

18/4/23

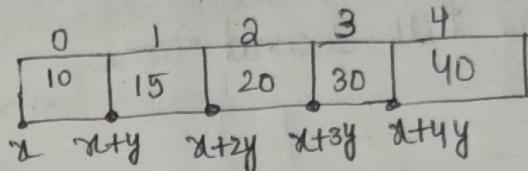
Array Data Structures

Introduction:

`arr[] = [10, 15, 20, 30, 40]`

`a = arr[0];`
`b = arr[2]; // a+2y`

Contiguous Memory:



$x \rightarrow$ Address where Array is stored
 $y \rightarrow$ size of an array element

Advantages:

- 1) Random Access \Rightarrow constant time.
- 2) Cache friendliness

Array D.S (Types)

- fixed sized Arrays
- Dynamic sized Arrays

C/C++

`int arr[100]`] stack Allocated

`int arr[n]`

 } heap Allocated

`int *arr = new int[n]`

`int arr[] = [10, 15, 30, 40]` } stack Allocated

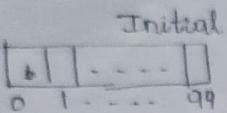
Dynamic Sized Arrays:

Resize Automatically.

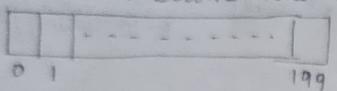
C++ : `vector`

Java : `ArrayList`

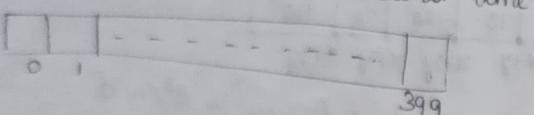
python : `list`



when Become full first time



when Become full Second time.



Vectors in C++

→ Problems with Simple arrays:

```
int arr[100];
int arr[n]
int *arr = new int[n];
```

Vector advantages:-

- Dynamic Size
- Rich library function
 - find
 - erase
 - insert etc.
- easy to know size →
 - Arrays:


```
int n = sizeof(arr)/sizeof(arr[0]);
```
 - Vectors:


```
int n = V.size();
```
- no need to pass size →
 - Arrays:


```
int fun(int *arr, int n)
```
 - Vectors:


```
int fun(vector<int> v)
```

→ can be returned from a function

```
int *fun()
{
    int arr[100];
    return arr;
}
```

simply array will be deallocated after returning

→ `vector<int> fun()`

```
{ vector<int> v;
```

```
return v;
```

→ By default initialised with default values.

→ we can copy a Vector to other.

$$V_1 = V_2;$$

Operations on Arrays

Search (unsorted Array) :-

Iff: $arr[] = \{20, 5, 7, 25\}$ | Iff: $arr[] = \{20, 5, 7, 25\}$
 $n = 5$ | $n = 5$

Opp: -1 | Opp: -1

`int search(int arr[], int n, int x)`

```
{
    for (int i=0; i<n; i++)
    {
        if (arr[i] == x)
            return i;
    }
    return -1;
}
```

$$T.C = O(n)$$



Insert:

$\Rightarrow \text{IP: arr[]} = \{5, 10, 20, -1, -3\}$

$x=7$

$pos=2$

$\text{DIP: arr[]} = \{5, 7, 10, 20, -3\}$

$\Rightarrow \text{IP: arr[]} = \{5, 7, 10, 20, -3\}$

$x=3$

$pos=2$

$\text{DIP: arr[]} = \{5, 3, 7, 10, 20\}$

int insert (int arr[], int n, int x, int cap, int pos)

{
 if ($n == cap$)

 return n;
 int rdm = pos-1;

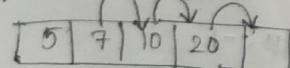
 for (int i=n-1; i>rdm; i--)

 arr[i+1] = arr[i];

 arr[rdm] = x;

 return (n+1);

 pos=2 x=3 rdm=1



$\boxed{5 \downarrow 7 \uparrow 10 \downarrow 20}$

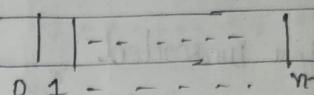
Time Complexity : $O(n)$

Insert At the End : $O(1)$

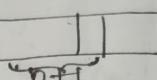
Insert At the Beginning : $O(n)$

Insert at the end for Dynamic Sized Array:

Initial capacity : n



\boxed{n}



Time Complexity of every insert for first n inserts : $\Rightarrow O(1)$

Average Time Complexity for $(n+1)$ = $\frac{D(1) + D(1) + \dots + D(1) + O(n)}{n+1}$
Inserts at the end

$= O(1)$

Deletion:

$\Rightarrow \text{IP: arr[]} = \{3, 8, 12, 5, 6\} \Rightarrow \text{IP: arr[]} = \{3, 8, 12, 5, 6\}$

$x=12$

$x=6$

$\text{DIP: arr[]} = \{3, 8, 5, 6, -3\} \quad \text{DIP: arr[]} = \{3, 8, 12, 5, -3\}$

int deleteEle (int arr[], int n, int x)

{
 int i;

 for (i=0; i<n; i++)

 if (arr[i] == x)

 break;

 if (i==n)

 return n;

 for (int j=i; j<n-1; j++)

 arr[j] = arr[j+1];

} return (n-1);



Operations on Arrays:

Insert : $O(n)$

Search : $O(n)$ for unsorted

$O(\log n)$ for sorted

Delete : $O(n)$

Get i-th Element : $O(1)$

Update i-th Element : $O(1)$

NOTE: Insert at the end and delete from the end can be done in $O(1)$ time.

Largest Element in an Array:

I/P: arr[] = {10, 5, 20, 8}

O/P: 2 // Index of 20.

I/P: arr[] = {40, 8, 50, 100}

O/P: 8 // Index of 100

Naive Approach

int getLargest(int arr[], int n)

{ for (int i=0; i<n; i++)

{ bool flag = true;

for (int j=0; j<n; j++)

{ if (arr[j] > arr[i])

{ flag = false;

break;

}

{ if (flag == true)

{ return i;

return -1;

Efficient Approach:

$a_0 \ a_1 \ a_2 \ a_3 \ \dots \ a_{i-1} \ a_i \ \dots \ a_{n-1}$

(Unsorted) \rightarrow (Arranged)

largest number

$a_i \leq a_{\text{largest}} \Rightarrow \text{Ignore}$

$a_i > a_{\text{largest}} \Rightarrow \text{largest} = i$

int getLargest (int arr[], int n)

{ int res = 0;

for (int i=1; i<n; i++)

{ if (arr[i] > arr[res])

{ res = i; }

return res; }

}

$\Rightarrow \Theta(n)$

arr[] = {5, 8, 20, 10}

res = 0; $i=1$

$i=1: res=1$

$i=2: res=2$

$i=3: res=2$

$i=4$

$\approx 5 \approx 10$

Second Largest Element

I/P: arr[] = {10, 5, 8, 20}

O/P: 0 // Index of 10

I/P: arr[] = {20, 10, 20, 8, 12}

O/P: 4 // Index of 12

I/P: arr[] = {10, 10, 10}

O/P: -1 // No second largest



Naive Approach:

```

int SecondLargest(int arr[], int n)
{
    int largest = getLargest(arr, n);
    int res = -1;
    for (int i=0; i<n; i++)
    {
        if (arr[i] != arr[largest])
        {
            if (res == -1)
                res = i;
            else if (arr[i] > arr[res])
                res = i;
        }
    }
    return res;
}

int getLargest(int arr[], int n)
{
    int largest = 0;
    for (int i=1; i<n; i++)
    {
        if (arr[i] > arr[largest])
            largest = i;
    }
    return largest;
}

int arr[] = {5, 20, 12, 20, 10};
int n = 5;
int result = SecondLargest(arr, n);
cout << "Second Largest element is: " << result;

```

$\Rightarrow \text{arr}[] = \{5, 20, 12, 20, 10\} \Rightarrow \text{arr}[0] = 5, \text{arr}[1] = 20, \text{arr}[2] = 12, \text{arr}[3] = 20, \text{arr}[4] = 10$

largest = 1
res = 0
res = 2

Efficient Approach:

```

int SecondLargest(int arr[], int n)
{
    int res = -1, largest = 0;
    for (int i=1; i<n; i++)
    {
        if (arr[i] > arr[largest])
        {
            res = largest;
            largest = i;
        }
        else if (arr[i] == arr[largest])
            res = i;
        else if (res == -1 || arr[i] > arr[res])
            res = i;
    }
    return res;
}

```

$$\boxed{\begin{array}{l} T_C = \Theta(n) \\ A.S = \Theta(1) \end{array}}$$

largest res.

$a[i] > a[\text{largest}] : \text{res} = \text{largest}, \text{largest} = i$

$a[i] == a[\text{largest}] : \text{Ignore} \Rightarrow \underbrace{5, 8, 12, 12}_{a_{i-1}} \underbrace{a_i}$

$a[i] < a[\text{largest}]$

- $\rightarrow \text{res} = -1 : \text{res} = i$
- $\rightarrow a[i] < a[\text{res}] : \text{Ignore}$
- $\rightarrow a[i] > a[\text{res}] : \text{res} = i$

\Rightarrow

$\begin{array}{c} 5, 8, 12, 12, 10 \\ \text{res} = 1, \text{largest} = 2 \\ a_{i-1} \quad a_i \end{array}$

$a[i] > a[\text{largest}] \Rightarrow \text{res} = \text{largest} \Rightarrow$

$\text{largest} = 1$

$\Rightarrow 5, 8, 12, 12$

$\hookrightarrow \underbrace{12, 12, 12, 7}_{a_{i-1}} \Rightarrow \text{res} = -1$

\downarrow

$\hookrightarrow \underbrace{12, 8, 12, 7}_{a_{i-1}} \Rightarrow \text{res} = 1$

$(\text{on}) 12, 7, 12, 7$

$a[i] < a[\text{res}] : \text{Ignore}$

$\Rightarrow 12, 7, 12, 7$

$a[i] > a[\text{res}] \Rightarrow \text{res} = i$

$\Rightarrow \{5, 20, 12, 20, 8\}$

largest = 0, res = -1

i=1: res = 0, largest = 1

i=2: res = 2

i=3:

i=4:



check if an array is sorted

I.P: arr[] = {8, 12, 15}

O.P: yes

I.P: arr[] = {8, 10, 10, 12}

O.P: yes

I.P: arr[] = {100}

O.P: yes

I.P: arr[] = {100, 20, 200}

O.P: no

I.P: arr[] = {200, 100}

O.P: no

Naive Method

bool isSorted (int arr[], int n)

```

{ for (int i=0; i<n; i++)
    for (int j=i+1; j<n; j++)
        if (arr[j] < arr[i])
            return false;
    return true;
}
  
```

T.C = $O(n^2)$

Efficient Method

{ $a_0, a_1, a_2, a_3, \dots, a_{i-1}, a_i, \dots, a_{n-1}$ }
 ↓
 1 - traversal 0 : 2nd 1 : 3rd
 2 - traversal 0 : 2nd 1 : 3rd
 3 - traversal 0 : 2nd 1 : 3rd
 ...
 : 8 : 9

bool isSorted (int arr[], int n)

{ for (int i=1; i<n; i++)

if (arr[i] < arr[i-1])

return false;

return true;

}

T.C = $O(n)$

A.S = $\Theta(1)$

{ 5, 12, 30, 2, 35 }

i=1: 5 < 12

i=2: 12 < 30

i=3: 30 < 2 (return false)

Reverse an Array

I.P: arr[] = {10, 5, 7, 30}

O.P: arr[] = {80, 7, 5, 10}

I.P: arr[] = {80, 20, 5}

O.P: arr[] = {5, 20, 80}

void reverse (int arr[], int n)

{ int low=0, high=n-1; (high: 3, low: 0) }

while (low < high)

{ int temp = arr[low]; (temp: 10, arr[low]: 10)

arr[low] = arr[high];] Swap arr[low] and arr[high]

arr[high] = temp;] + 1

low++;] + 1

high--;] + 1

{ 10, 5, 7, 30 }
low=0 high=3

T.C = $\Theta(n)$

A.S = $\Theta(1)$

{ 30, 5, 7, 10 }

{ 30, 7, 5, 10 }

{ 30, 5, 7, 10 }

{ 30, 7, 5, 10 }

{ 30, 7, 5, 10 }



Remove Duplicates from a Sorted Array

I/P: arr[] = {10, 20, 20, 30, 30, 30, 30} ; size = 7

O/P: arr[] = {10, 20, 30, - , - , - } ; size = 3

I/P: arr[] = {10, 10, 10} ; size = 3

O/P: arr[] = {10, - , - } ; size = 1

Noise Approach

int remDups (int arr[], int n)

{ int temp[n];

temp[0] = arr[0];

int res = 1;

for (int i=1; i<n; i++) {

{ if (temp[res-1] != arr[i]) {

temp[res] = arr[i]; } }

res++; }

for (int i=0; i<res; i++) {

arr[i] = temp[i]; }

return res;

T.C = O(n)

A.S = O(n)

arr[] = {10, 20, 20, 30, 30, 30}

i=1 : temp[] = {10, - , - , - , - , - } ; res = 1

i=2 : temp[] = {10, 20, - , - , - , - } ; res = 2

i=3 :

i=4 :

i=5 :

0

arr[] = {10, 20, 30, - , - , - }

Efficient Solution:

[O(n) Time]

[O(1) Space]

int remDups (int arr[], int n)

{ int res = 1;

for (int i=1; i<n; i++)

{ if (arr[i] != arr[res-1])

{ arr[res] = arr[i]; }

res++; }

}

return res;

arr[] = {10, 20, 20, 30, 30, 30}

res = 1

i=1 : arr[] = {10, 20, 20, 30, 30, 30} ; res = 2

i=2 : arr[] = {10, 20, 20, 30, 30, 30} ; res = 2

i=3 : arr[] = {10, 20, 30, 30, 30, 30} ; res = 3

i=4 : arr[] = {10, 20, 30, 30, 30, 30} ; res = 3

i=5 : arr[] = {10, 20, 30, 30, 30, 30} ; res = 3



Move all zeros to end

D.P: arr[] = {8, 5, 0, 10, 0, 20}

O.P: arr[] = {8, 5, 10, 20, 0, 0}

D.P: arr[] = {0, 0, 0, 10, 0}

O.P: arr[] = {10, 0, 0, 0}

D.P: arr[] = {10, 20}

O.P: arr[] = {10, 20}

Naive Solution

void moveToEnd (int arr[], int n)

```

{
    for (int i=0; i<n; i++)
    {
        if (arr[i] == 0)
        {
            for (int j=i+1; j<n; j++)
                if (arr[j] != 0)
                    Swap(arr[i], arr[j])
        }
    }
}

```

T.C = O(n²)

Efficient Solution

void moveZeros (int arr[], int n)

```

{
    int count=0;
    for (int i=0; i<n; i++)
    {
        if (arr[i] != 0)
            Swap(arr[i], arr[count]);
        count++;
    }
}

```

{10, 5, 0, 0, 10, 20}

i=0 : 0

i=1 : 0

i=2 : Swap(arr[1], arr[0])

j=4

i=3 : Swap(arr[3], arr[1])

j=6

i=4 :

i=5 :

i=6 :

i=7 :

i=8 :

i=9 :

i=10 :

i=11 :

i=12 :

i=13 :

i=14 :

i=15 :

i=16 :

i=17 :

i=18 :

i=19 :

i=20 :

{10, 8, 0, 0, 12, 0} Count = 0

i=0 : {10, 8, 0, 0, 12, 0} Count = 1

i=1 : - - - - , Count = 2

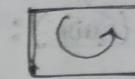
i=2 : - - - - , Count = 2

i=3 : - - - - , Count = 2

i=4 : {10, 8, 12, 0, 0, 0} Count = 3

i=5 : - - - - , Count = 3

Left Rotate arr Array by one



D.P: arr[] = {1, 2, 3, 4, 5}

O.P: arr[] = {2, 3, 4, 5, 1}

D.P: arr[] = {30, 5, 20}

O.P: arr[] = {5, 20, 30}

void lRotateOne (int arr[], int n)

```

{
    int temp = arr[0];
    for (i=1; i<n; i++)
        arr[i-1] = arr[i];
    arr[n-1] = temp;
}

```

arr[0] = {1, 2, 3, 4, 5}
 temp = 1
 arr[1] = {2, 3, 4, 5, 1}
 arr[2] = {2, 3, 4, 5, 1}

T.C = Θ(n)

A.S = Θ(1)



Scanned with OKEN Scanner

Left Rotate an Array by d

I/P: arr[] = {1, 2, 3, 4, 5} (→ Counter clockwise
d=2)

O/P: arr[] = {3, 4, 5, 1, 2}

I/P: arr[] = {10, 5, 30, 15}

d=3

O/P: arr[] = {15, 10, 5, 30}

We may assume $d \leq$ No of elements in the array.

method 1 (Naive): $T.C = \Theta(nd)$
 $\Theta(1)$ space

Void leftrotateOne (int arr[], int n)

{ int temp = arr[0];

for (int i=1; i<n; i++)

arr[i-1] = arr[i];

arr[n-1] = temp;

Void leftrotate (int arr[], int n, int d)

{ for (int i=0; i<d; i++)

leftrotateOne (arr, n);

arr[] = {1, 2, 3, 4, 5}

d=2

$i=0$
arr[] = {2, 3, 4, 5, 1}

$i=1$
arr[] = {3, 4, 5, 1, 2}

(0) 0 = 0.1

(1) 0 = 0.1

method - 2 (Better):

I/P: arr[] = {1, 2, 3, 4, 5}

d=2

O/P: arr[] = {3, 4, 5, 1, 2}

$T.C = \Theta(d + n - d + d)$
 $= \Theta(n + d)$
 $= \Theta(n)$
 $A.S = \Theta(d)$

Void leftrotate (int arr[], int n, int d)

{ int temp[d];

for (int i=0; i<d; i++)

temp[i] = arr[i];

for (int i=d; i<n; i++)

arr[i-d] = arr[i];

for (int i=0; i<d; i++)

arr[n-d+i] = temp[i];

temp[] = {1, 2}

arr[] = {3, 4, 5, 4, 5}

arr[] = {3, 4, 5, 1, 2}

method - 3 (Reversal algorithm):

I/P: arr[] = {1, 2, 3, 4, 5}

d=2

O/P: arr[] = {3, 4, 5, 1, 2}

$T.C = \Theta(d + n - d + n)$
 $= \Theta(2n) = \Theta(n)$

A.S = $\Theta(1)$

Void leftrotate (int arr[], int n, int d)

{ reverse (arr, 0, d-1);

reverse (arr, d, n-1);

reverse (arr, 0, n-1);

Void reverse (int arr[], int low, int high)

{ while (low < high)

{ swap (arr[low], arr[high]);

low++;

high--;

arr[] = {1, 2, 3, 4, 5}

After 1st Reverse

arr[] = {2, 1, 3, 4, 5}

After 2nd Reverse

arr[] = {2, 1, 5, 4, 3}

After 3rd Reverse

arr[] = {3, 4, 5, 1, 2}



Leaders in an array [Leader has to be greater than all elements on right side and equals one root element]

I/P: arr[] = {7, 10, 4, 8, 6, 5, 2} {7, 10, 4, 10, 6, 5, 2}

O/P: 10 6 5 2

I/P: arr[] = {10, 20, 30} {10, 20, 30}

O/P: 30

I/P: arr[] = {30, 20, 10}

O/P: 30, 20, 10

Naive Solution $\Rightarrow O(n^2)$

```

void Leaders(int arr[], int n)
{
    for (int i=0; i<n; i++)
    {
        bool flag = false;
        for (int j=i+1; j<n; j++)
        {
            if (arr[i] < arr[j])
                flag = true;
            break;
        }
        if (flag == false)
            cout << arr[i];
    }
}

```

Efficient Solution:

I/P: arr[] = {7, 10, 4, 10, 6, 5, 2}

O/P: 2 5 6 10 (Right to left O/P getting)

```

void Leader (int arr[], int n)
{
    int curr_ldr = arr[n-1];
    cout << curr_ldr;
    for (i=n-2; i>=0; i--)
    {
        if (curr_ldr < arr[i])
            curr_ldr = arr[i];
        cout << curr_ldr;
    }
}

```

Maximum Difference:

maximum value of $arr[j] - arr[i]$ such that $j \geq i$

I/P: arr[] = {2, 3, 10, 6, 4, 8, 1}

O/P: 8

I/P: arr[] = {7, 9, 5, 6, 3, 2}

O/P: 2

I/P: arr[] = {10, 20, 30}

O/P: 20

I/P: arr[] = {20, 10, 8, 2}

O/P: -2 (Error)

$$8 = (20 - 12) \text{ max - min} = 8$$

$$8 = (20 - 18) \text{ max - min} = 2$$

$$8 = (20 - 8) \text{ max - min} = 12$$

$$8 = (20 - 2) \text{ max - min} = 18$$



Naive:

$$\begin{array}{|c|} \hline T.C = O(n^2) \\ A.S = O(1) \\ \hline \end{array}$$

IP: arr[] = {2, 3, 10, 6, 4, 8, 13}

OP: 8

int maxDiff (int arr[], int n)

$$\{ \text{int res} = arr[1] - arr[0];$$

for (int i=0; i<n; i++)

for (int j=i+1; j<n; j++)

$$res = \max(res, arr[j] - arr[i]);$$

}

return res;

}

Efficient:

$$\begin{array}{|c|} \hline T.C = O(n) \\ A.S = O(1) \\ \hline \end{array}$$

int maxDiff (int arr[], int n)

$$\{ \text{int res} = arr[1] - arr[0], minVal = arr[0];$$

for (int j=1; j<n; j++)

$$\{ \text{res} = \max(res, arr[j] - minVal);$$

$$minVal = \min(minVal, arr[j]);$$

}

return res;

}

arr[] = {2, 3, 10, 6, 4, 8, 13}

$$res = arr[1] - arr[0] = 1$$

$$minVal = arr[0] = 2$$

$$j=1: res = \max(1, 3-2) = 1$$

$$minVal = \min(2, 3) = 2$$

$$j=2: res = \max(1, 10-2) = 8$$

$$minVal = \min(2, 10) = 2$$

$$j=3: res = \max(8, 6-2) = 8$$

$$minVal = \min(2, 6) = 2$$

Frequencies in a Sorted Array:

IP: arr[] = {10, 10, 10, 25, 30, 30}

OP: 10 3
25 1
30 2

IP: arr[] = {10, 10, 10, 10}

OP: 10 4

IP: arr[] = {10, 20, 30}

OP: 10 1
20 1
30 1

Void printFreq (int arr[], int n)

{ int freq=1, i=1; // pseudo code

while (i<n), will be stopped at some time

{ while (i<n) & arr[i] == arr[i-1])

{ freq++;

i++;

}

print(arr[i-1] + " " + freq);

i++;

freq=1;

}

{ (n==1 || arr[n-1] != arr[n-2])

print(arr[n-1] + " " + 1);

}

$$T.C = O(n)$$



Scanned with OKEN Scanner

$arr[] = \{10, 10, 10, 30, 30, 40\}$

Initially: freq = 1

i=1 : freq = 2

i=2 : freq = 3

i=3 : print(10, 3), freq = 1 $\Rightarrow 10^3$

i=4 : freq = 2

i=5 : print(30, 2), freq = 1 $\Rightarrow 30^2$

After loop:

print(40, 1)

$\Rightarrow arr[] = \{40, 50, 50, 50\}$

i=1 : print(40, 1), freq = 1

i=2 : freq = 2

i=3 : freq = 3

i=4 : print(50, 3), freq = 1

(freq in Unsorted Array?)

Stock Buy and Sell (Naive Solution)

B S

Ip: $arr[] = [1, 5, 3, 8, 12]$

B ↑ S ↑

+ (-3) + 8 + (-12) + 18

Op: 13

Ip: $arr[] = [30, 20, 10]$

Op: 0

Ip: $arr[] = [10, 20, 30]$

Op: 20

Ip: $arr[] = [1, 5, 3, 1, 2, 8]$

B ↑ S ↑
4 ↑ 7 ↑

Op: 11

s=0 e=n+1

int maxProfit (int price[], int start, int end)

{ if (end <= start)

 return 0;

 int profit=0;

 for (int i=start; i<end; i++)

 { for (int j=i+1; j<=end; j++)

 if (price[j] > price[i])

 int curr-profit = price[j]-price[i] +

 maxProfit (price, start, i-1) +

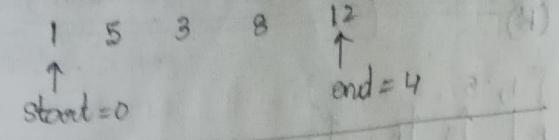
 maxProfit (price, j+1, end);

 profit = max (profit, curr-profit);

 } }

 return profit;





i=0 j=1 $\text{curr_profit} = (5-1) + \text{maxProfit}(\text{arr}, 0, -1) + \text{maxProfit}(\text{arr}, 2, 4)$
 $= 4 + 0 + 9$
 $= 13$ ✓

i=0 j=2 $\text{curr_profit} = (3-1) + \text{maxProfit}(\text{arr}, 0, -1) + \text{maxProfit}(\text{arr}, 3, 4)$
 $= 2 + 0 + 4$
 $= 6$

D.P.: $\text{arr}[] = [1, 5, 3, 8, 12]$

D.P.: 13

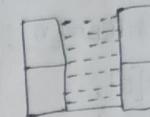
`int maxProfit(int price[], int n){
 int profit = 0;
 for (int i=1; i<n; i++)
 if (price[i] > price[i-1])
 profit += (price[i] - price[i-1]);
 return profit;`

1	5	3	8	12
Profit = 0				
i=1 : profit = 0 + (5-1) = 4				
i=2 :				
i=3 : profit = 4 + (8-3) = 9				
i=4 : profit = 9 + (12-8) = 13				

Trapping Rain water

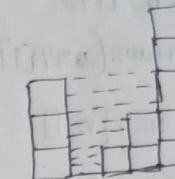
D.P.: $\text{arr}[] = [2, 0, 2]$

D.P.: 2



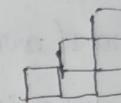
D.P.: $\text{arr}[] = [3, 0, 1, 2, 5]$

D.P.: 6



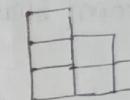
D.P.: $\text{arr}[] = [1, 2, 3]$

D.P.: 0



D.P.: $\text{arr}[] = [3, 2, 1]$

D.P.: 6



Naive Solution:

`int getWater (int arr[], int n)`

{ int res = 0;

 for (int i=1; i<n-1; i++)

 int lMax = arr[i];

 for (int j=0; j<i; j++)

 lMax = max(lMax, arr[j]);

 int rMax = arr[i];

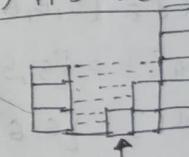
 for (int j=i+1; j<n; j++)

 rMax = max(rMax, arr[j]);

 res = res + (min(lMax, rMax) - arr[i]);

 return res;

$T.C = \Theta(n^2)$ A.S = $\Theta(1)$



LMax = 3

rMax = 5

$\min(LMax, rMax) = 3$

- arr[2]

int getWaves(int arr[], int n)

{ int res = 0;

int lmax[n], rmax[n];

lmax[0] = arr[0];

for (int i=1; i<n; i++)

rmax[i] = max(arr[i], lmax[i-1])

rmax[n-1] = arr[n-1];

for (int i=n-2; i>=0; i--)

rmax[i] = max(arr[i], rmax[i+1]);

for (int i=1; i<n-1; i++)

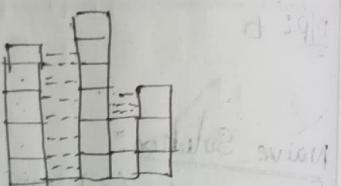
res = res + (min(lmax[i], rmax[i]) - arr[i]);

return res;

arr[] = [5, 0, 6, 2, 3]

lmax[] = [5, 5, 6, 6, 6]

rmax[] = [6, 6, 6, 3, 3]



$$res = \min(5, 5) - 0 + \min(5, 6) - 0 + \min(6, 6) - 6$$

Efficient

TC = $\Theta(n)$

AS = $\Theta(n)$

Maximum Consecutive 1's

I/P: arr[] = [0, 1, 1, 0, 1, 0]

O/P: 2

I/P: arr[] = [1, 1, 1, 1]

O/P: 4

I/P: arr[] = [0, 0, 0]

O/P: 0

I/P: arr[] = [1, 0, 1, 1, 1, 0, 1, 1]

O/P: 4

Naive Solution :-

TC = $O(n^2)$
AS = $O(1)$

int maxConsecutiveOnes(bool arr[], int n)

{ int res = 0;

for (int i=0; i<n; i++)

{ int curr = 0;

for (int j=0; j<n; j++)

{ if (arr[j] == 1) curr++;

else break;

} res = max(res, curr);

} return res;

arr[] = [0, 1, 1, 1, 0, 1, 1]

i=0, curr=0

i=1, curr=1, res=1

i=2, curr=2

i=3, curr=1

i=4, curr=0

i=5, curr=2

i=6, curr=1



Efficient Solution:

T.C = $\Theta(n)$
A.S = O(1)

int maxConsecutiveOnes (int arr[], int n)

```

{ int res = 0;
  for (int i=0; i<n; i++)
  {
    if (arr[i] == 0)
      curri = 0;
    else
    {
      curri++;
      res = max(res, curri);
    }
  }
  return res;
}
  
```

arr[] = [0, 1, 1, 0, 1, 1, 1]

i=0, curri=0

i=1, curri=1, res=1

i=2, curri=2, res=2

i=3, curri=0, res=2

i=4, curri=1, res=2

i=5, curri=2, res=2

i=6, curri=3, res=3

22. Maximum Subarray Sum

what is a Subarray?

Subarrays of [1, 2, 3] are [1], [2], [3], [1, 2], [2, 3], [1, 2, 3]

I.P: arr[] = [2, 3, -8, 7, 1, 2, 3]

O.P: 11

I.P: arr[] = [5, 8, 3]

O.P: 16

I.P: arr[] = [-6, -1, -8]

O.P: -1

T.C:	$O(n^2)$
A.S:	$O(1)$

int maxSum (int arr[], int n)

```

{ int res = arr[0];
  for (int i=0; i<n; i++)
  {
    int curri = 0;
    for (int j=i; j<n; j++)
    {
      curri = curri + arr[j];
      res = max(res, curri);
    }
  }
  return res;
}
  
```

[1, -2, 3, -1, 2]

res=1

i=0:

curri=0

curri=1

curri=-1 = -1

curri=-1+3 = 2, res=2

curri=2 = 1

curri=1+2 = 3, res=3

i=1:

curri=0

curri=-2

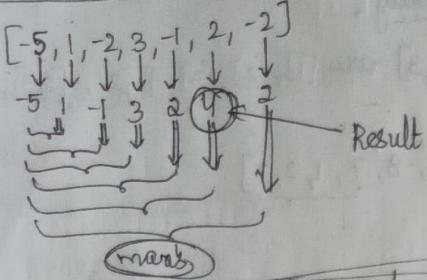
curri=-2+3 = 1

curri=1-1 = 0

curri=0+2 = 2



Efficient Solution: $T.C = O(n)$ $A.S = O(1)$



$$\text{maxEnding}(i) = \max(\text{maxEnding}(i-1) + \text{arr}[i], \text{arr}[i])$$

\Rightarrow int maxEvenSum (int arr[], int n)

```

    {
        int res = arr[0];
        int maxEnding = arr[0];
        for (int i=1; i<n; i++)
        {
            maxEnding = max(maxEnding + arr[i], arr[i]);
            res = max(res, maxEnding);
        }
        return res;
    }
  
```

$[-3, 8, -2, 4, -5, 6]$

$res = -3$, $\text{maxEnding} = -3$

$i=1$: $\text{maxEnding} = \max(-3+8, 8) = 8$

$res = 8$

$i=2$: $\text{maxEnding} = \max(8-2, -2) = 6$

$i=3$: $\text{maxEnding} = \max(6+4, 4) = 10$

$res = 10$

$i=4$: $\text{maxEnding} = \max(10-5, -5) = 5$

$i=5$: $\text{maxEnding} = \max(5+6, 6) = 11$

$res = 11$

23. Longest Even Odd Subarray

D.P.: $[10, 12, 14, 7, 8]$

O.P.: 3

D.P.: $[7, 10, 13, 14]$

D.P.: 4

D.P.: $[10, 12, 8, 4]$

D.P.: 1

Naive Solution:

$T.C = O(n^2)$
 $A.S = O(1)$

~~int maxEvenOdd (int arr[], int n)~~

~~int res = 1;~~

~~for (int i=0; i<n; i++)~~

~~{ int curr=1;~~

~~for (int j=i+1; j<n; j++)~~

~~{ if ((curr%2==0) && (arr[j-1]%2!=0) || (arr[j]%2==0))~~

~~&& arr[j-1]%2==0)~~

~~curr++;~~

~~else~~

~~break;~~

~~}~~

~~res = max(res, curr);~~

~~return res;~~

~~}~~

$arr[] = [5, 10, 20, 6, 3, 8]$

$res = 1$

$i=0$: $curr = 2$, $res = 2$

$i=1$: $curr = 1$

$i=2$: $curr = 1$

$i=3$: $curr = 3$, $res = 3$

$i=4$: $curr = 2$

$i=5$: $curr = 1$



Efficient Solution: (Explains algorithm)

$$T.C = O(n)$$

$$A.S = O(1)$$

int maxEvenOdd (int arr[], int n)

```

{ int res=1;
  int curr=1;
  for (i=1; i<n; i++)
  {
    if ((arr[i] % 2 == 0 && arr[i-1] % 2 != 0) ||
        (arr[i] % 2 != 0 && arr[i-1] % 2 == 0))
    {
      curr++;
      res = max (curr, res);
    }
    else
      curr=1;
  }
  return res;
}
  
```

$$\text{arr}[] = [5, 10, 20, 6, 3, 8]$$

$$i=1 : \text{curr}=2, \text{res}=2$$

$$i=2 : \text{curr}=1$$

$$i=3 : \text{curr}=1$$

$$i=4 : \text{curr}=2$$

$$i=5 : \text{curr}=3, \text{res}=3$$

24: Maximum Circular Subarray Sum

[10, 5, -5]

All circular subarrays are
 Normal subarrays [10] [5] [-5] $\boxed{[10, 5]}$ $\boxed{[5, -5]}$ $\boxed{[-5, 10]}$ [10, 5, -5]
 Only circular subarrays [5, -5, 10] [-5, 10] [-5, 10, 5]

$$\Rightarrow \underline{\text{IP:}} \text{ arr}[] = [5, -2, 3, 4] \Rightarrow \underline{\text{IP:}} \text{ arr}[] = [8, \underline{7}, \underline{6}]$$

$$\underline{\text{OP:}} 12 \quad \underline{\text{OP:}} 13$$

$$\Rightarrow \underline{\text{IP:}} \text{ arr}[] = [2, \underline{3}, -4] \Rightarrow \underline{\text{IP:}} \text{ arr}[] = [3, \underline{4}, \underline{5}, 6, -8, \underline{7}]$$

$$\underline{\text{OP:}} 5 \quad \underline{\text{OP:}} 17$$

$$\Rightarrow \underline{\text{IP:}} \text{ arr}[] = [8, -4, 3, -5, \underline{4}] \Rightarrow \underline{\text{IP:}} \text{ arr}[] = [3, \underline{4}, \underline{6}, -2]$$

$$\underline{\text{OP:}} 12 \quad \underline{\text{OP:}} 10$$

Naive Solution:

$$T.C = O(n^2) \quad A.S = \Theta(1)$$

int maxCircularSum (int arr[], int n)

{ int res = arr[0];

for (int i=0; i<n; i++)

{ int curr_max = arr[i];

int curr_sum = arr[i];

for (int j=0; j<n; j++)

{ int index = (i+j)%n;

curr_sum = arr[index];

curr_max = max (curr_max, curr_sum);

res = max (res, curr_max);

return res;



$\text{arr}[] = [5, -2, 3, 4]$ [for every element, n Subarrays
 are there including current]

$i=0 : \text{arr_max} = 10, \text{res} = 10$
 $i=1 : \text{arr_max} = 10$
 $i=2 : \text{arr_max} = 12, \text{res} = 12$
 $i=3 : \text{arr_max} = 10$

Efficient Solution: O(n)

Idea: take the maximum of the following two

① maximum sum of a Normal Subarray
 (Easy: Kadane)

② maximum sum of a circular Subarray
 also ~~modified~~ (How to find this?)

$$[a_0, a_1, \dots, a_i, a_{i+1}, \dots, a_j, a_{j+1}, \dots, a_{n-1}]$$

circular
 (connecting last element
 with the first element)

$$\Rightarrow \text{arr}[] = [5, -2, 3, 4]$$

Ideas:
 maximum sum of a circular Subarray = sum of all the elements of array - minimum of Normal Subarray (Modified Kadane)

$$\Rightarrow \text{arr}[] = [8, -4, 3, -5, 4]$$

$$\Rightarrow \text{arr}[] = [3, -4, 5, 6, -8, 7]$$

standardized

```

int normalMaxSum(int arr[], int n)
{
  int res = arr[0], maxEnding = arr[0];
  for (int i=1; i<n; i++)
  {
    maxEnding = max(maxEnding + arr[i], arr[i]);
    res = max(res, maxEnding);
  }
  return res;
}
  
```

$\text{int overallMaxSum(int arr[], int n)}$
 $\{ \text{int max_normal} = \text{normalMaxSum}(arr, n); \Rightarrow \text{Normal sum}$
 $\text{if (max_normal < 0)}$
 $\text{return max_Normal;}$

```

int overallSum = 0;
for (int i=0; i<n; i++)  $\Rightarrow$  circular sum
{
  overallSum += arr[i];
  arr[i] = -arr[i];
}
int max_circular = overallSum + maxNormal(arr, n);
return max(max_normal, max_circular);
  
```

$\text{arr}[] = [8, -4, 3, -5, 4]$
 $\text{max_normal} = 8$
 $\text{arr_sum} = 6$
After inversion:
 $\text{arr}[] = [-8, 4, -3, 5, -4]$
 $\text{max_circular} = 6 + 6 = 12$
 $\text{res} = \max(\text{max_normal}, \text{max_circular})$
 $= \max(8, 12)$
 $= 12$



$$\text{arr}[] = [-5, -3]$$

$$\text{max-normal} = -3$$

$$\begin{matrix} -5 & -3 \\ \downarrow & \downarrow \\ -5 & -3 \end{matrix}$$

If we remove the condition (if $\text{max-normal} < 0$)

$$\text{arr-Sum} = -11$$

$$\text{arr}[] = [8, 3]$$

$$\text{max.circular} = -11 + 8$$

$$= 0$$

$$\text{res} = \text{max}(3, 0)$$

$$= 0$$

it returns 0, so that's why condition is wrong

$$T.C = \Theta(n+n+n) = \Theta(3n) = \Theta(n)$$

25. Majority Element

↳ elements appear in the array more than $n/2$ times

⇒ IP: $\text{arr}[] = [8, 3, 4, 8, 8]$

dp: 0 OR 3 OR 4 (any index of 8)

⇒ IP: $\text{arr}[] = [3, 7, 4, 7, 7, 5]$

dp: -1 (no majority)

⇒ IP: $\text{arr}[] = [20, 20, 40, 30, 50, 50, 50]$

dp: 3 OR 4 OR 5 OR 6 (any index of 50)

Noise: $T.C = O(n^2)$
 $A.S = \Theta(1)$

int findMajority (int arr[], int n)

```
{ for (int i=0; i<n; i++)
    {
        int count=1;
        for (int j=i+1; j<n; j++)
        {
            if (arr[i] == arr[j])
                count++;
            if (count > n/2)
                return i;
        }
    }
    return -1;
}
```

$\text{arr}[] = [8, 7, 6, 8, 6, 6, 6, 6]$
 $n=8 \quad n/2=8/2=4$
 $i=0 \quad \text{count}=2$
 $i=1 \quad \text{count}=1$
 $i=2 \quad \text{count}=5$

dp: 2



$$\text{efficiency: } \begin{cases} T.C = \Theta(n) \\ A.S = O(1) \end{cases}$$

(moore's Voting Algorithm)

int findMajority (int arr[], int n)

```

    int res=0, count=1;
    for (int i=1; i<n; i++) // find a
        if (arr[i] == arr[res]) // candidate
            count++;
        else
            count--;
        if (count == 0) {res=i, count=1}
    }
}

```

```

    count=0;
    for (int i=0; i<n; i++)
        if (arr[res] == arr[i]) // check if the
            count++; // candidate is
        if (count <= n/2) // actually a
            res=-1; // majority
    return res;
}

```

$$arr[] = [8, 8, 6, 6, 6, 4, 6]$$

Initially: count=1, res=0

i=1 : count=2

i=2 : count=1

i=3 : count=0

Count=1, res=3

i=4 : Count=2

i=5 : Count=1

i=6 : Count=2

$$arr[] = [6, 8, 4, 6, 8]$$

Initially : count=1, res=0

i=1 : count=0

Count=1, res=1

i=2 : Count=0

Count=1, res=2

i=3 : Count=0

Count=1, res=3

i=4 : Count=2

Working of algorithm

Traversing L → R & maintaining count & majority Var's

whole traversing
some elements increment count,
some elements decrement count,
→ elements decrement the
count will cancel the votes
of others

To show majority: dividing elements into 2 sets
cancel each other

ex-1: 6, 8, 7, 6, 6, 6
✓ X ✓ X ✓ X ✓
increment count decrement count
not cancel each other

ex-2: 8, 8, 6, 6, 6, 6
✓ X X ✓ X ✓
cancel each other

8, 8
6, 6

To prove its not a majority
set contains pairs of elements
& every set contain diff pairs

n (8, 8, 6, 6, 6, 6)
no element can appear more than $\frac{n}{2}$ times



26. Minimum Consecutive Flips

Minimum Group Flips to Make Same

I/P: arr[] = [1, 1, 0, 0, 0, 1]

O/P: from 2 to 4

I/P: arr[] = [1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 1]

O/P: from 1 to 3

from 5 to 6

I/P: arr[] = [1, 1, 1]

O/P:

I/P: arr[] = [0, 1]

O/P: from 0 to 0

from 1 to 1

~~Same~~ ~~TC O(n^2)~~

Group Counts Differ by One:

11 000 111 0 0 1 \Rightarrow 1s (group count) = 3, 3 - 2 = 1
 0 0 11000 1100 1 \Rightarrow 0s (group count) = 2, 2 - 1 = 1
 1 1 1 1
 0 0 0 0

Group Counts are Same:

0 0 1110000 1 1
 1 1 000011110

Only Two Possibilities:

Idea:
 If array 1st element & last element are same \Rightarrow then Group Count differ by 1
 " " " " " different \Rightarrow differ by same

Always flip second group elements

Efficient Solution:

$$\begin{cases} TC = O(n) \\ AS = O(1) \end{cases}$$

Void printGroups (bool arr[], int n)

{ for (int i=1; i<n; i++)

{ if (arr[i] != arr[i-1])

{ if (arr[i] != arr[0])

cout << "From " << i << " to ";

else

cout << (i-1) << endl;

}

if (arr[n-1] != arr[0])

cout << (n-1) << endl;

}

0 0 1 1 0 0 1 1 0 1

i=1:

i=2: from 2 to

i=3:

i=4: 3 (newline)

i=5:

i=6: from 6 to

i=7:

i=8: 7 (newline)

i=9: from 9 to 9

O/P: From 2 to 3

From 6 to 7

From 9 to 9



27. Window Sliding technique

Given an array of integers and a number k, find the maximum sum of k consecutive elements.

$$\text{D.P.: arr}[] = [1, 8, 30, -5, 20, 7] \quad k=3$$

$$\text{D.P.: } 45$$

$$\text{D.P.: arr}[] = [-5, 10, 6, 9, 2, 3] \quad k=2$$

$$\text{D.P.: } 96$$

Naive:

```
int maxSum = INT_MIN; => -∞
for (int i=0; i+k-1 < n; i++)
{
    int sum = 0;
    for (int j=0; j < k; j++)
        sum += arr[i+j];
    maxSum = max(sum, maxSum);
}
return maxSum;
```

$$T.C = O(n \cdot k) \times k$$

$$\text{Imagine } \Rightarrow k = \frac{n}{2}$$

$$O\left(\frac{(n-n)}{2} \cdot \frac{n}{2}\right)$$

$$O\left(\frac{n^2}{4}\right) \Rightarrow O(n^2)$$

$T.C = O(n^2)$

$$\text{D.P.: arr}[] = [1, 8, 30, -5, 20, 7] \quad k=3$$

$39 = 38 + (-5) = 33$

D.P.: yes

$$[10, 5, -2, 20, 1]$$

$$k=3$$

$$i=0, \text{curr} = 13, \text{res} = 13$$

$$i=1, \text{curr} = 23, \text{res} = 23$$

$$i=2, \text{curr} = 19$$

Sliding Window:

Compute the Sum of current window Using sum of previous window in $O(1)$ Time

$$[1 | 8 | 30 | -5 | 20 | 7] \quad k=4$$

Initially: curr = 1+8+30-5 = 34, res = 34

1st slide: curr = 34+20-1 = 53, res = 53

Adding last of current window
Removing first of previous window.

$$[1 | 8 | 30 | -5 | 20 | 7]$$

2nd slide: curr = 53+7-8 = 52

Adding last of current window
Removing first of previous window.

$$[1 | 8 | 30 | -5 | 20 | 7]$$



int maxksum (int arr[], int n, int k)

```
{  
    int curr = 0;  
    for (int i=0; i<k; i++)  
        curr += arr[i];  
    int res = curr;  
    for (int i=k; i<n; i++)  
    {  
        curr = curr - arr[i-k];  
        res = max(res, curr);  
    }  
    return res;  
}
```

[1, 8, 30, -5, 20, 7]

K=4

After 1st loop: curr = 34
res = 34

2nd loop 1st iteration i=4

$$\begin{aligned} curr &= 34 + 20 - 1 \\ &= 53 \end{aligned}$$

res = 53

2nd Iteration i=5
curr = 53 + 7 - 8
= 52

$$\boxed{T.C = O(n)} \\ A.S = O(1)}$$

Subarray with Given Sum : {there are two negative elements in the array}

IP: [1, 4, 20, 3, 10, 5]
Sum = 23

OP: Yes

IP: [1, 4, 0, 0, 3, 10, 5]
Sum = 7

OP: Yes

IP: [2, 4]. Sum = 3

OP: No

Naive Solution:

$$\boxed{T.C = O(n^2)} \\ A.S = O(1)}$$

function isSubSum (int arr[], int n, int sum)

```
{  
    for (i=0; i<n; i++)
```

```
{  
    int curr = 0;
```

for (j=i; j<n; j++)

i=0; j=0; curr = 0

curr += arr[j];

j=1; curr = 5

j=2; curr = 10

j=3; curr = 15

j=4; curr = 20

j=5; curr = 25

j=6; curr = 30

j=7; curr = 35

j=8; curr = 40

j=9; curr = 45

j=10; curr = 50

j=11; curr = 55

j=12; curr = 60

j=13; curr = 65

j=14; curr = 70

j=15; curr = 75

j=16; curr = 80

j=17; curr = 85

j=18; curr = 90

j=19; curr = 95

j=20; curr = 100

j=21; curr = 105

j=22; curr = 110

j=23; curr = 115

j=24; curr = 120

j=25; curr = 125

j=26; curr = 130

j=27; curr = 135

j=28; curr = 140

j=29; curr = 145

j=30; curr = 150

j=31; curr = 155

j=32; curr = 160

j=33; curr = 165

j=34; curr = 170

j=35; curr = 175

j=36; curr = 180

j=37; curr = 185

j=38; curr = 190

j=39; curr = 195

j=40; curr = 200

j=41; curr = 205

j=42; curr = 210

j=43; curr = 215

j=44; curr = 220

j=45; curr = 225

j=46; curr = 230

j=47; curr = 235

j=48; curr = 240

j=49; curr = 245

j=50; curr = 250

j=51; curr = 255

j=52; curr = 260

j=53; curr = 265

j=54; curr = 270

j=55; curr = 275

j=56; curr = 280

j=57; curr = 285

j=58; curr = 290

j=59; curr = 295

j=60; curr = 300

j=61; curr = 305

j=62; curr = 310

j=63; curr = 315

j=64; curr = 320

j=65; curr = 325

j=66; curr = 330

j=67; curr = 335

j=68; curr = 340

j=69; curr = 345

j=70; curr = 350

j=71; curr = 355

j=72; curr = 360

j=73; curr = 365

j=74; curr = 370

j=75; curr = 375

j=76; curr = 380

j=77; curr = 385

j=78; curr = 390

j=79; curr = 395

j=80; curr = 400

j=81; curr = 405

j=82; curr = 410

j=83; curr = 415

j=84; curr = 420

j=85; curr = 425

j=86; curr = 430

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j=106; curr = 530

j=107; curr = 535

j=108; curr = 540

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j=111; curr = 555

j=112; curr = 560

j=113; curr = 565

j=114; curr = 570

j=115; curr = 575

j=116; curr = 580

j=117; curr = 585

j=118; curr = 590

j=119; curr = 595

j=120; curr = 600

j=121; curr = 605

j=122; curr = 610

j=123; curr = 615

j=124; curr = 620

j=125; curr = 625

j=126; curr = 630

j=127; curr = 635

j=128; curr = 640

j=129; curr = 645

j=130; curr = 650

j=131; curr = 655

j=132; curr = 660

j=133; curr = 665

j=134; curr = 670

j=135; curr = 675

j=136; curr = 680

j=137; curr = 685

j=138; curr = 690

j=139; curr = 695

j=140; curr = 700

j=141; curr = 705

j=142; curr = 710

j=143; curr = 715

j=144; curr = 720

j=145; curr = 725

j=146; curr = 730

j=147; curr = 735

j=148; curr = 740

j=149; curr = 745

j=150; curr = 750

j=151; curr = 755

j=152; curr = 760

j=153; curr = 765

j=154; curr = 770

j=155; curr = 775

j=156; curr = 780

j=157; curr = 785

j=158; curr = 790

j=159; curr = 795

j=160; curr = 800

j=161; curr = 805

j=162; curr = 810

j=163; curr = 815

j=164; curr = 820

j=165; curr = 825

j=166; curr = 830

j=167; curr = 835

j=168; curr = 840

j=169; curr = 845

j=170; curr = 850

j=171; curr = 855

j=172; curr = 860

j=173; curr = 865

j=174; curr = 870

j=175; curr = 875

j=176; curr = 880

j=177; curr = 885

j=178; curr = 890

j=179; curr = 895

j=180; curr = 900

j=181; curr = 905

j=182; curr = 910

j=183; curr = 915

j=184; curr = 920

j=185; curr = 925

j=186; curr = 930

j=187; curr = 935

j=188; curr = 940

j=189; curr = 945

j=190; curr = 950

j=191; curr = 955

j=192; curr = 960

j=193; curr = 965

j=194; curr = 970

j=195; curr = 975

j=196; curr = 980

j=197; curr = 985

j=198; curr = 990

j=199; curr = 995

j=200; curr = 1000

j=201; curr = 1005

j=202; curr = 1010

j=203; curr = 1015

j=204; curr = 1020

j=205; curr = 1025

j=206; curr = 1030

j=207; curr = 1035

j=208; curr = 1040

j=209; curr = 1045

j=210; curr = 1050

j=211; curr = 1055

j=212; curr = 1060

j=213; curr = 1065

j=214; curr = 1070

j=215; curr = 1075

j=216; curr = 1080

j=217; curr = 1085

j=218; curr = 1090

Idea for efficient solution:

We use window sliding technique with a window of variable size.

$$[1, 4, 2, 3, 10, 5]$$

$$\text{Sum} = 33$$

$$S=0, e=0, \text{curr}=1$$

$$S=0, e=1, \text{curr}=5$$

$$S=0, e=2, \text{curr}=25$$

$$S=0, e=3, \text{curr}=28$$

$$S=0, e=4, \text{curr}=38$$

$$S=1, e=4, \text{curr}=87$$

$$S=2, e=4, \text{curr}=83$$

while curr is smaller than sum, extend the window by increasing e

$$\begin{cases} T.C = O(n) \\ A.S = O(1) \end{cases}$$

Efficient Solution:

function isSubSum(arr, sum)

{ int s=0, curr=0;

for (int i=0; i<arr.length; i++)

{ curr+=arr[i];

while (sum < curr)

{ curr-=arr[g];

g++;

Removing & adding
an item takes
O(1) time &
every element
is removed & added

if (curr == sum)
return true;

return false;

$$\text{sum} = 17$$

$$S=0, e=0, \text{curr}=4$$

$$e=1, \text{curr}=12$$

$$e=2, \text{curr}=24$$

$$S=1, e=2, \text{curr}=20$$

$$S=2, e=2, \text{curr}=12$$

$$S=2, e=3, \text{curr}=17$$

return true;

Prefix Sum:

Given a fixed array and multiple range sum queries, how to answer the queries efficiently.

$$\text{IP: arr} = [2, 8, 3, 9, 6, 5, 4]$$

Queries: getSum(0, 2)

getSum(1, 3)

getSum(2, 6)

OP: 13 20 27

Naive Solution:

$$\begin{cases} T.C = \Theta(n-l+1) \\ \text{C.O.R} \\ O(n) \\ A.S = O(1) \end{cases}$$

int getSum (int l, int n)

{ int res=0;

for (int i=l; i<=n; i++)

res+=arr[i];

return res;

$$\text{arr} = [2, 8, 3, 9, 6, 5, 4]$$

getSum(1, 3);

Initially: res=0

i=1: res=8

i=2: res=11

i=3: res=20

Idea for getSum() in O(1) Time

$$\text{arr} = [2, 8, 3, 9, 6, 5, 4]$$

① precompute prefixSum Array

$$\text{pSum} = [2, 10, 13, 22, 28, 33, 37]$$

$$\text{pSum}[i] = \sum_{j=0}^i \text{arr}[j]$$

② $\text{getSum}(l, n) = \{\text{pSum}[n] \text{ if } l=0\}$

$$\{\text{pSum}[n]-\text{pSum}[l-1] \text{ otherwise}\}$$

$$\begin{aligned} \text{getSum}(0, 2) &= \text{pSum}[2] \\ &= 13 \end{aligned}$$

$$\begin{aligned} \text{getSum}(1, 3) &= \text{pSum}[3]-\text{pSum}[0] \\ &= 22-2 = 20 \end{aligned}$$

$$\begin{aligned} \text{getSum}(2, 6) &= \text{pSum}[6]-\text{pSum}[1] \\ &= 37-10 = 27 \end{aligned}$$



```

int ps[n];
preprocessing
ps[0] = arr[0];
for (int i=1; i<n; i++)
    ps[i] = ps[i-1] + arr[i];
int getSum(int l, int r)
{
    if (l==0) // O(1) time
        return ps[r];
    return ps[r] - ps[l-1];
}

```

$$\begin{aligned}
arr &= [2, 3, 5, 4, 6, 1] \\
ps[0] &= arr[0] \\
ps[1] &= ps[0] + arr[1] \\
&= 2+3 \\
&= 10 \\
ps[6] &= ps[5] + arr[6] \\
&= 8+4 \\
&= 12 \\
getSum(2, 4) &= ps[4] - ps[1] \\
&= 12 - 10 \\
&= 2
\end{aligned}$$

Exercise problems:

- ① How to answer the below weighted sum queries efficiently.

$$arr = [2, 3, 5, 4, 6, 1]$$

$$\text{getwSum}(0, 2) = 23$$

$$[1*arr[0] + 2*arr[1] + 3*arr[2]]$$

$$\text{getwSum}(2, 3) = 13$$

$$[1*arr[2] + 2*arr[3]]$$

$$\begin{aligned}
\text{getwSum}(l, r) &= 1*arr[l] + 2*arr[l+1] + \dots + (r-l+1)*arr[r] \\
&= \sum_{i=l}^{r-1} (i-l+1)*arr[i]
\end{aligned}$$

$$\begin{aligned}
\text{getwSum}(l, r) &= \sum_{i=l}^{r-1} (i-l+1)*arr[i] \\
&= \sum_{i=l}^{r-1} i*arr[i] - (l-1) \sum_{i=1}^{r-1} arr[i] \rightarrow \text{psum}[i]
\end{aligned}$$

We need to build two prefix sum arrays

$$\text{pwsum}[i] = i*arr[i] + \text{pwsum}[i-1]$$

Equilibrium point:

$$\text{IP: } arr = [3, 4, 8, -9, 20, 6]$$

OP: True

$$\text{IP: } arr = [4, 2, -2]$$

OP: True

$$\text{IP: } arr = [4, 2, 2]$$

OP: False

Naive solution :

$T.C = O(n^2)$
$AS = O(1)$

bool epoint (int arr[], int n)

{ for (int i=0; i<n; i++)
 { int ls=0, rs=0;
 for (int j=0; j<i; j++)
 ls += arr[j];
 for (int k=i+1;
 k=n; k++)
 rs += arr[k];
 }
}

$$arr[] = [3, 4, 8, -9, 20, 6]$$

$$i=0 : ls=0, rs=20$$

$$i=1 : ls=3, rs=15$$

$$i=2 : ls=8, rs=9$$

return true

$$rs = arr[k];$$

$$if (ls == rs)$$

return true;

}

return false;

}



Basic idea for the efficient solution:

$$\text{arr} = [3, 4, 8, -7, 9, 7]$$

① Compute prefix sum

$$ps = [3, 7, 15, 6, 15, 22]$$

② Compute suffix sum

$$ss = [22, 19, 15, 7, 16, 7]$$

③ for every element (except corner ones),
check if $ps[i-1]$ is same as $ss[i+1]$

Efficient Solution: $\begin{cases} TC = \Theta(n) \\ AS = O(1) \end{cases}$

bool epoint (int arr[], int n)

{ int ns = 0;

for (int i=0; i < n; i++)

$$ns += arr[i];$$

int ls = 0;

for (int i=0; i < n; i++)

{ ~~ns~~ ns -= arr[i];

if (ls == ns)

return true;

$$ls += arr[i];$$

}

return false;

}

Extending
problem:

Given an array, check if it can be partitioned into
three different parts with equal sum.

I/P: arr = [5, 2, 6, 1, 1, 1, 4]

O/P: True

I/P: arr = [3, 2, 5, 1, 1, 5]

O/P: False.



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Sum of odd length / even length Subarrays

arr = [1, 4, 2, 5, 3]

$$\Rightarrow [1], [4], [2], [5], [3], [1, 4, 2], [4, 2, 5], [2, 5, 3], [1, 4, 2, 5, 3]$$

$\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$

$2 + 4 + 2 + 5 + 3 + 7 + 11 + 10 + 15$

Dns = 58

Naive approach:

Consider element included in all odd length subarrays

[1]	[1, 4, 2]	[1, 4, 2, 5, 3]	[4]	[4, 2, 5]	[2]	[2, 5, 3]	[5]	[3]
\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
$2 + 7 + 19 = 23$			$4 + 11 = 15$		$2 + 10 = 12$		5	3

T.C = O(n²)

A.S = O(1)

```

for(i=0; i<n; i++)
    sum = 0
    for(j=i; j<n; j++)
        sumt = arr[j];
        if((j-i)+1 % 2 == 0)
            res += sum;
    return res
  
```

$$23 + 15 + 12 + 5 + 3 = 58$$

[1]	[1, 4, 2]	[1, 4, 2, 5, 3]	i=0: j=0: sum=1, res=1 j=1: sum=5 j=2: sum=7, res=7 j=3: sum=12 j=4: sum=15, res=15	i=1: j=1: sum=1, res=19 j=2: sum=6 j=3: sum=11, res=30 j=4: sum=14	i=2: j=2: sum=2, res=32 j=3: sum=7 j=4: sum=10, res=42
i=3: j=3: sum=5 res=47 j=4: sum=					

Efficient approach:

T.C = O(n)

A.S = O(1)

length	even	odd	Total
[1] $\leftarrow 1$	0	1	1
[1, 4] $\leftarrow 2$	1	1	2
[1, 4, 2] $\leftarrow 3$	1	2	3
[1, 4, 2, 5] $\leftarrow 4$	2	2	4

$\frac{1}{2} = O(\frac{n(n+1)}{2}) = O(n^2)$

even = total/2

odd = (total+1)/2

T \rightarrow left & right
 $\rightarrow (i+1) \times (n-i)$

Sum = Sum + ((i+1) * (n-i) + 1) / 2 * arr[i]

(even) Sum = Sum + ((i+1) * (n-i)) / 2 * arr[i]

Sum = 0
 for(i=0; i<n; i++)

Sum += $\frac{(i+1)(n-i+1)}{2} * arr[i]$

