CS2020A Discrete Mathematics

Tutorial 12 | 10/Nov/2025

1. What is the best strategy to win a 5-door Monty Hall problem? What is the winning probability of your strategy?

   5-Door Monty Hall Problem. A car is equally likely behind one of the five doors on stage. A contestant is asked to choose one door, following which the host opens one of the other doors behind which there is no car. The contestant now has an option to make a second (final) guess.

2. A fair coin is tossed n times. What is the probability that you get a sequence in which no head follows a tail?

3. You have three six-sided fair dice A, B, C with the following numbers on their faces.

$$
\begin{aligned}
A &= (2,6,7,2,6,7) \\
B &= (1,5,9,1,5,9) \\
C &= (3,4,8,3,4,8)
\end{aligned}
$$

   You can choose any one of the dice and then I will choose another. Then we throw the dice (fairly) and who ever gets the larger number will win. Which dice will you choose and what is your winning probability (assuming that the second player is your probability teacher)?

4. *Inclusion-Exclusion Principle.* Prove that for any sequence of n events $A_1, \ldots, A_n$, in some probability space $(\Omega, \mathbb{P})$,

$$
\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{k=1}^{n} \left((-1)^{k-1} \sum_{\substack{I \subseteq \{1,\ldots,n\} \\ |I|=k}} \mathbb{P}(A_I) \right),
$$

   where the last sum runs over all subsets I of the indices 1 to n which contain exactly k elements, and $A_I := \bigcap_{i \in I} A_i$ denotes the intersection of all those A_i with index $i \in I$.

   Proof Strategy 1. Let x be any member of Ω. Depending on how many sets among $A_1, \ldots, A_n$ contain x, calculate the contribution of $\mathbb{P}(x)$ on the RHS of the above equation.

   Proof Strategy 2. Try induction on n.