Imperial College London
BUSINESS SCHOOL

Exercises 4

(1) **In-Class:** The coach of the US national men's basketball team has to select a "dream team" of 12 players from the following list of 20 candidate players:

#	Avg. rebounds/game	Avg. assists/game	Height	Avg. points/game	Defensive ability
1	1	7	5'11"	10	10
2	2	14	6'0"	14	9
3	3	12	6'4"	19	8
4	4	4	6'0"	18	6
5	5	9	6'3"	20	8
6	7	6	6'5"	21	10
7	7	8	6'8"	23	10
8	4	2	6'5"	13	5
9	8	2	6'10"	17	8
10	5	5	6'4"	25	8
11	10	6	6'10"	20	9
12	8	8	6'9"	30	10
13	10	2	7'3"	24	9
14	9	5	6'10"	15	7
15	6	3	6'10"	17	6
16	16	2	6'9"	3	6
17	11	1	7'4"	27	9
18	12	5	7'2"	26	10
19	11	1	7'3"	21	9
20	9	1	7'0"	14	8

Players 1-5 are playmakers, players 4-11 are shooting guards, players 9-16 are forwards, and players 16-20 are centres. The coach would like to select the team with the highest average number of points/game, subject to the following restrictions:

1. The team should consist of at least 3 playmakers, 4 shooting guards, 4 forwards and 3 centres.
2. Players #4, #8, #15 and #20 come from the National Collegiate Athletic Association (NCAA), and at least two of them should be selected.
3. The average number of rebounds/game among the selected players should at least be 7, the average number of assists/game at least 6, the average number of points/game at least 18, and the average defensive ability at least 8.5.
4. The average height among the selected players should at least be 6'7" (note: 1 foot = 12 inches).
5. Player 5 would turn down the invitation if player 9 is selected.
6. Players 2 and 19 would only accept if both of them are selected.
7. At most three of the players #1, #7, #12, #16 should be selected as they all belong to the same franchise team.

(a) Construct a binary optimisation model that finds the optimal assignment.

(b) Use Excel to determine the optimal solution.

(2) **In-Class:** Consider the following multivariate linear regression problem: We are given m observations $(x_{11}, \ldots, x_{1n}; y_1), \ldots, (x_{m1}, \ldots, x_{mn}; y_m)$ where $x_{i1}, \ldots, x_{in}$ are the values of the n explanatory variables and y_i is the value of the response variable of the i-th observation, respectively, where $i = 1, \ldots, m$. We wish to find the slopes $b_1, \ldots, b_n$ as well as the intercept b_0 that minimise the sum of squared errors between the responses y_i and the regression line $b_0 + b_1 x_{i1} + \ldots + b_n x_{in}$, where the sum is taken over $i = 1, \ldots, m$.

(a) Write down the problem as a nonlinear optimisation problem.

(b) Is your problem from part (1) convex? Justify your response!

(c) How can you incorporate the following additional constraints:

- Each of the slopes $b_1, \ldots, b_n$ should be between -10 and 10.
- The slope b_1 needs to be at least twice as big as the slope b_2.
- The slopes b_3 and b_4 must be the same.
- The slope b_5 must either be ≤ 1 or ≥ 2.
- The sum of all absolute values of the slopes $|b_1|, \ldots, |b_n|$ must be 10 or less.
- At most 5 of the slopes $b_1, \ldots, b_n$ should be nonzero.

For each constraint, argue whether your optimisation problem remains convex!

(3) **Homework:** The table below lists 10 prominent components of the FTSE 100 share index:

Company name	Ticker	Sector
HSBC Holdings	HSBA.L	Banking
Royal Dutch Shell	RDSA.L	Oil & Gas
Rio Tinto	RIO.L	Oil & Gas
BP	BP.L	Oil & Gas
GlaxoSmithKline	GSK.L	Pharmaceuticals
Diageo	DGE.L	Beverages
AstraZeneca	AZN.L	Pharmaceuticals
Vodafone Group	VOD.L	Telecomms
Glencore	GLEN.L	Mining
Unilever	ULVR.L	Food

(a) Download the last two years of weekly price data for each of those stocks (you can use Google Finance or Yahoo Finance, for example). Make sure you use the "Adjusted Close" data (which are the daily closing prices, adjusted for dividends and stock splits). Present your return data in a table (e.g. in Excel).

(b) Using the data from part (a), calculate the mean weekly return for each stock, as well as the covariances of the weekly returns between all pairs of stocks. Present your results in a table. (You can use the `sample' or the `population' statistics.)

(c) By solving a sequence of Markowitz problems, produce a graph that shows the efficient frontier. Present the optimal portfolio composition for three selected portfolios (low return/low risk, medium return/medium risk and high return/high risk). Also plot each of the aforementioned 10 individual assets in the graph. Provide the Markowitz problem that you used for computing the portfolios; the problem should be implemented in AMPL.

(d) How would your optimisation problem change if you are only allowed to invest in at most one company per sector? You do not have to solve the new optimisation problem, but do provide a formulation for the problem!