

# The Cost of Capital and Misallocation in the United States

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**Goal:** measure how dispersion in the cost of capital affects its allocation

## Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

## Empirical Results (US):

- Low levels of misallocation in normal times ( $\approx 0.5\%$  of GDP)
- Losses from misallocation increased to 1.1% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

# Outline

1. Model
2. Welfare and misallocation
3. Measurement with credit registry data
4. Empirical results for the US

## 1. Model

# Model

## Borrowers

- Produce output  $f(k_i, z_i)$
- Invest in capital  $k_i$
- Long-term debt  $b_i$
- Limited liability

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- Borrower-lender match
- $\rho_i \sim$  match efficiency
- Heterogeneity in  $\rho_i$

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**Key question:** how do heterogeneity in  $\rho_i$  and financial frictions distort the allocation of capital?



# Firm's problem

**Value of repayment:**

$$V_i(k_i, b_i, z_i) = \max_{k'_i, b'_i} \pi_i(k_i, b_i, z_i, k'_i, b'_i) + \beta \mathbb{E} \left[ \overbrace{\max \{V_i(k'_i, b'_i, z'_i), 0\}}^{\text{Limited liability}} \mid z_i \right]$$

**Profits:**

$$\pi_i(k_i, b_i, z_i, k'_i, b'_i) = f(k_i, z_i) + (1 - \delta) k_i - k'_i - \theta b_i + Q_i(k'_i, b'_i, z_i) (b'_i - (1 - \theta_i) b_i)$$

**Price of debt:**

$$Q_i(k'_i, b'_i, z_i) = \frac{\mathbb{E} \left[ \mathcal{P}_i(k'_i, b'_i, z'_i) (\theta_i + (1 - \theta_i) Q_i(k''_i, b''_i, z'_i)) + (1 - \mathcal{P}_i(k'_i, b'_i, z'_i)) \frac{\phi_i k'_i}{b'_i} \mid k'_i, b'_i, z_i \right]}{\underbrace{1 + \rho_i}_{\text{lender discount rate / match efficiency}}}$$

## Firm's cost of capital

Define the implicit interest rate paid by the firm as

$$1 + r_i^{firm} = \frac{\mathbb{E}[\mathcal{P}'_i(\theta_i + (1 - \theta_i)Q'_i) | k'_i, b'_i, z_i]}{Q_i}$$

### Lemma 1 (Firm cost of capital)

*The firm cost of capital is:*

$$1 + r_i^{firm} = \frac{1 + \rho_i}{1 + \Lambda_i}$$

$$\Lambda_i := \frac{\mathbb{E}[(1 - \mathcal{P}'_i) \phi_i(k'_i)/b'_i | k'_i, b'_i, z_i]}{\mathbb{E}[\mathcal{P}'_i(\theta + (1 - \theta_i)Q'_i) | k'_i, b'_i, z_i]}$$

▷ *Proof*

$\Lambda_i$ : **financial frictions wedge** that arises due to limited liability and partial recovery  $\phi_i$

- $\phi_i = 0$ : no recovery after default, then  $r_i^{firm} = \rho_i$
- If  $\phi_i > 0$ , then  $\Lambda_i > 0$  and  $r_i^{firm} < \rho_i$ : borrower only takes into account repayment states

# Marginal revenue product of capital (MRPK)

$$\underbrace{(1 + r_i^{firm})\mathcal{M}_i}_{\text{cost of capital}} = \underbrace{\mathbb{E}[\mathcal{P}'_i(f_k(k'_i, z'_i) + 1 - \delta) | k'_i, b'_i, z_i]}_{\text{expected marginal revenue product of capital}} \quad (1)$$

where  $\mathcal{M}_i$  captures the *price impact* of the firm's actions

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b'_i}{k'_i} \times \frac{\partial \log Q_i}{\partial \log k'_i}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b'_i}}, \quad \gamma_i := \frac{b'_i - (1 - \theta_i)b_i}{b'_i}$$

- Heterogeneity in  $r_i^{firm} \rightarrow$  heterogeneity in  $MRPK_i$
- **Approach:** measure  $r_i^{firm}$  by measuring  $\rho_i$  and  $\Lambda_i$

## 2. Welfare and misallocation

# Aggregate economy and welfare

## Decentralized Equilibrium:

$$Y^{DE} + (1 - \delta)K^{DE} = \int_0^1 \mathbb{E}_t [\mathcal{P}_{i,t+1}^{DE} (f(k_{i,t+1}^{DE}, z_{i,t+1}) + (1 - \delta)k_{i,t+1}^{DE}) + (1 - \mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^{DE}] di$$

# Aggregate economy and welfare

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## Planner's problem:

- **Inner problem:** redistribute  $\{k_{i,t+1}\}_i$  taking exit decisions and  $K^{DE}$  as given  $\triangleright$  full planner problem
- Lower bound on full misallocation:

$$\begin{aligned} & \max_{\{k_{i,t+1}^*\}_i} \int_0^1 \mathbb{E}_t [\mathcal{P}_{i,t+1}^{DE} (f(k_{i,t+1}^*, z_{i,t+1}) + (1 - \delta)k_{i,t+1}^*) + (1 - \mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^*] di \\ \text{s.t.} \quad & \int_0^1 k_{i,t+1}^* di = K_{t+1}^{DE} \end{aligned}$$

# Social return on capital

- In equilibrium:

$$(1 + r_{i,t}^{firm}) \mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

- Define the social marginal product of capital at firm  $i$ ,  $r_{i,t}^{social}(k_{i,t+1})$

$$1 + r_{i,t}^{social}(k_{i,t+1}) \equiv \mathbb{E} [\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}, z_{i,t+1}) + 1 - \delta) + (1 - \mathcal{P}_{i,t+1}^{DE}) \phi]$$

social return takes into account recovery in case of default

- Planner Optimality: at  $\{k_{i,t+1}^*\}$  the planner equalizes  $r_{i,t}^{social}(k_{i,t+1}^*)$  across firms
- Equilibrium: dispersion on  $r_{i,t}^{social}(k_{i,t+1}^{DE}) \rightarrow$  misallocation

# Misallocation

## Proposition 1 (Misallocation)

Misallocation can be measured with  $\mathbb{E}[r_i^{social}]$  and  $\text{Var}(r_i^{social})$  as

$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log \left( 1 + \frac{\text{Var}(r_i^{social})}{(\mathbb{E}[r_i^{social}] + \delta)^2} \right)$$

▷ *Proof*

- Extend Hughes and Majerovitz (2025) to a dynamic economy with default
- Set  $\mathcal{E} = \frac{1}{2}$  and  $\delta = 0.06$
- **Next:** we show how to measure  $r_i^{social}$  using credit registry data

▷ Calibration



### 3. Measurement with credit registry data

# Data: FR Y-14Q (Schedule H.1)

▷ summary stats. ▷ time series

- Quarterly loan-level panel on universe of loan facilities  $> \$1\text{M}$
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies
- Cannot consider credit lines due to lack of information on fees.

## Pricing term loans

The **break-even** condition for a lender with discount rate  $\rho_i$  is

$$1 = \sum_{t=1}^{T_i} \left[ \frac{P_i^t \mathbb{E}_0[r_{i,t}] + P_i^{t-1} (1 - P_i) (1 - LGD_i)}{(1 + \rho_i)^t} \right] + \frac{P_i^{T_i}}{(1 + \rho_i)^{T_i}} \quad (2)$$

- $T_i$ : maturity
- $\mathbb{E}_0[r_{i,t}]$ : fixed interest rate or fixed spread over floating benchmark rate ▷ forward rates
- $P_i$ : repayment probability (constant over time)
- $LGD_i$ : loss given default (constant over time)
- $\Rightarrow$  Solve for lender's discount rate:  $\rho_i$
- Fixed contractual rate:  $1 + \rho_i = P_i (1 + r_i) + (1 - P_i) (1 - LGD_i)$  ▷ fixed rate

## Firm cost of capital

### Lemma 2 (Firm cost of capital)

We can solve for  $\Lambda_i$  as

$$\Lambda_i = \frac{(1 - P_i)(1 - LGD_i)}{1 + \rho_i - (1 - P_i)(1 - LGD_i)}$$

and write the firm cost of capital as

$$1 + r_i^{firm} = (1 + \rho_i) - (1 - P_i)(1 - LGD_i)$$

▷ *Proof*

- $(1 - P_i)(1 - LGD_i) \simeq$  prob. of default event that does not result in a loss for the lender
- Measures pricing wedge between borrower and lender
- For fixed interest rate loans, use  $(1 + \rho_i)$  as in Lemma 2 to write  $1 + r_i^{firm} = (1 + r_i) P_i$

# Social cost of capital

## Lemma 3 (Social cost of capital)

*The social cost of capital can be written as:*

$$\begin{aligned} 1 + r_i^{\text{social}} &= (1 + r_i^{\text{firm}}) \mathcal{M}_i + (1 - P_i)(1 - LGD_i) \text{lev}_i \\ &= \underbrace{(1 + \rho_i) \mathcal{M}_i}_{\text{lender discount rate}} + \underbrace{(\text{lev}_i - \mathcal{M}_i) \cdot (1 - P_i) \cdot (1 - LGD_i)}_{\text{wedge due to financial frictions}} \end{aligned}$$

- social cost of capital  $\simeq$  lender discount rate + wedge due to financial frictions
- Wedge due to financial frictions:
  - **Lenders** care about average recovery per dollar of debt:  $\phi_i(k_i)/b_i = \mathcal{M}_i(1 - LGD_i)$
  - **Planner** cares about the marginal recovery:  $\phi'_i(k_i) = (1 - LGD_i) \times \text{lev}_i$
  - Coincide when  $\text{lev}_i = \mathcal{M}_i$

## Sufficient statistic for misallocation

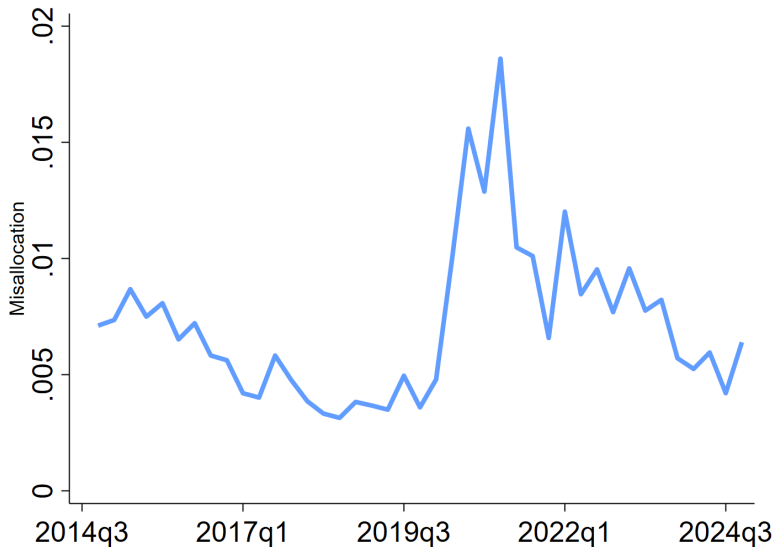
$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log \left( 1 + \frac{\text{Var}(r_i^{\text{social}})}{(\mathbb{E}[r_i^{\text{social}}] + \delta)^2} \right)$$
$$1 + r_i^{\text{social}} = (1 + \rho_i) \mathcal{M}_i + (\text{lev}_i - \mathcal{M}_i) \cdot (1 - P_i) \cdot (1 - \text{LGD}_i)$$

- Set  $\mathcal{M}_i = 1$ ; reasonable approximation given our model
- Can measure misallocation directly with credit registry data!
- Dispersion in  $r_i^{\text{social}}$  comes from:
  1. Dispersion in lender's discount rate,  $\rho_i$
  2. Dispersion in financial frictions wedge
  3. Covariance between  $\rho_i$  and financial frictions wedge

▷ Estimate  $\mathcal{M}$

## 4. Empirical results

## Misallocation in the US, 2014-2024



- About 0.5% before 2020
- ↑ to 1.1% in 2020-2021
- ↓ to 0.8% in 2022-2024

▷ Average Rates



## The 2020–2021 increase in misallocation

1. Predominantly explained by dispersion in  $\rho_i$ , rather than financial frictions wedge [▷ Details.](#)
2. Sharp rise in the coefficient of variation of  $\rho_i$  [▷ Details.](#)
3.  $\rho_i$  dispersion traced to changes in distribution of contractual rates (not  $P_i$  or  $LGD_i$ ) [▷ Details.](#)
4. Driven by underpricing of very risky loans

## The 2020-21 increase: underpricing of risky loans

- **Very risky loans**—offered with unusually favorable contractual rates
- These loans have **low implied**  $\rho_i$ , increasing overall dispersion

### Our hypothesis:

- Broad fiscal and monetary interventions (PPP, MSLP, PMCCF, SMCCF) supported distressed firms
- Lenders **inferred explicit and implicit government guarantees** for risky loans
- Moral hazard/zombie lending

### Implication:

- Risk was mispriced, leading to **credit misallocation**
- Absent guarantees, risk would have been priced more accurately, improving allocative efficiency.

# Robustness & Extensions

- Risk Premia & Aggregate Shocks

▷ [Details.](#)

- Cross-country comparison

▷ [Details.](#)

# Conclusions

- Develop a framework to measure misallocation using credit registry data
  1. Standard macrofinance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Inputs: standard credit registry variables ( $r$ ,  $P$ ,  $LGD$ ,  $T$ , etc.)
- Application to U.S. credit registry data (FR Y-14Q)
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 0.5% in normal times
  3. Sharp rise in 2020-21, possible tied to credit market interventions

# Appendices

- **Measuring misallocation:**

- Seminal work by Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
- **Contribution:** use **heterogeneity in funding costs** to measure **dispersion in MPK**

- **Heterogeneity in the cost of capital:**

- Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
- US: Gilchrist, Sim, and Zakrajsek (2013), David, Schmid, and Zeke (2022), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
- **Contribution:**
  - Estimate firm cost of capital using **credit registry data**, correcting for maturity, default, etc.
  - Derive and estimate **sufficient statistic** for misallocation

$$\begin{aligned}\mathbb{E}_t \left[ \frac{\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})}{Q_t} \right] &= (1 + \rho) \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})] + \mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']} \\ &= (1 + \rho) \left( 1 + \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]} \right)^{-1} \\ &= (1 + \rho) (1 + \Lambda)^{-1}\end{aligned}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}$$

$$\begin{aligned}
 U^* &= \max_{\{\{k_{i,t}(S^{t-1}), \omega_{i,t}(S^t)\}_i\}_{t=1}^\infty} \sum_{t=0}^\infty \beta^t \cdot u(Y_t - I_t) \\
 \text{s.t.} \quad &\omega_{i,t}(S^t) \in \{0, 1\} \forall i \\
 &\omega_{i,t+1}(S^{t+1}) \geq \omega_{i,t}(S^t) \quad \forall S^t \subset S^{t+1}, \forall i
 \end{aligned}$$

Can separate into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\{K_t, \{\omega_{i,t}(S^t)\}_{i \in [0,1]}\}_{t=1}^\infty} \sum_{t=0}^\infty \beta^t \cdot u \left( \left( \max_{\{\{k_{i,t}(S^{t-1})\}_{i \in [0,1]}\}_{t=1}^\infty} Y_t \right) - I_t \right)$$

Rewrite inner problem as:

$$\begin{aligned}
 Y_t^* \left( K_t, \{\omega_{it}\}_{i \in [0,1]} \right) &= \max_{\{k_{i,t}\}_{i \in [0,1]}} \int_0^1 \mathbb{E}_{t-1} [\omega_{it} \cdot f(k_{it}; z_{it}) - (1 - \omega_{it}) \cdot ((1 - \delta) k_{it} - \phi(k_{it}))] di \\
 \text{s.t.} \quad &K_t = \int_0^1 k_{it} di
 \end{aligned}$$



- Formally, planner's problem is now the same as solving  $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i) di$ , where  $f_i(k_i)$  is now expected output
- Apply Hughes and Majerovitz (2024), noting  $\frac{dY}{dk} = r^{social} + \delta$

$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log \left( 1 + \frac{\text{Var}(r^{social})}{(\mathbb{E}[r^{social}] + \delta)^2} \right)$$

- $\mathcal{E}$  is (negative) elasticity of output w.r.t. cost of capital ( $r^{social} + \delta$ )

- $\mathcal{E}_i$  is the elasticity of expected output with respect to the cost of capital
- Assume that  $f(k, z) = z \cdot k^\alpha$  and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1 - \alpha}$$

- $\alpha = \frac{1}{3}$  implies  $\mathcal{E} = \frac{1}{2}$

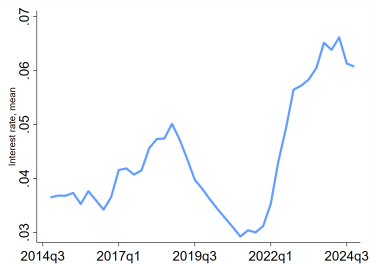
Table: Summary Statistics

	mean	sd	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
$\rho$ (%)	3.75	1.69	2.05	3.69	5.88
$r^{firm}$ (%)	2.82	2.75	0.87	3.04	5.26
$r^{social}$ (%)	3.54	1.88	1.77	3.53	5.71
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.81	1.11	2.55	22.64
Sales (M)	1,254.73	5,923.53	2.17	58.80	1,556.58
Assets (M)	1,770.83	8,956.78	1.06	35.52	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	22.61	29.05	4.68	15.56	44.04
N Loans	62687				
N Firms	38587				
N Fixed Rate	31540				
N Variable Rate	31147				

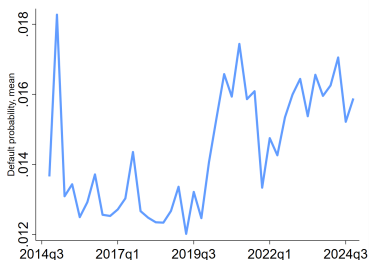
# Time series for averages: Contractual Rate, Default, LGD

▷ back

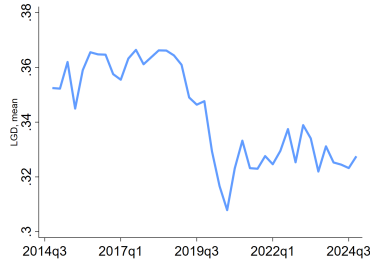
Contractual rate (fixed only)



Default Probability



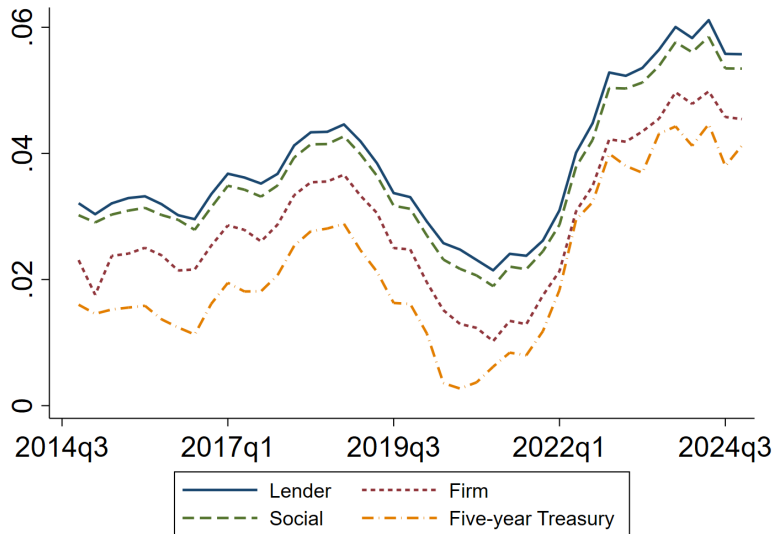
LGD



- 2020-2021: Increase in default probability
- Modest decline in losses given default (better recovery)

# Average Discount Rate, Firm and Social Cost of Capital

▷ back



- Rates follow 5y UST

- Financial frictions:

$$\mathbb{E}[r_i^{social}] > \mathbb{E}[r_i^{firm}]$$

- $\mathbb{E}[r_i^{social}] \approx \mathbb{E}[\rho_i]$

# Data Cleaning and Sample Construction

**Sample period:** We use FR Y-14Q Schedule H.1 data from 2014Q4 onward **Borrower Filters:**

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

# Data Cleaning and Sample Construction

## Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only “vanilla” term loans (Facility type = 7)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or  $> 50\%$ )
  - Missing or invalid PD/LGD values (outside  $[0, 1]$ )
  - PD = 1 (flagged as in default)

# Lender's discount rate

▷ back

## Fixed contractual rate:

### Lemma 4 (Lender's discount rate)

*For a fixed contractual rate loan:*

$$1 + \rho_i = P_i (1 + r_i) + (1 - P_i) (1 - LGD_i)$$

▷ *Proof*

- $\rho_i$  is independent of maturity  $T_i$  for fixed rate loans
- **Floating rate:** numerical solution of (2)



# Forward interest rate expectations

▷ back

To estimate  $\rho$  for floating rate loans, we need estimates of  $\mathbb{E}_0[r_t]$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out  $\mathbb{E}_0[r_t]$  for each loan, using treasury forward rate plus loan's spread

## Lender's discount rate: fixed rate

▷ back

$$1 = \sum_{t=1}^T \left( \frac{P}{1+\rho} \right)^t \left[ r + \frac{(1-P)}{P} (1-LGD) \right] + \left( \frac{P}{1+\rho} \right)^T$$

Let  $x = \frac{P}{1+\rho}$  so

$$1 = \left( r + \frac{(1-P)}{P} (1-LGD) \right) \frac{x}{1-x} (1-x^T) + x^T$$

Guess that  $1 + \rho = (1 + r) P + (1 - P) (1 - LGD)$

$$\frac{1-x}{x} = \frac{1}{x} - 1 = \frac{(1+r)P + (1-P)(1-LGD)}{P} - 1 = r + \frac{1-P}{P} (1-LGD)$$

And, therefore

$$1 = 1 (1 - x^T) + x^T$$

which validates the guess.

$$Q_t = \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1}) + (1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{1 + \rho}$$

Note that

$$\begin{aligned} Q_t &= Q_t^P + Q_t^D \\ Q_t^P &= \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{1 + \rho} \\ Q_t^D &= \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{1 + \rho} \end{aligned}$$

That is, we strip the bond into the payment in repay ( $Q_t^P$ ) and the payment in default ( $Q_t^D$ ). Then:

$$\Lambda = \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]} = \frac{Q_t^D}{Q_t^P}$$

## Firm cost of capital: measurement

▷ back

The firm defaults with probability  $(1 - P)$  and the lender recovers  $(1 - LGD)$ . Hence

$$Q_t^{D,data} = \frac{(1 - P)(1 - LGD)}{1 + \rho}$$

For the payment portion notice that at issuance we have the following condition

$$\begin{aligned} 1 &= \sum_{s=1}^T \left[ \frac{P^s \mathbb{E}_t[r_{t+s}] + P^{s-1}(1 - P)(1 - LGD)}{(1 + \rho)^s} \right] + \frac{P^T}{(1 + \rho)^T} \\ 1 &= \frac{(1 - P)(1 - LGD)}{1 + \rho} + P \frac{\mathbb{E}_t[r_{t+1}]}{1 + \rho} + \left( \sum_{s=2}^T \left[ \frac{P^s \mathbb{E}_t[r_{t+s}] + P^{s-1}(1 - P)(1 - LGD)}{(1 + \rho)^s} \right] + \frac{P^T}{(1 + \rho)^T} \right) \end{aligned}$$

So, we can define  $Q_t^{P,data}$  as  $1 = Q_t^{P,data} + Q_t^{D,data}$  so  $Q_t^{P,data} = 1 - Q_t^{D,data}$ . Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1 - P)(1 - LGD)}{1 + \rho - (1 - P)(1 - LGD)}$$

# Decomposing misallocation

**Counterfactual I:** What if all lenders have the same  $\bar{\rho}$ ?

$$1 + r_{social}^{cf,I} = \overline{(1 + \rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in  $r_{social}^{cf} \rightarrow$  Misallocation due to financial frictions

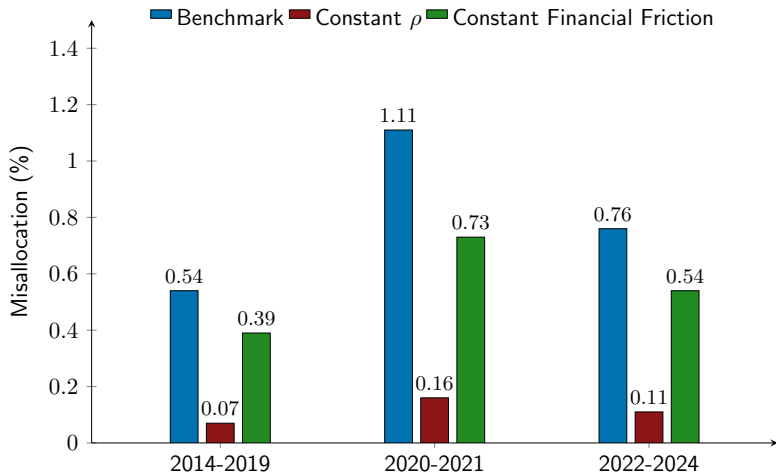
**Counterfactual II:** what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1 + \rho)\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in  $r_{social}^{cf} \rightarrow$  Misallocation due to heterogeneous cost of capital

# 1. The 2020-21 increase: sources of misallocation

▷ back



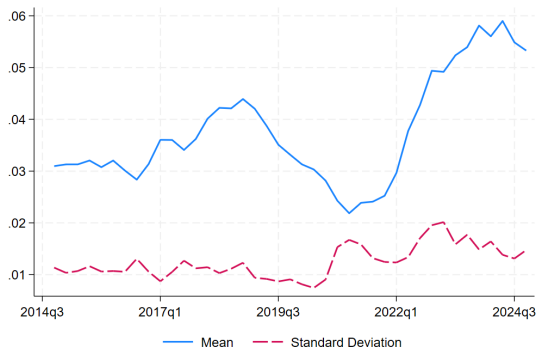
- Mostly driven by heterogeneity in  $\rho_i$
- Interaction between  $\rho_i$  and financial frictions ( $0.54 > 0.07 + 0.39$ )

⇒ 1. Predominantly explained by dispersion in  $\rho_i$

## 2. The 2020-21 increase: dispersion in $\rho_i$

▷ back

Heterogeneity in  $\rho_i$  is the most important driver of increase in misallocation during 2020-21



- As policy rates decreased in 2020-21, so did the mean  $\rho_i$
- The standard deviation of  $\rho_i$  increased during this period

⇒ 2. Sharp rise in the coefficient of variation of  $\rho_i$

### 3. The 2020-21 increase: role of contractual rates

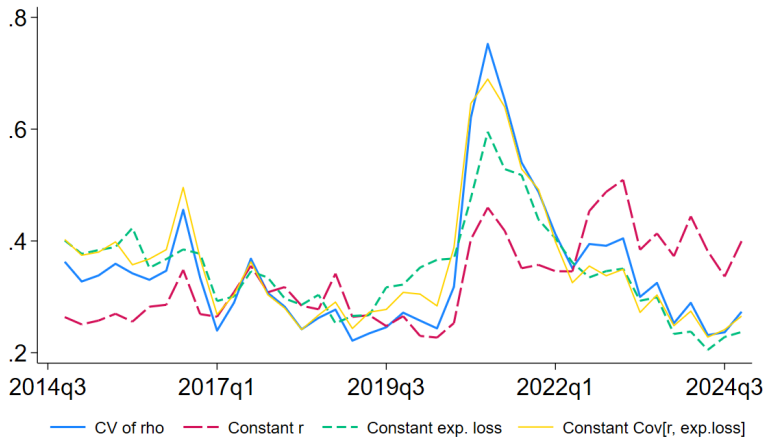
- Approximate  $\rho_i \approx r_i - (1 - P_i)LGD_i$
- The coefficient of variation depends on: (i)  $r_i$ , (ii)  $(1 - P_i)LGD_i$  and (iii) their covariance

$$\frac{\mathbb{V}[\rho_i]^{0.5}}{\mathbb{E}[\rho_i]} \approx \frac{(\mathbb{V}[r_i] + \mathbb{V}[(1 - P_i)LGD_i] - 2\mathbb{COV}[r_i, (1 - P_i)LGD_i])^{0.5}}{\mathbb{E}[r_i] - \mathbb{E}[(1 - P_i)LGD_i]}$$



### 3. Decomposition of the coefficient of variation of $\rho_i$

▷ back



⇒ 3. Dispersion in  $\rho_i$  is traced to changes in the distribution of contractual rates  $r_i$  (not  $P_i$  or  $LGD_i$ )

## Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	71.88	1.63	13.45	13.04
Lender discount rate, $\rho$	61.94	3.08	14.02	20.96
Firm cost of capital, $r^{firm}$	33.23	4.25	20.12	42.4
Social cost of capital, $r^{social}$	53.84	3.87	16.21	26.08
N Firms	1681			
N Loans	14738			

Table: Variance decomposition of interest rates and cost of capital ( $\rho$ ,  $r^{firm}$ , and  $r^{social}$ )

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function  $Q$ ,  $\gamma$ , and firm leverage  $Qb'/k'$  we can compute  $\mathcal{M}$

1. Loans are modeled as perpetuities that decay at a geometric rate  $\theta$ , we can write  $Q$  as the present value of all future payments, discounted at the contractual interest rate  $r$ :

$$Q = \frac{\theta + (1 - \theta)Q}{1 + r} = \frac{\theta}{r + \theta}$$

$r$  is directly observed in the data, and we can approximate  $\theta = 1/T$

2. Guess a functional approximation  $Q(z, k, b, \rho)$
3. Estimate  $\log \hat{Q}(z, k, b, \rho)$  for every loan origination; compute partial derivatives
4. At steady state,  $\gamma = \theta = 1/T$

## Estimating $\mathcal{M}$ : $Q$ elasticities

▷ back

- We approximate (the log of)  $Q$  as a polynomial of investment, borrowing, productivity and  $\rho$
- Investment: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets (Hsieh and Klenow, 2009)
- Approximation:

$$\begin{aligned}\log Q_i = & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ & + \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ & + \beta_{b,b} (\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ & + \beta_{z,z} (\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 + \epsilon_i\end{aligned}$$

- Compute the partial derivatives of  $\log Q$  with respect to investment and borrowing.

- The distribution is extremely concentrated around 1.
- The mean is equal to 0.996 and the median to 0.997, with a standard deviation of 0.006.
- The two measures of misallocation are extremely similar
- Taken together, these results suggest that our assumption that  $\mathcal{M} = 1$  is a good one.

- **Alternative hypothesis:** Rise in  $\rho$  reflects higher **risk premia** as lenders demand extra compensation amid extreme uncertainty (e.g. COVID-19).
- Firms differ in exposure to aggregate shocks  $\Rightarrow$  heterogeneous risk premia need not imply misallocation (David et al., 2022).
- Our framework is steady-state  $\Rightarrow$  cannot model time-varying aggregate shocks or risk-premium spikes.
- **Data contradict the risk-premia story:**
  - Average  $\rho$  **falls** from 3.6% (2014-19) to 2.7% (2020-21).
  - Skewness becomes **more negative**:  $-2.6 \rightarrow -3.5$  (left tail thickens).
- **Interpretation:** Risk premia likely **declined**, perhaps owing to explicit/implicit policy guarantees.

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mu(r_i)$ , %	78.7	14.1	83.0	16.8	3.9
$\sigma(r_i)$ , %	38.1	2.9	93.3	5.2	1.5
$\mu(1 - P_i)$ , %	2.7	16.9	4.0	8.9	1.4
$\mu(1 - LGD_i)$ , % (World Bank)	42.8	42.8	18.2	63.9	81.0
Implied misallocation, %	4.9	2.2	21.5	1.7	0.6

- **Developing countries:** higher mean and standard deviation of contractual rates
- **U.S.:** lower mean and standard deviation of contractual rates, **higher recovery**
- **Brazil:** most extreme misallocation: 21.5%.
- Misallocation in the U.S. small but non-trivial: **0.6%**.

- Recovery rates from the World Banks Doing Business report
- Approximate  $r^{social}$  with  $\rho$  in the SS for misallocation
- Use the fixed rate formula for  $\rho$  and assume that  $(P, LGD)$  are constant across firms
- Approximated cost of misallocation for the US is similar to the actual cost



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