CS2020A Discrete Mathematics

Problem Set 02 | Oct 2025

1. In the eight strings below, m denotes a domain constant, x, y, z denote domain variables, f denotes a unary function, and S, B denote binary predicates. Which of the following strings are well formed first order formulae? Specify a reason for failure for strings which are invalid. Draw parse trees of all strings which are valid.

   (a) $S(m, x)$
   (b) $B(m, f(m))$
   (c) $f(m)$
   (d) $B(B(m, x), y)$
   (e) $S(B(m), z)$
   (f) $(B(x, y) \rightarrow (\exists z S(z, y)))$
   (g) $(S(x, y) \rightarrow S(y, f(f(x))))$
   (h) $(B(x) \rightarrow B(B(x)))$

2. Write a first order formula ψ which is true if and only if a set P of pigeons can occupy a set H of holes such that every pigeon in P occupies a hole in H and every hole in H is occupied by at most 1 pigeon. Let $A(x, y)$ denote the predicate "x is occupying y".

   Hint. You can use constructs like $\forall i_{\in P}$, $\exists j_{\in H}$ etc.

3. Rewrite the above formula if the domain of discourse D contains the pigeons and the holes and you are not allowed to quantify over subsets of D (you cannot use constructs like $\forall i_{\in P}$, $\exists j_{\in H}$ etc.) However, you can use the following two additional predicates

   - P(x) : x is a pigeon
   - H(x) : x is a hole

4. Draw the parse tree for the above formula and put a back arrow from each bound variable occurrence to the quantifier that binds it.

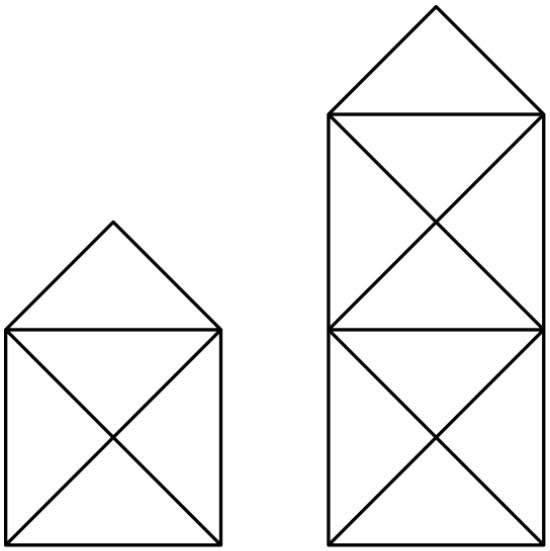
5. Which of the following properties a graph G on vertex set $V = \{1, 2, \ldots, 20\}$ are graph invariants?

   a) The shortest cycle in G has length 10.
   b) Every cycle in G has an even numbered vertex.
   c) Every cycle in G has an even degree vertex.
   d) Every odd-numbered vertex in G has an odd degree
   e) G has an odd number of odd degree vertices.
   f) Two edges of G are crossing each other.

g) There is a drawing of G such that no two edges cross each other.
h) G has three connected components
i) G has chromatic number 5

6. Show that the number of odd degree vertices in any simple undirected graph is an even number.

7. Draw these two figures without taking your pencil off the paper and drawing each line exactly once.



House 1 House 2

Figure 1: House

8. Prove. In every finite simple undirected graph, there exist two vertices of the same degree.

9. In IIT Palakkad campus, each male student is friends with 10.5 female students on average and each female student is friends with 52.5 male students on average. (Assume that friendships are always mutual). Find the percentage of female students in campus.

10. An undirected graph $G = (V, E)$ is called k-partite if its vertex set V can be partitioned into k parts $V_1, \ldots, V_k$ such that every edge in E is between two different parts. Show that a graph G is k-partite if and only if $\chi(G) \leq k$.

11. Draw a triangle-free graph whose chromatic number is at least 4

12. Prove that every edge included in a closed walk W on a tree T is repeated an even number of times in W.

13. Prove that every trail in a tree is a path.