T.C.

İSTANBUL ÜNİVERSİTESİ

MÜHENDİSLİK FAKÜLTESİ

| Sınav Sonucu | |
|---|---|
| İmza | İmza |

Fakülte Numarası :

Adı ve Soyadı :

Yarıyıl :

Ders :

Bölümü :

Tarih :

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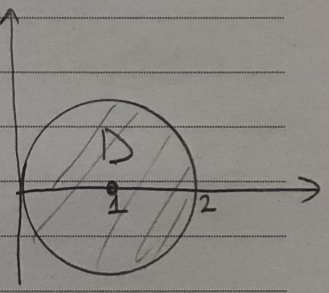
4) The region of integration, say D, has equation $x^2+y^2=2x$ or, after completing the square

$$x^2-2x+y^2=0$$

$$\Rightarrow \quad x^2-2x+1+y^2=0$$

$$(x-1)^2+y^2=0$$




In polar coordinates we have $x=r\cos\theta$, $y=r\sin\theta$ and

$$x^2+y^2=2x \Rightarrow (r\cos\theta)^2+(r\sin\theta)^2=2r\cos\theta$$

$$\Rightarrow \quad r=2\cos\theta$$

So we have

$$V=\iint_D (x^2+y^2)\,dA=\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r^2\, r\,dr\,d\theta=\int_{-\pi/2}^{\pi/2}\frac{r^4}{4}\Big|_0^{2\cos\theta}\,d\theta$$

$$=4\int_{-\pi/2}^{\pi/2}\cos^4\theta\,d\theta=4\int_{-\pi/2}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$=4\left[\int_{-\pi/2}^{\pi/2}\frac{1}{4}\cdot\left(1+2\cos 2\theta+(\cos 2\theta)^2\right)d\theta\right.$$

$$=\left(\theta+\sin 2\theta\Big|_{-\pi/2}^{\pi/2}+\int_{-\pi/2}^{\pi}\frac{1+\cos 4\theta}{2}\,d\theta\right.=\pi+\frac{1}{2}\left(\theta+\frac{\sin 4\theta}{4}\Big|_{-\pi/2}^{\pi/2}\right.$$

$$=\pi+\frac{1}{2}\pi=\frac{3}{2}\pi \;//$$