Imperial College London
BUSINESS SCHOOL

MSc Business Analytics 2019/20
Optimisation and Decision Models
Wolfram Wiesemann

Exercises 3

(1) **In-Class:** WCBN-TV, a TV broadcasting company, wishes to plan the schedule of TV shows for next Wednesday evening. Of the 9 possible 30-minute shows listed in the table below, exactly 5 shows must be scheduled for the time period from 8:00pm to 10:30pm. WCBN-TV wishes to determine a revenue-maximising schedule of TV shows; however, they must be mindful of the following considerations:

1. There must be at least as many shows scheduled that are categorised as public interest as there are shows scheduled that are categorised as containing violence.
2. If "Focus on Science — The Fusion Issue" is scheduled, then either "Jake" or "L. A. Law" (or both) must be scheduled as well.
3. WCBN-TV cannot schedule both "Focus on Science" and "Urban Action for Education", as both of these shows are considered a bit on the dry side.
4. If WCBN-TV schedules two or more shows in the comedy category, then they must schedule at least one show in the drama category.
5. If WCBN-TV schedules more than three shows in the "contains violence" category, they will lose an estimated £4 million in advertising revenues from family-oriented sponsors.

TV Show	Advertising revenue (£m)	Public interest	Contains violence	Comedy	Drama
Cheers	6			yes	yes
Dynasty	10		yes		yes
L. A. Law	9	yes	yes		yes
Jake	4		yes		yes
Bob Newhart	5			yes	
News Special — the Middle East	2	yes	yes		
Focus on Science: The Fusion Issue	6	yes			yes
Magnificent Beaches	7			yes	
Urban Action for Education	8	yes			

(a) Construct a binary optimisation model that solves WCBN-TV's scheduling problem.

(b) Use Excel to determine the optimal solution. What is the revenue-maximising TV schedule?

(2) **In-Class:** The Belmont Bank is considering placing ATM machines in the town centres of some of the following six communities: Arlington, Belmont, Cambridge, Lexington, Somerville and Winchester. The bank would like to purchase the minimum number of ATM machines needed to ensure that at least one ATM machine within a ten-minute drive from the centre of each of these six communities. The times required to drive between the communities are as follows:

Town	Arlington	Belmont	Cambridge	Lexington	Somerville	Winchester
Arlington	0	5	10	15	20	15
Belmont	5	0	8	10	15	12
Cambridge	10	8	0	15	20	10
Lexington	15	10	15	0	10	12
Somerville	20	15	20	10	0	12
Winchester	15	12	10	12	12	0

(a) Construct a binary optimisation model of the problem faced by Belmont Bank.

(b) Solve your model using AMPL. What is the optimal number of ATM machines that Belmont Bank needs to purchase? What is the optimal placement of these ATM machines?

(3) **Homework:** The R&D Division of the Progressive Company has been developing four possible new product lines. The board of the company must now make a decision as to which of these four products actually will be produced and at what levels. A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. The company has already undertaken a market research, which has led to estimates of the maximum demand for each product (third row). Assume that the company wants to find the product mix that maximises the total profit (i.e., the total net revenues [using the unit revenues from the second row] minus the start-up costs).

	Product line			
	1	**2**	**3**	**4**
Start-up costs	£50,000	£40,000	£70,000	£60,000
Net revenues/unit sold	£70	£60	£90	£80
Maximum demand (units)	10,000	15,000	12,500	9,000

Moreover, the production plan is restricted by the following considerations:

1. At most two of the product lines should be produced.
2. Product 3 can only be produced if either product 1 or product 2 (or both) is produced.
3. Either the cumulative production quantities of products 1 and 2 are less than or equal to 20,000, or the cumulative production quantities of products 3 and 4 are less than or equal to 20,000 (or both).

(a) Formulate an optimisation model that determines the profit-maximising production mix.

(b) Solve your model using Excel. What is the profit-maximising production mix?

(4) **Homework:** In this exercise we consider once more the Graduation Admissions dataset from Assignment 2, Question 4. Instead of running a "normal" 1-norm regression, however, we run a *sparse* 1-norm regression where we wish to control how many of the slopes corresponding to the independent variables GRE, TOEFL, ..., Res are allowed to be nonzero. Sparse regression is often used in practice to obtain interpretable models (which independent variables are most indicative for the value of the dependent variable?) or to avoid overfitting to the available data (more on that in the Machine Learning course).

(a) Construct a mixed binary linear optimisation model that solves the 1-norm regression problem in such a way that at most K of the slopes corresponding to the independent variables are allowed to be nonzero. (Here, K is a parameter to be chosen by the decision maker.) Note that the intercept of your regression line should always be present and not count towards the quota K.

(b) Solve your problem on the reduced Kaggle dataset (provided again as data file in addition to this assignment) in AMPL. As before, make sure you are using CPLEX as a solver. Solve the problem for $K = 0, 1, \ldots$ and plot a graph with K on the x-axis and the estimation error (i.e., your optimal objective value from the regression problem) on the y-axis.