Scarring Body and Mind: The Long-Term Belief-Scarring Effects of COVID-19

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Abstract

The largest economic cost of the COVID-19 pandemic could arise if it changed behavior long after the immediate health crisis is resolved. A common explanation for such a long-lived effect is the scarring of beliefs. We show how to quantify the extent of such belief changes and determine their impact on future economic outcomes. We find that the long-run cost of the COVID crisis for the U.S. economy is many times higher than the estimates of the short-run cost. Even if a vaccine cures everyone in a year, fears of future pandemics will change economic choices in ways that have costs for many years to come.

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One of the most pressing economic questions of the day is what long-run cost will arise from the COVID-19 pandemic. While the virus will eventually pass, vaccines will be developed, and workers will return to work, an event of this magnitude could leave lasting effects on the nature of economic activity. Economists are actively debating whether the recovery will be V-shaped, U-shaped or L-shaped.[1] Much of this discussion revolves around confidence, fear and the ability of firms and consumers to rebound to their old investment and spending patterns. Our goal is to formalize this discussion and quantify these effects, both in the short- and long-run. To explore these conjectures about the extent to which the economy will rebound from this COVID-induced downturn, we use a standard economic and epidemiology framework, with one novel channel: a "scarring effect." Scarring is a persistent change in beliefs about the probability of an extreme, negative shock to the economy. We use a version of Kozlowski et al. (2020), to formalize this scarring effect and quantify its the long-run economic consequences, under different scenarios for the dynamics of the crisis.

We start from a simple premise: No one knows the true distribution of shocks in the economy. Consciously or not, we all estimate the distribution using economic data, like an econometrician would. Tail events are those for which we have little data. Scarce data makes new tail event observations particularly informative. Therefore, tail events trigger larger belief revisions. Furthermore, because it will take many more observations of non-tail events to convince someone that the tail event really is unlikely, changes in tail risk beliefs are particularly persistent.

We have seen the scarring effect in action before. Before 2008, few people entertained the possibility of financial collapse. Today, more than a decade after the Great Recession and financial crsis, the possibility of another run on the financial sector is raised frequently, even though the system today is probably much safer. Likewise, businesses will make future decisions with the risk of another pandemic in mind. Observing the pandemic has taught us that the risks were greater than we thought. It is this new-found knowledge that has long-lived effects on economic choices.

To explore tail risk in a meaningful way, we need to use an estimation procedure that does not constrain the shape of the distribution's tail. Therefore, we allow our agents to learn about the distribution of aggregate shocks non-parametrically. Each period, agents observe one more piece of data and update their estimates of the distribution. Section 1 shows how this process leads to long-lived responses of beliefs to transitory events, especially extreme, unlikely ones. The mathematical foundation for such persistence is the martingale property of beliefs. The logic is that once observed, the event remains in agents' data set. Long after the direct effect of the shock has passed, the knowledge of that tail event affects beliefs and therefore, continues to restrain economic activity.

[1] See e.g., Summers (FT, 2020), Krugman (2020), Reinhart and Rogoff (2020) and Cochrane (2020).

Our analysis shows that a policy that prevents capital depreciation or obsolescence, even if it has only modest immediate effects on output, can have substantial long-run benefits, on the order of eight times larger than the short-run benefit that policy makers typically consider. No policy can prevent people from believing that future pandemics are more likely than they originally thought. Policy can however affect how the ongoing crisis affects capital returns. By changing that mapping, the costs of belief scarring can be mitigated. For example, widespread bankruptcies can lead to destruction of specific investments and a permanent erosion in the value of certain types of capital. While policy makers understood from the start that preventing bankruptcies was important, neglecting changing beliefs leads one to drastically underestimate just how important that mission is.

To illustrate the economic importance of these belief dynamics, Section 2 embeds our belief updating tool in a macroeconomic model with an epidemiology event that erodes the value of capital. This framework is designed to link tail events like the current crisis to macro outcomes in a quantitatively plausible way and has been used – e.g. by Gourio (2012) and Kozlowski et al. (2020) – to study the 2008-09 Great Recession. It features, among other elements, bankruptcy risk and elevated capital depreciation from social distancing, which separates labor from capital. This set of economic assumptions is not our main contribution. It is simply a laboratory we employ to illustrate the persistent economic effects from observing extreme events. Section 3 describes the data we feed into the model to discipline our belief estimates. Section 4 combines model and data and uses the resulting predictions to show how belief updating can generate large, permanent losses. We compare our results to those from the same economic model, but with agents who have full knowledge of the distribution, to pinpoint belief updating as the source of the persistence.

Our main insight about why the economic effect of COVID-19 is likely to persist does not rest on the specific economic structure of the Gourio (2012) model, or on the use of a particular epidemiology model or shock process as a driving force. We use a SIR model, with a policy response that results in a temporary loss of productivity and a faster rate of capital depreciation. We use this structure because it generates large asset price fluctuations, of the order observed at the onset of the pandemic, because it captures the economic cost of social distancing, and because it allows us to employ existing frameworks to explore the economics of tail risk. In our analysis, we consider different scenarios – different combinations of permanent and transitory shocks. Our point is not that these are the right forecast of the coming year's events. The point is that whatever you think will happen over the next year, the costs of this pandemic are much larger than your short-run calculations suggest.

The second ingredient is a belief updating process that uses new data to estimate the distribution of shocks, including the probability of extreme events. It is not crucial that the estimation is frequentist.[2] It is important that the distribution does not impose thin tails. We use data on the aggregate market value of capital to measure the capital depreciation shocks and estimate their frequency, prior to the COVID-19 crisis.

The third necessary ingredient is an economic model that links the risk of extreme events to real output. The model in Gourio (2012, 2013) has the necessary curvature (non-linearity in policy functions) to deliver a sizeable output response from changes in disaster risk. However, when agents do not learn from new data, the same model succeeds in producing a large initial output drop, but fails to produce any long-run scarring effects.

Finally, data on interest rates are also consistent with an increase in tail risk. Others point to low interest rates as a potential cause of stagnation. Our story complements this low interest rate trap narrative by demonstrating how heightened tail risk makes safe assets more attractive, depressing riskless rates in a persistent fashion.

Comparison to the literature There are many new studies of the impact of the COVID-19 pandemic on the U.S. economy. Dingel and Neiman (2020) classify the feasibility of working at home for all occupations: About 34% of US jobs, accounting for 44% of overall wages, can plausibly be performed at home. Similarly, Koren and Pető (2020) provide theory-based measures of the reliance of U.S. businesses on human interaction, detailed by industry and geographic location. Leibovici et al. (2020) investigates the extent to which the shock on contact-intensive industries may propagate to the rest of the economy. A 51% drop in the final demand for goods and services from contact-intensive industries implies a 13% decline in the gross output of low contact-intensive industries and a 24% drop in aggregate gross output. Eichenbaum et al. (2020) and Farboodi et al. (2020) explore the economic costs of the disease and the job dislocation that has resulted from it. Ludvigson et al. (2020) use VARs to estimate the cost of the pandemic over the next few months. What our paper adds to this discussion is the long-term effects of changes in behavior that persist long after the disease is gone.

Other papers share our focus on long-run persistent effects, but use other mechansisms, like financial frictions to propagate the shock. Elenev et al. (2020) and Krishnamurthy and Li (2020) use changes in beliefs, but propagate the shock primarily through financial balance sheet effects. Mostly closely related, Jorda et al. (2020) study rates of return on assets using a dataset stretching back to the 14th century, focusing on 15 major pandemics where more than 100,000 people died. Significant macroeconomic after-effects of the pandemics persist for about 40 years, with real rates of return substantially depressed. In a more informal discussion, Cochrane (2020) explores whether the recovery from the COVID-shock will be V, U or L shaped.

[2] For an example of Bayesian estimation of tail risks in a setting without an economic model, see Orlik and Veldkamp (2014).

This work formalizes many of the ideas in that discussion.

Outside of economics, biologists and socio-biologists have noted long ago that epidemics change the behavior of both humans and animals for generations. Loehle (1995) explore the social barriers to transmission in animals as a mode of defense against pathogen attack. Past disease events have effects on mating strategies, social avoidance, group size, group isolation, and other behaviors for generations. Gangestad and Buss (1993) find evidence of similar behavior among human communities.

In the economics realm, a small number of uncertainty-based theories of business cycles also deliver persistent effects from other sorts of transitory shocks. In Straub and Ulbricht (2013) and Van Nieuwerburgh and Veldkamp (2006), a negative shock to output raises uncertainty, which feeds back to lower output, which in turn creates more uncertainty. To get even more persistence, Fajgelbaum et al. (2014) combine this mechanism with an irreversible investment cost, a combination which can generate multiple steady-state investment levels. These uncertainty-based explanations are difficult to embed in quantitative DSGE models and to discipline with macro and financial data.

Our belief formation process is similar to the parameter learning models by Johannes et al. (2015), Cogley and Sargent (2005) and Kozeniauskas et al. (2014) and is advocated by Hansen (2007). However, these papers focus on endowment economies and do not analyze the potential for persistent effects in a setting with production.[3] The most important difference is that our non-parametric approach allows us to incorporate beliefs about tail risk.

1 Belief Formation

Before laying out the underlying economic environment, we begin by explaining how we formalize the notion of belief scarring. After we explain the non-standard, but most crucial part of the model, we embed it in an economic environment, to quantify the effect of belief changes form the COVID-19 pandemic on the US economy.

No one knows the true distribution of shocks to the economy. We estimate such distributions, updating our beliefs as new data arrives. The first step is to choose a particular estimation procedure. A common approach is to assume a normal or other parametric distribution and estimate its parameters. The normal distribution, with its thin tails, is unsuited to thinking about changes in tail risk. Other distributions raise obvious concerns about the sensitivity of

[3]Other learning papers in this vein include papers on news shocks, such as, Beaudry and Portier (2004), Lorenzoni (2009), Veldkamp and Wolfers (2007), uncertainty shocks, such as Jaimovich and Rebelo (2006), Bloom et al. (2014), Nimark (2014) and higher-order belief shocks, such as Angeletos and La'O (2013) or Huo and Takayama (2015).

results to the specific distributional assumption used. To minimize such concerns, we take a non-parametric approach and let the data inform the shape of the distribution.

Specifically, we employ a kernel density estimation procedure, one of most common approaches in non-parametric estimation. Essentially, it approximates the true distribution function with a smoothed version of a histogram constructed from the observed data. By using the widely-used normal kernel, we impose a lot of discipline on our learning problem but also allow for considerable flexibility. We also experimented with a handful of other kernels.

Setup Consider a shock ϕ_t whose true density g is unknown to agents in the economy. The agents do know that the shock ϕ_t is i.i.d. Their information set at time t, denoted $\mathcal{I}_t$, includes the history of all shocks ϕ_t observed up to and including t. They use this available data to construct an estimate $\hat{g}_t$ of the true density g. Formally, at every date, agents construct the following normal kernel density estimator of the pdf g

$$\hat{g}_t(\phi) = \frac{1}{n_t \kappa_t} \sum_{s=0}^{n_t-1} \Omega\left(\frac{\phi - \phi_{t-s}}{\kappa_t}\right)$$

where $\Omega(\cdot)$ is the standard normal density function, κ_t is the smoothing or bandwidth parameter and n_t is the number of available observations of at date t. As new data arrives, agents add the new observation to their data set and update their estimates, generating a sequence of beliefs $\{\hat{g}_t\}$.

The key mechanism in the paper is the persistence of belief changes induced by transitory ϕ_t shocks. This stems from the martingale property of beliefs - i.e. conditional on time-t information ($\mathcal{I}_t$), the estimated distribution is a martingale. Thus, on average, the agent expects her future belief to be the same as her current beliefs. This property holds exactly if the bandwidth parameter κ_t is set to zero and holds with tiny numerical error in our application.[4] In line with the literature on non-parametric assumption, we use the optimal bandwidth.[5] As a result, any changes in beliefs induced by new information are expected to be approximately permanent. This property plays a central role in generating long-lived effects from transitory shocks.

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[4]As $\kappa_t \to 0$, the CDF of the kernel converges to $\hat{G}_t^0(\phi) = \frac{1}{n_t}\sum_{s=0}^{n_t-1} \mathbf{1}\{\phi_{t-s} \leq \phi\}$. Then, for any ϕ, $j \geq 1$

$$\mathbb{E}_t\left[\hat{G}_{t+j}^0(\phi)\middle|\mathcal{I}_t\right] = \mathbb{E}_t\left[\frac{1}{n_t+j}\sum_{s=0}^{n_t+j-1} \mathbf{1}\{\phi_{t+j-s} \leq \phi\}\middle|\mathcal{I}_t\right] = \frac{n_t}{n_t+j}\hat{G}_t^0(\phi) + \frac{j}{n_t+j}\mathbb{E}_t\left[\mathbf{1}\{\phi_{t+1} \leq \phi\}|\mathcal{I}_t\right]$$

Thus, future beliefs are, in expectation, a weighted average of two terms - the current belief and the distribution from which the new draws are made. Since our best estimate for the latter is the current belief, the two terms are exactly equal, implying $\mathbb{E}_t\left[\hat{G}_{t+j}^0(\phi)\middle|\mathcal{I}_t\right] = \hat{G}_t^0(\phi)$.

[5]See Hansen (2015).

2 Economic and Epidemiological Model

To gauge the magnitude of the scarring effect of the COVID-19 pandemic on long-run economic outcomes, we need to embed it in an economic environment in which tail risk and belief changes have meaningful effects. From this perspective, our model needs two key features. First, it should have the potential for 'large' shocks, that have both transitory and lasting effects. The former would include lost productivity from stay-at-home orders preventing services from reaching consumers. But, for this shock to look like the extreme event it is to investors, the model must also allow for the possibility of a more persistent loss of productive capital. The interior of the restaurant that went bankrupt, the unused capacity of the hotel that will not fill again for many years to come. When stay-at-home policies force consumers to find other ways to fulfilling their needs, tastes, habits, and consumption patterns may change permanently, rendering some capital obsolete. One might think this is hard-wiring persistence in the model. Yet, as we will show, this loss of capital by itself, has a short lived effect and typically triggers an investment boom, as the economy rebuilds capital better suited to the new consumption normal. We explore two possible scenarios that highlight the enormous importance of preventing capital obsolescence, because of the scarring of beliefs.

The second key feature is sufficient policy function curvature, which serves to make economic activity sensitive to the probability of extreme large shocks. Two key ingredients – namely, Epstein-Zin preferences and costly bankruptcy – combine to generate significant non-linearity in policy functions. Similarly, preferences that shut down wealth effects on labor avoid a surge in hours in response to crises.

None of these ingredients guarantees persistent effects. Absent belief revisions, shocks, no matter how large, do not change the long-run trajectory of the economy. Similarly, the non-linear responses induced by preferences and debt influence the size of the economic response, but by themselves do not generate any internal propagation. They simply govern the magnitude of the impact, both in the short and long run.

To this setting, we add belief scarring. We model beliefs using the non-parametric estimation described in the previous section and show how to discipline this procedure with observable macro data, avoiding free parameters. This belief updating piece is not there to generate the right size reaction to the initial shock. Instead, belief updating adds the persistence, which considerably inflates the cost.

2.1 The Disease Environment

This block of the model serves to generates a time path for disruption to economic activity, which will then be mapped into transitory productivity shock and capital obsolescence . Of course, we could have directly created scenarios for the shocks and arrived at the same predictions. The explicit modeling of the spread of disease allows us to see how different social distancing policies map into shocks and ultimately into long-term economic costs from belief scarring. Given this motivation, we build on a very simple SEIR model, which is a discrete-time version of Atkeson (2020) or Stock (2020), who build on work in the spirit of Kermack and McKendrick (1927). To this model, we add two ingredients: 1) a behavioral / policy rule that imposes capital idling when the infection rate increase. This rule could represent optimal behavior or government policy; and 2) a higher depreciation rate of unused capital. While we normally think of capital utilization depreciating capital, this is a different circumstance where habits, technologies and norms are changing more rapidly than normal. Unused capital may be restaurants whose customers find new favorites, old conferencing technologies replaced with new online technology or office space that will be replaced with home offices. This higher depreciation rate represents a speeding up of capital obsolescence.

Disease and shutdowns Time is discrete and infinite. For the disease part of the model, we will count time in days, indexed by $\tilde{t}$. Later, to describe long-run effects, we will change the measure of time to t, which represents years. There are N agents in the economy. At date 1, the first person gets infected. Let S represent the number of people susceptible to the disease, but not currently exposed, infected, dead or recovered. At date 1, that susceptible number is $S(1) = N - 1$. Let E be the number of exposed persons and I be the number infected. We start with $E(1) = 0$ and $I(1) = 1$. Finally, D represents the number who are either recovered or dead, where $D(1) = 0$. The following four equations describe the dynamics of the disease.

$$S(\tilde{t}+1) = S(\tilde{t}) - \tilde{\beta}_{\tilde{t}} S(\tilde{t}) I(\tilde{t})/N \tag{1}$$

$$E(\tilde{t}+1) = E(\tilde{t}) + \tilde{\beta}_{\tilde{t}} S(\tilde{t}) I(\tilde{t})/N - \sigma_E E(\tilde{t}) \tag{2}$$

$$I(\tilde{t}+1) = I(\tilde{t}) + \sigma_E E(\tilde{t}) - \gamma_I I(\tilde{t}) \tag{3}$$

$$D(\tilde{t}+1) = D(\tilde{t}) + \gamma_I I(\tilde{t}) \tag{4}$$

The parameter γ_I is the rate at which people exit infection and become deceased or recovered. Thus, the expected duration of infection is approximately $1/\gamma_I$, and the number of contacts an infected person has with a susceptible person is $\tilde{\beta}$ times the fraction of the population that is susceptible $S(\tilde{t})/N$. The initial reproduction rate, often referred to as R_0 is therefore $\tilde{\beta}/\gamma_I$.

We put a t subscript on $\tilde{\beta}_{\tilde{t}}$ because behavior and policy can change it. When the infection rate rises, people reduce infection risk by staying home. The reduces the number of social contacts, reducing $\tilde{\beta}$. Lockdown policies also work by reducing $\tilde{\beta}$. We capture this relationship by assuming that $\tilde{\beta}$ can vary between a maximum of $\gamma_I R_0$ and a minimum of $0.8R_{min}$, where R_{min} is the estimated U.S. reproduction rate for regions under lockdown. Where on the spectrum the contact rate lies depends on the last 30-day change in infection rates, measured with a 15-day lag.[6] If ΔI_t is the difference between the average 15-30 day past infections and the average of 30-45 day infections, then policy and individual behavior achieves a frequency of social contact:

$$\tilde{\beta}_{\tilde{t}} = \gamma_I \times min(R_0, max(R_{min}, R_0 - \zeta * \Delta I_t)) \tag{5}$$

The key part of the epidemic from a belief-scarring perspective is that reducing the contact rate requires separating labor from capital. In other words, capital is idle. No capital is idled (full capacity) when no mitigation efforts are underway, i.e. when $\tilde{\beta}_{\tilde{t}} = \gamma_I R_0$. But as $\tilde{\beta}_{\tilde{t}}$ falls, capital idling (K^-) rises. We formalize that relationship as

$$K_t^- = \tilde{\theta} * (R_0 - \gamma \tilde{\beta}_{\tilde{t}}). \tag{6}$$

Idle capital depreciates as a rate $\tilde{\delta}$. As mentioned before, this is not physical deterioration of the capital stock. Instead, it represent a loss of value from accelerated obsolescence due to changes in tastes, habits and technologies. It could also represent a loss in value because of persistent upstream or downstream supply chain constraints.

2.2 The Economy

Preferences and technology: To describe long-term economic consequences, we switch from the daily time index $\tilde{t}$ to an annual time index t. An infinite horizon, discrete time economy has a representative household, with preferences over consumption (C_t) and labor supply (L_t):

$$U_t = \left[(1-\beta)\left(C_t - \frac{L_t^{1+\gamma}}{1+\gamma}\right)^{1-\psi} + \beta E_t \left(U_{t+1}^{1-\eta}\right)^{\frac{1-\psi}{1-\eta}}\right]^{\frac{1}{1-\psi}} \tag{7}$$

where ψ is the inverse of the intertemporal elasticity of substitution, η indexes risk-aversion, γ is inversely related to the elasticity of labor supply, and β represents time preference.[7]

The economy is also populated by a unit measure of firms, indexed by i and owned by the representative household. Firms produce output with capital and labor, according to a standard Cobb-Douglas production function $z_t k_{it}^{\alpha} l_{it}^{1-\alpha}$.

[6] This is consistent with the U.S. official policy on re-opening (CDC, 2020). Note that individual optimal choice to social distance are also included in this "policy." These optimal choices look similar. See Kaplan et al. (2020).

[7] This utility function rules out wealth effects on labor, as in Greenwood et al. (1988).

Aggregate uncertainty is captured by a single random variable, ϕ_t, which is i.i.d. over time and drawn from a distribution $g(\cdot)$. The i.i.d. assumption is made in order to avoid an additional, exogenous, source of persistence. [8] This shock has both permanent and transitory effects on production. The permanent component works as follows: A firm that enters the period t with capital $\hat{k}_{it}$ has effective capital $k_{it} = \phi_t \hat{k}_{it}$. In other words, ϕ reflects a long-lasting change in the economic value of a piece of capital. A realization of $\phi < 1$ captures the loss of specific investments or other forms of lasting damage from a prolonged disruption, e.g. the lost value of cruise ships that will never sail again, restaurants that do not re-open, airplanes that will be grounded due to a lasting drop in business travel, or office buildings space that will stay empty for a while as working from home becomes more the norm. Of course, all these forms of capital will still have some residual value: but, a sizeable portion of their value was specific to that purpose and it will require costly investments to repurpose them. As we will see, this investment is exactly what happens in a standard model, and what the scarring effect of beliefs prevents.

In addition to this permanent component, the shock ϕ_t also has a temporary effect, through the TFP term $z_t = \phi_t^\nu$. The parameter ν governs the relative strength of the transitory component. This specification allows us to capture both permanent and transitory disruptions with only one source of uncertainty ϕ. By varying ν, we can capture a range of scenarios with having to introduce additional shocks.

Firms are also subject to an idiosyncratic shock v_{it}. These shocks scale up and down the total resources available to each firm (before paying debt, equity or labor)

$$\Pi_{it} = v_{it} \left[z_t k_{it}^{\alpha} l_{it}^{1-\alpha} + (1-\delta) k_{it} \right] \tag{8}$$

where δ is the rate of capital depreciation. The shocks v_{it} are i.i.d. across time and firms and are drawn from a known distribution, F.[9] The mean of the idiosyncratic shock is normalized to be one: $\int v_{it} \quad di = 1$. The primary role of these shocks is to induce an interior default rate in equilibrium, allowing a more realistic calibration, particularly of credit spreads.

Finally, firms hire labor in a competitive market at a wage W_t. We assume that this decision is made after observing the aggregate shock but before the idiosyncratic shocks are observed,

[8] The i.i.d. assumption also has empirical support. In the next section, we use macro data to construct a time series for ϕ_t. We estimate an autocorrelation of 0.15, statistically insignificant. In previous work, we showed that this generates almost no persistence in the economic response.

[9] This is a natural assumption - with a continuum of firms and a stationary shock process, firms can learn the complete distribution of any idiosyncratic shocks after one period.

i.e. labor choice is solves the following static problem:

$$\max_{l_{it}} \quad z_t(\phi_t \hat{k}_{it})^{\alpha} l_{it}^{1-\alpha} - W_t l_{it}$$

Credit markets and default: Firms have access to a competitive non-contingent debt market, where lenders offer bond price (or equivalently, interest rate) schedules as a function of aggregate and idiosyncratic states, in the spirit of Eaton and Gersovitz (1981). A firm enters period $t+1$ with an obligation, b_{it+1} to bondholders The shocks are then realized and the firm's shareholders decide whether to repay their obligations or default. Default is optimal for shareholders if, and only if,

$$\Pi_{it+1} - b_{it+1} + \Gamma_{t+1} < 0$$

where Γ_{t+1} is the present value of continued operations. Thus, the default decision is a function of the resources available to the firm (output plus undepreciated capital less wages Π_{it+1}) and the obligations to bondholders (b_{it+1}). Let $r_{it+1} \in \{0, 1\}$ denote the default policy of the firm.

In the event of default, equity holders get nothing. The productive resources of a defaulting firm are sold to an identical new firm at a discounted price, equal to a fraction $\theta < 1$ of the value of the defaulting firm. The proceeds are distributed *pro-rata* among the bondholders.[10]

Let q_{it} denote the bond price schedule faced by firm i in period t, i.e. the firm receives q_{it} in exchange for a promise to pay one unit of output at date $t+1$. Debt is assumed to carry a tax advantage, which creates incentives for firms to borrow. A firm which issues debt at price q_{it} and promises to repay b_{it+1} in the following period, receives a date-t payment of $\chi q_{it} b_{it+1}$, where $\chi > 1$. This subsidy to debt issuance, along with the cost of default, introduces a trade-off in the firm's capital structure decision, breaking the Modigliani-Miller theorem.[11]

For a firm that does not default, the dividend payout is its total available resources, minus its payments to debt and labor, minus the cost of building next period's capital stock (the undepreciated current capital stock is included in Π_{it}), plus the proceeds from issuing new debt, including its tax subsidy

$$d_{it} = \Pi_{it} - b_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1}. \tag{9}$$

Importantly, we do not restrict dividends to be positive, with negative dividends interpreted as (costless) equity issuance. Thus, firms are not financially constrained, ruling out another

[10] In our baseline specification, default does not destroy resources - the penalty is purely private. This is not crucial - it is straightforward to relax this assumption by assuming that all or part of the cost of the default represents physical destruction of resources.

[11] The subsidy is assumed to be paid by a government that finances it through a lump-sum tax on the representative household.

potential source of persistence.

Bankruptcy amplifies obsolescence . We would like to have a role for financial policy in the model as well. Finance cannot eliminate the cost of a pandemic. But it can amplify it, or not. To capture this role, we assume that the ϕ_t shock that reduces temporary productivity and erodes the value of capital has a component $\tilde{\phi}_t$, which comes from the direct effect of disease/shutdown and a components that depends on d_t, the default rate. We use a flexible form for this relationship:

$$\ln \phi_t = \ln \tilde{\phi}_t + \kappa_0 d_t^{1-\varpi}. \tag{10}$$

Timing and value functions:

1. Firms enter the period with a capital stock $\hat{k}_{it}$ and outstanding debt b_{it},
2. The epidemiology dynamics run for one year and determine the aggregate capital depreciation shock ϕ_{2020}. Thereafter, $\phi \sim g(\phi)$ is drawn randomly. Labor choice and production take place. The firm-specific profit shocks v_{it} are realized.
3. The firm decides whether to default or repay ($r_{it} \in \{0, 1\}$) its bond claims.
4. The firm makes capital $\hat{k}_{it+1}$ and debt b_{it+1} choices for the following period.

In recursive form, the problem of the firm is

$$V\left(\Pi_{it}, b_{it}, S_t\right) = \max\left[0, \max_{d_{it}, \hat{k}_{it+1}, b_{it+1}} d_{it} + \mathbb{E}_t M_{t+1} V\left(\Pi_{it+1}, b_{it+1}, S_{t+1}\right)\right] \tag{11}$$

subject to

$$\text{Dividends:} \quad d_{it} \leq \Pi_{it} - b_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1} \tag{12}$$

$$\text{Resources:} \quad \Pi_{it} = v_{it+1}\left[\max_{l_{it}} \; z_t(\phi_t \hat{k}_{it})^{\alpha} l_{it}^{1-\alpha} - W_t l_{it} + (1-\delta)\phi_t \hat{k}_{it}\right] \tag{13}$$

$$\text{Bond price:} \quad q_{it} = \mathbb{E}_t M_{t+1}\left[r_{it+1} + (1 - r_{it+1})\frac{\theta \tilde{V}_{it+1}}{b_{it+1}}\right] \tag{14}$$

The first max operator in (11) captures the firm's option to default. The expectation $\mathbb{E}_t$ is taken over the idiosyncratic and aggregate shocks, given beliefs about the aggregate shock distribution. The value of a defaulting firm is simply the value of a firm with no external obligations, i.e. $\tilde{V}_{it} = V\left(\Pi_{it}, 0, S_t\right)$.

The aggregate state S_t consists of $(\Pi_t, L_t, \mathcal{I}_t)$ where $\Pi_t \equiv z_t A K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t$ is the aggregate resources available, and $\mathcal{I}_t$ is the economy-wide information set. Equation (14) reveals that bond prices are a function of the firm's capital $\hat{k}_{it+1}$ and debt b_{it+1}, as well as the aggregate state S_t. The firm takes the aggregate state and the function $q_{it} = q\left(\hat{k}_{it+1}, b_{it+1}, S_t\right)$ as given, while recognizing that its firm-specific choices affect its bond price.

Information, beliefs and equilibrium The set $\mathcal{I}_t$ includes the history of all shocks ϕ_t observed up to and including time-t. For now, we specify a general function, denoted Ψ, which maps $\mathcal{I}_t$ into an appropriate probability space. The expectation operator $\mathbb{E}_t$ is defined with respect to this space. In the following section, we make this more concrete using the kernel density estimation procedure outlined in section 1 to map the information set into beliefs.

For a given belief function Ψ, a recursive equilibrium is a set of functions for (i) aggregate consumption and labor that maximize (7) subject to a budget constraint, (ii) firm value and policies that solve (11) , taking as given the bond price function (14) and the stochastic discount factor is such that (iii) aggregate consumption and labor are consistent with individual choices.

2.3 Characterization

The equilibrium of the economic model is a solution to the following set of non-linear equations. First, we use the solution to the labor choice problem

$$l_t = \left[\frac{(1-\alpha)z_t}{W_t}\right]^{\frac{1}{\alpha}} \phi_t \hat{k}_t$$

and the fact that the constraint on dividends (12) will bind at the optimum to substitute for d_{it} and l_{it} in the firm's problem (11). This leaves us with 2 choice variables $(\hat{k}_{it+1}, b_{it+1})$ and a default decision. Optimal default is characterized by a threshold rule in the idiosyncratic output shock v_{it}:

$$r_{it} = \begin{cases} 0 & \text{if} \quad v_{it} \; < \; \underline{v}\,(S_t) \\ 1 & \text{if} \quad v_{it} \; \geq \; \underline{v}\,(S_t) \end{cases}$$

It turns out to be more convenient to redefine variables and cast the problem as a choice of $\hat{k}_{it+1}$ and leverage, $lev_{it+1} \equiv \frac{b_{it+1}}{\hat{k}_{it+1}}$. We relegate detailed derivations and the full characterization to the Appendix. Since all firms make symmetric choices for these objects, in what follows, we suppress the i subscript. The optimality condition for $\hat{k}_{t+1}$ is:

$$1 = \mathbb{E}[M_{t+1}R^k_{t+1}] + (\chi - 1)lev_{t+1}q_t - (1-\theta)\mathbb{E}[M_{t+1}R^k_{t+1}h(\underline{v})] \tag{15}$$

$$\text{where} \qquad R^k_{t+1} = \frac{z_t\phi^\alpha_{t+1}\hat{k}^\alpha_{t+1}l^{1-\alpha}_{t+1} - W_{t+1}l_{t+1} + (1-\delta)\,\phi_{t+1}\hat{k}_{t+1}}{\hat{k}_{t+1}} \tag{16}$$

The term R^k_{t+1} is the *ex-post* per-unit, pre-wage return on capital, while $\underline{v} \equiv \frac{lev_{t+1}}{R^k_{t+1}}$ $h(\underline{v}) \equiv \int_{-\infty}^{\underline{v}} vf(v)dv$ is the default-weighted expected value of the idiosyncratic shock.

The first term on the right hand side of (15) is the usual expected direct return from investing, weighted by the stochastic discount factor. The other two terms are related to debt. The second term reflects the indirect benefit to investing arising from the tax advantage of debt - for each unit of capital, the firm raises $\frac{b_{t+1}}{\hat{k}_{t+1}}q_t$ from the bond market and earns a subsidy of $\chi - 1$ on the proceeds. The last term is the cost of this strategy - default-related losses, equal to a fraction $1 - \theta$ of available resources.

Next, the firm's optimal choice of leverage, lev_{t+1} is

$$(1-\theta)\,\mathbb{E}_t\left[M_{t+1}\frac{lev_{t+1}}{R^k_{t+1}}f\left(\frac{lev_{t+1}}{R^k_{t+1}}\right)\right] = \left(\frac{\chi-1}{\chi}\right)\mathbb{E}_t\left[M_{t+1}\left(1 - F\left(\frac{lev_{t+1}}{R^k_{t+1}}\right)\right)\right]. \tag{17}$$

The left hand side is the marginal cost of increasing leverage - it raises the expected losses from the default penalty (a fraction $1-\theta$ of the firm's value). The right hand side is the marginal benefit - the tax advantage times the value of debt issued.

The optimality conditions, (15) and (17), along with those from the household side, form the system of equations we solve numerically.

3 Measurement, Calibration and Solution Method

This section describes how we use macro data to estimate beliefs and parameterize the model, as well as our computational approach. A strength of our theory is that we can use observable data to estimate beliefs at each date.

Measuring past shocks We model the COVID-19 shock as an inability to pair workers with previously productive capital. Some of that inability is temporary. But some of it represents a more permanent loss of value. A helpful feature of capital depreciation and productivity shocks is that their mapping to available data is straightforward. A unit of capital installed in period $t-1$ (i.e. as part of $\hat{K}_t$) is, in effective terms, worth ϕ_t units of consumption goods in period t. Thus, the change in its market value from $t-1$ to t is simply ϕ_t.

We apply this measurement strategy to annual data on non-residential capital held by US corporates. Specifically, we use two time series Non-residential assets from the Flow of Funds, one evaluated at market value and the second, at historical cost.[12] We denote the two series by NFA_t^{MV} and NFA_t^{HC} respectively. To see how these two series yield a time series for ϕ_t, note that, in line with the reasoning above, NFA_t^{MV} maps directly to effective capital in the model. Formally, letting P_t^k the nominal price of capital goods in t, we have $P_t^k K_t = NFA_t^{MV}$. Investment X_t can be recovered from the historical series, $P_{t-1}^k X_t = NFA_t^{HC} - (1-\delta)\, NFA_{t-1}^{HC}$. Combining, we can construct a series for $P_{t-1}^k \hat{K}_t$:

$$\begin{aligned} P_{t-1}^k \hat{K}_t &= (1-\delta) P_{t-1}^k K_{t-1} + P_{t-1}^k X_t \\ &= (1-\delta) NFA_{t-1}^{MV} + NFA_t^{HC} - (1-\delta)\, NFA_{t-1}^{HC} \end{aligned}$$

Finally, in order to obtain $\phi_t = \frac{K_t}{\hat{K}_t}$, we need to control for nominal price changes. To do this, we proxy changes in P_t^k using the price index for non-residential investment from the National Income and Product Accounts (denoted $PINDX_t$).[13] This yields:

$$\begin{aligned} \phi_t = \frac{K_t}{\hat{K}_t} &= \left(\frac{P_t^k K_t}{P_{t-1}^k \hat{K}_t}\right)\left(\frac{PINDX_{t-1}^k}{PINDX_t^k}\right) \\ &= \left[\frac{NFA_t^{MV}}{(1-\delta) NFA_{t-1}^{MV} + NFA_t^{HC} - (1-\delta)\, NFA_{t-1}^{HC}}\right]\left(\frac{PINDX_{t-1}^k}{PINDX_t^k}\right) \end{aligned} \tag{18}$$

Using the measurement equation (18), we construct an annual time series for capital depreciation shocks for the US economy since 1950. The left panel of Figure 2 plots the resulting series. The mean and standard deviation of the series over the entire sample are 1 and 0.03 respectively. The autocorrelation is statistically insignificant at 0.15.

Calibration A period t is interpreted as a year. We choose the discount factor β and depreciation δ to target a steady state capital-output ratio of 3.5 (this is taken from Cooley and Prescott (1995)) and an investment-output ratio of 0.12 (this is the average ratio of non-residential investment to output during 1950-2019 from NIPA accounts).[14] The share of capital in the production, α, is 0.40, which is also taken from Cooley and Prescott (1995). The recovery rate upon default, θ, is set to 0.70, following Gourio (2013). The distribution for the idiosyncratic shocks, v_{it} is assumed to be lognormal, i.e. $\ln v_{it} \sim N\left(-\frac{\hat{\sigma}^2}{2}, \hat{\sigma}^2\right)$ with $\hat{\sigma}^2$ chosen

[12]These are series FL102010005 and FL102010115 from Flow of Funds.

[13]Our results are robust to alternative measures of nominal price changes, e.g. computed from the price index for GDP or Personal Consumption Expenditure.

[14]This leads to values for β and δ of 0.91 and 0.03 respectively. These are lower than other estimates in the literature.

to target a default rate of 0.02.[15] The labor supply parameter, γ, is set to 0.5, in line with Midrigan and Philippon (2011), corresponding to a Frisch elasticity of 2.

For the parameters governing risk aversion and intertemporal elasticity of substitution, we use standard values from the asset pricing literature and set $\psi = 0.5$ (or equivalently, an IES of 2) and $\eta = 10$. The tax advantage parameter χ is chosen to match a leverage target of 0.50, the ratio of external debt to capital in the US data - from Gourio (2013)).

Epidemiology parameters. Following Wang et al. (2020)'s study of infection in Hubei, China, we calibrate $\sigma_E = 1/5.2$ and $\gamma_I = 1/18$ to the average duration of exposure (5.2 days) and infection (18 days). We use an initial reproduction number of $R_0 = 3.5$, based on more recent estimates of higher antibody prevalence and more asymptomatic infection than originally thought (Center for Disease Control, 2020). This implies that the initial number of contacts per period must be $\tilde{\beta} = \gamma R_0$.

Our scenario 1 uses accelerated depreciation of idle capital at the rate of 6.5% per month: $\tilde{\delta} = 0.065/30$. That matches the recent 10% drop in the value of commercial real estate ?? (CPP), as well as the estimates of 25% annualized output loss during the lockdown in Hubei .

We lower this depreciation rate to 3% per month in scenarios 2 and 3. The relationship between capital idling and contact rates is $\tilde{\theta} = 1/3$ because estimates of capital idling during the period of complete lockdown where the reproduction number fell to 0.8 was about 50%. That level of idling is consistent with the 25% drop in output, estimated during the lockdown period in Hubei province, China.

The relationship between infection increases and capital idling, $\zeta = 300$, is chosen to match a 2-month initial lockdown. However, this is not the only lockdown predicted by the model. As predicted by the CDC, we also predict waves of re-infection and new lockdowns. Scenario 3 considers a much less aggressive social distancing policy by setting $\zeta = 50$.

Table 1 summarizes the resulting parameter choices.

Numerical solution method Because curvature in policy functions is an important feature of the economic environment, our algorithm solves equations (15) – (17) with a non-linear collocation method. Appendix A.2 describes the iterative procedure. In order to keep the computation tractable, we need one more approximation. The reason is that date-t decisions (policy functions) depend on the current estimated distribution ($\hat{g}_t(\phi)$) and the probability distribution h over next-period estimates, $\hat{g}_{t+1}(\phi)$. Keeping track of $h(\hat{g}_{t+1}(\phi))$, (a compound lottery) makes a function a state variable, which renders the analysis intractable. However, the

[15] This is in line with the target in Khan et al. (2014), though a bit higher than the one in Gourio (2013). We verified that our quantitative results are not sensitive to this target.

Parameter	Value	Description
Preferences:		
β	0.91	Discount factor
η	10	Risk aversion
ψ	0.50	1/Intertemporal elasticity of substitution
γ	0.50	1/Frisch elasticity
Technology:		
α	0.40	Capital share
δ	0.03	Depreciation rate
$\hat{\sigma}$	0.25	Idiosyncratic volatility
Debt:		
χ	1.06	Tax advantage of debt
θ	0.70	Recovery rate
κ_0	0.2	Default-obsolescence feedback
ϖ	0.5	Default-obsolescence elasticity
Disease / Policy:		
R_0	3.5	Initial disease reproduction rate
σ_E	0.2	Exposure to infection transition rate
γ_I	0.05	Recovery / death rate
ζ	300 (50)	Lockdown policy sensitive to past infections
$\tilde{\theta}$	0.19	Capital idling required to reduce transmission
$\tilde{\delta}$	0.065 (0.03)	Depreciation of idled capital
ρ_I	0.95	Persistence of policy rule

Table 1: **Parameters** Numbers in parentheses are used in scenarios 2 or 3.

approximate martingale property of $\hat{g}_t$ discussed in Section 1 offers an accurate and computationally efficient approximation to this problem. The martingale property implies that the average of the compound lottery is $E_t[\hat{g}_{t+1}(\phi)] \approx \hat{g}_t(\phi)$, $\forall \phi$. Therefore, when computing policy functions, we approximate the compound distribution $h(\hat{g}_{t+1}(\phi))$ with the simple lottery $\hat{g}_t(\phi)$, which is today's estimate of the probability distribution. We use a numerical experiment to show that this approximation is quite accurate. The reason for the small approximation error is that $h(\hat{g}_{t+1})$ results in distributions centered around $\hat{g}_t(\phi)$, with a small standard deviation. Even 30 periods out, $\hat{g}_{t+30}(\phi)$ is still quite close to its mean $\hat{g}_t(\phi)$. For 1-10 quarters ahead, where most of the utility weight is, this standard error is tiny.

To compute our benchmark results, we begin by estimating $\hat{g}_{2019}$ using the data on ϕ_t described above. Given this $\hat{g}_{2019}$, we compute the stochastic steady by simulating the model for 1000 periods, discarding the first 500 observations and time-averaging across the remaining periods. This steady state forms the starting point for our results. Subsequent results are in log deviations from this steady state level. Then, we subject the model economy to two possible additional adverse realizations for 2020, one at a time. Using the one additional data point for each scenario, we re-estimate the distribution, to get $\hat{g}_{2020}$. To see how persistent economic responses are, we need a long future time series. We don't know what distribution future shocks will be drawn from. Given all the data available to us, our best estimate is also $\hat{g}_{2020}$. Therefore, we simulate future paths by drawing many sequences of future ϕ shocks from the $\hat{g}_{2020}$ distribution and plot the mean future path of various aggregate variables, in the results that follow.

4 Main Results

Our contribution is to quantify the long run effect of the COVID crises, stemming from the scarring effect on beliefs of learning that pandemics are more likely than we thought. We formalize and measure belief effects, using the assumption that people do not know the true distribution of aggregate economic shocks and learn about it statistically. This is the source of the long-run economic effects. Comparing a model with real-time estimation (learning) to the same model with full knowledge of the distribution (no learning) reveals the extent to which beliefs matter.

But first, we briefly describe the disease spread, the policy reaction and the economic shocks these policies generate.

Epidemiology and economic shutdown. A major hurdle to quantifying the long-run effects is the lack of data and estimates of the current impact. While this will surely be sorted out in months, for now, with the crisis still raging and policy still being set, the impact is uncertain. More importantly for us, the nature of the economic shock is uncertain. It may be a temporary closure with furloughs, or it could involve widespread bankruptcies and changes in habits that permanently separate workers from capital or make the existing stock of capital ill-suited to the new consumption demands. Since it is too early to know this, we present teo possible scenarios, chosen to illustrate the interaction between learning and the type of shock we experience. All three involve enormous losses in the short term but their long-term effects and their lifetime welfare effects for the economy are drastically different.

Figure 1 illustrates the spread of disease, in both scenarios, as well as the policy response, which results in capital idling. Scenario 2 explores a weaker policy response with $\zeta = 50$. That represents a policy that is six times less responsive to changes in the infection rate than the $\zeta = 300$ policy in scenario 1. That second scenario has significantly less idle capital, but also a faster spike in infection rates.

For our purposes, the sufficient statistics for each scenario is a realization for ϕ_{2020}. The target for this initial, transitory impact follow common forecasts: an 10% or 6.5% annual

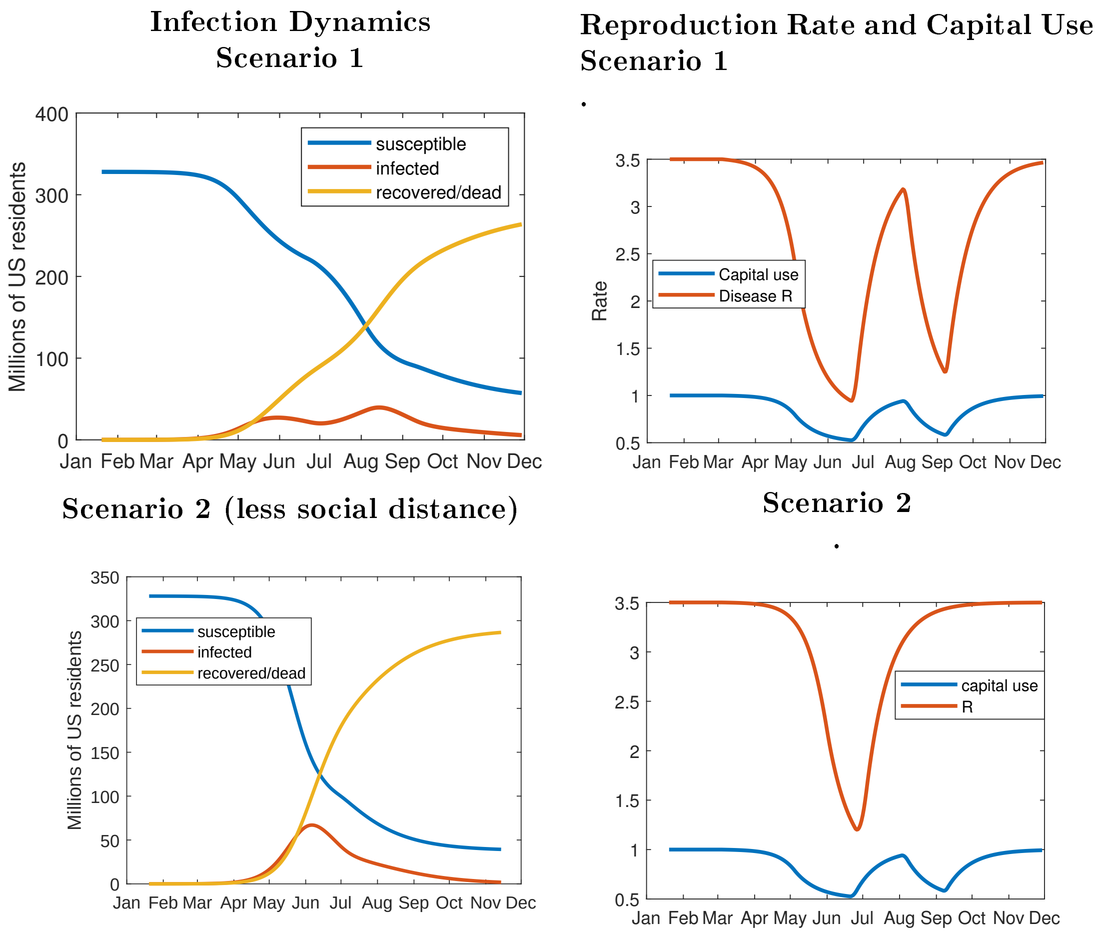

Figure 1: **Disease spread and capital dynamics.**
Parameters listed in Table 1. Scenario 1 uses an aggressive lockdown policy $\zeta_I = 300$*, while scenario 2 uses a more relaxed policy of* $\zeta_I = 50$.

decline in GDP. This is likely a conservative estimate for Q2 2020, but more extreme than some forecasts for the entire year.

In scenario 1, the shock makes 10% of the capital stock obsolete: $\phi_t = 0.9$. In scenario 2, only 5% of capital becomes obsolete: $\phi_t = 0.95$.

How much belief scarring? We apply our kernel density estimation procedure to the capital return time series and our two scenarios to construct a sequence of beliefs. In other words, for each t, we construct $\{\hat{g}_t\}$ using the available time series until that point. The resulting estimates for 2019 and 2020 are shown in Figure 2. The differences are subtle. Spotting them requires close inspection where the dotted and solid lines diverge, around 0.75, 0.85 and 0.95, in scenarios 1, 2 and 3 respectively. They show that the COVID-19 pandemic induces an increase in the perceived likelihood of extreme negative shocks. In scenario 1, the estimated density for 2019 implies near zero (less than 10^{-5}%) chance of a $\phi = 0.75$ shock; the 2020 density attaches a 1-in-70 or 1.42% probability to a similar event recurring.

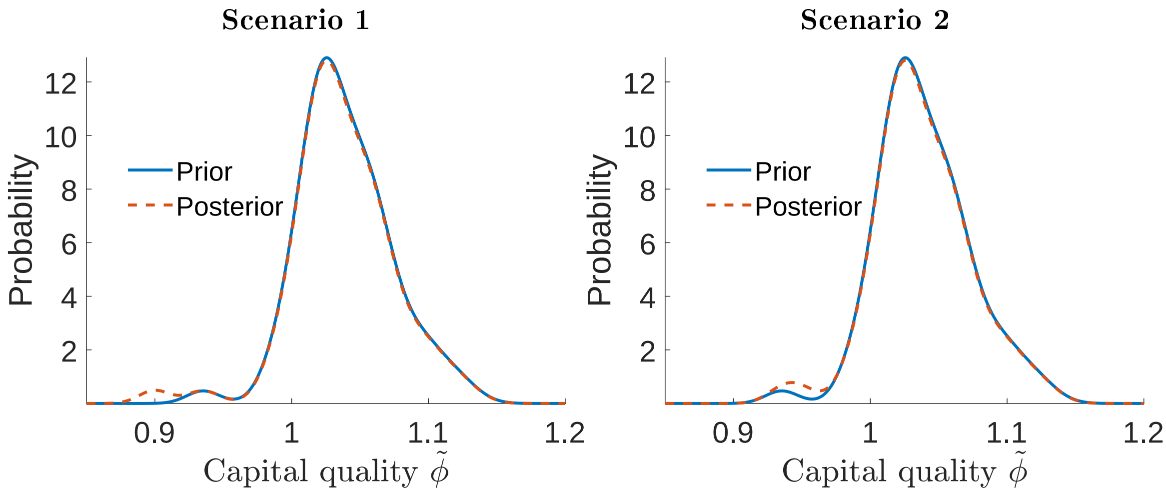

Figure 2: **Beliefs about the probability distribution of outcomes, plotted before and during the COVID-19 crisis.**
Both panels show the same estimated kernel densities in 2019 (solid line). They show different estimated distributions of shocks in 2020 (dashed line), depending on the scenario. The subtle changes in the left tail represent the scarring effect of COVID-19.

As the graph shows, for most of the sample period, the shock realizations are in a relatively tight range around 1, but we saw two large adverse realizations during the Great Recession: 0.93 in 2008 and 0.84 in 2009. These reflect the large drops in the market value of non-residential capital stock. The COVID shock is now a third extreme realization of negative capital returns in the last 20 years. It makes such an event appear much more likely.

Long-run output loss Observing a tail event like the COVID-19 pandemic changes beliefs and outcomes in a persistent way. Figure 3 compares the predictions of our model for total output (GDP) to an identical model without learning. The units are log changes, relative to the pre-crisis steady-state. In the model without learning, agents know the true probability of pandemics. When they see the COVID crisis, they do not update the distribution, because they know it to be true with certainty. This corresponds to what is typically called "rational expectations." The model with learning, which uses our real-time kernel density estimation to inform beliefs, generates similar short-term reactions, but worse long-term effects.

When agents learn, the scenarios correspond to what economist might call a V-shaped or tilted-V recession. The size on impact is just a result of calibration targeting that size drop. It is an assumption, not a result. The result is what happens subsequently. In the long run, it is not the size of the initial impact that matters, as much as its persistence.

Of course, such a temporary drop in productivity is also a rare event, also learned about by

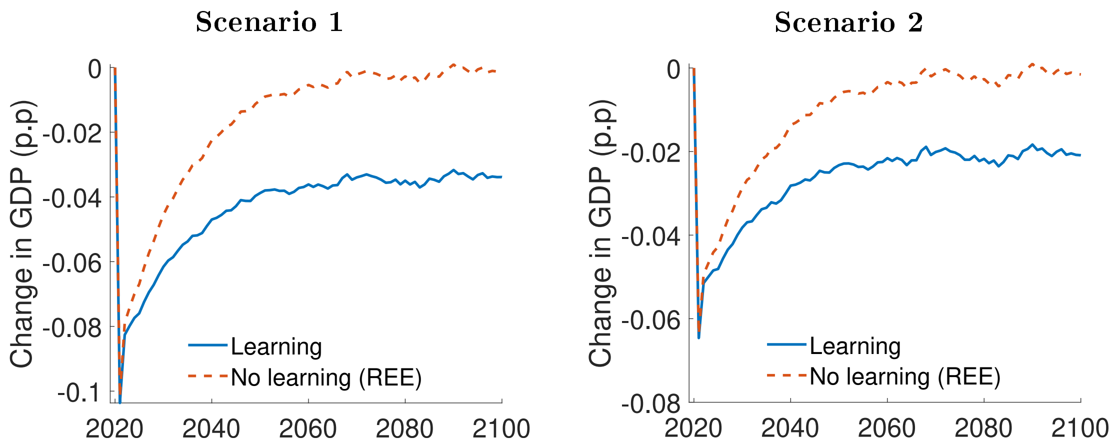

Figure 3: **Output with scarring of beliefs (solid line) and without (dashed line).** *Units are percentage changes, relative to the pre-crisis steady-state, with 0 being equal to steady state and* -0.1 *meaning 10% below steady state. Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1:* $\tilde{\phi}_{2020} = 0.90$ *Scenario 2:* $\tilde{\phi}_{2020} = 0.95$.

our agents, and also scars their beliefs going forward. But the scarring is much less, producing only a 2% loss in long-run annual output. The reason that the productivity shock has much less effect is that it impairs the return to capital by less. Tail risk mostly affects the risk premium required on capital investments. Labor also contracts, but that is a reaction to the loss of available capital that can be paired with labor. When a chunk of capital becomes mal-adapted and worthless, that is an order of magnitude more costly to the investor than the temporary decline in capital productivity. From the point of few of the distribution of capital returns, the ϕ shock that operates directly on the capital stock is much more a an outlier for capital returns than is the z shock that is derived from the same ϕ realization.

Turning off belief updating When agents do not learn, both scenarios involve quick and complete recoveries, even with this large initial impact. Without the scarring of beliefs, facilities are retrofitted, workers find new jobs, and while the transition is painful to many, the economy does return to its pre-crisis trajectory. With learning, this is no longer true.

To demonstrate the role of learning, we plot average simulated outcomes from an otherwise identical economy where agents know the final distribution $\hat{g}_{2020}$ with certainty, from the very beginning (dashed line in each figure). Now, by assumption, agents do not revise their beliefs after the Great Recession. This corresponds to a standard rational expectations econometrics approach, where agents are assumed to know the true distribution of shocks hitting the economy and the econometrician estimates this distribution using all the available data. The post-2020 paths are simulated as follows: each economy is assumed to be at its stochastic steady state in 2019 and is subjected to the same 2020 ϕ and z shocks; subsequently, sequences of shocks

Scenario	2020 GDP drop	NPV(Belief Scarring)	NPV(Obsolete capital)
I. Tough	-10%	-16%	-78%
II. Lite	- 6%	-9%	-48%

Table 2: New present value costs in percentages of 2019 GDP.

drawn from the estimated 2020 distribution, the same as in the main results.

In the absence of belief revisions, the negative shocks lead to an investment boom, as the economy replenishes the lost effective capital. While the curvature in utility moderates the speed of this transition to an extent, the overall pattern of a steady recovery back to the original steady state is clear. Since the no-learning economy is endowed with the same end-of-sample beliefs as the learning model, they both ultimately converge to the same *levels*. But, they start at different steady states (normalized to 0 for each series). This shows that learning is what generates long-lived reductions in economic activity.

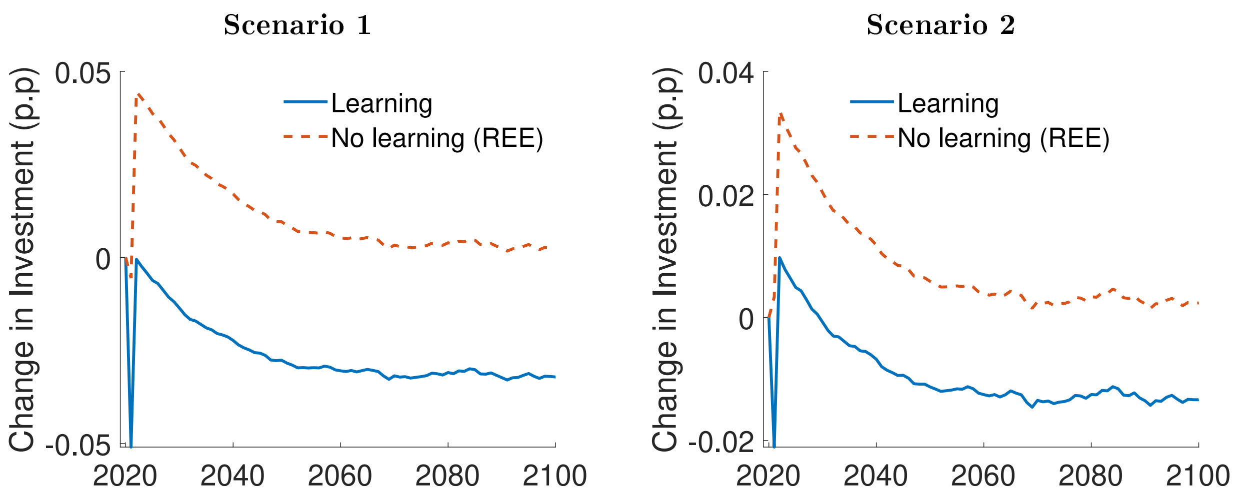

Figure 4: **Without belief scarring, investment surges.**
Results show average aggregate investment, with scarring of beliefs (solid line) and without (dashed line). Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1: $\tilde{\phi}_{2020} = 0.90$ *Scenario 2:* $\tilde{\phi}_{2020} = 0.95$.

Assessing long-run loss To assess the size of the long-run loss, we need to discount future output. We do that using the stochastic discount factor implied by this model.

Note that the 1-year loss during the pandemic is 10% of GDP. The cost of belief scarring is one and a half times as large. The cost of obsolete capital is nearly eight times as large as the damage done during the pandemic. Figure 5 illustrates the losses each year from the capital obsolescence and belief changes. The area of each of these regions, discounted as one moves to the right in time, is the NPV calculation in the table above. The one-year cost is a tiny fraction

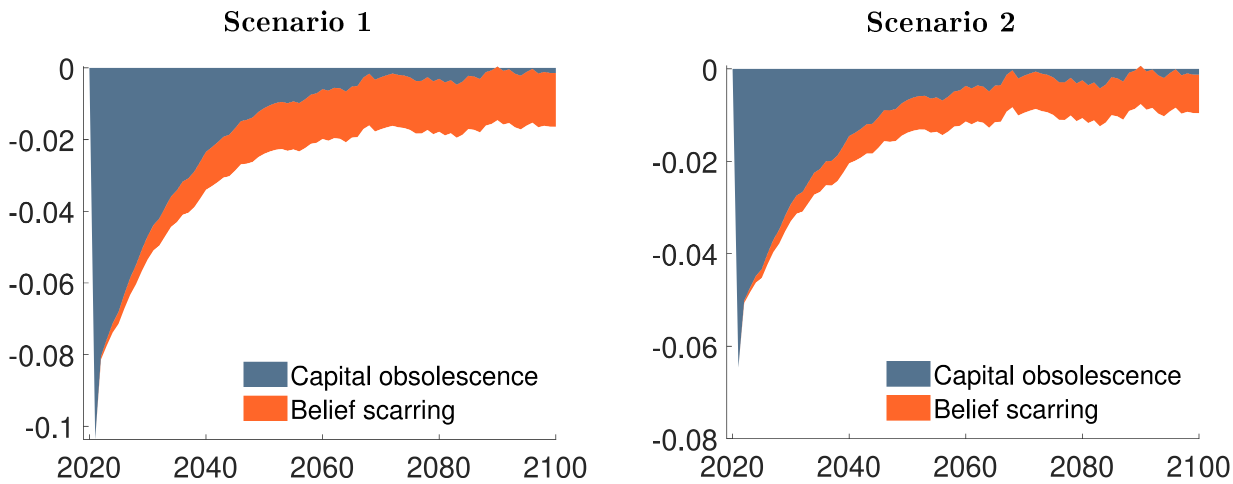

Figure 5: Long-term costs of the pandemic.

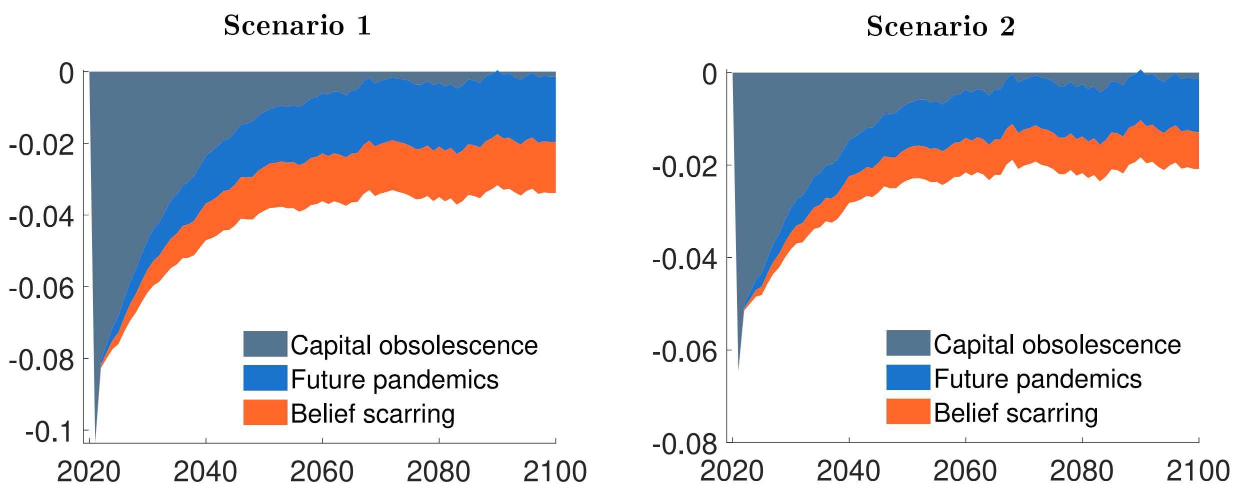

Figure 6: Long-term costs of with future pandemics.

of this total area.

Of course, that calculation misses an important aspect of what we've learned – that pandemics will recur. Since our agents have 70 years of data, during which they've seen one pandemic, they assess the future risk of pandmics to be 1-in-70 initially. That probability declines slowly as time goes on and other pandemics are not observed. But there is also the risk there will be more pademics, like this. This is not really a result of this pandemic. But that risk of future pandemics is what we should consider if we think about the benefits of public health investments. The pandemic cost going forward, in a world where a pandemic has a 1/70th probability of occuring each year is given in Figure 6

Note that the risk of future pandemics costs the economy as much as belief scarring, with a present discounted value of more than 1.5 times as much as the one-year cost during the COVID crisis.

Investment and Labor. The behavior in investment in Figure 4 further reinforces this point. It is the combination of the capital-return depressing small ϕ, along with learning, that has large investment effects. When agents do not learn, investment surges or dips for only one period. When agents learn, but the shock is less detrimental to capital returns, there is a quick fall in investment during the period when productivity is low and then a rebound, to a higher investment level than before, to make up for the lower capital stock.

One reason that we focus on shocks directly to capital is because we saw such large equity price responses. The initial 30% drop in the value of equity is nearly impossible to reconcile with any temporary productivity shock, since more than half of the value of equity derives from its cash flows more than one year ahead. The large initial reaction suggests that investors had substantial concerns for losses that were more permanent.

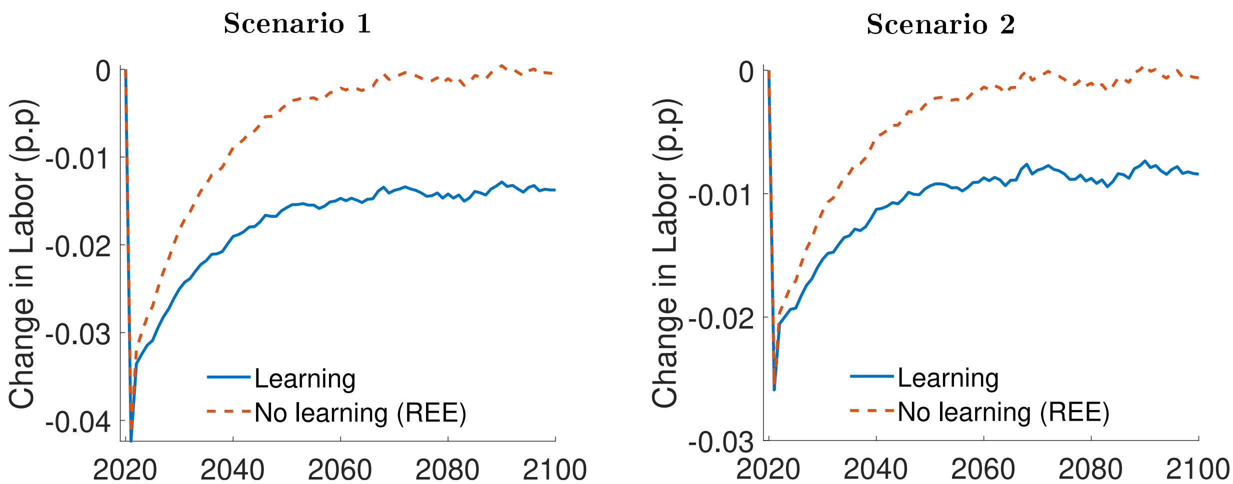

Figure 7: **Labor with scarring of beliefs (solid line) and without (dashed line).** *Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1:* $\tilde{\phi}_{2020} = 0.90$ *Scenario 2:* $\tilde{\phi}_{2020} = 0.95$..

In figure 7, we see that the initial reaction of labor is a little more mild than for investment. But the bigger difference is in the second period reaction. When the transitory shock passes, investment surges, to higher than its initial level, to compensate for the lost mal-adapted capital. But labor returns to a lower than initial level. Labor demand is depressed by the lower capital stock. Labor demands continues to respond gradually, as the capital stock is rebuilt.

What belief scarring does is to make the rebound slower and less. By deterring investment, with the knowledge of the greater possibility of low investment returns, the scarring effect lowers the capital stock and thereby lowers the complementary demand for labor. While labor is not the main mechanism of the effect, it matters immensely for welfare, equity and human happiness.

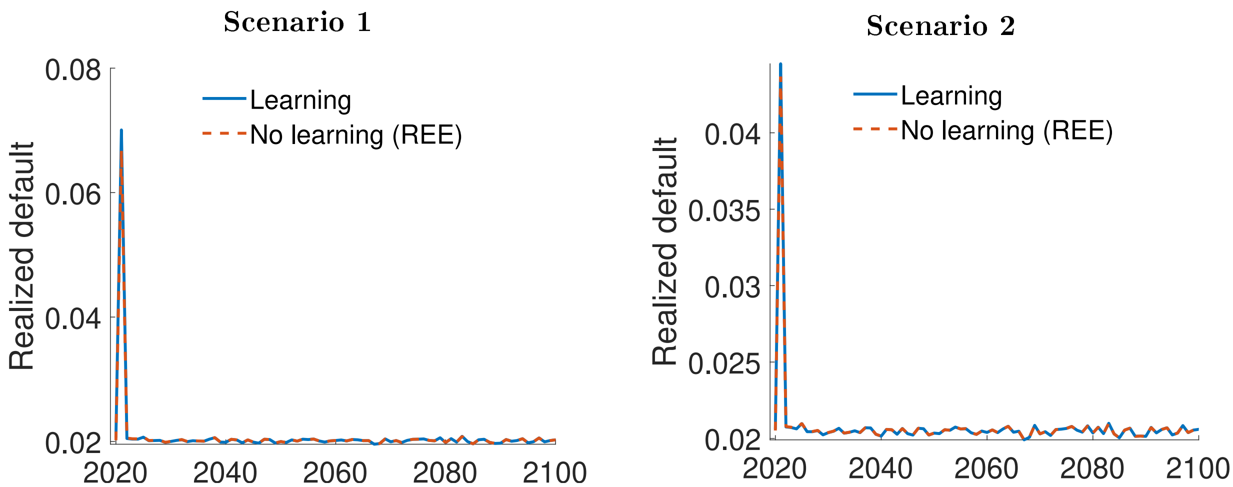

Figure 8: **Realized default does not respond much to beliefs.**
Results show with scarring of beliefs (solid line) and without (dashed line), often with the two lines on top of each other. Common parameters listed in Table 1. Scenario-specific parameters are: Scenario 1: $\tilde{\phi}_{2020} = 0.90$ *Scenario 2:* $\tilde{\phi}_{2020} = 0.95$

Defaults and Interest Rates. One of the ways in which these scenarios differ is in their default rates. Default happens only in the first period, when the shock hits. But the 15% default rate in scenario 1 scars investors far more than the 7% and 3% default rates of scenarios 2 and 3, with their more transitory shocks. This result suggest that, to prevent long-term scarring effects, policy needs to work hard to prevent business default. While this has been a focus of recent policy, and is common sense, we had little quantitative evidence to evaluate the benefits of such policy. This work suggests that the benefits of avoiding widespread defaults is a nearly-permanent 11%-17% annual gain in GDP, in perpetuity. That gain is far more than one might have imagined.

Finally, we show that incorporating learning affects long-run interest rate trajectories.

5 Conclusion

No one knows the true distribution of shocks to the economy. Economists typically assume that agents in their models know this distribution, as a way to discipline beliefs. But assuming that agents do the same kind of real-time estimation that an econometrician would do is equally disciplined and more plausible. For many applications, assuming full knowledge has little effect on outcomes and offers tractability. But for outcomes that are sensitive to tail probabilities, the difference between knowing these probabilities and estimating them with real-time data can be large. The estimation error can introduce new, persistent dynamics into a model with otherwise transitory shocks. The essence of the persistence mechanism is this: Once observed, a shock (a

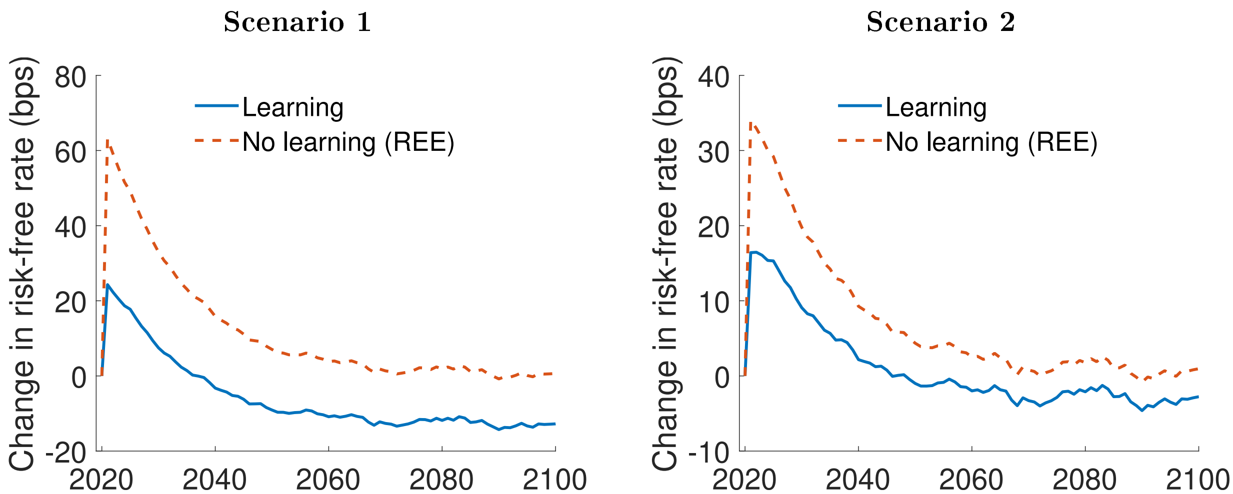

Figure 9: **Belief scarring lowers riskless rate in the long-run.**
Results show the return on a riskless asset, with scarring of beliefs (solid line) and without (dashed line). Common parameters listed in Table 1. Scenario-specific parameters are– Scenario 1: $\tilde{\phi}_{2020} = 0.90$ *Scenario 2:* $\tilde{\phi}_{2020} = 0.95$.

piece of data) stays in one's data set forever and therefore persistently affects belief formation. The less frequently similar data is observed, the larger and more persistent the belief revision.

When we quantify this mechanism and use capital price and quantity data to directly estimate beliefs, our model's predictions tell us that preventing bankruptcies or permanent separation of labor and capital, could have enormous consequences for the value generated by the U.S. economy for decades to come. What sound policy might avoid is the prospect that, after seeing how fragile our economy is to a pandemic, firms will be scarred and will never think about tail risk in the same way again.

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A Solution

A.1 Equilibrium Characterization

Thus, an equilibrium is the solution to the following system of equations:

$$
\begin{aligned}
1 &= \mathbb{E}M_{t+1}\left[R^k_{t+1}\right]J^k(\underline{v}) && (19)\\
R^k_{t+1} &= \frac{z_t\phi^{\alpha}_{t+1}\hat{K}^{\alpha}_{t+1}L^{1-\alpha}_{t+1} - W_{t+1}L_{t+1} + (1-\delta)\,\phi_{t+1}\hat{K}_{t+1}}{\hat{K}_{t+1}} && (20)\\
L_t &= \left[\frac{(1-\alpha)z_t}{L^{\gamma}_t}\right]^{\frac{1}{\alpha}}\phi_t\hat{K}_t = \left[\frac{(1-\alpha)z_t}{W_t}\right]^{\frac{1}{\alpha}}\phi_t\hat{K}_t && (21)\\
(1-\theta)\,\mathbb{E}_t\left[M_{t+1}\underline{v}f\left(\underline{v}\right)\right] &= \left(\frac{\chi-1}{\chi}\right)\mathbb{E}_t\left[M_{t+1}\left(1-F\left(\underline{v}\right)\right)\right] && (22)\\
C_t &= z_t\phi^{\alpha}_t\hat{K}^{\alpha}_tL^{1-\alpha}_t + (1-\delta)\,\phi_t\hat{K}_t - \hat{K}_{t+1} && (23)\\
U_t &= \left[(1-\beta)\left(u\left(C_t,L_t\right)\right)^{1-\psi} + \beta\mathbb{E}\left(U^{1-\eta}_{t+1}\right)^{\frac{1-\psi}{1-\eta}}\right]^{\frac{1}{1-\psi}} && (24)
\end{aligned}
$$

where

$$
\begin{aligned}
\underline{v} &= \frac{lev_{t+1}}{R^k_{t+1}}\\
J^k(\underline{v}) &= 1 + (\chi-1)\,\underline{v}\left(1-F\left(\underline{v}\right)\right) + (\chi\theta-1)\,h\left(\underline{v}\right)\\
M_{t+1} &= \left(\frac{dU_t}{dC_t}\right)^{-1}\frac{dU_t}{dC_{t+1}} = \beta\left[\mathbb{E}\left(U^{1-\eta}_{t+1}\right)\right]^{\frac{\eta-\psi}{1-\eta}}U^{\psi-\eta}_{t+1}\left(\frac{u\left(C_{t+1},L_{t+1}\right)}{u\left(C_t,L_t\right)}\right)^{-\psi}
\end{aligned}
$$

A.2 Solution Algorithm

To solve the system described above at any given date t (i.e. after any observed history of ϕ_t), we recast it in recursive form with grids for the aggregate state ($\hat{K}$) and the shocks ϕ. We then use an iterative procedure:

- Estimate $\hat{g}$ on the available history using the kernel estimator.
- Start with a guess (in polynomial form) for $U(\hat{K},\phi), C(\hat{K},\phi)$.
- Solve (19)-(22) for $\hat{K}', lev'(\Pi, L)$ using a non-linear solution procedure.
- Verify/update the guess for U, C using (23)-(24) and iterate until convergence.