CS2020A Discrete Mathematics

Tutorial 09 | 13/Oct/2025

1. Define *walk*, *tour* and *cycle* in a directed graph.
2. An *Euler Circuit* in a directed graph G is a circuit (closed trail) which includes every edge of G. Equivalently, it is a tour (closed walk) which includes every edge exactly once. Complete the following theorem statement and prove it.

   Theorem (Directed Euler). A directed graph G has an Euler Circuit if and only if G is _____ and _____

Prove or give a counterexample to the following.

3. **Theorem 3.** Any simple undirected graph which contains a tour of positive length contains a cycle.
4. **Theorem 4.** Any simple undirected graph with minimum degree at least 3 contains an even cycle.
5. **Theorem 5.** Let G be a simple undirected graph on at least three vertices. If $d(u) + d(v) \geq n$ for every pair of non-adjacent vertices u and v, then G has a Hamiltonian cycle.

   Note. $d(v)$ is the degree of v and n is the number of vertices in G. Two vertices are non-adjacent if they do not form an edge.