Free fall of an Object:-

$$y = y(t)$$

$$v = \frac{dy}{dt}$$

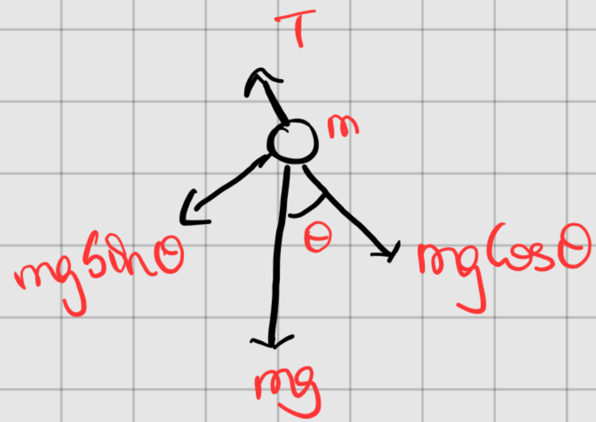
$$a = \frac{d^2y}{dt^2}$$

$$\left.\begin{aligned} F &= mg \\ F &= m\frac{d^2y}{dt^2} \end{aligned}\right\} \boxed{\frac{d^2y}{dt^2} = g}$$

→ Order : 2

→ Linear

Oscillating Pendulum:-



$$s = l\theta$$

$$v = l\frac{d\theta}{dt}$$

$$a = l\frac{d^2\theta}{dt^2}$$

$$\left.\begin{aligned} F &= -mg\sin\theta \\ F &= l\frac{d^2\theta}{dt^2} \end{aligned}\right\} \boxed{-g\sin\theta = l\frac{d^2\theta}{dt^2}}$$

→ Order : 2

→ Non-linear.

* To make solving them easy, DEs are classified into two types:

ODE — Ordinary Diff. Eq^n.

PDE — Partial Diff. Eq^n.

⇒ We use order & linearity to classify them.

Order: Order of a D.E is the order of the highest derivative in that equation.

* ODE of order n is of the form $F(x, y, y', \ldots, y^n) = 0$

Linearity: If F is linear in $y, y', \ldots, y^n$; then we say that F is linear.

* The general form of F will be;

$$\boxed{a_n y^n + a_{n-1} y^{n-1} + \ldots + a_1 y' + a_0 y = b(x)}$$

Here, $a_0, a_1, \ldots, a_n, b$ can be functions of x.; $a_n \neq 0$

⇒ If $b = 0$; the DE will be Homogenous

$\Rightarrow$ If $a_0, \ldots\ldots, a_n$ are all constant, it is said to be a linear ODE with const. coeff.

Solution of a DE:

A fn. $y = y(x)$ is called a solution to

$F(x, y', y'', \ldots\ldots, y^n) = 0$ on an interval I, — ①

if y is differentiable as many times as the order of the eq^n on I and y can be substi-tuted in ①

e.g: $(1-x)y' - y = 0$

$$y = \frac{C}{1-x} \quad \text{is a sol}^n \text{ on } (-\infty, 1) \cup (1, \infty)$$

i.e; y is not a solution in any interval containing 1.

Implicit Solution: The dependent var can't be expressed in terms of indep. var. w/o ambiguity

$\boxed{h(x,y) = 0}$

$\boxed{F(x,y)=0}$

eg: $x^2+y^2=25$.

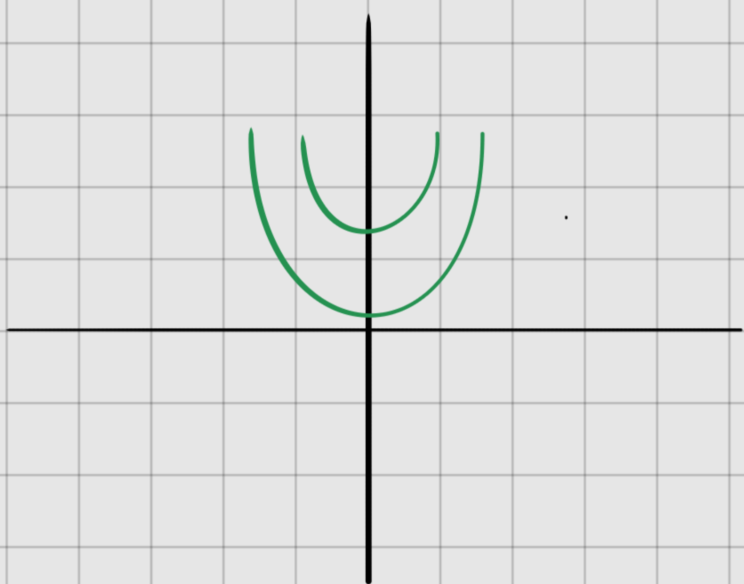
NOTE:- The equation must be valid for it to be considered a solⁿ. For example, $x^2+y^2+25=0$ is not a solⁿ.

$\Rightarrow$ $y' = x$

$y = \frac{x^2}{2} + c$ } This is a family of solutions

c is called a "parameter".

As it is dependent only on one parameter, it is said to be a "One-parameter family of solⁿ".



$\Rightarrow$ $y'' = x$

$$y' = \frac{x^2}{2} + c_1$$

$$y = \frac{x^3}{6} + c_1 x + c_2 \Big\} \text{ 2 parameter family of sol}^n.$$

* In general, an n^{th} order ODE will give an n-par family of sol^n.

Exception:- In some case, the family will not contain all solutions

e.g.:- $y' = 4x(y-1)^{1/2}$

$$y = (x^2+c)^2 + 1$$

But $y = 1$ is also a sol^n. Such solutions are called Singular Solutions

* A solution for a DE is also called Solution Curve / Integral Curve.

* In most of the case, the n-parameter family of sol^n gives the set of all solutions. Then we

call it the General solution.

* If specific values are assigned to the parameters in a General solⁿ of a DE, then it is called a particular solⁿ.

* Linear D.E will always have a general solution.

---

* Suppose the n-parameter family is $\{f(x,c_1,\dots,c_n)=0\}$

$$f(x, c_1, \dots\dots, c_n) = 0$$

Differentiating it n-times gives you $n+1$ eqⁿs

Eliminate parameters using these eqⁿs.

We will get the n^{th} order DE.

---

e.g:- Find the DE of the following 2 parameter family of eqⁿs.

$$y = c_1 \cos 2x + c_2 \sin 2x$$

Solⁿ

$$y' = -2c_1 \sin 2x + 2c_2 \cos 2x$$

$$y'' = -4c_1\cos 2x - 4c_2\sin 2x$$

$$y'' = -4y$$

$$\boxed{y'' + 4y = 0}$$