlier, the signal is band limited to 3.4 KHz by the low pass filter.
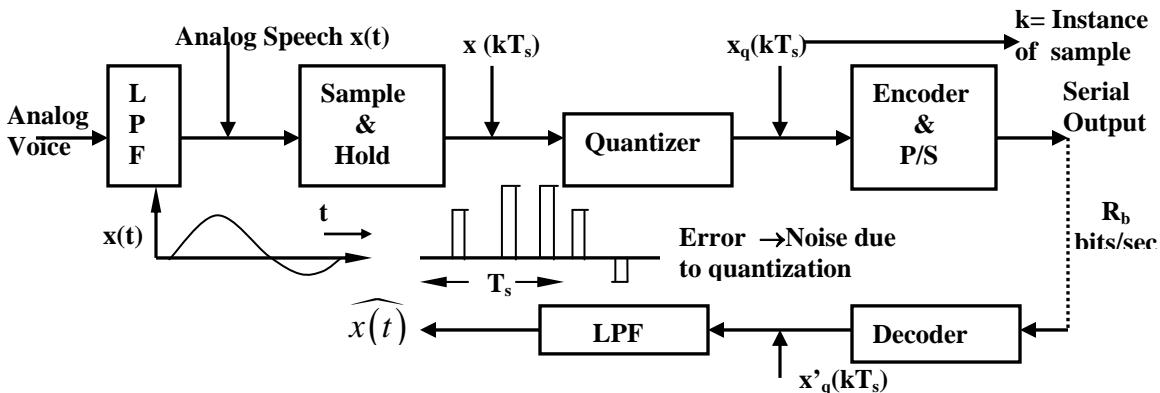

Fig. 3.11.1   *Schematic diagram of a PCM coder – decoder*

Let x (t) denote the filtered telephone-grade speech signal to be coded. The process of analog to digital conversion primarily involves three operations: (a) Sampling of x (t), (b) Quantization (i.e. approximation) of the discrete time samples, $x(kT_s)$ and (c) Suitable encoding of the quantized time samples $x_q(kT_s)$. T_s indicates the sampling interval where $R_s = 1/T_s$ is the sampling rate (samples /sec). A standard sampling rate for speech signal, band limited to 3.4 kHz, is 8 Kilo-samples per second ($T_s = 125\mu$ sec), thus, obeying Nyquist's sampling theorem. We assume instantaneous sampling for our discussion. The encoder in **Fig 3.11.1** generates a group of bits representing one quantized sample. A parallel–to–serial (P/S) converter is optionally used if a serial bit stream is desired at the output of the PCM coder. The PCM coded bit stream may be taken for further digital signal processing and modulation for the purpose of transmission.

The PCM decoder at the receiver expects a serial or parallel bit-stream at its input so that it can decode the respective groups of bits (as per the encoding operation) to generate quantized sample sequence [$x'_q(kT_s)$]. Following Nyquist's sampling theorem for band limited signals, the low pass filter produces a close replica $\hat{x}(t)$ of the original speech signal x (t).

If we consider the process of sampling to be ideal (i.e. instantaneous sampling) and if we assume that the same bit-stream as generated by PCM encoder is available at PCM decoder, we should still expect $\hat{x}(t)$ to be somewhat different from x (t). This is

solely because of the process of quantization. As indicated, quantization is an approximation process and thus, causes some distortion in the reconstructed analog signal. We say that quantization contributes to "noise". The issue of quantization noise, its characterization and techniques for restricting it within an acceptable level are of importance in the design of high quality signal coding and transmission system. We focus a bit more on a performance metric called SQNR (Signal to Quantization Noise power Ratio) for a PCM codec. For simplicity, we consider uniform quantization process. The input-output characteristic for a uniform quantizer is shown in **Fig 3.11.2(a).** The input signal range (± V) of the quantizer has been divided in eight equal intervals. The width of each interval, δ, is known as the step size. While the amplitude of a time sample x (kT_s) may be any real number between +V and −V, the quantizer presents only one of the allowed eight values (±δ, ±3δ/2, …) depending on the proximity of x (kT_s) to these levels.
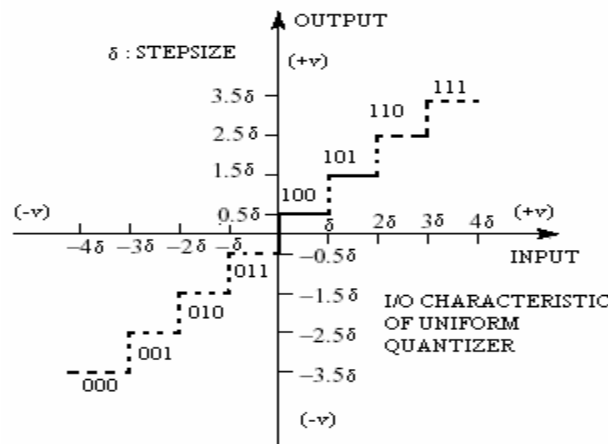

Fig 3.11.2(a) *Linear or uniform quantizer*

The quantizer of **Fig 3.11.2(a)** is known as "mid-riser" type. For such a mid-riser quantizer, a slightly positive and a slightly negative values of the input signal will have different levels at output. This may be a problem when the speech signal is not present but small noise is present at the input of the quantizer. To avoid such a random fluctuation at the output of the quantizer, the "mid-tread" type uniform quantizer [**Fig 3.11.2(b)**] may be used.
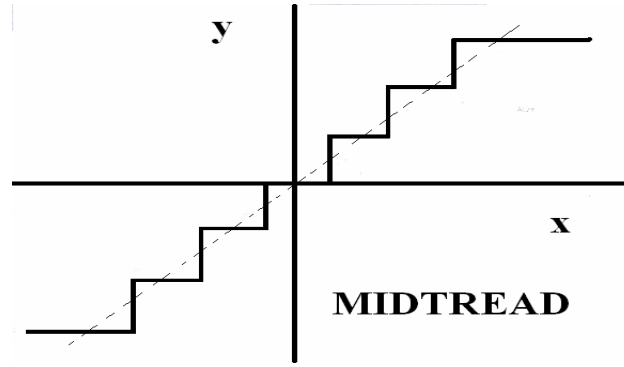
Fig 3.11.2(b) *Mid-tread type uniform quantizer characteristics*

SQNR for uniform quantizer

In **Fig.3.11.1** $x(kT_s)$ represents a discrete time $(t = kT_s)$ continuous amplitude sample of $x(t)$ and $x_q(kT_s)$ represents the corresponding quantized discrete amplitude value. Let e_k represents the error in quantization of the kth sample i.e.

$$e_k = x_q(kT_s) - x(kT_s) \qquad\qquad 3.11.1$$

Let,

M = Number of permissible levels at the quantizer output.

N = Number of bits used to represent each sample.

±V = Permissible range of the input signal x (t).

Hence,

$M = 2^N$ and,

$M.\delta \cong 2.V$ [Considering large M and a mid-riser type quantizer]

Let us consider a small amplitude interval dx such that the probability density function (pdf) of $x(t)$ within this interval is $p(x)$. So, $p(x)dx$ is the probability that $x(t)$ lies in the range $(x - \dfrac{dx}{2})$ and $(x + \dfrac{dx}{2})$. Now, an expression for the mean square quantization error $\overline{e^2}$ can be written as:

$$\overline{e^2} = \int_{x_1-\delta/2}^{x_1+\delta/2} p(x)(x-x_1)^2 dx + \int_{x_2-\delta/2}^{x_2+\delta/2} p(x)(x-x_2)^2 dx + ... \qquad 3.11.2$$

For large M and small δ we may fairly assume that p(x) is constant within an interval, i.e. $p(x) = p_1$ in the 1st interval, $p(x) = p_2$ in the 2nd interval, ..., $p(x) = p_k$ in the kth interval.

Therefore, the previous equation can be written as

$$\overline{e^2} = (p_1 + p_2 + ...) \int_{-\delta/2}^{\delta/2} y^2 dy$$

Where, y = x-x_k for all 'k'.

So,

$$\overline{e^2} = (p_1 + p_2 + ...)\frac{\delta^3}{12}$$

$$= [(p_1 + p_2 + ...)\delta]\frac{\delta^2}{12}$$

Now, note that $(p_1 + p_2 + \ldots + p_k + \ldots)\delta = 1.0$

$$\therefore \overline{e^2} = \frac{\delta^2}{12}$$

The above mean square error represents power associated with the random error signal. For convenience, we will also indicate it as N_Q.

Calculation of Signal Power (S_i)

After getting an estimate of quantization noise power as above, we now have to find the signal power. In general, the signal power can be assessed if the signal statistics (such as the amplitude distribution probability) is known. The power associated with x (t) can be expressed as

$$S_i = \overline{x^2(t)} = \int_{-V}^{+V} x^2(t)p(x)\,dx$$

where p (x) is the pdf of x (t). In absence of any specific amplitude distribution it is common to assume that the amplitude of signal x (t) is uniformly distributed between $\pm V$.

In this case, it is easy to see that

$$S_i = \overline{x^2(t)} = \int_{-V}^{+V} x^2(t)\frac{1}{2V}\,dx = \left[\frac{x^3}{3.2V}\right]_{-V}^{+V} = \frac{V^2}{3} = \frac{(M\delta)^2}{12}$$

Now the SNR can be expressed as,

$$\frac{S_i}{N_Q} = \frac{\dfrac{V^2}{3}}{\dfrac{\delta^2}{12}} = \frac{\dfrac{(M\delta)^2}{12}}{\dfrac{\delta^2}{12}} = M^2$$

It may be noted from the above expression that this ratio can be increased by increasing the number of quantizer levels N.

Also note that S_i is the power of x (t) at input of the sampler and hence, may not represent the SQNR at the output of the low pass filter in PCM decoder. However, for large N, small δ and ideal and smooth filtering (e.g. Nyquist filtering) at the PCM

decoder, the power S_o of desired signal at the output of the PCM decoder can be assumed to be almost the same as S_i i.e.,

$$S_o \simeq S_i$$

With this justification the SQNR at the output of a PCM codec, can be expressed as,

$$SQNR = \frac{S_o}{N_Q} \simeq M^2 = \left(2^N\right)^2 = 4^N$$

and in dB,

$$\left.\frac{S_o}{N_Q}\right|_{dB\triangleright} = 10\log_{10}\left(\frac{S_o}{N_Q}\right) \simeq 6.02 NdB$$

A few observations

(a) Note that if actual signal excursion range is less than $\pm V$, $S_o / N_o < 6.02NdB$.

(b) If one quantized sample is represented by 8 bits after encoding i.e., $N = 8$,
$SQNR \simeq 48dB$.

(c) If the amplitude distribution of x (t) is not uniform, then the above expression may not be applicable.

Problems

Q3.11.1)   If a sinusoid of peak amplitude 1.0V and of frequency 500Hz is sampled at 2 k-sample /sec and quantized by a linear quantizer, determine SQNR in dB when each sample is represented by 6 bit.

Q3.11.2)   How much is the improvement in SQNR of problem 3.11.1 if each sample is represented by 10 bits?

Q3.11.3)   What happens to SQNR of problem 3.11.2 if each sampling rate is changed to 1.5 k-samples/ sec?