1. Show that a set of vectors $\{u_1, u_2, \ldots, u_k\}$ is linearly dependent *iff* one of the vectors from the set is a linear combination of the remaining vectors.

2. Show that the span of any set of vectors is a subspace.

3. **(Dimension)** Show that all bases of a given vector space contain the same number of vectors.

4. Show that if $\{u_1, u_2, \ldots, u_k\}$ is a basis of V, then any element $a \in V$ can be represented *uniquely* as a linear combination
$$a = \alpha_1 u_1 + \ldots + \alpha_k u_k$$
with $\alpha_i \in \mathbb{R},\ i \in [k]$.

5. **(Cauchy-Schwarz inequality)** Prove the following:
$$|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$$
with equality iff $x = \alpha \cdot y$ for some $\alpha \in \mathbb{R}$.

6. **(Triangle inequality)** Show that for a norm $\|\cdot\|$ and $x, y \in \mathbb{R}^n$,
$$\|x + y\| \leq \|x\| + \|y\|.$$

7. Let $v_1, \ldots, v_k \in \mathbb{R}^n$ be non-zero vectors orthogonal to each other. Show that $v_1, \ldots, v_k$ are linearly independent.

8. Show that for any two vectors $x, y \in \mathbb{R}^n$ and a norm $\|\cdot\|$,
$$\|x - y\| \geq \big|\|x\| - \|y\|\big|.$$
Hint: Write $x = (x - y) + y$; use triangle inequality.