6.4.46 – 6.4.58)   к 15.06.2020

Повторение

① Второй замечательный предел

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

② Следствие

1) $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

2) $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$

3) $\lim_{x\to 0}\frac{e^x-1}{x} = 1$

6.4.46

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x, \quad k\in R$$

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \left[\left(1+\frac{k}{\infty}\right)^\infty = (1+0)^\infty = 1^\infty\right] =$$

$$= \lim_{x\to\infty}\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^x = \lim_{x\to\infty}\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^{\frac{x}{k}\cdot k} =$$

$$= \left[a^{m\cdot n} = (a^m)^n\right] = \lim_{x\to\infty}\left(\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^{\frac{x}{k}}\right)^k =$$

$$= \left[\lim_{x\to x_0}(f(x))^k = \left(\lim_{x\to x_0}(f(x))\right)^k\right] =$$

$$= \left(\lim_{x\to\infty}\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^{\frac{x}{k}}\right)^{k} = \left[y=\frac{x}{k},\ x\to\infty,\ k\in R \Rightarrow y\to\infty\right.$$

$$= \left(\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{y}\right)^{k} = \left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e\right] =$$

$$= e^{k}$$

2) $$\lim_{x\to0}\sqrt[x]{1+5x} = \lim_{x\to0}(1+5x)^{\frac{1}{x}} = \left[(1+5\cdot0)^{\frac{1}{0}} =\right.$$

$$\left.= 1^{\infty}\right] = \left[\lim_{x\to0}(1+x)^{\frac{1}{x}} = e\right] = \left[y=5x,\ x\to0 \Rightarrow y\to0,\right.$$

$$\left. x=\frac{y}{5}\right] = \lim_{y\to0}(1+y)^{\frac{1}{\frac{y}{5}}} = \lim(1+y)^{\frac{1}{\frac{y}{5}}} =$$

$$= \left(\lim_{y\to0}(1+y)^{\frac{1}{y}}\right)^{5} = e^{5}$$

3) $$\lim_{x\to\infty}\left(\frac{x+3}{x-2}\right)^{x} = \left[\left(\frac{\infty}{\infty}\right)^{\infty}\right] = \lim_{x\to\infty}\left(\frac{x\cdot\left(1+\frac{3}{x}\right)}{x\cdot\left(1-\frac{2}{x}\right)}\right)^{x}$$

$$= \lim_{x\to\infty}\left(\frac{1+\frac{3}{x}}{1-\frac{2}{x}}\right)^{x} = \left[\left(\frac{a}{b}\right)^{n} = \frac{a^{n}}{b^{n}}\right] = \lim_{x\to\infty}\frac{\left(1+\frac{3}{x}\right)^{x}}{\left(1-\frac{2}{x}\right)^{x}} =$$

$$= \frac{\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}}{\lim_{x\to\infty}\left(1+\frac{(-2)}{x}\right)^{x}} = \left[\begin{matrix}\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e \\ \lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{x} = e^{k}\end{matrix}\right] =$$

$$= \frac{e^{3}}{e^{-2}} = \left[\frac{a^{n}}{a^{m}} = a^{n-m}\right] = e^{3-(-2)} = e^{3+2} = e^{5}$$

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4) $\lim\limits_{x\to 0} \frac{e^{2x}-1}{7x} = \left[\frac{e^{2\cdot 0}-1}{7\cdot 0} = \frac{e^{0}-1}{0} = \frac{1-1}{0} = \frac{0}{0}\right] =$

$= \left[\lim\limits_{x\to 0} \frac{e^{x}-1}{x} = 1;\ \ y = 2x \Rightarrow x = \frac{y}{2};\ \ x \to 0 \Rightarrow y \to 0\right] =$

$= \lim\limits_{y\to 0} \frac{e^{y}-1}{7\cdot\frac{y}{2}} = \lim\limits_{y\to 0} \frac{e^{y}-1}{\frac{7}{2}\cdot y} = \lim\limits_{y\to 0} \frac{2}{7}\cdot\frac{e^{y}-1}{y} =$

$= \frac{2}{7}\cdot\lim\limits_{y\to 0} \frac{e^{y}-1}{y} = \frac{2}{7}\cdot 1 = \frac{2}{7}$