Liquidity and Investment in General Equilibrium

Nicolas Caramp
UC Davis

Julian Kozlowski
St. Louis Fed

Keisuke Teeple
U Waterloo

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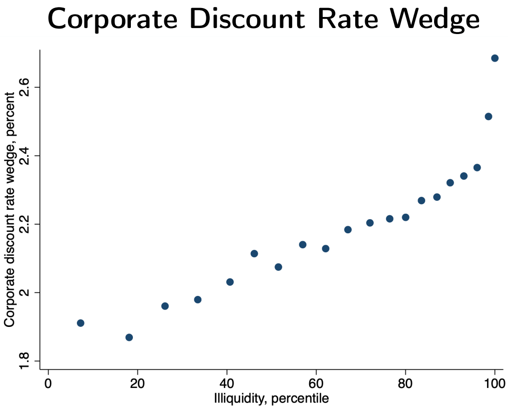
SED, UTDT

What is the SDF in an economy with incomplete markets and illiquid assets?



Discount rate wedge: Gap between discount rate and cost of capital (Gormsen Huber 2024). Relative spreads from CRSP.

Fact: Illiquid firms have higher SDF wedges

This paper:

- Rationalize this fact
- Implication for investment

Liquidity and investment in general equilibrium

Model

- Aiyagari production economy with liquid and illiquid assets in general equilibrium
- Firms take into account that ownership shares trade in frictional asset markets

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Results

1. Theory: the problem of the firm is time inconsistent
   - firms' SDF as if firms have $\beta - \delta$ discounting
   - result from frictions in financial markets

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Results

1. Theory: the problem of the firm is time inconsistent
   - firms' SDF as if firms have $\beta - \delta$ discounting
   - result from frictions in financial markets
2. Quantitative: trading frictions & aggregate distortions
   - Trading frictions have adverse effects on capital without commitment
   - Counterfactual with commitment: trading frictions have little effect on capital
3. Empirics: rationalize facts on the cross-section of liquidity, SDF, and investment

Model

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Aiyagari production economy with liquid and illiquid assets

Households

- idiosyncratic labor risk h
- incomplete markets:
  - liquid bond b, borrowing limit $b \geq \underline{b}$
  - illiquid stock θ, transaction costs $\mathcal{T}$

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Firms

- DRS technology $y = \left(h^{\gamma} k^{1-\gamma}\right)^{\psi}$
- capital accumulation $k_{t+1} = i_t + (1-\delta)k_t \longleftarrow$ firms solve a **dynamic** problem

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Stationary equilibrium

- interest rate r, stock price q, and wage w such that markets clear:

$$\mathbb{E}[b] = 0 \qquad \mathbb{E}[\theta] = 1 \qquad \mathbb{E}[h] = H$$

Household problem

$$V(\theta, b, h) = \max_{c, b', \Delta^+, \Delta^-} u(c) + \beta \mathbb{E}\left[V\left(\theta', b', h'\right)\right]$$

subject to

$$\begin{aligned}
c + b' + q\Delta^+ &\leq wh + b(1+r) + d\theta + q\left(\Delta^- - \mathcal{T}\left(\Delta^-\right)\right) \\
\theta' &= \theta + \Delta^+ - \Delta^- \\
\Delta^- &\leq \theta \;\leftarrow\; \text{short-selling constraint} \\
b' &\geq \underline{b} \;\leftarrow\; \text{borrowing constraint} \\
\mathcal{T}\left(\Delta^-\right) &= \frac{\phi}{2}\left(\Delta^-\right)^2 \;\leftarrow\; \text{Transaction costs for sellers (e.g., Heaton Lucas 96)} \\
\Delta^+, \Delta^- &\geq 0
\end{aligned}$$

Shareholder's valuation

- Let $\tilde{q}(\theta, b, h)$ be the shareholder's valuation in units of the consumption good

$$\tilde{q}(\theta, b, h) \equiv \frac{V_\theta(\theta, b, h)}{u'(c)}$$

where V_θ is the marginal valuation of stocks.

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- **Lemma:** The shareholder's valuation is

$$\tilde{q}(\theta, b, h) = d + \left(1 - \phi\Delta^{-}(\theta, b, h)\right) q$$

- Buyers, $\Delta^{-} = 0$: agree the value of the firm is $\tilde{q}(\theta, b, h) = d + q$
- Sellers: Heterogeneous valuations, depend on marginal transaction cost $\phi\Delta^{-}$

$\rightarrow$ Disagreement among owners on the valuation of the firm

Firm's problem

Assumption 1: Firm maximizes an ownership-weighted valuation:

$$\int_{\theta,b,h} \theta \underbrace{\left[d + (1 - \phi\Delta^{-}(\theta,b,h))q\right]}_{\text{shareholder's valuation}} d\Gamma(\theta,b,h)$$

In spirit of Grossman Hart 1979 (paper also considers Dreze 1974 and DeMarzo 1993).

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Define $\bar{\Phi}$ as the weighted average marginal transaction cost

$$\bar{\Phi} \equiv \phi \int_{\theta,b,h} \theta\Delta^{-}(\theta, b, h) d\Gamma(\theta, b, h)$$

The firm maximizes

$$d + \left(1 - \bar{\Phi}\right) q$$

The frictionless case $\phi = 0$

- The firm's objective is to maximize $d + q$
- The price is equal to $q = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t d_t$
- Standard time-consistent problem
- Maximize the NPV of dividends, discounted at the risk-free rate

$\rightarrow$ deviations from *exponential discounting* come from transaction costs, $\phi > 0$

▷ Time inconsistency in a three-period model

Euler equation

$$(1-\phi\Delta_t^-)q_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right](d_{t+1}+(1-\Phi_t)\,q_t)+\eta_t$$

where η_t is the Lagrange multiplier on $\Delta^- \leq \theta$ and

$$\Phi_t \equiv \mathbb{E}_t\left[\phi\Delta_{t+1}^-\right]+\phi\frac{\mathrm{cov}_t\left(u'(c_{t+1}),\Delta_{t+1}^-\right)}{\mathbb{E}_t\left[u'(c_{t+1})\right]}$$

Φ captures liquidity frictions:

1. Expected marginal transaction costs: $\phi\Delta_{t+1}^- \rightarrow$ lower asset prices
2. Positive covariance if sell in bad times $\rightarrow$ further depress asset prices

The liquidity premium

- Focus on unconstrained buyers: $\Delta_t^- = 0$, $\Delta_t^+ > 0$, $b_{t+1} > \underline{b}$
- Asset price:

$$q_t = \frac{d_{t+1} + \left(1 - \Phi^B\right) q_{t+1}}{1 + r_t}$$

- The liquidity premium is $\Phi^B = r^\theta - r$, where r^θ is the yield of the stock

Assumption 2: The firm takes $\bar{\Phi}$ and Φ^B as given.

Firm's problem

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} d_t + (1 - \bar{\Phi})q_t$$

subject to

$$q_t = \frac{d_{t+1} + (1 - \Phi)\, q_{t+1}}{1 + r}$$

where $d_t = F(k_t, k_{t+1}) = zk_t^{\alpha} + (1 - \delta)k_t - k_{t+1}$

▷ static labor choice

$\beta - \delta$ discounting and time consistency

$\beta - \delta$ discounting

Proposition: *we can cast the firm's problem as if it has $\beta - \delta$ discounting*

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

where

- $\tilde{\delta} = \frac{1-\Phi^B}{1+r}$ exponential discounting with liquidity premium
- $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi^B}$ time-inconsistency
- $\beta - \delta$ discounting iff $\Phi^B \neq \bar{\Phi}$
- present bias (i.e., $\tilde{\beta} < 1$) iff $\bar{\Phi} > \Phi^B$

Time inconsistency & present bias

Proposition: *the difference $\Phi^B - \bar{\Phi}$ is equal to persistence and risk premium effects:*

$$\Phi^B - \bar{\Phi} = \underbrace{\phi\left(\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right] \middle\| \text{buyer}\right] - \tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]\right)}_{\text{persistence effect}} + \underbrace{\phi\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]} \middle\| \text{buyer}\right]}_{\text{risk premium}}$$

$\tilde{\mathbb{E}}$ is the cross-sectional expectation, weighted by stock shares θ'

No transaction costs: If $\phi = 0$ then $\Phi^B = \bar{\Phi} = 0$, so $\tilde{\beta} = 1$, time consistent problem.

Intuition: persistence and risk premium

$$\text{Persistence effect:} \quad \phi\left(\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right] \middle\| \text{buyer}\right] - \tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]\right)$$

- difference on average transaction costs for buyers and owners
- smaller for buyers than owners $\rightarrow$ negative term

$$\text{Risk premium:} \quad \phi\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]} \middle\| \text{buyer}\right]$$

- if sell in bad times $\rightarrow$ positive covariance

- quantitatively the persistence effect dominates, so $\tilde{\beta} < 1$
- the problem is time inconsistent and the firm has present bias

Solution with and without commitment

Solution with with and without commitment

With commitment

$$\max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

Steady state

- SDF: $\tilde{\delta}$
- Capital

$$k^C = \left(\frac{(1-\gamma)\,\psi\tilde{\delta}}{1-\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Without commitment

- Markov perfect equilibrium

$$\max_{k'} F(k, k') + \tilde{\beta}\tilde{\delta} W(k')$$

$$W(k') = F(k', g(k')) + \tilde{\delta} W(g(k'))$$

Steady state

- SDF: $\tilde{\beta}\tilde{\delta}$
- Capital

$$k^N = \left(\frac{(1-\gamma)\,\psi\tilde{\beta}\tilde{\delta}}{1-\tilde{\beta}\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Incomplete markets, transaction costs, and commitment

Classic results

1. **Complete markets**
   - $\beta(1+r)=1$, firms discount at rate $\frac{1}{1+r}=\beta$

2. **Aiyagari 94:** incomplete markets without transactions costs
   - $\tilde{\beta}=1$, no problems of commitment
   - firms discount at rate $\frac{1}{1+r}$
   - GE: precautionary savings, $\beta(1+r)<1$, *more capital* than in complete markets

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New

1. **Transactions costs, with commitment**
   - firms discount at rate $\tilde{\delta} = \frac{1-\Phi^B}{1+r}$
   - PE: Liquidity premium $\Phi^B \rightarrow$ more discounting, less capital than in Aiyagari 94

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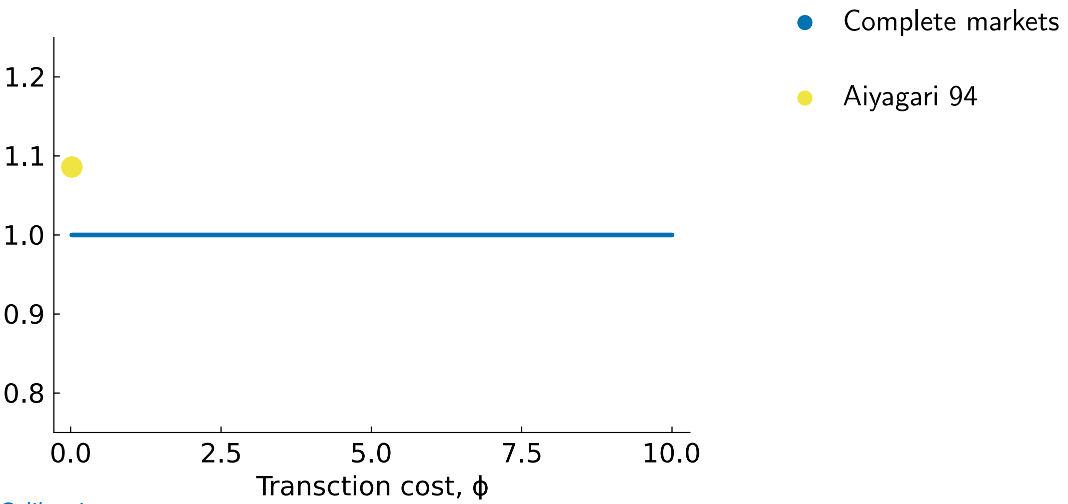
2. **Transactions costs, without commitment**
   - firms discount at rate $\tilde{\beta}\tilde{\delta}$, present bias $\tilde{\beta} < 1$
   - PE: less capital than with commitment: $k^n < k^c$

Caveat: for 3. and 4., in GE, r and Φ^B also change $\rightarrow$ quantitative evaluation

Quantitative evaluation

Capital, relative to complete markets



▷ Calibration

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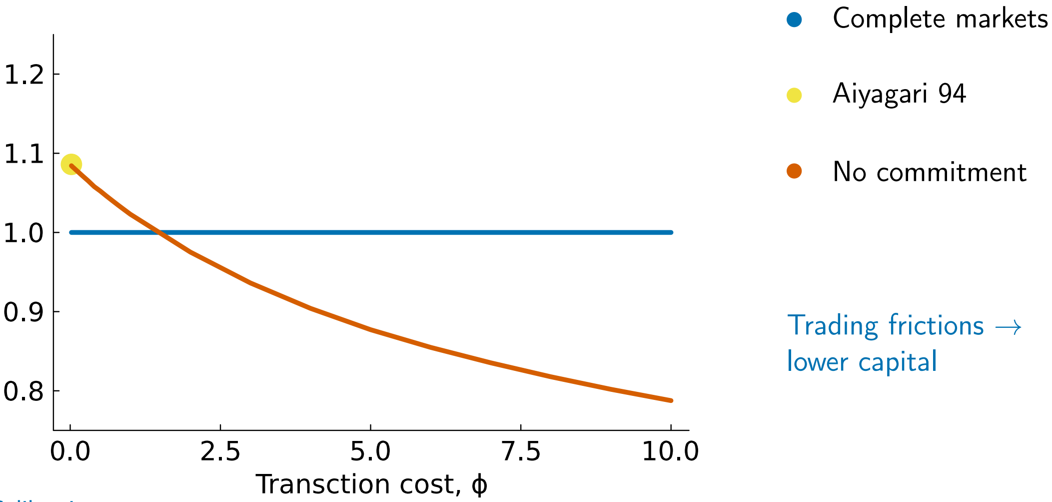
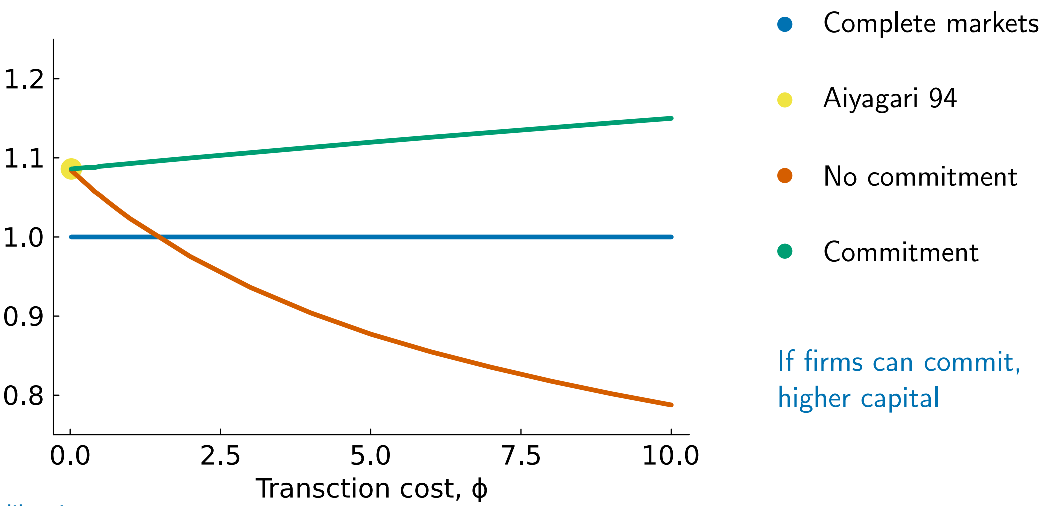


Trading frictions → lower capital

▷ Calibration

Capital, relative to complete markets



If firms can commit, higher capital

▷ Calibration

Transmission of trading frictions to investment depends on commitment

With commitment

- SDF: $\tilde{\delta} = \frac{1-\Phi^B}{1+r}$
- PE: trading frictions depress asset prices ($\uparrow \Phi^B$) $\rightarrow$ lower level of capital
- GE: higher precautionary savings ($\downarrow r$) $\rightarrow$ larger level of capital
- Quantitatively: moderate increase in capital

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Without commitment

- Present bias: strong force towards more discounting ($\downarrow \tilde{\beta}$) and lower capital

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Elasticity of capital to the liquidity: An increase of 10 bps in the liquidity premium

- reduces capital by about 7% without commitment
- increases capital by less than 1% with commitment

▷ How does the model work?

Extensions & applications

Extensions & applications

1. Corporate discount rate wedge ▷ Wedge
2. Capital structure: Robust to include corporate bonds ▷ Corporate bonds
3. Demand of liquidity: Increase in idiosyncratic uncertainty ▷ Demand
4. Supply of liquidity: Introduce government bonds ▷ Supply
5. Short-termism ▷ Short-termism
6. Heterogeneous firms: Public vs Private ▷ Heterogeneous firms

Conclusions

- Aiyagari production economy, with liquid and illiquid assets in general equilibrium
- The problem of the firm is time inconsistent
  - result from frictions in financial markets
  - the discount factor of firms is as if they have $\beta - \delta$ discounting
- Aggregate distortions due to trading frictions depend on commitment
- Rationalize empirical regularities on liquidity and investment

Appendix

Related Literature

- Incomplete markets & firm insurance: Diamond (1967), Dreze (1974), **Grossman Hart (1979)**, Aiyagari Gertler (1991), Heaton Lucas (1996), Magill Quinzii (1996), Espino Kozlowski Sanchez (2018)
  New: Trading frictions and/or GE

- Illiquid assets & macro: Kaplan Violante (2014), Cui Radde (2019), Jeenas Lagos (2020)
  New: Dynamic firm's problem with liquidity frictions

- $\beta - \delta$ discounting: Krusell Smith (2003), Azzimonti (2011), Amador (2012), Cao Werning (2018)
  New: $\beta - \delta$ discounting as a result

- Short-termism: Graham Harvey Rajgopal (2005), Terry (2023)
  New: Don't need additional constraints

Firm: static labor choice

- Static labor choice

$$\max_{l} \left(l^{\gamma} k^{1-\gamma}\right)^{\psi} - wl$$

with labor demand $l = \psi\gamma\frac{y}{w}$

- In equilibrium $w = \psi\gamma k^{(1-\gamma)\psi}$
- Dividends are

$$d_t = F(k_t, k_{t+1}) = zk_t^{\alpha} + (1-\delta)k_t - k_{t+1}$$

where $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$

▷ back

Time Inconsistency in a Three-Period Model

Three-period model

Simplified model to show the time inconsistency problem

- Three periods: $t \in \{0, 1, 2\}$
- No income risk, two type of households with income $\{H, L, H\}$ and $\{L, H, L\}$
- No bonds

Three-period model: Euler equations & firm's value

Euler equations:

$$\left(1-\phi\Delta_0^{j-}\right)q_0=\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1+\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}\left(1-\phi\Delta_1^{j-}\right)q_1$$

$$\left(1-\phi\Delta_1^{j-}\right)q_1=\beta\frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)}d_2$$

Firm's value:

$$\sum_{j\in\{l,h\}}\frac{\theta_0^j}{2}\left[d_0+(1-\phi\Delta_0^{j-})q_0\right]$$

$$\sum_j\frac{\theta_0^j}{2}\left[d_0+\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1+\beta^2\frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)}d_2\right]$$

Time consistency in the three-period model

Problem in period 0

$$\max_{k_1,k_2\geq 0}\sum_j \frac{\theta_0^j}{2}\left[d_0+\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1+\beta^2\frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)}d_2\right]$$

Problem in period 1

$$\max_{k_2\geq 0}\sum_j \frac{\theta_1^j}{2}\left[d_1+\beta\frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)}d_2\right]$$

The problem is **time consistent** iff the discounting between period 1 and 2 coincides

$$\underbrace{\frac{\sum_j \frac{\theta_0^j}{2}\beta^2\frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2}\beta\frac{u'(c_1^j)}{u'(c_0^j)}}}_{\substack{t=0 \text{ discount between}\\ t=1 \text{ and } t=2}} = \underbrace{\sum_j \frac{\theta_1^j}{2}\beta\frac{u'(c_2^j)}{u'(c_1^j)}}_{\substack{t=1 \text{ discount between}\\ t=1 \text{ and } t=2}}$$

Three-period model, frictionless case $\phi = 0$

The Euler equation implies equalization of marginal rates of substitution across agents:

$$\beta \frac{u'\left(c^j_{t+1}\right)}{u'\left(c^j_t\right)} = \frac{q_t}{d_{t+1} + q_{t+1}}$$

Hence

$$\underbrace{\frac{\sum_j \frac{\theta^j_0}{2} \beta^2 \frac{u'(c^j_2)}{u'(c^j_0)}}{\sum_j \frac{\theta^j_0}{2} \beta \frac{u'(c^j_1)}{u'(c^j_0)}}}_{\substack{t=0 \text{ discount between} \\ t=1 \text{ and } t=2}} = \underbrace{\frac{\frac{q_0}{d_1+q_1}\frac{q_1}{d_2+q_2}}{\frac{q_0}{d_1+q_1}}}_{\text{use Euler equation}} = \frac{q_1}{d_2+q_2} = \underbrace{\sum_j \frac{\theta^j_1}{2} \beta \frac{u'(c^j_2)}{u'(c^j_1)}}_{\substack{t=1 \text{ discount between} \\ t=1 \text{ and } t=2}}$$

- The problem is time consistent when $\phi = 0$

Three-period model with trading frictions, $\phi > 0$

With transaction costs:

$$\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} \neq \sum_j \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}$$

- The intertemporal marginal rates of substitution are **not** equalized across agents
- The problem is time inconsistent

▷ Back

Calibration: Transaction costs & Liquidity Premium

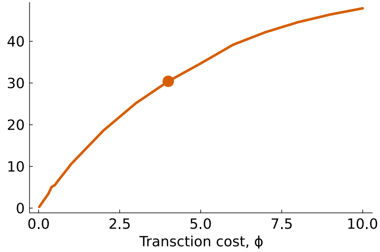
Most of the parameters follow a standard calibration

Transaction costs:

Target a liquidity premium of 35-37 bps
(van Binsbergen Diamond Grotteria 2022)
Inferred from call-put parity on S&P 500 options.

Consider $\phi \in [0, 10]$
Liquidity premium between 0 and 50 bps

Model: Liquidity premium



Calibration

Parameter	Value
Discount factor β	0.95
Risk aversion σ	2.00
Depreciation δ	0.05
Production weight on labor γ	0.80
Returns to scale ψ	0.95
Borrowing limit $\underline{b}$	-1.00
Labor persistence ρ_h	0.50
Labor st dev σ_h	0.03
Transaction cost ϕ	4.00

Most of the parameters are standard

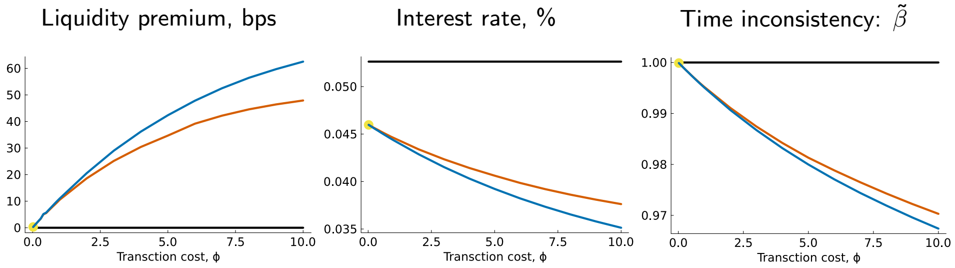
Transaction cost: liquidity premium of 40 bps (van Binsbergen Diamond Grotteria 2022)

Non-Targeted Moments

	Model	Data
Corporate discount rate wedge, percent	1.5	2.1
Variance log consumption / variance log income	0.2	0.3
Mean illiquid assets	3.5	2.9
Mean liquid assets	0.5	0.3
Frac. with $b > 0$	0.5	0.5
Stock owners at the borrowing constraint, percent	5.4	5.7

▷ Back

Commitment: constant discounting



- Higher ϕ → bonds *better* than stocks → higher liquidity premium & lower r
- Capital with commitment about constant, recall $\tilde{\delta} = \frac{1-\Phi^B}{1+r}$

▷ Back

Corporate discount rate wedge

- Gormsen Huber (2024) decompose the firm's discount factor Λ

$$\Lambda = \underbrace{r^{fin}}_{\text{financial cost}} + \underbrace{\kappa}_{\text{discount rate wedge}}$$

- Model without commitment:

$$r^{fin} \equiv \log\left(\frac{1}{\tilde{\delta}}\right) \approx r + \Phi^B, \quad \text{and} \quad \kappa \equiv \log\left(\frac{1}{\tilde{\beta}}\right) \approx \overline{\Phi} - \Phi^B.$$

- Present bias generates the discount rate wedge

	Model	**Data**
Corporate discount rate wedge, percent	1.5	2.1

The model explains about 70% of the wedge

Liquidity and the corporate discount rate wedge

More illiquid firms have higher wedges

$$\kappa_{it} = \alpha_t + \delta_i + \beta \text{ liquidity}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

Liquidity	0.228***	0.184***	0.230***	0.181***
	(0.016)	(0.012)	(0.016)	(0.012)
Observations	27163	27158	27163	27158
R-squared	0.266	0.668	0.266	0.669
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: Firm-quarter data,2002Q1 to 2021Q4. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. Liquidity is measured with relative spreads from CRSP. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase.

- Iliquid firms have higher discount rate wedges
- Model suggests that present bias is a factor behind this empirical finding

Empirics: More illiquid firms have higher discount rates

Relative spread	0.509***	0.281***	0.497***	0.278***
	(0.026)	(0.016)	(0.027)	(0.016)
Observations	27163	27158	27163	27158
R-squared	0.236	0.805	0.238	0.805
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: The dataset is at the firm-quarter level and runs from 2002 to 2021. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase. The specification includes fixed effects for time, or time and firm. Statistical significance is denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

▷ Back

Corporate bonds

Firms can borrow at interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$ up to a limit

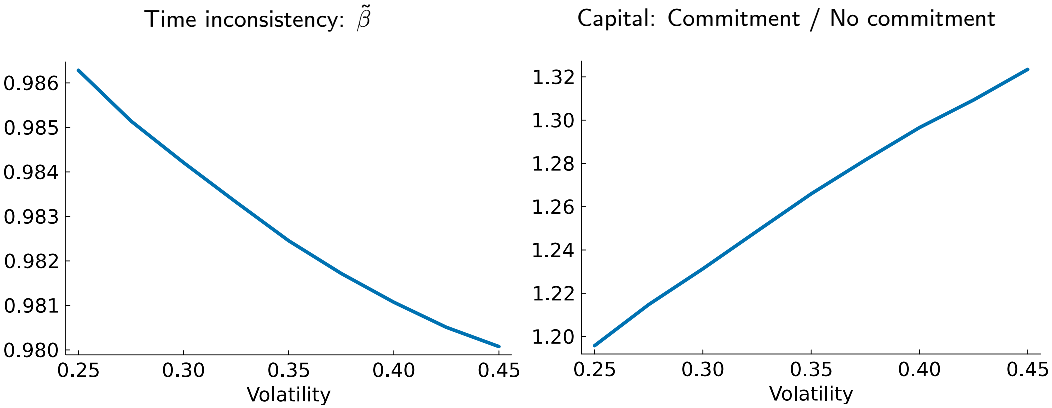
- If $\tilde{\phi} < \Phi^B$ the firm always borrows to the limit independently of its commitment.
- If $\Phi^B < \tilde{\phi} < \overline{\Phi}$ only the firm without commitment borrows up to the limit.

Implications:

- can alter financing but not investment and the time-inconsistency problem
- firms borrow even if bonds are more illiquid than stocks due to present bias
- rationalize corporate debt that does not rely on the tax advantage of debt

▷ Back

Demand of liquidity: increase idiosyncratic volatility



- Without commitment: more time inconsistency $\rightarrow$ less capital
- With commitment: more precautionary savings $\rightarrow$ more capital

▷ Back

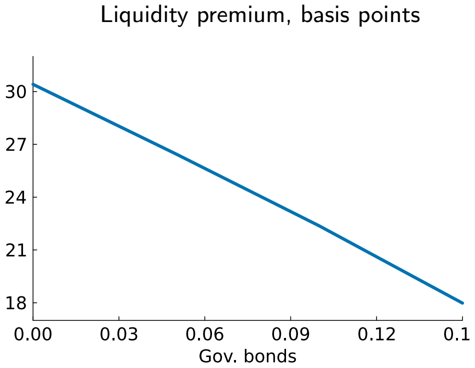
Government bonds

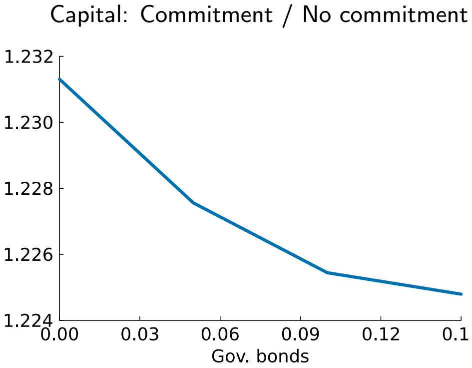
- Introduce government bonds
- Lump-sum taxes to pay for the debt services
- Bonds market clearing

$$\int b'(\theta, b, h) d\Gamma(\theta, b, h) = B^g$$

- As B^g increases: more liquid assets

Supply of liquidity & government bonds





- Capital closer to complete markets
- Without commitment: less time inconsistency $\rightarrow$ more capital
- With commitment: less precautionary savings $\rightarrow$ less capital

▷ Back

Short-termism

Evidence on short-termism:

- an excessive focus on short-term results at the expense of long-term interests (Graham et al. 05, Terry 23, Fink 15)
- public firms distort their investment to meet short-term targets (Graham et al., 05).

Model: short-termism as a result of (i) trading frictions, and (ii) lack of commitment.

▷ Back

Heterogeneous Firms: Public vs private firms

- Asker et al. (2015) finds that public firms invest substantially less than private firms.
- We add private firms to the benchmark equilibrium. Private firms are owned by only one household and are not traded in financial markets.
- The investment decisions of private firms are independent of ϕ, while investment in public firms decreases with the transaction cost.
- For most values of ϕ private firms invest more than public firms, consistent with the empirical evidence.