

CS2020A Discrete Mathematics

Tutorial 02 | 11/Aug/2025

Prove the following or give a counterexample.

Theorem 1. For every positive integer n , the product $n(n + \frac{1}{2})(n + 1)$ is a multiple of 3.

Theorem 2. The product of any k consecutive integers is a multiple of $k!$ (k -factorial)
(Attempt to find a one line proof for Theorem 2)

Theorem 3. For every two positive integers n and k , $n^k - n$ is a multiple of k .

Theorem 4. Every positive integer can be expressed as a product of prime numbers.

Proof. Suppose Theorem 4 is false. Then there exists positive integers which cannot be expressed as a product of primes. Let n be the **smallest** such counterexample.

Complete the above proof.

Theorem 5. There are infinitely many prime numbers.

Theorem 6. Every prime number larger than 3 is just one away from 3!.

Task 7. (Homework) Read this blog post

[https://gowers.wordpress.com/2011/11/13/
why-isnt-the-fundamental-theorem-of-arithmetic-obvious/](https://gowers.wordpress.com/2011/11/13/why-isnt-the-fundamental-theorem-of-arithmetic-obvious/)

(Timothy Gowers is a Field's Medalist. So if he doesn't find it obvious, we shouldn't either).