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Mathema...

CS2020A Discrete Mathematics

Tutorial 5 Submission

Submitted By

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Problem 1. Identify the atomic propositions in the following statements, denote them by distinct propositional variables and express the whole statement as a propositional formula.

a) You can be happy without being rich

b) You cannot be happy unless you are rich

c) You cannot be both happy and rich

d) A happy place is a healthy place

e) If this statement is false then it is true

Solution.

a) **p**: You can be happy.
q: You are rich.
Formula: Tautology(T).

b) **p**: You can be happy.
q: You are rich.
Formula: $\neg q \to \neg p$

c) **p**: You can be happy.
q: You are rich.
Formula: $\neg p \wedge \neg q$

c) The correct answer is ¬(p ∧ q).
We can infer from the formula you have written that they are not happy and they are not rich, but the question is only saying that they cannot be happy and rich simultaneously.ie, (Happy, not rich), (Not happy, rich) should be True.

d) **p**: A place is happy.
q: A place is healthy.
Formula: $p \to q$

e) **p**: The statement is true.
Formula: $\neg p \to p$

Problem 2. Identify the atomic propositions in the following statements, denote them by distinct propositional variables and express the intended meaning of the statement as a propositional formula.

a) You will catch your bus if you run. (Answer given: • p: "you run" • q: "you will catch your bus" • intended meaning: $p \to q$)

b) You will catch your bus only if you run.

c) You will not catch your bus unless you run

d) You may not catch your bus unless you run

e) To catch the bus, it is necessary that you run

f) To catch the bus, it is sufficient that you run

g) You will catch the bus if and only if you run.

Group the logically equivalent statements into the same group. How many groups did you get?

Solution.

Let

$$p : \text{"you run"}, \quad q : \text{"you will catch your bus"}.$$

d) This is a tautology since "may" expresses both the possibilities of something happening or not happening.

a) You will catch your bus if you run. $p \to q$ ✓

b) You will catch your bus only if you run. $q \to p$ ✓

c) You will not catch your bus unless you run. $\neg p \to \neg q$ ✓

d) You may not catch your bus unless you run. $p \to q$ ✗

e) To catch the bus, it is necessary that you run. $q \to p$ ✓

f) To catch the bus, it is sufficient that you run. $p \to q$ ✓

g) You will catch the bus if and only if you run. $p \leftrightarrow q$ ✓

Problem 3. Negate the following propositions and write the result in plain English.

a) Both India and Pakistan can beat Australia in cricket.

b) If India can beat England, then Pakistan can also beat England.

c) India can beat England only if Pakistan can beat England.

d) Neither India nor Pakistan can beat Australia in cricket.

Solution.

a) One of India and Pakistan can't beat Australia in Cricket.

b) India can beat England, but Pakistan can't beat England.

c) India can beat England, but Pakistan cant beat England.

d) Atleast one of India ~~and~~ or Pakistan can beat Australia in Cricket.

a) Either India cannot beat Australia, or Pakistan cannot beat Australia (or both).

Problem 4. Establish the following Logical Equivalences using Truth Tables.

a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

c) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

d) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

e) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

f) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

g) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

h) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Solution.

a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	**q**	**r**	**p**	$\wedge$	**(q $\vee$ r)**	**(p $\wedge$ q)**	$\vee$	**(p $\wedge$ r)**
T	T	T	T	**T**	T	T	**T**	T
T	T	F	T	**T**	T	T	**T**	F
T	F	T	T	**T**	T	F	**T**	T
T	F	F	T	**F**	F	F	**F**	F
F	T	T	F	**F**	T	F	**F**	F
F	T	F	F	**F**	T	F	**F**	F
F	F	T	F	**F**	T	F	**F**	F
F	F	F	F	**F**	F	F	**F**	F

b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	**q**	**r**	**p**	$\vee$	**(q $\wedge$ r)**	**(p $\vee$ q)**	$\wedge$	**(p $\vee$ r)**
T	T	T	T	**T**	T	T	**T**	T
T	T	F	T	**T**	F	T	**T**	T
T	F	T	T	**T**	F	T	**T**	T
T	F	F	T	**T**	F	T	**T**	T
F	T	T	F	**T**	T	T	**T**	T
F	T	F	F	**F**	F	T	**F**	F
F	F	T	F	**F**	F	F	**F**	T
F	F	F	F	**F**	F	F	**F**	F

c) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q
T	T
T	F
F	T
F	F

¬ (p ∧ q)
F
T
T
T

¬ p	∨	¬ q
F	**F**	F
F	**T**	T
T	**T**	F
T	**T**	T

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d) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q
T	T
T	F
F	T
F	F

¬ (p ∨ q)
F
F
F
T

¬ p	∧	¬ q
F	**F**	F
F	**F**	T
T	**F**	F
T	**T**	T

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e) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(p → q)	∧	(p → r)
T	**T**	T
T	**F**	F
F	**F**	T
F	**F**	F
T	**T**	T
T	**T**	T
T	**T**	T
T	**T**	T

p	→	(q ∧ r)
T	**T**	T
T	**F**	F
T	**F**	F
T	**F**	F
F	**T**	T
F	**T**	F
F	**T**	F
F	**T**	F

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f) $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(p → r)	∧	(q → r)
T	**T**	T
F	**F**	F
T	**T**	T
F	**F**	T
T	**T**	T
T	**F**	F
T	**T**	T
T	**T**	T

(p ∨ q)	→	r
T	**T**	T
T	**F**	F
T	**T**	T
T	**F**	F
T	**T**	T
T	**F**	F
F	**T**	T
F	**T**	F

g) $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(p → q)	∨	(p → r)
T	**T**	T
T	**T**	F
F	**T**	T
F	**F**	F
T	**T**	T
T	**T**	T
T	**T**	T
T	**T**	T

p	→	(q ∨ r)
T	**T**	T
T	**T**	T
T	**T**	T
T	**F**	F
F	**T**	T
F	**T**	T
F	**T**	T
F	**T**	F

h) $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(p → r)	∨	(q → r)
T	**T**	T
F	**F**	F
T	**T**	T
F	**T**	T
T	**T**	T
T	**T**	F
T	**T**	T
T	**T**	T

(p ∧ q)	→	r
T	**T**	T
T	**F**	F
F	**T**	T
F	**T**	F
F	**T**	T
F	**T**	F
F	**T**	T
F	**T**	F

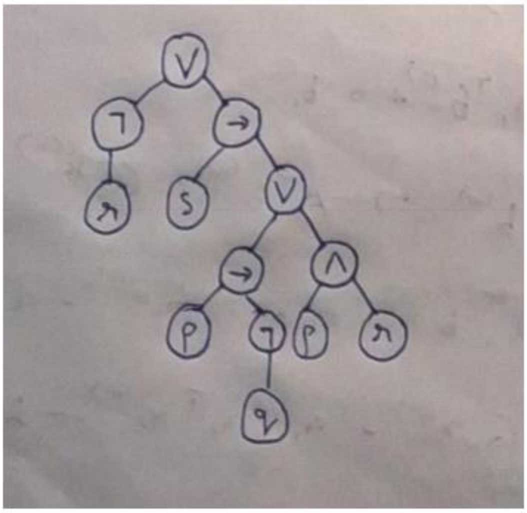
Problem 5. For each of the propositional formulae below, draw its parse tree and list all the subformulae (Every node of the parse tree gives a subformula).

a) $\neg(s \to (\neg(p \to (q \vee \neg s))))$ (You'll get 9 distinct subformulae)

b) $((p \to \neg q) \vee (p \wedge r) \to s) \vee \neg r$ (You'll get 11 distinct subformulae)

Solution.

a) $\neg(s \to (\neg(p \to (q \vee \neg s))))$



(b)

Subformulae: The 9 distinct subformulae are:

a) s

b) p

c) q

d) $\neg s$

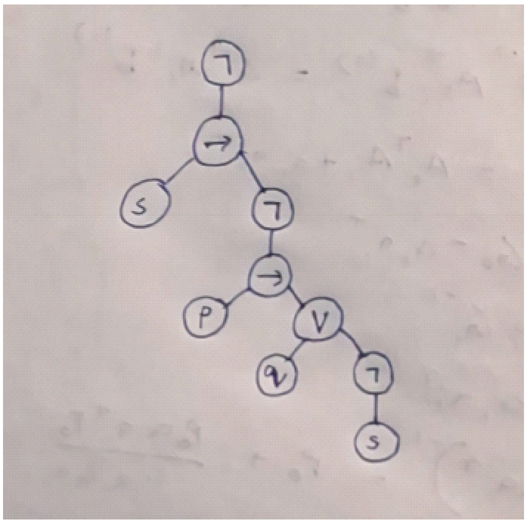
e) $q \vee \neg s$

f) $p \to (q \vee \neg s)$

g) $\neg(p \to (q \vee \neg s))$

h) $s \to (\neg(p \to (q \vee \neg s)))$

i) $\neg(s \to (\neg(p \to (q \vee \neg s))))$



(a)

b) $((p \to \neg q) \vee (p \wedge r) \to s) \vee \neg r$

Subformulae: The 11 distinct subformulae are:

a) p

b) q

c) r

d) s

e) $\neg q$

f) $\neg r$

g) $p \to \neg q$

h) $p \wedge r$

i) $(p \to \neg q) \vee (p \wedge r)$

j) $((p \to \neg q) \vee (p \wedge r)) \to s$

k) $(((p \to \neg q) \vee (p \wedge r)) \to s) \vee \neg r$