**Enrollment No.:** _____

**Darshan Institute of Engineering & Technology**

**B.Tech. | Sem-4 | Winter-2023**

| | | | | | |
|---|---|---|---|---|---|
| **Course Code** | : | 2101HS402 | **Date** | : | 16-10-2023 |
| **Course Name** | : | Probability & Statistics | **Duration** | : | 150 Minutes |
| | | | **Total Marks** | : | 70 |

**Instructions:**

1. Attempt all the questions.
2. Figures to the right indicates maximum marks.
3. Make suitable assumptions wherever necessary.

**Q.1** **(A)** If A and B are independent events with $P(A) = 0.26$, and $P(B) = 0.45$, find (i) $P(\overline{A} \cap \overline{B})$ (ii) $P(\overline{A} \cup \overline{B})$ (ii) $P(A/B)$ (iii)$P(B/A)$ . **4**

**(B)** The Probability distribution of a random variable X is given below. Find (i) $E(X)$, (ii) $V(X)$. **3**

| X | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.2 | 0.1 | 0.3 | 0.3 | 0.1 |

**OR**

A card is drawn from well-shuffled pack of 52 cards. What is the probability that it is either spade or an ace?

**(C)** State Bayes' theorem. In a bolt factory, three machines A, B and C manufacture 25%, 35% and 40% of the total product respectively. Out of these outputs 5%, 4% and 2% respectively, are defective bolts. A bolt is picked up at random and found to be defective. What are the probabilities that it was manufactured by machine A, B, and C? **7**

**OR**

Write the condition of probability density function (PDF). also,

Show that the function $f(x)$ defined by $f(x) = \begin{cases} \frac{1}{9}x^2 ; & 0 < x < 3 \\ 0 ; & \text{otherwise} \end{cases}$ is PDF and find probability $P(1 < X < 2)$.

**Q.2** **(A)** The probability that an item produced by a machine will be defective is $\frac{1}{10}$. If 12 such items are produced, find the probability that (i) Exactly one will be defective, (ii) at least two will be defective. **4**

**(B)** If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Find the probability that out of 2000 individuals, (i) more than 2 individuals. **3**

**OR**

The mean and variance of a binomial distribution are 4 and 2. Find $P(X \geq 2)$.

**(C)** Define Standard normal variate. **7**

The lifetime of a certain kind of batteries has a mean life of $400$ hours and the standard deviation as $45$ hours. Assuming the distribution of lifetime to be normal. Find The probability of batteries with lifetime (i) at least $490$ hours, (ii) between $385$ and $490$ hours (iii) at most $490$ hours

[Use: $P(0 < Z < 2) = 0.4772, P(0 < Z < 0.33) = 0.1293$ and $P(0 < Z < 1.65) = 0.45$ and ]

**OR**

The average time it takes to serve a customer at a petrol pump is $6$ minutes. The service time follows exponential distribution. Calculate the probability that

(i) A customer will take less than $2$ minutes to complete the service.

(ii) A customer will take between $4$ and $5$ minutes to get service.

(iii) A customer will take more than $10$ minutes for his service.

**Q.3** **(A)** Calculate the variance of the following distribution: **4**

| Class | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
|---|---|---|---|---|---|
| f | 6 | 14 | 10 | 8 | 3 |

**(B)** Calculate the mean of the following distribution: **3**

| Class | $10-25$ | $25-40$ | $40-55$ | $55-70$ | $70-85$ |
|---|---|---|---|---|---|
| f | 2 | 3 | 7 | 6 | 6 |

**OR**

Calculate the median of the following distribution:

| Class | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
|---|---|---|---|---|---|
| f | 10 | 14 | 19 | 7 | 13 |

**(C)** Find the $\beta_2$ of the following distribution: **7**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| f | 5 | 10 | 15 | 20 | 25 | 20 | 15 | 10 | 5 |

**OR**

Find Karl Pearson's coefficient of skewness for the following data:

| class | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
|---|---|---|---|---|---|
| f | 13 | 20 | 30 | 25 | 12 |

**Q.4** **(A)** Obtain regression line of x on y from the following data: **4**

| x | 65 | 66 | 67 | 68 | 69 | 70 |
|---|---|---|---|---|---|---|
| y | 67 | 68 | 65 | 72 | 72 | 69 |

**(B)** If $57$ out of $150$ patients suffering from certain disease are cured by allopathy and $33$ out of $100$ patients with the same disease are cured by homeopathy, is there reason to believe that allopathy is better than homeopathy at $0.05$ level of significance? $(Z_{0.05} > 1.645)$ **3**

**OR**

Test the significance of the difference between the means of the two normal population with the same standard deviation from the following data: $(Z_{0.05} = 1.96)$

| | Size | Mean | SD |
|---|---|---|---|
| Sample – I | 100 | 64 | 6 |
| Sample – II | 200 | 67 | 8 |

**(C)** (i) Compute Karl Pearson's coefficient of correlation between x and y for the following data: **7**

| x | 100 | 98 | 78 | 85 | 110 | 93 | 80 |
|---|---|---|---|---|---|---|---|
| y | 85 | 90 | 70 | 72 | 95 | 81 | 74 |

(ii) Ten participants in a contest are ranked by two judges as follows:

| x | 1 | 3 | 7 | 5 | 4 | 6 | 2 | 10 | 9 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| y | 3 | 1 | 4 | 5 | 6 | 9 | 7 | 8 | 10 | 2 |

Calculate the rank correlation coefficient.

**OR**

Fit a parabola $y = c + bx + ax^2$ to the following data:

| x | 1 | 2 | 3 | 5 | 6 |
|---|---|---|---|---|---|
| y | 1.1 | 5.8 | 17.5 | 55.9 | 86.7 |

**Q.5** **(A)** To examine the hypothesis that the husbands are more intelligent than the wives, an investigator took a sample of $10$ couples and administrated them a test which measures the IQ. The results are as follows. **4**

| Husband | 117 | 105 | 97 | 105 | 123 | 109 | 86 | 78 | 103 | 107 |
|---|---|---|---|---|---|---|---|---|---|---|
| Wives | 106 | 98 | 87 | 104 | 116 | 95 | 90 | 69 | 108 | 85 |

Test the hypothesis with a reasonable test at the level of significance of $0.05$ . $(t_{0.05}(v = 18) = 1.734)$ .

**(B)** A random sample of fifteen paired observations from a bivariate population gives a correlation coefficient of $-0.5$. does this signify the existence of correlation in the sample population? $(t_{0.05}(v = 13) = 2.160)$ **3**

**OR**

The height of 8 males of a given locality are found to be $168, 175, 155, 170, 160, 165, 152, 170$ cm. Based on this sample, find the $95\%$ confidence limit for the heights of males in the locality. $(t_{0.05}(v = 7) = 2.365)$

**(C)** Two random sample drawn from two normal populations are as follows: **7**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | 17 | 27 | 18 | 25 | 27 | 29 | 13 | 17 |
| B | 16 | 16 | 20 | 27 | 26 | 25 | 21 | – |

Test whether the samples are drawn from the two normal population.
$(t_{0.05}(v = 13) = 2.160\ , F_{0.05}(7,6) = 4.21)$

**OR**

From the following data, find whether there is any significant linking in the habit of taking soft drinks among the categories of employees. $(\chi^2_{0.05}(v = 4) = 9.49)$

| Soft drinks | Employees | | |
|---|---|---|---|
| | Clerks | Teachers | Officers |
| Pepsi | 10 | 25 | 65 |
| Thumsup | 15 | 30 | 65 |
| Fanta | 50 | 60 | 30 |

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