QUESTION (A)

Decision Variables

The task is to find a representative daily schedule for 56 days to maximise average daily profit. This means that in each hour of a 24-hour cycle, we need to decide what proportion of the hour is used for pumping, generating and idling.

Let P_t be a variable which takes on a positive value if the storage facility is pumping and 0 if the facility is not pumping, for t = 1, 2, ..., 24 representing each hour in a 24-hour period.

Let G_t be a variable which takes on a positive value if the storage facility is generating and 0 if the facility is not generating, for t = 1, 2, ..., 24 representing each hour in a 24-hour period.

Let L_t be a variable which represents the level of water in the storage facility at the end of each period t, for t = 0, 1, …, 24 representing each hour in a 24-hour period. L_0 and L_{24} take on value of 5,000,000.

Objective Function

We imported the dataset into and wrangled the data using R programming into the following format as shown in Table 1 below.

To determine the average daily profit, we calculated the average prices of each hour for a 24-hour period over 56 days and fitted them into our objective function.

Table 1

Day	**1st hour (00:00:00)**	**2nd hour (01:00:00)**	**…**	**24th hour (23:00:00)**
Day 1	Price in Euro	Price in Euro	…	Price in Euro
Day 2	Price in Euro	Price in Euro	…	Price in Euro
…	Price in Euro	Price in Euro	…	Price in Euro
Day 56	Price in Euro	Price in Euro	…	Price in Euro
Average Day	mean of 1st hour prices	mean of 2nd hour prices		mean of 24th hour prices

Maximise profit: $\sum$-300 * $\sum\frac{Price(t)}{56}$ * P_t + $\sum$270 * $\sum\frac{Price(t)}{56}$ * G_t , for i = 1, 2 , …, 24

⇨ $\sum$-300 * $avg\_hourly\_price_t$ * P_t + $\sum$270 * $avg\_hourly\_price_t$ * G_t

Subject to Constraints:

1. Hourly constraints: $P_t + G_t <= 1$, for t = 1, 2, ..., 24

The hourly constraints restrict the total time spent on pumping, generating or idling to sum up to an hour for each hour in the 24-hour period.

2a. Level constraints: $L_t = L_{t-1} + P_t * 14{,}000*60 – G_t * 16{,}000*60$ for t = 1, 2, …, 24

2b. Capacity constraints $L_t <= 10{,}000{,}000$ for t = 1, 2, ..., 24

2c. L_0 and L_{24} take on value of 5,000,000

The level constraints illustrate that the level in the storage facility at the end of each hour is the balance from previous hour and including any increases from P_t and decreases from G_t during the hour t.

The capacity constraints ensure that the water level does not go above the facility's capacity of 10,000,000 Litres.

3. Non-negativity constraints:
   $0 =< P_t <= 1$,
   $0 =< G_t <= 1$,
   $0 =< L_t$, for $t = 1, 2, \ldots, 24$

The non-negativity constraints prevent an unbounded optimisation problem and restrict the variables P and G to a fraction of an hour.

Solution

We used AMPL, a free-to-use software for optimisation problems. The result from the AMPL linear programming is appended in the table below and the optimal solution is around €7982.62.

G1 = 0	P1 = 1
G2 = 0	P2 = 1
G3 = 0	P3 = 1
G4 = 0	P4 = 1
G5 = 0	P5 = 1
G6 = 0	P6 = 0
G7 = 1	P7 = 0
G8 = 1	P8 = 0
G9 = 0.25	P9 = 0
G10 = 0	P10 = 0
G11 = 0	P11 = 0
G12 = 0	P12 = 0
G13 = 0	P13 = 0
G14 = 0	P14 = 0
G15 = 0	P15 = 0
G16 = 0	P16 = 0
G17 = 1	P17 = 0
G18 = 1	P18 = 0
G19 = 1	P19 = 0
G20 = 0	P20 = 0
G21 = 0	P21 = 0
G22 = 0	P22 = 0
G23 = 0	P23 = 0
G24 = 0	P24 = 1

The AMPL model and run script can be found in the accompanying text files.

We translated the result into the table below which shows the proportion of action for each hour (t). If P_t and $G_t = 0$, we know that the plant is in an idle state as the proportion of idling can be deduced.

00:00:00 (1)	**01:00:00 (2)**	**02:00:00 (3)**	**03:00:00 (4)**	**04:00:00 (5)**	**05:00:00 (6)**
pump	pump	pump	pump	pump	idle
06:00:00 (7)	**07:00:00 (8)**	**08:00:00 (9)**	**09:00:00 (10)**	**10:00:00 (11)**	**11:00:00 (12)**
generate	generate	0.25 => 15 minutes used to generate	idle	idle	idle
12:00:00 (13)	**13:00:00 (14)**	**14:00:00 (15)**	**15:00:00 (16)**	**16:00:00 (17)**	**17:00:00 (18)**
idle	idle	idle	idle	generate	generate
18:00:00 (19)	**19:00:00 (20)**	**20:00:00 (21)**	**21:00:00 (22)**	**22:00:00 (23)**	**23:00:00 (24)**
generate	idle	idle	idle	idle	pump

From these results, it can be seen that the Tantalus Hydro storage facility, on average, should pump water into the raised reservoir, during the largely non-operational hours of homes and businesses (overnight) when the power costs are low. This constitutes the “dormant” hours between 23:00 and 05:00, when many businesses are closed and most of the population is asleep.

Tantalus Hydro should generally switch its operations to generate power in the “active” hours of the day which are associated with high load demand. These are namely the hours between 06:00 and 19:00. It must be mentioned however, that within this period, for the plant to maximise efficiency, there may be a need for a long idle period from 09:00-16:00 which may be associated with the “comedown” from the morning peak, and again after the evening peak from 19:00-23:00.

Due to the losses in efficiency of 30MW on average, as more power is required to pump the water into the raised storage facility than release it, the plant is also idle for a significant period of time, more than 12 hours per day – more than 50% of the time. This could be due to enough power generated and to prevent further efficiency losses. Tantalus Hydro is also placed on idle for one hour as the plant switches from “pumping hours” to “generation hours”. This may be to minimise operational costs and allow the facility to switch on the necessary equipment to switch to a different functionality of the plant.

QUESTION (B)

Decision Variables

The task is to determine an optimal linear threshold policy to maximise the average profit made over the 56 days of data. This linear threshold takes the following general form:

$$Z_t = x_t + \beta y_t$$

Where x_t is analogous to L_t the reservoir level at the end of period *t*.

AM variables

Let βa_t be a variable which is the coefficient of y_t, the price at period *t*, for *t* = 1, 2, ..., 12 during the first 12 hours in a 24-hour period.

Let Ca_t be a threshold variable to generate, for *t* = 1, 2, ..., 12 during the first 12 hours in a 24-hour period. If the water level in the storage facility is below *Ca* at time *t-1*, the facility will pump at the next period, *t*.

Let Da_t be a threshold variable to generate, for *t* = 1, 2, ..., 12 during the first 12 hours in a 24-hour period. If the water level in the storage facility is above *Da* at time *t-1*, the facility will generate at the next period, *t*.

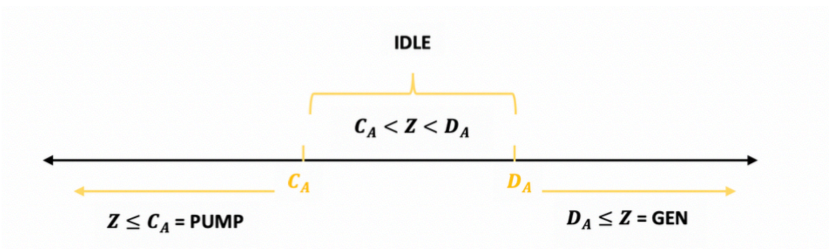
PM variables

Let βp_t be a variable which is the coefficient of y_t, the price at period *t*, for *t* = 13, 14, ..., 24 during the next 12 hours in a 24-hour period.

Let Cp_t be a threshold variable to generate, for *t* = 13, 14, ..., 24 during the next 12 hours in a 24-hour period. If the water level in the storage facility is below *Cp* at time *t-1*, the facility will pump at the next period, *t*.

Let Dp_t be a threshold variable to generate, for *t* = 1, 2, ..., 12 during the first 12 hours in a 24-hour period. If the water level in the storage facility is above Dp_t at time *t-1*, the facility will generate at the next period, *t*.

The linear threshold variables are illustrated more explictly below with the corresponding threshold values for whether the facility should pump, generate or remain idle.



Other variables

We retain the variables P_t, G_t, and L_t from Question (A). However, while P_t and G_t were **continuous** variables previously, they are now **binary** variables in this question. The reason is because the question specified that the storage facility needs to decide what to do for the next hour (pump, generate or idle).

Objective Function

Maximise profit: $\sum -300 * \sum \frac{Price(t)}{56} * P_t + \sum 270 * \sum \frac{Price(t)}{56} * G_t$, for i = 1, 2, …, 24

⇨ $\sum -300 * avg\_hourly\_price * P_t + \sum 270 * avg\_hourly\_price * G_t$

Subject to:

Same constraints as Question (A)

1. Hourly constraints: $P_t + G_t <= 1$, for t = 1, 2, …, 24

2a. Level constraints: $L_t = L_{t-1} + P_t * 14{,}000*60 - G_t * 16{,}000*60$ for t = 2, 3, …, 24

2b. Capacity constraints $L_t <= 10{,}000{,}000$ for t = 1, 2, …, 24

2c. L_0 and L_{24} take on value of 5,000,000

3. Non-negativity constraints:

   $0 =< P_t <= 1$,

   $0 =< G_t <= 1$,

   $0 =< L_t$, for t = 1, 2, …, 24

New constraints

4. 1st period constraint: $L_1 = 5{,}000{,}000 + 14{,}000*1*60$

From our answer in Question (A), the fixed policy for first hour of the day (i.e. t = 1) is to pump. Hence $P_1 = 1$.

5. Policy constraints (applies for both AM and PM):

$P_{t+1} = 1$ if $L_t + \beta_t * avg\_hourly\_price_t <= C_t$ else $P_{t+1} = 0$

$G_{t+1} = 1$ if $L_t + \beta_t * avg\_hourly\_price_t >= D_t$ else $G_{t+1} = 0$

The rationale behind the policy constraints are explained in the decision variables paragraph above.

Explanation on Modified Policy

Capacity and non-negativity constraints were added to the optimisation problem to ensure that L_t lies in the region between the values 0 and 10,000,000 and return to 5,000,000 at L_{24}.

This is achieved by adding Constraints 2a, 2b and 2c in AMPL, using the MINOS algorithm for non-linear optimisation. However, the team faced some technical difficulties implementing Policy Constraint 5 using if-else operators in AMPL.

As such, the team used the optim() function in R programming with the "L-BFGS-B" optimisation algorithm. To implement Policy Constraint 5, we incorporated them into the lower and upper bound arguments of the optim() function. The white dashed box in the "upper bound" argument below caps the level of the reservoir at 10,000,000 for 23 hours and 5,000,000 for the last hour.

```
lower_bound <- c(-Inf,-Inf,0,0,0,0,rep(0,24),rep(0,24),rep(0,24))
upper_bound <- c(+Inf,+Inf,10*10^6,10*10^6,10*10^6,10*10^6,rep(1,24),rep(1,24),rep(10*10^6,23),5000000)
```

The optim() solver is greedy as it depends heavily on the initial values of the parameters of the optimisation problem. To mitigate this issue, we tried multiple starting points. The results are appended in the table below.

Starting values of β	**high**	**middle**	**low**
β (am)	1^e07	5^e05	1^e05
β (pm)	1^e07	5^e05	1^e05
C (am)	6^e06	6^e06	6^e06
C (pm)	8^e06	8^e06	8^e06
D (am)	6^e06	6^e06	6^e06
D (pm)	8^e06	8^e06	8^e06
Avg Profit	**€4637.14**	**€3533.95**	**€-696.98**

QUESTION (C)

Since each hour of the day in **(B)** is used either for pumping or for generation, we made use of binary decision variables.

We can make some improvements here according to the policy threshold equation. Namely, we can make decisions of whether to pump, generate or to remain idle using continuous decision variables like in **(A)** for P_t, G_t, and L_t, rather than restricting operational decisions to binary choices. This facilitates greater operational flexibility and could achieve a higher objective value at the local optimum.

In particular, it may help us take advantage of the price variations over the course of the day, as well as the recursive nature of reservoir level to maximise efficiency (e.g. $L_t = L_{t-1}$ + some action in t). In general, using continuous decision variables allows for more powerful “constraint satisfaction”, as it enables us to take advantage of more global information. Thus in practice, the head of operations at the facility may be able to make optimal decisions of whether to pump, generate or remain idle for periods of the next hour, by continually checking the constraints and value of the variables at optimality in order to maximise profits.