CS2020A Discrete Mathematics

Tutorial 11 | 03/Nov/2025

Definition. A set A is *countable* if there exists an injective function from A to $\mathbb{N}$.

Prove or give a counterexample to the following.

1. **Theorem 1.** The union of any two countable sets is countable.
2. **Theorem 2.** The Cartesian product of any two countable sets is countable.
3. **Theorem 3.** The union of countably many countable sets is countable.

   Equivalently, if $A_0, A_1, A_2, \ldots$ is a sequence of countable sets, then $A = \bigcup_{i \in \mathbb{N}} A_i$ is also countable.
4. **Theorem 4.** Set of all binary strings of finite length is countable.
5. **Theorem 5.** If there exists an injective function from a set A to a set B, then there exists a surjective function from B to A.
6. **Theorem 6.** There is no surjective function from $\mathbb{R}$ to $P(\mathbb{R})$.

   Note. Do not assume Cantor's general theorem. This is obviously a special case of that theorem.