Total Marks: 20 **Duration: 50 min** **Pages: 1**

MA2020: Differential Equations
Test 1
Department of Mathematics, Indian Institute of Technology, Palakkad

Date: 16/09/2025 Time: 08:00 − 08:50 AM

Instructions:

- Cell phones are **not** allowed within the exam hall.

- Please state all the results which you use and justify your answers.

1. Solve the differential equation

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}.$$

(4)

2. Consider the differential equation $M(x,y)dx + N(x,y)dy = 0$ on the domain $D = \{(x,y) : \frac{1}{4} < x^2 + y^2 < 4\}$, where $M(x,y) = \frac{-y}{x^2+y^2}$ and $N(x,y) = \frac{x}{x^2+y^2}$. Check whether this is exact. Justify. (4)

3. Show that the first order ODE

$$(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$$

is not exact. Find an integrating factor for it and thereby solve the above ODE. (5)

4. For what values of the initial value $y(0) = y_0$ of the solution to the ODE:

$$y'(t) = y(t) + 1 + 3\sin(t),$$

does the solution remain bounded for all time? Justify. (4)

5. Find the orthogonal trajectories of the 1-parameter family of the curves given by

$$y^2 = cx^3.$$

(3)

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