ДЗ 1.4.5 – 1.4.8 (к отчету от 20.05.)

1.4.5.

$A = \begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix}$ $A\Delta = \begin{vmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{vmatrix} = -5\cdot 2\cdot 3 - 3\cdot 2\cdot 5 - 4\cdot 7 + 5\cdot 5 +$
$+ 5\cdot 5 + 4\cdot 3\cdot 2 + 2\cdot 7\cdot 2 = -30 - 30 -$
$+ 25 + 36 + 28 = 1 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1}\cdot 2 \begin{vmatrix} -5 & 2 \\ -7 & 3 \end{vmatrix} = (-5\cdot 3 + 7\cdot 2) = (-1) = -1$

$A_{12} = (-1)^{1+2}\cdot \begin{vmatrix} 4 & 2 \\ 5 & 3 \end{vmatrix} = -(4\cdot 3 - 5\cdot 2) = -2$

$A_{13} = (-1)^{1+3}\cdot (1) \begin{vmatrix} 4 & -5 \\ 5 & -7 \end{vmatrix} = (4\cdot(-7) + 5\cdot 5) = -28 + 25 = -3$

$A_{21} = (-1)^{2+1}\cdot \begin{vmatrix} -3 & 1 \\ -7 & 3 \end{vmatrix} = -(-3\cdot 3 + 7\cdot 1) = 2$

$A_{22} = (-1)^{2+2}\cdot \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = (2\cdot 3 - 5) = 1$

$A_{23} = (-1)^{2+3}\cdot \begin{vmatrix} 2 & -3 \\ 5 & -7 \end{vmatrix} = -(-7\cdot 2 + 5\cdot 3) = -1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 1 \\ -5 & 2 \end{vmatrix} = (-3\cdot 2 + 5\cdot 1) = -6 + 5 = -1$

$A_{33} = (-1)^{3+3}\cdot \begin{vmatrix} 2 & -3 \\ 4 & -5 \end{vmatrix} = (-5\cdot 2 + 3\cdot 4) = -10 + 2 = 2$

3) $\tilde{A} = \begin{pmatrix} -1 & 2 & -3 \\ -2 & 1 & 1 \\ -1 & 0 & 2 \end{pmatrix}^T = \begin{pmatrix} -1 & -2 & -1 \\ 2 & -1 & 0 \\ -3 & -1 & 2 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\Delta A}\cdot \tilde{A} = \frac{1}{1}\cdot \begin{pmatrix} -1 & 2 & -1 \\ -2 & 1 & 0 \\ -3 & -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 & -1 \\ -2 & 1 & 0 \\ -3 & -1 & 2 \end{pmatrix}$

1.4.6.

1) $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3\cdot 3\cdot 3 + 2\cdot 2 + 2\cdot 1 - 2\cdot 3 - 3$
$- 2\cdot 2\cdot 3 = 12$

2) $A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 3\cdot 3 - 1\cdot 1 = 8$

$A_{12} = (1)^{1+2} \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = -(2\cdot 3 - 2) = -4$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2\cdot 1 - 3\cdot 2 = 2 - 6 = -4.$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -(2\cdot 3 - 1\cdot 1) = -5$

$A_{22} = (-1)^{2+2}\cdot \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 3\cdot 3 - 2 = 7$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -(3\cdot 1 - 2\cdot 2) = 1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = (2\cdot1 - 3\cdot1) = -1$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = -(3\cdot1 - 2\cdot1) = -1$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = (3\cdot3 - 2\cdot2) = 5$

3) $\tilde{A} = \begin{pmatrix} 8 & -4 & -4 \\ -5 & 7 & 1 \\ -1 & -1 & 5 \end{pmatrix}^T = \begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\Delta A} \cdot \tilde{A} = \frac{1}{12} \begin{pmatrix} -8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix} = \begin{pmatrix} \frac{8}{2} & -\frac{5}{12} & -\frac{1}{12} \\ -\frac{4}{2} & \frac{7}{12} & -\frac{1}{12} \\ -\frac{4}{12} & \frac{1}{12} & \frac{5}{12} \end{pmatrix}$

1.4.7.

$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\cdot5\cdot9 + 2\cdot6\cdot7 + 4\cdot8\cdot3 - 7\cdot5\cdot3 - 4\cdot2\cdot9 - 8\cdot6\cdot1 = 0$

$\Rightarrow \nexists A^{-1}$

1.4.8.

$A = \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix}$ $\Delta A = \begin{vmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{vmatrix} = -3\cdot5 + 3\cdot2\cdot5 + 2 - 5\cdot3 - 3 + 2\cdot2\cdot5 = 19 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1} \begin{vmatrix} -3 & -2 \\ 2 & 1 \end{vmatrix} = -3 + 4 = 1$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & -2 \\ -5 & 1 \end{vmatrix} = -(1 - 5\cdot2) = 9$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -3 \\ -5 & 2 \end{vmatrix} = 1\cdot2 - (-5)\cdot(-3) = -13$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = -(3\cdot1 - 2\cdot1) = -1$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 1 \\ -5 & 1 \end{vmatrix} = (5\cdot1 - (-5)\cdot1) = 10$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ -5 & 2 \end{vmatrix} = -(5\cdot2 - (5\cdot3)) = -(10 + 15) = -25$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 1 \\ -3 & -2 \end{vmatrix} = 3(-2) - (-3)\cdot1) = -3$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 5 & 1 \\ 1 & -2 \end{vmatrix} = -(5\cdot(-2) - 1\cdot1) = 11$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 5 & 3 \\ 1 & -3 \end{vmatrix} = 5\cdot(-3) - 3\cdot1 = -18.$

3) $\tilde{A} = \begin{pmatrix} 1 & 9 & -13 \\ -1 & 10 & -25 \\ -3 & 11 & -18 \end{pmatrix}^T = \begin{pmatrix} 1 & -1 & -3 \\ 9 & 10 & 11 \\ -13 & -25 & -18 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\Delta A} \cdot \tilde{A} = \frac{1}{19} \cdot \begin{pmatrix} 1 & -1 & -3 \\ 9 & 10 & 11 \\ -13 & -25 & -18 \end{pmatrix} =$

$= \begin{pmatrix} 1/19 & -1/19 & -3/19 \\ 9/19 & 10/19 & 11/19 \\ -13/19 & -25/19 & -18/19 \end{pmatrix}$