

# Demand Forecasting for a Fast-Food Restaurant Chain

## Logistics and Supply Chain Analytics - Individual Project

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### Solutions

We first load the necessary libraries.

We have a dataset, which includes daily sales for lettuce at a store in New York from a fast-food restaurant chain from 20 March 2015 to 15 June 2015. Each observation includes two values: day pair, and sales in that particular day.

Then, we load and split the data set into train and test set.

```
# read csv file
data <- read.csv(file = "NewYork2_final.csv", header = TRUE, stringsAsFactors = FALSE)

# convert column date of data set to type date
data$date <- as.Date(data$date)

# convert sales into a time series object
lettuce <- ts(data[, 2], frequency = 7, start = c(12, 2)) # 12th week 2nd day

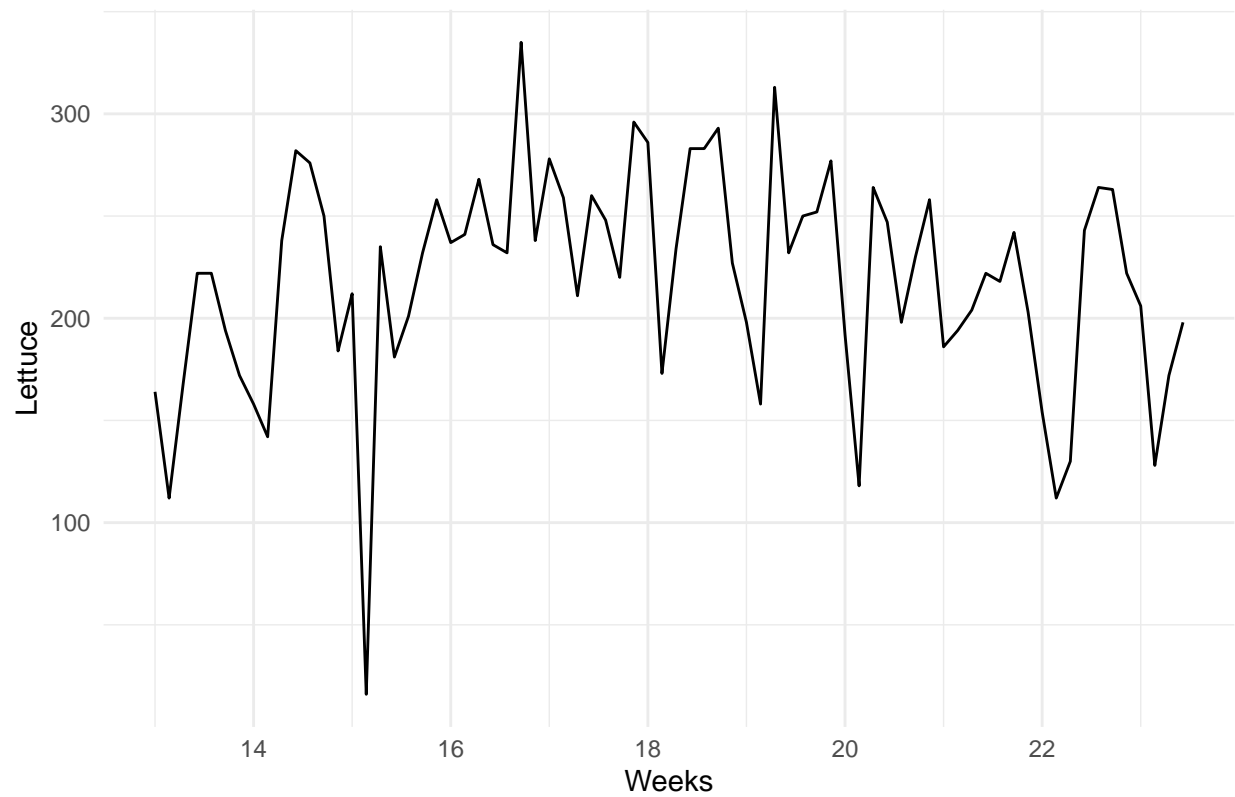
# split data set into train and test set
lettuce_train <- subset(lettuce, end = 80, start = 7) # ignore first 6 days due to strange data
lettuce_test <- subset(lettuce, start = 81) # last 14 lines-days
```

### ARIMA

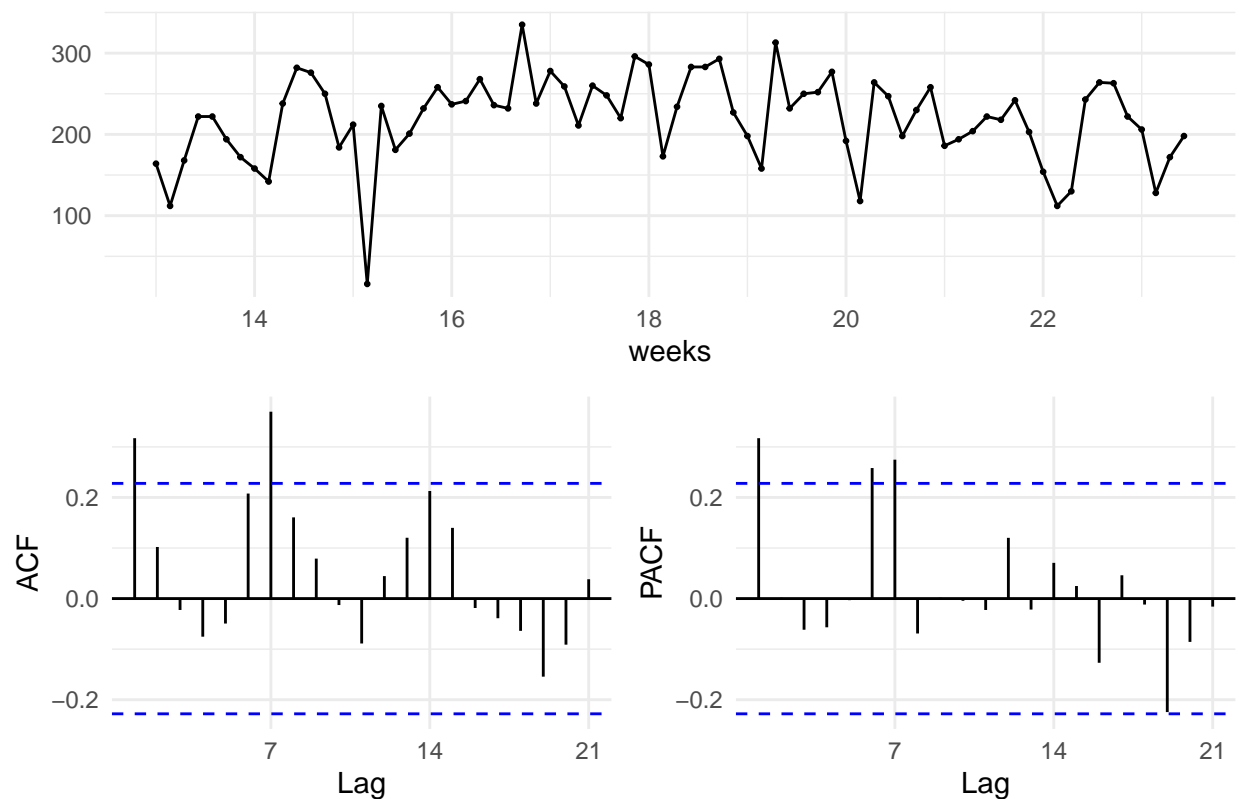
We visually inspect the time series.

```
autoplot(lettuce_train, xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +
ggtitle("New York 2 Store (20974) - Time series plot")
```

New York 2 Store (20974) – Time series plot



```
ggtsdisplay(lettuce_train, xlab = "weeks", theme = theme_minimal())
```



We plot the time series, and observe that there is no seasonality and appears to be stationary. We run ADF, PP and KPSS tests to formally test the stationarity of time series and all suggest that the time series is stationary.

```
# stationary test
adf.test(lettuce_train)
```

```
##
## Augmented Dickey-Fuller Test
##
## data: lettuce_train
## Dickey-Fuller = -3.5721, Lag order = 4, p-value = 0.04182
## alternative hypothesis: stationary
```

```
pp.test(lettuce_train)
```

```
## Warning in pp.test(lettuce_train): p-value smaller than printed p-value
```

```
##
## Phillips-Perron Unit Root Test
##
## data: lettuce_train
## Dickey-Fuller Z(alpha) = -48.35, Truncation lag parameter = 3, p-value
## = 0.01
## alternative hypothesis: stationary
```

```
kpss.test(lettuce_train)
```

```
## Warning in kpss.test(lettuce_train): p-value greater than printed p-value
```

```
##  
## KPSS Test for Level Stationarity  
##  
## data: lettuce_train  
## KPSS Level = 0.25367, Truncation lag parameter = 3, p-value = 0.1
```

The two automatic functions, `ndiffs()` and `nsdiffs()` tell us how many first-order differences, and how many seasonal differences, respectively, we need to take to make the time series stationary. We use those functions below:

```
ndiffs(lettuce_train)
```

```
## [1] 0
```

```
# seasonal stationarity  
nsdiffs(lettuce_train)
```

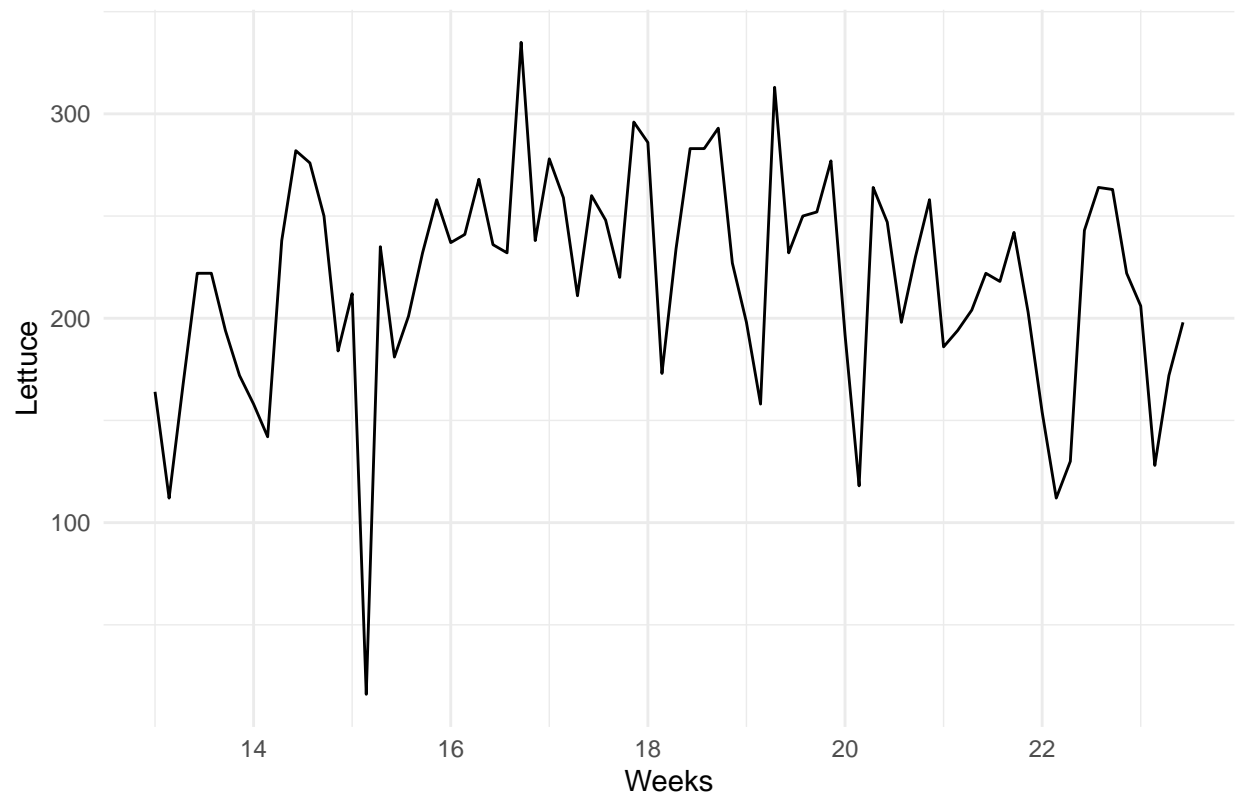
```
## [1] 0
```

We do not need to differentiate.

We plot again the time series to see if stationary.

```
autoplot(lettuce_train, xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +  
ggtitle("New York 2 Store (20974) - Time series plot")
```

New York 2 Store (20974) – Time series plot



We can easily observe a concave pattern. This forms a trend.

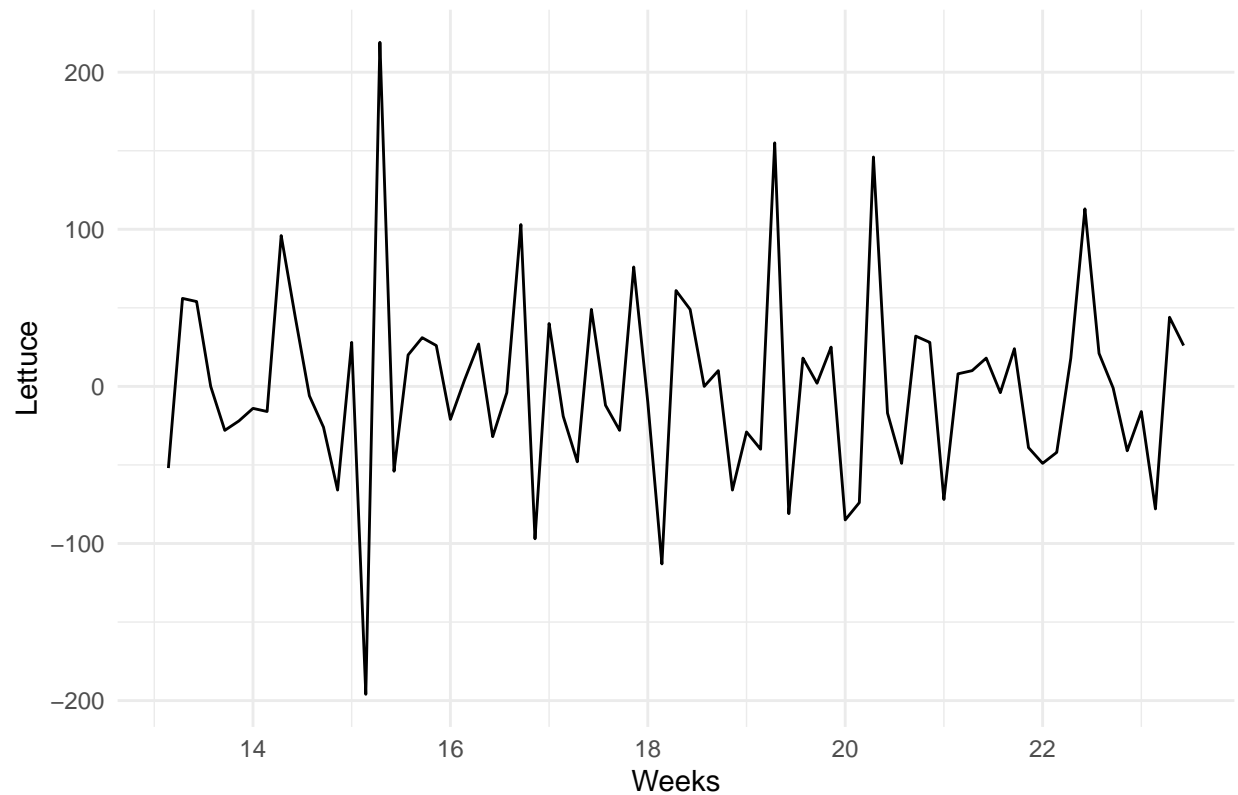
As a result, we have to differentiate as follows:

```
lettuce_train.diff1 <- diff(lettuce_train, differences = 1)
```

We plot again the time series to see if stationary.

```
autoplot(lettuce_train.diff1, xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +  
ggtitle("New York 2 Store (20974) - Time series plot")
```

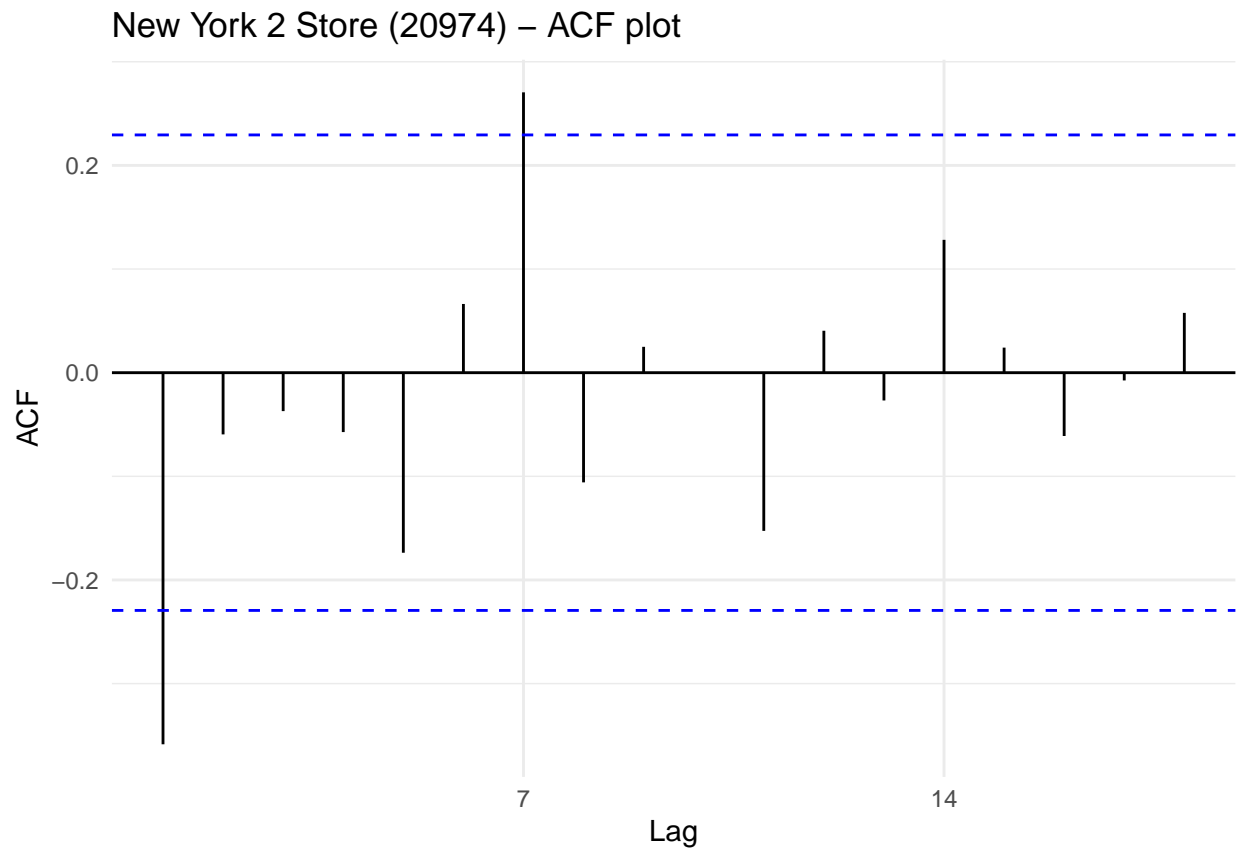
New York 2 Store (20974) – Time series plot



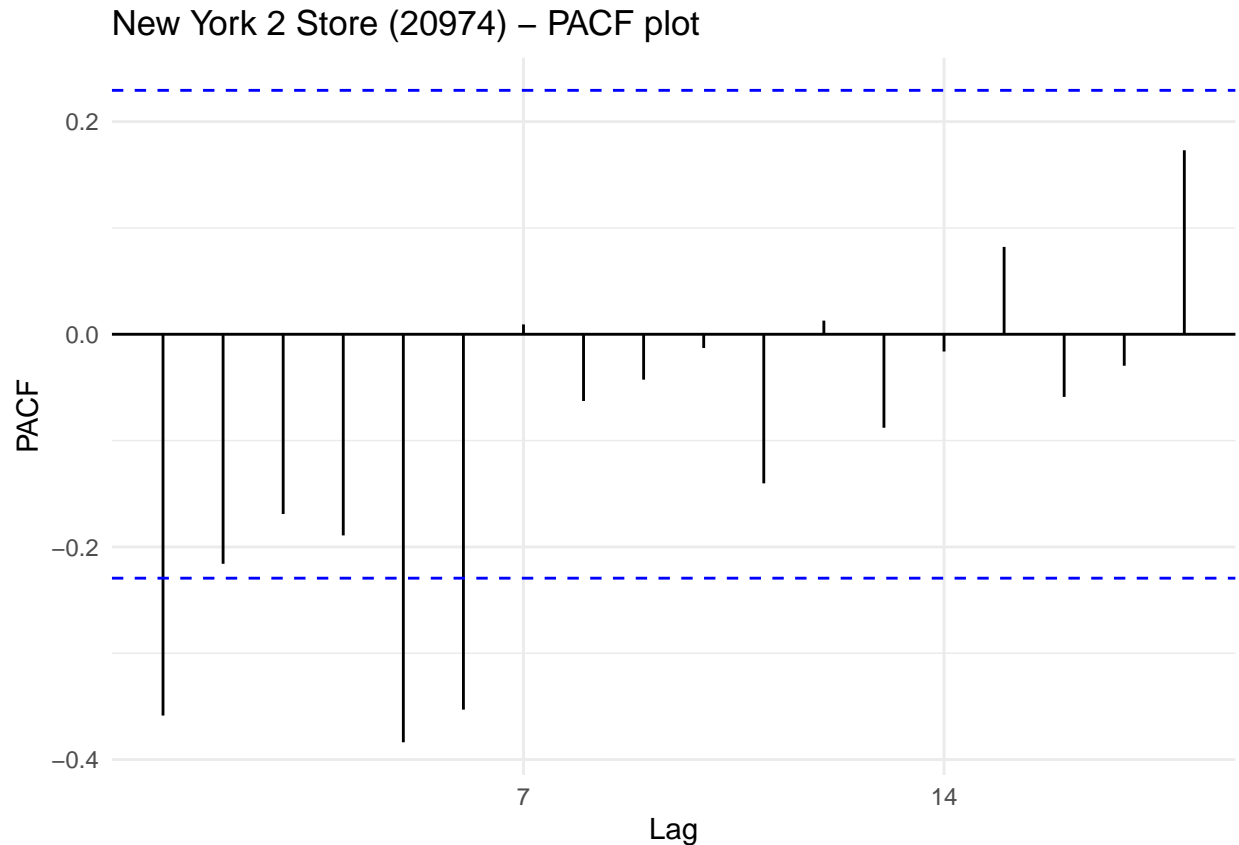
Looks stationary.

The next step is to determine the optimal orders of MA and AR components. We first plot the ACF and PACF of the time series.

```
# acf plot  
ggAcf(lettuce_train.diff1) + theme_minimal() + ggtitle("New York 2 Store (20974) - ACF plot")
```



```
# pacf plot  
ggPacf(lettuce_train.diff1) + theme_minimal() + ggtitle("New York 2 Store (20974) – PACF plot")
```



Next we use `auto.arima()` to search for the best ARIMA models.

The default procedure uses some approximations to speed up the search. These approximations can be avoided with the argument `approximation = FALSE`. It is possible that the minimum AIC model will not be found due to these approximations, or because of the stepwise procedure. A much larger set of models will be searched if the argument `stepwise = FALSE` is used. We also use `d = 1` and `D = 0` since we had only first differencing.

```
auto.arima(lettuce_train, trace = TRUE, ic = 'aic', approximation = FALSE, stepwise = FALSE, d=1, D=0)
```

```
##
## ARIMA(0,1,0) : 813.2385
## ARIMA(0,1,0) with drift : 815.2344
## ARIMA(0,1,0)(0,0,1)[7] : 810.6353
## ARIMA(0,1,0)(0,0,1)[7] with drift : 812.6345
## ARIMA(0,1,0)(0,0,2)[7] : 811.7488
## ARIMA(0,1,0)(0,0,2)[7] with drift : 813.7482
## ARIMA(0,1,0)(1,0,0)[7] : 809.7859
## ARIMA(0,1,0)(1,0,0)[7] with drift : 811.7854
## ARIMA(0,1,0)(1,0,1)[7] : Inf
## ARIMA(0,1,0)(1,0,1)[7] with drift : Inf
## ARIMA(0,1,0)(1,0,2)[7] : Inf
## ARIMA(0,1,0)(1,0,2)[7] with drift : Inf
## ARIMA(0,1,0)(2,0,0)[7] : 811.3921
## ARIMA(0,1,0)(2,0,0)[7] with drift : 813.3916
## ARIMA(0,1,0)(2,0,1)[7] : Inf
```



```

## ARIMA(0,1,0)(2,0,1)[7] with drift : Inf
## ARIMA(0,1,0)(2,0,2)[7] : Inf
## ARIMA(0,1,0)(2,0,2)[7] with drift : Inf
## ARIMA(0,1,1) : 790.9536
## ARIMA(0,1,1) with drift : 792.9134
## ARIMA(0,1,1)(0,0,1)[7] : 786.5134
## ARIMA(0,1,1)(0,0,1)[7] with drift : 788.4282
## ARIMA(0,1,1)(0,0,2)[7] : 785.0547
## ARIMA(0,1,1)(0,0,2)[7] with drift : 787.003
## ARIMA(0,1,1)(1,0,0)[7] : 784.0596
## ARIMA(0,1,1)(1,0,0)[7] with drift : 785.9729
## ARIMA(0,1,1)(1,0,1)[7] : Inf
## ARIMA(0,1,1)(1,0,1)[7] with drift : Inf
## ARIMA(0,1,1)(1,0,2)[7] : Inf
## ARIMA(0,1,1)(1,0,2)[7] with drift : Inf
## ARIMA(0,1,1)(2,0,0)[7] : 784.0994
## ARIMA(0,1,1)(2,0,0)[7] with drift : 786.0462
## ARIMA(0,1,1)(2,0,1)[7] : Inf
## ARIMA(0,1,1)(2,0,1)[7] with drift : Inf
## ARIMA(0,1,1)(2,0,2)[7] : Inf
## ARIMA(0,1,1)(2,0,2)[7] with drift : Inf
## ARIMA(0,1,2) : 789.8214
## ARIMA(0,1,2) with drift : 791.7907
## ARIMA(0,1,2)(0,0,1)[7] : 785.9105
## ARIMA(0,1,2)(0,0,1)[7] with drift : 787.8476
## ARIMA(0,1,2)(0,0,2)[7] : 785.0438
## ARIMA(0,1,2)(0,0,2)[7] with drift : 787.0003
## ARIMA(0,1,2)(1,0,0)[7] : 783.8495
## ARIMA(0,1,2)(1,0,0)[7] with drift : 785.7834
## ARIMA(0,1,2)(1,0,1)[7] : Inf
## ARIMA(0,1,2)(1,0,1)[7] with drift : Inf
## ARIMA(0,1,2)(1,0,2)[7] : Inf
## ARIMA(0,1,2)(1,0,2)[7] with drift : Inf
## ARIMA(0,1,2)(2,0,0)[7] : 784.3942
## ARIMA(0,1,2)(2,0,0)[7] with drift : 786.3473
## ARIMA(0,1,2)(2,0,1)[7] : Inf
## ARIMA(0,1,2)(2,0,1)[7] with drift : Inf
## ARIMA(0,1,3) : 791.6816
## ARIMA(0,1,3) with drift : 793.6516
## ARIMA(0,1,3)(0,0,1)[7] : 787.4216
## ARIMA(0,1,3)(0,0,1)[7] with drift : 789.3617
## ARIMA(0,1,3)(0,0,2)[7] : 786.6606
## ARIMA(0,1,3)(0,0,2)[7] with drift : 788.6181
## ARIMA(0,1,3)(1,0,0)[7] : 785.2878
## ARIMA(0,1,3)(1,0,0)[7] with drift : 787.2256
## ARIMA(0,1,3)(1,0,1)[7] : Inf
## ARIMA(0,1,3)(1,0,1)[7] with drift : Inf
## ARIMA(0,1,3)(2,0,0)[7] : 786.0388
## ARIMA(0,1,3)(2,0,0)[7] with drift : 787.9935
## ARIMA(0,1,4) : 792.8385
## ARIMA(0,1,4) with drift : 794.7918
## ARIMA(0,1,4)(0,0,1)[7] : 789.4088
## ARIMA(0,1,4)(0,0,1)[7] with drift : 791.3501
## ARIMA(0,1,4)(1,0,0)[7] : Inf

```

```

## ARIMA(0,1,4)(1,0,0)[7] with drift : Inf
## ARIMA(0,1,5) : 793.5211
## ARIMA(0,1,5) with drift : 795.4626
## ARIMA(1,1,0) : 805.2092
## ARIMA(1,1,0) with drift : 807.1968
## ARIMA(1,1,0)(0,0,1)[7] : 799.9844
## ARIMA(1,1,0)(0,0,1)[7] with drift : 801.9769
## ARIMA(1,1,0)(0,0,2)[7] : 800.4492
## ARIMA(1,1,0)(0,0,2)[7] with drift : 802.4452
## ARIMA(1,1,0)(1,0,0)[7] : 798.2373
## ARIMA(1,1,0)(1,0,0)[7] with drift : 800.2332
## ARIMA(1,1,0)(1,0,1)[7] : Inf
## ARIMA(1,1,0)(1,0,1)[7] with drift : Inf
## ARIMA(1,1,0)(1,0,2)[7] : Inf
## ARIMA(1,1,0)(1,0,2)[7] with drift : Inf
## ARIMA(1,1,0)(2,0,0)[7] : 799.6011
## ARIMA(1,1,0)(2,0,0)[7] with drift : 801.5986
## ARIMA(1,1,0)(2,0,1)[7] : Inf
## ARIMA(1,1,0)(2,0,1)[7] with drift : Inf
## ARIMA(1,1,0)(2,0,2)[7] : Inf
## ARIMA(1,1,0)(2,0,2)[7] with drift : Inf
## ARIMA(1,1,1) : 789.8301
## ARIMA(1,1,1) with drift : 791.7999
## ARIMA(1,1,1)(0,0,1)[7] : 785.605
## ARIMA(1,1,1)(0,0,1)[7] with drift : 787.5462
## ARIMA(1,1,1)(0,0,2)[7] : 784.7885
## ARIMA(1,1,1)(0,0,2)[7] with drift : 786.747
## ARIMA(1,1,1)(1,0,0)[7] : 783.5071
## ARIMA(1,1,1)(1,0,0)[7] with drift : 785.4459
## ARIMA(1,1,1)(1,0,1)[7] : Inf
## ARIMA(1,1,1)(1,0,1)[7] with drift : Inf
## ARIMA(1,1,1)(1,0,2)[7] : Inf
## ARIMA(1,1,1)(1,0,2)[7] with drift : Inf
## ARIMA(1,1,1)(2,0,0)[7] : 784.1796
## ARIMA(1,1,1)(2,0,0)[7] with drift : 786.1346
## ARIMA(1,1,1)(2,0,1)[7] : Inf
## ARIMA(1,1,1)(2,0,1)[7] with drift : Inf
## ARIMA(1,1,2) : 791.7503
## ARIMA(1,1,2) with drift : 793.7202
## ARIMA(1,1,2)(0,0,1)[7] : 787.5884
## ARIMA(1,1,2)(0,0,1)[7] with drift : 789.5299
## ARIMA(1,1,2)(0,0,2)[7] : Inf
## ARIMA(1,1,2)(0,0,2)[7] with drift : Inf
## ARIMA(1,1,2)(1,0,0)[7] : Inf
## ARIMA(1,1,2)(1,0,0)[7] with drift : Inf
## ARIMA(1,1,2)(1,0,1)[7] : Inf
## ARIMA(1,1,2)(1,0,1)[7] with drift : Inf
## ARIMA(1,1,2)(2,0,0)[7] : 786.1127
## ARIMA(1,1,2)(2,0,0)[7] with drift : 788.0688
## ARIMA(1,1,3) : 793.6015
## ARIMA(1,1,3) with drift : 795.571
## ARIMA(1,1,3)(0,0,1)[7] : 787.7978
## ARIMA(1,1,3)(0,0,1)[7] with drift : 789.7134
## ARIMA(1,1,3)(1,0,0)[7] : Inf

```

```

## ARIMA(1,1,3)(1,0,0)[7] with drift : Inf
## ARIMA(1,1,4) : 790.5554
## ARIMA(1,1,4) with drift : 792.5279
## ARIMA(2,1,0) : 803.7339
## ARIMA(2,1,0) with drift : 805.7225
## ARIMA(2,1,0)(0,0,1)[7] : 798.5767
## ARIMA(2,1,0)(0,0,1)[7] with drift : 800.5664
## ARIMA(2,1,0)(0,0,2)[7] : 798.2416
## ARIMA(2,1,0)(0,0,2)[7] with drift : 800.2372
## ARIMA(2,1,0)(1,0,0)[7] : 796.352
## ARIMA(2,1,0)(1,0,0)[7] with drift : 798.3457
## ARIMA(2,1,0)(1,0,1)[7] : Inf
## ARIMA(2,1,0)(1,0,1)[7] with drift : Inf
## ARIMA(2,1,0)(1,0,2)[7] : Inf
## ARIMA(2,1,0)(1,0,2)[7] with drift : Inf
## ARIMA(2,1,0)(2,0,0)[7] : 797.2045
## ARIMA(2,1,0)(2,0,0)[7] with drift : 799.2015
## ARIMA(2,1,0)(2,0,1)[7] : Inf
## ARIMA(2,1,0)(2,0,1)[7] with drift : Inf
## ARIMA(2,1,1) : 791.6627
## ARIMA(2,1,1) with drift : 793.6323
## ARIMA(2,1,1)(0,0,1)[7] : 787.573
## ARIMA(2,1,1)(0,0,1)[7] with drift : 789.5143
## ARIMA(2,1,1)(0,0,2)[7] : Inf
## ARIMA(2,1,1)(0,0,2)[7] with drift : Inf
## ARIMA(2,1,1)(1,0,0)[7] : 785.3313
## ARIMA(2,1,1)(1,0,0)[7] with drift : 787.2705
## ARIMA(2,1,1)(1,0,1)[7] : Inf
## ARIMA(2,1,1)(1,0,1)[7] with drift : Inf
## ARIMA(2,1,1)(2,0,0)[7] : 786.0649
## ARIMA(2,1,1)(2,0,0)[7] with drift : Inf
## ARIMA(2,1,2) : 788.3468
## ARIMA(2,1,2) with drift : 790.3279
## ARIMA(2,1,2)(0,0,1)[7] : Inf
## ARIMA(2,1,2)(0,0,1)[7] with drift : Inf
## ARIMA(2,1,2)(1,0,0)[7] : Inf
## ARIMA(2,1,2)(1,0,0)[7] with drift : Inf
## ARIMA(2,1,3) : Inf
## ARIMA(2,1,3) with drift : Inf
## ARIMA(3,1,0) : 803.7635
## ARIMA(3,1,0) with drift : 805.7533
## ARIMA(3,1,0)(0,0,1)[7] : 798.5321
## ARIMA(3,1,0)(0,0,1)[7] with drift : 800.5183
## ARIMA(3,1,0)(0,0,2)[7] : 798.7036
## ARIMA(3,1,0)(0,0,2)[7] with drift : 800.6969
## ARIMA(3,1,0)(1,0,0)[7] : 796.5931
## ARIMA(3,1,0)(1,0,0)[7] with drift : 798.5837
## ARIMA(3,1,0)(1,0,1)[7] : Inf
## ARIMA(3,1,0)(1,0,1)[7] with drift : Inf
## ARIMA(3,1,0)(2,0,0)[7] : 797.7101
## ARIMA(3,1,0)(2,0,0)[7] with drift : 799.7051
## ARIMA(3,1,1) : 792.3296
## ARIMA(3,1,1) with drift : 794.2906
## ARIMA(3,1,1)(0,0,1)[7] : 789.0712

```

```
## ARIMA(3,1,1)(0,0,1)[7] with drift : 791.0056
## ARIMA(3,1,1)(1,0,0)[7] : 787.1708
## ARIMA(3,1,1)(1,0,0)[7] with drift : Inf
## ARIMA(3,1,2) : 790.0768
## ARIMA(3,1,2) with drift : 792.0609
## ARIMA(4,1,0) : 803.2391
## ARIMA(4,1,0) with drift : 805.2291
## ARIMA(4,1,0)(0,0,1)[7] : 799.4988
## ARIMA(4,1,0)(0,0,1)[7] with drift : 801.4827
## ARIMA(4,1,0)(1,0,0)[7] : 797.7698
## ARIMA(4,1,0)(1,0,0)[7] with drift : 799.7577
## ARIMA(4,1,1) : 791.444
## ARIMA(4,1,1) with drift : 793.3827
## ARIMA(5,1,0) : 793.6175
## ARIMA(5,1,0) with drift : 795.5889
##
##
## Best model: ARIMA(1,1,1)(1,0,0)[7]
```

```
## Series: lettuce_train
## ARIMA(1,1,1)(1,0,0)[7]
##
## Coefficients:
##          ar1          ma1          sar1
##          0.2115   -0.9259   0.3408
## s.e.    0.1354    0.0693   0.1148
##
## sigma^2 estimated as 2442:  log likelihood=-387.75
## AIC=783.51   AICc=784.1   BIC=792.67
```

```
# Best model: ARIMA(1,1,1)(1,0,0)[7] (AIC=783.51)
# Second best: ARIMA(0,1,2)(1,0,0)[7] (AIC=783.85)
# Third best: ARIMA(0,1,1)(1,0,0)[7] (AIC=784.06)
```

Based on the output of `auto.arima()`, a couple of models have similar AICs. Now suppose that we choose the three models with the lowest AICs, namely `ARIMA(1,1,1)(1,0,0)[7]` with `AIC=783.51`, `ARIMA(0,1,2)(1,0,0)[7]` with `AIC=783.85` AND `ARIMA(0,1,1)(1,0,0)[7]` with `AIC=784.06`, as the candidate models that we would like to evaluate further.

```
# three candidate models
lettuce.m1 <- Arima(lettuce_train, order = c(1, 1, 1),
                    seasonal = list(order = c(1, 0, 0), period = 7))
lettuce.m2 <- Arima(lettuce_train, order = c(0, 1, 2),
                    seasonal = list(order = c(1, 0, 0), period = 7))
lettuce.m3 <- Arima(lettuce_train, order = c(0, 1, 1),
                    seasonal = list(order = c(1, 0, 0), period = 7))
```

Now we evaluate the in-sample performance/fit of the model with `accuracy()` function, which summarizes various measures of fitting errors.

A couple of functions are proved to be useful for us to evaluate the in-sample performance/fit of the model. One is `accuracy()` function, which summarizes various measures of fitting errors. In the post-estimation

analysis, we would also like to check out the residual plots, including time series, ACFs and etc, to make sure that there is no warning signal. In particular, residuals shall have a zero mean, constant variance, and distributed symmetrically around mean zero. ACF of any lag greater 0 is expected to be statistically insignificant.

```
# in-sample one-step forecasts model 1
accuracy(lettuce.m1)
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 3.111878 48.05772 37.70491 -16.0591 31.88792 0.8042754 -0.02117438
```

```
# in-sample one-step forecasts model 2
accuracy(lettuce.m2)
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 2.848727 48.17716 37.764 -16.37333 32.14221 0.8055358 0.002492967
```

```
# in-sample one-step forecasts model 3
accuracy(lettuce.m3)
```

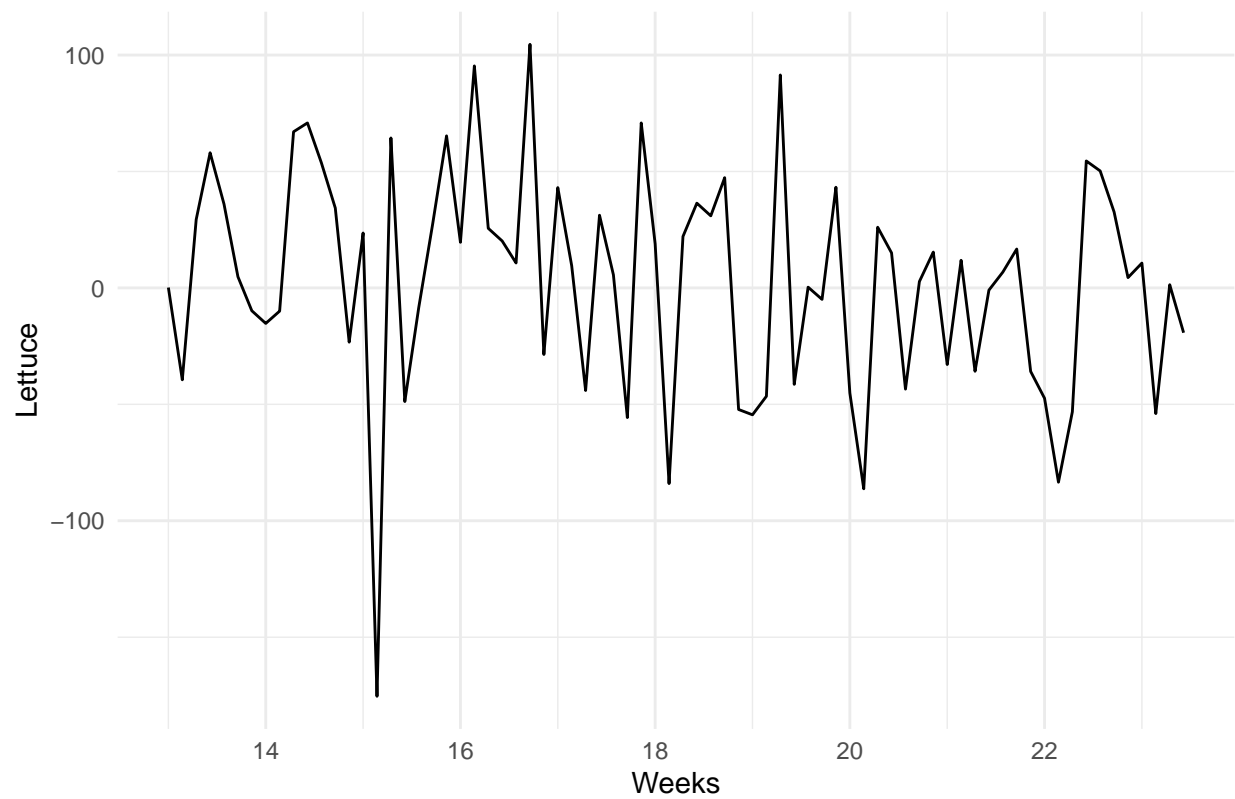
```
##               ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 2.568306 48.86888 38.11537 -16.67426 32.46622 0.8130309 0.1550136
```

The first model even though it has both the lowest AIC score as well as the lowest RMSE.

Now we proceed with the residual analysis of the three models.

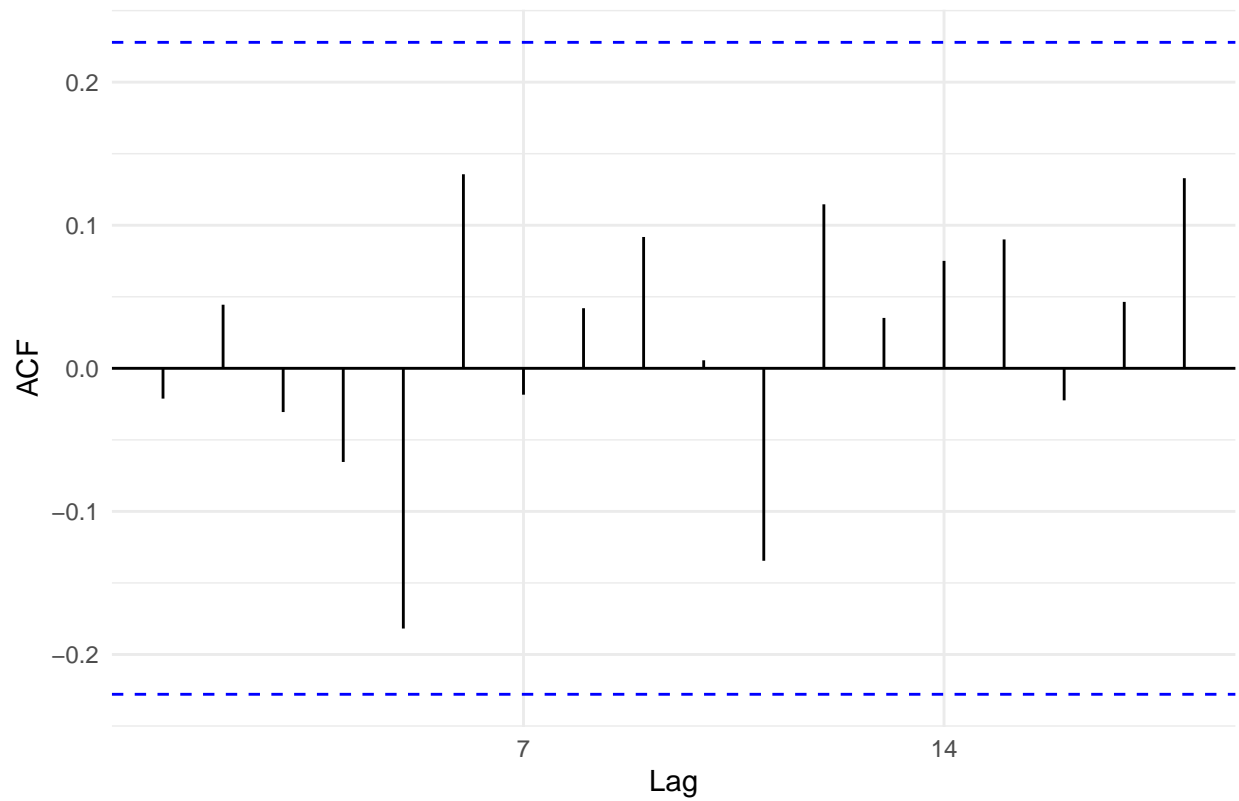
```
# residual analysis model 1
autoplot(lettuce.m1$residuals, xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +
ggtitle("New York 2 Store (20974) - Residuals model 1 plot")
```

New York 2 Store (20974) – Residuals model 1 plot

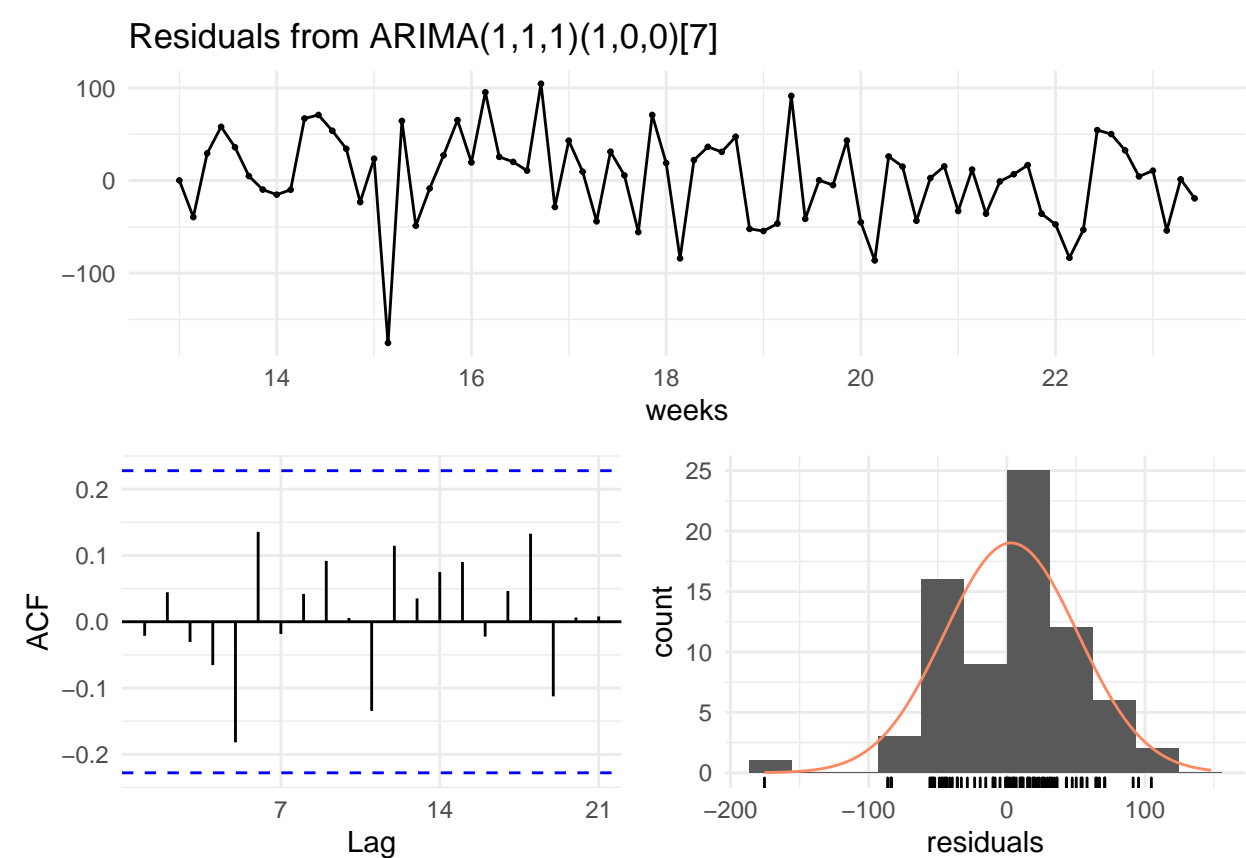


```
ggAcf(lettuce.m1$residuals) + theme_minimal() +  
ggtitle("New York 2 Store (20974) - ACF residuals plot model 1")
```

New York 2 Store (20974) – ACF residuals plot model 1



```
checkresiduals(lettuce.m1, xlab = "weeks", theme = theme_minimal())
```

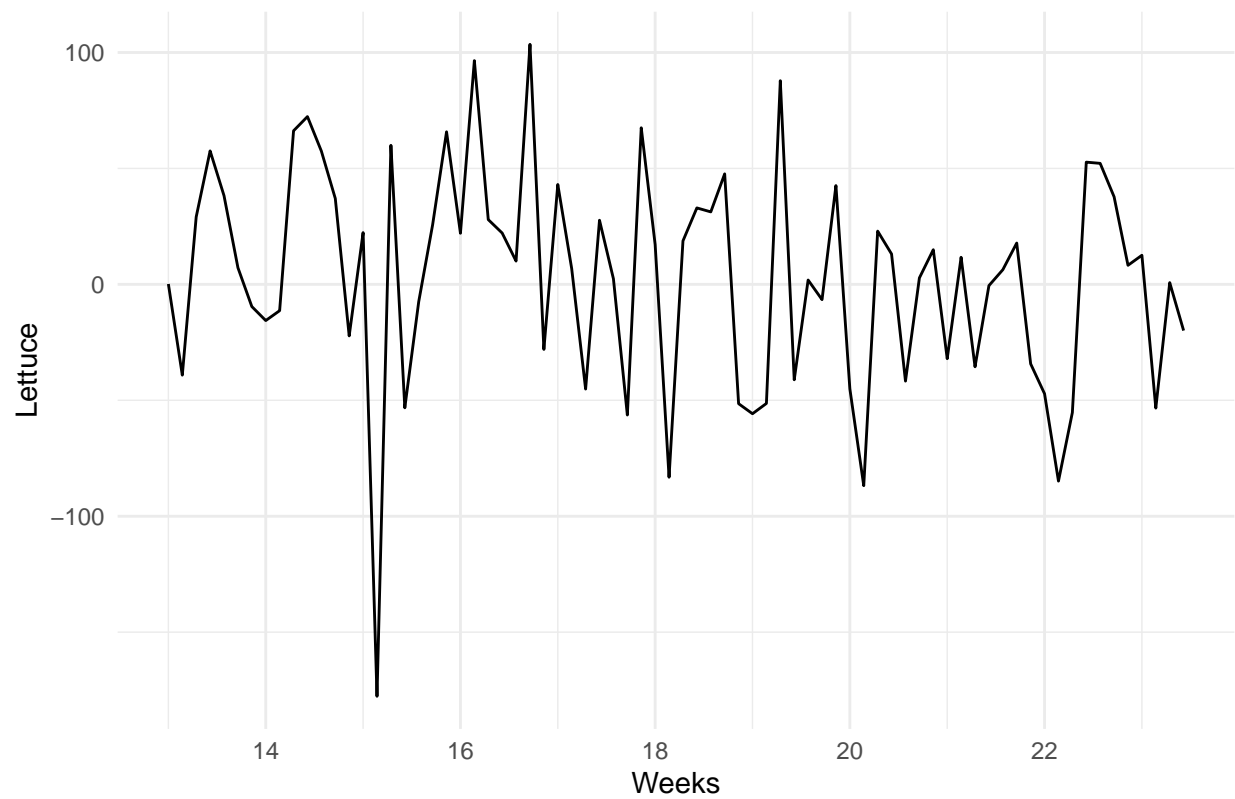


```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(1,1,1)(1,0,0)[7]
## Q* = 9.185, df = 11, p-value = 0.6048
##
## Model df: 3.   Total lags used: 14
```

```
# residual analysis model 2
autoplot(lettuce.m2$residuals, xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +
ggtitle("New York 2 Store (20974) - Residuals model 2 plot")
```

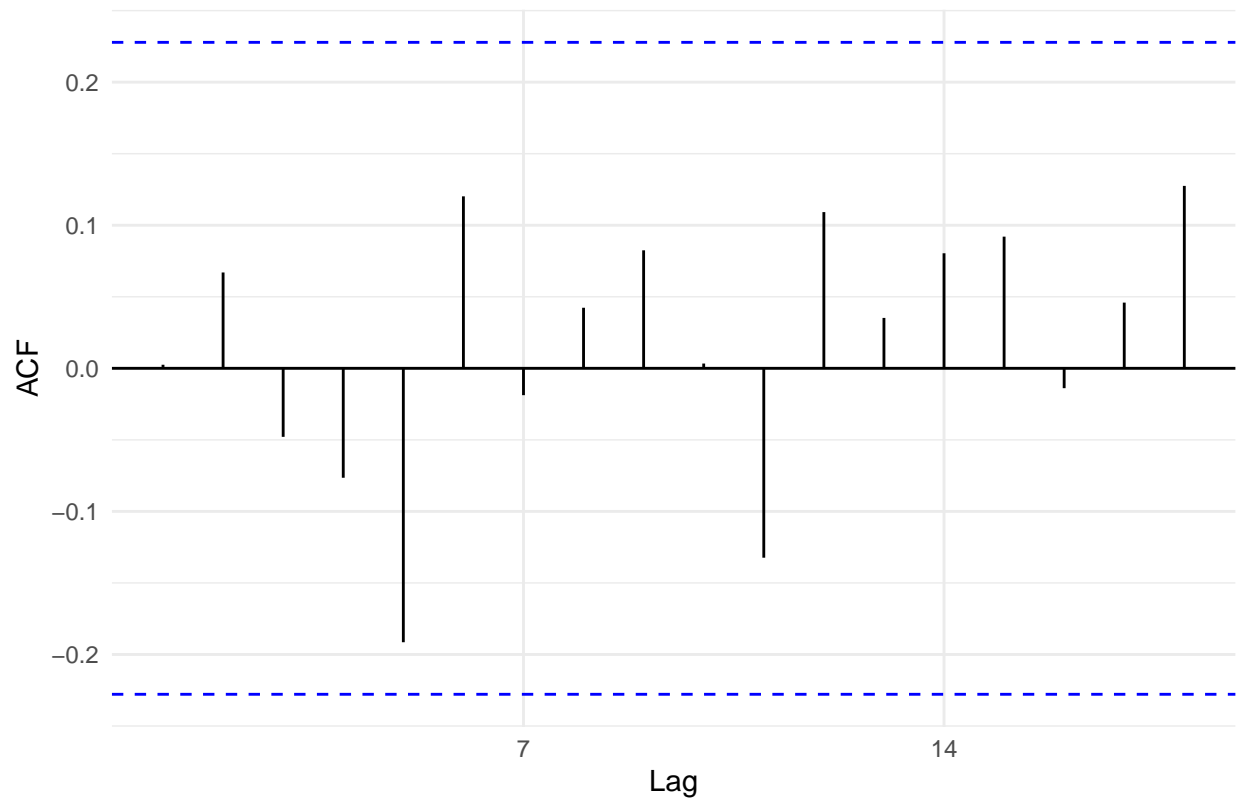


New York 2 Store (20974) – Residuals model 2 plot

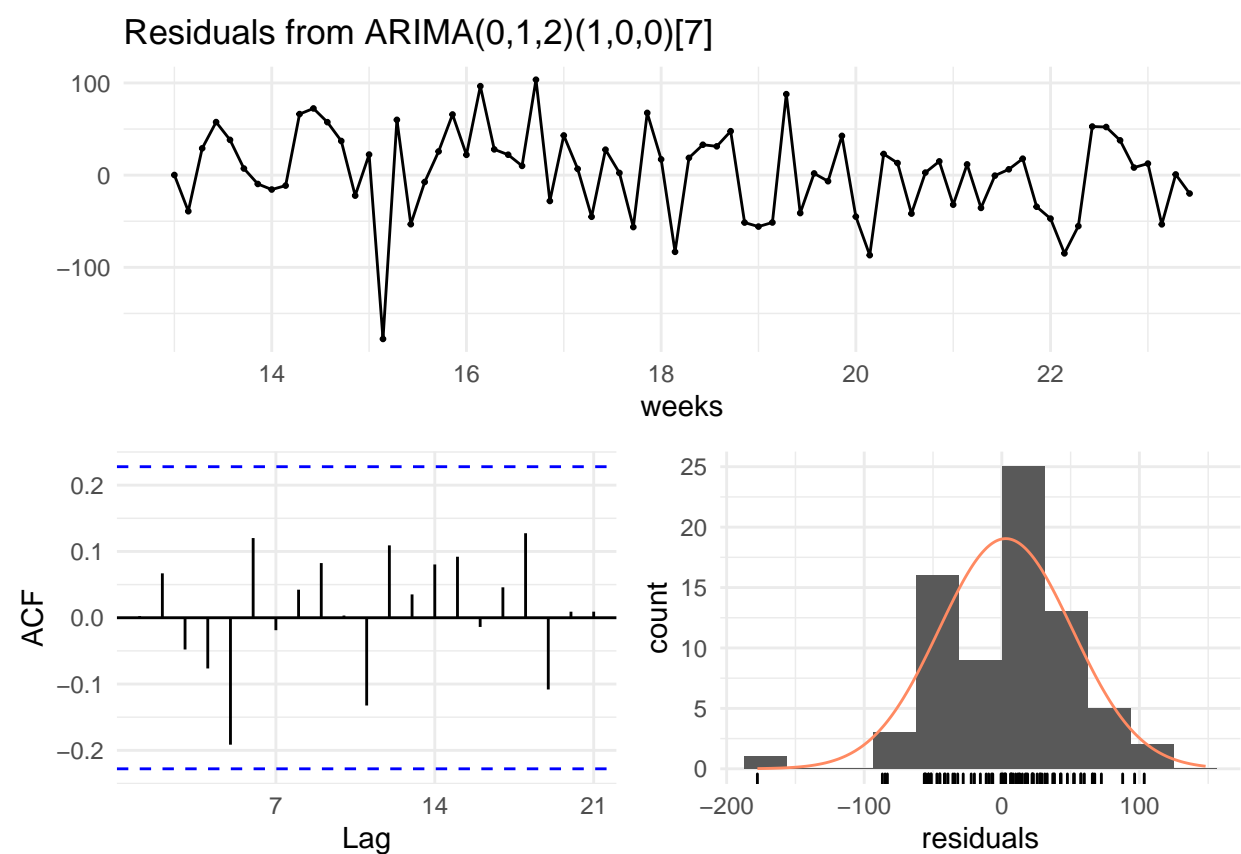


```
ggAcf(lettuce.m2$residuals) + theme_minimal() +  
ggtitle("New York 2 Store (20974) - ACF residuals plot model 2")
```

New York 2 Store (20974) – ACF residuals plot model 2



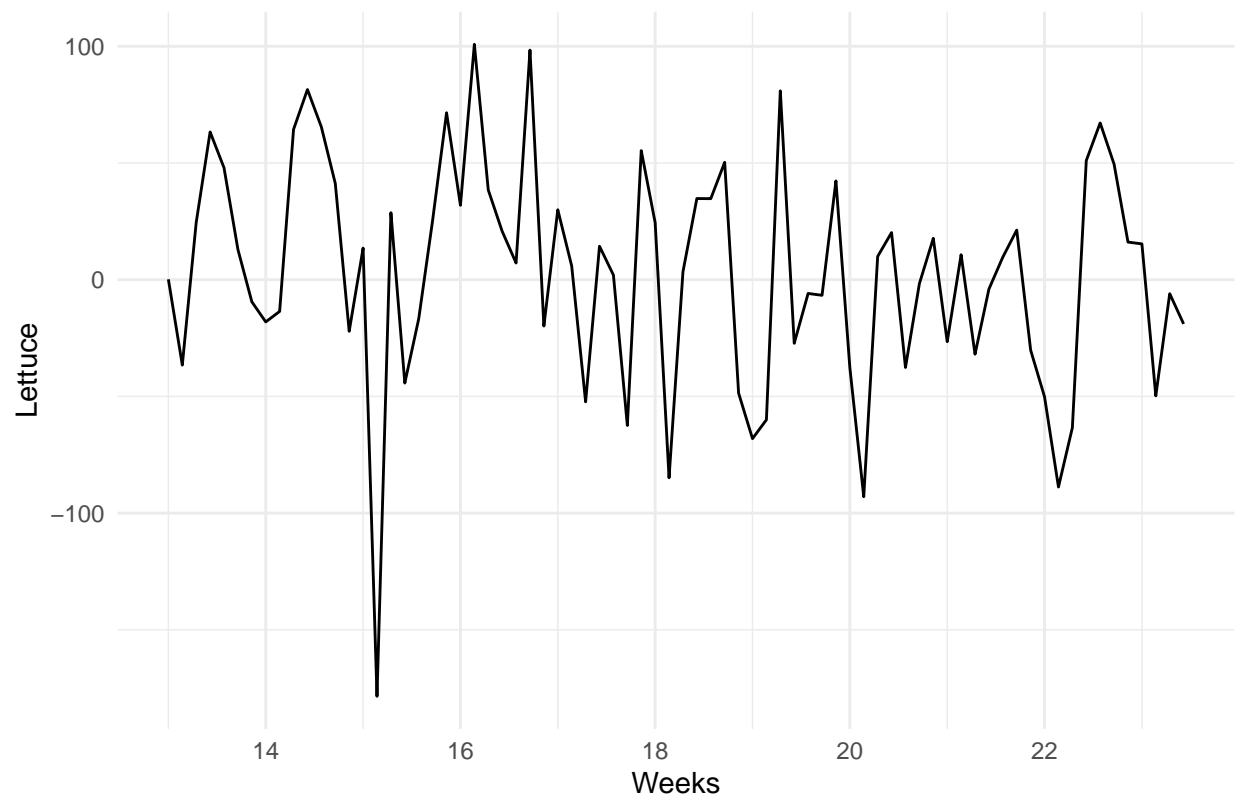
```
checkresiduals(lettuce.m2, xlab = "weeks", theme = theme_minimal())
```



```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,2)(1,0,0)[7]
## Q* = 9.3227, df = 11, p-value = 0.5921
##
## Model df: 3.   Total lags used: 14
```

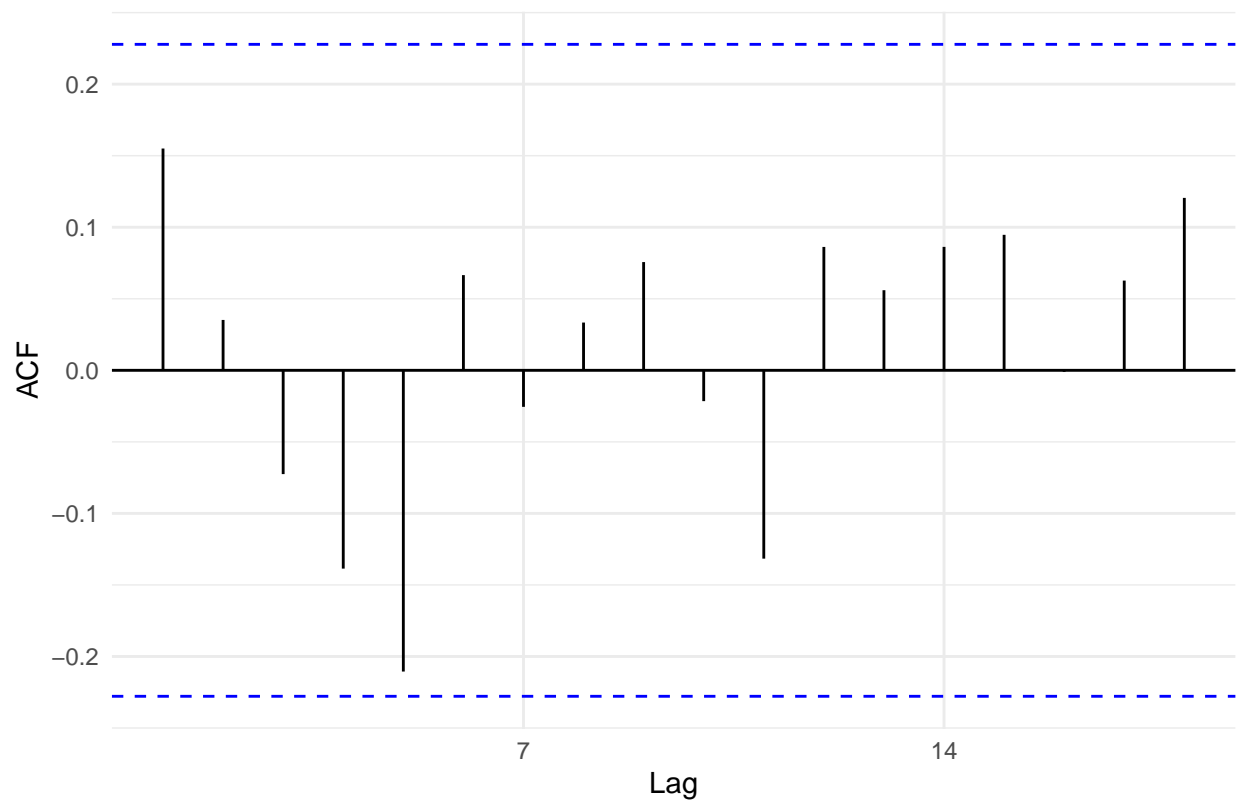
```
# residual analysis model 3
autoplot(lettuce.m3$residuals, xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +
ggtitle("New York 2 Store (20974) - Residuals model 3 plot")
```

New York 2 Store (20974) – Residuals model 3 plot

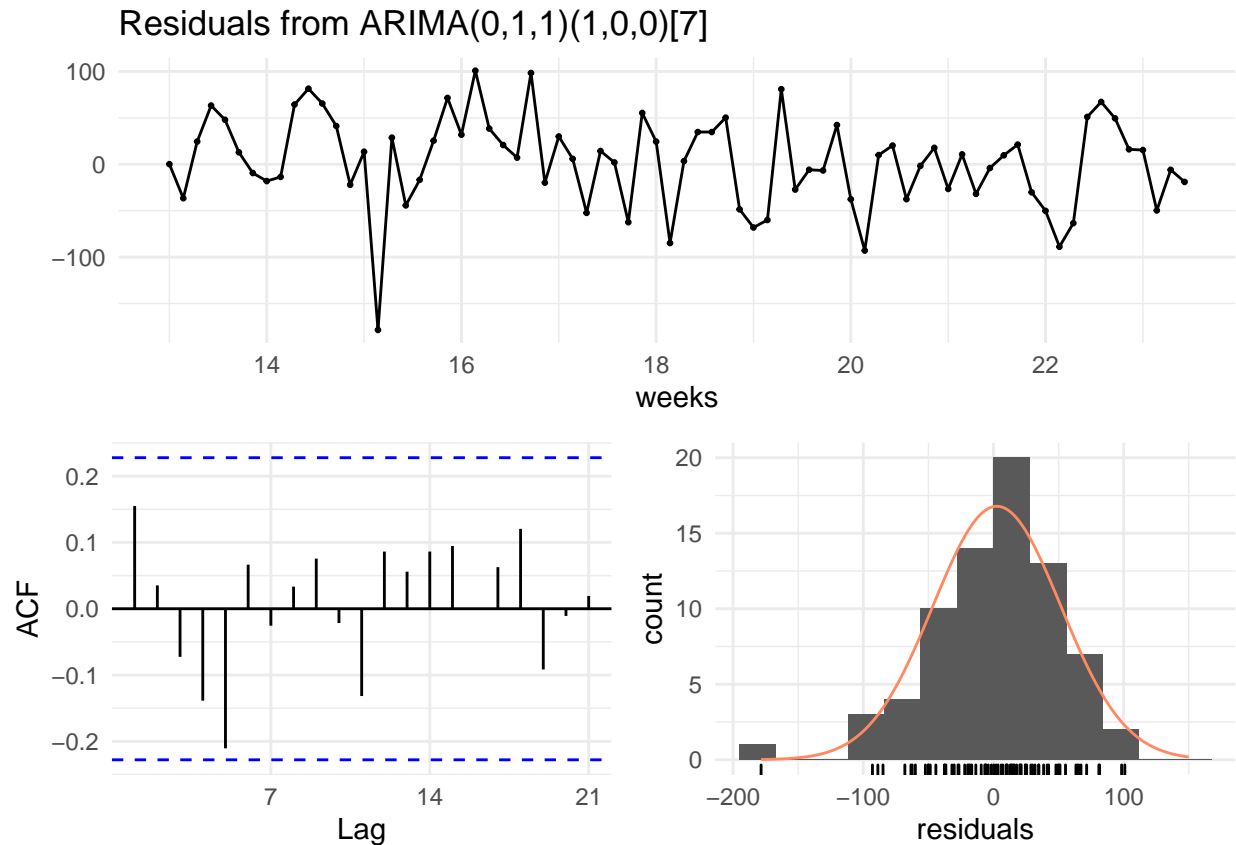


```
ggAcf(lettuce.m3$residuals) + theme_minimal() +  
ggtitle("New York 2 Store (20974) - ACF residuals plot model 3")
```

New York 2 Store (20974) – ACF residuals plot model 3



```
checkresiduals(lettuce.m3, xlab = "weeks", theme = theme_minimal())
```

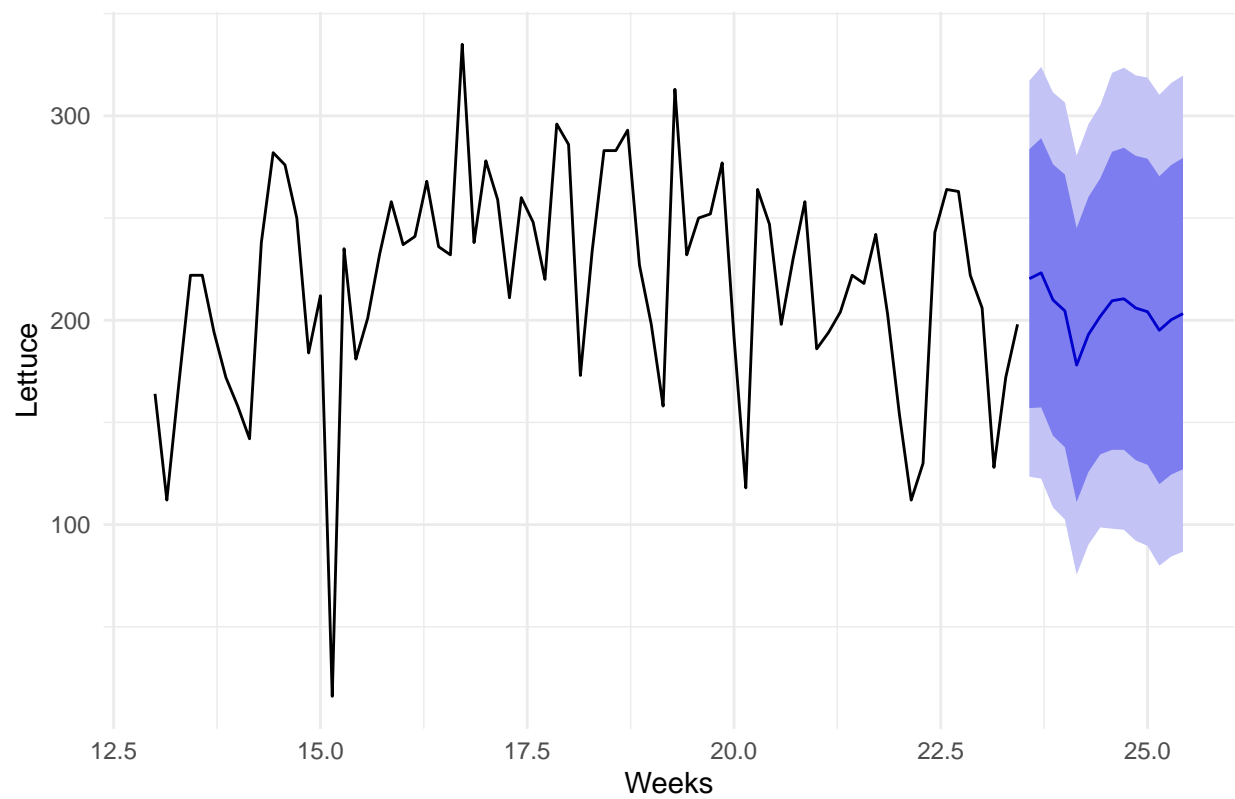


```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,1)(1,0,0)[7]
## Q* = 11.78, df = 12, p-value = 0.4635
##
## Model df: 2.   Total lags used: 14
```

Now we continue with the forecasting part for the three candidate models:

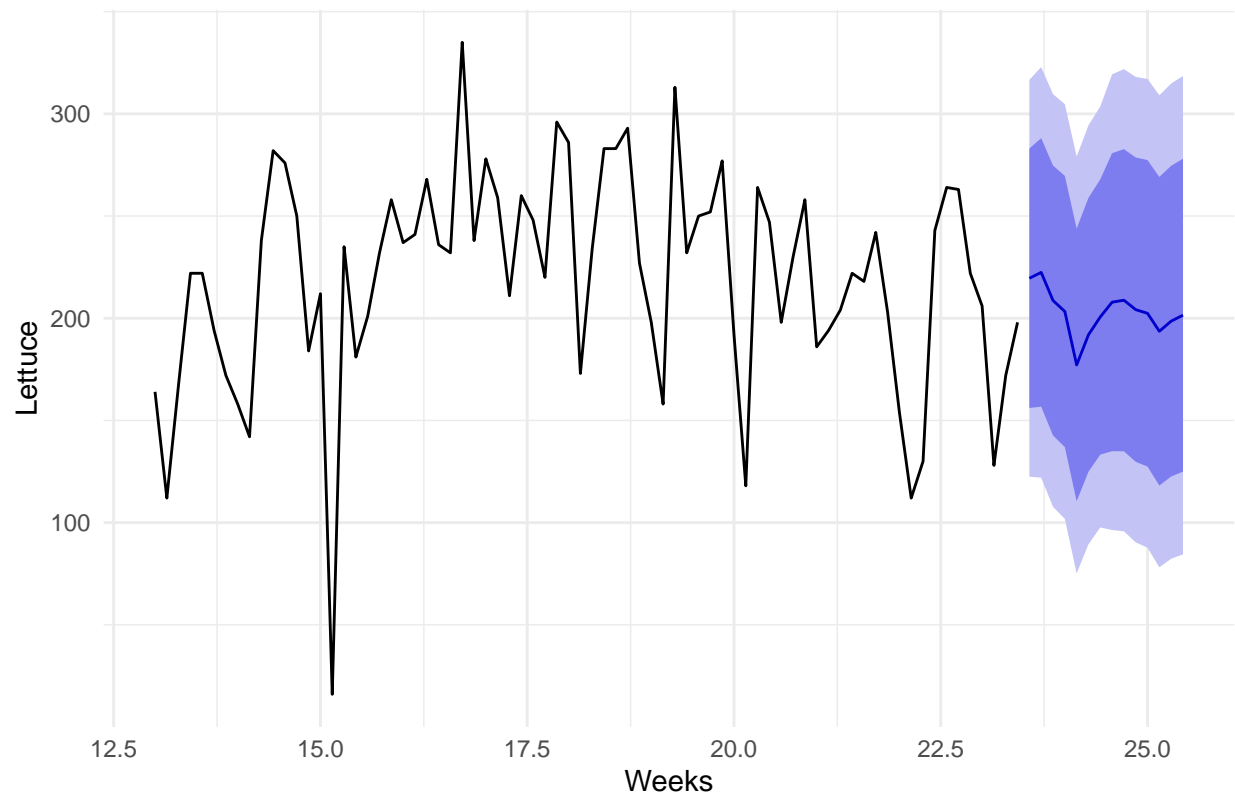
```
#Forecasting part model 1
lettuce.f1 <- forecast(lettuce.m1, h = 14)
autoplot(lettuce.f1, xlab = "Weeks", ylab = "Lettuce") + theme_minimal()
```

Forecasts from ARIMA(1,1,1)(1,0,0)[7]



```
#Forecasting part model 2  
lettuce.f2 <- forecast(lettuce.m2, h = 14)  
autoplot(lettuce.f2, xlab = "Weeks", ylab = "Lettuce") + theme_minimal()
```

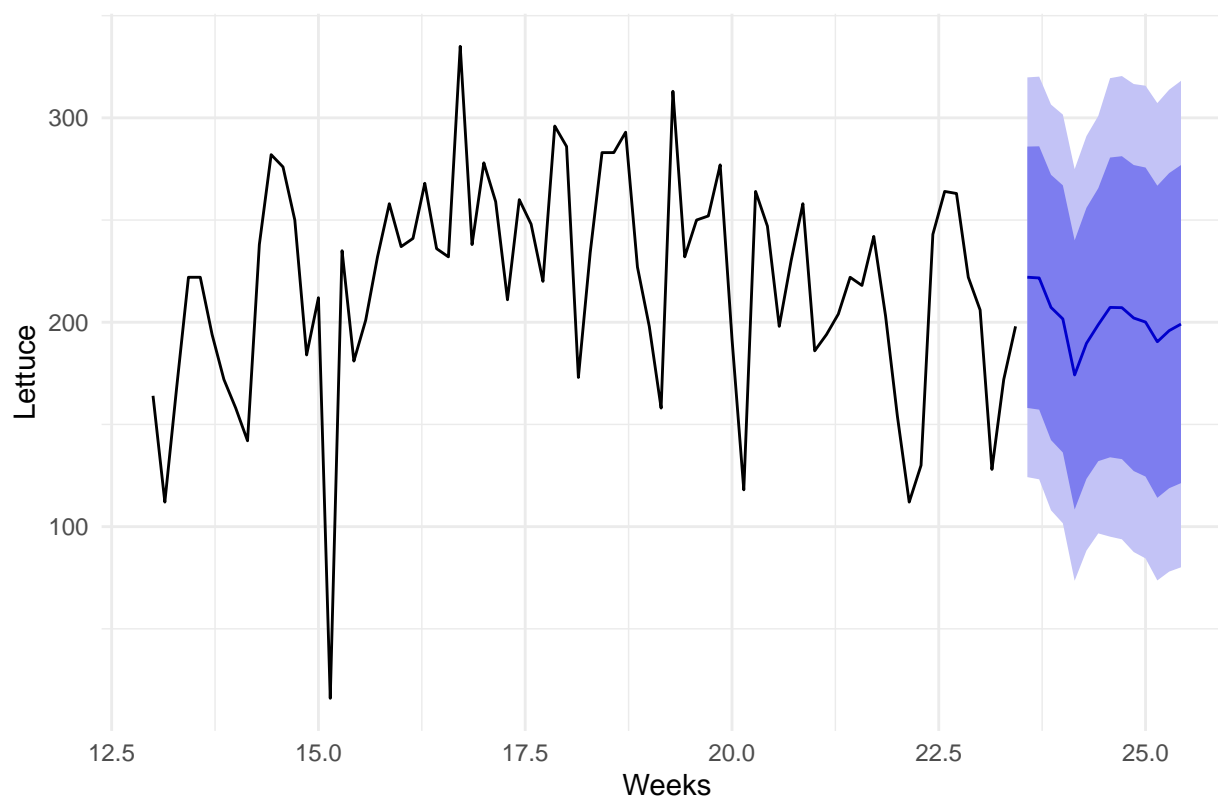
Forecasts from ARIMA(0,1,2)(1,0,0)[7]



```
#Forecasting part model 3  
lettuce.f3 <- forecast(lettuce.m3, h = 14)  
autoplot(lettuce.f3, xlab = "Weeks", ylab = "Lettuce") + theme_minimal()
```



## Forecasts from ARIMA(0,1,1)(1,0,0)[7]



Now we need to test how our models performs for test set. Earlier observations are used for training, and more recent observations are used for testing. Suppose we use the first 80 days of data for training and the last 14 for test. Based on `auto.arima()`, we choose two candidate models with the lowest AICs.

```
### model evaluation
# Apply fitted model to later data
# Accuracy test for candidate model 1
accuracy.m1 <- accuracy(forecast(lettuce.m1, h = 14), lettuce_test)
accuracy.m1
```

```
##
## Training set  3.111878 48.05772 37.70491 -16.059101 31.88792 0.8042754
## Test set     19.080288 55.30611 33.94545  4.536817 13.27772 0.7240832
##
##              ACF1 Theil's U
## Training set -0.02117438    NA
## Test set     -0.11246459  0.801241
```

```
# Accuracy test for candidate model 2
accuracy.m2 <- accuracy(forecast(lettuce.m2, h = 14), lettuce_test)
accuracy.m2
```

```
##
## Training set  2.848727 48.17716 37.76400 -16.373326 32.14221 0.8055358
## Test set     20.453128 55.88913 34.51513  5.176498 13.44700 0.7362348
##
##              ACF1 Theil's U
```

```
## Training set 0.002492967 NA
## Test set -0.108567016 0.8119853
```

```
# Accuracy test for candidate model 3
```

```
accuracy.m3 <- accuracy(forecast(lettuce.m3, h = 14), lettuce_test)
accuracy.m3
```

```
##           ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 2.568306 48.86888 38.11537 -16.67426 32.46622 0.8130309
## Test set    22.125811 56.56734 34.81456  5.99483 13.45120 0.7426220
##           ACF1 Theil's U
## Training set 0.1550136      NA
## Test set    -0.1130414 0.8256285
```

Thus we pick the first model, since it performs better on the test set.

Now we train the first model on the whole date set as follows:

```
# Training on both train and test set
```

```
lettuce.f.both <- Arima(lettuce, order = c(1, 1, 1),
                        seasonal = list(order = c(1, 0, 0), period = 7))
```

Lastly, we forecast lettuce demand for the next 2 weeks.

```
# Forecast for next 14 days
```

```
lettuce.f.final <- forecast(lettuce.f.both, h = 14)
lettuce.f.final
```

```
##           Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 25.57143      235.5873 164.7033 306.4713 127.17952 343.9950
## 25.71429      248.4292 171.6683 325.1901 131.03351 365.8248
## 25.85714      294.1569 213.8089 374.5050 171.27518 417.0387
## 26.00000      217.3068 133.7452 300.8685  89.51031 345.1034
## 26.14286      230.0948 143.4674 316.7222  97.60964 362.5800
## 26.28571      216.2299 126.6454 305.8144  79.22229 353.2375
## 26.42857      251.4221 158.9756 343.8685 110.03736 392.8068
## 26.57143      240.6110 136.4357 344.7863  81.28861 399.9334
## 26.71429      245.1760 135.8092 354.5428  77.91389 412.4381
## 26.85714      261.4312 147.6837 375.1788  87.46933 435.3931
## 27.00000      234.1127 116.2210 352.0044  53.81288 414.4125
## 27.14286      238.6585 116.7733 360.5438  52.25116 425.0659
## 27.28571      233.7299 107.9792 359.4805  41.41080 426.0489
## 27.42857      246.2399 116.7393 375.7405  48.18579 444.2940
```

We present our forecast through ARIMA(1,1,1)(1,0,0) model for each of the next 14 days.

```
forecast_data <- as.data.frame(lettuce.f.final)
next2weeks <- data.frame(day = seq(1, 14))
final_forecast_NewYork2_arma <- cbind(next2weeks, forecast_data$`Point Forecast`)
final_forecast_NewYork2_arma
```

##	day	forecast_data\$`Point Forecast`
## 1	1	235.5873
## 2	2	248.4292
## 3	3	294.1569
## 4	4	217.3068
## 5	5	230.0948
## 6	6	216.2299
## 7	7	251.4221
## 8	8	240.6110
## 9	9	245.1760
## 10	10	261.4312
## 11	11	234.1127
## 12	12	238.6585
## 13	13	233.7299
## 14	14	246.2399

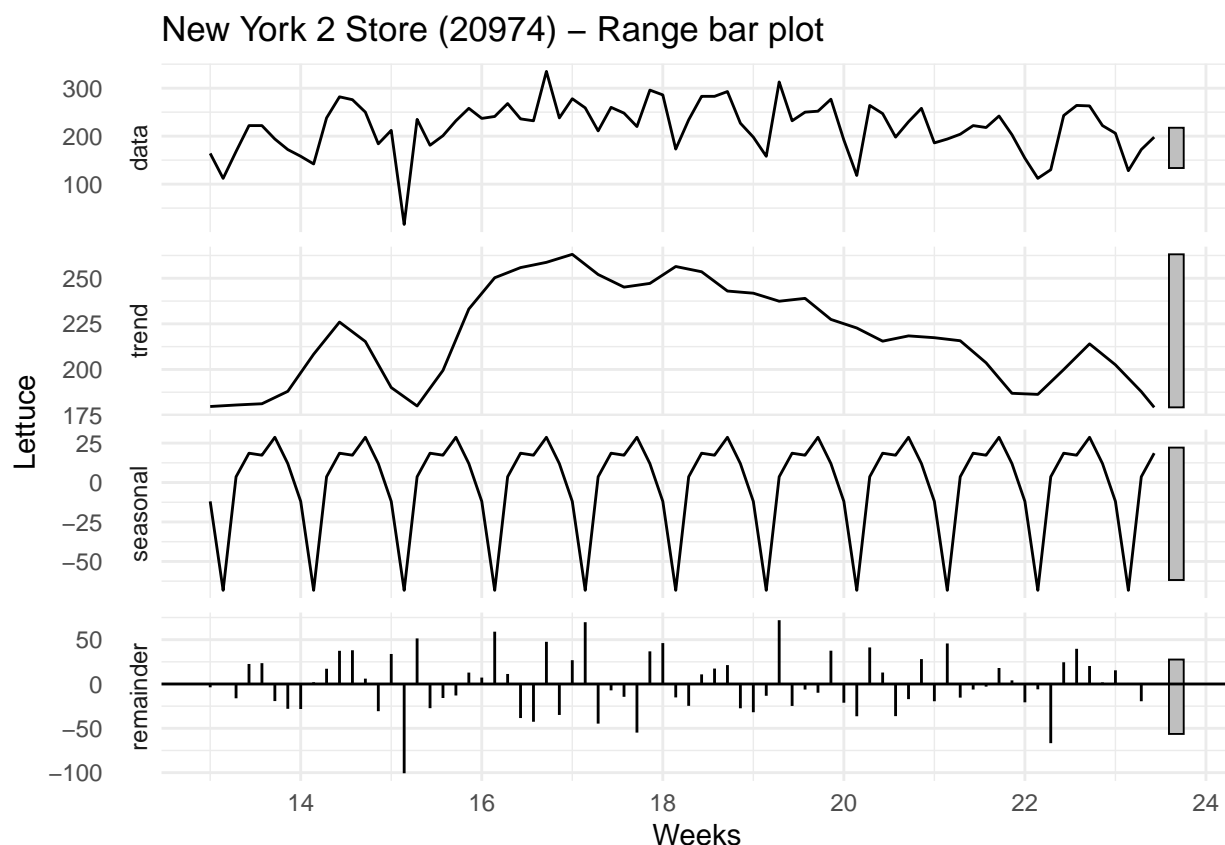
## Holt-Winters

Now we will use another model to forecast lettuce demand. Our goal is to pick the model with the most accurate predictions.

We will forecast the lettuce demand for next two weeks using Holt-Winters model.

For time series analysis, the first step is always to visually inspect the time series. In this regard, the `stl()` function is quite useful. It decomposes the original time series into trend, seasonal factors, and random error terms. The relative importance of different components are indicated by the grey bars in the plots.

```
lettuce_train %>% stl(s.window = "period") %>%
autoplot(xlab = "Weeks", ylab = "Lettuce") + theme_minimal() +
ggtitle("New York 2 Store (20974) - Range bar plot")
```



For this data set, the grey bar of the trend panel is significantly larger than that on the original time series panel, which indicates that the contribution of the trend component to the variation in the original time series is marginal.

The grey bar of the seasonal panel is also large, and larger than the grey bar of random error term, which indicates that the contribution of seasonal factors to the variation in the original time series is marginal too. In other words, it indicates that there is no seasonality in the data.

With `ets()`, initial states and smoothing parameters are jointly estimated by maximizing the likelihood function. We need to specify the model in `ets()` using three letters. The way to approach this is: (1) check out time series plot, and see if there is any trend and seasonality; (2) run `ets()` with `model = "ZZZ"`, and see whether the best model is consistent with your expectation; (3) if they are consistent, it gives us confidence that our model specification is correct; otherwise try to figure out why there is a discrepancy.

We now use `ets` function as previously indicated to find our best model:

```
# using ets
lettuce.ets2 <- ets(lettuce_train, model = "ZZZ")
lettuce.ets2
```

```
## ETS(A,N,A)
##
## Call:
## ets(y = lettuce_train, model = "ZZZ")
##
## Smoothing parameters:
##   alpha = 0.1905
##   gamma = 1e-04
```

```
##
## Initial states:
## l = 209.6333
## s = 12.0706 29.4123 17.4096 18.8407 4.0938 -65.3266
## -16.5004
##
## sigma: 43.4163
##
## AIC AICc BIC
## 886.9883 890.4803 910.0289
```

Our best model is the ETS(A,N,A).

```
# using ets
lettuce.ets <- ets(lettuce_train, model = "ANA", ic = 'aic')
lettuce.ets
```

```
## ETS(A,N,A)
##
## Call:
## ets(y = lettuce_train, model = "ANA", ic = "aic")
##
## Smoothing parameters:
## alpha = 0.1905
## gamma = 1e-04
##
## Initial states:
## l = 209.6333
## s = 12.0706 29.4123 17.4096 18.8407 4.0938 -65.3266
## -16.5004
##
## sigma: 43.4163
##
## AIC AICc BIC
## 886.9883 890.4803 910.0289
```

After estimation, we can use `accuracy()` function to determine in-sample fit and `forecast()` function to generate forecast.

Similarly with ARIMA model, we use AIC to determine our best model in terms of best in-sample performance.

```
# in-sample one-step forecast
accuracy(lettuce.ets)
```

```
## ME RMSE MAE MPE MAPE MASE
## Training set -0.9560356 40.69057 33.6634 -13.46909 26.18756 0.7180669
## ACF1
## Training set 0.08618948
```

We present the in-sample forecast part for the ets model as follows:

```
# best model
lettuce.ets.f <- forecast(lettuce.ets, h = 14)
lettuce.ets.f
```

```
##          Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 23.57143      213.5625 157.92223 269.2027 128.46806 298.6569
## 23.71429      225.5651 168.92369 282.2064 138.93956 312.1905
## 23.85714      208.2230 150.59786 265.8480 120.09298 296.3529
## 24.00000      179.6553 121.06295 238.2476  90.04606 269.2644
## 24.14286      130.8193  71.27544 190.3631  39.75486 221.8837
## 24.28571      200.2432 139.76289 260.7236 107.74653 292.7399
## 24.42857      214.9915 153.58779 276.3952 121.08264 308.9003
## 24.57143      213.5625 151.25019 275.8748 118.26406 308.8609
## 24.71429      225.5651 162.35723 288.7729 128.89704 322.2331
## 24.85714      208.2230 144.13212 272.3138 110.20448 306.2414
## 25.00000      179.6553 114.69340 244.6171  80.30467 279.0058
## 25.14286      130.8193  64.99791 196.6406  30.15420 231.4843
## 25.28571      200.2432 133.57348 266.9130  98.28064 302.2058
## 25.42857      214.9915 147.48298 282.5000 111.74614 318.2368
```

After the forecast, we continue with the in and out of sample accuracy of the two ets models.

```
# Out of sample accuracy
# best model
accuracy.ets <- accuracy(lettuce.ets.f, lettuce_test)
accuracy.ets
```

```
##          ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.9560356 40.69057 33.66340 -13.46909 26.18756 0.7180669
## Test set     27.2057558 55.99135 33.87778   9.46673 13.38155 0.7226398
##          ACF1 Theil's U
## Training set  0.08618948      NA
## Test set     -0.29493593 0.8497462
```

We now train our best model - ETS(A,N,A) on the whole data set as indicated below:

```
# final model
lettuce.ets <- ets(lettuce, model = "ANA", ic = 'aic')
lettuce.ets
```

```
## ETS(A,N,A)
##
## Call:
## ets(y = lettuce, model = "ANA", ic = "aic")
##
## Smoothing parameters:
##   alpha = 0.44
##   gamma = 1e-04
##
## Initial states:
##   l = 114.9789
##   s = -10.1021 18.4806 24.8543 9.6888 15.7566 -0.3719
```

```
##          -58.3063
##
##   sigma:  53.2174
##
##      AIC      AICc      BIC
## 1184.794 1187.444 1210.227
```

We now present the out-of-sample forecast for the next 14 days (2 weeks) as seen below:

```
lettuce.ets.f <- forecast(lettuce.ets, h = 14)
lettuce.ets.f
```

##	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
## 25.57143	239.2185	171.01765	307.4193	134.91430	343.5227
## 25.71429	254.3795	179.86998	328.8891	140.42701	368.3321
## 25.85714	248.0047	167.68037	328.3289	125.15926	370.8500
## 26.00000	219.4262	133.68063	305.1718	88.28965	350.5628
## 26.14286	171.2160	80.37199	262.0599	32.28210	310.1498
## 26.28571	229.1470	133.47598	324.8180	82.83080	375.4632
## 26.42857	245.2863	145.01829	345.5543	91.93962	398.6329
## 26.57143	239.2185	134.55720	343.8798	79.15286	399.2841
## 26.71429	254.3795	145.50206	363.2570	87.86581	420.8933
## 26.85714	248.0047	135.06829	360.9410	75.28340	420.7259
## 27.00000	219.4262	102.57188	336.2806	40.71292	398.1395
## 27.14286	171.2160	50.57079	291.8611	-13.29491	355.7268
## 27.28571	229.1470	104.82654	353.4674	39.01527	419.2787
## 27.42857	245.2863	117.39452	373.1780	49.69272	440.8798

We present our forecast for each of the next 14 days.

```
forecast_data <- as.data.frame(lettuce.ets.f)
next2weeks <- data.frame(day = seq(1, 14))
final_forecast_NewYork2_ets <- cbind(next2weeks, forecast_data$`Point Forecast`)
final_forecast_NewYork2_ets
```

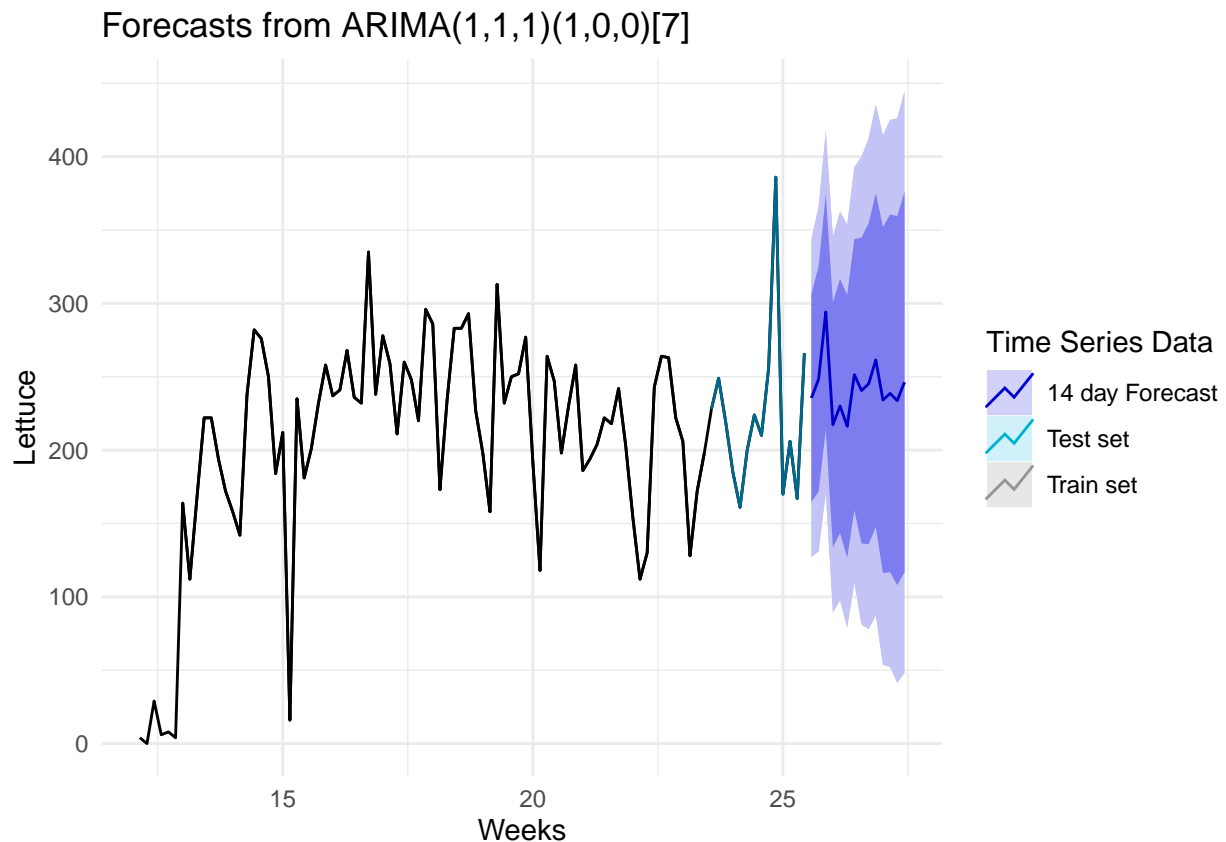
##	day	forecast_data\$`Point Forecast`
## 1	1	239.2185
## 2	2	254.3795
## 3	3	248.0047
## 4	4	219.4262
## 5	5	171.2160
## 6	6	229.1470
## 7	7	245.2863
## 8	8	239.2185
## 9	9	254.3795
## 10	10	248.0047
## 11	11	219.4262
## 12	12	171.2160
## 13	13	229.1470
## 14	14	245.2863

## Comparison

Now we will compare the two best models for New York 2 Store (20974).

We plot time series data for train and test set and also the forecasts from our two models as indicated below:

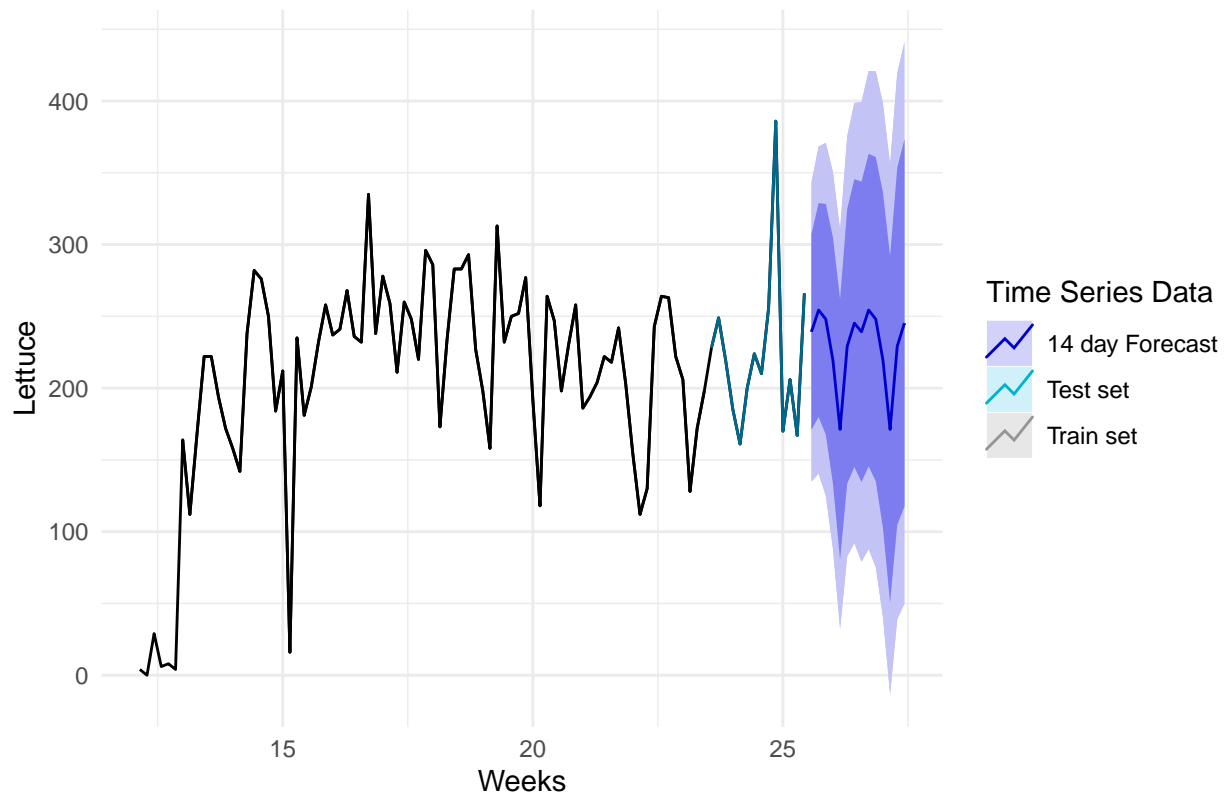
```
colours <- c("blue", "deepskyblue4", "black")
autoplot(lettuce.f.final, xlab = "Weeks", ylab = "Lettuce") +
  autolayer(lettuce_train, series = "Train set") +
  autolayer(lettuce_test, series = "Test set") +
  autolayer(lettuce.f.final, series = "14 day Forecast") +
  guides(colour = guide_legend(title = "Time Series Data")) +
  scale_colour_manual(values = colours) + theme_minimal()
```



```
autoplot(lettuce.ets.f, xlab = "Weeks", ylab = "Lettuce") +
  autolayer(lettuce_train, series = "Train set") +
  autolayer(lettuce_test, series = "Test set") +
  autolayer(lettuce.ets.f, series = "14 day Forecast") +
  guides(colour = guide_legend(title = "Time Series Data")) +
  scale_colour_manual(values = colours) + theme_minimal()
```



## Forecasts from ETS(A,N,A)



In order to decide which of the two models  $ARIMA(0,1,2)(1,0,0)$  or  $ETS(A,N,A)$  to choose, we will check their RMSE in the test set.

```
# best ets model
# ETS(A,N,A)
accuracy.ets
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE
## Training set -0.9560356 40.69057 33.66340 -13.46909 26.18756 0.7180669
## Test set     27.2057558 55.99135 33.87778   9.46673 13.38155 0.7226398
##               ACF1 Theil's U
## Training set  0.08618948      NA
## Test set      -0.29493593 0.8497462
```

```
# best arima model
# ARIMA(1,1,1)(1,0,0)
accuracy.m2
```

```
##               ME      RMSE      MAE      MPE      MAPE      MASE
## Training set  2.848727 48.17716 37.76400 -16.373326 32.14221 0.8055358
## Test set      20.453128 55.88913 34.51513   5.176498 13.44700 0.7362348
##               ACF1 Theil's U
## Training set  0.002492967      NA
## Test set      -0.108567016 0.8119853
```

We can observe that ARIMA(1,1,1)(1,0,0) has a better (lower) RMSE (41.21143 vs 55.99135) respectively. Therefore, we choose the ARIMA(1,1,1)(1,0,0) for New York (20974) store. Hence, our forecast for lettuce demand of next 2 weeks for that store is the following:

```
final_forecast_NewYork2_arima
```

```
##      day forecast_data$`Point Forecast`
## 1      1                235.5873
## 2      2                248.4292
## 3      3                294.1569
## 4      4                217.3068
## 5      5                230.0948
## 6      6                216.2299
## 7      7                251.4221
## 8      8                240.6110
## 9      9                245.1760
## 10     10                261.4312
## 11     11                234.1127
## 12     12                238.6585
## 13     13                233.7299
## 14     14                246.2399
```