1. Let $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$. Let
$$\Omega = \{x \in \mathbb{R}^n : Ax = b\}.$$
Show that $d \in \mathbb{R}^n$ is a feasible direction at $x \in \Omega$ if and only if d is in the null space of A.

2. Consider the problem
$$\min f \text{ subject to } x \in \Omega$$
where $x = [x_1 \;\; x_2]^\top$ and $f : \mathbb{R}^2 \to \mathbb{R}$ is given by
$$f(x) = 4x_1^2 - x_2^2$$
and
$$\Omega = \{x : x_1^2 - 2x_1 - x_2 \geq 0,\; x_1 \geq 0,\; x_2 \geq 0\}.$$
   a. Does the point $x^\star = 0 = [0 \;\; 0]^\top$ satisfy the first-order necessary condition?
   b. Does the point $x^\star = 0$ satisfy the second-order necessary condition?
   c. Is the point $x^\star = 0$ a local minimizer of the given problem?

3. Consider the following function $f : \mathbb{R}^2 \to \mathbb{R}$ given by
$$f(x) = x^\top \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} x + x^\top \begin{bmatrix} 3 \\ 5 \end{bmatrix} + 6.$$
   a. Find the gradient and Hessian of f at the point $[1 \;\; 1]^\top$.
   b. Using Taylor's theorem, write down the first order approximation of f at $[1 \;\; 1]^\top$. Verify that the approximation error matches the error given by the theorem, orderwise.
   c. Find the directional derivative of f at $[1 \;\; 1]^\top$ with respect to a unit vector in the direction of maximal rate of increase.
   d. Find a point that satisfies the FONC (interior case) for f. Does this point satisfy the SONC (for a minimizer)?

4. Compute the gradient and Hessian of the following functions:
   i. $f(x_1, x_2) = 2x_1 + 3x_2 + 4x_1^2 - 5x_1x_2 + 6$
   ii. $f(x, y, z) = x + y + z + xy + yz + xz + x^2y^2 + z^2 + xyz + 8$

5. Prove or disprove:
   i. $\sin x = O(x)$
   ii. If $f(x) = o(g(x))$, then $f(x) = O(g(x))$
   iii. If $f(x) = O(g(x))$, then $f(x) = o(g(x))$
   iv. $x^3 = o(x^2)$
   v. $x^2 + 2x + 1 = o(x)$

6. Write down the Taylor series expansion of $f : \mathbb{R}^2 \to \mathbb{R}$ about the given point $x^{(0)} \in \mathbb{R}^2$. Neglect terms of order three or higher.
   i. $f(x) = x_1 e^{-x_2} + x_2 + 1$; $\quad ; x^{(0)} = [1 \;\; 0]^\top$
   ii. $f(x) = x_1^4 + x_1^2 x_2^2 + x_2^4$; $\quad ; x^{(0)} = [1 \;\; 1]^\top$
   iii. $f(x) = e^{x_1 - x_2} + e^{x_1 + x_2} + x_1 + x_2 + 1$ $\quad ; x^{(0)} = [1 \;\; 0]^\top$

7. Problem 6.1 in textbook.

8. Problem 6.9 in textbook.