1. Find the dimension and construct a basis for the row space, null space, and column space of

   i. $A = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

   ii. $B = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 3 & 2 & 5 \end{bmatrix}$

   iii. $C = \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ 1 & 1 & 0 \end{bmatrix}$

2. Prove: The system $Ax = b$ has a solution *iff* rank A = rank $[A,\ b]$.

3. Let $A \in \mathbb{R}^{m \times n}$ be a matrix. Show that

   a. Row space of A is a subspace of $\mathbb{R}^n$

   b. Left null space of A is a subspace of $\mathbb{R}^m$.

4. Show that $x^T y = \|x\| \cdot \|y\|$ *iff* x is a scalar multiple of y.

5. Show that given a vector space $V \subseteq \mathbb{R}^n$, every $x \in \mathbb{R}^n$ can be represented uniquely as

$$x = x_1 + x_2 \qquad\qquad x_1 \in V,\ x_2 \in V^\perp.$$

6. Diagonalize the following matrices if possible.

   i. $A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$

   ii. $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

   iii. $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

   iv. $B = \begin{bmatrix} 6 & 2 & 1 \\ 0 & 6 & 1 \\ 0 & 0 & 6 \end{bmatrix}$

7. Consider the statement: a real symmetric matrix with positive diagonal entries is positive definite. Prove or provide a counterexample.

8. Prove: If Q is a symmetric matrix, then

$$Q \text{ is positive definite} \iff \text{all eigenvalues of } Q \text{ are positive.}$$