

# The Cost of Capital and Misallocation in the United States

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# Cost of capital and misallocation

**Research question:** How does dispersion in the cost of capital affect misallocation?

## Traditional misallocation approach:

- Strong assumptions about production functions (homogeneous Cobb-Douglas)
- Measure heterogeneity in marginal products from cross-sectional input data
- Estimate capital misallocation

## Our macrofinance approach:

- Main idea: cost of capital equals marginal product of capital
- Combine credit registry data with model to carefully measure cost of capital
- Use dispersion in cost of capital to quantify welfare losses from imperfect credit markets

# The cost of capital and misallocation in the US

## Methodological contribution:

- Adapt a standard **dynamic corporate finance model** for measurement with **loan-level data**
- Derive a **sufficient statistic** for misallocation based on **credit registry data**

## Empirical results for the US:

- Average cost of capital tracks treasury rates, with a spread
- Measures of cost of capital correlate with traditional measures of  $ARPK_i$  at the firm level
- Credit markets seem quite efficient in normal times—misallocation losses of 0.9% of GDP
- Losses from misallocation increased to 1.8% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

## Related literature

- **Measuring misallocation:**

- Seminal work: Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
- Challenge: Standard methods rely on strong assumptions (Haltiwanger et al., 2018).
- Recent advances: Experimental/quasi-experimental methods to recover marginal products directly (Carrillo et al., 2023; Hughes and Majerovitz, 2025).
- **Contribution:** use **heterogeneity in funding costs** to measure **dispersion in MRPK**

- **Heterogeneity in the cost of capital:**

- Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
- US: Gilchrist, Sim, and Zakrajsek (2013), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
- **Contribution:**
  - Estimate firm cost of capital using **credit registry data**, correcting for loan characteristics, etc.
  - Derive and estimate **sufficient statistic** for misallocation

# Outline

Model

Welfare and Misallocation

Measurement with credit registry data

Misallocation in the US

Extensions & Robustness

Model

## Borrower-Lender model

- Discrete time, infinite horizon
- Continuum of firms, each matched with a lender
- No aggregate risk (for now - work in progress!)

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- Invest in capital  $k_i$
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- Limited liability

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- Recover  $\phi_i k_i$  in default
- Lenders price loans to break even

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**Key question:** how do heterogeneity in  $\rho_i$  and financial frictions distort the allocation of capital?

# Model

**Firm value function:**

$$V_i(k_i, b_i, z_i) = \max_{k'_i, b'_i} \pi_i(k_i, b_i, z_i, k'_i, b'_i) + \beta \mathbb{E} \left[ \overbrace{\max \{V_i(k'_i, b'_i, z'_i), 0\}}^{\text{Limited liability}} \mid z_i \right]$$

**Firm profits:**

$$\pi_i(k_i, b_i, z_i, k'_i, b'_i) = f(k_i, z_i) + (1 - \delta)k_i - k'_i - \theta b_i + Q_i(k'_i, b'_i, z_i) [b'_i - (1 - \theta_i)b_i]$$

**Price of debt:**

$$Q_i(k'_i, b'_i, z_i) = \frac{\mathbb{E} \left\{ \overbrace{\mathcal{P}_i(k'_i, b'_i, z'_i)}^{\text{repayment prob.}} [\theta_i + (1 - \theta_i) Q_i(k''_i, b''_i, z'_i)] + (1 - \mathcal{P}_i(k'_i, b'_i, z'_i)) \overbrace{\frac{\phi_i k'_i}{b'_i}}^{\text{recovery}} \mid z_i \right\}}{1 + \rho_i}$$

lender discount rate

## Firm optimality: cost of capital vs. expected MRPK

▷ details

- **Cost of capital:**

$$\underbrace{\frac{\mathbb{E}[\mathcal{P}'_i(\theta_i + (1 - \theta_i)Q'_i) | z_i]}{Q_i}}_{1 + r_i^{\text{firm}}} \times \underbrace{\left[ \frac{1 - \frac{\partial Q_i}{\partial k'_i} [b'_i - (1 - \theta_i)b_i]}{1 + \frac{\partial Q_i}{\partial b'_i} \frac{[b'_i - (1 - \theta_i)b_i]}{Q_i}} \right]}_{\mathcal{M}_i}$$

- $1 + r_i^{\text{firm}}$ : implied interest rate perceived by the firm.
- $\mathcal{M}_i$ : price-impact term capturing how  $(k'_i, b'_i)$  affects the debt price  $Q_i$ .

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$$\underbrace{\frac{\mathbb{E}[\mathcal{P}'_i(\theta_i + (1 - \theta_i)Q'_i) | z_i]}{Q_i}}_{1+r_i^{\text{firm}}} \times \underbrace{\left[ \frac{1 - \frac{\partial Q_i}{\partial k'_i} [b'_i - (1 - \theta_i)b_i]}{1 + \frac{\partial Q_i}{\partial b'_i} \frac{[b'_i - (1 - \theta_i)b_i]}{Q_i}} \right]}_{\mathcal{M}_i}$$

- $1 + r_i^{\text{firm}}$ : implied interest rate perceived by the firm.
- $\mathcal{M}_i$ : price-impact term capturing how  $(k'_i, b'_i)$  affects the debt price  $Q_i$ .
- **Optimality:** Optimal investment equates the firms financing cost to the expected MRPK

$$(1 + r_i^{\text{firm}}) \mathcal{M}_i = \underbrace{\mathbb{E}[\mathcal{P}'_i(f_k(k'_i, z'_i) + 1 - \delta) | z_i]}_{\text{expected MRPK}}$$

- **Measurement idea:** dispersion in  $r_i^{\text{firm}}$  (from loan data)  $\Rightarrow$  dispersion in MRPK  $\Rightarrow$  misallocation.

# Firm's cost of capital

## Lemma 1 (Firm's cost of capital)

*The firm's cost of capital is:*

$$1 + r_i^{\text{firm}} = \frac{1 + \rho_i}{1 + \Lambda_i}$$

$$\Lambda_i := \frac{\mathbb{E}[(1 - \mathcal{P}'_i) \phi_i k'_i / b'_i | k'_i, b'_i, z_i]}{\mathbb{E}[\mathcal{P}'_i (\theta + (1 - \theta_i) Q'_i) | k'_i, b'_i, z_i]}$$

▷ *Proof*

- $\Lambda_i$  is a wedge due to financial frictions, positive if lender recovers in default.
- In general,  $r_i^{\text{firm}} < \rho_i$ , since lender recovers some in default, but firm pays zero.

## Welfare and Misallocation

## Aggregate economy and welfare

- Aggregate resource constraint:

$$Y_{t+1} + (1 - \delta)K_{t+1} = \int_0^1 \mathbb{E}_t [\mathcal{P}_{i,t+1} (f(k_{i,t+1}, z_{i,t+1}) + (1 - \delta)k_{i,t+1}) + (1 - \mathcal{P}_{i,t+1}) \cdot \phi_i k_{i,t+1}] di$$

# Aggregate economy and welfare

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- Let  $\omega_{i,t}(S^t) \in \{0, 1\}$  denote whether a firm operates or not
- Assume that existing firms are replaced by identical ones
- Planner's problem:

$$\begin{aligned} U^* &= \max_{\{ \{k_{i,t}(S^{t-1}), \omega_{i,t}(S^t) \}_{i \in [0,1]} \}_{t=1}^\infty} \sum_{t=0}^\infty \beta^t \cdot u(C_t) \\ \text{s.t.} \quad K_t &= \int_0^1 k_{i,t}(S^{t-1}) di \\ C_t + K_{t+1} &= Y_t + (1 - \delta)K_t \\ \omega_{i,t+1}(S^{t+1}) &\leq \omega_{i,t}(S^t) \quad \forall S^t \subset S^{t+1}, \forall i \end{aligned}$$

# Aggregate economy and welfare

- Can separate planner's problem into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\{K_t, \{\omega_{i,t}(S^t)\}_{i \in [0,1]}\}_{t=1}^\infty} \sum_{t=0}^{\infty} \beta^t \cdot u \left( \left( \max_{\{\{k_{i,t}(S^{t-1})\}_{i \in [0,1]}\}_{t=1}^\infty} Y_t \right) - I_t \right)$$

- Rewrite inner problem as:

$$Y_t^* \left( K_t, \{\omega_{it}\}_{i \in [0,1]} \right) = \max_{\{k_{i,t}^*\}_{i \in [0,1]}} \int_0^1 \mathbb{E}_{t-1} \{ \omega_{it} \cdot f(k_{it}^*; z_{it}) - (1 - \omega_{it}) \cdot [(1 - \delta) k_{it}^* - \phi_i k_{it}^*] \} di$$

s.t.  $K_t = \int_0^1 k_{it}^* di$

# Aggregate economy and welfare: Intensive-margin misallocation

- Redistribute capital taking exit decisions and aggregate capital as given.
- Misallocation on the intensive margin is the main focus of the misallocation literature (e.g. Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008).
- Redistribute  $\{k_{i,t+1}\}_i$  taking exit decisions  $\{\mathcal{P}_{i,t+1}^{DE}\}_{i \in [0,1]}$  and  $K_{t+1}^{DE}$  as given

$$\begin{aligned} & \max_{\{k_{i,t+1}^*\}_i} \int_0^1 \mathbb{E}_t \left[ \mathcal{P}_{i,t+1}^{DE} \left( f(k_{i,t+1}^*, z_{i,t+1}) + (1 - \delta) k_{i,t+1}^* \right) + (1 - \mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^* \right] di \\ \text{s.t. } & \int_0^1 k_{i,t+1}^* di = K_{t+1}^{DE} \end{aligned}$$

# Social return on capital

- Define the **social marginal product of capital at firm  $i$** ,  $r_{i,t}^{social}(k)$

$$1 + r_{i,t}^{social}(k) \equiv \mathbb{E} [\mathcal{P}_{i,t+1}^{DE} (f_k(k, z_{i,t+1}) + 1 - \delta) + (1 - \mathcal{P}_{i,t+1}^{DE}) \phi_i]$$

- Social return includes expected recovery in default, which is not taken into account by the firm.
- Planner Optimality:** The planner chooses  $k_{i,t+1}^*$  to **equalize**  $r_{i,t}^{social}(k_{i,t+1}^*)$  across firms.
- Equilibrium:** Dispersion in  $r_{i,t}^{social}(k_{i,t+1}^{DE}) \rightarrow$  misallocation.

# Misallocation in the decentralized equilibrium

- In the decentralized equilibrium:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

- Hence:

$$1 + r_{i,t}^{social}(k_{i,t+1}^{DE}) = (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - \mathcal{P}_{i,t+1}^{DE})\phi_i$$

- **Measurement idea:** dispersion in  $r_{i,t}^{firm}$  (from loan data)  $\Rightarrow$  dispersion in  $r_{i,t}^{social} \Rightarrow$  misallocation.

# Sufficient statistic for misallocation

## Proposition 1 (Misallocation)

Misallocation can be measured with  $\mathbb{E}[r_i^{social}]$  and  $\text{Var}(r_i^{social})$  as

$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}(r_i^{social})}{(\mathbb{E}[r_i^{social}] + \delta)^2}\right)$$

▷ *Proof*

- Extends Hughes and Majerovitz (2025) to a dynamic economy with default
- Measures intensive-margin misallocation.
- Calibrate  $\mathcal{E} = \frac{1}{2}$  (elasticity of output w.r.t.  $r^{social} + \delta$ ) and  $\delta = 0.06$
- **Next:** show how to measure  $r_i^{social}$  using credit registry data

▷ Calibration

Measurement with credit registry data

- Data: FR Y-14Q (Schedule H.1)
- Universe: all C&I loans  $\geq$  \$1M (2014Q4 - 2024Q4)
- Coverage: top 40 BHCs ( $\approx$  91 % of C&I lending)
- Variables: interest rate, spread, PD, LGD, maturity, type.
- Focus on term loans issued to non-government, non-financial US firms

## Pricing term loans

For a loan  $i$  originated at  $t$ , the **break-even** condition for a lender with discount rate  $\rho_{i,t}$  is

$$1 = \sum_{s=1}^{T_{i,t}} \left[ \frac{(P_{i,t})^s \cdot \mathbb{E}_t(r_{i,t,s}) + (P_{i,t})^{s-1} \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}{(1 + \rho_{i,t})^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{(P_{i,t})^{T_{i,t}}}{(1 + \rho_{i,t})^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- $T_{i,t}$ : maturity
- $\mathbb{E}_t[r_{i,t,s}]$ : fixed rate or spread over benchmark rate (Gürkaynak et al., 2007) ▷ forward rates
- $P_{i,t}$ : repayment probability (constant over time)
- $LGD_{i,t}$ : loss given default (constant over time)
- $\mathbb{E}_t(\Pi_{t,s})$ : total expected inflation from  $t$  to  $s$  (Cleveland Fed)
- $\Rightarrow$  Solve for lender's discount rate:  $\rho_{i,t}$  ▷ fixed real rate

# Measuring firm and social cost of capital

## Lemma 2 (Firm and social cost of capital)

*We can write the firm cost of capital as*

$$1 + r_{i,t}^{firm} = (1 + \rho_{i,t}) - (1 - P_{i,t})(1 - LGD_{i,t})$$

*The social cost of capital can be written as:*

$$\begin{aligned} 1 + r_{i,t}^{social} &= (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - P_{i,t})(1 - LGD_{i,t})lev_{i,t} \\ &= \underbrace{(1 + \rho_{i,t})\mathcal{M}_{i,t}}_{\text{lender discount rate}} + \underbrace{(lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}_{\text{wedge due to financial frictions}} \end{aligned}$$

▷ *Proof*

$r^{firm} \leq r^{social} \leq \rho$ : firms face lower perceived cost of capital because lenders recover in default

## Wedge due to financial friction

$$(lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})$$

- **Lenders** care about average recovery per dollar of debt:  $\phi_i(k_i)/b_i = \mathcal{M}_i(1 - LGD_i)$
- **Planner** cares about the marginal recovery:  $\phi'_i(k_i) = (1 - LGD_i) \times lev_i$
- Coincide when  $lev_i = \mathcal{M}_i$

## Sufficient statistic for misallocation

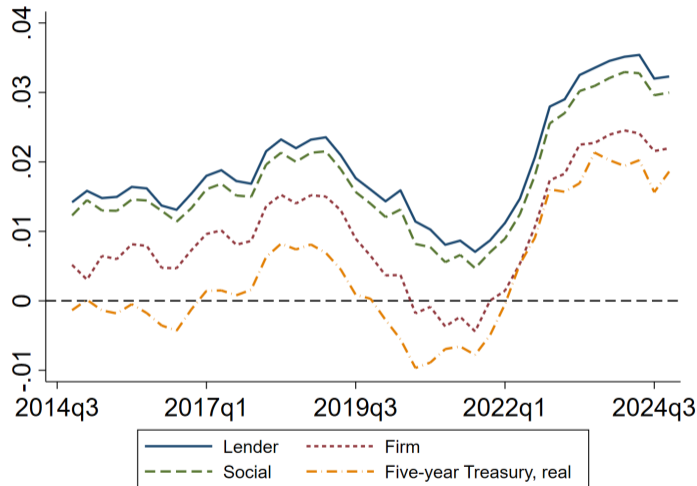
$$\log(Y_t^*/Y_t^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log \left( 1 + \frac{\text{Var}(r_{i,t}^{social})}{(\mathbb{E}[r_{i,t}^{social}] + \delta)^2} \right)$$
$$1 + r_{i,t}^{social} = (1 + \rho_{i,t}) \mathcal{M}_{i,t} + (lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})$$

- Set  $\mathcal{M}_{i,t} = 1$ ; reasonable approximation given our model
- Can measure misallocation directly with credit registry data!
- Dispersion in  $r_{i,t}^{social}$  comes from:
  1. Dispersion in lender's discount rate,  $\rho_{i,t}$
  2. Dispersion in financial frictions wedge
  3. Covariance between  $\rho_{i,t}$  and financial frictions wedge

▷ Estimate  $\mathcal{M}$

## Misallocation in the US

# Time series for average discount rate, firm and social cost of capital



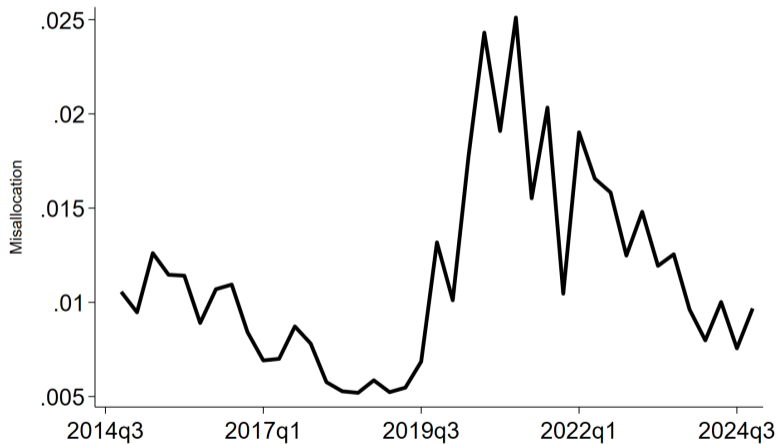
- Rates follow 5y UST
- Financial frictions:  
 $\mathbb{E}[r_i^{social}] > \mathbb{E}[r_i^{firm}]$
- $\mathbb{E}[r_i^{social}] \approx \mathbb{E}[\rho_i]$

## Estimates for lender discount rate, firm and social cost of capital

	Mean	SD	p10	p50	p90
$\rho$ (%)	1.87	1.55	0.41	1.88	3.62
$r^{firm}$ (%)	0.92	2.80	-0.86	1.26	3.03
$r^{social}$ (%)	1.66	1.78	0.12	1.73	3.47

- Financial frictions/recovery:  $\mathbb{E}[r_{i,t}^{firm}] < \mathbb{E}[r_{i,t}^{social}], \mathbb{E}[\rho_{i,t}]$

## Output losses from capital misallocation



- About 0.9% before 2020
- ↑ to 1.8% in 2020-2021
- ↓ to 1.2% in 2022-2024

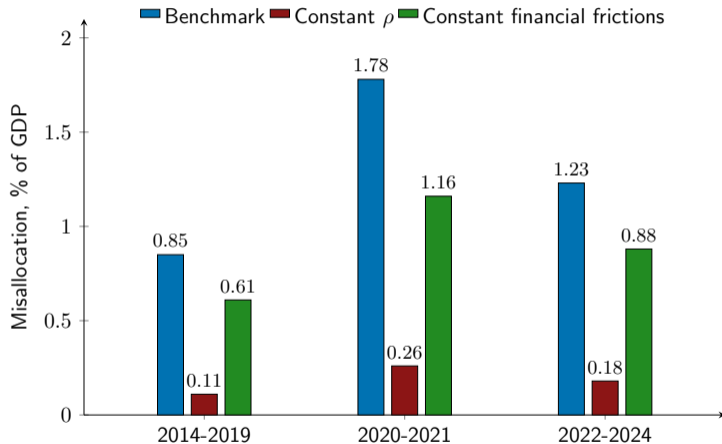
## The 2020–2021 increase in misallocation

1. Driven by dispersion in lender discount rates  $\rho_i$ , not financial frictions.
2. Sharp rise in the coefficient of variation of  $\rho_i$ .
3. Variance of  $\rho_i$  increases due to increased dispersion of expected losses.

⇒ Likely linked to policy-induced underpricing of risky loans and implicit guarantees.

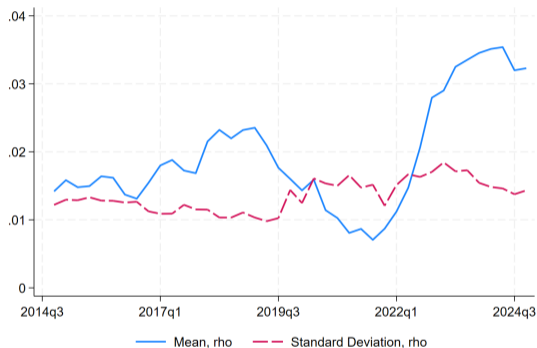
# 1. The 2020-21 rise in misallocation was driven by $\{\rho_i\}$

▷ details



- Main driver: dispersion in lender discount rates
- Interaction between  $\rho_i$  and financial frictions ( $0.85 > 0.11 + 0.61$ )

## 2. The CV of $\rho_i$ increased during 2020-21



- As policy rates decreased in 2020-21, so did mean  $\rho_i$
- Standard deviation of  $\rho_i$  increased during this period

⇒ 2. Sharp rise in the coefficient of variation of  $\rho_i$

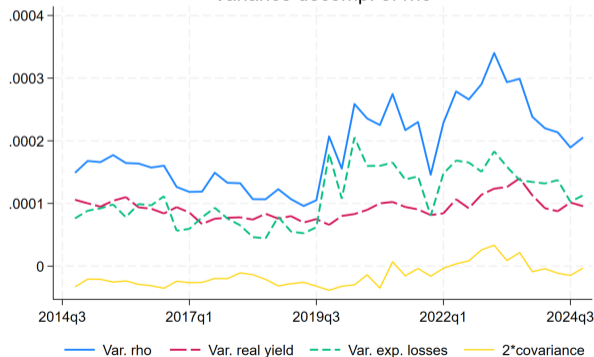
### 3. Variance of $\rho$ related to variance of expected losses

- Compute **real yield**  $\rho_{i,t}^*$ : the rate implied if no default

▷ real yield

- Decomposition:  $\rho_i = \underbrace{\rho_i^*}_{\text{real yield}} + \underbrace{[\rho_i - \rho_i^*]}_{\text{exp. losses}}$

Variance decomp. of rho



Variance of  $\rho_i$ :

$$\mathbb{V}[\text{yield}] + \mathbb{V}[\text{exp. losses}] + 2\mathbb{C}[\text{yield}, \text{exp. losses}]$$

- Increase in variance explained by exp. losses
- Likely linked to policy-induced underpricing of risky loans and implicit guarantees.

## Extensions & Robustness

## Variance decomposition

	Time	Bank	Firm	Loan
Contractual rate	69	2	15	15
Real rate	49	4	25	22
$\rho$	43	4	23	30
$r^{firm}$	17	4	31	49
$r^{social}$	35	4	25	36

Notes: 1,844 firms and 16,088 loans. Sample restricted to firms with at least five securities.

Within-period dispersion of  $r^{social}$ :

- Bank 6%
- Firm 38%
- Loan 55%

Large dispersion even within a quarter-bank-firm relationship.

# Extensions & Robustness

1. Estimate heterogeneous price-impact term  $\mathcal{M}$ .

▷ [heterogeneous  \$\mathcal{M}\$](#)

2. Validate  $r^{social}$  using firm-level ARPK measures.

▷ [details](#)

3. Application to cross-country data.

▷ [details](#)

## Work in progress

1. Aggregate risk
2. Quantitative model

# Conclusion

- Framework to measure misallocation from credit registry data.
  1. Standard dynamic corporate finance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Uses standard credit registry variables ( $r, P, LGD, T, \dots$ )
- Application to U.S. credit registry data
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 1% in normal times
  3. Rise in 2020-21, driven by increase in variance of expected losses

Credit markets in the US appear efficient, but crisis interventions can amplify misallocation.

# Appendices

Firm FOCs:

$$[k'_i] : \quad -1 + \frac{\partial Q_i(k'_i, b'_i, z_i)}{\partial k'_i} [b'_i - (1 - \theta_i)b_i] + \beta \mathbb{E} \{ \mathcal{P}_i(k'_i, b'_i, z'_i) [f_k(k'_i, z'_i) + 1 - \delta] | z_i \} = 0$$

$$[b'_i] : \quad \frac{\partial Q_i(k'_i, b'_i, z_i)}{\partial b'_i} [b'_i - (1 - \theta_i)b_i] + Q_i(k'_i, b'_i, z_i) - \beta \mathbb{E} \{ \mathcal{P}_i(k'_i, b'_i, z'_i) [\theta_i + (1 - \theta_i)Q_i(k''_i, b''_i, z'_i)] | z_i \} \\ = 0$$

## Lemma 3 (Firm's cost of capital)

*The firm's cost of capital is:*

$$1 + r_i^{\text{firm}} = \frac{1 + \rho_i}{1 + \Lambda_i}$$

$$\Lambda_i := \frac{\mathbb{E}[(1 - \mathcal{P}'_i) \phi_i k'_i / b'_i | k'_i, b'_i, z_i]}{\mathbb{E}[\mathcal{P}'_i (\theta + (1 - \theta_i) Q'_i) | k'_i, b'_i, z_i]}$$

▷ *Proof*

- $\Lambda_i$  is a wedge due to financial frictions, positive if lender recovers in default.
- In general,  $r_i^{\text{firm}} < \rho_i$ , since lender recovers some in default, but firm pays zero.

$$\begin{aligned}\frac{1}{Q_t} \mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})] &= \frac{(1 + \rho) \mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})] + \mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']} \\ &= (1 + \rho) \left( 1 + \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]} \right)^{-1} \\ &= (1 + \rho) (1 + \Lambda)^{-1}\end{aligned}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}$$

- Formally, planner's problem is now the same as solving  $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i) di$ , where  $f_i(k_i)$  is now expected output
- Apply Hughes and Majerovitz (2024), noting  $\frac{dY}{dk} = r^{social} + \delta$

$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}(r^{social})}{(\mathbb{E}[r^{social}] + \delta)^2}\right)$$

- $\mathcal{E}$  is (negative) elasticity of output w.r.t. cost of capital ( $r^{social} + \delta$ )

- $\mathcal{E}_i$  is the elasticity of expected output with respect to the cost of capital
- Assume that  $f(k, z) = z \cdot k^\alpha$  and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1 - \alpha}$$

- $\alpha = \frac{1}{3}$  implies  $\mathcal{E} = \frac{1}{2}$

# Summary Statistics

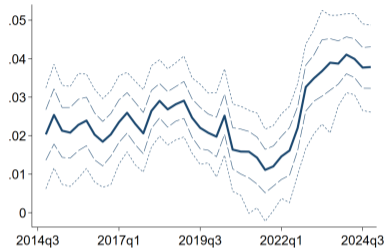
[▷ back](#)

	Mean	St. Dev.	p10	p50	p90
Interest rate	4.18	1.69	2.21	3.94	6.60
Maturity (yrs)	6.83	4.65	3.00	5.00	10.25
Real interest rate	2.39	1.24	0.88	2.33	4.00
Prob. Default (%)	1.45	2.53	0.19	0.85	2.88
LGD (%)	34.41	13.17	16.00	35.60	50.00
Loan amount (M)	10.75	67.58	1.11	2.57	22.92
Sales (M)	1,269.93	6,051.48	2.16	58.50	1,560.10
Assets (M)	1,760.37	8,894.15	1.07	35.55	1,782.22
Leverage (%)	72.17	24.68	42.68	71.29	100.00
Return on assets (%)	27.60	58.51	4.56	15.76	47.81
N Loans	65,284				
N Firms	38,751				
N Fixed Rate	32,592				
N Variable Rate	32,692				

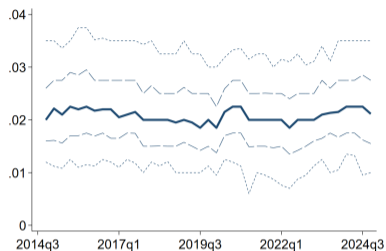
# Time series for averages: real interest rate, PD, LGD

[▷ back](#)

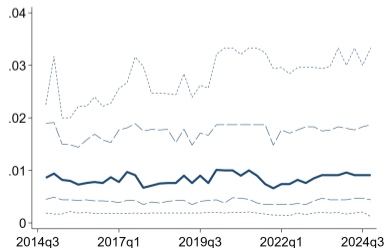
## Real interest rate



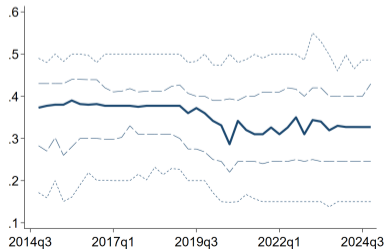
## Interest rate spread (var.)



## Probability of default



## Loss given default



# Data cleaning and sample construction

▷ back

We use FR Y-14Q Schedule H.1 data from 2014Q4 to 2024Q4.

## Borrower Filters:

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

# Data cleaning and sample construction, cont'd

▷ back

## Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only “vanilla” term loans (Facility type = 7)
- Drop loans with:
  - Mixed-interest rate structures
  - Maturity less than 1 year or longer than 10 years
  - Implausible interest rates or spreads (outside 1st - 99th percentile)
  - Missing or invalid PD/LGD values (outside  $[0, 1]$ )
  - PD = 1 (flagged as in default)

# Forward interest rate expectations

▷ back

To estimate  $\rho_i$  for floating rate loans, need estimates of  $\mathbb{E}_0[r_t] + s_i$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Average spread between SOFR and Treasury rates 2018-2025  $\simeq 2$  basis points
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out  $\mathbb{E}_0[r_t] + s_i$  for each loan, using treasury forward rate plus loan's spread

$$Q_t = \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1}) + (1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{1 + \rho}$$

Note that

$$\begin{aligned} Q_t &= Q_t^P + Q_t^D \\ Q_t^P &= \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{1 + \rho} \\ Q_t^D &= \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{1 + \rho} \end{aligned}$$

That is, we strip the bond into the payment in repay ( $Q_t^P$ ) and the payment in default ( $Q_t^D$ ). Then:

$$\Lambda = \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]} = \frac{Q_t^D}{Q_t^P}$$

## Firm cost of capital: measurement

▷ back

The firm defaults with probability  $(1 - P)$  and the lender recovers  $(1 - LGD)$ . Hence

$$Q_t^{D,data} = \frac{(1 - P)(1 - LGD)}{1 + \rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^T \left[ \frac{P^s \mathbb{E}_t[r_{t+s}] + P^{s-1} (1 - P)(1 - LGD)}{(1 + \rho)^s} \right] + \frac{P^T}{(1 + \rho)^T}$$
$$1 = \frac{(1 - P)(1 - LGD)}{1 + \rho} + P \frac{\mathbb{E}_t[r_{t+1}]}{1 + \rho} + \left( \sum_{s=2}^T \left[ \frac{P^s \mathbb{E}_t[r_{t+s}] + P^{s-1} (1 - P)(1 - LGD)}{(1 + \rho)^s} \right] + \frac{P^T}{(1 + \rho)^T} \right)$$

So, we can define  $Q_t^{P,data}$  as  $1 = Q_t^{P,data} + Q_t^{D,data}$  so  $Q_t^{P,data} = 1 - Q_t^{D,data}$ . Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1 - P)(1 - LGD)}{1 + \rho - (1 - P)(1 - LGD)}$$

# Decomposing misallocation

▷ back

**Counterfactual I:** What if all lenders have the same  $\bar{\rho}$ ?

$$1 + r_{social}^{cf,I} = \overline{(1 + \rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in  $r_{social}^{cf}$  → Misallocation due to financial frictions

**Counterfactual II:** what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1 + \rho) \mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in  $r_{social}^{cf}$  → Misallocation due to heterogeneous cost of capital

- The “real yield” is the implied  $\rho_{i,t}^*$  when  $P_{i,t} = 1$

$$1 = \sum_{s=1}^{T_{i,t}} \left[ \frac{\mathbb{E}_t(r_{i,t,s})}{\left(1 + \rho_{i,t}^*\right)^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{1}{\left(1 + \rho_{i,t}^*\right)^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- Real yield independent of  $P_{i,t}$ ,  $LGD_{i,t}$
- Only affected by losses through the contractual rate  $r$

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \frac{\partial \log Q}{\partial \log b'}}$$

Need  $Q$ ,  $\gamma$ , and firm leverage  $Qb'/k'$  to compute  $\mathcal{M}$

1. To compute  $Q$ , assume that loans are perpetuities that decay at a geometric rate  $\theta$ , discounted at the loan's real interest rate  $r$ :

$$Q = \frac{\theta + (1 - \theta)Q}{1 + r} = \frac{\theta}{r + \theta}$$

$r$  is directly observed in the data, and we can approximate  $\theta = 1/T$

2. Guess a functional approximation  $Q(z, k, b, \rho)$
3. Estimate  $\log \hat{Q}(z, k, b, \rho)$  for every loan origination; compute partial derivatives
4. At steady state,  $\gamma = \theta = 1/T$

## Estimating $\mathcal{M}$ : $Q$ elasticities

▷ back

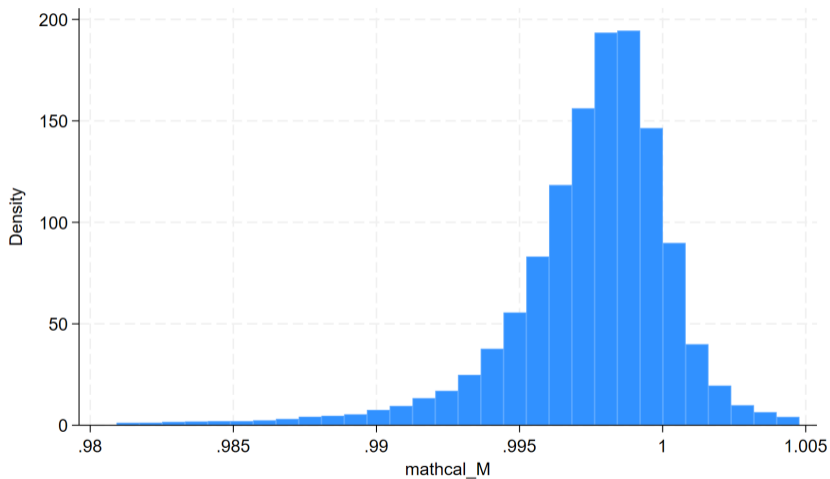
- We approximate (the log of)  $Q$  as a polynomial of firm capital, borrowing, productivity and  $\rho$

$$\begin{aligned}\log Q_i = & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ & + \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ & + \beta_{b,b} (\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ & + \beta_{z,z} (\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 + \epsilon_i\end{aligned}$$

- Capital: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets
- This allows us to compute  $\frac{\partial \log Q}{\partial \log k'}$  and  $\frac{\partial \log Q}{\partial \log b'}$

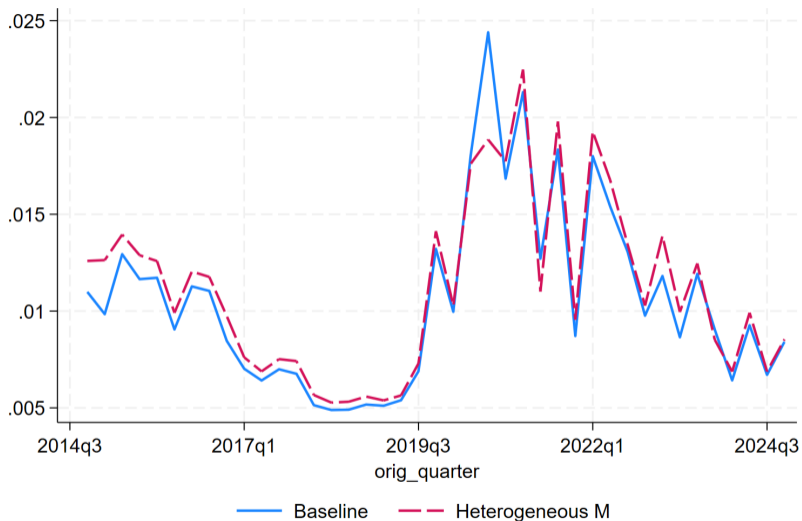
# Estimating $\mathcal{M}$ : results

[▷ back](#)



# Misallocation with heterogeneous $\mathcal{M}$

▷ [back](#)



# $r^{social}$ correlates with standard measures of ARPK

[▷ back](#)

	(1)	(2)	(3)	(4)	(5)
	$\log(ARPK)$ , Sales	$\log(ARPK)$ , EBITDA	$\log(ARPK)$ , Sales	$\log(ARPK)$ , EBITDA	$\log(ARPK)$ , VA
$\log(r^{social} + \delta)$	0.15*** (0.03)	0.24*** (0.04)	0.16** (0.07)	0.15* (0.08)	0.39*** (0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat

Robust standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

# ARPK-based misallocation

▷ other measures

Focus on Compustat firms to make measures comparable

	$r^{social} + \delta$	$\frac{\text{Sales}}{\text{Capital}}$	$\frac{\text{EBITDA}}{\text{Capital}}$	$\frac{\text{Value Added}}{\text{Capital}}$
$Var(\log)$	0.01	0.19	0.24	0.21
Misallocation (%)	0.36	4.75	6.20	5.28

- Our measure looks only at misallocation coming from heterogeneity in the cost of capital
- ...but does not require detailed data on firm financials (i.e., value added)
- $\implies$  directly applicable to most existing credit registries

## Cross-country comparison, approximation

[▷ details](#)

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
Mean real rate, %	66.8	8.00	83.0	12.4	1.4
SD real rate, %	38.1	2.9	93.3	5.2	1.2
Mean def. prob., %	2.7	16.9	4.0	8.9	1.5
Mean recovery rate, %	42.8	42.8	18.2	63.9	66.6
Implied misallocation, %	6.5	13.5	21.5	2.8	0.8

- **Developing countries:** higher mean and standard deviation of real interest rates
- **U.S.:** lower mean and standard deviation of interest rates, **higher recovery**
- **Brazil:** most extreme misallocation: 21.5%.

	(1)	(2)	(3)	(4)	(5)
	log <i>ARPK</i> , Sales	log <i>ARPK</i> , EBITDA	log <i>ARPK</i> , Sales	log <i>ARPK</i> , EBITDA	log <i>ARPK</i> , VA
$\log(r^{social} + \delta)$	0.15*** (0.03)	0.24*** (0.04)	0.16** (0.07)	0.15* (0.08)	0.39*** (0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat
Var(log <i>ARPK</i> )	1.97	1.52	0.19	0.24	0.21
Misalloc., <i>ARPK</i> , %	63.63	46.08	4.75	6.20	5.28
Var(log( $r^{social} + \delta$ ))	0.04	0.04	0.01	0.01	0.01
Misalloc., $r^{social} + \delta$ , %	0.96	0.96	0.36	0.36	0.36

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## Details on cross-country comparison

[▷ back to loan pricing](#) [▷ back to cross-country](#)

- For a fixed real interest rate  $r_{i,t}$ ,  $\rho$  has a closed-form:

$$1 + \rho_{i,t} = P_{i,t} (1 + r_{i,t}) + (1 - P_{i,t}) (1 - LGD_{i,t})$$

- Assume all loans have the same maturity:
  - Obtain mean real rate by subtracting average realized inflation from mean nominal rate
  - Inflation should not affect standard deviation of nominal rates (or spreads)
- Assume all loans have the same  $P_{i,t}$ ,  $LGD_{i,t}$ , equal to the average
- Recovery rates and inflation rates from the World Bank
- Approximate  $r_{i,t}^{social} \simeq \rho_{i,t}$  and compute misallocation using our formula:

$$\log(Y_t^*/Y_t^{DE}) = \frac{1}{2} \mathcal{E} \log \left( 1 + \frac{\text{Var}(\rho_{i,t})}{(\mathbb{E}[\rho_{i,t}] + \delta)^2} \right)$$