$$Y_i^* = Y_i\,[M_i=0] + \overset{\text{constant}}{c}\,[M_i=1]$$

$$P(M_i=1) = \text{expit}(\beta_0 + \beta_1 Y_i)$$

$$E(Y_i^*) = E\left[Y_i[M_i=0] + c\,[M_i=1]\right]$$

$$= E_Y\left[E_{M|Y}\{Y_i[M_i=0] + c\,[M_i=1]\}\right]$$

$$= E_Y\Big[\,Y_i\cdot\overset{P(M=0)}{\underline{(1-\text{expit}(\beta_0+\beta_1 Y_i))}} + c\cdot\overset{P(M=1)}{\underline{\text{expit}(\beta_0+\beta_1 Y_i)}}\,\Big]$$

$$= \int y\cdot\{1-\text{expit}(\beta_0+\beta_1 y)\}\,f_Y(y)\,dy + c\cdot\int \text{expit}(\beta_0+\beta_1 y)\,f_Y(y)\,dy$$

Assume $Y \sim \text{normal}(\mu, \sigma^2)$; $f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$

$$= \int_{-\infty}^{\infty} y\cdot\{1-\text{expit}(\beta_0+\beta_1 y)\}\cdot\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}dy + c\cdot\int_{-\infty}^{\infty}\text{expit}(\beta_0+\beta_1 y)\,\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}dy$$

Let $y = \mu + \sigma\sqrt{2}t$, $dy = \sigma\sqrt{2}\,dt$ $\Rightarrow$ $\frac{y-\mu}{\sigma\sqrt{2}} = t$, $\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2 = t^2$

Also $dy = \sigma\sqrt{2}\,dt$ so, $\sigma\sqrt{2\pi}$ in the denominator would now be $\sqrt{\pi}$

$$= \int_{-\infty}^{\infty}\underbrace{(\mu+\sigma\sqrt{2}t)\{1-\text{expit}(\beta_0+\beta_1(\mu+\sigma\sqrt{2}t))\}\cdot\frac{1}{\sqrt{\pi}}}_{f_1(t_i)}\,e^{-t^2}dt + c\int_{-\infty}^{\infty}\underbrace{\text{expit}(\beta_0+\beta_1(\mu+\sigma\sqrt{2}t))\frac{1}{\sqrt{\pi}}}_{f_2(t_i)}\cdot e^{-t^2}dt$$

$\approx \sum_{i=1}^{n} w_i\cdot f(t_i)$ by Gauss-Hermite quadrature

If we let $k = \beta_0 + \beta_1(\mu+\sigma\sqrt{2}t)$, $f_1(t_i) = (\mu+\sigma\sqrt{2}t_i)\{1-\text{expit}(k)\}\frac{1}{\sqrt{\pi}}$, $f_2(t_i) = c\cdot\text{expit}(k)\cdot\frac{1}{\sqrt{\pi}}$

We can calculate numerical bias if we plug in parameter values $\beta_0, \beta_1, \mu, \sigma, c$

Also, t_i and w_i can be computed with R

Note ***

If we let $P[M_i=1] = \pi_i$

Then, $E(Y_i^*) = E_Y[Y_i\cdot(1-\pi_i) + c\cdot\pi_i] = (1-\pi_i)E_Y(Y_i) + \pi_i\cdot c$ if MCAR ($Y_i \perp\!\!\!\perp \pi_i$)

π_i Missingness not related to any other values (including Y_i)

$$= \mu - \mu\pi_i + c\pi_i = \mu\cdot(1-\pi_i) + c\cdot\pi_i = \mu + (c-\mu)\cdot\pi_i$$

bias term added (under MCAR)

$\Rightarrow$ bias increases with the increase in absence rate π_i

$\Rightarrow$ but, our study assumes MAR (missingness depends on observed)