Darshan Institute of Engineering & Technology

B.Tech. | Sem-4 | Winter-2024

Course Code	: 2101HS402		**Date**	: 14-11-2024
Course Name	: Probability & Statistics		**Duration**	: 150 Minutes
			Total Marks	: 70

Instructions:
1. Attempt all the questions.
2. Figures to the right indicates maximum marks.
3. Make suitable assumptions wherever necessary.

Q.1	**(A)**	Probability distribution of a random variable X is given below. Find $E(X), V(X), \sigma(X), E(3X + 2), V(3X + 2)$.	**4**

X	1	2	3	4
P(X = x)	0.1	0.2	0.5	0.2

	(B)	If A and B are independent events, with $P(A) = \frac{3}{8}, P(B) = \frac{7}{8}$. Find $P(A \cup B), P(A/B)$ and $P(B/A)$.	**3**

OR

If $P(x) = \frac{2x+1}{48}, x = 1, 2, 3, 4, 5, 6$. Verify whether $P(x)$ is probability function or not.

	(C)	I. There are three boxes. Box I contain 10 light bulbs of which 4 are defective. Box II contain 6 light bulbs of which 1 is defective. Box III contain 8 light bulbs of which 3 are defective. A box is chosen and a bulb is drawn. Find the probability that the bulb is non-defective.	**7**

II. From pack of 52 cards 3 cards are drawn at random. Find the probability that
   a) none is pictured
   b) all three will be aces or all three kings

OR

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax & ; 0 \leq x < 1 \\ a & ; 1 \leq x < 2 \\ 3a - ax & ; 2 \leq x < 3 \\ 0 & ; \text{otherwise} \end{cases}.$$

Find the value of a and also find c.d.f of X.

Q.2	**(A)**	Define Poisson Distribution. Also write properties of it.	**4**
	(B)	Find the binomial distribution for $n = 4$ and $p = 0.3$.	**3**

OR

Find the probability that in five tosses of a fair die 3 will appear twice.

(C) A sample of 100 dry battery cell tested & found that average life is 12 hours & standard deviation 3 hours. Assuming data to be normally distributed what % of battery cells are expected to have life   **7**
(a) more than 15 hrs.?
(b) less than 6 hrs.?
(c) between 10 & 14 hrs.?
$[P(z = 1) = 0.3413 \; ; \quad P(z = 2) = 0.4772 \; ; \quad P(z = 0.67) = 0.2486]$

OR

In a large corporate computer network, user log-on to the system can be modeled as a Poisson process with a mean of 25 log-on per hours.
(a) What is the probability that there are no log-on in an interval of six min.?
(b) What is the probability that time until next log-on is between 2 & 3 min.?

Q.3 **(A)** Find μ'_2 for the data: $6.48, 6.51, 6.47, 6.48, 6.52, 6.50, 6.46$ about $a = 6.5$.   **4**

(B) Write formula to find out Mean deviation.   **3**

OR

Explain Kurtosis in detail.

(C) Find Mean Median and Mode for following data.   **7**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students	5	10	40	20	25

OR

Find the standard deviation and variance for the following distribution:

x	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
f	6	14	10	8	1	3	8

Q.4 **(A)** Obtain the lines of regression for the following data: **4**

Sales (No. of tablets)	190	240	250	300	310	335	300
Advertising expense (Rs.)	5	10	12	20	20	30	30

(Detailed table is not required.)

(B) A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specification. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test this claim at 5% level of significance. $(Z_{0.05} = -1.645)$ **3**

OR

The mean IQ of a sample of 1600 children was 99. Is it likely that this was a random sample from a population with mean IQ 100 and SD 15? $(|Z_{0.05}| = 1.96)$

(C) For 10 randomly selected observations, the following data were recorded.   **7**

Observation Number	1	2	3	4	5	6	7	8	9	10
Overtime Hours (x)	1	1	2	2	3	3	4	5	6	7
Additional units (y)	2	7	7	10	8	12	10	14	11	14

Determine the coefficient of regression using the non-linear $y = a + b_1 x + b_2 x^2$.

OR

Fit a curve of the form $y = a\,e^{bx}$ to the following data by the method of list squares:

x	1	2	3	4
y	1.65	2.70	4.50	7.35

Q.5 **(A)** Two types of batteries are tested for their length of life and the following data are obtained: **4**

	No. of samples	Mean life in hours	Variance
Type A	9	600	121
Type B	8	640	144

Is there a significant difference in the two means? $\left(\left| T_{0.05,15} \right| = 2.1314 \right)$

(B) A machine is designed to produce insulting washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024 cm wih S.D. of 0.002 cm. Test the significance of the deviation. $\left(\left| t_{0.05,9} \right| = 2.2622 \right)$ **3**

OR

Suppose that a die is tossed 120 times and the recorded data is as follows:

Face Observed(x)	1	2	3	4	5	6
Frequency	20	22	17	18	19	24

Test the hypothesis that the die is unbiased at $\alpha = 0.05.$ $\left(\chi^2_{0.05,5} = 11.070 \right)$

(C) Test the hypothesis at 5% level of significance that the presence or absence of hypertension is independent of smoking habits from the following data of 180 persons. $\left(\chi^2_{0.05,2} = 5.991 \right)$ **7**

	Non smokers	Moderate smokers	Heavy smokers
HT	21	36	30
No HT	48	26	19

OR

Two random samples gave the following data:

Sample no.	Size	Mean	Variance
I	16	9.6	40
II	25	16.5	42

Can we conclude that the two samples have been drawn from the same normal population? $\left(F_{0.05}(24, 15) = 2.29 \; ; \left| t_{0.05,39} \right| = 2.0227 \right)$

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