Kinematic Modeling and Study of Gough Stewart Platform for Flight Simulator

Koyal Bhartia

Final Project for ENPM 662

Robot Modeling

University of Maryland, College Park

MD,USA

Contents

List of Figures

1 Motivation

The Gough-Stewart platform mechanism, introduced by Gough and Whitehall and Stewart, was originally used as a universal six degree of freedom (6-DOF) mechanism in a tire test machine and a flight simulator. A Parallel Robot has multiple kinematic chains connecting their base with a moving platform (end effector). Generally, each of these kinematic chains has a series of links connected by joints. This leads to advantages such as high force to weight ratio, low inertia, accuracy, high speeds, acceleration and rigidity which sets them apart from conventional serial manipulators. Hence they are perfectly suitable for industrial high-speed applications and in many fields such as aviation, manufacturing, entertainment, and medical.

I wish to focus on its application as a flight simulator, for the benefit of the project. The flight simulator is an application which makes generic use of all the different positions and orientations of the stewart platform (all 6 degrees of freedom). It minimizes time loss in training pilots. A parallel mechanism placed under the simulator provides the translational and rotational movements that the pilot would be exposed to when flying with a real aircraft.

Figure 1: A Stewart platform in use by Lufthansa

As shown in the picture, the stewart platform is extensively gaining in popularity. Here, the payload is a replica cockpit and a visual display system to aid in the training of the aircraft crew. With an interest in air vehicles I have chosen the above as my project topic not just to get an hands on experience on the designing and simulation of a full-fledged parallel manipulator but to also get a deeper understanding on the applications and advantages of a 6DOF robot.

2 Description of the robot

The Stewart-Gough platform is a 6 DOF parallel mechanism; that consists of:

- A rigid body top plate or mobile plate, connected to,
- A fixed base plate, through,
- 6 independent kinematic legs - These legs are identical chains, composed of,
- A universal joint

- A linear electrical actuator, and
- A spherical joint.

Hence it has 6 actuators to control whole mechanism.

3 Degrees of Freedom of the Mechanism

$$F = \lambda(n - j - i) + \sum_{i=1}^{n} f_i \tag{1}$$

where, F : DoF of the robot.
λ : DoF of the space in which the robot will work.
n : number of links in the mechanism, including the base.
j : number of joints in the mechanism.
f_i : degrees of relative motion permitted in the joint i.
Here for the above Stewart Gough platform (6UPS), $\lambda = 6$,n $= 14$,$j_1 = 6$,$j_2 = 6$,$j_3 = 6$. $Substitutingthisvaluesinto(1), weobtai$
$6(14181) + (61 + 62 + 63) = 6DoF$

4 Schematic of a 6-3 stewart parallel manipulator

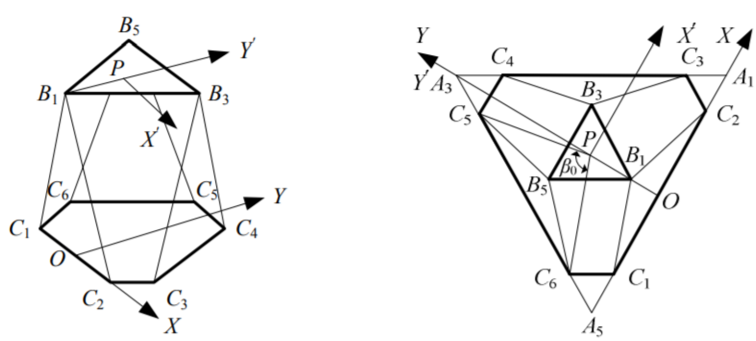


Figure 2: Side view and top view of the Stewart Platform

Here (B_1, B_3, B_5) is the moving triangular platform and $(C_1C_2....C_6)$ is the fixed hexagon one. Each leg contains a precision ball-screw assembly and a DC- motor. Thus the motion of the moving platform can be performed by controlling the variable lengths of the legs. Here rotary motion is converted to linear drive which enable us to have precise motion. Encoder values can have exact position of desired links. Which on the other hand helps in control and feedback mechanism.

The 6 DOF of the upper plate of the manipulator lets it compensate with all kinds of motions:
3 Translational: Surge(longitudinal), sway(lateral), and heave(vertical) and;
3 Rotational: Roll, Pitch and Yaw. Thus the acceleration forces of the moveable plate can easily emulate the physical feeling of piloting an aircraft in forward, backward, or turning motions.

5 Representation of the robot as flight simulator

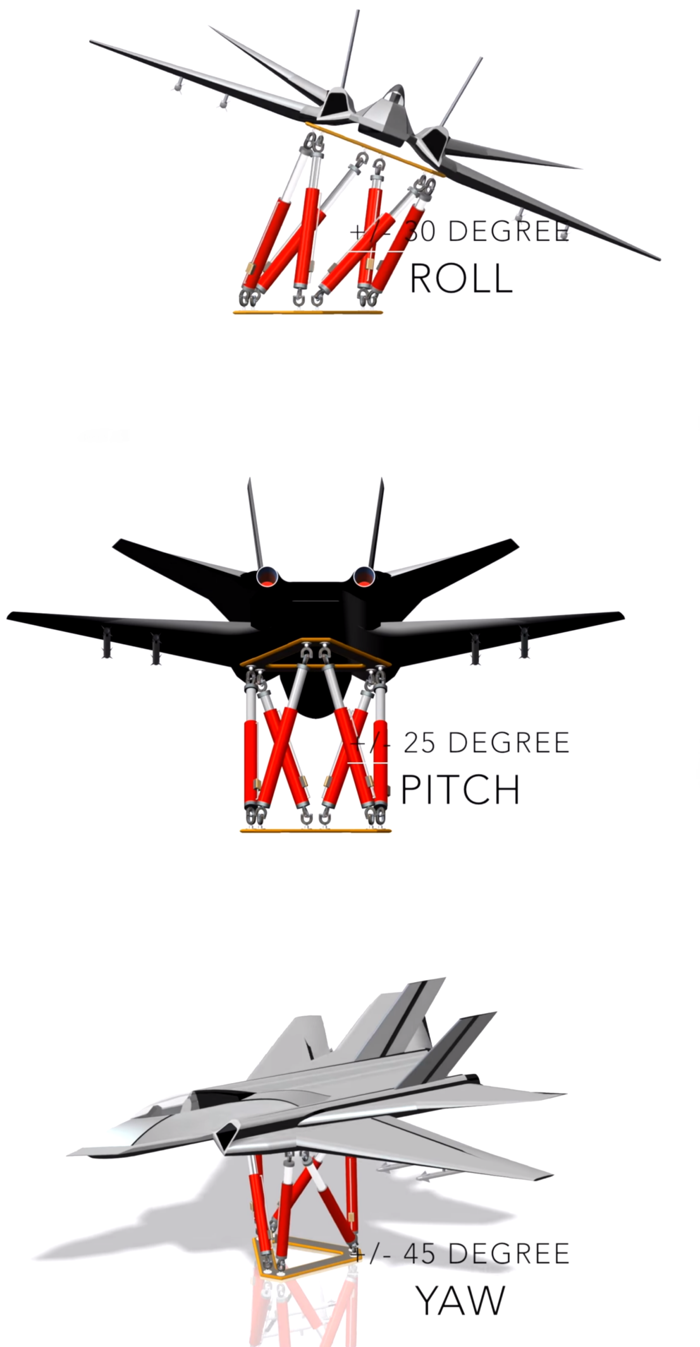
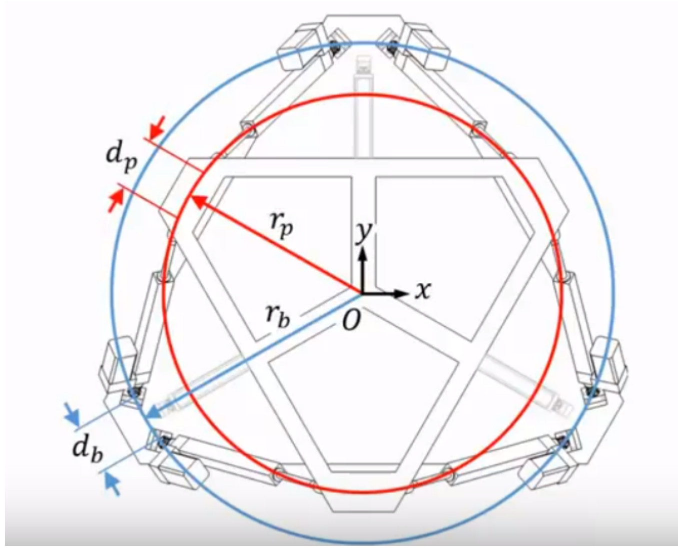
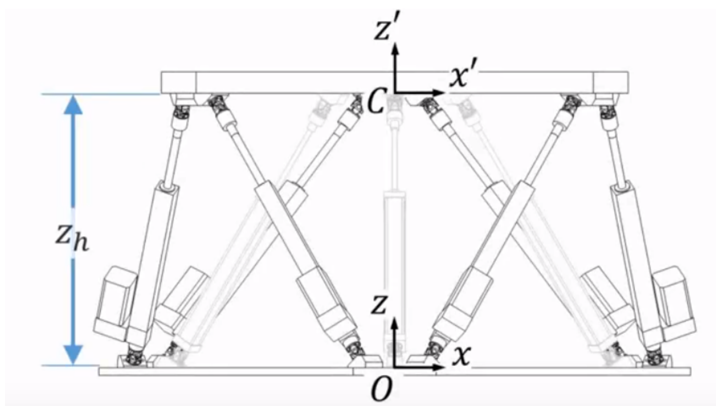


Figure 3: Roll Pitch and Yaw of the robot as a Simulator

The stewart platform can be fully defined by 7 parameters: including 5 geometrical parameters and 2 contructional ones. The geometrical parameters are: Neutral robot height (Zh) Radius of circle passing through loci of universal joints in base platform (rb) Distance between two adjacent universal joints in base platform (db) Radius of circle passing through loci of spherical joints in top platform (rp) Distance between two adjacent spherical joints in top platform (dp)





The constructional parameters are:
Stroke = Max. Actuator length - Min.
Actuator Length Dead = Min. Actuator Length - Stroke

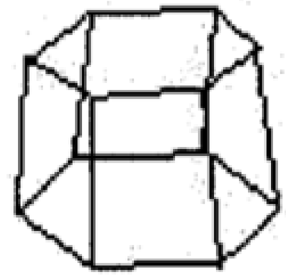
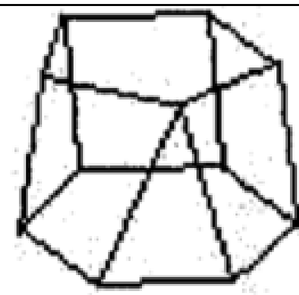
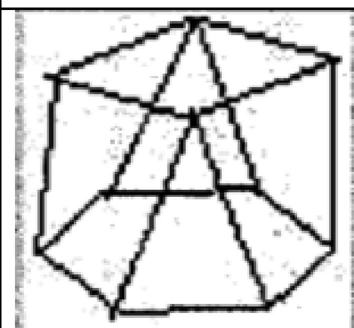
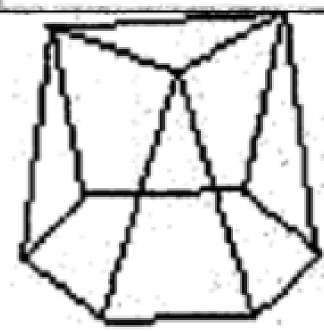
6 Classification of Stewart Platforms

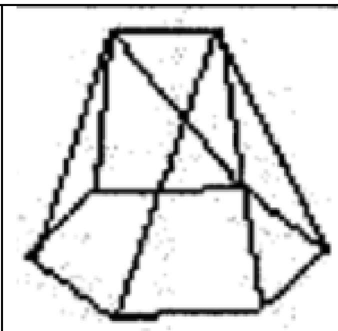
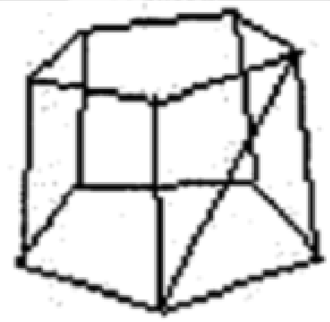
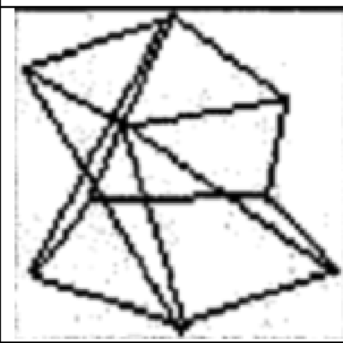
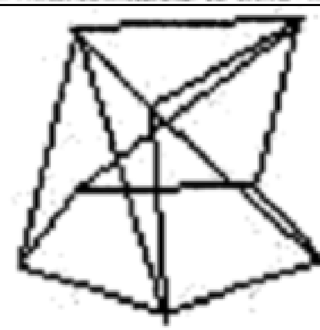
The rotational joints on base and moving platform enables the classification of the different categories of the stewart platform based on the shape. Basically there are five types of geometric shapes: hexagon, pentagon, quadrangle, triangle and line. A sixth type i.e point, has been excluded as here all the rotational joints coincide together on platform or base, leading to the reduction of the DOF to zero, which is meaningless. For the base, first four shapes are practicable but the line shape is unusable for its un-stability.

- **Hexagon Base:** All types of moving platform shapes can be applied to it. **Pentagon base and hexagon moving platform**: has the exactly same calculating condition as the 6-6 type.

- **Quadrangle base**: 5 available topologies are practicable.
- **Traingular Base**

Table 3.1: Hexagon Base Topologies

Conf	Conditions	Demo
6-6	Com:[ls] 1/2, 2/3, 3/4, 4/5, 5/6, 6/1 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: None	
6-5	Com:[ls] 2/3, 3/4, 4/5, 5/6, 6/1 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2	
6-4	Com:[ls] 2/3. 3/4, 5/6, 6/1 [2s] 1/3, 2/4, 3/5,46. 5 1. 6 2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 4/5	
6-3	Com:[ls] 2/3, 4/5, 6/1 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 3/4, 5/6	

6-2	Com:[Is] 3/4, 6/1 [2s] 2/4, 3/5, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 2/3,4/5, 5/6	
5-5	Com:[Is] 2/3, 3/4, 4/5, 5/6 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2	
5-4	Com:[Is] 2/3, 3/4, 5/6 [2s] 1/3, 2/4, 3/5.4/6, 5/1, 6/2 [3s] 1/4. 2/5, 3/6 Coi: 1/2,4/5	
5-3	Com:[Is] 2/3, 4/5 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 3/4, 5/6	

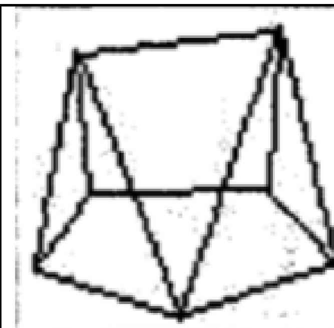
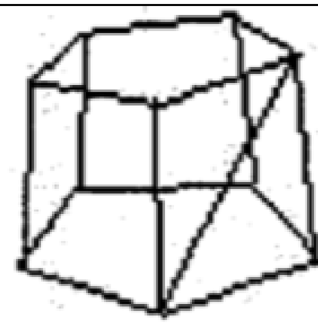
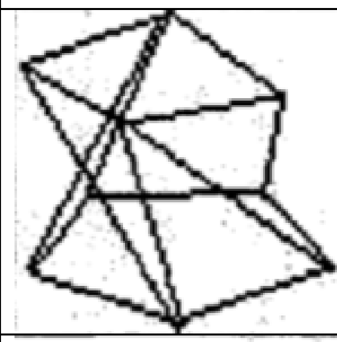
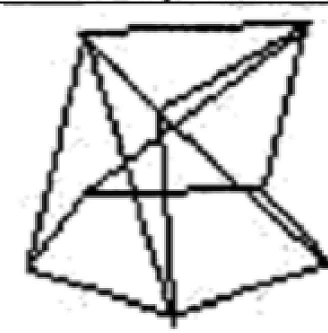
5-2	Com:[Is] 3/4 [2s] 2/4, 3/5. 5/1. 6/2 [3s] 1/4. 2/5. 3/6 Coi: 1/2.2/3,4/5. 5/6	

Table 3.2 : Pentagon Base Topologies

Conf	Conditions	Demo
5-5	Com:[Is] 2/3, 3/4, 4/5, 5/6 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2	
5-4	Com:[ls] 2/3, 3/4, 5/6 [2s] 1/3, 2/4, 3/5.4/6, 5/1, 6/2 [3s] 1/4. 2/5, 3/6 Coi: 1/2,4/5	
5-3	Com:[Is] 2/3, 4/5 [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 3/4, 5/6	

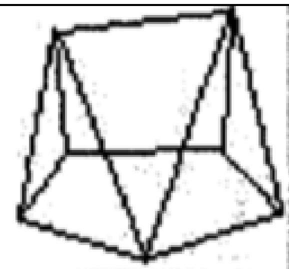
5-2	Com:[ls] 3/4 [2s] 2/4, 3/5. 5/1. 6/2 [3s] 1/4. 2/5. 3/6 Coi: 1/2.2/3,4/5. 5/6	

Table 3.3 : Quadrangle Base Topologies

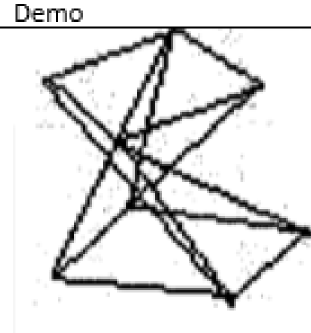
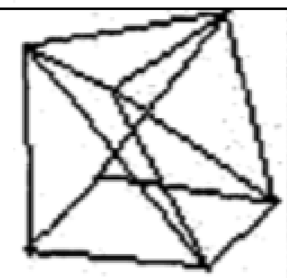
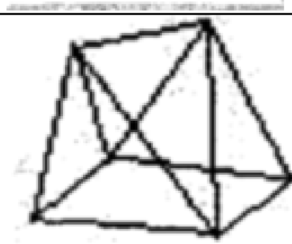
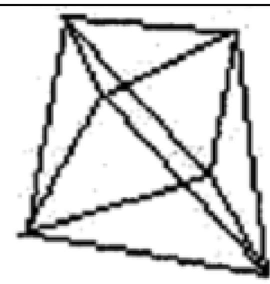
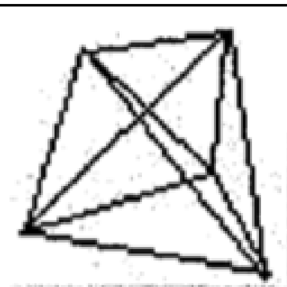
Conf	Conditions	Demo
4-4	Com:[ls] 2/3, 5/6 [2s] 1/3,2/4, 3/5,4/6, 5/1,6/2 [3s] 1/4,2/5, 3/6 Coi: 1/2, 4/5	
4-3	Com:[ls] 4/5 [2s] 1/3, 2/4. 3/5. 4/6, 5/1, 6/2 [3s] 1/4. 2/5. 3/6 Coi: 1/2.3/4, 5/6	
4-2	Com:[ls] N/A [2s] 2/4, 3/5, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 2/3, 4/5, 5/6	

Table 3.3 : Traingular Base Topologies

Conf	Conditions	Demo
3-3	Com:[Is] N/A [2s] 1/3, 2/4, 3/5, 4/6, 5/1, 6/2 [3s] 1/4, 2/5, 3/6 Coi: 1/2, 3/4, 5/6	
3-2	Com:[Is] 1/2. 3/4, 5/6 [2s] None [3s] 1/4, 2/5, 3/6 Coi: None	

7 3D Model

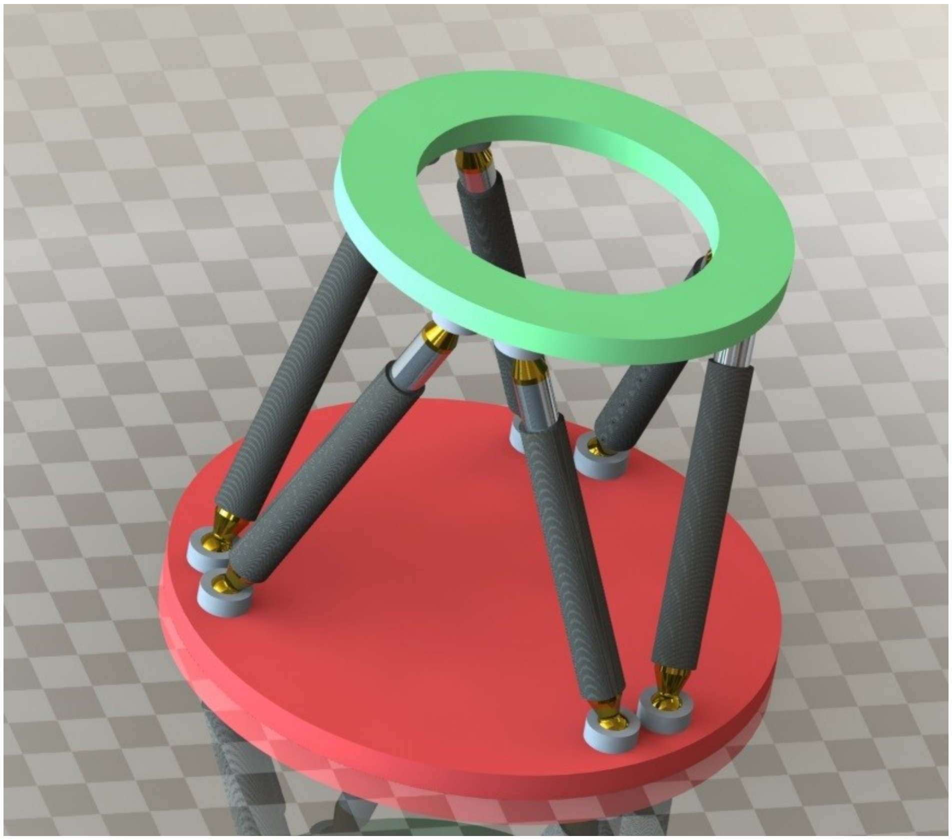


Figure 4: 3D Model generated and rendered using Solidworks

8 Forward Kinematics - Different Approaches

Forward kinematics of a parallel manipulator is basically finding the position and orientation of the mobile platform when the strut lengths are known. In the stewart platform, due to the existence of multi-closed kinematic loops between the six links and the moving and the base platforms, the direct kinematics is highly complicated. In real time environments different combinations of mathematical representations of FK and optimization algorithms are used to find a solution. The FK relations can be mathematically formulated in several ways; each having its own advantages and disadvantages when a different optimization algorithm is applied.

Mentioning of few of the different approaches that can be implemented:

1. **Polynomial-based approach:** This method results in 12 equations with 12 unknowns, where each equation is of second degree, resulting in 4096 solutions. However, 40 real solutions can be found after taking some considerations. The resulting constraint equations can be reduced into a univariate high-order polynomial by the elimination method and the roots of the polynomial can be found using any root-solver. **Reason for complexity:** This requires extremely complicated formulation procedures. Determining the actual solution among all the roots is a very challenging task.

2. **Numerical iterative approach:** Much faster than the Polynomial based approach. **Reason for complexity:** 3 unknown angles of the moving platform orientation is used resulting in problems due to the calculation of the partial derivative matrix and its inverse.

3. **Tetrahedron approach:**A tetrahedron is first identified based on the geometric structure of the linkages and the relationship between the moving and the base platforms,and then using it as a basis for identifying and piling up the next tetrahedrons.

4. **Some of the other approaches include:**A 40th-degree univariate polynomial found using the greatest common divisor of intermediate polynomials of degree 320. **Grobner Sylvester hybrid method**: This gives a 40th degree polynomial from 68 x 68 Sylvester's matrix formed by 68 equations of calculated Grobner basis. **Elimination method:** A univariate polynomial of degree 40 derived from a 28 x 28 matrix

9 Proposed Approach of Forward Kinematics

The approaches mentioned in the above section provide algorithms to obtain all solutions, but are not intended for real-time applications. I think project, I have taken the reference of **** paper to help in the development of an algorithm to provide all the FK solutions analysis of the 6-6 Stewart platform in real-time. In the algorithm proposed, 20th-degree univariate polynomial has been derived from the determinant of the final 4x4 Sylvester's matrix.

9.1 Kinematic Constraint Equations

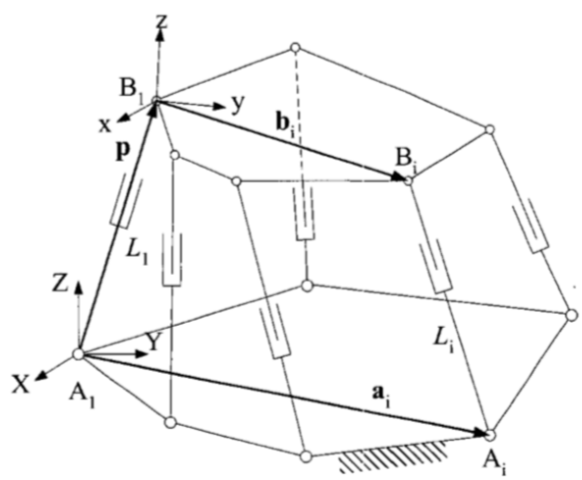


Figure 5: Kinematic Model of 6-6 Stewart Platform

Consider the Kinematic Model of the 6-6 Stewart Platform with the planar base and moving platform shown above. The locomotion and orientation of the upper platform are controlled by the 6 leg length inputs from each prismatic joint. Let the base coordinate frame X-Y-Z be chosen as shown at A_1 with the z axis perpendicular to the base. Similarly, let the frame x-y-z be attached at B_1 to the moving platform. Let a_i and b_i denote the position vectors in the 2 frames respectively. The position vector p denotes the location of frame x-y-z wrt X-Y-Z. The kinematic constraint equations considering the length of each leg as constant are as follows:

$$(\mathbf{p} + \mathbf{R}\mathbf{b}_\mathrm{i} - \mathbf{a}_\mathrm{i})^\mathrm{T}(\mathbf{p} + \mathbf{R}\mathbf{b}_\mathrm{i} - \mathbf{a}_\mathrm{i}) = L_\mathrm{i}^2, \quad \mathrm{i} = 2, \ldots, 6 \tag{2}$$

$$\mathbf{p}^\mathrm{T}\mathbf{p} = L_1^2 \tag{3}$$

where L_i is the ith leg length and $\mathbf{R}$ is the rotation matrix. Using Cayleys' formula, R can be expressed as:

$$\mathbf{R} = [\mathbf{I} - \mathbf{C}]^{-1}[\mathbf{I} + \mathbf{C}] \tag{4}$$

where I is the 3 x 3 identity matrix and C is an arbitrary 3 x 3 skew symmetric matrix with three independent parameters which can be expressed as follows:

$$\mathbf{C} = \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \end{bmatrix} \tag{5}$$

Substituting the above in eqn 4 gives:

$$\mathbf{R} = \Delta^{-1} \begin{bmatrix} 1 + c_1^2 - c_2^2 - c_3^2 & 2(c_1c_2 - c_3) & 2(c_3c_1 + c_2) \\ 2(c_1c_2 + c_3) & 1 - c_1^2 + c_2^2 - c_3^2 & 2(c_2c_3 - c_1) \\ 2(c_3c_1 - c_2) & 2(c_2c_3 + c_1) & 1 - c_1^2 - c_2^2 + c_3^2 \end{bmatrix} \tag{6}$$

where $\Delta = 1 + (c_1)^2 - (c_2)^2 + (c_3)^2$. Now using Eqn(3) to expand Eqn(1), following is obtained:

$$-2\mathbf{a}_\mathrm{i}^\mathrm{T}\mathbf{p} + 2\mathbf{b}_\mathrm{i}^\mathrm{T}\mathbf{R}^\mathrm{T}\mathbf{p} - 2\mathbf{a}_\mathrm{i}^\mathrm{T}\mathbf{R}\mathbf{b}_\mathrm{i} + \mathbf{a}_\mathrm{i}^\mathrm{T}\mathbf{a}_\mathrm{i} + \mathbf{b}_\mathrm{i}^\mathrm{T}\mathbf{b}_\mathrm{i} - L_\mathrm{i}^2 + L_1^2 = 0, \mathrm{i} = 2, \ldots, 6 \tag{7}$$

The above equations contain the unknown translation vector p and the vector $c = [c_1, c_1, c_3]^T$ (rotational parameters). Given the six leg lengths, these variables can be computed to determine the postures of the moving platform.

9.2 Modification of constraint equations

Consider the following $\mathbf{q}$ as another translation vector to increase efficiency of solution.

$$\mathbf{q} = \mathbf{R}^\mathrm{T}\mathbf{p} \tag{8}$$

Thus, eqn(7) can be rewritten as:

$$-\mathbf{a}_\mathrm{i}^\mathrm{T}\mathbf{p} + \mathbf{b}_\mathrm{i}^\mathrm{T}\mathbf{q} - \mathbf{a}_\mathrm{i}^\mathrm{T}\mathbf{R}\mathbf{b}_\mathrm{i} + \frac{1}{2}\left(\mathbf{a}_\mathrm{i}^\mathrm{T}\mathbf{a}_\mathrm{i} + \mathbf{b}_\mathrm{i}^\mathrm{T}\mathbf{b}_\mathrm{i} - L_\mathrm{i}^2 + L_1^2\right) = 0, \qquad \mathrm{i} = 2, \ldots, 6 \tag{9}$$

Also this will be true from eqn(4) and eqn(8):

$$[\mathbf{I} - \mathbf{C}]\mathbf{p} - [\mathbf{I} + \mathbf{C}]\mathbf{q} = 0 \tag{10}$$

Substituting $p = [p_x, p_y, p_z]^T$ and $q = [q_x, q_y, q_z]^T$ into the above eqn gives:

$$p_{\mathrm{x}} + c_3 p_{\mathrm{y}} - c_2 p_{\mathrm{z}} - q_{\mathrm{x}} + c_3 q_{\mathrm{y}} - c_2 q_{\mathrm{z}} = 0 \tag{11}$$

$$-c_3 p_{\mathrm{x}} + p_{\mathrm{y}} + c_1 p_{\mathrm{z}} - c_3 q_{\mathrm{x}} - q_{\mathrm{y}} + c_1 q_{\mathrm{z}} = 0 \tag{12}$$

$$c_2 p_{\mathrm{x}} - c_1 p_{\mathrm{y}} + p_{\mathrm{z}} + c_2 q_{\mathrm{x}} - c_1 q_{\mathrm{y}} - q_{\mathrm{z}} = 0 \tag{13}$$

Thus the 3 unknown vectors p,q and c can be determined from the nine scalar equations.

9.3 Intermediate polynomials in c_1, c_2 and c_3 only

Eqn 9 can be expressed in matrix form as follows:

$$\mathbf{Mu} = \begin{bmatrix} -a_{2\mathrm{x}} & -a_{2\mathrm{y}} & b_{2\mathrm{x}} & b_{2\mathrm{y}} & F_2 \\ -a_{3\mathrm{x}} & -a_{3\mathrm{y}} & b_{3\mathrm{x}} & b_{3\mathrm{y}} & F_3 \\ -a_{4\mathrm{x}} & -a_{4\mathrm{y}} & b_{4\mathrm{x}} & b_{4\mathrm{y}} & F_4 \\ -a_{5\mathrm{x}} & -a_{5\mathrm{y}} & b_{5\mathrm{x}} & b_{5\mathrm{y}} & F_5 \\ -a_{6\mathrm{x}} & -a_{6\mathrm{y}} & b_{6\mathrm{x}} & b_{6\mathrm{y}} & F_6 \end{bmatrix} \begin{bmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \\ q_{\mathrm{x}} \\ q_{\mathrm{y}} \\ 1 \end{bmatrix} = 0 \tag{14}$$

where

$$F_{\mathrm{i}} = -\mathbf{a}_{\mathrm{i}}^{\mathrm{T}} \mathbf{R} \mathbf{b}_{\mathrm{i}} + \frac{1}{2}\left(\mathbf{a}_{\mathrm{i}}^{\mathrm{T}} \mathbf{a}_{\mathrm{i}} + \mathbf{b}_{\mathrm{i}}^{\mathrm{T}} \mathbf{b}_{\mathrm{i}} - L_{\mathrm{i}}^2 + L_1^2\right), \quad \mathrm{i} = 2, \ldots, 6 \tag{15}$$

and $a_i = [a_{ix}, a_{iy}, 0]^T$ and $b_i = [b_{ix}, b_{iy}, 0]^T$ $(i = 2, ..., 6)$. Now,

$$\det(\mathbf{M}) = 0 \tag{16}$$

as eqn 14 is over constrained, there should be linear dependency in the system. From eqn 15 we get:

$$\Phi_1(c_1, c_2, c_3) \equiv g_1 + g_2 c_3 + g_3 c_3^2 + g_4 c_1^2 + g_5 c_1 c_2 + g_6 c_2^2 = 0 \tag{17}$$

where $g_i (i = 1, ..., 6)$ are the given real coefficients. Summing c_1 x eqn 11 to c_1 x eqn 12 gives:

$$(c_1 - c_2 c_3) p_{\mathrm{x}} + (c_2 + c_1 c_3) p_{\mathrm{y}} - (c_1 + c_2 c_3) q_{\mathrm{x}} - (c_2 - c_1 c_3) q_{\mathrm{y}} = 0 \tag{18}$$

Another linear system can be obtained as follows if the last row of matrix in eqn 14 is replaced with eqn 18.

$$\mathbf{M}'\mathbf{u} = \begin{bmatrix} -a_{2\mathrm{x}} & -a_{2\mathrm{y}} & b_{2\mathrm{x}} & b_{2\mathrm{y}} & F_2 \\ -a_{3\mathrm{x}} & -a_{3\mathrm{y}} & b_{3\mathrm{x}} & b_{3\mathrm{y}} & F_3 \\ -a_{4\mathrm{x}} & -a_{4\mathrm{y}} & b_{4\mathrm{x}} & b_{4\mathrm{y}} & F_4 \\ -a_{5\mathrm{x}} & -a_{5\mathrm{y}} & b_{5\mathrm{x}} & b_{5\mathrm{y}} & F_5 \\ c_1 - c_2 c_3 & c_2 + c_1 c_3 & -c_1 - c_2 c_3 & -c_2 + c_1 c_3 & 0 \end{bmatrix} \begin{bmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \\ q_{\mathrm{x}} \\ q_{\mathrm{y}} \\ 1 \end{bmatrix} = 0 \tag{19}$$

Ia a similar way, eqn. 19 can have a solution if and only if

$$\det(\mathbf{M}') = 0 \tag{20}$$

Upon calcution, eqn 20 leads to another polynomial in c_1, c_2 and c_3 as follows:

$$\Phi_2(c_1, c_2, c_3) \equiv T_1 c_1 + T_2 c_2 + \sum_{\mathrm{j}=0}^{3} T_{3\mathrm{j}} c_1^{3-\mathrm{j}} c_2^{\mathrm{j}} = 0 \tag{21}$$

where

$$T_{\mathrm{i}}(c_3) = \sum_{\mathrm{k}=0}^{3} t_{\mathrm{i,k}} c_3^{\mathrm{k}}, \qquad \mathrm{i} = 1, 2 \tag{22}$$

$T_{3\mathrm{j}}(c_3) = \sum_{\mathrm{k}=0}^{1} t_{3\mathrm{j,k}} c_3^{\mathrm{k}}, \quad \mathrm{j} = 0, \ldots, 3.$
These are coefficients determined by the input data only. So we have two eqns (17 and 21) in 3 unknowns, c_1, c_2 and c_3. Hence we need 1 or more equations similar to eqn 3 as follows:

$$p_x^2 + p_y^2 + p_z^2 - L_1^2 = 0 \tag{23}$$

Following is the minor system of eqn 13, the solution $[\bar{p}_x, \bar{p}_y, \bar{q}_x, \bar{q}_y]$ for $[p_x, p_y, q_x, q_y]$ is determined in terms of c_1, c_2 and c_3.

$$\begin{bmatrix} -a_{2\mathrm{x}} & -a_{2\mathrm{y}} & b_{2\mathrm{x}} & b_{2\mathrm{y}} & F_2 \\ -a_{3\mathrm{x}} & -a_{3\mathrm{y}} & b_{3\mathrm{x}} & b_{3\mathrm{y}} & F_3 \\ -a_{4\mathrm{x}} & -a_{4\mathrm{y}} & b_{4\mathrm{x}} & b_{4\mathrm{y}} & F_4 \\ -a_{5\mathrm{x}} & -a_{5\mathrm{y}} & b_{5\mathrm{x}} & b_{5\mathrm{y}} & F_5 \end{bmatrix} \begin{bmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \\ q_{\mathrm{x}} \\ q_{\mathrm{y}} \\ 1 = 0 \end{bmatrix} \tag{24}$$

Let $\mathbf{v}[v_1, v_2, v_3, v_4]$=$[\bar{p}_x, \bar{p}_y, \bar{q}_x, \bar{q}_y]$. Thus eqn 24 can be written as:

$$v_{\mathrm{i}} = \frac{s_{\mathrm{i}1} + s_{\mathrm{i}2} c_3 + s_{\mathrm{i}3} c_3^2 + s_{\mathrm{i}4} c_1^2 + s_{\mathrm{i}5} c_1 c_2 + s_{\mathrm{i}6} c_2^2}{1 + c_1^2 + c_2^2 + c_3^2}, \quad \mathrm{i} = 1, \ldots, 4 \tag{25}$$

The solution for $[\bar{p}_{z1}, \bar{q}_{z1}]$ for $[p_{xz}, q_z]$ can be found by substituting v in eqn 11 and 12. Similarly, solution for $[\bar{p}_{z2}, \bar{q}_{z2}]$ is obtained using eqn 11 and 13. $\bar{p}_{z1}$ and $\bar{p}_{z2}$ for p_z is expressed as shown:

$$p_{\mathrm{z}1} = \frac{N_1(c_1, c_2, c_3)}{D_1(c_1, c_2, c_3)} = \frac{N_{1,1} + \sum_{\mathrm{j}=0}^{2} N_{1,2\mathrm{j}} c_1^{2-\mathrm{j}} c_2^{\mathrm{j}} + \sum_{\mathrm{j}=0}^{4} n_{1,3\mathrm{j}} c_1^{4-\mathrm{j}} c_2^{\mathrm{j}}}{c_1(1 + c_1^2 + c_2^2 + c_3^2)} \tag{26}$$

$$p_{\mathrm{z}2} = \frac{N_2(c_1, c_2, c_3)}{D_2(c_1, c_2, c_3)} = \frac{N_{2,1} + \sum_{\mathrm{j}=0}^{2} N_{2,2\mathrm{j}} c_1^{2-\mathrm{j}} c_2^{\mathrm{j}} + \sum_{\mathrm{j}=0}^{4} n_{2,3\mathrm{j}} c_1^{4-\mathrm{j}} c_2^{\mathrm{j}}}{c_2(1 + c_1^2 + c_2^2 + c_3^2)} \tag{27}$$

where

$$N_{\mathrm{i},1}(c_3) = \sum_{\mathrm{k}=0}^{3} n_{\mathrm{i},1\mathrm{k}} c_3^{\mathrm{k}}, \qquad \mathrm{i} = 1, 2 \tag{28}$$

$N_{\mathrm{i,2j}}(c_3) = \sum_{\mathrm{k=0}}^{2} n_{\mathrm{i,2jk}} c_3^{\mathrm{k}}, \quad \mathrm{j} = 0, 1, 2$ where n's are data inputs. The solution for p_{z3} can be ontained as:

$$p_{\mathrm{z3}} = \frac{N_1(c_1, c_2, c_3) - N_2(c_1, c_2, c_3)}{D_1(c_1, c_2, c_3) - D_2(c_1 c_2 c_3)} \tag{29}$$

Substituting the three solutions for P, i.e. $[\bar{p}_x, \bar{p}_y, \bar{p}_{z1}]$, $[\bar{p}_x, \bar{p}_y, \bar{p}_{z2}]$ and $[\bar{p}_x, \bar{p}_y, \bar{p}_{z3}]$ into Eq. (22) one by one, after rationalizing, the following three polynomials are produced:

$$Phi_{\mathrm{i+2}}(c_1, c_2, c_3) = U_{\mathrm{i,1}} + \sum_{\mathrm{j=0}}^{2} U_{\mathrm{i,2j}} c^{2-\mathrm{j}} c_2^{\mathrm{j}} + \sum_{\mathrm{j=0}}^{4} U_{\mathrm{i,3j}} c_1^{4-\mathrm{j}} c_2^{\mathrm{j}} + \sum_{\mathrm{j=0}}^{6} U_{\mathrm{i,4j}} c_1^{6-\mathrm{j}} c_2^{\mathrm{j}} + \sum_{\mathrm{j=0}}^{8} u_{\mathrm{i,5j}} c_1^{8-\mathrm{j}} c_2^{\mathrm{j}}, \quad \mathrm{i} = 1, 2, 3 \tag{30}$$

where

$$U_{\mathrm{i,1}}(c_3) = \sum_{\mathrm{k=0}}^{6} u_{\mathrm{i,1k}} c_3^{\mathrm{k}}, \qquad U_{\mathrm{i,2j}}(c_3) = \sum_{\mathrm{k=0}}^{5} u_{\mathrm{i,2jk}} c_3^{\mathrm{k}}, U_{\mathrm{i,3j}}(c_3) = \sum_{\mathrm{k=0}}^{4} u_{i,3\mathrm{jk}} c_3^{\mathrm{k}}, \quad U_{\mathrm{i,4j}}(c_3) = \sum_{\mathrm{k=0}}^{2} u_{1,4\mathrm{jk}} c_3^{\mathrm{k}} \tag{31}$$

u:real coefficients

Next step id to derive the 5 intermediate polynomials ϕ(i=1,...,5) of eqn 17, 21 and 30.

9.4 Modification of the polynomials $\phi(i = 3, 4, 5)$

We need to derive a univariate equation in c3, for which the 2 unknowns c1 and c2 needs to be eliminated from the equations $\phi(i = 1, ..., 5)$. But these contain high order degree terms which is very difficult to solve. Hence, here a method is produced to decrease the power which reduces the size of the final Sylvester's matrix to 4 4.

Using the following in matrix form: $\phi_1 c_1^2, \phi_1 c_1 c2, \phi_1 c_2^2, \phi_2 c_1 and \phi_1 c_2$

$$\begin{bmatrix} g_4 & g_5 & g_6 & 0 & 0 \\ 0 & g_4 & g_5 & g_6 & 0 \\ 0 & 0 & g_4 & g_5 & g_6 \\ T_{30} & T_{31} & T_{32} & T_{33} & 0 \\ 0 & T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} c_1^4 \\ c_1^3 c_2 \\ c_1^2 c_2^2 \\ c_1 c_2^3 \\ c_2^4 \end{bmatrix} = - \begin{bmatrix} (g_1 + g_2 c_3 + g_3 c_3^2) c_1^2 \\ (g_1 + g_2 c_3 + g_3 c_3^2) c_1 c_2 \\ (g_1 + g_2 c_3 + g_3 c_3^2) c_2^2 \\ T_1 c_1^2 + T_2 c_1 c_2 \\ T_1 c_1 c_2 + T_2 c_2^2 \end{bmatrix} \tag{32}$$

The following expression is obtained while solving eqn 32, for $c_1^{4-j} c_2^j (j = 0, ..., 4)$ as linear equations:

$$c_1^{4-\mathrm{j}} c_2^{\mathrm{j}} = \frac{H_{\mathrm{j1}} c_1^2 + H_{\mathrm{j2}} c_1 c_2 + H_{\mathrm{j3}} c_2^2}{E}, \quad \mathrm{j} = 0, \ldots, 4 \tag{33}$$

where,

$$H_{\mathrm{jk}}(c_3) = \sum_{\mathrm{m=0}}^{4} h_{\mathrm{jkm}} c_3^{\mathrm{m}}, \quad \mathrm{j} = 0, \ldots, 4, \mathrm{k} = 1, 2, 3 \tag{34}$$

$E(c_3) = d_0 + d_1 c_3 + d_2 c_3^2 \; h_{jkm}$ and $d_i (i = 0, 1, 2)$ are input constants.

Using eqn 32, successive substitutions can be done, and the terms $c_1^{4-j} c_2^j (j = 0, ..., 4), c_1^{6-j} c_2^j (j = 0, ..., 6)$ and $c_1^{8-j} c_2^j (j = 0, ..., 8)$ of eqn 30 can be changed into $c_1^{2-j} c_2^j (j = 0, 1, 2)$. Thus $\phi_1 (i = 3, 4, 5)$ gets transformed as:

$$\Phi'_{\mathrm{i+2}}(c_1, c_2, c_3) = \frac{W_{\mathrm{i1}} + W_{\mathrm{i2}}c_1^2 + W_{\mathrm{i3}}c_1c_2 + W_{\mathrm{i4}}c_2^2}{E^3}, \mathrm{i} = 1, 2, 3 \tag{35}$$

where

$$W_{\mathrm{i1}}(c_3) = \sum_{\mathrm{k=0}}^{12} w_{\mathrm{ilk}}c_3^{\mathrm{k}}, \quad \mathrm{i} = 1, 2, 3 \tag{36}$$

$W_{\mathrm{ij}}(c_3) = \sum_{\mathrm{k=0}}^{10} w_{\mathrm{ijk}}c_3^{\mathrm{k}}, \quad \mathrm{i} = 1, 2, 3, \mathrm{j} = 2, 3, 4$ Thus in eqn 35, only $c_1^{2-j}c_2^j(j = 0, 1, 2)$ and $1(= c_1^0c_2^0)$ remains.

9.5 Deriving a univariate polynomial in c_3

The following linear system can be obtained with the combination of ϕ_1 of eqn 17 with $\phi_1^{'(i=3,4,5)}$.

$$\mathbf{S}\hat{\mathbf{c}} = \begin{bmatrix} G_1 & g_4 & g_5 & g_6 \\ \frac{W_{11}}{E^3} & \frac{W_{12}}{E^3} & \frac{W_{13}}{E^3} & \frac{W_{14}}{E^3} \\ \frac{W_{21}}{E^3} & \frac{W_{22}}{E^3} & \frac{W_{23}}{E^3} & \frac{W_{24}}{E^3} \\ \frac{W_{31}}{E^3} & \frac{W_{32}}{E^3} & \frac{W_{33}}{E^3} & \frac{W_{34}}{E^3} \end{bmatrix} \begin{bmatrix} 1 \\ c_1^2 \\ c_1c_2 \\ c_1^2 \end{bmatrix} = 0 \tag{37}$$

where $G_1 = g_1 + g_2c_3 + g3c_3^2$. E and W_{ij} are as given in the eqns 33 and 35. The system 36 has a soln if and only if:

$$\det(\mathbf{S}) = 0 \tag{38}$$

After factorization of eqn 38 we get the following:

$$\frac{1}{E^3}\sum_{\mathrm{i=0}}^{20} s_ic_3^{\mathrm{i}} = 0 \tag{39}$$

where $s_i(1 = 0, ..., 20)$ are input constants. 20 roots of $c_{3i}(i = 1, ..., 20)$ for c_3 in the complex domain is obtained after solving eqn 39.

9.6 Back Substitution for the other unknowns

The values $(c_1^2)_i$, $(c_1c_2)_i$ and $(c_2^2)_i$ for c_1^2, (c_1c_2) and c_2^2 is obtained by linearly solving the system. This is obtained by replacing c_3 with $c_{3i}(i = 1, ..., 20)$ in any row of matrix s of eqn 37. The values c_{1i} and c_{2i} for c_1 and c_2 can be easily computed in the complex domain as follows

$$\mathrm{if}(c_1^2)_{\mathrm{i}} \neq 0: \; c_{1\mathrm{i}} = \pm\sqrt{(c_1^2)_{\mathrm{i}}}, \quad c_{2\mathrm{i}} = \frac{(c_1c_2)_{\mathrm{i}}}{c_{1\mathrm{i}}}\mathrm{if}(c_1^2)_{\mathrm{i}}: \quad c_{1\mathrm{i}} = 0, \quad c_{2\mathrm{i}} = \pm\sqrt{(c_2^2)_{\mathrm{i}}} \tag{40}$$

Thus there exists 2 sets of solutions for c_{3i}; i.e if $[c_{1i}, c_{2i}$ is a soln corresponding to c_{3i}, $[-c_{1i}, -c_{2i}]$ is also a solution.

Thus, the moving platform can have 40 solutions for a given set if leg lengths, though the degree of the univariate is only 20. The values for p_{xi} and p_{xi} for p_x and p_y of the translational parameter p are obtained from eqn 25 and p_{zi} for p_z is determined by:

{cases if $c_{1i} \neq 0:$ byEq. (26)

if $c_{1i} = 0, c_{2i} \neq 0:$ by Eq. (27)

if $c_{1i}, c_{2i} = 0 : p_{zi} = \pm\sqrt{L_1^2 - p_{xi}^2 - p_{yi}^2}$ (41)

Thus 40 sets of solns of the necessary parameters to describe the posture of the moving platform can be determined.

10 Inverse Kinematics

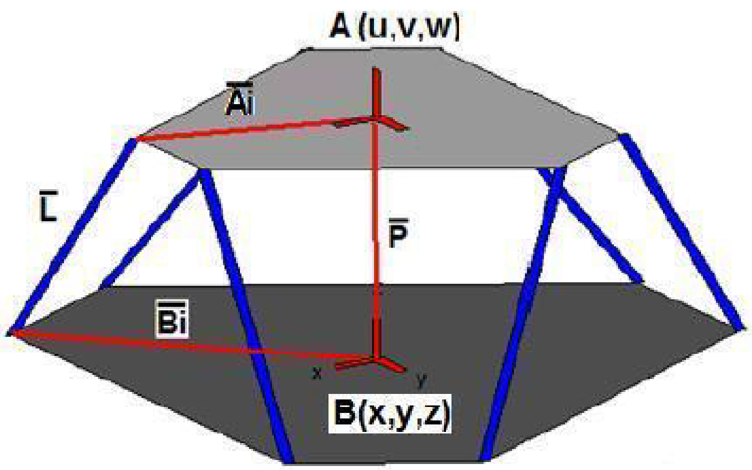


Figure 6: 3D Model generated and rendered using Solidworks

In inverse kinematics the pods' lengths do not change linearly with a linear path travelled by the upper platform. The linear movements of the joints can be expressed as a function of position and orientation due to the fixed coordinate system linked at the base of the platform, i.e $q_i = f(P)$ where $q_i = (L_1, L_2, L_3, L_4, L_5, L_6)$ represents the linear position of the joints and $P = [XYZ]^T$ represents the position-orientation of a platform's point.

Here A(u,v,w) and B(x,y,z) are the coordinate systems attached to the mobile platform and the base. Description of the transformation from the mobile platform's centroid to the base can be done using the position vector P and the rotation matrix R_B^A, which can be expressed as: $[r11r12r13; r21r22r23; r31r32r33]$ which using Euler angles can be expressed as:

$$R_B^A = \begin{bmatrix} c\alpha c\beta & c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha c\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma - c\alpha c\gamma & s\alpha s\beta c\gamma + c\alpha s\gamma \\ -c\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} \tag{42}$$

This rotation matrix can now be used to write the vector-loop equation for the ith actuator of the manipulator which is given by: $L_i = R.A_i + P - B_i$ Thus the equation for each actuator can be obtained as follows: $l_i^2 = x^2 + y^2 + z^2 + r_p^2 + r_b^2 + 2(r_11.A_ix + r_12A_11)(x - B_ix) + 2(r_21.A_ix + r_22A_iy)(y - B_iy) + 2(r_31.A_ix + r_23.A_iy)z - 2(x.B_ix + y..B_iy)fori = 1, 2, 3, 4, 5, 6$

This describes the motion of the platform wrt the base. The parameters r_p and r_b are the radius of the platform and the base respectively.

11 Assumptions

- 6-3 configuration of the stewart parallel manipulator is being used amongest from all other possible variants of the topology such as 6-6, 6-5, 6-4, 6-3, 5-4, 5-3, 4-3, and 3-3
- All links are of desired strength to withstand given load
- All links are ridged.
- All joints are perfectly spherical (No limit on joint).
- Friction between the links is minimal.
- Stalling torque of individual joint motors is more than maximum torque required for robot at extreme position.
- Safety feedback is assumed to be complete for some critical situations of flight simulation.
- width is not considered to the link to have geometric constraint.

12 Challenges faced in implementation of Stewart

- The calculation of the kinematics of a parallel platform like this involves heavy computations burden and is a time-consuming task.
- Not all solutions that are obtained is physically feasible.
- The mechanical challenges include
  - Compliance of the ball screws in the prismatic joints
  - High forces that the passive joints have to resist.
  - Strict tolerances with which the joints have to be manufactured and assembled.

13 Validation

The above approach taken for Forward Kinematics has been implemented in the MATLAB code attached, along with inverse kinematics.

Below is a numerical example of the same.

I have referred to the following paper for the approach taken and also as a means for verification and validation. "Algebraic elimination-based real-time forward kinematics of the 6-6 Stewart platform with planar base and platform"

14 Numerical Example

The geometric parameters of the platform is taken as:

$\mathbf{a}_1 = [0, 0, 0]^{\mathrm{T}}, \qquad \mathbf{a}_2 = [62, 0, 0]^{\mathrm{T}}, \qquad \mathbf{a}_3 = [62, 11, 0]^{\mathrm{T}},$

$\mathbf{a}_4 = [42, 38, 0]^{\mathrm{T}}, \quad \mathbf{a}_5 = [32, 39, 0]^{\mathrm{T}}, \quad \mathbf{a}_6 = [7, 13, 0]^{\mathrm{T}},$

$\mathbf{b}_1 = [0, 0, 0]^{\mathrm{T}}, \qquad \mathbf{b}_2 = [14, 0, 0]^{\mathrm{T}}, \qquad \mathbf{b}_3 = [47, 13, 0]^{\mathrm{T}},$

$\mathbf{b}_4 = [46, 27, 0]^{\mathrm{T}}, \quad \mathbf{b}_5 = [23, 45, 0]^{\mathrm{T}}, \qquad \mathbf{b}_6 = [16, 42, 0]^{\mathrm{T}}$

The predetermined configuration used for the computation of leg lengths are:

$\mathbf{p} = [12, 23, 96]^T,$

c=[1.0, -1.2,0.8]^T.

The exact values of leg lengths are:

$L_1 = 99.4434512675420 L_2 = 122.382476638755$

$L_3 = 156.014956547975 L_4 = 153.949953670971$

$L_5 = 136.270060584725 L_6 = 117.805089939638$

Using a_i, b_i and $L_i (i = 1, ..., 6)$ as inputs, 40 sets of solutions can be obtained.

Thus the exact values obtained by the implemented algorithm in this case are:

$\mathbf{p} = [11.9999999975, 23.0000000219, 96.0000000247]^{\mathrm{T}}$

$\mathbf{c} = [1.00000000060, -1.20000000113, 0.80000000018]^T.$

15 Replicability/Implementation in MATLAB

Attached is the GUI based Stewart Simulation. I have used the same for the purpose of Verification and Validation. The MATLAB implementation takes in the following inputs:

- 6 Coordinates of Base Frame.
- 6 Coordinates of the Platform Frame
- Min and Max constraints on the 3D space coordinates and the Euler angles.

Forward Kinematics takes in the input of the lengths of the 6 legs or actuators between the 2 platforms. Inverse Kinematics takes in the Orientation (Euler Angles) as inputs and the positions of the complete model i.e X,Y,Z

Following is the screenshot of the GUI plot.

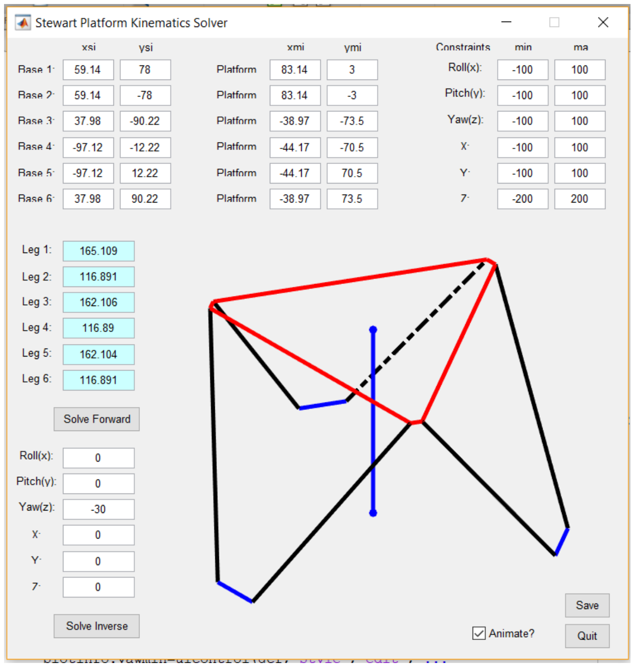


Figure 7: GUI for Stewart Platform Kinematics Solver

16 Conclusion

In practical purposes, it is usually the IK that is made use in Gough-Stewart Mechanism. So in this way we studied the geometry, inverse kinematics and forward kinematics of Gough-Stewart robot which is an example of parallel configuration of robotics. This helped me gain a complete understanding of robotics modelling course which was based series manipulator and project gave me understanding of parallel manipulators. Also, not all the solutions of Forward Kinematics are physically feasible; whereas the Inverse Kinematics is easy to solve.

17 References

1. Algebraic elimination-based real-time forward kinematics of the 6-6 Stewart platform with planar base and platform - Tae-Young Lee and Jae-Kyung Shim

2. A new Approach for the Forward Kinematics of General Stewart-Gough Platforms - Manuel Cardona, Senior Member, IEEE

3. A Platform with six degrees of freedom - By D. Stewart

4. Reconfigurable kinematics of General Stewart Platform and simulation interface - Anqi Wang