The Cost of Capital and Misallocation in the United States

Miguel Faria-e-Castro
FRB St. Louis

Julian Kozlowski
FRB St. Louis

Jeremy Majerovitz
University of Notre Dame

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The Cost of Capital and Misallocation in the United States

▷ Related Lit.

Goal: measure how dispersion in the cost of capital affects its allocation

Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

Empirical Results (US):

- Low levels of misallocation in normal times ($\approx 0.5\%$ of GDP)
- Losses from misallocation increased to 1.1% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

Outline

1. Model
2. Welfare and misallocation
3. Measurement with credit registry data
4. Empirical results for the US

1. Model

Model

Borrowers 🏭

- Produce output $f(k_i, z_i)$
- Invest in capital k_i
- Long-term debt b_i
- Limited liability

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Lenders 💰

- Discount rate ρ_i
- Competitive pricing
- Recover $\phi_i k_i$ in default

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Matching 🤝

- Borrower-lender match
- $\rho_i \sim$ match efficiency
- Heterogeneity in ρ_i

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Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Firm's problem

Value of repayment:

$$V_i\left(k_i, b_i, z_i\right) = \max_{k_i', b_i'} \; \pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) + \beta \mathbb{E} \overbrace{\left[\max\left\{V_i\left(k_i', b_i', z_i'\right), 0\right\} \middle| \, z_i\right]}^{\text{Limited liability}}$$

Profits:

$$\pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) = f\left(k_i, z_i\right) + \left(1 - \delta\right) k_i - k_i' - \theta b_i + Q_i\left(k_i', b_i', z_i\right)\left(b_i' - \left(1 - \theta_i\right) b_i\right)$$

Price of debt:

$$Q_i\left(k_i', b_i', z_i\right) = \frac{\mathbb{E}\left[\mathcal{P}_i\left(k_i', b_i', z_i'\right)\left(\theta_i + \left(1 - \theta_i\right) Q_i\left(k_i'', b_i'', z_i'\right)\right) + \left(1 - \mathcal{P}_i\left(k_i', b_i', z_i'\right)\right) \frac{\phi_i k_i'}{b_i'} \,\middle|\, k_i', b_i', z_i\right]}{\underbrace{1 + \rho_i}_{\text{lender discount rate / match efficiency}}}$$

Firm's cost of capital

Define the implicit interest rate paid by the firm as

$$1 + r_i^{firm} = \frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}{Q_i}$$

Lemma 1 (Firm cost of capital)

The firm cost of capital is:

$$1 + r_i^{firm} = \frac{1+\rho_i}{1+\Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1-\mathcal{P}_i')\,\phi_i(k_i')/b_i' \middle| k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i'\,(\theta + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}$$

▷ *Proof*

Λ_i: financial frictions wedge that arises due to limited liability and partial recovery ϕ_i

- $\phi_i = 0$: no recovery after default, then $r_i^{firm} = \rho_i$
- If $\phi_i > 0$, then $\Lambda_i > 0$ and $r_i^{firm} < \rho_i$: borrower only takes into account repayment states

Marginal revenue product of capital (MRPK)

$$\underbrace{(1 + r_i^{firm})\mathcal{M}_i}_{\text{cost of capital}} = \underbrace{\mathbb{E}[\mathcal{P}_i'(f_k(k_i', z_i') + 1 - \delta) |\, k_i', b_i', z_i]}_{\text{expected marginal revenue product of capital}} \tag{1}$$

where $\mathcal{M}_i$ captures the *price impact* of the firm's actions

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b_i'}{k_i'} \times \frac{\partial \log Q_i}{\partial \log k_i'}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b_i'}}, \qquad \gamma_i := \frac{b_i' - (1 - \theta_i) b_i}{b_i'}$$

- Heterogeneity in r_i^{firm} → heterogeneity in $MRPK_i$
- Approach: measure r_i^{firm} by measuring ρ_i and Λ_i

2. Welfare and misallocation

Aggregate economy and welfare

Decentralized Equilibrium:

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^{DE}_{i,t+1}, z_{i,t+1}) + (1-\delta)k^{DE}_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi k^{DE}_{i,t+1}\right] \mathrm{d}i$$

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Planner's problem:

- Inner problem: redistribute $\{k_{i,t+1}\}_i$ taking exit decisions and K^{DE} as given ▷ full planner problem

- Lower bound on full misallocation:

$$\max_{\left\{k^*_{i,t+1}\right\}_i} \quad \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi k^*_{i,t+1}\right] \mathrm{d}i$$

$$\text{s.t.} \quad \int_0^1 k^*_{i,t+1} \mathrm{d}i = K^{DE}_{t+1}$$

Private vs. social optimality

Private optimality:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

Planner optimality:

- Define the social marginal product of capital at firm i, $r_{i,t}^{social}$

$$1 + r_{i,t}^{social} \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k_{i,t+1}, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi\right]$$

- Takes into account recovery in case of default
- Optimality: planner **equalizes** $r_{i,t}^{social}$ across firms at $\{k_{i,t+1}^*\}_i$

Misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

▷ *Proof*

- Extend Hughes and Majerovitz (2025) to a dynamic economy with default
- Set $\mathcal{E} = \frac{1}{2}$ and $\delta = 0.06$ ▷ Calibration
- **Next:** we show how to measure r_i^{social} using credit registry data

3. Measurement with credit registry data

Data: FR Y-14Q (Schedule H.1)

▷ summary stats. ▷ time series

- Quarterly loan-level panel on universe of loan facilities $> \$1$M
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies
- Cannot consider credit lines due to lack of information on fees.

Pricing term loans

The break-even condition for a lender with discount rate ρ_i is

$$1 = \sum_{t=1}^{T_i} \left[\frac{P_i^t \mathbb{E}_0 \left[r_{i,t} \right] + P_i^{t-1} (1 - P_i) \left(1 - LGD_i \right)}{\left(1 + \rho_i \right)^{t}} \right] + \frac{P_i^{T_i}}{\left(1 + \rho_i \right)^{T_i}} \tag{2}$$

- T_i: maturity
- $\mathbb{E}_0[r_{i,t}]$: fixed interest rate or fixed spread over floating benchmark rate ▷ forward rates
- P_i: repayment probability (constant over time)
- LGD_i: loss given default (constant over time)
- $\Rightarrow$ Solve for lender's discount rate: ρ_i
- Fixed contractual rate: $1 + \rho_i = P_i \left(1 + r_i \right) + \left(1 - P_i \right) \left(1 - LGD_i \right)$ ▷ fixed rate

Firm cost of capital

Lemma 2 (Firm cost of capital)

We can solve for Λ_i as

$$\Lambda_i = \frac{(1 - P_i)(1 - LGD_i)}{1 + \rho_i - (1 - P_i)(1 - LGD_i)}$$

and write the firm cost of capital as

$$1 + r_i^{firm} = (1 + \rho_i) - (1 - P_i)(1 - LGD_i)$$

▷ *Proof*

- $(1 - P_i)(1 - LGD_i) \simeq$ prob. of default event that does not result in a loss for the lender
- Measures pricing wedge between borrower and lender
- For fixed interest rate loans, use $(1 + \rho_i)$ as in Lemma 2 to write $1 + r_i^{firm} = (1 + r_i)\, P_i$

Social cost of capital

Lemma 3 (Social cost of capital)

The social cost of capital can be written as:

$$\begin{aligned} 1 + r_i^{social} &= (1 + r_i^{firm})\mathcal{M}_i + (1 - P_i)(1 - LGD_i)lev_i \\ &= \underbrace{(1+\rho_i)\,\mathcal{M}_i}_{\textit{lender discount rate}} + \underbrace{(lev_i - \mathcal{M}_i)\cdot(1 - P_i)\cdot(1 - LGD_i)}_{\textit{wedge due to financial frictions}} \end{aligned}$$

- social cost of capital $\simeq$ lender discount rate $+$ wedge due to financial frictions
- Wedge due to financial frictions:
  - **Lenders** care about average recovery per dollar of debt: $\phi_i(k_i)/b_i = \mathcal{M}_i(1 - LGD_i)$
  - **Planner** cares about the marginal recovery: $\phi_i'(k_i) = (1 - LGD_i) \times lev_i$
  - Coincide when $lev_i = \mathcal{M}_i$

Sufficient statistic for misallocation

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\mathsf{Var}\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

$$1+r_i^{social} = (1+\rho_i)\,\mathcal{M}_i + (lev_i-\mathcal{M}_i)\cdot(1-P_i)\cdot(1-LGD_i)$$

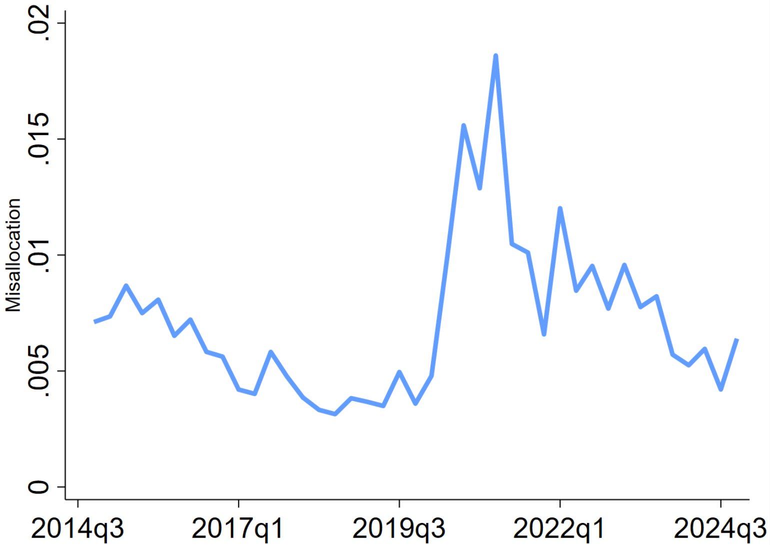
- Set $\mathcal{M}_i = 1$; reasonable approximation given our model

▷ Estimate $\mathcal{M}$

- Can measure misallocation directly with credit registry data!
- Dispersion in r_i^{social} comes from:
  1. Dispersion in lender's discount rate, ρ_i
  2. Dispersion in financial frictions wedge
  3. Covariance between ρ_i and financial frictions wedge

4. Empirical results

Misallocation in the US, 2014-2024



- About 0.5% before 2020
- ↑ to 1.1% in 2020-2021
- ↓ to 0.8% in 2022-2024

▷ Average Rates

The 2020–2021 increase in misallocation

1. Predominantly explained by dispersion in ρ_i, rather than financial frictions wedge ▷ Details.

2. Sharp rise in the coefficient of variation of ρ_i ▷ Details.

3. ρ_i dispersion traced to changes in distribution of contractual rates (not P_i or LGD_i) ▷ Details.

4. Driven by underpricing of very risky loans

The 2020-21 increase: underpricing of risky loans

- Very risky loans—offered with unusually favorable contractual rates
- These loans have low implied ρ_i, increasing overall dispersion

Our hypothesis:

- Broad fiscal and monetary interventions (PPP, MSLP, PMCCF, SMCCF) supported distressed firms
- Lenders inferred explicit and implicit government guarantees for risky loans
- Moral hazard/zombie lending

Implication:

- Risk was mispriced, leading to credit misallocation
- Absent guarantees, risk would have been priced more accurately, improving allocative efficiency.

Robustness & Extensions

- Risk Premia & Aggregate Shocks ▷ Details.
- Cross-country comparison ▷ Details.

Conclusions

- Develop a framework to measure misallocation using credit registry data
  1. Standard macrofinance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Inputs: standard credit registry variables (r, P, LGD, T, etc.)
- Application to U.S. credit registry data (FR Y-14Q)
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 0.5% in normal times
  3. Sharp rise in 2020-21, possible tied to credit market interventions

Appendices

Related literature

▷ back

- **Measuring misallocation:**
  - Seminal work by Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Contribution: use heterogeneity in funding costs to measure dispersion in MPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013), David, Schmid, and Zeke (2022), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for maturity, default, etc.
    - Derive and estimate sufficient statistic for misallocation

Proof: firm cost of capital

▷ back

$$\mathbb{E}_t\left[\frac{\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)}{Q_t}\right]=(1+\rho)\,\frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}$$

$$=(1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1}$$

$$=(1+\rho)\,(1+\Lambda)^{-1}$$

where

$$\Lambda\equiv\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}$$

Full planner problem

▷ back

$$U^* = \max_{\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_i\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(Y_t - I_t\right)$$

$$\text{s.t.} \qquad \omega_{i,t}\left(S^t\right) \in \{0,1\}\ \forall i$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \geq \omega_{i,t}\left(S^t\right)\ \forall S^t \subset S^{t+1}, \forall i$$

Can separate into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t,\{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\{k_{i,t}\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left[\omega_{it} \cdot f\left(k_{it}; z_{it}\right) - \left(1-\omega_{it}\right) \cdot \left(\left(1-\delta\right) k_{it} - \phi\left(k_{it}\right)\right)\right] di$$

$$\text{s.t.} \qquad K_t = \int_0^1 k_{it} di$$

Proof of Misallocation

▷ back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i) \mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^* / Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\mathrm{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital

- Assume that $f(k, z) = z \cdot k^{\alpha}$ and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1 - \alpha}$$

- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Summary Statistics

▷ back

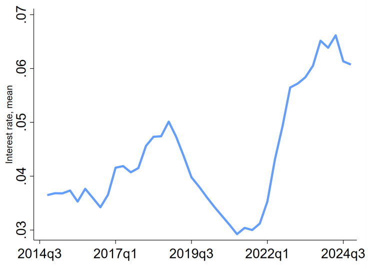
Table: Summary Statistics

	mean	sd	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
ρ (%)	3.75	1.69	2.05	3.69	5.88
r^{firm} (%)	2.82	2.75	0.87	3.04	5.26
r^{social} (%)	3.54	1.88	1.77	3.53	5.71
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.81	1.11	2.55	22.64
Sales (M)	1,254.73	5,923.53	2.17	58.80	1,556.58
Assets (M)	1,770.83	8,956.78	1.06	35.52	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	22.61	29.05	4.68	15.56	44.04
N Loans	62687				
N Firms	38587				
N Fixed Rate	31540				
N Variable Rate	31147				

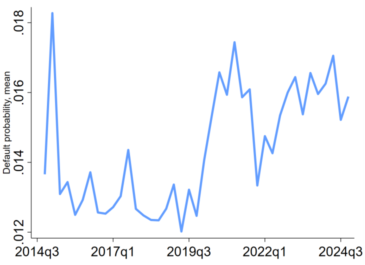
Time series for averages: Contractual Rate, Default, LGD

▷ back

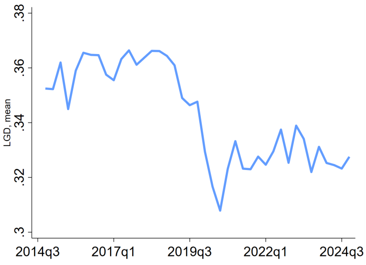
Contractual rate (fixed only)



Default Probability



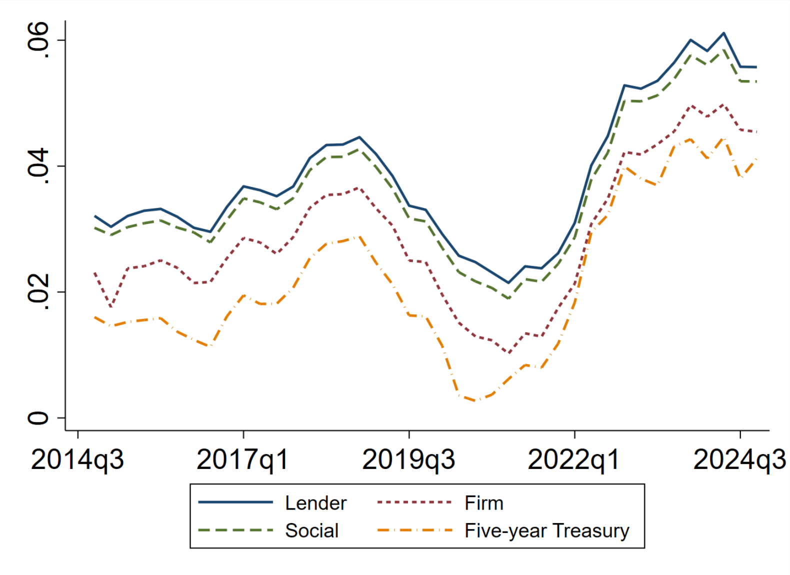
LGD



- 2020-2021: Increase in default probability
- Modest decline in losses given default (better recovery)

Average Discount Rate, Firm and Social Cost of Capital

▷ back



- Rates follow 5y UST
- Financial frictions:
  $\mathbb{E}\left[r_i^{social}\right] > \mathbb{E}\left[r_i^{firm}\right]$
- $\mathbb{E}\left[r_i^{social}\right] \approx \mathbb{E}\left[\rho_i\right]$

Data Cleaning and Sample Construction

Sample period: We use FR Y-14Q Schedule H.1 data from 2014Q4 onward **Borrower Filters:**

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data Cleaning and Sample Construction

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type $= 7$)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or $> 50\%$)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Lender's discount rate

▷ back

Fixed contractual rate:

Lemma 4 (Lender's discount rate)

For a fixed contractual rate loan:

$$1 + \rho_i = P_i (1 + r_i) + (1 - P_i)(1 - LGD_i)$$

▷ *Proof*

- ρ_i is independent of maturity T_i for fixed rate loans
- **Floating rate:** numerical solution of (2)

Forward interest rate expectations

▷ back

To estimate ρ for floating rate loans, we need estimates of $\mathbb{E}_0[r_t]$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0[r_t]$ for each loan, using treasury forward rate plus loan's spread

Lender's discount rate: fixed rate

▷ back

$$1 = \sum_{t=1}^{T} \left(\frac{P}{1+\rho} \right)^{t} \left[r + \frac{(1-P)}{P} (1-LGD) \right] + \left(\frac{P}{1+\rho} \right)^{T}$$

Let $x = \frac{P}{1+\rho}$ so

$$1 = \left(r + \frac{(1-P)}{P} (1-LGD) \right) \frac{x}{1-x} \left(1 - x^{T}\right) + x^{T}$$

Guess that $1 + \rho = (1+r)\,P + (1-P)\,(1-LGD)$

$$\frac{1-x}{x} = \frac{1}{x} - 1 = \frac{(1+r)\,P + (1-P)\,(1-LGD)}{P} - 1 = r + \frac{1-P}{P} (1-LGD)$$

And, therefore

$$1 = 1\left(1 - x^{T}\right) + x^{T}$$

which validates the guess.

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\, Q_{t+1}\right) + \left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho}$$

Note that

$$\begin{aligned} Q_t &= Q_t^P + Q_t^D \\ Q_t^P &= \frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\, Q_{t+1}\right)\right]}{1+\rho} \\ Q_t^D &= \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{1+\rho} \end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k_{t+1}/b_{t+1}\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta + (1-\theta)\, Q_{t+1}\right)\right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t\left[r_{t+1}\right]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

Decomposing misallocation

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r_{social}^{cf,I} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to financial frictions

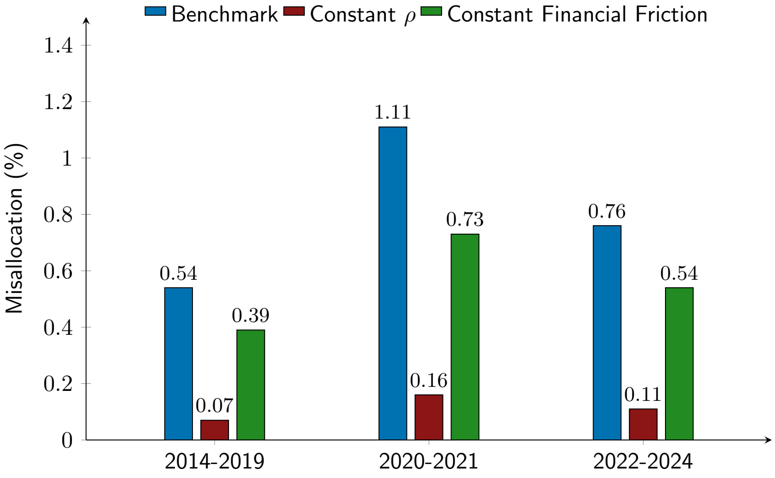
Counterfactual II: what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r_{social}^{cf} $\rightarrow$ Misallocation due to heterogeneous cost of capital

1. The 2020-21 increase: sources of misallocation

▷ back



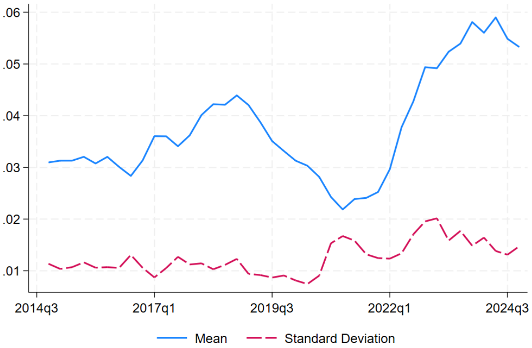
- Mostly driven by heterogeneity in ρ_i
- Interaction between ρ_i and financial frictions $(0.54 > 0.07 + 0.39)$

$\Rightarrow$ 1. Predominantly explained by dispersion in ρ_i

2. The 2020-21 increase: dispersion in ρ_i

▷ back

Heterogeneity in ρ_i is the most important driver of increase in misallocation during 2020-21



- As policy rates decreased in 2020-21, so did the mean ρ_i
- The standard deviation of ρ_i increased during this period

⇒ 2. Sharp rise in the coefficient of variation of ρ_i

3. The 2020-21 increase: role of contractual rates

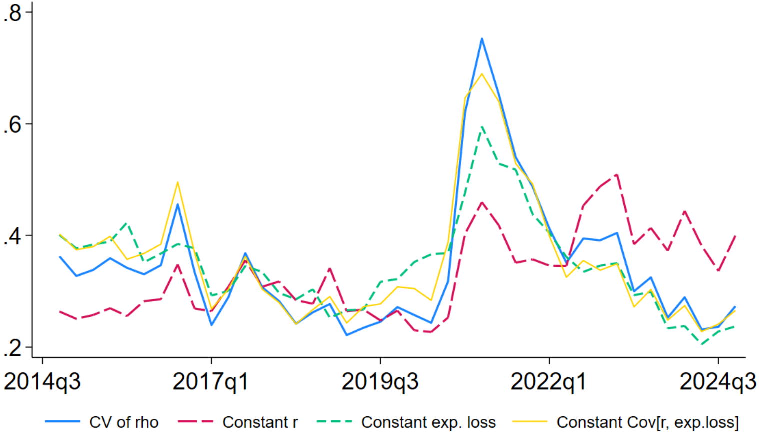
- Approximate $\rho_i \approx r_i - (1 - P_i) LGD_i$

- The coefficient of variation depends on: (i) r_i, (ii) $(1 - P_i) LGD_i$ and (iii) their covariance

$$\frac{\mathbb{V}\left[\rho_i\right]^{0.5}}{\mathbb{E}\left[\rho_i\right]} \approx \frac{\left(\mathbb{V}\left[r_i\right] + \mathbb{V}\left[(1 - P_i) LGD_i\right] - 2\,\mathbb{COV}\left[r_i, (1 - P_i) LGD_i\right]\right)^{0.5}}{\mathbb{E}\left[r_i\right] - \mathbb{E}\left[(1 - P_i) LGD_i\right]}$$

3. Decomposition of the coefficient of variation of ρ_i

▷ back



⇒ 3.Dispersion in ρ_i is traced to changes in the distribution of contractual rates r_i (not P_i or LGD_i)

Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	71.88	1.63	13.45	13.04
Lender discount rate, ρ	61.94	3.08	14.02	20.96
Firm cost of capital, r^{firm}	33.23	4.25	20.12	42.4
Social cost of capital, r^{social}	53.84	3.87	16.21	26.08
N Firms	1681			
N Loans	14738			

Table: Variance decomposition of interest rates and cost of capital (ρ, r^{firm}, and r^{social})

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function Q, γ, and firm leverage Qb'/k' we can compute $\mathcal{M}$

1. Loans are modeled as perpetuities that decay at a geometric rate θ, we can write Q as the present value of all future payments, discounted at the contractual interest rate r:

$$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

- We approximate (the log of) Q as a polynomial of investment, borrowing, productivity and ρ
- Investment: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets (Hsieh and Klenow, 2009)
- Approximation:

$$\begin{aligned}\log Q_i = &\ \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ &+ \beta_{k,k}(\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ &+ \beta_{b,b}(\log b_i)^2 \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ &+ \beta_{z,z}(\log z_i)^2 \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho}(\rho_i)^2 \epsilon_i\end{aligned}$$

- Compute the partial derivatives of $\log Q$ with respect to investment and borrowing.

Estimating $\mathcal{M}$: results

▷ back

- The distribution is extremely concentrated around 1.
- The mean is equal to 0.996 and the median to 0.997, with a standard deviation of 0.006.
- The two measures of misallocation are extremely similar
- Taken together, these results suggest that our assumption that $\mathcal{M} = 1$ is a good one.

Risk Premia & Aggregate Shocks

▷ back

- Alternative hypothesis: Rise in ρ reflects higher **risk premia** as lenders demand extra compensation amid extreme uncertainty (e.g. COVID-19).
- Firms differ in exposure to aggregate shocks $\Rightarrow$ heterogeneous risk premia need not imply misallocation (David et al., 2022).
- Our framework is steady-state $\Rightarrow$ cannot model time-varying aggregate shocks or risk-premium spikes.
- Data contradict the risk-premia story:
    - Average ρ **falls** from 3.6% (2014-19) to 2.7% (2020-21).
    - Skewness becomes **more negative**: $-2.6 \rightarrow -3.5$ (left tail thickens).
- Interpretation: Risk premia likely **declined**, perhaps owing to explicit/implicit policy guarantees.

Cross-country comparison

▷ back

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mu(r_i)$, %	78.7	14.1	83.0	16.8	3.9
$\sigma(r_i)$, %	38.1	2.9	93.3	5.2	1.5
$\mu(1-P_i)$, %	2.7	16.9	4.0	8.9	1.4
$\mu(1-LGD_i)$, % (World Bank)	42.8	42.8	18.2	63.9	81.0
Implied misallocation, %	4.9	2.2	21.5	1.7	0.6

- Developing countries: higher mean and standard deviation of contractual rates
- U.S.: lower mean and standard deviation of contractual rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.
- Misallocation in the U.S. small but non-trivial: 0.6%.

Approximating misallocation

▷ back

- Recovery rates from the World Banks Doing Business report
- Approximate r^{social} with ρ in the SS for misallocation
- Use the fixed rate formula for ρ and assume that (P, LGD) are constant across firms
- Approximated cost of misallocation for the US is similar to the actual cost

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