Imperial College London
BUSINESS SCHOOL

MSc Business Analytics 2019/20
Optimisation and Decision Models
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Exercises 1

(1) **In-Class:** The Magnetron Company manufactures two types of microwave ovens: full-size and compact. Each full-size oven requires 2 hours of general assembly and 2 hours of electronic assembly, whereas each compact oven requires 1 hours of general assembly and 3 hours of electronic assembly. For the current production period, there are 500 hours of general assembly labour and 800 hours of electronic assembly labour available. The company estimates that it can sell up to 220 full-size and 180 compact ovens with an earnings contributions of £120 per full-size oven and £130 per compact oven. Magnetron wants to find a production plan that maximising earnings.

   (a) Formulate the above problem as a linear program.

   (b) Solve the linear program graphically. What is the optimal solution, and what is its objective value? Which constraints are binding at the optimal solution?

   (c) Verify the exact value of your optimal solution from part (b) by solving algebraically for the simultaneous solution of the relevant equations.

   (d) Using overtime, Magnetron is considering to increase the number of general assembly labour hours by 10. What hourly rate should Magnetron be willing to pay at most?

(2) **In-Class:** Nature's Best Frozen Foods company produces 4 mixes of frozen ready-to-eat vegetables. The mixes consists of five different vegetables: carrots, mushrooms, green peppers, broccoli and corn. The mixes are "Stir Fry", "Barbecue", "Hearty Mushrooms" and "Veggie Crunch", and each mix is sold in 250g bags. Each bag of "Stir Fry", "Barbecue", "Hearty Mushrooms" and "Veggie Crunch" contributes £0.22, £0.20, £0.18 and £0.18 to the company's earnings, respectively. The compositions of the mixes and the monthly supplies of the ingredients are given in the following table:

Ingredients	Mixes				Monthly supply of ingredients
	Stir Fry	Barbecue	Hearty Mushrooms	Veggie Crunch	
Carrots	62.5g	50g	0g	62.5g	3,750kg
Mushrooms	75g	0g	100g	0g	2,000kg
Green peppers	62.5g	50g	75g	62.5g	3,375kg
Broccoli	50g	75g	75g	62.5g	3,500kg
Corn	0g	75g	0g	62.5g	3,750kg

(a) Assuming that the company can sell all the mixes that they produce, formulate a linear program that determines a production plan which maximises the total earnings.

(b) Solve the linear program using AMPL. What is the optimal solution, and what is its objective value? Which constraints are binding at the optimal solution? Make sure you submit the AMPL model!

(c) What is the value of an extra 100kg of green peppers?

(3) **Homework:** The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is £5/unit on special risk insurance and £2/unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximise total expected profit. The work requirements are as follows:

Department	Work-hours per unit		Work-hours available
	Special risk	Mortgage	
Underwriting	3	2	2,400
Administration	0	1	800
Claims	2	0	1,200

(a) Formulate a linear program for this problem.

(b) Use the graphical method to solve this problem. What is the optimal solution, and what is its objective value? Which constraints are binding at the optimal solution?

(c) Verify the exact value of your optimal solution from part (b) by solving algebraically for the simultaneous solution of the relevant equations.

(d) Solve the model in Excel using Excel's add-in solver. Make sure you submit the Excel spreadsheet!

(4) **Homework:** We now consider a variant of Gemstone Tool Company's problem where the company optimises its productions for each of the four quarters of the financial year, and where the company can keep its excess production on inventory. To this end, we make the following assumptions:

	Wrenches	Pliers	Availability per quarter
Required steel (lbs.)	1.5	1.0	27,000 lbs.
Required molding machine time (h)	1.0	1.0	21,000 h
Required assembly machine time (h)	0.3	0.5	9,000 h
Contributions to earnings ($/unit)	US$0.13	US$0.10	

Moreover, we assume that the demands for wrenches and pliers are going to be as follows:

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Demand for wrenches	6,000	10,000	12,000	15,000
Demand for pliers	5,000	8,000	10,000	16,000

(a) Formulate a linear program where the company decides how much to produce in each of the four quarters ***so as to maximise the overall earnings across the four quarters***, and where the company can decide to keep some of the production in the first three quarters for sales in a later quarter.

Hint: *Introduce the decision variables PW_1, …, PW_4 and PP_1, …, PP_4 for the production quantities of wrenches and pliers in each of the four quarters; the decision variables SW_1, …, SW_4 and SP_1, …, WP_4 for the sales quantities of wrenches and pliers in each of the four quarters; the decision variables IW_1, …, IW_4 and IP_1, …, IP_4 for the wrenches and pliers on inventory at the beginning of each quarter (before anything is produced or sold). Make sure you enforce that the inventories in the first quarter are all zero! (Depending on your formulation, you may even need variables IW_5 and IP_5.)*

(b) Solve the problem in AMPL. Make sure you submit your AMPL file! Explain the optimal solution: What should we produce, sell and keep on inventory in every quarter, and what are the optimal earnings?

Hint: *As mentioned in class, we can avoid explicit non-negativity constraints by declaring a variable accordingly. For the variable PW_1, for example, we can use the command "var PW1 >= 0;" (without quotes).*