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NETWORK ANALYTICS

ASSIGNMENT 1 28/02/2020

INTRODUCTION

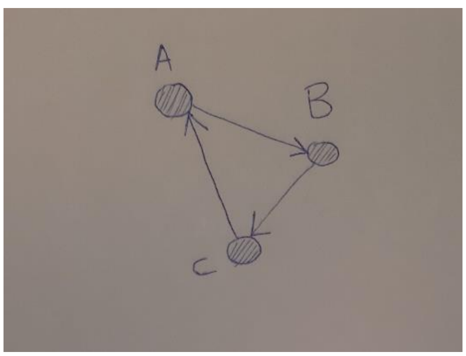
In this assignment solutions for homework 1 of "Network Analytics MSc Business Analytics 2019/20" Exercises 1 are provided. The solutions include handwritten calculations for proving graph theory in the first part, and Jupyter html file using Python programming and NetworkX library in the second part.

QUESTION 1

1.

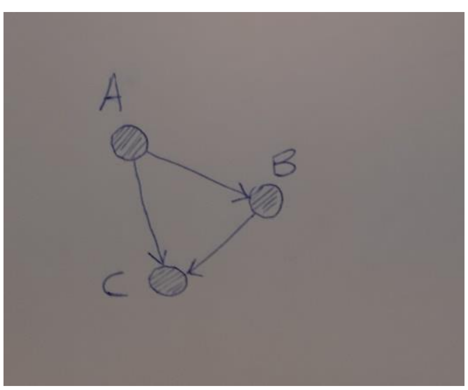
We know that a directed graph is strongly connected if there is a (directed) path from every node to every other node.

Let's think of a directed and strongly connected graph with more than one node as that of the following picture (picture 1). The indegree and the outdegree of node A equals 1, as those of node B and node C.



Picture 1: Graph 1

Let's now think of a directed graph, which is not strongly connected as that of the following picture (picture 2). At that case, the indegree of A equals 0 and the outdegree equals 2. Similarly, the indegree of node B is 1 and it's outdegree is 1, whereas the indegree and outdegree of node C are 2 and 0 respectively.



Picture 2: Graph 2

As a result, in the second graph, which is not strongly connected, we cannot reach all the nodes starting from any other node crossing edges in the direction(s) indicated by them.

In other words, if a node has a zero indegree (outdegree) then there is no path to (from) that node. Hence, the node is not strongly connected.

Therefore, in a directed strongly connected graph containing more than one node, no node can have a zero indegree or a zero outdegree.

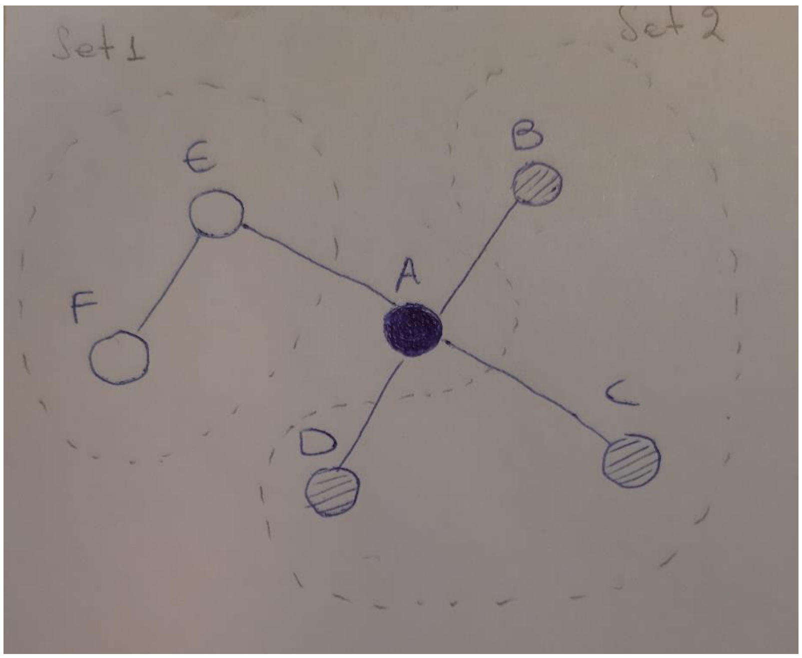
2.

We know that a graph is bipartite if it has no cycles (of odd length).

Trees have no cycles. Consequently, trees are bipartite graphs. This is true since if we remove an edge from a tree then it will be separated into 2 components.

In other words, the nodes of a bipartite graph can be divided into 2 disjoint sets “1” and “2”, such that each link connects a node of set “1” to a node of set “2”.

The above are also seen in the following picture (picture 3), where we see the two different sets. The left one (Set 1) with a length of path of 2 and the right one (Set 2) with a length of path of 1.



Picture 3: Graph 3

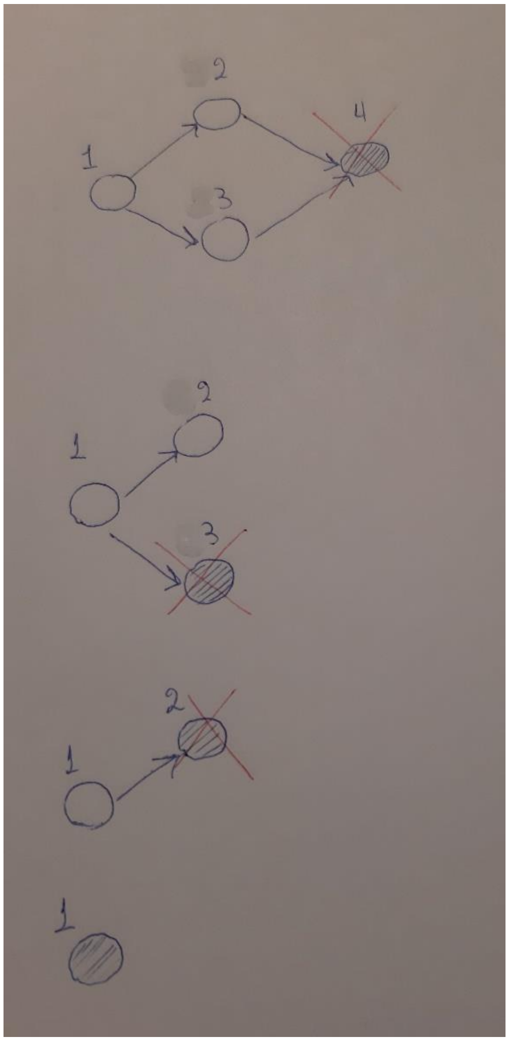
3.

We know that a DAG is a (finite) directed graph with no directed cycles.

We have to show that a DAG is a directed graph that has a topological ordering, which is a sequence of nodes such that every edge is directed from former to later in that sequence.

In other words, according to that sequence, the start node of the edge appears earlier in the sequence than the end node of that edge.

Let's imagine a directed acyclic graph as that of the following picture (picture 4). We have a DAG so our graph must have a sink node (node with outdegree equals 0).



Picture 4: Graph 4

Let that sink node be the last one, in our graph node 4. If we delete that node, then because we need to have a DAG, we need to have another sink node, let's say 3. If we delete node 3, for the same reason, we have node 2 as sink node. Deleting node 2 will result in only having node 1.

Consequently, all outgoing arcs that were deleted led to a decreasing labeling ordering in nodes.

Therefore, a DAG gas a labeling of nodes such that every arc goes from a lower-numbered node to a higher-numbered one.

Conclusion

In this assignment solutions for homework 1 of “Network Analytics MSc Business Analytics 2019/20” Assignment 1 were provided. The Jupyter html file is attached separately.