1. Find necessary and sufficient conditions on the reals a and b under which the linear program

$$\begin{aligned} &\text{maximize } x_1 + x_2 \\ &\text{subject to} \\ &\qquad ax_1 + bx_2 \leq 1 \\ &\qquad x_i \geq 0,\ i = 1, 2. \end{aligned}$$

   (a) is infeasible.

   (b) is unbounded.

   (c) has a unique optimal solution.

2. Find necessary and sufficient conditions on the reals a and b under which the linear program

$$\begin{aligned} &\text{minimize } x_1 + 2x_2 + 3x_3 \\ &\text{subject to} \\ &\qquad ax_1 + bx_2 + x_3 \geq 4 \\ &\qquad x_i \geq 0,\ i = 1, 2, 3. \end{aligned}$$

   (a) is infeasible.

   (b) is unbounded.

   (c) has a unique optimal solution.

3. **(IAC 7.0)** Mr. Manjunath is preparing to sell freshly brewed tea and homemade *pazhamporis* at the weekend industry-academia conclave. He wants to decide how much of each item to prepare and sell in order to maximize profit. Manjunath earns:

   - Rs. 8 profit for every cup of tea sold,
   - Rs. 5 profit for every pazhampori sold.

   However, he faces the following practical constraints:

   - Due to limited milk and tea leaves, no more than 40 cups of tea can be prepared.
   - The stall has limited storage space: both tea and decorative items (such as cups, plates, signage, and packaging) are stored in crates. Each crate can hold either tea or decorations, and only 60 crates are available in total. So the combined number of tea cups and crates of decorations cannot exceed this limit.
   - Preparation time is also limited. Preparing a cup of tea takes 3 minutes, while frying a pazhampori takes 6 minutes. The vendor has at most 240 minutes (4 hours) to prepare everything before the conclave opens.

   Manjunath must determine the optimal number of tea cups (x_1), pazhamporis (x_2), and decorative items (x_3) to bring to maximize the total profit, while staying within all time and space limitations.

   Pose this as an LP and convert it to standard form.

4. Solve graphically and also using the simplex method (basic version done in class):

$$\begin{aligned}&\text{minimize } 4x_1 + 3x_2\\&\text{subject to}\\&\qquad 5x_1 + x_2 \geq 11\\&\qquad 2x_1 + x_2 \geq 8\\&\qquad x_1 + 2x_2 \geq 7\\&\qquad\qquad x_i \geq 0,\ i = 1, 2.\end{aligned}$$

5. Solve graphically and also using the simplex method (basic version done in class):

$$\begin{aligned}&\text{maximize } 3x_1 + 2x_2\\&\text{subject to}\\&\qquad 2x_1 + x_2 \leq 18\\&\qquad x_1 + 3x_2 \leq 21\\&\qquad\qquad x_i \geq 0,\ i = 1, 2.\end{aligned}$$

6. Solve graphically and also using the simplex method (basic version done in class):

$$\begin{aligned}&\text{maximize } x_1\\&\text{subject to}\\&\qquad 2x_1 - x_2 \leq 3\\&\qquad -x_1 + 3x_2 \leq 5\\&\qquad\qquad x_i \geq 0,\ i = 1, 2.\end{aligned}$$

7. **(Simplex tableau)** Consider the linear program

$$\begin{aligned}&\text{maximize } 7x_1 + 6x_2\\&\text{subject to}\\&\qquad 2x_1 + 4x_2 \leq 16\\&\qquad 3x_1 + 2x_2 \leq 12\\&\qquad x_i \geq 0,\ i = 1, 2.\end{aligned}$$

   Convert the problem to standard form and solve it using the simplex method.

8. Solve using the simplex tableau:

   maximize $c^\top x$ subject to $Ax \leq b$ where

$$c = [1 \ \ 4]^\top, \qquad A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 1 & 3 \end{bmatrix}, \qquad b = [3 \ \ 9 \ \ 5]^\top$$

   and $x = [x_1 \ \ x_2]^\top \geq 0$.