§3 Ранг матрицы.

1.3.17

$$A = \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ -2 & 1 & 3 & 3 & -9 \\ -4 & -3 & 11 & -19 & 17 \end{pmatrix}_{III-2I} = \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ -2 & 1 & 3 & 3 & -9 \\ 0 & -5 & 5 & -25 & 35 \end{pmatrix}_{III+5II} \sim$$

$$\sim \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ 0 & -5 & 5 & -25 & 35 \\ 0 & -5 & 5 & -25 & 35 \end{pmatrix}_{III-II} \sim \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ 0 & -5 & 5 & -25 & 35 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad r(A) = 2.$$

1.3.18.

$$A = \begin{pmatrix} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 3 & 8 & 2 & -10 \end{pmatrix}_{\substack{II-3I \\ III-3I}} \sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & 2 & -10 & -10 \end{pmatrix}_{III+2II} \sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & -22 & 0 \end{pmatrix}$$

$r(A) = 3$

1.3.19

$$A = \begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 5 & -3 & 2 & 3 & 4 \\ 3 & -1 & 3 & 2 & 5 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}_{\substack{II-5I \\ III-3I \\ IV-7I}} \sim$$

$$\sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 0 & 12 & 27 & 3 & 39 \\ 0 & 8 & 18 & 2 & 26 \\ 0 & 16 & 36 & 4 & 50 \end{pmatrix}_{IV-2III} \sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 0 & 12 & 27 & 3 & 39 \\ 0 & 8 & 18 & 2 & 26 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}_{\substack{:3 \\ :2}} \sim$$

$$\sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 0 & 4 & 9 & 1 & 13 \\ 0 & 4 & 9 & 1 & 13 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}_{\updownarrow} \sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 0 & 4 & 9 & 1 & 13 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 4 & 9 & 1 & 13 \end{pmatrix}_{IV-II} \sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 0 & 4 & 9 & 1 & 13 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$r(A) = 3$

1.3.20

$$A = \begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix}_{V+2III} \sim \begin{pmatrix} 24 & 19 & 36 & 72 & 0 \\ 49 & 40 & 73 & 147 & 0 \\ 73 & 59 & 98 & 219 & 0 \\ 47 & 36 & 71 & 141 & 0 \end{pmatrix}_{IV-\frac{1}{3}II} \sim$$

$$\sim \begin{pmatrix} 24 & 19 & 36 & 0 & 0 \\ 49 & 40 & 73 & 0 & 0 \\ 73 & 59 & 98 & 0 & 0 \\ 47 & 36 & 71 & 0 & 0 \end{pmatrix} \quad r(A) = 3$$

1.3.21.

$$A = \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}_{II-III-IV} \sim \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}_{III-IV} \sim$$

$$\sim \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 0 & 12 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}_{V-2I+III} \sim \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 0 & 12 \\ 4 & 3 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{IV-I} \sim$$

$$\sim \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 0 & 12 \\ 0 & 0 & 6 & 0 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{IV+\frac{2}{3}III} \sim \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{\updownarrow} \sim \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & -9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad r(A) = 2$$

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1.3.22.

$$A = \begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 12 & 13 & 25 & -55 & -68 \end{pmatrix} \begin{matrix} \\ \\ \\ \\ V - I - III \end{matrix} \sim \begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 0 & 48 & -48 & -48 & -96 \end{pmatrix} : 48 \sim$$

$$\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 0 & 1 & -1 & -1 & -2 \end{pmatrix} \begin{matrix} I + 28V \\ II + 37V \\ III + 7V \\ IV - 12V \\ \\ \end{matrix} \sim \begin{pmatrix} 17 & 0 & 17 & -17 & -17 \\ 24 & 0 & 24 & -24 & -24 \\ 25 & 0 & 25 & -25 & -25 \\ 31 & 0 & 31 & -31 & -31 \\ 0 & 1 & -1 & -1 & -2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 17 & 0 & 0 & -17 & -17 \\ 24 & 0 & 0 & -24 & -24 \\ 25 & 0 & 0 & -25 & -25 \\ 31 & 0 & 0 & -31 & -31 \\ 0 & 1 & 0 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} 17 & 0 & 0 & 0 & 0 \\ 24 & 0 & 0 & 0 & 0 \\ 25 & 0 & 0 & 0 & 0 \\ 31 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad r(A) = 2.$$

1.3.23

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

1. $|2| = 2 \neq 0 \Rightarrow r(A) \geq 1.$

2. $\begin{vmatrix} -1 & 2 \\ -3 & 3 \end{vmatrix} = -1 \cdot 3 + 6 = +3 \neq 0 \Rightarrow r(A) \geq 2.$

3. $\begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 0 - 3 + 24 + 6 + 0 - 27 = 0 \Rightarrow r(A) = 2.$

$\begin{vmatrix} -1 & 2 \\ -3 & 3 \end{vmatrix}$ - Базисный минор.

1.3.24.

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

1) $|2| = (см. 1.3.23) \Rightarrow r(A) \geq 1.$

2) $\begin{vmatrix} -1 & 2 \\ -3 & 3 \end{vmatrix}$ (см. 1.3.23) $\Rightarrow r(A) \geq 2.$

3. $\begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{vmatrix} = +18 - 3 + 24 + 6 + 8 - 27 = -10 \Rightarrow r(A) = 3$

1.3.25.

$$A = \begin{pmatrix} 2 & -1 & 5 & 6 \\ 1 & 1 & 3 & 5 \\ 1 & -5 & 1 & -3 \end{pmatrix}$$

1. $|2| = 2 \neq 0 \Rightarrow r(A) \geq 1$

2. $\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 + 1 = 3 \Rightarrow r(A) \geq 2.$

3. $\begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & 3 \\ 1 & -5 & 1 \end{vmatrix} = 2 - 3 - 25 - 5 + 1 + 30 = 0$

$\begin{vmatrix} 2 & -1 & 6 \\ 1 & 1 & 5 \\ 1 & -5 & -3 \end{vmatrix} = -6 - 5 - 30 - 6 + 50 + 3 = -50 + 50 = 0.$

$\begin{vmatrix} 2 & 5 & 6 \\ 1 & 3 & 5 \\ 1 & 1 & -3 \end{vmatrix} = -18 + 6 + 25 - 18 - 10 + 15 = 0$

3. $\begin{vmatrix} -1 & 5 & 6 \\ 1 & 3 & 5 \\ -5 & 1 & -3 \end{vmatrix} = 9 + 6 - 125 + 90 - 5 + 15 = -10 \Rightarrow r(A) \leq 3$

4. $\begin{vmatrix} -1 & 5 \\ 1 & 3 \end{vmatrix} = -3 \neq 0 \Rightarrow r(A) = 3$

Базисный минор $\begin{vmatrix} -1 & 5 & 6 \\ 1 & 3 & 5 \\ -5 & 1 & -3 \end{vmatrix}$

1.3.26

$$A = \begin{pmatrix} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 1 & 3 & 0 & -3 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{pmatrix}$$

1) $\begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix} = (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 1 & -1 & 1 \\ 3 & 0 & -3 \\ -7 & 3 & 1 \end{vmatrix}$

$+ 0 + (-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} -2 & 3 & -4 \\ 1 & -1 & 1 \\ -7 & 3 & 1 \end{vmatrix} + 0 = (0 - 21 + 9 - 0 + 3 + 9) +$

$+ (20 - 21 - 12 + 28 - 3 + 6) = 10 \neq 0 \Rightarrow r(A) = 4.$

1.3.27.

$$A = \begin{pmatrix} 1 & -2 & 1 & -1 & 1 \\ 2 & 1 & -1 & 2 & -3 \\ 3 & -2 & -1 & 1 & -2 \\ 2 & -5 & 1 & -2 & 2 \end{pmatrix}$$

1. $\begin{vmatrix} -2 & 1 & -1 & 1 \\ 1 & -1 & 2 & -3 \\ -2 & -1 & 1 & -2 \\ -5 & 1 & -2 & 2 \end{vmatrix} = (-1)^{1+1} \cdot (-2) \begin{vmatrix} -1 & 2 & -3 \\ -1 & 1 & -2 \\ 1 & -2 & 2 \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} 1 & 2 & -3 \\ -2 & 1 & -2 \\ -5 & -2 & 2 \end{vmatrix}$

$+ (-1)^{1+3} \cdot (-1) \cdot \begin{vmatrix} 1 & -1 & -3 \\ -2 & -1 & -2 \\ -5 & 1 & 2 \end{vmatrix} + (-1)^{1+4} \cdot 1 \cdot \begin{vmatrix} 1 & -1 & 2 \\ -2 & -1 & 1 \\ -5 & 1 & -2 \end{vmatrix} = (-2 - 6 - 4$

$+ 3 + 4 + 4) - (2 - 12 + 10 - 15 - 4 + 8) - (-2 + 6 - 10 + 15 + 2 - 4) -$

$- (2 - 4 + 5 - 10 - 1 + 4) = -1 + 1 - 7 + 4 = (-2)(-1) + 1 - 7 + 4 = 0.$

$\begin{vmatrix} 1 & 1 & -1 & 1 \\ 2 & -1 & 2 & -3 \\ 3 & -1 & 1 & -2 \\ 2 & 1 & -2 & 2 \end{vmatrix} = (-1)^{1+1} \cdot 1 \begin{vmatrix} -1 & 2 & -3 \\ -1 & 1 & -2 \\ 1 & -2 & 2 \end{vmatrix} + (-1)^{1+2} \cdot 1 \begin{vmatrix} 2 & 2 & -3 \\ 3 & 1 & -2 \\ 2 & -2 & 2 \end{vmatrix} + (-1)^{1+3} \cdot (-1)$

$\begin{vmatrix} 2 & -1 & -3 \\ 3 & -1 & -2 \\ 2 & 1 & 2 \end{vmatrix} + (-1)^{1+4} \cdot 1 \begin{vmatrix} 2 & -1 & 2 \\ 3 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = (-2 - 6 - 4 + 3 + 4 + 4) - (4 + 18 - 8$

$+ 6 - 8 - 12) - (-4 - 9 + 4 - 6 + 4 + 6) - (4 + 6 - 2 + 4 - 2 - 6) = -1 + 5 - 4 = 0$

$\begin{vmatrix} 1 & -2 & -1 & 1 \\ 2 & 1 & 2 & -3 \\ 3 & -2 & 1 & -2 \\ 2 & -5 & -2 & 2 \end{vmatrix} = (-1)^{1+1} \cdot 1 \begin{vmatrix} 1 & 2 & -3 \\ -2 & 1 & -2 \\ -5 & -2 & 2 \end{vmatrix} + (-1)^{1+2} \cdot (-2) \begin{vmatrix} 2 & 2 & -3 \\ 3 & 1 & -2 \\ 2 & -2 & 2 \end{vmatrix} +$

$+(-1)^{1+3}\cdot(-1)\begin{vmatrix} 2 & 1 & -3 \\ 3 & -2 & -2 \\ 2 & -5 & 2 \end{vmatrix} + (-1)^{1+4}\cdot 1\cdot\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & 1 \\ 2 & -5 & -2 \end{vmatrix} = (2 - 12 + 24$

$- 15 - 4 + 8) + 2(4 + 18 - 8 + 6 - 8 - 12) - (-8 + 45 - 4 - 12 - 20 - 6) -$

$(8 - 30 + 2 + 8 + 10 + 6) = -1 + 5 - 4 = 0$

$\begin{vmatrix} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & -3 \\ 3 & -2 & -1 & -2 \\ 2 & -5 & 1 & 2 \end{vmatrix} = (-1)^{1+1}\cdot 1\cdot\begin{vmatrix} 1 & -1 & -3 \\ -2 & -1 & -2 \\ -5 & 1 & 2 \end{vmatrix} + (-1)^{1+2}(-2)\cdot\begin{vmatrix} 2 & -1 & -3 \\ 3 & -1 & -2 \\ 2 & 1 & 2 \end{vmatrix} +$

$+(-1)^{1+3}\cdot 1\cdot\begin{vmatrix} 2 & 1 & -3 \\ 3 & -2 & -2 \\ 2 & -5 & 2 \end{vmatrix} + (-1)^{1+4}\cdot 1\begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & -1 \\ 2 & -5 & 1 \end{vmatrix} = (-2 + 6 - 10 + 6$

$+2 - 4) + 2(-4 - 9 + 4 \cdot 6 + 4 + 6) + (-8 + 45 - 4 - 12 - 20 - 6) - (-4 + 15 - 2$

$-4 - 10 - 3) = 7 + 2\cdot(-5) - 5 + 8 = 7 + 8 - 15 = 0.$

$\begin{vmatrix} 1 & -2 & 1 & -1 \\ 2 & 1 & -1 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & -5 & 1 & -2 \end{vmatrix} = (-1)^{1+1}\cdot\begin{vmatrix} 1 & -1 & 2 \\ -2 & -1 & 1 \\ -5 & 1 & -2 \end{vmatrix} + (-1)^{1+2}\cdot(-2)\begin{vmatrix} 2 & -1 & 2 \\ 3 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix}$

$+(-1)^{1+3}\cdot 1\cdot\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & 1 \\ 2 & -5 & -2 \end{vmatrix} + (-1)^{1+4}\cdot(-1)\begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & -1 \\ 2 & -5 & 1 \end{vmatrix} = (2 - 4 + 5 - 10 - 1 + 4)$

$+2(4 + 6 - 2 + 4 - 2 - 6) - (8 - 30 + 2 + 8 + 10 + 6) - (-4 + 15 - 2 - 4 - 10 - 3)$

$= -4 + 8 + 4 - 8 = 0$

$\Rightarrow r(A) < 4$

2) $\begin{vmatrix} -2 & 1 & -1 \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = 2 - 4 + 1 + 2 - 1 - 4 = -4 \neq 0 \Rightarrow r(A) \leq 3$

3) $\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 2 - (-1)\cdot(-1) = 1 \neq 0 \Rightarrow r(A) \geq 2$

4) $|-1| = -1 \neq 0 \Rightarrow r(A) \geq 1 \Rightarrow r(A) = 3$

Базисный минор:

$\begin{vmatrix} -2 & 1 & -1 \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix}$

3. 28

$$A = \begin{pmatrix} 2 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -2 \\ 3 & 3 & -3 & -3 & 4 \\ 4 & 5 & -5 & -5 & 7 \end{pmatrix}$$

1) $|2| = 2 \neq 0 \Rightarrow r(A) \geq 1$

2) $\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3 \neq 0 \Rightarrow r(A) \geq 2$

3) $\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix} = 6 + 3 - 3 - 3 + 3 - 6 = 0$

$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix}$ = из прошлого опр. = 0.

$\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 3 & 3 & 4 \end{vmatrix} = -8 + 3 - 6 + 3 + 12 - 4 = 0$

$\begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -5 & -5 \end{vmatrix} = -10 + 5 - 4 + 4 + 10 - 5 = 0$

$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \\ 4 & -5 & 7 \end{vmatrix} = 14 - 5 + 8 - 4 - 20 + 7 = 0$

$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \\ 3 & -3 & 4 \end{vmatrix} = 8 - 3 + 6 - 3 - 12 + 4 = 0$

$\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ 3 & -3 & -3 \end{vmatrix} = -3 - 3 - 3 + 3 + 3 - 3 = 0$

$\begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & -2 \\ 3 & -3 & 4 \end{vmatrix} = 4 + 3 + 6 - 3 - 6 - 4 = 0$

$\begin{vmatrix} -1 & -1 & 1 \\ 1 & 1 & -2 \\ -3 & -3 & 4 \end{vmatrix} = -4 - 3 - 6 + 3 + 6 + 4 = 0$

$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 4 & 5 & -5 \end{vmatrix} = 10 - 5 + 4 - 4 - 10 + 5 = 0$

$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 3 & -3 \\ 4 & 5 & -5 \end{vmatrix} = -30 - 15 - 12 + 12 + 30 + 15 = 0$

$\Rightarrow r(A) < 3 \Rightarrow r(A) = 2$

Базисный минор: $\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$

$$A = \begin{pmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & a \\ 1 & -2 & 0 & 1 \end{pmatrix}$$

1) $\begin{vmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & a \\ 1 & -2 & 0 & 1 \end{vmatrix} = (-1)^{1+1} \cdot 1 \begin{vmatrix} -3 & -1 & 3 \\ -6 & -1 & a \\ -2 & 0 & 1 \end{vmatrix} + (-1)^{1+3} \cdot 2 \cdot \begin{vmatrix} 2 & -1 & 3 \\ 3 & -6 & a \\ 1 & -2 & 1 \end{vmatrix}$

$+ (-1)^{1+4} \cdot (-3) \cdot \begin{vmatrix} 2 & -1 & 3 \\ 3 & -1 & a \\ 1 & 0 & 1 \end{vmatrix} = (3 + 0 + 2a - 6 - 0 - 6) - 2(-12 - 18$

$-3a + 18 + 4a + 9) + 3(-2 + 0 - a + 3 + 0 + 3) = 0$

$2a - 9 - 3a + 12 - 2a + 6 = 0$

$-3a + 9 = 0$

$a = 3$

2) $\begin{vmatrix} 1 & -3 & 2 \\ 2 & -3 & -1 \\ 3 & -6 & -1 \end{vmatrix} = 3 - 24 + 9 + 18 - 6 - 6 = -6 \neq 0 \Rightarrow r(A) \geq 3$

3) $\begin{vmatrix} 1 & -3 \\ 2 & -3 \end{vmatrix} = -3 + 6 = 3 \neq 0 \Rightarrow r(A) \geq 2$

4) $|1| = 1 \neq 0 \Rightarrow r(A) = 3$, при $a = 3$ $r(A) = 4$ при $a \neq 3$

1.3.30

$$A = \begin{pmatrix} 3 & 1 & 1 & 4 \\ a & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$$

1) $\begin{vmatrix} 3 & 1 & 1 & 4 \\ a & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{vmatrix} \leftarrow = (-1)^{1+1} \cdot 3 \cdot \begin{vmatrix} 4 & 10 & 1 \\ 7 & 17 & 3 \\ 2 & 4 & 3 \end{vmatrix} + (-1)^{1+2} \cdot$

$\cdot 1 \cdot \begin{vmatrix} a & 10 & 1 \\ 1 & 7 & 3 \\ 2 & 4 & 3 \end{vmatrix} + (-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} a & 4 & 1 \\ 1 & 7 & 3 \\ 2 & 2 & 3 \end{vmatrix} + (-1)^{1+4} \cdot 4 \cdot \begin{vmatrix} a & 4 & 10 \\ 1 & 7 & 17 \\ 2 & 2 & 4 \end{vmatrix} =$

$= 3(204 + 28 + 60 - 34 - 48 - 210) - (5a + 4 + 60 - 34 - 12a - 30) +$

$(21a + 2 + 14 - 14 - 6a - 12) - 4(23a + 20 + 136 - 140 - 34a - 16) = 0$

$3 \cdot 0 - 39a + 15a + 24a = 0 \Rightarrow r(A) < 4$

2) $\begin{vmatrix} 3 & 1 & 1 \\ a & 4 & 10 \\ 2 & 2 & 4 \end{vmatrix} = 0 \quad 48 + 2a + 20 - 8 - 60 - 4a = 0$

$-2a = 0$

$a = 0$

3) $\begin{vmatrix} 4 & 1 \\ 4 & 10 \end{vmatrix} = 40 - 4 = 36 \neq 0 \Rightarrow r(A) \geq 2$ $r(A) = 3$ при $a \neq 0$

4) $|10| = 10 \neq 0 \Rightarrow r(A)$ $r(A) \geq 1$ $r(A) < 2$ при $a = 0$

3. 31

$$A = \begin{pmatrix} \lambda & 1 & 1 & 1 \\ 1 & \lambda & 1 & 1 \\ 1 & 1 & \lambda & \lambda^2 \end{pmatrix}$$

1) $\begin{vmatrix} 1 & 1 & 1 \\ \lambda & 1 & 1 \\ 1 & \lambda^2 & \lambda^2 \end{vmatrix} = 0$

$\lambda^2 + \lambda^2 + 1 - 1 - \lambda - \lambda^3 = 0$

$-\lambda^3 + 2\lambda^2 - \lambda = 0$

$(-\lambda^2 + 2\lambda - 1)\lambda = 0$

$D = 4 - 4 = 0$

$\lambda_1 = \frac{2}{2} = 1$

$\lambda = 0; 1$

2) $\begin{vmatrix} 1 & 1 \\ \lambda & \lambda^2 \end{vmatrix} = \lambda^2 - \lambda$

2) $|1| = 1 \neq 0 \Rightarrow r(A) = 1$

Ответ: при $\lambda \in (-\infty; 0)(0; 1)(1; +\infty)$

$r(A) = 3$.

при $\lambda = 0$ и $\lambda = 1$

$r(A) = 1$