1. Find the dimension and construct a basis for the row space, null space, and column space of

   i. $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

   ii. $B = \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0 \end{bmatrix}$

   iii. $C = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

2. Show that the eigenvectors associated with distinct eigenvalues of a symmetric matrix $A \in \mathbb{R}^{n \times n}$ are linearly independent. As a corollary, conclude that if all the eigenvalues of A are distinct, then A is diagonalizable. Is the converse true? Prove or give a counterexample.

3. Let $A \in \mathbb{R}^{m \times n}$ be a matrix. Show that

   a. Column space of A is a subspace of $\mathbb{R}^m$

   b. Null space of A is a subspace of $\mathbb{R}^n$.

4. Prove that the orthogonal complement of a vector space V is a subspace.

5. Show that for a matrix A, $\text{Row}(A)^\perp = \mathcal{N}(A)$.

6. Show that a symmetric p.d. matrix is diagonalizable.

7. Can you diagonalize the following matrices?

   i. $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

   ii. $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

8. (**Rayleigh's inequalities**) Prove: For a symmetric p.d. matrix Q,

   $$\lambda_{\min} \leq x^T Q x \leq \lambda_{\max}$$

   for any x with $\|x\| = 1$ where $\lambda_{\min}$ and $\lambda_{\max}$ are the smallest and the largest eigenvalue of Q, respectively.