1) Козырьков Игорь 18 Вариант.

$f(x) = 4x^2+6x-5$

$f(x) = 4x^2+6x+\frac{9}{4}-\frac{9}{4}-5$

$f(x) = (2x+\frac{3}{2})^2 - 7,25$

т.к $(2x+\frac{3}{2})^2 \geq 0$

$\Rightarrow [-7,25; +\infty)$

2) $x_n = n^2 + \frac{n^2-8n+3}{(-1)^n}$

при $n=1$ $x_1 = 1 + \frac{1-8+3}{-1} = 5$

при $n=2$ $x_2 = 4 + \frac{4-16+3}{1} = 4 + (-\frac{9}{1}) = -5$

при $n=3$ $x_3 = 9 + \frac{9-8\cdot 3+3}{-1} = 9+12 = 21$

при $n=4$ $x_4 = 16 + \frac{16-8\cdot 4+3}{(-1)^4} = 16-13 = 3$

при $n=5$ $x_5 = 25 + \frac{25-8\cdot 5+3}{-1} = 37$

3) Найти пределы

1. $\lim\limits_{x\to -2} \frac{5x^3+40}{36x^3+36x^2+108x+360} = \lim\limits_{x\to -2} \frac{5(x^3+8)}{36(x^3+x^2+3x+10)} =$

$= \lim\limits_{x\to -2} \frac{5(x^3+8)}{36(x+2)(x^2-x+5)} = \lim\limits_{x\to -2} \frac{5\cancel{(x+2)}(x^2-2x+4)}{36\cancel{(x+2)}(x^2-x+5)} = \lim\limits_{x\to -2} \frac{5(x^2-2x+4)}{36(x^2-x+5)} =$

$= \lim\limits_{x\to -2} \frac{5(4-2\cdot(-2)+4)}{36(4+2+5)} = \frac{60}{396} = \frac{5}{33}$

2. $\lim\limits_{x\to -2} \frac{\sqrt{x+11}-3}{\sqrt{2-x}-2} = \left[\frac{0}{0}\right] = \lim\limits_{x\to -2} \frac{(\sqrt{x+11}-3)'}{(\sqrt{2-x}-2)'} =$

$= \lim\limits_{x\to -2} \frac{\frac{1}{2\sqrt{x+11}}}{\frac{1}{2\sqrt{2-x}}} = \lim\limits_{x\to -2} \frac{\sqrt{2-x}}{\sqrt{x+11}} = -\frac{2}{3}$

3. $\lim\limits_{x \to 0} \dfrac{\sin^2(x) - tg(x) \cdot \sin(x)}{x \cdot \sin^3(x)} = \left[\dfrac{0}{0}\right] = \lim\limits_{x \to 0} \dfrac{\sin^2(x) - \frac{\sin(x)}{\cos(x)} \cdot \sin(x)}{x \cdot \sin^3(x)} =$

$= \lim\limits_{x \to 0} \dfrac{\sin^2(x)\left(1 - \frac{1}{\cos(x)}\right)}{x \cdot \sin^3(x)} = \lim\limits_{x \to 0} \dfrac{1 - \frac{1}{\cos(x)}}{x \cdot \sin(x)} = \lim\limits_{x \to 0} \dfrac{\left(1 - \frac{1}{\cos(x)}\right)'}{(x \cdot \sin(x))'} =$

$= \lim\limits_{x \to 0} \dfrac{-\frac{\sin(x)}{\cos(x)^2}}{x \cdot \cos(x) + \sin(x)} = \left[\dfrac{-\frac{0}{1}}{0}\right] = \left[\dfrac{0}{0}\right] = \lim\limits_{x \to 0} \dfrac{\left(-\frac{tg(x)}{\cos(x)}\right)'}{(x \cdot \cos(x) + \sin(x))'} =$

$= \lim\limits_{x \to 0} \dfrac{-\sin(x) \cdot \frac{tg(x)}{\cos^2(x)} - \frac{1}{\cos(x)^3}}{-x \cdot \sin(x) + 2 \cdot \cos(x)} = \left[\dfrac{0 - \frac{1}{1^3}}{0 + 2}\right] = -\dfrac{1}{2}$

4. $\lim\limits_{x \to \infty} \left(\dfrac{2 - x}{x + 11}\right)^{7x} = \left[\left(\dfrac{\infty}{\infty}\right)^{\infty}\right] = \lim\limits_{x \to \infty} \left(\dfrac{x\left(\frac{2}{x} - 1\right)}{x\left(1 + \frac{11}{x}\right)}\right)^{7x} =$

$= \lim\limits_{x \to \infty} \left(\dfrac{\left(\frac{2}{x} - 1\right)}{\left(1 + \frac{11}{x}\right)}\right)^{7x} = \lim\limits_{x \to \infty} \left(\dfrac{-\left(1 - \frac{2}{x}\right)}{\left(1 + \frac{11}{x}\right)}\right)^{7x} = \left(-\dfrac{e^{-2}}{e^{11}}\right)^{7} = \left(-e^{-13}\right)^{7} = -e^{-91}$

4) Найти производные

$ctg(5x + y) + \dfrac{4x^3 - x}{5y} = 8x$

$ctg(5x + y) + \dfrac{4x^3 - x}{5y} - 8x = 0$

ф-ция в неявном виде $\Rightarrow$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dF(x,y)}{dx} \Big/ \dfrac{dF(x,y)}{dy}$

$\dfrac{dF(x,y)}{dx} = y \cdot \left(12\dfrac{x^2}{5} - \dfrac{1}{5}\right) - 5\, ctg(5x + y)^2 - 13$

$\dfrac{dF(x,y)}{dy} = y \cdot \dfrac{x^3}{5} - \dfrac{x}{5} - ctg(5x + y)^2 - 1$

$\Rightarrow$

$\sin^2(x) - tg(x) \sin(x) \quad \ldots \quad \sin^2(x) - \sin(x) \cdot \sin(x)$

$$\frac{dy}{dx} = \frac{y\left(12 \cdot \frac{x^2}{5} - \frac{1}{5}\right) - 5\,ctg(5x+y)^2 - 13}{4\frac{x^3}{5} - \frac{x}{5} - ctg(5x+y)^2 - 1}$$

2. $y = (\sqrt{x+9})^{arctg(x^2-11)}$

$$\left((\sqrt{x+9})^{arcctg(x^2-11)}\right)' = (x+9)^{arcctg(x^2-11)/2} \cdot \left(-\frac{x \cdot \ln(x+9)}{(x^2-11)^2+1} + \frac{arcctg(x^2-11)}{2\cdot(x+9)}\right)$$

Прологарифмируем

$$\frac{y'}{y} = \left(\frac{\ln(x+9) \cdot arcctg(x^2-11)}{2}\right)'$$

$$y' = (x+9)^{(arcctg(x^2-11))/2}\left(\frac{\ln(x+9) \cdot arcctg(x^2-11)}{2}\right)'$$

$$\left(\frac{\ln(x+9) \cdot arcctg(x^2-11)}{2}\right)' = \frac{1}{2} \cdot \left((arcctg(x^2-11))' \cdot \ln(x+9) + arcctg(x^2-11) \cdot (\ln(x+9))'\right) = \frac{1}{2}\left(\left(-\frac{2x}{(x^2-11)^2+1}\right) \cdot \ln(x+9) + arcctg(x^2-11) \cdot \frac{1}{x+9}\right)$$

$$(arcctg(x^2-11))' = \frac{-2x}{(x^2-11)^2+1}$$

$$(\ln(x+9))' = (\ln(x+9))'(x+9)' = \frac{1}{x+9} \cdot 1 = \frac{1}{x+9}$$

$$(x+9)^{(arcctg(x^2-11))/2} \cdot \left(-\frac{x \cdot \ln(x+9)}{(x^2-11)^2+1} + \frac{arcctg(x^2-11)}{2(x+9)}\right)$$ – Ответ.