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OPTIMISATION AND DECISION MODELS

ASSIGNMENT 1 15/11/2019

INTRODUCTION

In this assignment solutions for homework 3 and 4 of "Optimisation and Decision Models MSc Business Analytics 2019/20" Exercises 1 are provided. The solutions include hand written calculations, as well as using Microsoft excel and AMPL program for solving linear problems.

QUESTION 1

(a)

The linear program is the following:

<u>Decision Variables:</u>

$$x_1 = number\ of\ units\ on\ special\ risk\ insurance$$
$$x_2 = number\ of\ units\ on\ mortgage$$

<u>Objective Function:</u>

$$maximise\ 5X_1 + 2X_2$$

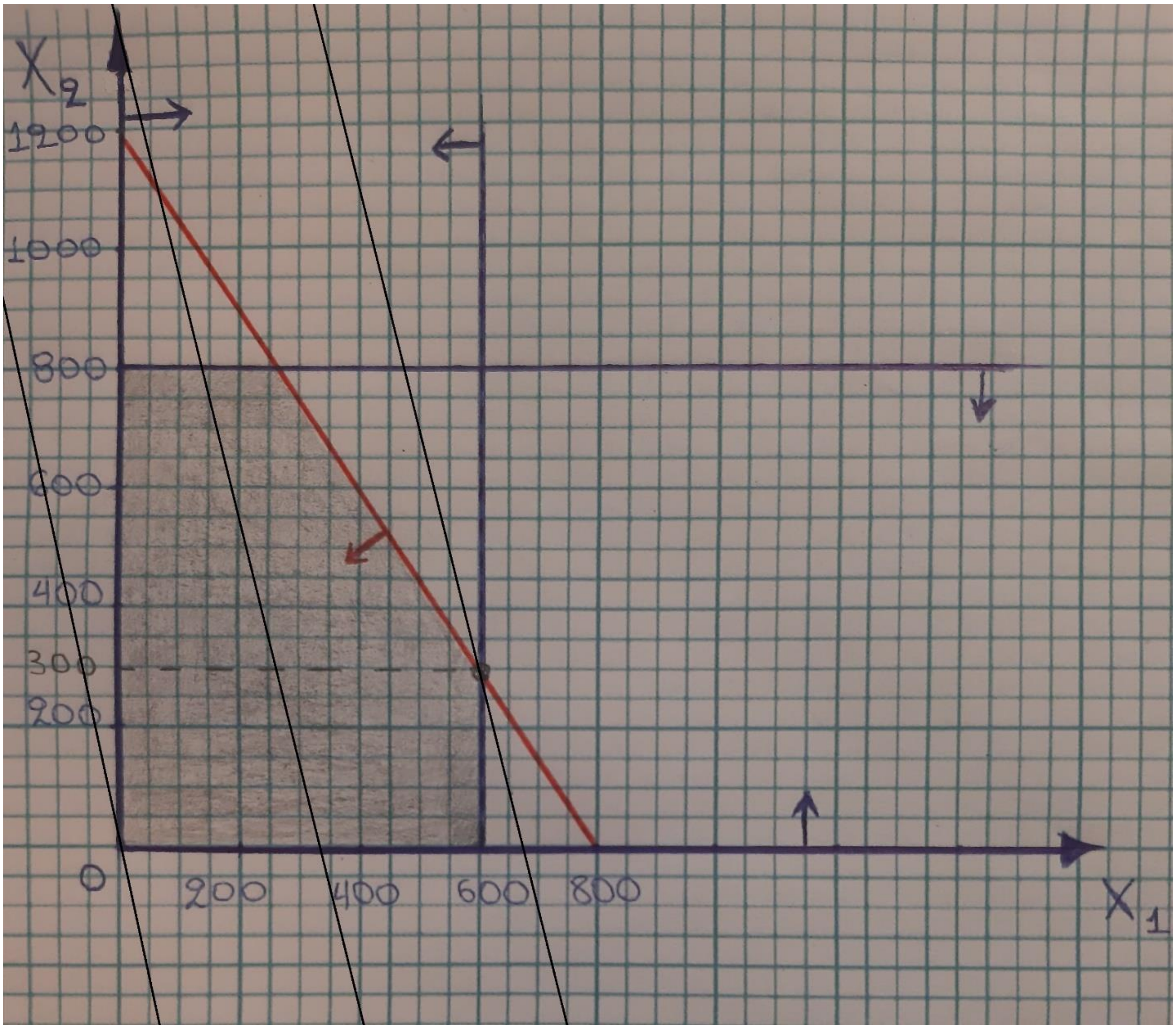
<u>Constraints:</u>

$$3x_1 + 2x_2 \leq 2,400$$
$$2x_1 \leq 1,200$$
$$x_2 \leq 800$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

(b)

We first plot x_1 and x_2 in the x and y – axis respectively. In order to find the intersects of the objective function equation line with those axes, we set $x_1 = 0$ and $x_2 = 0$ in the equation $3x_1 + 2x_2 = 2,400$, so we have that $x_2 = 1200$ and $x_1 = 800$, respectively. We then apply the two constraint equation lines for $x_1 \leq 600$ and $x_2 \leq 800$ and choose the positive region due to $x_1 \geq 0$ and $x_2 \geq 0$ and plot the graph.

The graphical solution looks as follows:



The optimal solution seems to be something like $x_2 \approx 300$ and $x_1 \approx 600$ with an objective value of approximately 5 * 600 + 2 * 300 = 3,600. In order to find it we move the multiple parallel random isoquants in the direction of improving objective values until they just touch the feasible region starting from point O (0,0) and finally find the point of the line that crosses the objective value line. That line, as well as the constraint for x_1 and the objective function equation line are binding at the optimal solution.

(c)

We know that for the optimal solution we have $x_1 = 600$, as a result we need to solve the two equations

$$3x_1 + 2x_2 = 2,400$$

$$x_1 = 600$$

in the unknown x_2. Using standard linear algebra, we see that the solution is $x_1 = 600$ $x_2 = 300$ with an objective value of 5*600 + 2*300 = 3,600.

(d)

A screenshot of the solution via Excel is the following:

	A	B	C	D
1		**X1**	**X2**	
2	**Price (£/unit)**	5	2	
3	**Underwriting**	3	2	
4	**Administration**	0	1	
5	**Claims**	2	0	
6				
7	**Limits**			
8	**Underwriting**	2,400		
9	**Administration**	800		
10	**Claims**	1,200		
11				
12		**Decision Variables**		
13		**X1**	**X2**	
14	**Purchase Amount**	600	300	
15				
16	**Objective Function Value**			
17	**Overall profit**	3,600		
18				
19		**Constraints**		
20		**Actual Value**	**Limit**	
21	**Underwriting**	2400	2,400	
22	**Administration**	300	800	
23	**Claims**	1200	1,200	
24	**Nonnegativity X1**	600	0	
25	**Nonnegativity X2**	300	0	
26				

Please find attached the solution in the Excel Spreadsheet (“Konstantinos_Paganopoulos_Assingment_1.xlsx”) for more details.

QUESTION 2

(a)

The linear program is the following:

Decision Variables:

PW_1	=	number of wrenches to produce in quarter 1		
PW_2	=	number of wrenches to produce in quarter 2		
PW_3	=	number of wrenches to produce in quarter 3		
PW_4	=	number of wrenches to produce in quarter 4		
PP_1	=	number of pliers to produce in quarter 1		
PP_2	=	number of pliers to produce in quarter 2		
PP_3	=	number of pliers to produce in quarter 3		
PP_4	=	number of pliers to produce in quarter 4		
IW_1	=	number of wrenches on inventory at the beginning of quarter 1	=	0
IW_2	=	number of wrenches on inventory at the beginning of quarter 2		
IW_3	=	number of wrenches on inventory at the beginning of quarter 3		
IW_4	=	number of wrenches on inventory at the beginning of quarter 4		
IW_5	=	number of wrenches on inventory at the beginning of quarter 5		
IP_1	=	number of pliers on inventory at the beginning of quarter 1	=	0
IP_2	=	number of pliers on inventory at the beginning of quarter 2		
IP_3	=	number of pliers on inventory at the beginning of quarter 3		
IP_4	=	number of pliers on inventory at the beginning of quarter 4		
IP_5	=	number of pliers on inventory at the beginning of quarter 5		
SW_1	=	number of wrenches to sell in quarter 1	=	$PW_1 - IW_2$
SW_2	=	number of wrenches to sell in quarter 2	=	$PW_2 + IW_2 - IW_3$
SW_3	=	number of wrenches to sell in quarter 3	=	$PW_3 + IW_3 - IW_4$
SW_4	=	number of wrenches to sell in quarter 4	=	$PW_4 + IW_4 - IW_5$
SP_1	=	number of pliers to sell in quarter 1	=	$PP_1 - IP_2$
SP_2	=	number of pliers to sell in quarter 2	=	$PP_2 + IP_2 - IP_3$
SP_3	=	number of pliers to sell in quarter 3	=	$PP_3 + IP_3 - IP_4$
SP_4	=	number of pliers to sell in quarter 4	=	$PP_4 + IP_4 - IP_5$

Objective Function:

$$maximise\, 0.13\,(SW_1+SW_2+SW_3+SW_4)+0.10\,(SP_1+SP_2+SP_3+SP4)$$

Constraints:

$$1.5PW_1 + PP_1 \leq 27,000$$
$$1.5PW_2 + PP_2 \leq 27,000$$
$$1.5PW_3 + PP_3 \leq 27,000$$
$$1.5PW_4 + PP_4 \leq 27,000$$
$$PW_1 + PP_1 \leq 21,000$$
$$PW_2 + PP_2 \leq 21,000$$
$$PW_3 + PP_3 \leq 21,000$$
$$PW_4 + PP_4 \leq 21,000$$
$$0.3PW_1 + 0.5PP_1 \leq 9,000$$
$$0.3PW_2 + 0.5PP_2 \leq 9,000$$
$$0.3PW_3 + 0.5PP_3 \leq 9,000$$
$$0.3PW_4 + 0.5PP_4 \leq 9,000$$
$$SW_1 \leq 6,000$$
$$SW_2 \leq 10,000$$
$$SW_3 \leq 12,000$$
$$SW_4 \leq 15,000$$
$$SP_1 \leq 5,000$$
$$SP_2 \leq 8,000$$
$$SP_3 \leq 10,000$$
$$SP_4 \leq 16,000$$
$$PW_1, PW_2, PW_3, PW_4, PP_1, PP_2, PP_3, PP_4 \geq 0$$
$$IW_1, IW_2, IW_3, IW_4, IW_5, IP_1, IP_2, IP_3, IP_4, IP_5 \geq 0$$

(b)

The AMPL model could look like this:

```
# define the variables
var PW1 >= 0;
var PW2 >= 0;
var PW3 >= 0;
var PW4 >= 0;

var PP1 >= 0;
var PP2 >= 0;
var PP3 >= 0;
var PP4 >= 0;

param IW1 = 0;
var IW2 >= 0;
var IW3 >= 0;
var IW4 >= 0;
var IW5 >= 0;

param IP1 = 0;
var IP2 >= 0;
var IP3 >= 0;
var IP4 >= 0;
var IP5 >= 0;

var SW1 = PW1 - IW2;
var SW2 = PW2 + IW2 - IW3;
var SW3 = PW3 + IW3 - IW4;
var SW4 = PW4 + IW4 - IW5;

var SP1 = PP1 - IP2;
var SP2 = PP2 + IP2 - IP3;
var SP3 = PP3 + IP3 - IP4;
var SP4 = PP4 + IP4 - IP5;

# define the objective function
maximize earnings: 0.13 * (SW1 + SW2 + SW3 + SW4) + 0.10 * (SP1 + SP2 + SP3 + SP4);

# define the constraints
subject to steel1: 1.5 * PW1 + 1.0 * PP1 <= 27000;
subject to steel2: 1.5 * PW2 + 1.0 * PP2 <= 27000;
subject to steel3: 1.5 * PW3 + 1.0 * PP3 <= 27000;
subject to steel4: 1.5 * PW4 + 1.0 * PP4 <= 27000;

subject to molding1: 1.0 * PW1 + 1.0 * PP1 <= 21000;
subject to molding2: 1.0 * PW2 + 1.0 * PP2 <= 21000;
subject to molding3: 1.0 * PW3 + 1.0 * PP3 <= 21000;
subject to molding4: 1.0 * PW4 + 1.0 * PP4 <= 21000;

subject to assembly1: 0.3 * PW1 + 0.5 * PP1 <= 9000;
subject to assembly2: 0.3 * PW2 + 0.5 * PP2 <= 9000;
subject to assembly3: 0.3 * PW3 + 0.5 * PP3 <= 9000;
subject to assembly4: 0.3 * PW4 + 0.5 * PP4 <= 9000;

subject to demandW1: SW1 <= 6000;
subject to demandW2: SW2 <= 10000;
subject to demandW3: SW3 <= 12000;
subject to demandW4: SW4 <= 15000;

subject to demandP1: SP1 <= 5000;
subject to demandP2: SP2 <= 8000;
subject to demandP3: SP3 <= 10000;
subject to demandP4: SP4 <= 16000;
```

Please find attached the solution in the AMPL file ("Konstantinos_Paganopoulos_Assignment_1.mod") for more details.

The optimal solution is to: 1) produce 7,500 of wrenches, 13,500 of pliers, sell 6,000 of wrenches and 5,000 of pliers (with a zero inventory of wrenches and pliers) in the 1st quarter, 2) produce 8,500 of wrenches and 12,500 of pliers, sell 10,000 of wrenches and 8,000 of pliers (with a 1,500 inventory of wrenches and 8,500 of pliers) in the 2nd quarter, 3) produce 16,000 of wrenches, 3,000 of pliers, sell 12,000 of wrenches and 10,000 of pliers (with a zero inventory of wrenches and 13,000 of pliers) in the 3rd quarter, 4) produce 11,000 of wrenches, 10,000 of pliers, sell 15,000 of wrenches and 16,000 of pliers (with a 4,000 inventory of wrenches and 6,000 of pliers) in the 4th quarter, resulting in having no excess production (zero inventory) of wrenches and pliers at the end of this quarter.

```
ampl: model assignment1.mod;
ampl: solve;
MINOS 5.51: optimal solution found.
16 iterations, objective 9490
ampl: display PW1,PW2,PW3,PW4,PP1,PP2,PP3,PP4;
PW1 = 7500
PW2 = 8500
PW3 = 16000
PW4 = 11000
PP1 = 13500
PP2 = 12500
PP3 = 3000
PP4 = 10000

ampl: display IW1,IW2,IW3,IW4,IW5,IP1,IP2,IP3,IP4,IP5;
IW1 = 0
IW2 = 1500
IW3 = 0
IW4 = 4000
IW5 = 0
IP1 = 0
IP2 = 8500
IP3 = 13000
IP4 = 6000
IP5 = 0

ampl: display SW1,SW2,SW3,SW4,SP1,SP2,SP3,SP4;
SW1 = 6000
SW2 = 10000
SW3 = 12000
SW4 = 15000
SP1 = 5000
SP2 = 8000
SP3 = 10000
SP4 = 16000
```

Conclusion

In the provided coursework assignment, solutions to 2 linear problems through various ways are provided. The Microsoft Excel and AMPL files are attached separately.