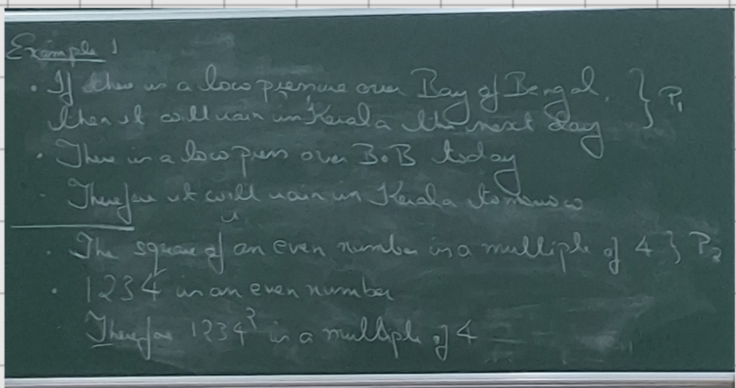
Logic :- (The structure of reason)

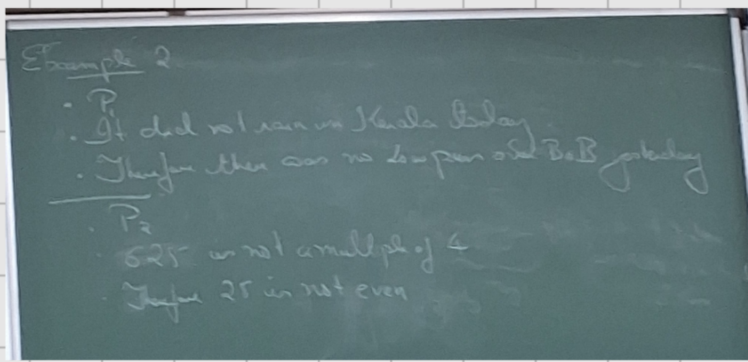
*** Rules of Inference :-**

→ Modus Ponens
→ Modus Tollens
→ Logical Syllogism
→ Disgentive Syllogism
→ Double negation elimination

*** Logical Fallacies**

→ Fallacy of the converse.



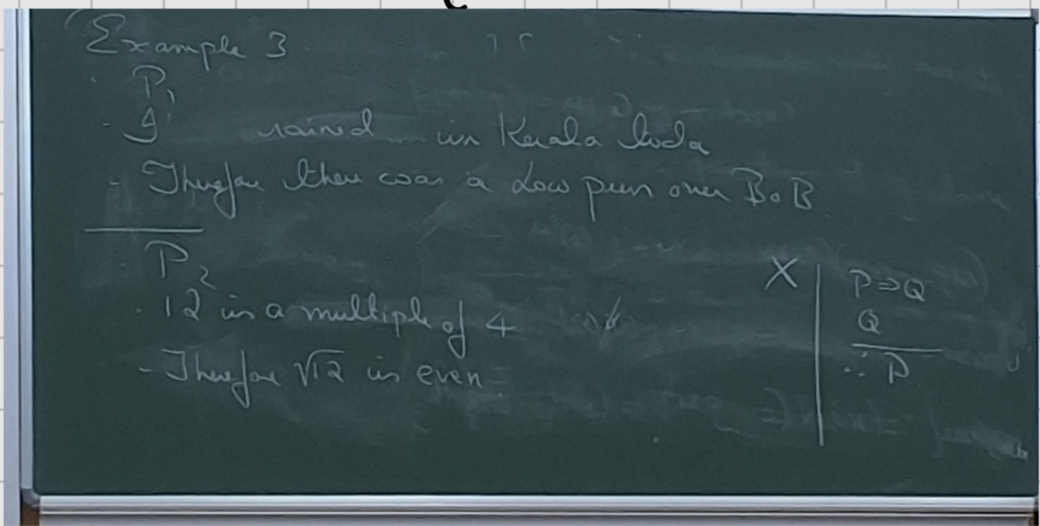


MODUS PONENS

$$\begin{array}{l} P \Rightarrow Q \\ \underline{\quad P \quad} \\ \therefore Q \end{array}$$

MODES TOLLENS

$$\begin{array}{l} P \Rightarrow Q \\ \underline{\quad \neg Q \quad} \\ \therefore \neg P \end{array}$$



FALLACY OF THE CONVERSE:

$$\begin{array}{l} P \Rightarrow Q \\ \underline{\quad Q \quad} \\ \therefore P \end{array}$$

X

$$\frac{\begin{array}{c}P \Rightarrow Q \\ Q \Rightarrow R\end{array}}{\therefore P \Rightarrow R}$$

LOGICAL SYLLOGISM

→ If a no. is even, it's sq. is a multiple of 4.
→ If a no. is a mul. of 4, it's sq. is a mul. of 16
→ If a no. is even, it's 4th power is a mul. of 16

$$\frac{\neg\neg P}{\therefore P}$$

DOUBLE NEG ELIM

DISJUNCTIVE SYLLOGISM

$$\frac{\begin{array}{c}P \vee Q \\ \neg P\end{array}}{\therefore Q}$$

Proposition:-

* Def: A proposition is a statement that is either true/false
* Use of variable to denote propositions and able to do operations on them

Axiom - Theorem:-

* Axiom is a proposition that is assumed to be true.
* Theorem is a " " " proved " " ".

Proof:-

* A seq. of statements, where the truth of each one

follows from one or more axioms or statements before it.

Operations on Propositions :-

* Negation ($\neg$)
* Conjunction ($\wedge$)
* Disjunction ($\vee$)
* Conditional ($\rightarrow$)
* BiConditional ($\leftrightarrow$)

Order of precedence.

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

$\Rightarrow \quad \neg(P \rightarrow Q) \equiv \neg(\neg P \vee Q) \equiv P \wedge \neg Q$

$\Rightarrow \quad P \rightarrow Q \equiv \neg Q \rightarrow \neg P$ Contrapositive

$Q \rightarrow P \equiv \neg P \rightarrow \neg Q$.

Converse Inverse.

Well formed propositional formula:

① T & F are WFPF.

② Any propositional variable is a WFPF

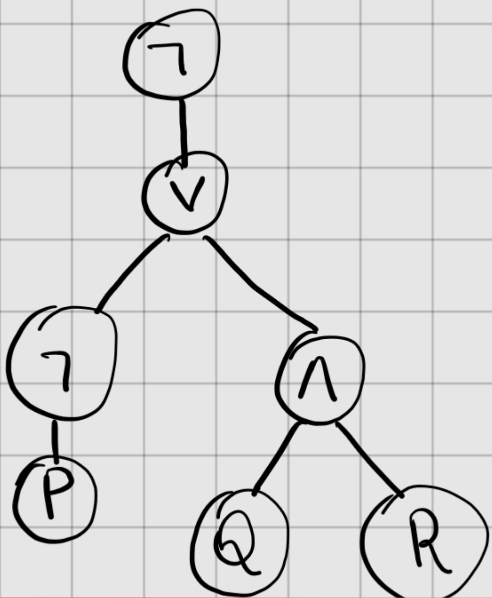
(2) Any propositional variable is a WFPF

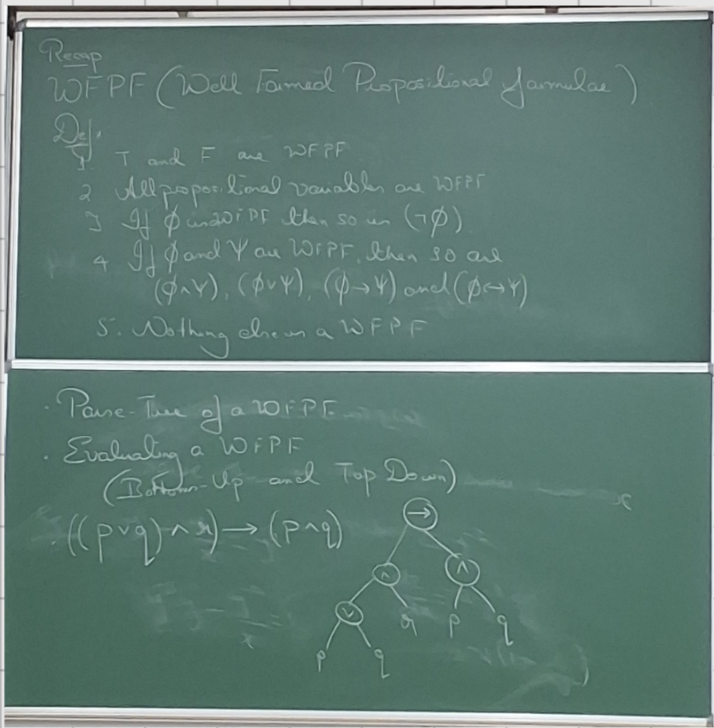
(3) If ϕ is a WFPF, $(\neg\phi)$ is also a WFPF.

(4) If ϕ, ψ are WFPF, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are also WFPF.

(5) Nothing else is WFPF.

e.g: $\neg(\neg P \vee Q \wedge R)$





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e.g: If you study hard (p) or a superhero comes in your dream (q), you get good marks (r).

Find negation.

Sol:

$$\phi : (p \vee q) \rightarrow r \equiv \neg(p \vee q) \vee r$$

$$\neg\phi : \neg(\neg(p \vee q) \vee r)$$

$\equiv (\neg\neg(p \vee q)) \wedge \neg r$

$\equiv (p \vee q) \wedge \neg r.$

If you study hard or a Sup. H comes to your dream, you will not get good marks

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Tautology

* A proposition that is always true is called a tautology

$\Rightarrow T,\ p \vee T,\ (p \wedge q) \rightarrow p$

Contradiction :- (Fallacy)

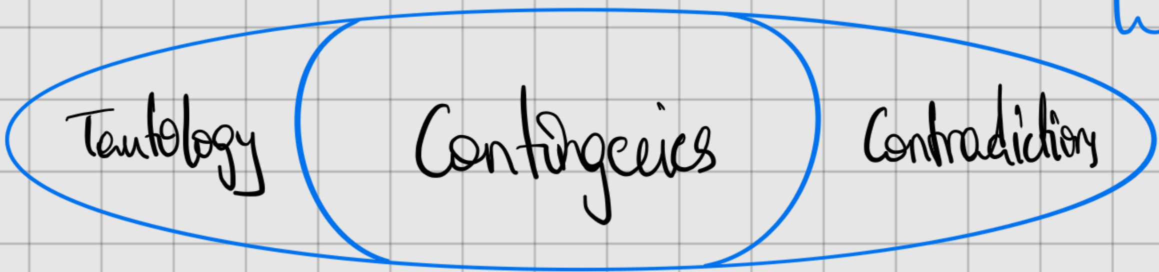
* Contradiction is the opposite of tautology

$\Rightarrow F,\ p \wedge F,\ p \wedge \neg q,\ (p \wedge q) \wedge (p \rightarrow \neg q)$

Contingencies :-

* It's value depends on it's inputs

WFPF



Satisfiable. Unsatisfiable

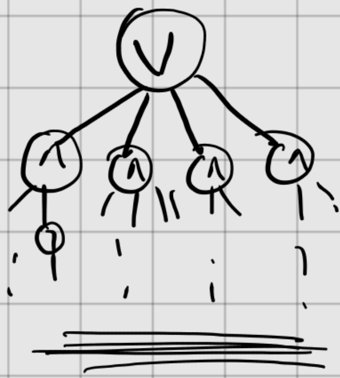
Unfalsifiable Falsifiable.

Disjenctive Normal form :- (DNF).

* A literal is either a proposition variable or it's own negation.
* A formula is in DNF if it is a disjunction of conjuction of literals

e.g

$\Rightarrow (p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2 \wedge p_3) \vee (\neg p_1)$



* Or of empty set is FALSE.
* And of empty set is TRUE.

e.g :-

① $p \rightarrow q \;=\; \neg p \vee q$

$(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

$(p \wedge q) \vee (\neg p \wedge \neg q)$

② $p \leftrightarrow q \quad (p \wedge q) \vee (\neg p \wedge \neg q)$

③ $(p_1 \vee q_1) \wedge (p_2 \vee q_2) \wedge \ldots \wedge (p_k \vee q_k)$.

$$\neg(\neg p_1 \wedge \neg q_1) \wedge \ldots \wedge \neg(\neg p_k \wedge \neg q_k)$$

$$\neg\left[(\neg p_1 \wedge \neg q_1) \vee \ldots \vee (\neg p_k \wedge \neg q)\right]$$

Conjunctive Normal Form:

A formula is said to be in CNF if it is a conjunction of disjunction of literals.

e.g: $\underbrace{(p_1 \vee q_1)}\wedge\underbrace{(p_2 \vee q_2)}\wedge\underbrace{(p_3 \vee \neg q_3)}$

are called clauses

⇒ CNF SAT:

Given a prop. formula in CNF, find a truth argument to the variables st the formula evaluated to TRUE.

e.g: $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$$

[illegible]