

7.3.19 - 7.3.22 (Koruny 05 11.05)

$$\begin{aligned} \lim_{x \rightarrow +\infty} x^2 \cdot e^{-x} &= [\infty \cdot \infty] = \lim_{x \rightarrow +\infty} (x^2)' \cdot (e^{-x})' \\ &= \lim_{x \rightarrow +\infty} 2x \cdot e^{-x} = \lim_{x \rightarrow +\infty} (2x)' \cdot (e^{-x})' \\ &= \lim_{x \rightarrow +\infty} 2 \cdot e^{-x} = 2 \cdot e^{-\infty} = 2 \cdot 0 = 0 \end{aligned}$$

7.3.20.

$$\begin{aligned} \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) &= [\infty - \infty] = \lim_{x \rightarrow 0} \frac{\sin x - x}{x \cdot \sin x} \\ &= \lim_{x \rightarrow 0} \frac{(\sin x - x)'}{(x \cdot \sin x)'} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{x \cdot \sin x + x \cdot (\sin x)'} \\ &= \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x + x \cdot \cos x} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)'}{(\sin x + x \cdot \cos x)'} \\ &= \lim_{x \rightarrow 0} \frac{-\sin x}{\cos x - \sin x} = \frac{-0}{1-0} = 0 \end{aligned}$$

7.3.21

$$\begin{aligned} \lim_{x \rightarrow \infty} x(e^{\frac{1}{x}} - 1) &= [\infty \cdot 0] = \lim_{x \rightarrow \infty} x'(e^{\frac{1}{x}} - 1)' \\ &= \lim_{x \rightarrow \infty} e^{\frac{1}{x}} = e^{\frac{1}{\infty}} = 1 \end{aligned}$$

7.3.22.

$$\lim_{x \rightarrow 1} \left(\frac{1}{1-x^3} - \frac{1}{1-x^2} \right) = [0 - 0]$$

$$= \lim_{x \rightarrow 1} \left(\frac{1 - x^2 - 1 + x^3}{(1 - x^3)(1 - x^2)} \right) = \lim_{x \rightarrow 1} \frac{(x^3 - x^2)}{(1 - x^3)(1 - x^2)} =$$

$$= \lim_{x \rightarrow 1} \frac{3x^2 - 2x}{-3x^2(1 - x^2) + 2x(1 - x^3)} = \lim_{x \rightarrow 1} \frac{3x^2 - 2x}{-3x^2 + 3x^4 - 2x + 2x^4} =$$

$$= \lim_{x \rightarrow 1} \frac{3x^2 - 2x}{5x^4 - 3x^2 - 2x} = \frac{3 \cdot 1 - 2 \cdot 1}{5 \cdot 1 - 3 \cdot 1 - 2 \cdot 1} = \frac{1}{0} = \infty$$