ДЗ/3 Обратная матрица. Козырьков Игорь.

1.4.37

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0,5 & 0 \end{pmatrix} \quad \det A = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0,5 & 0 \end{vmatrix} = -(-1)\cdot 2 \cdot 0,5 = 1 \neq 0 \Rightarrow \exists A^{-1}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & 2 \\ 0,5 & 0 \end{vmatrix} = -1$$

$$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 0,5 \end{vmatrix} = 0$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 0,5 & 0 \end{vmatrix} = 0$$

$$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0,5 \end{vmatrix} = -0,5$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} = 0$$

$$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} = -2$$

$$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$\tilde{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -0,5 \\ 0 & -2 & 0 \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -0,5 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} = \frac{1}{1} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -0,5 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -0,5 & 0 \end{pmatrix}$$

1.4.38.

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix} \quad \det A = \begin{vmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{vmatrix} = 9 + 8 + 24 - 12 - 6 = -1 \neq 0 \Rightarrow \exists A^{-1}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & -6 \\ -1 & 3 \end{vmatrix} = 15$$

$$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 8 & -6 \\ -4 & 3 \end{vmatrix} = 0$$

$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 8 & 3 \\ -4 & -1 \end{vmatrix} = 4$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ -4 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ -4 & 3 \end{vmatrix} = -1$$

$$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} = -3$$

$$A_{31} = (-1)^{1+3} \cdot \begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -3$$

$$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & -1 \\ 8 & -6 \end{vmatrix} = -2$$

$$A_{33} = \begin{vmatrix} 1 & 1 \\ 8 & 3 \end{vmatrix} = -5$$

$$\tilde{A} = \begin{pmatrix} 15 & 0 & 4 \\ 2 & -1 & -3 \\ -3 & -2 & -5 \end{pmatrix}^T = \begin{pmatrix} 15 & 2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{-1} \cdot \begin{pmatrix} 15 & 2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix} = \begin{pmatrix} -15 & -2 & 3 \\ 0 & 1 & 2 \\ -4 & 3 & 5 \end{pmatrix}$$

1.4.39

$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix}$ $\det A = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{vmatrix} = -4+4+8+8-2-8 = 6 \neq 0$

$\Rightarrow \exists A^{-1}$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -6$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 6$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -2$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = -4$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = 3$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 4$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 2$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$

$\tilde{A} = \begin{pmatrix} -6 & 0 & 6 \\ -2 & -4 & 3 \\ 4 & 2 & -3 \end{pmatrix}^T = \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix}$

$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{6} \cdot \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix}$

$= \begin{pmatrix} -\frac{6}{6} & -\frac{2}{6} & \frac{4}{6} \\ 0 & -\frac{4}{6} & \frac{2}{6} \\ \frac{6}{6} & \frac{3}{6} & -\frac{3}{6} \end{pmatrix} =$

$= \begin{pmatrix} -1 & -1/3 & \frac{2}{3} \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ 1 & 1/2 & -1/2 \end{pmatrix}$

1.4.40

$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{pmatrix}$ $\det A = \begin{vmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{vmatrix} = -12+20-12+8+45-8 = 41 \neq 0$

$\Rightarrow \exists A^{-1}$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -4 & -3 \\ 5 & 1 \end{vmatrix} = 11$

$A_{12} = (-1)^{2+1} \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 14$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} = 6$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = -11$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 4 \\ 1 & -4 \end{vmatrix} = -20$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = 13$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 4 \\ 2 & -4 \end{vmatrix} = -16$

$\tilde{A} = \begin{pmatrix} 11 & -5 & 14 \\ 6 & 1 & -11 \\ -20 & 13 & -16 \end{pmatrix}^T = \begin{pmatrix} 11 & 6 & -20 \\ -5 & 1 & 13 \\ 14 & -11 & -16 \end{pmatrix}$

$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{41} \cdot \begin{pmatrix} 11 & 6 & -20 \\ -5 & 1 & 13 \\ 14 & -11 & -16 \end{pmatrix}$

$= \begin{pmatrix} \frac{11}{41} & \frac{6}{41} & -\frac{20}{41} \\ \frac{-5}{41} & \frac{1}{41} & \frac{13}{41} \\ \frac{14}{41} & \frac{-11}{41} & \frac{-16}{41} \end{pmatrix}$

1.4.41

$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$ $\det A = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 0 - 3 + 24 + 6 - 27 - 0 = 0$

$\Rightarrow \nexists A^{-1}$

1.4.42

$A = \begin{pmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ $\det A = \begin{vmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -30 - 4 + 16 - 3 - 20 - 48 = -104$

$\neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -3 & 2 \\ 2 & 3 \end{vmatrix} = -13$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = -4$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 7$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 8 & -1 \\ 2 & 3 \end{vmatrix} = -26$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 16$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 8 \\ 1 & 2 \end{vmatrix} = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 8 & -1 \\ -3 & 2 \end{vmatrix} = 13$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 5 & -1 \\ 2 & 2 \end{vmatrix} = -12$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 5 & 8 \\ 2 & -3 \end{vmatrix} = -31$

$\hat{A} = \begin{pmatrix} -13 & -4 & 7 \\ -26 & 16 & -2 \\ 13 & -12 & -31 \end{pmatrix}^T = \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix}$

$A^{-1} = \frac{-1}{104} \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix} =$

$= \begin{pmatrix} +\frac{1}{8} & +1/4 & -\frac{1}{8} \\ +\frac{1}{26} & -2/13 & +3/26 \\ \frac{-7}{104} & 1/52 & -31/104 \end{pmatrix}$

1.4.43

$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ $\det A = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = 4 + 1 + 1 - 2 - 1 - 2 = 1 \neq 0$

$\Rightarrow \exists A^{-1}$

$\Gamma = \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ II + I \\ III + I \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)^{I + II + III} \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$ $A^{-1} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

1.4.44.

$A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}$ $\det A = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{vmatrix} = 54 + 45 + 28 - 27 - 40 - 63$

$= -3 \neq 0 \Rightarrow \exists A^{-1}$

$\Gamma = \left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 2 & 7 & 3 & 1 & 0 & 0 \end{array}\right)^{II - I - III} \sim$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & -2 & -1 & 1 & -1 \\ 2 & 7 & 3 & 1 & 0 & 0 \end{array}\right)_{III-2I} \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & -2 & -1 & 1 & -1 \\ 0 & -3 & -3 & 1 & 0 & -2 \end{array}\right)_{III-II}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & -2 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)_{II+2III} \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & 0 & 3 & -1 & -3 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)_{(:-3)}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1/3 & 1 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)^{I-5II-3III} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 4/3 & -1 \\ 0 & 1 & 0 & -1 & 1/3 & 1 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -1 & \frac{4}{3} & -1 \\ -1 & \frac{1}{3} & 1 \\ 2 & -1 & -1 \end{pmatrix}$$

1.4.45

$$A = \begin{pmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{pmatrix} \quad \det A = \begin{vmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{vmatrix} \leftarrow = (-1)^{3+1}\cdot 3 \begin{vmatrix} 2 & -2 & 4 \\ 6 & 1 & 0 \\ 4 & 5 & -4 \end{vmatrix}$$

$$+ 0 + (-1)^{3+3} \begin{vmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \\ -1 & 4 & -4 \end{vmatrix} + (-1)^{3+4}\cdot 2 \begin{vmatrix} 1 & 2 & -2 \\ 2 & 6 & 1 \\ -1 & 4 & 5 \end{vmatrix} = 3(-8+120+0-16-0-48) + (-24+32+0+24-0$$

$$+16) - 2(30-16-2-12-4-20) = 144+48+48 = 240 \neq 0 \Rightarrow \exists A^{-1}$$

$$\Gamma = \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 6 & 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 4 & 5 & -4 & 0 & 0 & 0 & 1 \end{array}\right)^{II-III-IV} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 4 & 5 & -4 & 0 & 0 & 0 & 1 \end{array}\right)_{IV-II} \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -3 & 0 & 9 & -12 & -2 & 0 & 0 & 2 \end{array}\right)_{III+IV} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 0 & 0 & 10 & -10 & -2 & 0 & 1 & 1 \\ -3 & 0 & 9 & -12 & -2 & 0 & 0 & 1 \end{array}\right)_{:(-3)}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 0 & 0 & 10 & -10 & -2 & 0 & 1 & 1 \\ 1 & 0 & -3 & 4 & \frac{2}{3} & 0 & 0 & -1/3 \end{array}\right)_{IV-I+II} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 0 & 0 & 10 & -10 & -2 & 0 & 1 & 1 \\ 0 & 0 & -6 & 2 & -\frac{1}{3} & 1 & -1 & -\frac{4}{3} \end{array}\right)_{\begin{subarray}{l} :10 \\ :2 \end{subarray}}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 & -\frac{1}{5} & 0 & 1/10 & 1/10 \\ 0 & 0 & -3 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & -\frac{2}{3} \end{array}\right)_{III+IV} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 & -11/30 & \frac{1}{2} & -\frac{1}{10} & -17/30 \\ 0 & 0 & -3 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & -\frac{2}{3} \end{array}\right)_{\begin{subarray}{l} :(-2) \\ IV-\left(\frac{3}{2}\right)III \end{subarray}}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & -5 & 2 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & \frac{11}{60} & -\frac{1}{4} & \frac{1}{20} & \frac{17}{60} \\ 0 & 0 & 0 & 1 & \frac{13}{60} & -\frac{1}{8} & \frac{1}{10} & \frac{11}{60} \end{array}\right)^{II+5III-2IV} \overset{I-II+2III-4IV}{\sim} \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1/5 & 1/4 & -\frac{1}{5} & 1/20 \\ 0 & 0 & 1 & 0 & 11/60 & -1/4 & 2/10 & 17/60 \\ 0 & 0 & 0 & 1 & \frac{13}{60} & -\frac{1}{4} & 1/10 & 11/60 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{11}{30} & \frac{1}{4} & \frac{1}{5} & -\frac{2}{5} \\ 0 & 2 & 0 & 0 & \frac{1}{5} & \frac{1}{4} & -1/5 & 1/20 \\ 0 & 0 & 1 & 0 & \frac{11}{60} & -\frac{1}{4} & \frac{2}{10} & \frac{17}{60} \\ 0 & 0 & 0 & 1 & \frac{41}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{array}\right)_{:2} \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{11}{30} & \frac{1}{4} & \frac{1}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & 0 & \frac{1}{20} & \frac{1}{8} & -\frac{1}{10} & \frac{1}{40} \\ 0 & 0 & 1 & 0 & \frac{11}{60} & -\frac{1}{4} & \frac{2}{10} & 17/60 \\ 0 & 0 & 0 & 1 & \frac{37}{60} & -1/4 & 1/10 & 11/60 \end{array}\right)$$

$\boxed{4\ \text{CTP}}$

$$A^{-1} = \begin{pmatrix} -\frac{11}{30} & \frac{1}{4} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{10} & \frac{1}{6} & -\frac{1}{10} & \frac{1}{40} \\ \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} & \frac{17}{60} \\ \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{pmatrix}$$

1.4.50

$$X \cdot \underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{B} \quad \det A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2 \neq 0 \Rightarrow \exists A^{-1}$$

$A_{11} = |4| = 4$ $\quad \tilde{A} = \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix}^T = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

$A_{12} = -|3| = -3$

$A_{21} = -|2| = -2$ $\quad A^{-1} = -\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3/2 & -0,5 \end{pmatrix}$

$A_{22} = |1| = 1$ $\quad X = A^{-1} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

1.4.51

$$X \cdot \underbrace{\begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{B} \quad \det A = \begin{vmatrix} 4 & 3 \\ -5 & -4 \end{vmatrix} = -16 + 15 = -1 \neq 0 \Rightarrow \exists A^{-1}$$

$A_{11} = |-4| = -4$ $\quad \tilde{A} = \begin{pmatrix} -4 & -5 \\ +3 & 4 \end{pmatrix}^T = \begin{pmatrix} -4 & -3 \\ +5 & 4 \end{pmatrix}$

$A_{12} = -|-5| = +5$

$A_{21} = -|3| = -3$ $\quad A^{-1} = \frac{1}{-1} \cdot \begin{pmatrix} -4 & -3 \\ +5 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}$

$A_{22} = |4| = 4$

$$X = \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4+0 & 0+3 \\ -5+0 & 0-4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}$$

1.4.52

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \det A = 0 \Rightarrow \nexists A^{-1}$$

1.4.53

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \det A = 0 \Rightarrow \nexists A^{-1}$$

1.4.54

$$\underbrace{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}}_{A} X \cdot \underbrace{\begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}}_{C} = \underbrace{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}}_{B} \quad X = A^{-1} \cdot B \cdot C^{-1}$$

$$\det A = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5 \neq 0 \Rightarrow \exists A^{-1}$$

$$\det C = \begin{vmatrix} -5 & 6 \\ -4 & 5 \end{vmatrix} = -25 + 24 = -1 \neq 0 \Rightarrow \exists C^{-1}$$

$A_{11} = |3| = 3$

$A_{12} = -|2| = -2$

$A_{21} = -|-1| = 1$

$A_{22} = |1| = 1$

$\tilde{A} = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}^T = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{5} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$

$C_{11} = |5| = 5$

$C_{12} = -|-4| = 4$

$C_{21} = -|6| = -6$

$C_{22} = |-5| = -5$

$\tilde{C} = \begin{pmatrix} 5 & 4 \\ -6 & -5 \end{pmatrix}^T = \begin{pmatrix} 5 & -6 \\ 4 & -5 \end{pmatrix}$

$C^{-1} = -1 \cdot \begin{pmatrix} 5 & -6 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}$

$X = A^{-1} \cdot B \cdot C^{-1}$

$X = \overset{A^{-1}}{\begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}} \cdot \overset{B}{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}} \overset{C}{\begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}} = \begin{pmatrix} \frac{3}{5} + \frac{2}{5} & -\frac{3}{5} + \frac{3}{5} \\ -\frac{2}{5} + \frac{2}{5} & +\frac{2}{5} + \frac{3}{5} \end{pmatrix} \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}$

1.4.55

$\overset{A}{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}} \cdot X \cdot \overset{C}{\begin{pmatrix} 2 & -2 \\ -4 & 5 \end{pmatrix}} = \overset{B}{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}}$ $\det A = 5 \neq 0 \Rightarrow \exists A^{-1}$

$\det C = 2 \neq 0 \Rightarrow \exists C^{-1}$

$A_{11} = |3| = 3$

$A_{12} = -|2| = -2$

$A_{21} = -|-1| = 1$

$A_{22} = |1| = 1$

$\tilde{A} = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}^T = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$

$C_{11} = |5| = 5$

$C_{12} = -|-4| = 4$

$C_{21} = -|-2| = 2$

$C_{22} = |2| = 2$

$\tilde{C} = \begin{pmatrix} 5 & 4 \\ 2 & 2 \end{pmatrix}^T = \begin{pmatrix} 5 & 2 \\ 4 & 2 \end{pmatrix}$

$C^{-1} = \begin{pmatrix} \frac{5}{2} & 1 \\ 2 & 1 \end{pmatrix}$

$X = A^{-1} \cdot B \cdot C^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} \frac{5}{2} & 1 \\ 2 & 1 \end{pmatrix} =$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{5}{2} & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} + 0 & 1 + 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2{,}5 & 1 \\ 2 & 1 \end{pmatrix}$

1.4.56

$$X \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}}_{B} \qquad \det A = 1 \cdot \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + 0 + 0 = 6 \neq 0 \Rightarrow \exists A^{-1}$$

$$\Gamma = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ :2 \\ :3 \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \qquad X = B \cdot A^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+\frac{1}{3} \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 3+0+0 & 0+0+0 & 0+0+0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1/3 \\ 0 & 1 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

1.4.57.

$$\underbrace{\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix}}_{A} X = \underbrace{\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}}_{B} \qquad \det A = 3+0-12-0+4-2 = -9+4-2 = -9+2 = -7 \neq 0 \Rightarrow \exists A^{-1}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = 1$$
$$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -2$$
$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} = -4$$
$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix} = -4$$
$$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1$$
$$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = 2$$
$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} = -7$$
$$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = 7$$
$$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 7$$

$$\tilde{A} = \begin{pmatrix} 1 & -2 & -4 \\ -4 & 1 & 2 \\ -7 & 7 & 7 \end{pmatrix}^T = \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix}$$

$$A^{-1} = \frac{-1}{7} \cdot \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix} = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{2}{7} - \frac{4}{7} + 3 \\ \frac{4}{7} + \frac{1}{7} - 3 \\ \frac{8}{7} + \frac{2}{7} - 3 \end{pmatrix} = \begin{pmatrix} \frac{-2-4+21}{7} \\ \frac{4+1-21}{7} \\ \frac{8+2-21}{7} \end{pmatrix} = \begin{pmatrix} \frac{15}{7} \\ -\frac{16}{7} \\ -\frac{11}{7} \end{pmatrix}$$

1.4.58

$$\underbrace{\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix}}_{A} \cdot X \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}}_{C} = \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}}_{B}$$

$$\det A = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{vmatrix} = 3-12+0-0-2+4 = -7 \neq 0 \Rightarrow \exists A^{-1}$$

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$$\det C = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 0+96+84-105-0-48 = 27 \neq 0 \Rightarrow \exists\, C^{-1}$$

$$\Gamma_A = \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \text{II}-2\text{I} \sim \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 7 & -7 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \text{II}:7 \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \text{III}+2\text{II} \sim \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & -2 & 1 & -\frac{4}{7} & \frac{2}{7} & 1 \end{array}\right) (-1)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & 1 & \frac{4}{7} & -\frac{2}{7} & -1 \end{array}\right) \text{II}+\text{III} \sim \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{7} & -\frac{1}{7} & 0 \\ 0 & 0 & 1 & -\frac{4}{7} & \frac{2}{7} & 1 \end{array}\right) \text{I}+2\text{II}-3\text{III}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{7} & \frac{4}{7} & 1 \\ 0 & 1 & 0 & \frac{2}{7} & -\frac{1}{7} & -1 \\ 0 & 0 & 1 & \frac{4}{7} & -\frac{2}{7} & -1 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$$

$$C_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} = -48$$

$$C_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 4 & 6 \\ 7 & 0 \end{vmatrix} = 42$$

$$C_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 48-35 = +13$$

$$C_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 2 & 3 \\ 8 & 0 \end{vmatrix} = 24$$

$$C_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 3 \\ 7 & 0 \end{vmatrix} = -21$$

$$C_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8-14 = 6$$

$$C_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -3$$

$$C_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 6$$

$$C_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = -3$$

$$\tilde{C} = \begin{pmatrix} -48 & 42 & 13 \\ 24 & -21 & 6 \\ -3 & 6 & -3 \end{pmatrix}^T = \begin{pmatrix} -48 & 24 & -3 \\ 42 & -21 & 6 \\ 13 & 6 & -3 \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} -\frac{48}{27} & \frac{24}{27} & -\frac{3}{27} \\ \frac{42}{27} & -\frac{21}{27} & \frac{6}{27} \\ \frac{13}{27} & \frac{6}{27} & -\frac{3}{27} \end{pmatrix}$$

$$X = A^{-1} \cdot B \cdot C^{-1}$$

$$X = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix} \begin{pmatrix} -\frac{48}{27} & \frac{24}{27} & -\frac{3}{27} \\ \frac{42}{27} & -\frac{21}{27} & \frac{6}{27} \\ \frac{13}{27} & \frac{6}{27} & -\frac{3}{27} \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{7}+\frac{16}{7}+\frac{49}{7} & -\frac{2}{7}+\frac{20}{7}+\frac{56}{7} & -\frac{3}{7}+\frac{24}{7}+0 \\ \frac{2}{7}-\frac{4}{7}-\frac{49}{7} & \frac{4}{7}-\frac{5}{7}-\frac{56}{7} & \frac{6}{7}-\frac{6}{7}+0 \\ \frac{4}{7}-\frac{8}{7}-\frac{49}{7} & \frac{8}{7}-\frac{10}{7}-\frac{56}{7} & \frac{12}{7}-\frac{12}{7}+0 \end{pmatrix} = \begin{pmatrix} \frac{64}{7} & \frac{84}{7} & 3 \\ -\frac{51}{7} & -\frac{57}{7} & 0 \\ -\frac{53}{7} & -\frac{58}{7} & 0 \end{pmatrix} \cdot C^{-1}$$

$$= \begin{pmatrix} \frac{103}{63} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$$

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