CS2020A Discrete Mathematics

Tutorial 07 | 22/Sep/2025

1. For each of the first order formula below over the domain $D = \{1, \ldots, 100\}$,

   a) Draw its parse tree
   b) Write a single python function using nested loops which evaluates the formula
   c) Write a python program using multiple functions each using a single loop to evaluate the formula

   (You can assume that all the predicates are implemented and you can call them by their names supplying the correct number of arguments)

   1. $\forall x \exists y \forall z S(x, y, z)$
   2. $\forall x(P(x) \to \exists y(Q(y) \wedge \forall z(R(z) \vee S(x, y, z))))$

2. *Definition 1.* A function $f : \mathbb{R} \to \mathbb{R}$ is called *continuous* if for every every real number $\epsilon > 0$ and for every $x \in \mathbb{R}$ there exists a real number $\delta > 0$ such that for every $y \in \mathbb{R}$ with $|x - y| < \delta$, we have that $|f(x) - f(y)| < \epsilon$.

   Definition 2. A function $f : \mathbb{R} \to \mathbb{R}$ is called *uniformly continuous* if for every real number $\epsilon > 0$ there exists a real number $\delta > 0$ such that for every $x, y \in \mathbb{R}$ with $|x - y| < \delta$, we have that $|f(x) - f(y)| < \epsilon$.

   (a) Express the two definitions above as first order sentences. Apart from the logical symbols and the function symbol f, you have to define all the other symbols that you use in the formulae. Do not use shorthands like $\forall(\epsilon > 0)$.

   (b) Draw parse-trees for these formulae side by side.

   (c) Which one do you think is a stronger requirement, that is does one of the coninuity defintions follow as a logical consequence of the other?

3. Let P be any ternary predicate, and

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\begin{aligned}
\alpha_1 &= \forall x \forall y \exists z P(x, y, z) \\
\alpha_2 &= \forall x \exists z \forall y P(x, y, z) \\
\alpha_3 &= \exists z \forall x \forall y P(x, y, z) \\
\alpha_4 &= \forall x \exists y \exists z P(x, y, z) \\
\alpha_5 &= \forall x \exists z \exists y P(x, y, z) \\
\alpha_6 &= \exists z \forall x \exists y P(x, y, z)
\end{aligned}
$$

   (a) Identify and **count** all ordered pairs (i, j) such that $\alpha_i \implies \alpha_j$ and justify why each of the logical consequences you identified is true.

   (b) Identify and **count** all ordered pairs (i, j) such that α_j is not a logical consequence of α_i and find a model for P in which α_i evaluates to True and α_j evaluates to False.