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OPTIMISATION AND DECISION MODELS

ASSIGNMENT 4 06/12/2019

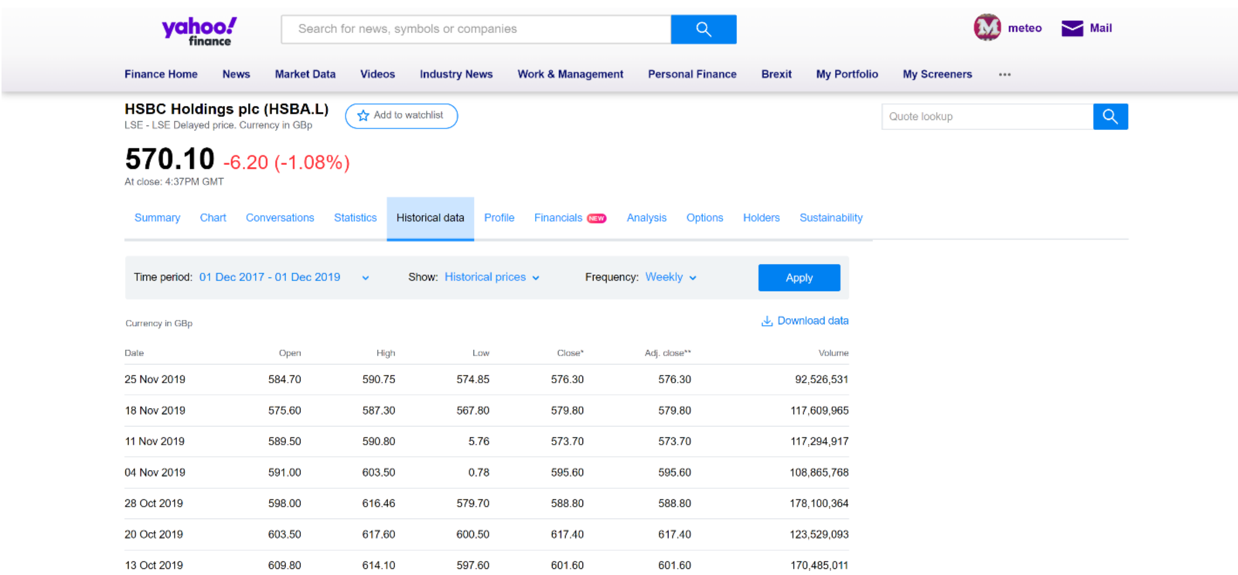
INTRODUCTION

In this assignment a solution for homework 3 of "Optimisation and Decision Models MSc Business Analytics 2019/20" Exercises 4 is provided. The solution includes hand written calculations, as well as using Microsoft Excel and AMPL program for solving a nonlinear (portfolio) optimisation problem.

QUESTION 3

(a)

We first download the last two years of weekly price data for each of the ten given stocks from Yahoo Finance as follows:



Date	Open	High	Low	Close*	Adj. close**	Volume
25 Nov 2019	584.70	590.75	574.85	576.30	576.30	92,526,531
18 Nov 2019	575.60	587.30	567.80	579.80	579.80	117,609,965
11 Nov 2019	589.50	590.80	5.76	573.70	573.70	117,294,917
04 Nov 2019	591.00	603.50	0.78	595.60	595.60	108,865,768
28 Oct 2019	598.00	616.46	579.70	588.80	588.80	178,100,364
20 Oct 2019	603.50	617.60	600.50	617.40	617.40	123,529,093
13 Oct 2019	609.80	614.10	597.60	601.60	601.60	170,485,011

Screenshot 1: Download the last two years of weekly price data for "HSBA.L" stock.

We use the "Adjusted Close" data (which are the daily closing prices, adjusted for dividends and stock splits).

Note that for "Royal Dutch Shell" company, we had some missing values at the first weeks. As a result, we replaced those data with another pound-dominated stock of the same company ("RDSB.L" instead of "RDSA.L" stock).

The above data are collected and presented in an Excel table as below (Screenshot 2).

	A	B	C	D	E	F	G	H	I	J	K
1	Date	HSBC Holdings	Royal Dutch Shell	Rio Tinto	BP	GlaxoSmithKline	Diageo	AstraZeneca	Vodafone Group	Glencore	Unilever
2	27-11-17	649.580933	2116.668945	3109.112305	435.044342	1162.215698	2449.762939	4429.964355	194.997162	303.792938	3860.651123
3	04-12-17	652.607117	2108.711914	3079.370605	435.70755	1159.963501	2517.095947	4445.762695	199.697983	309.846405	3923.040527
4	11-12-17	668.89563	2168.392334	3217.868652	446.716309	1166.720337	2535.719971	4535.904297	200.220306	325.316284	3936.644043
5	18-12-17	677.885437	2185.191162	3326.182129	457.327179	1173.027222	2568.669922	4592.591309	205.356384	336.660889	3872.847656
6	25-12-17	682.602905	2217.904541	3499.748779	462.190491	1191.496216	2602.574951	4758.936035	204.572906	349.754242	3870.501709
7	01-01-18	679.57666	2266.091309	3515.2854	468.291656	1226.182739	2551.956055	4836.067871	206.139862	348.498718	3839.541992
8	08-01-18	704.677063	2301.45752	3702.169434	472.889709	1218.975342	2522.348877	4769.158203	199.654465	365.493164	3737.748047
9	15-01-18	701.739685	2263.438965	3565.890625	450.872253	1223.479858	2503.724854	4686.450684	196.564102	365.13446	3854.08374
10	22-01-18	685.89624	2258.133789	3500.192627	456.531342	1220.776978	2444.032471	4734.774902	196.389999	360.426208	3747.129883
11	29-01-18	667.738586	2142.751709	3463.79248	432.789642	1180.955444	2400.576904	4679.945801	191.079788	343.252411	3789.348633
12	05-02-18	651.361084	1992.444946	3363.469727	415.635345	1163.116699	2327.991455	4432.752441	175.671539	323.746857	3549.172119
13	12-02-18	677.173401	2023.390503	3641.79834	418.597565	1187.982788	2392.45874	4404.40918	178.152527	347.960663	3566.52832
14	19-02-18	643.88446	2050.269775	3552.57373	427.437653	1167.621338	2366.194092	4541.648926	177.456131	355.359283	3557.66333
15	26-02-18	640.152344	2045.335083	3230.298096	416.664154	1182.752686	2297.421631	4526.357422	173.251511	326.885712	3512.251953
16	05-03-18	640.878784	2052.512939	3421.214111	429.322998	1224.928589	2360.126709	4573.666504	180.198257	331.638763	3684.435059
17	12-03-18	642.240784	2017.519165	3429.017822	425.327789	1213.376221	2349.997314	4651.558594	175.410385	345.718658	3582.733643
18	19-03-18	605.284485	2017.519165	3274.321777	415.18277	1206.224365	2275.71582	4576.055664	168.550659	321.504852	3533.065186
19	26-03-18	604.194824	2043.091919	3315.17627	430.265656	1278.106445	2326.844727	4678.797363	169.072998	317.289856	3742.144775
20	02-04-18	603.650085	2120.257324	3268.354004	447.458313	1304.328857	2404.502686	4769.591797	177.5867	314.913361	3797.489746
21	09-04-18	622.173645	2173.196289	3474.921631	452.755249	1301.945068	2404.020264	4788.228027	180.067703	310.115417	3717.547607
22	16-04-18	636.701843	2286.252686	3660.373291	465.234558	1311.113525	2377.973633	4755.733887	182.80983	341.369141	3663.62207
23	23-04-18	653.409424	2329.322266	3652.110352	482.472198	1343.387085	2467.690186	4857.99707	183.28862	330.921326	3832.493896
24	30-04-18	648.687744	2382.709961	3672.308105	501.056335	1347.054565	2568.983154	4974.59668	183.245102	329.269165	3766.269775
25	07-05-18	668.119263	2432.05957	3877.957275	506.802185	1344.487305	2585.865479	5045.320313	182.896912	353.141174	3949.205811
26	14-05-18	665.486023	2467.341064	3984.454102	533.314087	1382.236572	2603.712158	5009.00293	168.167633	347.88205	3947.774902
27	21-05-18	672.265503	2324.714844	3914.221191	504.933777	1393.196533	2664.970459	5235.512207	169.786804	345.275299	3990.229492
28	28-05-18	664.92627	2376.950684	3940.386475	524.581665	1414.373169	2620.112305	5181.035156	169.682358	348.750946	3955.407471

Screenshot 2: First 27 weeks for all stocks' Adjusted Close data.

For more details please refer to Excel Spreadsheet ("Konstantinos_Paganopoulos_Assingment_4.xlsx").

(b)

Using the data from part (a), we calculate the mean weekly return for each stock, as well as the covariances of the weekly returns between all pairs of stocks. We first calculate the weekly returns for each stock as follows:

L	M	N	O	P	Q	R	S	T	U
Weekly Returns									
HSBC Holdings	Royal Dutch Shell	Rio Tinto	BP	GlaxoSmithKline	Diageo	AstraZeneca	Vodafone Group	Glencore	Unilever
0.004658671	-0.003759223	-0.009565978	0.001524461	-0.001937848	0.02748552	0.003566245	0.024107125	0.019926293	0.016160332
0.024959141	0.028301837	0.044976089	0.025266395	0.005825042	0.007399012	0.020275847	0.002615565	0.049927573	0.003467595
0.013439775	0.007747135	0.033660006	0.023753039	0.005405653	0.012994318	0.0124974	0.025652133	0.03487254	-0.01620578
0.006959093	0.014970488	0.052181944	0.010634207	0.015744728	0.01319945	0.036220233	-0.003815211	0.038891815	-0.000605742
-0.00443339	0.021726259	0.004439353	0.013200542	0.029111736	-0.019449544	0.01620779	0.007659646	-0.003589732	-0.00799889
0.036935352	0.015606702	0.053163261	0.009818781	-0.005877914	-0.011601759	-0.013835552	-0.031461149	0.04876473	-0.026512002
-0.004168403	-0.016519338	-0.036810527	-0.046559389	0.00369533	-0.007383603	-0.017342163	-0.015478557	-0.000981425	0.031124541
-0.022577382	-0.002343856	-0.018424008	0.012551424	-0.002209174	-0.023841431	0.010311475	-0.000885731	-0.01289457	-0.027750787
-0.026472888	-0.051096211	-0.01039947	-0.052004535	-0.032619827	-0.017780274	-0.011580086	-0.027039111	-0.04764858	0.011266957
-0.024526817	-0.070146607	-0.028963269	-0.03963657	-0.01510535	-0.030236669	-0.052819706	-0.080637775	-0.056825687	-0.063382005
0.039628276	0.015531449	0.082750444	0.007126968	0.021378843	0.027692234	-0.006394055	0.01412288	0.074792405	0.004890211
-0.049158666	0.013284273	-0.024500151	0.021118345	-0.017139516	-0.010978099	0.031159627	-0.003908987	0.021262806	-0.002485608
-0.005796251	-0.00240685	-0.0907161	-0.025204843	0.012959123	-0.02906459	-0.00336695	-0.023693856	-0.080126149	-0.012764383
0.001134792	0.003509379	0.059101671	0.030381409	0.035659106	0.027293674	0.01045191	0.040096308	0.014540406	0.049023563
0.002125207	-0.017049234	0.002280977	-0.009305835	-0.009431054	-0.004291886	0.017030557	-0.026570024	0.042455517	-0.027602988
-0.057542747	0	-0.045113806	-0.023852236	-0.005894178	-0.031609183	-0.016231749	-0.039106727	-0.070039049	-0.013863285
-0.001800246	0.012675346	0.012477238	0.036328304	0.059592628	0.022467176	0.022452021	0.003099003	-0.01311021	0.05917796
-0.000901595	0.037768935	-0.014123613	0.039958237	0.02051661	0.033374792	0.019405507	0.050355184	-0.007489981	0.014789639
0.030685923	0.024968179	0.063202342	0.011837831	-0.001827598	-0.000200633	0.003907301	0.013970658	-0.015235759	-0.021051311
0.023350713	0.052023095	0.053368588	0.027563035	0.007042123	-0.010834614	-0.006786256	0.015228311	0.100780942	-0.014505675
0.026240824	0.018838504	-0.002257403	0.037051504	0.024615382	0.037728153	0.021503134	0.00261906	-0.030605622	0.046094227
-0.00722622	0.022919841	0.005530433	0.038518566	0.002730025	0.041047684	0.024001581	-0.000237429	-0.00499261	-0.017279642
0.02995512	0.020711547	0.055999977	0.011467473	-0.001905832	0.006571598	0.014216958	-0.001900133	0.072499983	0.048572207
-0.003941272	0.014506838	0.027462094	0.05231213	0.028077072	0.006901627	-0.007198231	-0.08053323	-0.014892412	-0.000362328
0.010187261	-0.057805636	-0.017626734	-0.053215002	0.00792915	0.023527294	0.045220432	0.009628315	-0.007493204	0.010754055
-0.010917164	0.022469784	0.006684672	0.038911812	0.015200035	-0.016832515	-0.010405295	-0.00061516	0.010066307	-0.008726822

Screenshot 3: First 27 weeks for all stocks' weekly returns.

We then calculate the man weekly returns simply by using the “average” function on the weekly returns of each company.

Mean weekly return									
-0.000878587	0.000725558	0.003551437	0.001304519	0.004319342	0.002698313	0.005491126	-0.001957841	-0.001191487	0.001905029
HSBC Holdings	Royal Dutch Shell	Rio Tinto	BP	GlaxoSmithKline	Diageo	AstraZeneca	Vodafone Group	Glencore	Unilever

Screenshot 4: Mean weekly returns for each stock.

We finally use the “covariance.p” function and calculate the covariances of the weekly returns between all pairs of stocks as indicated below:

	HSBC Holdings	Royal Dutch Shell	Rio Tinto	BP	GlaxoSmithKline	Diageo	AstraZeneca	Vodafone Group	Glencore	Unilever
HSBC Holdings	0.000541202	0.000236754	0.000383745	0.000211107	0.000141454	7.65344E-05	0.000151309	0.000291999	0.000528147	7.86057E-05
Royal Dutch Shell	0.000236754	0.000608622	0.000391736	0.000508494	0.000213039	0.000108517	0.000236717	0.000222367	0.000483724	0.000140311
Rio Tinto	0.000383745	0.000391736	0.001308269	0.000405735	0.000132278	0.000199635	3.31945E-05	4.78641E-05	0.000997124	0.000140407
BP	0.000211107	0.000508494	0.000405735	0.00070815	0.000199432	0.000134121	0.000237969	0.000180798	0.000403271	0.000122675
GlaxoSmithKline	0.000141454	0.000213039	0.000132278	0.000199432	0.000703991	0.000322037	0.000444943	0.000166537	9.32549E-06	0.000251853
Diageo	7.65344E-05	0.000108517	0.000199635	0.000134121	0.000322037	0.000467535	0.00031736	0.00012375	-6.8196E-05	0.000328969
AstraZeneca	0.000151309	0.000236717	3.31945E-05	0.000237969	0.000444943	0.00031736	0.000951899	0.000262645	3.00468E-06	0.0002459
Vodafone Group	0.000291999	0.000222367	4.78641E-05	0.000180798	0.000166537	0.00012375	0.000262645	0.00124752	0.000391711	0.000159946
Glencore	0.000528147	0.000483724	0.000997124	0.000403271	9.32549E-06	-6.8196E-05	3.00468E-06	0.000391711	0.001837356	-3.06329E-05
Unilever	7.86057E-05	0.000140311	0.000140407	0.000122675	0.000251853	0.000328969	0.0002459	0.000159946	-3.06329E-05	0.00053065

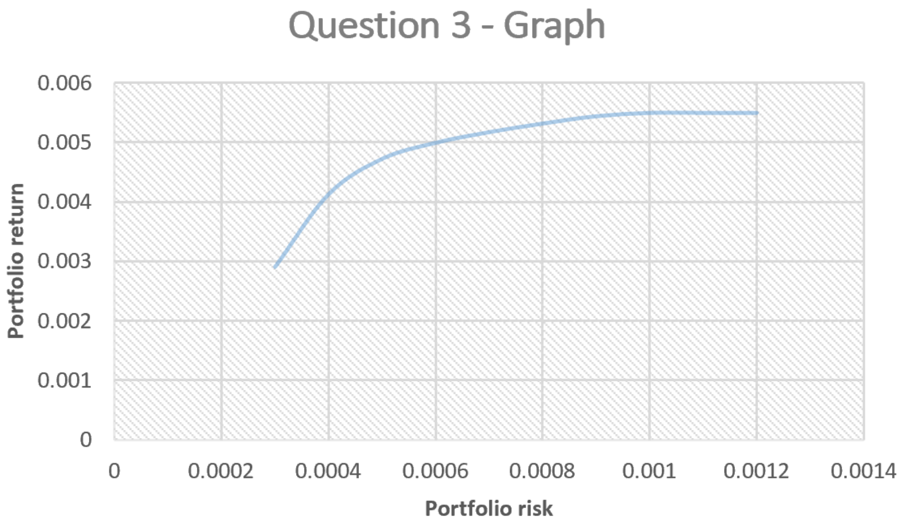
Screenshot 5: Covariances of the weekly returns between all pairs of stocks.

For more details please refer to Excel Spreadsheet (“Konstantinos_Paganopoulos_Assingment_4.xlsx”).

(c)

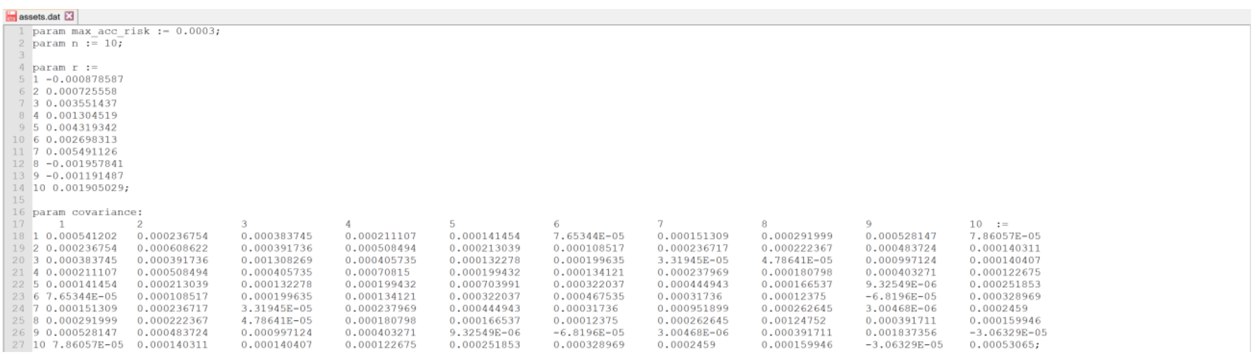
We solve the Markowitz problem in AMPL and produce a graph that shows the efficient frontiers as follows. The AMPL model could look like this:

```
# define the parameters
param max_acc_risk;
param n > 0;
param r {1 .. n};
param covariance {1 .. n, 1 .. n};

# define the variables
var x {1 .. n} >= 0;

# define the objective function
maximize Portfolio_return: sum {i in 1 .. n} r[i]*x[i];

# define the constraints
subject to maximum_acceptable_risk: sum {i in 1 .. n, j in 1 .. n}
covariance[i,j]*x[i]*x[j] <= max_acc_risk;
subject to percentage: sum {i in 1 .. n} x[i] = 1;
```



Screenshot 6: The efficient frontier formed by the solutions.

Note that in order to solve the problem, we had to create an "assets.dat" file, that includes all the data our model needs to run (such as mean weekly return and covariance). The "assets.dat" file for a low risk could look like the following:



assets.dat

```
param max_acc_risk := 0.0003;
param n := 10;

param r :=
1 -0.000878587
2 0.000725558
3 0.003551437
4 0.001304519
5 0.004319342
6 0.002698313
7 0.005491126
8 -0.001957841
9 -0.001191487
10 0.001905029;

param covariance:
    1             2             3             4             5             6             7             8             9             10  :=
1 0.000541202   0.000236754   0.000383745   0.000211107   0.000141454   7.65344E-05   0.000151309   0.000291999   0.000528147   7.86057E-05
2 0.000236754   0.000608622   0.000391736   0.000508494   0.000213039   0.000108517   0.000236717   0.000222367   0.000483724   0.000140311
3 0.000383745   0.000391736   0.001308269   0.000405735   0.000132278   0.000199635   3.31945E-05   4.78641E-05   0.000997124   0.000140407
4 0.000211107   0.000508494   0.000405735   0.00070815    0.000199432   0.000134121   0.000237969   0.000180798   0.000403271   0.000122675
5 0.000141454   0.000213039   0.000132278   0.000199432   0.000703991   0.000322037   0.000444943   0.000166537   9.32549E-06   0.000251853
6 7.65344E-05   0.000108517   0.000199635   0.000134121   0.000322037   0.000467535   0.00031736    0.00012375    -6.8196E-05   0.000328969
7 0.000151309   0.000236717   3.31945E-05   0.000237969   0.000444943   0.00031736    0.000951899   0.000262645   3.00468E-06   0.0002459
8 0.000291999   0.000222367   4.78641E-05   0.000180798   0.000166537   0.00012375    0.000262645   0.00124752    0.000391711   0.000159946
9 0.000528147   0.000483724   0.000997124   0.000403271   9.32549E-06   -6.8196E-05   3.00468E-06   0.000391711   0.001837356   -3.06329E-05
10 7.86057E-05  0.000140311   0.000140407   0.000122675   0.000251853   0.000328969   0.0002459     0.000159946   -3.06329E-05  0.00053065;
```

Screenshot 7: The "assets.dat" file.

The optimal value and the optimal portfolio composition for three selected portfolios (low return/low risk, medium return/medium risk and high return/high risk) are the following:

<u>low return/low risk (param max_acc_risk = 0.0003):</u>

```
ampl: model Konstantinos_Paganopoulos_Assignment_4.mod;
ampl: data assets.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.9.0.0: optimal solution; objective 0.002903263069
12 barrier iterations
No basis.
ampl: display x[1], x[2], x[3], x[4], x[5], x[6], x[7], x[8], x[9], x[10];
x[1] = 0.0930627
```

```
x[2] = 0.00940101
x[3] = 0.116758
x[4] = 0.0913263
x[5] = 0.160159
x[6] = 0.192604
x[7] = 0.164949
x[8] = 1.4711e-07
x[9] = 4.58706e-08
x[10] = 0.171741
```

<u>medium return/medium risk (param max_acc_risk = 0.0006):</u>

```
ampl: model Konstantinos_Paganopoulos_Assignment_4.mod;
ampl: data assets.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.9.0.0: optimal solution; objective 0.00498819351
15 barrier iterations
No basis.
ampl: display x[1], x[2], x[3], x[4], x[5], x[6], x[7], x[8], x[9], x[10];
x[1] = 4.98205e-08
x[2] = 7.2636e-08
x[3] = 0.158978
x[4] = 9.05996e-08
x[5] = 0.166039
x[6] = 1.93465e-07
x[7] = 0.674982
x[8] = 3.88066e-08
x[9] = 4.78879e-08
x[10] = 1.30184e-07
```

<u>high return/high risk (param max_acc_risk = 0.001):</u>

```
ampl: model Konstantinos_Paganopoulos_Assignment_4.mod;
ampl: data assets.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.9.0.0: optimal solution; objective 0.005491126
9 barrier iterations
No basis.
ampl: display x[1], x[2], x[3], x[4], x[5], x[6], x[7], x[8], x[9], x[10];
x[1] = 3.55259e-12
x[2] = 4.87514e-12
x[3] = 1.06825e-11
x[4] = 5.65564e-12
x[5] = 1.92694e-11
x[6] = 8.02916e-12
x[7] = 1
x[8] = 3.01404e-12
x[9] = 3.5551e-12
x[10] = 6.40674e-12
```

In other words, we can see that for example for a high risk, the Portfolio return equals 0.005491126 and the fractions/percentages of investing in each asset are 3.55259e-12 for asset 1, 4.87514e-12 for asset 2, etc.

(d)

If we were only allowed to invest in at most one company per sector, we would have the same problem as in part (c), with the only difference of having to add constraints for the sectors. As a result, we would have to solve the following problem:

Decision Variables:

We introduce the variables $x_1, \cdots, x_{10}$, where $x_i = fraction\ that\ we\ invest\ in\ asset\ i$

We now introduce the binary variables $Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}$, where $Y_i = \begin{cases} 1,\ if\ we\ invest\ in\ asset\ i \\ 0,\ otherwise \end{cases}$

For simplicity reasons, we assume that asset 1 is asset of "HSBC Holdings", asset 2 is that of "Royal Dutch Shell", asset 3 of "Rio Tinto", asset 4 of "BP", asset 5 of "GlaxoSmithKline", asset 6 of "Diageo", asset 7 of "AstraZeneca", asset 8 of "Vodafone Group", asset 9 of "Glencore" and asset 10 of "Unilever".

We also introduce the following parameters that are fed with data from our "assets.dat" file:

$maxaccrisk,\ which\ is\ the\ maximum\ acceptable\ risk$
$n,\ which\ is\ the\ number\ of\ assets$
$r_i,\ which\ is\ the\ expected\ return\ of\ asset\ i$
$covariance_{i,j},\ covariance\ of\ the\ weekly\ returns\ for\ all\ pairs\ of\ stocks_{i,j}$

Objective Function:

$maximise\ r_i x_i$

Constraints:

Due to maximum acceptable risk: $\sum_{i=1}^{n}\sum_{j=1}^{n} covariance[r_i, r_j] x_i x_j \leq maxaccrisk$

Due to total percentage of assets invested: $\sum_{i=1}^{n} x_i$

Now we add the constraint regarding the company sectors:

$Y_2 + Y_3 + Y_4 \leq 1$
$Y_5 + Y_7 \leq 1$

Note that we don't have to write the constraints for the rest sectors since the constraints will be useless, as only one firm consists of the sector and as a result we will always have $Y_i \leq 1$.

The last constraint that we should write down in order to implement the problem is one according to which the return x_i of every asset i in which we invest has a fraction that is not equal to zero. In other words:

$x_i \neq 0,\ if\ we\ invest\ in\ asset\ i$

As a result, we have the following for the binary variable: $Y_i \neq 0,\ if\ we\ invest\ in\ asset\ i$

Based on the Big-M-trick for a large number M we have the following constraint for each asset i:

$$\begin{cases} x_i \geq -MY_i \\ x_i \leq MY_i \end{cases}$$

Or in other words:

$$\begin{cases} x_1 \geq -MY_1 \\ x_1 \leq MY_1 \end{cases}$$

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$$\begin{cases} x_{10} \geq -MY_{10} \\ x_{10} \leq MY_{10} \end{cases}$$

CONCLUSION

In the given coursework assignment, a solution to a nonlinear (portfolio) optimisation problem is provided. The Excel spreadsheet, the AMPL, as well as the data file are attached separately.