1. Suppose that a wireless broadcast system has n transmitters. Transmitter j broadcasts at a power of $p_j > 0, j = 1, \ldots, n$. There are m locations where the broadcast is to be received. The path gain from transmitter j to location i is $g_{ij}, i = 1, \ldots, m$; that is, the power of the signal transmitted from transmitter j received at location i is $g_{ij}p_j$. The total power received at location i is the sum of the powers received from all the transmitters. Formulate the problem of finding the minimum sum of the powers transmitted subject to the requirement that the power received at each location is at least P.

2. Solve using the simplex method:

   maximize $c^T x$ subject to $Ax \leq b$ where

   $$c = [3 \;\; 2 \;\; 5]^T, \qquad A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 4 & 0 \end{bmatrix}, \qquad b = [430 \;\; 460 \;\; 420]^T$$

   and $x = [x_1 \;\; x_2 \;\; x_3]^T \geq 0$.

3. Consider the LP:

   $$\begin{aligned} \text{minimize } & x_1 + x_2 \\ \text{subject to } & \\ & x_1 + 2x_2 \geq 3 \\ & 2x_1 + x_2 \geq 3; \qquad x_1, x_2 \geq 0. \end{aligned}$$

   i. Solve the LP graphically.
   ii. Solve the dual using simplex method, and verify that the duality theorem holds.

4. Solve using the simplex tableau:

   $$\begin{aligned} \text{maximize } & -4x_1 - x_2 - 3x_3 - 5x_4 \\ \text{subject to } & \\ & 4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20 \\ & -3x_1 - 2x_2 + 4x_3 + x_4 \leq 10 \\ & -8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20 \end{aligned}$$

   where $x_i \geq 0$ for $i = 1, 2, 3, 4$.

5. Consider the primal problem:

   $$\begin{aligned} \text{minimize } & -(2x_1 + x_2 + 7x_3 + 4x_4) \\ \text{subject to } & \\ & x_1 + x_2 + x_3 + x_4 = 26 \\ & x_i \geq 0, i = 1, 2, 3, 4. \end{aligned}$$

   i. Formulate the dual and write down its solution.
   ii. Solve the primal using the solution to the dual and complementary slackness.

6. Consider the following LP:

$$\begin{aligned} &\text{minimize } 12x_1 + 9x_2 \\ &\text{subject to} \\ &\qquad 5x_1 + x_2 \geq 11 \\ &\qquad 2x_1 + x_2 \geq 8 \\ &\qquad x_1 + 2x_2 \geq 7 \\ &\qquad\qquad x_i \geq 0,\ i = 1, 2. \end{aligned}$$

   (i) Solve the LP graphically.

   (ii) Construct the dual of the problem.

   (iii) Check if
   $$y^* = [0 \;\; 5 \;\; 2]^\top$$
   is an optimal solution to the dual using complementary slackness.

   (iv) Check if
   $$y^* = [0 \;\; 5 \;\; 2]^\top$$
   is an optimal solution to the dual using the duality theorem.
   Caution: Do not forget to check feasibility.

7. Consider the LP:

$$\begin{aligned} &\text{minimize } x_1 + 2x_2 \\ &\text{subject to} \\ &\qquad -2x_1 + x_2 + x_3 \qquad\qquad = 2 \\ &\qquad -x_1 + 2x_2 \qquad + x_4 \qquad = 7 \\ &\qquad\; x_1 + \qquad\qquad\qquad + x_5 = 3 \\ &\qquad\; x_i \geq 0,\ i = 1, \ldots, 5. \end{aligned}$$

   It is known that the solution to the primal above is $x^* = [3 \quad 5 \quad 3 \quad 0 \quad 0]^\top$. Find the solution to the dual.

8. You are given the following LP:

$$\begin{aligned} &\text{maximize } c_1x_1 + \cdots + c_nx_n \\ &\text{subject to} \\ &\qquad x_1 + \cdots + x_n = 1 \\ &\qquad x_i \geq 0, i = 1, \ldots, n \end{aligned}$$

   where $c_1, \ldots, c_n \in \mathbb{R}^n$ are constants.

   i. Write down the dual linear program for the primal problem.

   ii. Suppose you know that $c_4 > c_i$ for all $i \neq 4$. Use this information to solve the dual.

   iii. Use part ii. to solve the given LP.