ASSIGNMENT FOR SUBMISSION

(1) Prove the 'recurrence relation' amongst the Bessel functions:

$$\frac{d}{dx}(x^p J_p(x)) = x^p J_{p-1}(x).$$

Use this to prove that the positive zeros of J_p and J_{p+1} occur alternatively. (Hint: Consider the function defined by $f_p(x) := x^p J_p(x)$. Let x_1, x_2 be two successive zeros of J_p, thereby of f_p as well. Note then that f_p is continuous on $[x_1, x_2]$, differentiable on $(x1, x2)$. So, Rolle's theorem may be applied to assert existence of a zero for the derivative f_p'. Now use the recurrence relation as given in the first part of the question to argue towards concluding the proof).

(2) Consider the following second-order, linear homogeneous ODE:

$$xy'' - (x + N)y' + Ny = 0,$$

where N is a positive integer. Show that it has a polynomial solution. Determine whether it has a second (linearly independent) Frobenius series solution.