# Exercises 2

(1) **In-Class:** Johnson Electric has three different plants, each of which produces electric motors for four different clients. The unit production costs and capacities for each plant are as follows.

| Plant | Production costs (£/motor) | Monthly capacity (# of motors) |
|---|---|---|
| **Aberdeen** | 17 | 800 |
| **Birmingham** | 20 | 600 |
| **Cardiff** | 24 | 700 |

For the next months, the clients have submitted the following orders:

| | Customer | | | |
|---|---|---|---|---|
| | **Onyx Inc.** | **Treble Co.** | **Hilton Appliances** | **Dean Electric** |
| **Demand** | 300 | 500 | 400 | 600 |

The unit transportation costs (£/motor) to each of the clients is shown in the following table:

| | Customer | | | |
|---|---|---|---|---|
| **Plant** | **Onyx Inc.** | **Treble Co.** | **Hilton Appliances** | **Dean Electric** |
| **Aberdeen** | 3 | 2 | 5 | 7 |
| **Birmingham** | 6 | 4 | 8 | 3 |
| **Cardiff** | 9 | 1 | 5 | 4 |

Johnson Electric must decide how many motors to produce in each plant, as well as how much of each customer's demand to supply from each plant, so as to minimise the overall costs (that is, the sum of production and transportation costs).

(a) Construct a linear program that determines the production and distribution plans that minimise the overall costs.

***Hint:*** *Introduce one decision variable for each plant-customer pair, such as AO for Aberdeen-Onyx or CT for Cardiff-Treble. The decision variable AO should determine how many motors Johnson Electric produces in Aberdeen that are to be shipped to Onyx Inc. You will then require constraints for the capacity of each plant as well as for the demand of each customer.*

(b) Solve the linear program using AMPL. What managerial actions would you recommend based on the optimal solution and the shadow price information?

(2) **In-Class:** Dualising linear programs.

(a) Using the "indirect method", find the dual to the linear program:

| | |
|---|---|
| minimise | $3\ x_1 + 5\ x_2 - x_3$ |
| subject to | $x_1 + x_3 = 4$ |
| | $x_2 - 2\ x_3 \leq 2$ |
| | $x_1, x_2 \geq 0$, $x_3$ unrestricted |

(b) Using the "direct method", find the dual to the linear program:

| | |
|---|---|
| maximise | $x_1 - x_3$ |
| subject to | $x_1 + x_2 = 4$ |
| | $x_3 \leq 2$ |
| | $x_1, x_2 \leq 0$, $x_3$ unrestricted |

(3) **Homework:** In class we have defined that the dual of the linear program

$$\begin{aligned} &\text{maximise } \mathbf{c}^T\mathbf{x} \\ &\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ &\qquad\qquad\quad \mathbf{x} \geq \mathbf{0} \end{aligned}$$

is

$$\begin{aligned} &\text{minimise } \mathbf{b}^T\mathbf{y} \\ &\text{subject to } \mathbf{A}^T\mathbf{y} \geq \mathbf{c} \\ &\qquad\qquad\quad \mathbf{y} \geq \mathbf{0} \end{aligned}$$

Show that this definition is symmetric, that is, the dual of the linear program

$$\begin{aligned} &\text{minimise } \mathbf{b}^T\mathbf{y} \\ &\text{subject to } \mathbf{A}^T\mathbf{y} \geq \mathbf{c} \\ &\qquad\qquad\quad \mathbf{y} \geq \mathbf{0} \end{aligned}$$

is

$$\begin{aligned} &\text{maximise } \mathbf{c}^T\mathbf{x} \\ &\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ &\qquad\qquad\quad \mathbf{x} \geq \mathbf{0} \end{aligned}$$

*Hint:* Bring the dual problem into the form of the primal problem (that is, change the objective into a maximisation and change the constraints), use the above definition to determine the dual of the resulting problem, and finally show that this dual is actually equivalent to the primal problem.

(4) **Homework:** The Graduate Admissions dataset from Kaggle (https://www.kaggle.com/mohansacharya/graduate-admissions) has the independent variables

| | |
|---|---|
| **GRE:** | GRE Scores (out of 340) |
| **TOEFL:** | TOEFL Scores (out of 120) |
| **Univ:** | University Rating (out of 5) |
| **SOP:** | Statement of Purpose Strength (out of 5) |
| **LOR:** | Letter of Recommendation Strength (out of 5) |
| **CGPA:** | Undergraduate GPA (out of 10) |
| **Res:** | Research Experience (either 0 or 1) |

as well as the dependent variable

| | |
|---|---|
| **Chance:** | Chance of Admission (ranging from 0 to 1) |

(a) Formulate the 1-norm regression problem for this data set (with Chance as the dependent variable and GRE, TOEFL, …, Res as independent variables) as a linear program. You can "index" the data by writing "GRE", "$\text{TOEFL}_i$" etc. in your model.

(Please also include an intercept in your regression problem!)

(b) Construct an AMPL file that solves the 1-norm regression problem. To this end, you can use the Graduate_Admissions.dat file in the exercises folder (which only involves the first 100 records, to comply with the size limitations of the AMPL demo version) and import the data as follows:

```
set NUM ordered;                    # candidate number

param GRE {NUM};                    # GRE Score
param TOEFL {NUM};                  # TOEFL Score
param Univ {NUM};                   # University Rating
param SOP {NUM};                    # Statement of Purpose Strength
param LOR {NUM};                    # Letter of Recommend. Strength
param CGPA {NUM};                   # Undergraduate GPA
param Res {NUM};                    # Research Experience
param Chance {NUM};                 # Chance of Admission

data Graduate_Admissions.dat;
```

In your model, you can access the i-th element of GRE via GRE[i]. Also, you can create variable vectors that are indexed by NUM (the candidate number) via

```
var x {NUM};
```

Similarly, the i-th component of x can be access via x[i]. You can sum over all elements of x via

```
sum {i in NUM} x[i]
```

Finally, you can create a set of constraints, one for each candidate NUM, via

```
subject to constr {i in NUM}: ...
```

You may want to consult the AMPL book (available online for free) for further information on sets and indexing.

Solve the problem. Make sure you use CPLEX as a solver (“option solver ‘cplex’;”), as other solvers have stricter size limitations in the demo version. What is your optimal objective value, and what are the regression coefficients?

(c) Formulate the infinity-norm regression problem for this data set (again with Chance as the dependent variable and GRE, TOEFL, …, Res as independent variables — plus an intercept) as a linear program. You can “index” your data as in part (a). Justify why your model gives the correct solution (similar to our argumentation for the 1-norm problem in the lecture)!

(d) Construct an AMPL file that solves the infinity-norm regression problem.

Solve the problem. Make sure you use CPLEX as a solver (“option solver ‘cplex’;”). What is your optimal objective value, and what are the regression coefficients?