1.4.1. $A^{-1} \cdot A = A \cdot A^{-1} = E$ 20.05.2020

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$$

1) ΔA (определитель) $\det A$ $= 7 \cdot \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - 8 \cdot \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + 0 =$

$= 7 \cdot (2 \cdot 6 - 3 \cdot 5) - 8(1 \cdot 6 - 3 \cdot 4) =$

$= 7 \cdot (-3) - 8 \cdot (-6) = -21 + 48 = 27 \neq 0$

$\Rightarrow \exists A^{-1}$

2) $A_{ij} = (-1)^{i+j} \cdot M_{ij}$
(алгебраичное дополнение)

$A_{1\times1} = (-1)^{1+1} \cdot \begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} = 1 \cdot (5 \cdot 0 - 6 \cdot 8) = -48$

$A_{1\times2} = (-1)^{1+2} \cdot \begin{vmatrix} 4 & 6 \\ 7 & 0 \end{vmatrix} = -1(4 \cdot 0 - 6 \cdot 7) = 42$

$A_{1\times3} = (-1)^{1+3} \cdot \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 1 \; (4 \cdot 8 - 5 \cdot 7) = -3$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 4 & 6 \\ 7 & 0 \end{vmatrix} = -1 \cdot (4 \cdot 0 - 6 \cdot 7) = 24$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 3 \\ 7 & 0 \end{vmatrix} = 1 \cdot (1 \cdot 0 - 3 \cdot 7) = -21$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = -1 \cdot (1 \cdot 8 - 2 \cdot 7) = 6$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 2 \cdot 6 - 5 \cdot 3 = 12 - 15 = -3$

$A_{32} = (-1)^{3+2} = \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -1(6 - 12 = 6$

$A_{33} = (-1)^{3+3} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 5 - 8 = -3$

3) $\tilde{A} = (A_{ij})^T$ – матрица алгебраических дополнений
(присоединенная матрица)

$$(A_{ij})^T = \begin{pmatrix} -48 & 42 & -3 \\ 24 & -21 & 6 \\ -3 & 6 & -3 \end{pmatrix}^T = \begin{pmatrix} -48 & 24 & -3 \\ 42 & -21 & 6 \\ -3 & 6 & -3 \end{pmatrix} = \tilde{A}$$

4) $A^{-1} = \frac{1}{\Delta A} \cdot \tilde{A}$

$$= \frac{1}{27} \cdot \begin{pmatrix} -48 & 24 & -3 \\ 42 & -21 & 6 \\ -3 & 6 & -3 \end{pmatrix} = \begin{pmatrix} -\frac{48}{27} & \frac{24}{27} & -\frac{3}{27} \\ \frac{42}{27} & -\frac{21}{27} & \frac{6}{27} \\ -\frac{3}{27} & \frac{6}{27} & -\frac{3}{27} \end{pmatrix} =$$

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$$= \begin{pmatrix} -\frac{16}{9} & \frac{6}{9} & -\frac{1}{9} \\ +\frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{pmatrix}$$

1.4.2

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad A^{-1} - ?$$

1) $\Delta A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0 + 0 = 1 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \cdot (1 \cdot 1 - 0 \cdot 0) = 1$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

3) $\tilde{A} = (A_{ij})^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

4) $\frac{1}{1} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

1.4.3.

$$A = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & 1/3 & -\frac{2}{3} \\ \frac{2}{3} & -2/3 & \frac{1}{3} \end{pmatrix}$$

1) $\det A = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\left(-\frac{2}{3}\right)\cdot\frac{2}{3} + \frac{2}{3}\cdot\left(-\frac{2}{3}\right)\cdot\frac{2}{3}$

$- \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} - \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} - \frac{1}{3}\cdot\left(-\frac{2}{3}\right)\cdot\left(-\frac{2}{3}\right) =$

$= \frac{1}{18} + \left(-\frac{8}{27}\right) - \frac{8}{27} - \frac{4}{27} - \frac{4}{27} - \frac{4}{27} =$

$= \cancel{\frac{486-8-8-4-4-4}{27}}\ \frac{1}{18} + \frac{-8-8-4-4-4}{27} = \frac{1}{27} - \frac{28}{27}$

$= -1$

$A_{11} = (-1)^{1+1}\cdot\begin{vmatrix} 1/3 & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{vmatrix} = 1\cdot\left(\frac{1}{3}\cdot\frac{1}{3} - \left(-\frac{2}{3}\right)\cdot\left(-\frac{2}{3}\right)\right) = -\frac{1}{3}$

$A_{12} = -\frac{2}{3}$

$A_{13} = -\frac{2}{3}$

$A_{21} = -\frac{2}{3}$

$A_{22} = -\frac{1}{3}$

$A_{23} = \frac{2}{3}$

$A_{31} = -\frac{2}{3}$

$A_{32} = \frac{2}{3}$

$A_{33} = -\frac{1}{3}$

2) $\tilde{A} = \begin{pmatrix} -1/3 & -2/3 & -2/3 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & 2/3 & -1/3 \end{pmatrix}^T =$

$= \begin{pmatrix} -\frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}\cdot\frac{1}{-1} = -1$

$= \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$

1.4.4.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix}$$

1) $\Delta A = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{vmatrix} = 2\cdot\begin{vmatrix} 2 & -3 \\ 2 & -4 \end{vmatrix} -$

$(-1)\cdot\begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} + 0 =$

$= 1.$

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$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 2 & -4 \\ -1 & 0 \end{vmatrix} = 0 + 4 = 4$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 3 & -4 \\ 2 & 0 \end{vmatrix} = 8$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -3 - 4 = -7$

$A_{21} = (-1)^{21} \cdot \begin{vmatrix} 2 & -3 \\ -1 & 0 \end{vmatrix} = -3$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix} = +6$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5 \cdot (-1) = 5$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 2 & -3 \\ 2 & -4 \end{vmatrix} = -6 + 8 = 2$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = -1(-4 + 9) = -5$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2 - 6 = -4.$

3) $\tilde{A} = \begin{pmatrix} 4 & 8 & -7 \\ -3 & 6 & 5 \\ -2 & -5 & -4 \end{pmatrix}^T = \begin{pmatrix} 4 & -3 & -2 \\ 8 & 6 & -5 \\ -7 & 5 & -4 \end{pmatrix}$

4) $A^{-1} = \begin{pmatrix} 4 & -3 & -2 \\ 8 & 6 & -5 \\ -7 & 5 & -4 \end{pmatrix}$