24/03/2020

Energy Analytics Forecasting Report

Alana McIntosh Clarke (01787324),
Chua Chun Kang (01787606),
Ilias Mylonas (01770605),
Konstantinos Paganopoulos (01769789),
Ujjwal (01814453)

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Introduction

This report outlines the team's forecasting methodologies used to estimate daily electricity demand in England and Wales for 8 separate days in March 2020. The report also aims to address the limitations in our modelling as we made improvements on the model each week. Forecasts for electricity demand depend on various factors such as seasonality, temperature, and the accessibility of available data. Throughout the forecasting process, we used proper scoring rules i.e. log score to measure the performance of our different forecasting models. This in turn allows us to rank the models and determine the best forecasting model for the given data.

Why forecast?

Energy is an important sector of the UK economy. In Great Britain, consumers spend about £55 billion each year on energy. Wholesale energy prices, with supplier margins, make up nearly 40% of the average annual domestic gas and electricity bill (Office of Gas and Electricity Markets - OFGEM, 2019). This is ahead of distribution and transmission charges, environmental charges or VAT.

Forecasting allows energy market participants to do forward planning. In this exercise, we focus on electricity demand. As electricity demand changes over the time due to seasonality and economic activity, understanding these trends allow market participants to match supply and demand of electricity. This in turn encourages price stability for electricity users.

In the shorter term, the National Grid provides grid balancing services where longer term contracts fail to match actual demand. In 2018/19, £1.19 billion was spent on balancing costs (OFGEM, 2019). If we can forecast daily demand accurately, these short term balancing costs could potentially be reduced.

The Data

First, in order to interpret our data, we plot electricity demand over time in Figure 1 below. This gives us the opportunity to understand if there is a clear pattern in the given data.

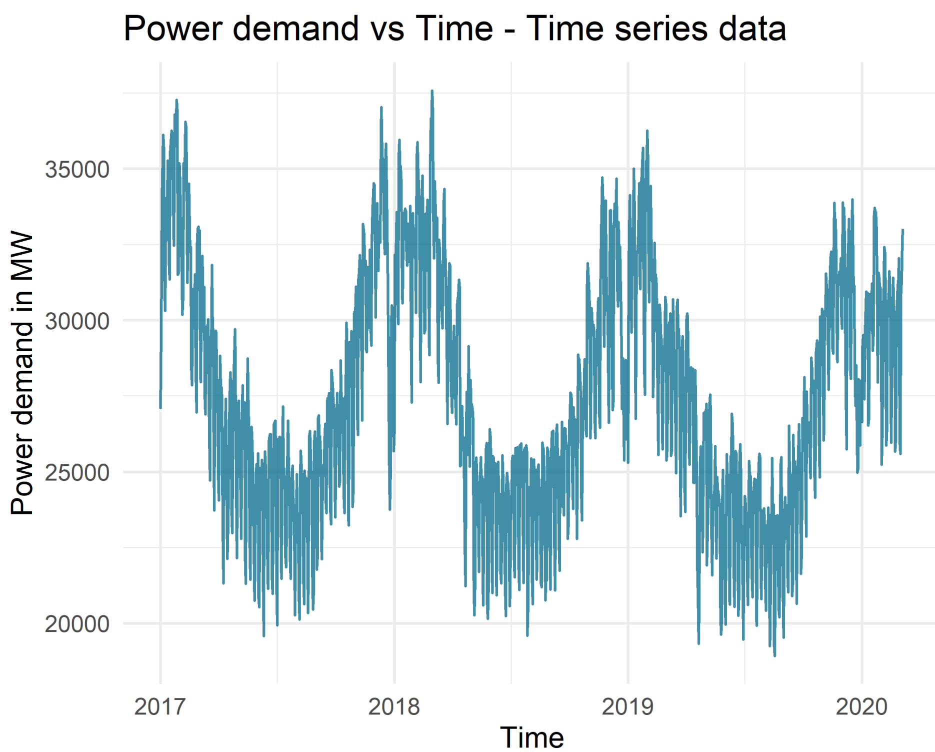


Figure 1: Time series data of load demand over time

Based on Figure 1, we can see that the electricity load follows a seasonal pattern. In general, there is higher demand during winter months, and lower during summer months. This seems reasonable, as UK is a country with negligible usage of air-conditioning during summer months, so most of the load, consumed domestically, is used for heating during winter.

We also observe that winter 2020 had a lower electricity load demand, which can be justified by the relatively mild winter. It is also worth mentioning that the electricity demand peak for the last 3 years is approximately at February of 2018, when there was a very high demand due to the "Beast from the East" cold wave. Finally, we observe that there is a relatively lower demand in electricity every Christmas New Year's Eve, probably since many firms are closed.

We now make a scatterplot with temperature and demand to see any patterns. Note that for temperature we use the mean temperature of the day, since it is the one that mostly affects load demand during the whole year. The latter is justified by calculating the correlations between demand and (i) minimum, (ii) mean and (iii) maximum temperature. The three correlations are (i) 0.7107297, (ii) 0.7844208 and (iii) 0.783007 respectively. As a result, if we base our decision on maximum absolute correlation, mean temperature appears to be the best choice.

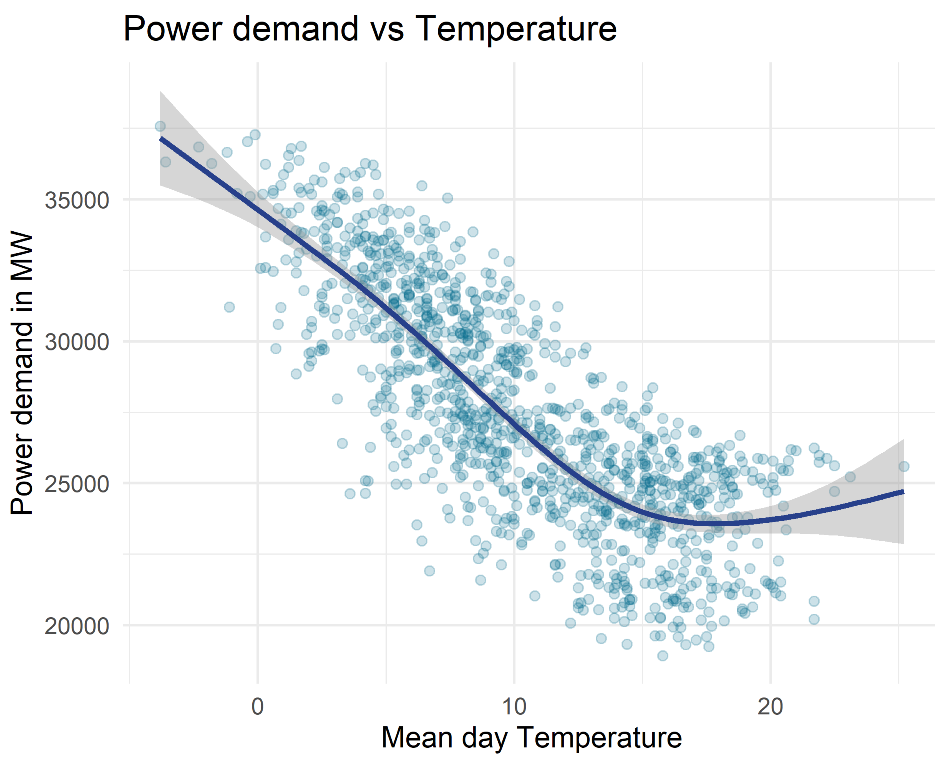


Figure 2: Power demand and its relationship with temperature

From the previous figure ("Figure 2") we can clearly confirm our previous assumptions about the seasonal factor. It is obvious that as the mean day temperature increases, the electricity demand decreases significantly, almost having a linear trend. However, we can mention again that there is a slight increase in electricity load demand for days with a relatively high mean temperature; this is probably due to some air-conditioning units.

Moreover, besides the seasonal factor that has to do with temperature, by examining our data in more detail, and in a smaller time interval, we can see that there is a higher demand for electricity load during weekdays. From the following boxplot ("Figure 3") we can see the effect of weekdays/weekends on the demand of power.

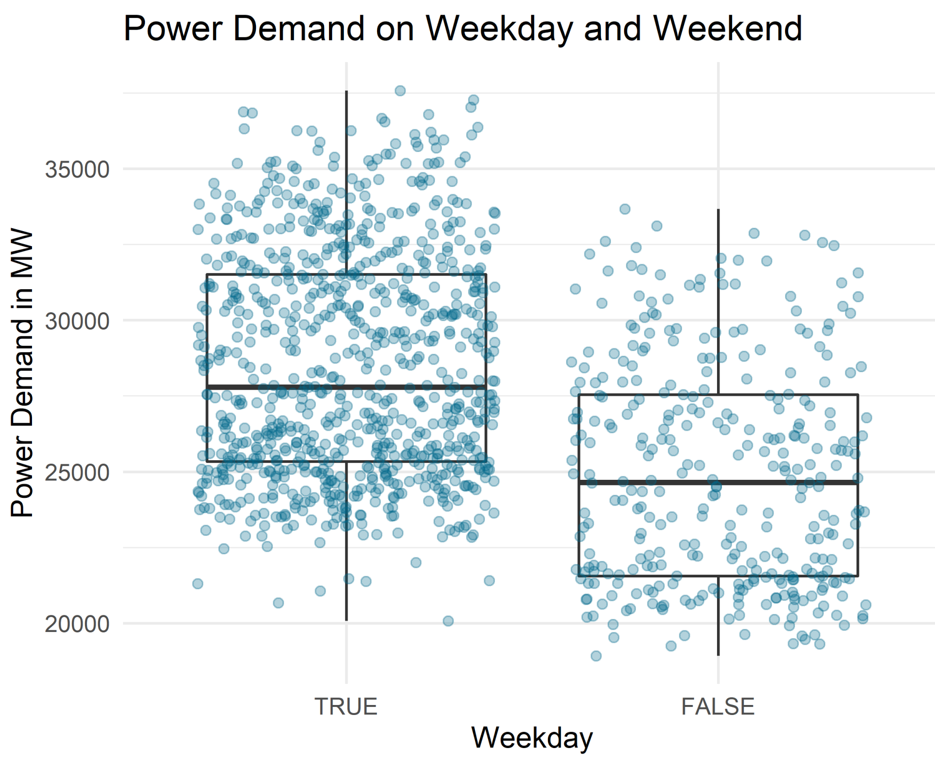


Figure 3: Power demand on weekdays and weekends

It is obvious that the average electricity load for weekdays is much higher (a little higher than 27,500 MW) than that of weekends (a bit lower than 25,000 MW). That can be clearly justified, as during weekends most of the firms are closed, so the electricity demand is significantly lower.

Forecasting Models

7 models were build, analysed and used throughout this report. They are outlined below. Following that, we will outline the methodology used and discuss the model form, assumptions, parameters, functional form and model performance and refinement for each of the seven models.

The models and their parameters are briefly listed below for the benefit of the reader. The dependent variable is always the natural logarithm of electricity demand.

- **Model 1**: weekday, average temperature
- **Model 2:** weekday, average temperature, average temperature squared
- **Model 3:** weekday, average temperature, average temperature squared, ln (demand t-1)
- **Model 4:** weekday, average temperature, average temperature squared, ln (demand t-1), ln (demand t-2)

- **Model 5:** weekday, ln (demand t-1), ln (demand t-2), piecewise linear weather function
- **Model 6:** ARIMA (1,1,2), weekday, average temperature, average temperature squared
- **Model 7:** ARIMA (1,1,2), weekday, piecewise linear weather function

Methodology

The objective for the forecasting exercise is to predict the next-day demand for 8 days; 6 of which are weekdays and 2 of which are weekends. We used the regression approach; this gives us the flexibility to model the different components of the time series as separate parameters. In addition, the regression approach has high explanatory power.

Our analysis of the data in the section before and our experience gained over the course of this exercise tells us that there are several key parameters that could be included in our model:

Weekly seasonality – This is observed in the large differences between weekday and weekend power demand

Link between temperature and power demand – We have historical temperature and power demand data. However, for the exercise, we do not have forward-looking HadCET temperature forecasts for prediction. We are given weather forecasts for London, Bristol and Leeds for a proxy. We therefore train our model on historical HadCET data and predict using the proxy weather forecasts.

Autocorrelation between consecutive days – In the immediate term, it is highly likely that the temperature between 2 consecutive days are similar as weather patterns do not change fluctuate frequently within 2 days in our geographical region of interest. As such, power demand does not differ significantly.

Our models below demonstrate how we build in and fine-tune these parameters over time. As the forecasting exercise progressed, we developed a piecewise linear function to model the knot in the relationship as the relationship between log_demand and temperature is non-linear. In addition, we enhanced the predictions with an ARIMA function to model the residual autocorrelation which our multivariate model did not pick up.

For all models, log_demand is used as the predicted value with log score being the proper scoring rule for model performance. Log_demand is useful for several reasons. In addition to enabling us to model multiplicative models, it also serves to smooth the data where log_demand can be interpreted as percentage change in demand. This is useful as our time series is non-stationary due to seasonality.

Model 1

Form:

In the first forecasting model that we built, we wanted to include the parameters that we thought have the greatest impact on the daily electricity demand. These parameters are the **weekday** and the **weather**.

Regarding the weekday, we needed to incorporate into our model the fact that during the weekends the electricity demand tends to fall compared to the working days of the week. As a result, we built a model where we used a binary variable Weekday which takes the value 1 if the day is a working day or 0 if the day is a Saturday or a Sunday. Therefore, the model takes this very initial form:

$$\ln(demand) = \beta_0 + \beta_1\, Weekday + \epsilon_t$$

Then, in this simple model, we would like to include also the information regarding the weather of the day we need to make a prediction. The data that we have available for the weather are related to the temperature of that particular day of the forecast and contain the values for the minimum temperature, the average temperature and the maximum temperature of this day.

From the calculations of the correlations between the minimum, the average and the maximum temperature of the day with the actual electricity demand of the day, we saw that the highest correlation was between the average temperature and the actual demand. As a result, we decided to use the average temperature as a decision variable for our model.

In the beginning, we added the average temperature factor with a linear relationship to the model:

$$\ln(demand) = \beta_0 + \beta_1 mean.temp + \beta_2\, Weekday + \epsilon_t$$

Assumptions:

Linear relationship between the average temperature and the natural logarithm of daily electricity demand.

Parameters:

In the first forecasting model that we built, we wanted to include the parameters that we thought have the greatest impact on the daily electricity demand. These parameters are the **weekday** and the **average temperature**.

Functional Form:

$$\ln(demand) = \beta_0 + \beta_1 mean.temp + \beta_2\, Weekday + \epsilon_t$$

Model Refinement and Model Performance:

We followed the methodology we mentioned before regarding the evaluation of the performance for the model and we saw that we got a log_score of 1.562.

Model 2

Form:

Now, by looking at the scatterplot of the daily electricity demand vs the average temperature of that day, we thought that the relationship may be not linear, we decided to use a quadratic form of the temperature and see the results.

Assumptions:

Quadratic relationship between the average temperature and the natural logarithm of daily electricity demand.

Parameters:

Weekday and the **average temperature** (same parameters as in model 1)

Functional Form:

$$\ln(demand) = \beta_0 + \beta_1 mean.temp + \beta_2 mean.temp^2 + \beta_3\, Weekday + \epsilon_t$$

Model Refinement and Model Performance:

We followed the methodology we mentioned before regarding the evaluation of the performance for the model and we saw that we got a log_score of 1.592.

Model 3

Form:

Model 3 is focuses on the changing behavior of power demand with mean temperature, mean temperature squared, whether the day is a weekday or weekend and yesterday's demand.

Assumptions:

An average temperature is assumed as temperature for this model as it provides a more wholistic view, then we are using a dummy variable for workday which is assumed to be as binary variable. Here we are assuming the time series effect, so including an object with lag t-1.

Parameters:

The parameters for model 3 are similar to model 1, with an extra one with the demand lagged by one day. In other words, the parameters include mean temperature, mean temperature squared, the dummy variable for weekday (weekday or weekend), yesterday's demand (lagged by t-1).

Functional Form:

$$\ln(demand) = \beta_0 + \beta_1 mean.temp + \beta_2 mean.temp^2 + \beta_3\, Weekday + \beta_4 \ln(demand)_{t-1} + \epsilon_t$$

Model Refinement and Model Performance:

Clearly the relationship between forecast error and model form is improved as model 3 forecasts with a higher accuracy. The results of the model 3 are better than that of the model 2 where we are not taking any time series into account. The forecasted values were better as compared to model 2, with a log score of 1.80578. However, there is still scope for further improvement.

Model 4

Form:

Model 4 incorporates the non-linearities in weather, specifically how the relationship with mean temperature and demand change over time.

Assumptions:

For temperature we use again the average temperature of the day, since we guess that it is more representative. We assume that workday variable is a binary variable. As in model 3, we use a timeseries object.

Parameters:

The parameters for model 4 are the same parameters for model 3, with an extra one with the demand lagged by two days. In other words, the parameters include mean temperature, mean temperature squared, the dummy variable for weekday (weekday or weekend), yesterday's demand (lagged by t-1) and two days ago demand (lagged by t-2).

Functional Form:

$$ln(demand) = \beta_0 + \beta_1 mean.temp + \beta_2 mean.temp^2 + \beta_3 Weekday + \beta_4 ln(demand)_{t-1} + \beta_5 ln(demand)_{t-2} + \epsilon_t$$

Model Refinement and Model Performance:

Clearly the relationship between forecast error and model form is improved as model 3 forecasts with a higher accuracy. The results of the model 4 (AR(2) dynamic regression model) are better than that of the model 3. The forecasted values were very close to the real values and the forecasts became more accurate as model's complexity increased.

Model 5

Form:

Model 5 builds upon the foundation laid by model 4. In doing so, model 5 adds to the complexity of the model, but again counting for the non-linearity between demand and weather but does so through a weather function utilizing the concept of knots.

Specifically, this weather function fits the weather data (mean temperatures) to the natural logarithm of the daily load. First, a function is built to estimate the average temperature, which in both cases are regression with parameters beta 0, beta 1, beta 2 and beta 3 estimated from the data. There are two functions one where average temperature is less than or equal to a specified "temp 0", which represents the "knot", and one where average temperature is greater than "temp 0" (see separate R markdown). This ultimately results in building an optimization problem, where the objective lies in minimizing the mean square error of the average temperature function, compared to electricity demand.

The optimization problem gives the result that the first knot be equal to 15.006 degrees Celsius. Figure 4 below shows the model fit of the weather function to the actual observed load and daily temperature. It is clear to see that the function does a pretty good job and estimates a knot in the function correctly around 15 degrees. This is intuitive, as in practice, this is generally a good threshold to which we would expect load to increase seasonally due to light and heating as temperatures below 15 degrees are associated with the colder months of the year which correspondingly have shorter days, and thus a greater electricity demand.

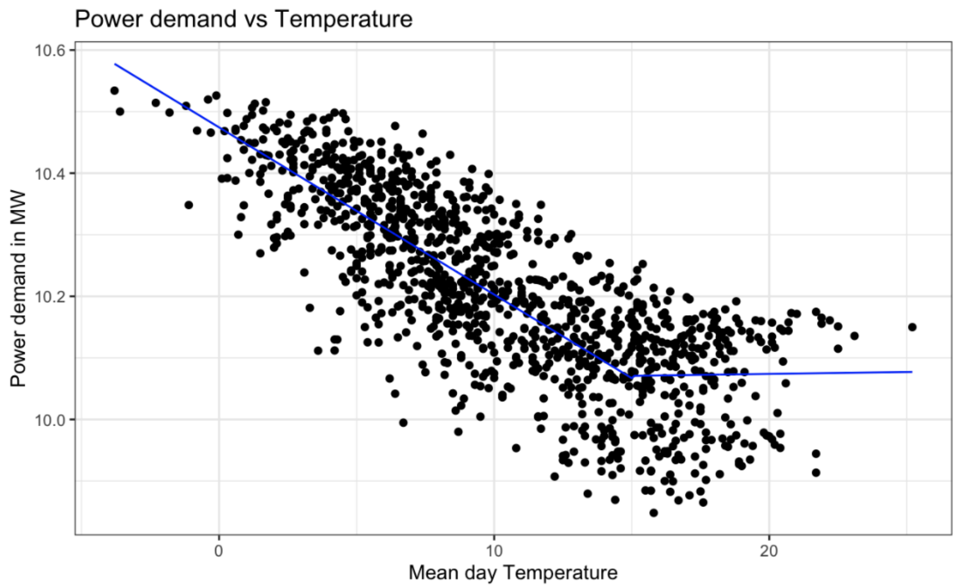


Figure 4: Graph showing the line of fit of the weather function to the log_demand and mean temperature

Assumptions:

As in model 4 we continue to use average temperature as a proxy for weather as average temperature again best captures the diurnal range in temperature for the three cities throughout a given day as an input in the weather function when estimating the optimization parameters. All other assumptions remain the same.

Parameters:

The parameters for model 5 are the same parameters for model 4, with the addition of a weather function. The weather function is an optimization function as above using one knot.

Functional Form:

$$\ln(demand) = \beta_0 + \beta_1 Weekday + \beta_2 \ln(demand)_{t-1} + \beta_3 \ln(demand)_{t-2} + f(w_t) + \epsilon_t$$

Model Refinement and Model Performance:

By including the weather function in our model specification, as described above in model form, we obtain a more sophisticated model that in fact has an improved log score and a smaller forecast error and root mean squared error in out of sample model performance compared to model 4.

Model 6

Form:

We took a different approach to developing model 6 as there were only slight improvements between models 3 and 5. Adding more parameters of larger lags or higher degrees would risk overfitting the model.

We decided to retain the core parameters of the regression model; namely, the link between temperature and its squared term, as well as the weekly seasonality index indicated by "weekday". This allows our model to continue relating these independent exogenous factors to power demand.

Furthermore, instead of explicitly modeling the autoregressive log_demand terms in previous models, we incorporated all autocorrelation using an ARIMA model. In doing so, we have a dynamic regression model that includes the effects of external factors such as temperature and the properties of the times series itself.

Assumptions:

All assumptions remain unchanged from previous models.

Parameters:

In addition to studying the autocorrelation function and the partial autocorrelation functions, we utilised the *auto.arima* function, using Bayes Information Criterion (BIC) as an optimisation criteria. This allows us to optimise the trade-off between the predictive power and complexity of the model. We found that ARIMA(2,1,2) is one of the better performing models. The numbers refer to autoregressive and moving averages terms of lags 1 and 2, after taking the first difference of the time series.

Functional Form:

$$ln(demand)' = \beta_0 + \beta_1 mean.temp' + \beta_2 mean.temp^{2'} + \beta_3 Weekday \ + \ \mathrm{n_t}$$

$$where \ \mathrm{n_t} \ = \ \emptyset_1 ln(demand)'_{\text{t-1}} \ + \ \emptyset_2 ln(demand)'_{\text{t-2}} \ + \ \gamma_1 \mathrm{u_{t-1}}' \ + \ \gamma_2 \mathrm{u_{t-2}}' \ + \ \mathrm{u_t}'$$

$$and \ ' \ represents \ the \ first - difference \ of \ the \ time \ series$$

Model Refinement and Model Performance:

The model showed a marked increase in performance in terms of much higher log score and lower standard error. It removes the risk of overfitting and presents a more accurate and parsimonious model.

Model 7

Form:

Can we do even better? We decided to go one step further and combine the optimised piecewise linear function developed in Model 5 with the ARIMA concept used in Model 6. This means replacing the quadratic function of temperature used in Model 5 with this optimised function.

Assumptions:

All assumptions remain unchanged from previous models. We also assume that the piecewise linear function with one knot developed in Model 5 remains unchanged for this model.

Parameters:

The parameters mean.temp and mean.temp.squared are replaced with the piecewise linear function. In addition, there is a need to re-run the ARIMA function as the model has changed. After studying the autocorrelation, partial autocorrelation functions, and the outputs from *auto.arima* using BIC, our analysis showed that the ARIMA(1,1,2) is one of the better performing models.

Functional Form:

$$ln(demand)' = \beta_0 + \beta_1 f(\mathrm{w_t})' + \beta_2 Weekday + \mathrm{n_t}$$

$$where\ \mathrm{n_t} = \emptyset_1 ln(demand)'_{\mathrm{t\text{-}1}} + \gamma_1 \mathrm{u_{t\text{-}1}}' + \gamma_2 \mathrm{u_{t\text{-}2}}' + \mathrm{u_t}'$$

$$and\ '\ represents\ the\ first - difference\ of\ the\ time\ series$$

Model Refinement and Model Performance:

The model showed almost negligible improvement in performance, measured in both log score and standard error. This is not surprising to us. The optimised piecewise linear function is only a slight improvement over the quadratic form used to model the relationship between temperature and power demand. Nonetheless, as predictive power is key in this exercise, we decided to go ahead with this final model.

Results

The results of the eight rounds of forecasting are shown in the table below. Submissions one to five use model form 4, which is the AR(2) dynamic regression model including the mean temperature, mean temperature squared, the dummy variable for weekday, yesterday's demand (lagged by t-1) and two days ago demand (lagged by t-2). The subsequent submissions, which include submissions six to eight use model 7.

Submission No.	Date of Forecasted Demand	Model Used	Forecast Mean (ln demand)	Forecast Standard Deviation (ln demand)
1	Tue - 3rd March	Model 4	10.32748	0.04855
2	Thu - 5th March	Model 4	10.39149	0.04855
3	Sat - 7th March	Model 4	10.20362	0.04855
4	Tue - 4th March	Model 4	10.16963	0.04855
5	Thu - 12th March	Model 4	10.31243	0.04855
6	Sat - 14th March	Model 7	10.11509	0.04116
7	Tue - 17th March	Model 7	10.26354	0.04121
8	Thu - 19th March	Model 7	10.27423	0.04121

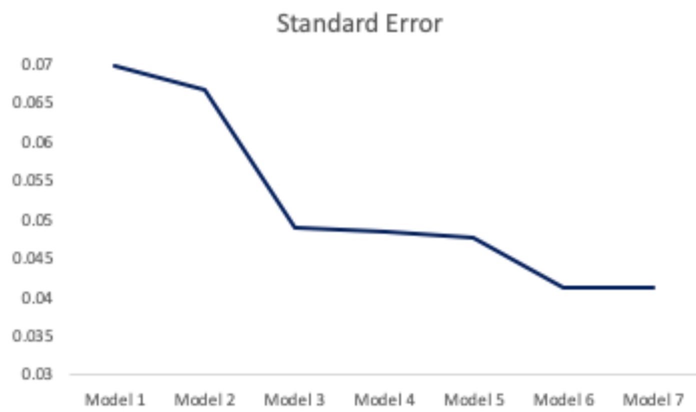
Table 1: Table showing the forecasted ln(mean demand) and corresponding standard deviation for each of the 8 forecasting rounds.

In this report, we have chosen our performance measures as the standard error of the model in the out of sample forecasting, and the log score of the model. While the forecast standard deviation is a widely understood concept, taken as the difference in the real value minus the forecasted value.

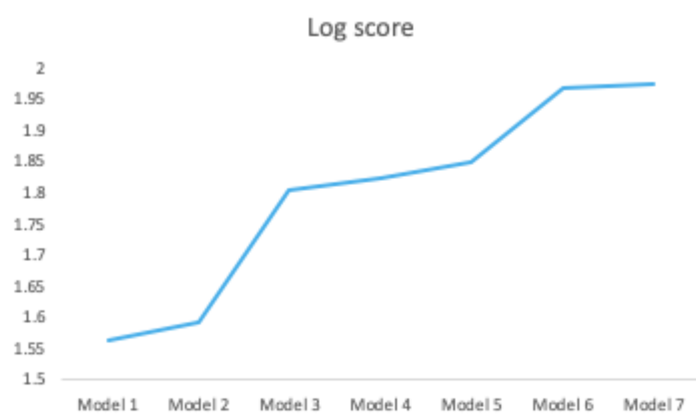
Calculating the log score is more involved and the formula used to compute the log score in R is as follows:

$$\log score = \ln\left(\frac{1}{\sqrt{2\pi}}\right) - \ln(forecast\ standard\ error) - \frac{1}{2}\left(\frac{actual\ observation\ - forecast\ mean}{forecast\ standard\ error}\right)^2$$

As we can see, the final model, model 7 provided the highest log score (see figure 6), so was therefore the best model by that measure, meanwhile the standard error was smallest for model 6 (see figure 5). This may be the case because the log score may be more sensitive to overfitting, meanwhile the standard error is more responsive to the number of parameters in the model, as more parameters results in more explanatory power.



<u>Figure 5: line graph showing the standard error for each of the six forecasting models</u>



<u>Figure 6: line graph showing the log score for each of the six forecasting models</u>

Hence, Model 7 which was the ARIMA(1,1,2) dynamic regression model including the dummy variable for weekday, and the weather optimisation function was chosen as the best model. In line with the actual observations for load, it was extremely interesting to see that much of the time our models performed well and responded well to the adjustments made to improve model fit and accuracy.

Discussion and Limitations

There are several limitations in our modelling assignment, particularly the incorporation of the non-linear relationship between weather and electricity demand into the forecasting model.

A first attempt was made to use smoothing splines to incorporate this non-linearity, by taking all the historical values for the HadCET mean temperature and calculating the quartiles. Then, the first, second and third quartiles 6.3 degrees Celsius, 9.7 degrees Celsius and 14.5 degrees Celsius respectively were used as the knots to create a weather function. Though these three temperatures are low enough for us to expect households to have their heating on, the functional form may not that accurate as the increase in temperature to 14.5 degrees and corresponding increase in electricity demand is not intuitive.

14.5 degrees is still too low for air-conditioning which could explain the increase in electricity demand. Moreover, air conditioning is not widely used in England and Wales' households. This means that the piecewise linear weather function with three knots may lead to both misleading results and overfitting. This was overcome by using the optimisation function for the knots weather function outlined in Model 5.

Another limitation is that the HadCET mean, max and min temperatures were estimated as an average of the recorded temperatures for London, Leeds and Bristol in the data set. This is an approximation, as we are not exactly sure what function HadCET uses to calculate the temperature variables. Nonetheless, even though they were not always perfectly accurate, they provided a good estimate with a relatively small error.

The forecasts predicted were relatively accurate over the course of the three weeks, and of course, could have been enhanced by having access to outside data. One of the biggest reflections of that, while doing these forecasts in real time, was the impact of the Novel Coronavirus, COVID-19 on peoples' lives, businesses and industry, not only in the UK but worldwide. For example, if we were permitted to use outside information, we could have perhaps enhanced the model by estimating the capacity utilisation of industry and offices, due to many of them having to close temporarily and the number of people working from home. Interestingly, we see a much higher electricity demand for the UK in the beginning of March before the effects of COVID-19 were so widespread in UK as compared to March 19th where the demand during the week is much lower. Being able to model these interesting and dynamic aspects of how demand for electricity changes so dramatically and is susceptible to global events would have made improvements in the accuracy of our models and thus the forecasts themselves.

Conclusion

Overall, each edit made to forecasting model improved the log score and reduced the standard error, which is reflected in our results. The best model to forecast electricity demand in the UK was model 7, an ARIMA(1,1,2) model of the natural logarithm of demand, which included a weather function to control for temperature and a dummy variable for weekday to control for the extra demand attributed to industry. We maintain that there is further scope for improvement in our models, which is largely due to accessibility to data on other exogenous factors that can be taken into consideration to improve accuracy of our predictions.

References

Office of Gas and Electricity Markets (2019) *State of the Energy Market 2019*. Available from: https://www.ofgem.gov.uk/system/files/docs/2019/11/20191030_state_of_energy_market_revised.pdf
[Accessed: March 22nd 2020].