27.04. 2020

1) $\Delta A = A \cdot \Delta x + \alpha(\Delta x) \cdot \Delta x$

2) dy – дифференциал

$df(x_0)$ – дифф в точке x_0

3) $dx = \Delta x$

4) $dy = f'(x)\,dx \quad x \in (a; b)$

5) ~~[illegible]~~ $dC = 0$, C – const

$d(\alpha u) = \alpha\, du$

$d(u \pm v) = du \pm dv$

$d(u \cdot v) = u\,dv + v\,du$

$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}, \; v(x) \neq 0$

7) $y = f(u(x))$

$$\left[\begin{array}{l} df(u) = f'(u)\,du \\ dy = y'_u\,du \end{array}\right.$$

8) $d^2 y = f''(x)\,(dx)^2 = f''(x) \cdot dx^2 = d(dy)$

$d^n y = d(d^{n-1} y) = f^{(n)}(x) \cdot (dx)^n = f^{(n)}(x) \cdot dx^n$

$f^{(n)}(x) = d^n y / dx^n$; $f''(x) = d^2 y / dx^2$

7.2.1

$y = e^{x^3}$ $\quad dy - ?$

$dy = y'_x \, dx = (e^{x^3})'_x \, dx = e^{x^3} \cdot (x^3)' dx$

$= 3x^2 \cdot e^{x^3}$

7.2.6.

$y = x^2 - 3x + 1$ $\quad \Delta x = 0{,}1$

$\Delta y, \; dy - ?$ $\quad x_0 = 2$

1) $\Delta y = y(x + \Delta x) - y(x) = ((x + \Delta x)^2 - 3(x + \Delta x) + 1)$

$- x^2 - 3x + 1 = \cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{3x} - 3\Delta x$

$+ \cancel{1} - \cancel{x^2} + \cancel{3x} - 1 = 2x\Delta x + \Delta x^2 - 3\Delta x$

$\underbrace{\Delta x \, (2x + \cancel{\Delta x} - 3)}_{dy} + \Delta x^2 \Rightarrow dy =$

$= (2x - 3) \, dx$

<u>2 сп</u>

$dy = f'(x) \, dx = (x^2 - 3x + 1)'_x \cdot dx =$

$= (2x - 3)_x \, dx$

$f^{(n)}(x)$

$\Delta y \big|_{\substack{x_0 = 2 \\ \Delta x = 0{,}1}} = (2x - 3)\Delta x + (\Delta x)^2 \Big|_{\substack{x_0 = 2 \\ \Delta x = 0{,}1}}$

$= (2 \cdot 2 - 3) \cdot 0,1 + (0,1)^2 = 0,1 + 0,01 =$

$= 0,11.$

$$dy\Big|_{\substack{x_0 = 2 \\ \Delta x = 0,1}} = ((2x-3)\,dx)\Big|_{\substack{x_0 = 2 \\ \Delta x = 0,1}} =$$

$$= ((2x-3)\,\Delta x)\Big|_{\substack{x_0 = 2 \\ \Delta x = 0,1}} = (2 \cdot 2 - 3)\, 0,1 =$$

$= 0,1$

7.2.9

1) $\ln 1,02$

$$f(x_0 + \underbrace{\Delta x}) \approx f(x_0) + f'(x_0) \cdot \Delta x$$

б.м по отношению к „x_0"

$\ln(1 + 0,02) = [x_0 = 1,\ \Delta x = 0,02$

можем найти $\ln 1 = f(x_0)$

$\approx \ln(1) + (\ln(1))'_x \cdot 0,02 =$

$= \left[(\ln x)'_x = \frac{1}{x} \right] = \frac{1}{\frac{1}{x_0}} \cdot \ln(1) \cdot 0,02$

$= 0,02 + \ln(1) = \boxed{0,02}$

$\ln 1,02 \approx 0,02.$

$\sqrt{24} = \left[\begin{array}{l} 24 = 16 + 8 \ — 1 \text{ способ} \\ 24 = 25 + (-1) \ — 2 \text{ способ} \end{array} \right.$

$x_0 = 25$, $dx = -1$ — бесконечно мало по отношению к x_0]

$= \sqrt{25 + (-1)} = \left[x_0 = 25 \quad \Delta x = -1 \right] \approx$

$\left[\sqrt{25} + (\sqrt{25})' \cdot (-1) = -\frac{1}{2\sqrt{25}} + \sqrt{25} = \right.$

$= -\frac{1}{\sqrt{100}} + \sqrt{25} = -0{,}1 + \sqrt{25} = 4{,}9$

7.2.13

$y = \sqrt[3]{x} \quad dy, d^2y, d^3y - ?$

$dy = y'_x dx = (\sqrt[3]{x})'_x dx = (x^{\frac{1}{3}})' dx =$

$= \frac{1}{3} x^{\frac{1}{3}-1} dx = \frac{1}{3} x^{-\frac{2}{3}} dx = \frac{dx}{3 \cdot \sqrt[3]{x^2}}$

$d^2y = d(dy) = d\left(\frac{dx}{3 \cdot \sqrt[3]{x^2}}\right) = \left(\frac{1}{3 \cdot \sqrt[3]{x^2}}\right)'_x d(dx)$

$= \frac{1}{3} \cdot (x^{-\frac{2}{3}})'_x \cdot dx^2 = \frac{1}{3} \cdot \left(-\frac{2}{3} \cdot x^{-\frac{2}{3}-1}\right) dx^2 =$

$= -\frac{2}{9} \cdot x^{-\frac{5}{3}} dx^2 = \frac{-2dx^2}{9\sqrt[3]{x^5}}$

$d^3y = d(d^2x) = d\left(\frac{-2dx^2}{9x\sqrt[3]{x^2}}\right) = \left(-\frac{2}{3x\sqrt[3]{x^2}}\right)'_x dx^3$

$= -\frac{2}{3} \cdot (x^{-\frac{5}{3}})'_x dx^3 = -\frac{2}{3} \cdot \left(-\frac{5}{3}\right) x^{-\frac{5}{3}-1} dx^3 =$

$\frac{10}{9} x^{-\frac{2}{3}} dx^3 = \frac{10 dx^3}{27 \cdot \sqrt[3]{x^2}} = \frac{10 \cdot dx^3}{27x^2 \cdot \sqrt[3]{x^2}}$