Производные от $f'(x)$ — второго
$f^{(n)}(x)$ — n-го порядка

VI Производная ф-ции, заданной параметрически

$y = f(x)$ определена $x = x(t)$ $y = y(t)$
Тогда если $x(t)$ и $y(t)$ имеют производные в точке t_0, причем $x'(t_0) \neq 0$
а ф-ция $y = f(x)$ имеет производную в точке $x_0 = x(t_0)$

$$y'(x_0) = \frac{y'_t(t_0)}{x'_t(t_0)} \quad \text{или} \quad y'_x = \frac{y'_t}{x'_t}$$

$y''(x)$

$$y''_{xx} = \frac{y''_t \cdot x'_t - x''_t \cdot y'_t}{(x'_t)^3}$$

7.!! Найти производную $y = f(x)$

$y = 3x^2 = 2 \cdot 3 \cdot x^{2-1} = 6x$

$y = \sin x \quad y' = \cos x$

$\Delta y = f(x+\Delta x) - f(x) \Rightarrow \Delta y = 3 \cdot (x+\Delta x)^2 - 3x^2$

$= 3(x^2 + 2x \cdot \Delta x + \Delta x^2) - 3x^2 = 3x^2 + 6x\Delta x + \Delta x^2 -$

$3x^2 = 6x \Delta x + \Delta x^2 = 3\Delta x(2x + \Delta x)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} = \frac{3 \cdot \Delta x(2x + \Delta x)}{\Delta x} =$$

$=$

$$\lim_{\Delta x\to 0} 3(2x+\Delta x) = 3\cdot(\lim_{\Delta x\to 0}(2x) + \lim_{\Delta x\to 0}\Delta x) =$$

$$= 3\cdot(2x+0) = 6x$$

$$y = \sin x$$

$$\Delta y = \sin(x+\Delta x) - \sin x = [\sin(\alpha+\beta) =$$

$$= \sin\alpha\cdot\cos\beta + \sin\beta\cdot\cos\alpha;\ \sin\alpha - \sin\beta =$$

$$= 2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2} = 2\sin\frac{x+\Delta x-x}{2}$$

$$\cdot\cos\frac{x+\Delta x+x}{2} = 2\cdot\sin\frac{\Delta x}{2}\cdot\cos\frac{2x+\Delta x}{2}$$

$$= 2\cdot\sin\frac{\Delta x}{2}\cdot\cos\left(x+\frac{\Delta x}{2}\right)$$

$$y' = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{2\cdot\sin\frac{\Delta x}{2}\cdot\cos\left(x+\frac{\Delta x}{2}\right)}{\Delta x} =$$

$$= \left[\cos\left(x+\frac{\Delta x}{2}\right) \xrightarrow[\Delta x\to 0]{} \cos(x+0) = \cos x\right] =$$

$$= \lim_{\Delta x\to 0}(\cos(x+\Delta x))\cdot\lim_{\Delta x\to 0}\frac{2\cdot\sin\frac{\Delta x}{2}}{\Delta x} =$$

$$= \left[\lim_{x\to 0}\frac{\sin x}{x} = 1;\ \frac{2\sin\left(\frac{\Delta x}{2}\right)}{\Delta x} = \frac{\sin\left(\frac{\Delta x}{2}\right)}{\Delta x} =\right.$$

$$= \frac{\sin\left(\frac{\Delta x}{2}\right)}{\Delta x} : \frac{1}{2} = \frac{2\sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$= \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} \quad \Delta x \to 0 \Rightarrow \frac{\Delta x}{2} \to 0$$

$$\lim_{\frac{\Delta x}{2} \to 0} = \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} = 1 \Big]$$

$= \cos x \cdot 1 = \cos x$

7.1.6

Найти $f'(x)$

1) $f(x) = \frac{9}{\sqrt[3]{x^2}} - 5^{x+1} = 9 \cdot x^{-\frac{2}{3}} - 5^{x+1}$

$= \Big[(x^{\alpha}) = \alpha \cdot x^{\alpha - 1}; \ (a^x)' = a^x \cdot \ln a; \ f'(\varphi(x)) = f'_{\varphi(x)} \cdot \varphi'_x; \ (u+v)' = u' + v' \Big]$

$= (9 \cdot x^{-\frac{2}{3}})'_x - (5^{x+1})'_x = 9 \cdot (x^{-\frac{2}{3}})'_x - (5^{x+1})'_{x+1} \cdot (x+1)'_x = 9 \cdot (-\frac{2}{3}) \cdot x^{-\frac{2}{3}-1} - 5^{x+1} \cdot \ln 5 \cdot (x' + 1') = -6 \cdot x^{-\frac{5}{3}} - 5^{x+1} \cdot \ln 5 \cdot (x^0 + 0) =$

$= -6x^{-\frac{5}{3}} - 5^{x+1} \cdot \ln 5$

2) $f(x) = (x^4 - x) \cdot (\operatorname{tg} x - 1) =$

$= f'(x) = u' \cdot v + u \cdot v' = (x^4 - x)' \cdot (\operatorname{tg} x - 1) + (x^4 - x) \cdot (\operatorname{tg} x - 1)' = ((x^4)' - (x)') \cdot (\operatorname{tg} x - 1) + (x^4 - x) \cdot ((\operatorname{tg} x)' - 1')$

$= (4x^3 - 1)(\operatorname{tg} x - 1) + (x^4 - x) \cdot \frac{1}{\cos^2 x} =$

№ 7.1.27.

$y = \sin^2 x$

$y = (\sin x \cdot \sin x)'$

$= \sin x' \cdot \sin x + \sin x \cdot \sin x'$

$= \cos x \cdot \sin x + \sin x \cdot \cos x$

$y'(x) = (U^2)'_x = 2u \cdot u' = 2\sin x \cdot \sin x' =$

$= 2\sin x \cdot \cos x = \sin 2x$

$y = \ln(\text{arctg}\, 3x)$

$(\ln(\text{arctg}\, 3x))'_x =$

$\frac{1}{\text{arctg}(3x)} \cdot \text{arctg}(3x)' =$

$= \frac{(\text{arctg}(3x))'}{\text{arctg}(3x)} = \frac{\frac{1}{1+x^2} \cdot (3x)'}{\text{arctg}(3x)} =$

$= \frac{3}{1+x^2} : \frac{\text{arctg}(3x)}{1}$

$= \frac{3}{(1+x^2)(\text{arctg}(3x))}$

7.1.58

$y = x^{\sin x}$

$(\ln y)'_x = \frac{y'}{y}$

$(y = x^{\sin x})' = \sin x \cdot x^{\sin x - 1} \cdot \sin x'$

$= \cos x \cdot \sin x \cdot x^{\sin x - 1}$

$y = x^{\sin x}$

$\log_a b^c = c \cdot \log_a b$

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

$(\ln y)' = (\sin x \cdot \ln x)'$

$\frac{y'}{y} = (\sin x)' \cdot \ln x + \sin x \cdot (\ln x)' =$

$= \frac{y'}{y} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$

$\frac{y'}{y} = \cos x \cdot \ln x + \frac{\sin x}{x}$

$y' = y \cdot (\cos x \cdot \ln x + \frac{\sin x}{x}) = x^{\sin x} \cdot (\cos \ln x + \frac{\sin x}{x})$

2) $y = \frac{(x-1)^3 \cdot \sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$

$\ln y = \ln \frac{(x-1)^3 \cdot \sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$

$$\ln y = \ln(x-1)^3 + \ln\sqrt{x+2} - \ln\sqrt[3]{(x+1)^2}$$

$$\ln y = 3\cdot\ln(x-1) + \frac{1}{2}\ln(x+2) - \frac{2}{3}\ln(x+1)$$

$$(\ln y)' = \left(3\ln(x-1) + \frac{1}{2}\ln(x+2) - \frac{2}{3}\ln(x+1)\right)'$$

$$\frac{y'}{y} = 3\cdot\frac{1}{x-1}(x-1)' + \frac{1}{2}\cdot\frac{1}{x+2}\cdot(x+2)' - \frac{2}{3}\cdot\frac{1}{x+1}\cdot(x+1)' = \frac{3}{x-1} + \frac{1}{2x+4} - \frac{2}{3x+3}$$

$$y' = y\cdot(\ldots)$$

$$y' = \frac{(x-1)^3\cdot\sqrt{x+2}}{\sqrt[3]{(x+1)^2}}(\ldots)$$