Direction Field.

$$\left.\frac{dy}{dx}\right|_P = f(x,y) \quad ; y = y(x) \qquad -①$$

$\Rightarrow$ Suppose $f(x,y)$ is defined on a region $D \subseteq \mathbb{R}^n$ and $y = y(x)$ is a solution to ①. Then ① tells you that at (x,y) on the solution curve $y = y(x)$, the slope is $f(x,y)$.

i.e; ① represents the slope of the sol^n curve at each point on the curve.

$\Rightarrow$ Such directions can be represented by a small line segment with arrows pointing to the direction given by the slope $f(x,y)$

$\Rightarrow$ Set of all such directed line segments is called the direction field.

e.g;

$*$ $H : D \to \mathbb{R}^2$, $D \subseteq \mathbb{R}^2$

$H(x,y) = (1, f(x,y))$ gives the Directional field.

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e.g :- $y' = x$

$f(x,y) = x.$

Take $f(x,y) = k$; $k = 1, 1/2, -1 \ldots$

Then trace the direction to get the solution curve.

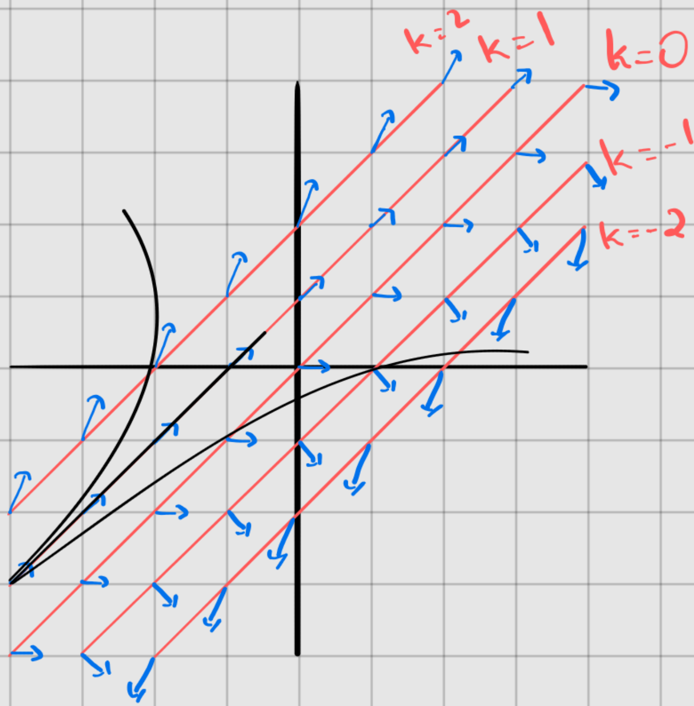
Isoclines :- Curves in $\mathbb{R}^2$, which have a const. slope at a particular point

e.g $y' = y - x$

look at $y - x = k$.

take $k = -2, -1, 0, 1, 2$.

$y = ce^x + x + 1$



Method :-

On the domain, we trace the curve $f(x,y) = k$.
The curve has a slope y' and is equal to the const. k. Such curves are called Isoclines

⇒ Using isoclines, we draw the direction field in the domain of f. To draw sol^n, we follow these dir. line sg.

Initial Value Problem:-

Problem of solving

$$\left.\begin{aligned} y' &= f(x,y) \\ y(x_0) &= y_0 \end{aligned}\right\} - (1)$$

* Let $p(x)$ & $q(x)$ be continuous functions on an open interval (a,b). Let $x_0 \in (a,b)$. The the first order ODE $y' + p(x)y = q(x)$ along with the initial condition $y(x_0) = y_0$ has a unique solution y on (a,b)

Picard's Theorem:-

Let $R = (a,b) \times (c,d)$ be an open rectangle with $(x_0, y_0) \in R$. Let f be defined on R.

(a) If f is continuous on R, then the initial value problem,

Peano's Thm

$y' = f(x,y)$, $y(x_0) = y_0$ has atleast one solution on some subinterval $(a_1, b_1) \subseteq (a,b)$ containing x_0

(b) If f and $\frac{\partial f}{\partial x}$ are continuous, then the Initial Value Problem has a solution in an interval containing x_0.

$\Rightarrow$ Picard's iterative method:

$$y' = f(x,y), \quad y(x_0) = y_0, \quad f : R \to \mathbb{R}. \quad -①$$

$$\Rightarrow y(x) = y_0 + \int_{x_0}^{x} f(t, y(t))\, dt. \quad -②$$

* Solutions of ① are precisely the continuous solution of ②

* We start with.

$$y_0(x) = y_0$$

$$y_1(x) = y_0 + \int_{x_0}^{x} f(t, y_0)\, dt$$

Iteration

$$\int^{x}$$

$$y_1(x) = y_0 + \int_{x_0} f(t, y_1(t))\,dt$$

$\vdots$

e.g. $y' = y$, $y(0,1)$

$$y_0(x) = 1$$

$$y_1(x) = 1 + \int_0^x dx = 1 + x$$

$$y_2(x) = 1 + \int_0^x (1+x)\,dt = 1 + x + \frac{x^2}{2}$$

$\vdots$

e^x Expansion