CS2020A Discrete Mathematics

Tutorial 08 | 29/Sep/2025

Defn. An undirected graph G is called *connected* if there is a path between any two vertices in G.

1. What is the minimum number of edges in a connected undirected graph on n vertices? Justify.

Defn. A simple undirected graph $G = (V, E)$ is called *bipartite* if the vertex set V can be partitioned into two parts V_1 and V_2 such that all the edges in E have one endpoint in V_1 and the other in V_2.

2. What is the maximum number of edges possible in a (simple undirected) bipartite graph on n vertices. Justify.

Defn. A simple undirected graph $G = (V, E)$ is called *k-colourable* if one can colour all the vertices in V with at most k different colours so that the colors of any two vertices u and v, such that $\{u, v\} \in E$ are different.

3. Consider the infinite graph $G = (\mathbb{Z}, E)$ where $\{i, j\} \in E$ iff $(i - j)$ is **not** a multiple of 5. What is the smallest k for which G is k-coloroable? Justify.

Defn. A *cycle* in a graph $G = (V, E)$ is a sequence $v_1, v_2, \ldots, v_k$ of vertices such that $v_k = v_1$ and for each i from 1 to $k - 1$, $\{v_i, v_{i+1}\} \in E$. The length of a cycle is the length of the above sequence.

4. Prove that a graph in which every vertex has degree at least two contains a cycle.

Defn. An n-cycle is a graph consisting of a single cycle of length n and nothing else.

5. For what values of n is the n-cycle 2-colorable?

Theorem 1. The following statements are equivalent for a simple undirected graph $G = (V, E)$.

a) G is bipartite.
b) G is 2-colorable
c) G does not contain a cycle of odd length.

6. Prove or give a counter example to the above theorem.