# Geometrijske transformacije i interpolacija

# Sadržaj

- Osnovne globalne geometrijske transformacije
  - Translacija
  - Skaliranje
  - Rotiranje
  - Nakošenje
  - Opća afina transformacija
  - Projekcijska transformacija
- Homogene koordinate
- Unaprijedna i unatražna transformacija
- Interpolacija
  - Najbližim susjedom
  - Bilinearna
  - Bikubična
- Radijalna distorzija
- Deformacijska polja

# Slikovne transformacije

Ulazna slika: $f(x,y)$
Izlazna slika: $g(x,y)$

a) Transformacije intenziteta:

$g(x,y) = Tr(f(x,y))$

- mijenja kodomenu slike (engl. range) – intenzitete piksela

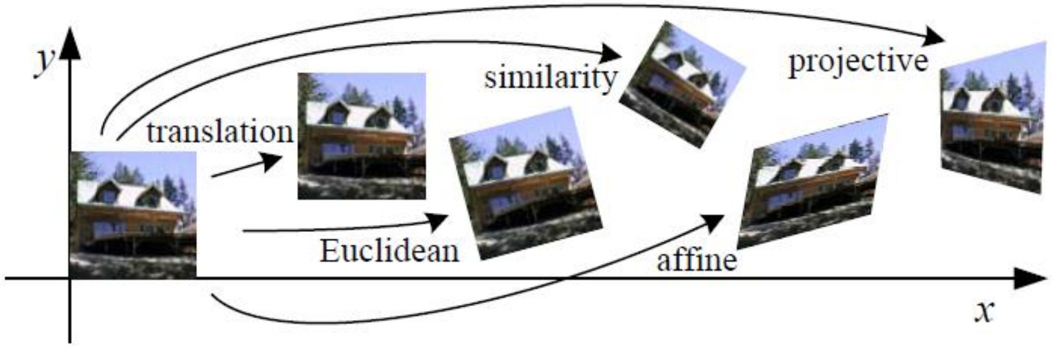
-> ne proučavamo ovdje, njima se bavi područje obrade slike (≠ računalni vid)

b) Geometrijske transformacije (engl. warping):

$g(x,y) = f(Td(x,y))$

- mijenja domenu (engl. domain) – lokacije piksela / oblik slike
- ovdje se bavimo njima

# Hijerarhija geometrijskih transformacija




| Transformation | Matrix | # DoF | Preserves | Icon |
|---|---|---|---|---|
| translation | $\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2\times 3}$ | 2 | orientation | |
| rigid (Euclidean) | $\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2\times 3}$ | 3 | lengths | |
| similarity | $\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2\times 3}$ | 4 | angles | |
| affine | $\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2\times 3}$ | 6 | parallelism | |
| projective | $\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3\times 3}$ | 8 | straight lines | |

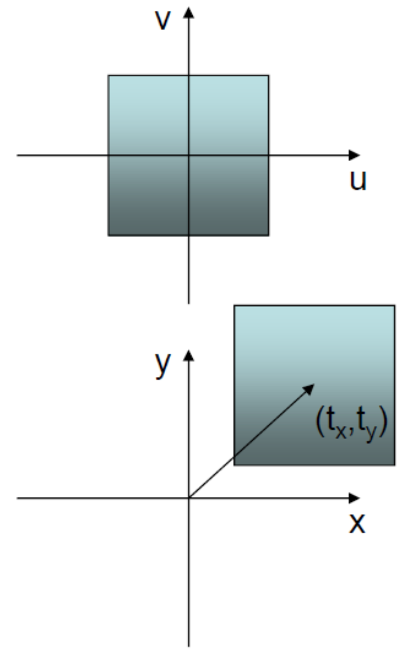
(Szeliski 2022)

# Translacija

$x' = x + tx$

$y' = y + ty$

- $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$

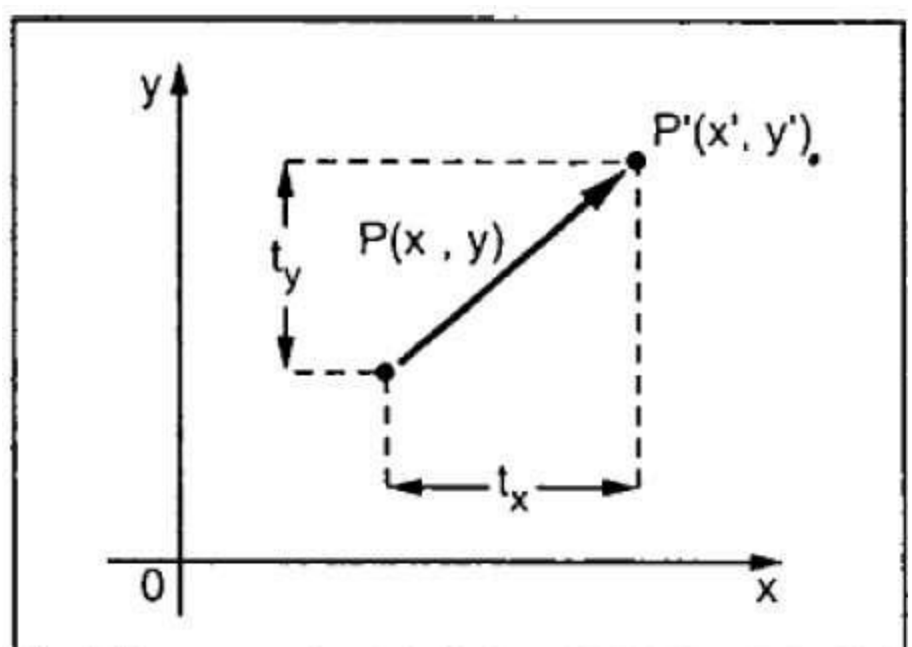



Inverzna translacija:

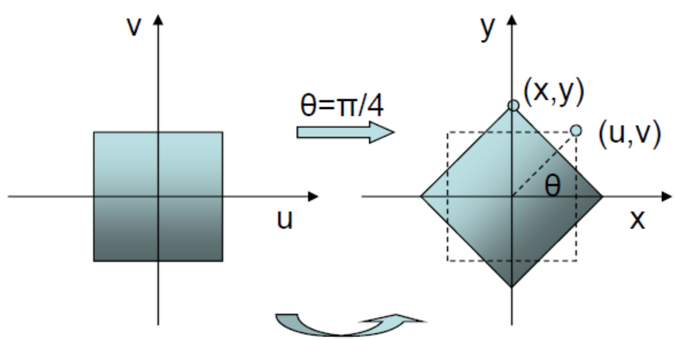
$x = x' - tx$

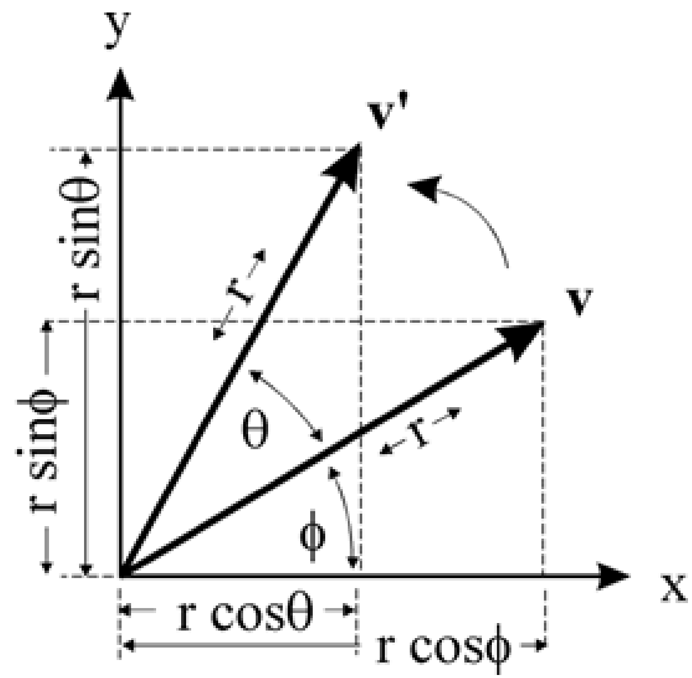
$y = y' - ty$

- $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} - \begin{bmatrix} tx \\ ty \end{bmatrix}$



# Rotacija




(https://math.stackexchange.com/questions/346672/2d-rotation-of-point-about-origin)

*Početna točka: v=(x,y)*

$x = r\cos\phi; y = r\sin\phi$

*Konačna točka: v'=(x',y')*

$x' = r\cos(\phi+\theta)$

$x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta$

$x' = x\cos\theta - y\sin\theta$

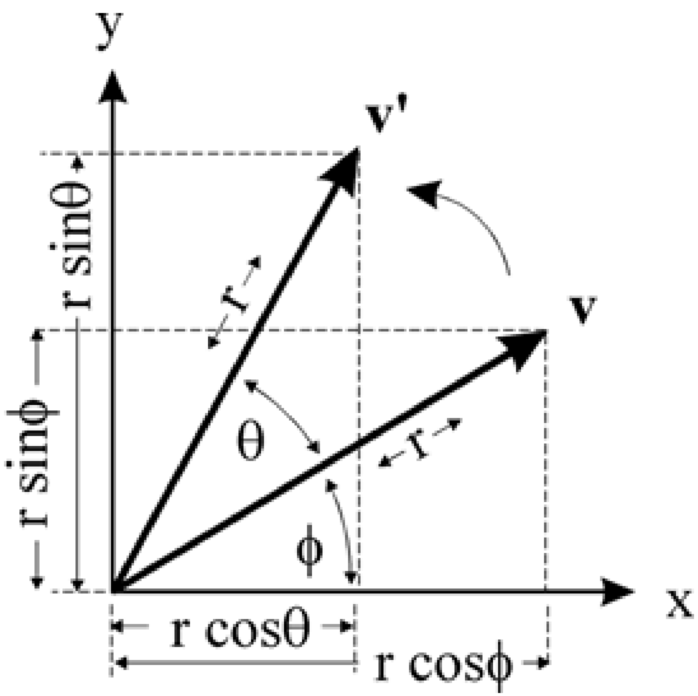
$y' = r\sin(\phi+\theta)$

$y' = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$y' = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Inverzna rotacija




(https://math.stackexchange.com/questions/346672/2d-rotation-of-point-about-origin)

Inverzna rotacija = rotacija za –θ

Vrijedi:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

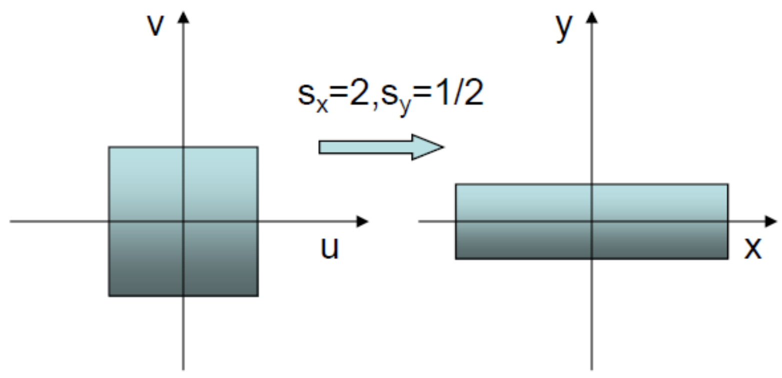
pa je :

$x = x'\cos\theta + y'\sin\theta$

$y = -x'\sin\theta + y'\cos\theta$

- $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$

# Skaliranje




$x' = Sx*x$
$y' = Sy*y$

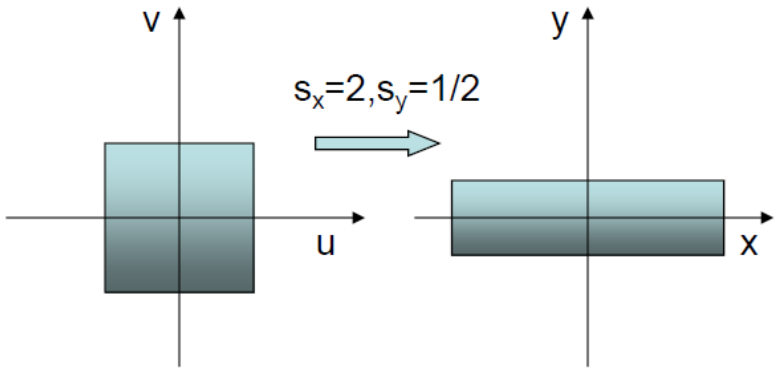
- Opći slučaj:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Jednoliko u oba smjera:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Inverzno skaliranje




*x = (1/Sx)*x'*
*y = (1/Sy)*y'*

- Opći slučaj:

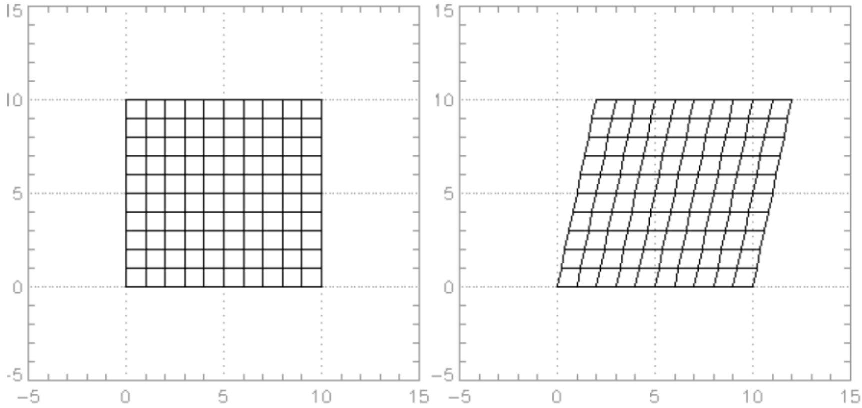
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/Sx & 0 \\ 0 & 1/Sy \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- Jednoliko:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/S & 0 \\ 0 & 1/S \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

# Nakošenje

- (engl. skew, shear)




- Horizontalno:

  $x' = x + a*y$
  $y' = y$

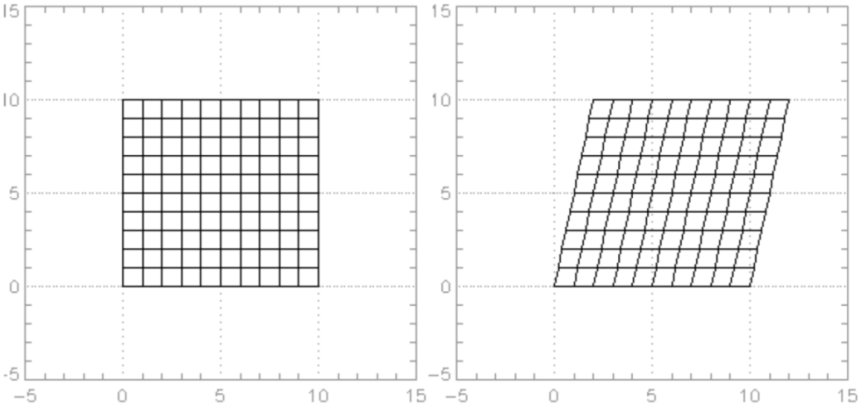
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Vertikalno:

  $x' = x$
  $y' = y + a*x$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Inverzno nakošenje




- Horizontalno:

  $x' = x - a*y$

  $y' = y$

  $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- Vertikalno:

  $x' = x$

  $y' = y - ax$

  $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -a & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

# Opća afina transformacija

- Kombinacija skaliranja, rotacije, nakošenja i translacije

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

Možemo pisati:

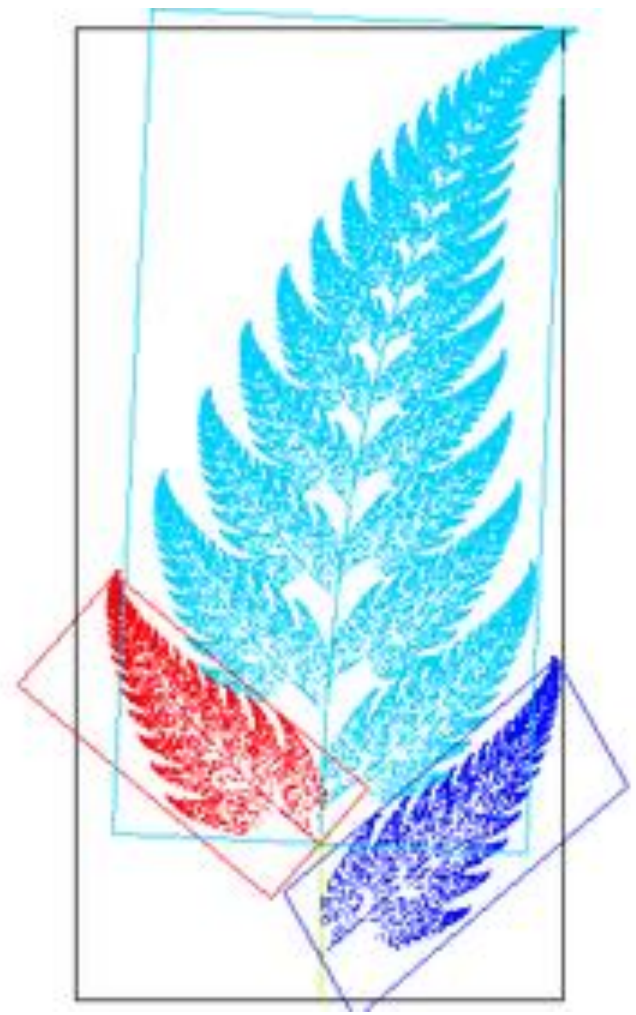
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a11 & a12 & tx \\ a21 & a22 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = A \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} RSW & | T \\ 0\ 0 & |1 \end{bmatrix}$$

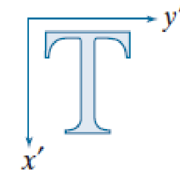
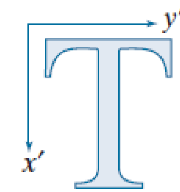
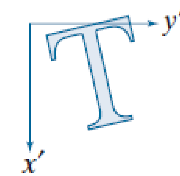
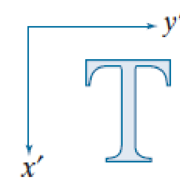
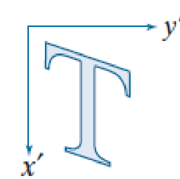
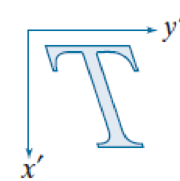
(R, S i W su matrice, redom, rotacije, skaliranja i nakošenja, a T vektor pomaka (translacije))

Inverz:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = A^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$



# Afine transformacije

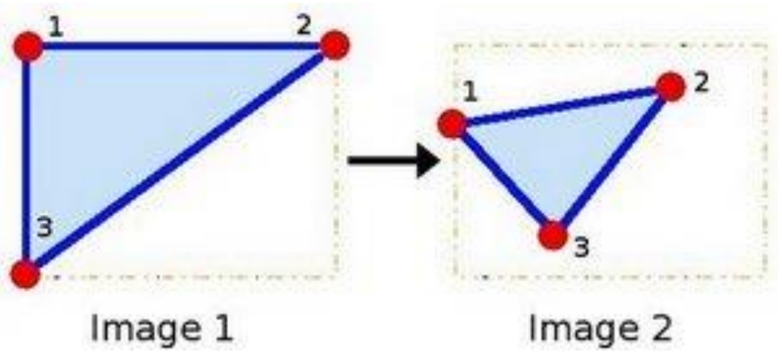
| Transformation Name | Affine Matrix, A | Coordinate Equations | Example |
|---|---|---|---|
| Identity | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x$<br>$y' = y$ |  |
| Scaling/Reflection (For reflection, set one scaling factor to –1 and the other to 0) | $\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = c_x x$<br>$y' = c_y y$ |  |
| Rotation (about the origin) | $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x\cos\theta - y\sin\theta$<br>$y' = x\sin\theta + y\cos\theta$ |  |
| Translation | $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x + t_x$<br>$y' = y + t_y$ |  |
| Shear (vertical) | $\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x + s_v y$<br>$y' = y$ |  |
| Shear (horizontal) | $\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x$<br>$y' = s_h x + y$ |  |

(Gonzales & Woods, 2018)

# Određivanje afine transformacije

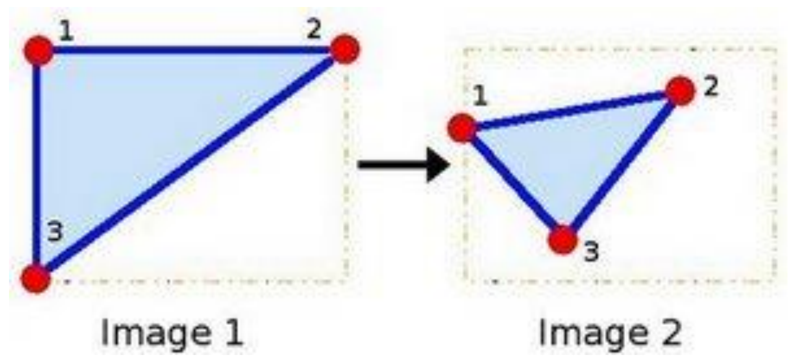
- $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$
- Potrebno odrediti 6 nepoznanica:
  **a11, a12, a21, a22, tx i ty**
- Uz poznata 3 para korespodentnih točaka (3 nekolinearne točke u izvornoj slici i odgovarajuće 3 točke u transformiranoj slici) možemo postaviti sustav 6 jednadžbi s 6 nepoznanica:

$$\begin{aligned} a11x1 + a12y1 + tx = x1' \\ a21x1 + a22y1 + ty = y1' \\ a11x2 + a12y2 + tx = x2' \\ a21x2 + a22y2 + ty = y2' \\ a11x3 + a12y3 + tx = x3' \\ a21y3 + a22y3 + ty = y3' \end{aligned}$$



# Određivanje afine transformacije

$$
\begin{aligned}
a11x1 + a12y1 + tx &= x1' \\
a21x1 + a22y1 + ty &= y1' \\
a11x2 + a12y2 + tx &= x2' \\
a21x2 + a22y2 + ty &= y2' \\
a11x3 + a12y3 + tx &= x3' \\
a21y3 + a22y3 + ty &= y3'
\end{aligned}
$$



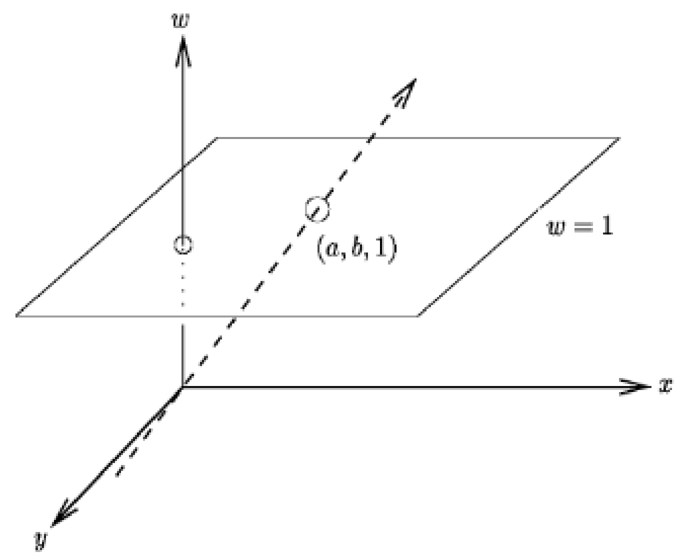

Matrični zapis:

$$
\begin{bmatrix}
x1 & y1 & 0 & 0 & 1 & 0 \\
0 & 0 & x1 & y1 & 0 & 1 \\
x2 & y2 & 0 & 0 & 1 & 0 \\
0 & 0 & x2 & y2 & 0 & 1 \\
x3 & y3 & 0 & 0 & 1 & 0 \\
0 & 0 & x3 & y3 & 0 & 1
\end{bmatrix}
\begin{bmatrix} a11 \\ a12 \\ a21 \\ a22 \\ tx \\ ty \end{bmatrix}
=
\begin{bmatrix} x1' \\ y1' \\ x2' \\ y2' \\ x3' \\ y3' \end{bmatrix}
$$

Ax = b => npr. **`x = np.linalg.solve(A,b)`**

# Homogene koordinate

- $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \widetilde{w}x \\ \widetilde{w}y \\ \widetilde{w} \end{bmatrix} \rightarrow \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \widetilde{w} \end{bmatrix}$
- $Nazad\ u\ nehomogene$: $x = \frac{\tilde{x}}{\widetilde{w}}; y = \frac{\tilde{y}}{\widetilde{w}}$
- Geometrijska interpretacija
  - Prelazimo u 3D prostor $(\tilde{x}, \tilde{y}, \widetilde{w})$
  - Ravnina $\widetilde{w} = 1$ predstavlja izvornu 2D ravninu
  - Sve točke na pravcu iz ishodišta 3D koord. sustava kroz točku u ravnini $\widetilde{w} = 1$ smatraju se ekvivalentnima
- Omogućuju matrični zapis (3x3 matricama) složenijih transformacija (projektivna tr. – homografija); prikaz točaka u beskonačnosti

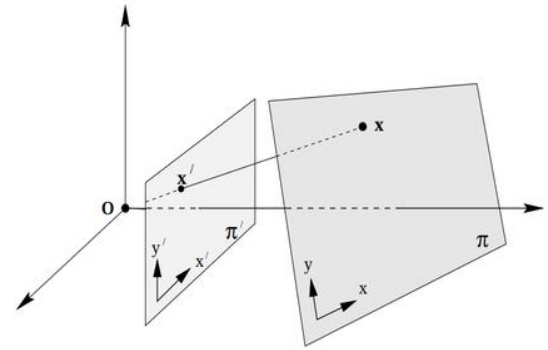


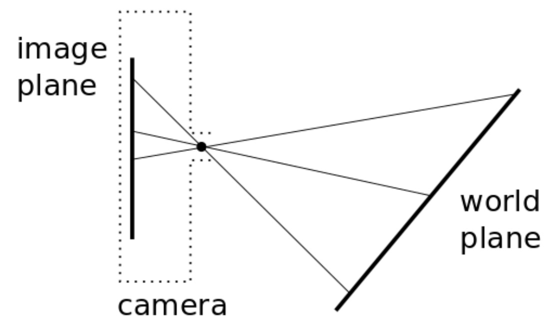
# Projekcijska transformacija (homografija) u 2D

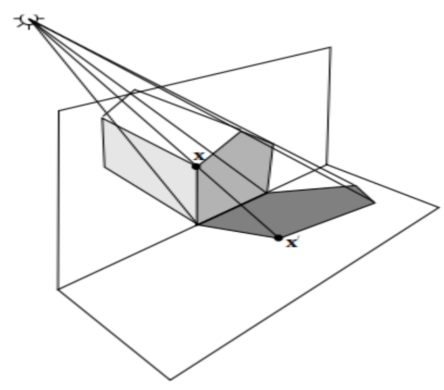
- *Preslikavanje između dviju ravnina u prostoru* ***kroz jednu točku***
- *Opisana 3x3 matricom u homogenim koordinatama:*

$$\begin{bmatrix} \widetilde{x}' \\ \widetilde{y}' \\ \widetilde{w}' \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- 8 stupnjeva slobode (**ne** 9)
  (zbog ekvivalencije skaliranih homogenih koordinata, možemo podijeliti obje strane jednadžbe proizvoljnom konstantom i fiksirati jedan od elemenata $h_{ij}$ na proizvoljnu vrijednost (npr. $h_{33}$ na 1)

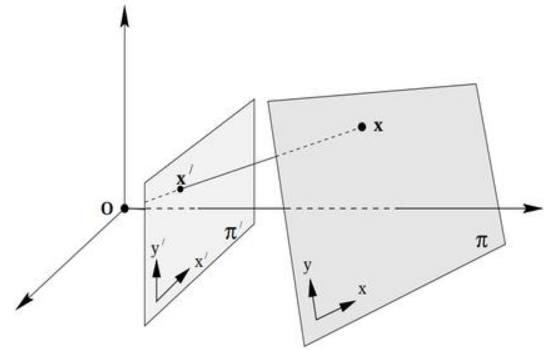


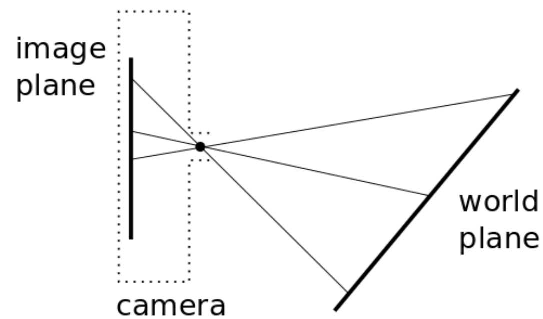



# Projekcijska transformacija (homografija) u 2D

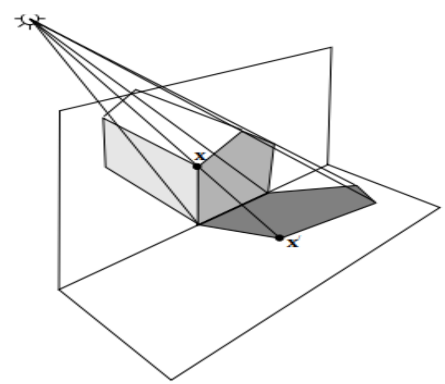
$$\begin{bmatrix} \widetilde{x'} \\ \widetilde{y'} \\ \widetilde{w'} \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$




- $\widetilde{x'} = h11x + h12\,y + h13$
- $\widetilde{y'} = h21x + h22\,y + h23$
- $\widetilde{w'} = h31x + h32\,y + h33$




- U nehomogenim koordinatama:




- $x' = \dfrac{h11x+h12y+h13}{h31x+h32y+h33}$
- $y' = \dfrac{h21x+h22y+h23}{h31x+h32y+h33}$

# Projekcijska transformacija (homografija) u 2D

$$\begin{bmatrix} \widetilde{x'} \\ \widetilde{y'} \\ \widetilde{w'} \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Pravci ostaju pravci
- Nije sačuvana paralelnost pravaca
- Nisu sačuvani kutovi, orijentacija, duljina

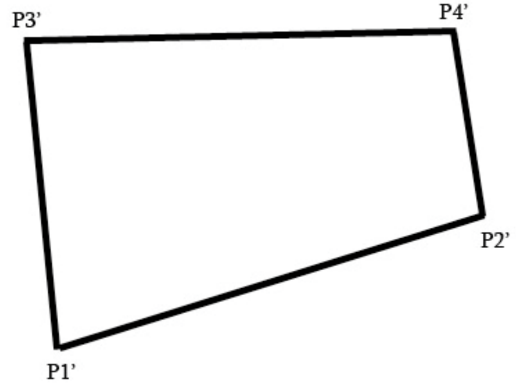
# Inverzna projekcijska transformacija u 2D

- $\begin{bmatrix} x \\ y \\ w \end{bmatrix} = H^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$
- H je matrica izvorne projekcijske transformacije

# Određivanje 2D projecijske transformacije

- $$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- 8 nepoznanica –> potrebno min. 8 jednadžbi
- Potrebno ustanoviti korespodencije četiriju parova točaka (pri čemu nikoje 3 nisu kolinearne)

# Određivanje 2D projecijske transformacije

*Za 1 par korespodentnih točaka vrijedi:*

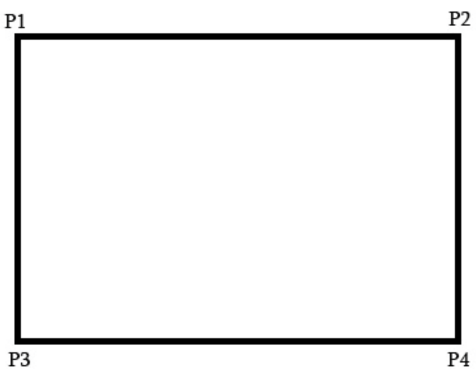
$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & h33 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

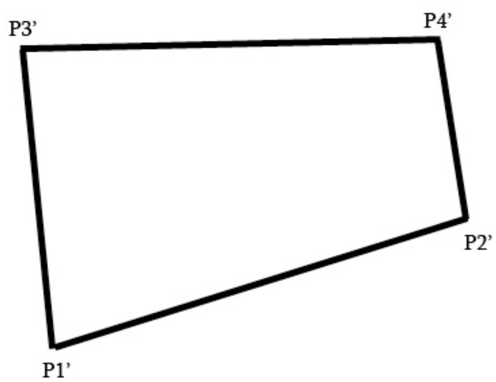
$$x' = \frac{h11x + h12y + h13}{h31x + h32y + h33}$$

$$y' = \frac{h21x + h22y + h23}{h31x + h32y + h33}$$

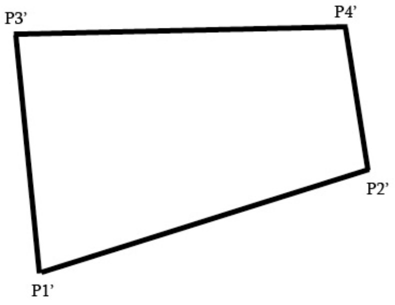
$$(h31x + h32y + h33)x' = h11x + h12y + h13$$
$$(h31x + h32y + h33)y' = h21x + h22y + h23$$

$$h31xx' + h32yx' + h33x' = h11x + h12y + h13$$
$$h31xy' + h32yy' + h33y' = h21x + h22y + h23$$



# Određivanje 2D projecijske transformacije


$$h31xx' + h32yx' + h33x' = h11x + h12y + h13$$
$$h31xy' + h32yy' + h33y' = h21x + h22y + h23$$

- Sve korespodencije mogu se napisati u matričnom obliku (homogeni sustav):

$$\begin{bmatrix} -x1 & -y1 & -1 & 0 & 0 & 0 & x1x1' & y1x1' & x1' \\ 0 & 0 & 0 & -x1 & -y1 & -1 & x1y1' & y1y1' & y1' \\ -x2 & -y2 & -1 & 0 & 0 & 0 & x2x2' & y2x2' & x2' \\ 0 & 0 & 0 & -x2 & -y2 & -1 & x2y2' & y2y2' & y2' \\ -x3 & -y3 & -1 & 0 & 0 & 0 & x3x3' & y3x3' & x3' \\ 0 & 0 & 0 & -x3 & -y3 & -1 & x3y3' & y3y3' & y3' \\ -x4 & -y4 & -1 & 0 & 0 & 0 & x4x4' & y4x4' & x4' \\ 0 & 0 & 0 & -x4 & -y4 & -1 & x4y4' & y4y4' & y4' \end{bmatrix} \begin{bmatrix} h11 \\ h12 \\ h13 \\ h21 \\ h22 \\ h23 \\ h31 \\ h32 \\ h33 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

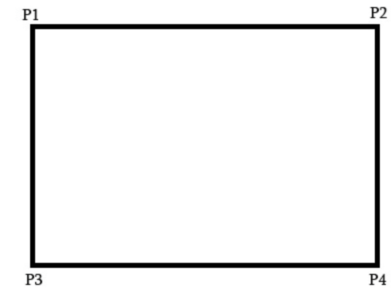
# Određivanje 2D projecijske transformacije

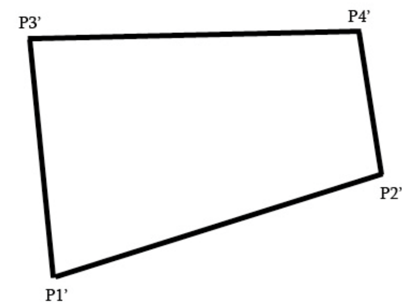
$$\begin{bmatrix} -x1 & -y1 & -1 & 0 & 0 & 0 & x1x1' & y1x1' & x1' \\ 0 & 0 & 0 & -x1 & -y1 & -1 & x1y1' & y1y1' & y1' \\ -x2 & -y2 & -1 & 0 & 0 & 0 & x2x2' & y2x2' & x2' \\ 0 & 0 & 0 & -x2 & -y2 & -1 & x2y2' & y2y2' & y2' \\ -x3 & -y3 & -1 & 0 & 0 & 0 & x3x3' & y3x3' & x3' \\ 0 & 0 & 0 & -x3 & -y3 & -1 & x3y3' & y3y3' & y3' \\ -x4 & -y4 & -1 & 0 & 0 & 0 & x4x4' & y4x4' & x4' \\ 0 & 0 & 0 & -x4 & -y4 & -1 & x4y4' & y4y4' & y4' \end{bmatrix} \begin{bmatrix} h11 \\ h12 \\ h13 \\ h21 \\ h22 \\ h23 \\ h31 \\ h32 \\ h33 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Zanima nas **netrivijalno** rješenje (trivijalno rješenje je svi hij=0)
- Netrivijalno rješenje nije jedinstveno (8 jednadžbi, 9 nepoznanica) no sva se razlikuju samo za faktor skale te su ekvivalentna (homogene koordinate koje se razlikuju za faktor skale predstavljaju jednake nehomogene koordinate)

# Određivanje 2D projekcijske transformacije

Drugi način formulacije – fiksirajmo jedan parametar (npr. h33) => nehomogeni sustav jednadžbi 8x8




$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} h11 & h12 & h13 \\ h21 & h22 & h23 \\ h31 & h32 & \mathbf{1} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$h31xx' + h32yx' + x' = h11x + h12y + h13$$
$$h31xy' + h32yy' + y' = h21x + h22y + h23$$

$$\begin{bmatrix} -x1 & -y1 & -1 & 0 & 0 & 0 & x1x1' & y1x1' \\ 0 & 0 & 0 & -x1 & -y1 & -1 & x1y1' & y1y1' \\ -x2 & -y2 & -1 & 0 & 0 & 0 & x2x2' & y2x2' \\ 0 & 0 & 0 & -x2 & -y2 & -1 & x2y2' & y2y2' \\ -x3 & -y3 & -1 & 0 & 0 & 0 & x3x3' & y3x3' \\ 0 & 0 & 0 & -x3 & -y3 & -1 & x3y3' & y3y3' \\ -x4 & -y4 & -1 & 0 & 0 & 0 & x4x4' & y4x4' \\ 0 & 0 & 0 & -x4 & -y4 & -1 & x4y4' & y4y4' \end{bmatrix} \begin{bmatrix} h11 \\ h12 \\ h13 \\ h21 \\ h22 \\ h23 \\ h31 \\ h32 \end{bmatrix} = \begin{bmatrix} -x1' \\ -y1' \\ -x2' \\ -y2' \\ -x3' \\ -y3' \\ -x4' \\ -y4' \end{bmatrix}$$

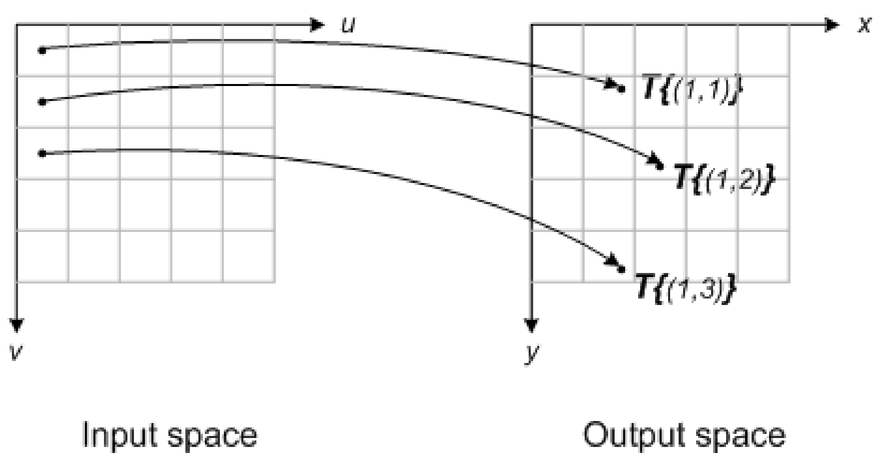
# Primjena geometrijskih transformacija

- Unaprijedna transformacija (engl. forward warp)

**procedure** *forwardWarp*$(f, \mathbf{h}, \textbf{out}\ g)$:

For every pixel $\mathbf{x}$ in $f(\mathbf{x})$

1. Compute the destination location $\mathbf{x}' = \mathbf{h}(\mathbf{x})$.
2. Copy the pixel $f(\mathbf{x})$ to $g(\mathbf{x}')$.




Problemi:

(1) transformirani piksel upada „između piksela" odredišne slike (necjelobrojne koordinate)

(2) Susjedni pikseli izvorišne slike preslikavaju se u ne-susjedne piksele u odredišnoj slici (ostaju **praznine** u odredišnoj slici)

(https://blogs.mathworks.com/steve/2006/04/28/spatial-transforms-forward-mapping/)

# Unaprijedna transformacija - primjer



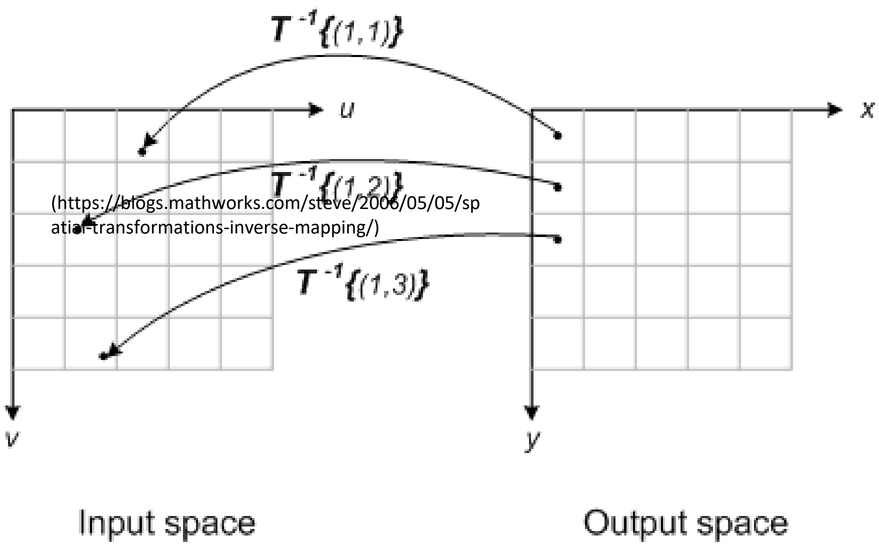
# Primjena geometrijskih transformacija

- Unatražna transformacija (engl. backward warp)

**procedure** *inverseWarp*$(f, \mathbf{h}, \mathbf{out}\ g)$:

For every pixel $\mathbf{x}'$ in $g(\mathbf{x}')$

1. Compute the source location $\mathbf{x} = \hat{\mathbf{h}}(\mathbf{x}')$
2. Resample $f(\mathbf{x})$ at location $\mathbf{x}$ and copy to $g(\mathbf{x}')$



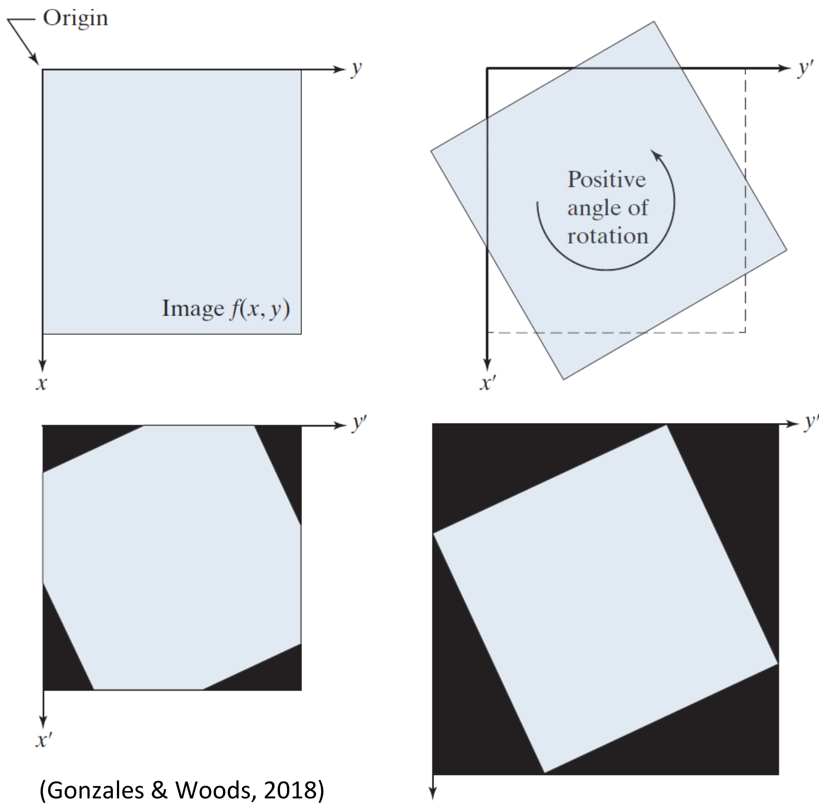

(https://blogs.mathworks.com/steve/2006/05/05/spatial-transformations-inverse-mapping/)

- I dalje moguć problem necjelobrojnih koordinata – lako se rješava interpolacijom
- Nema praznina među pikselima odredišne slike

# Unatražna transformacija - primjer



# Primjena geometrijskih transformacija




(Gonzales & Woods, 2018)

Dimenzije transformirane slike:

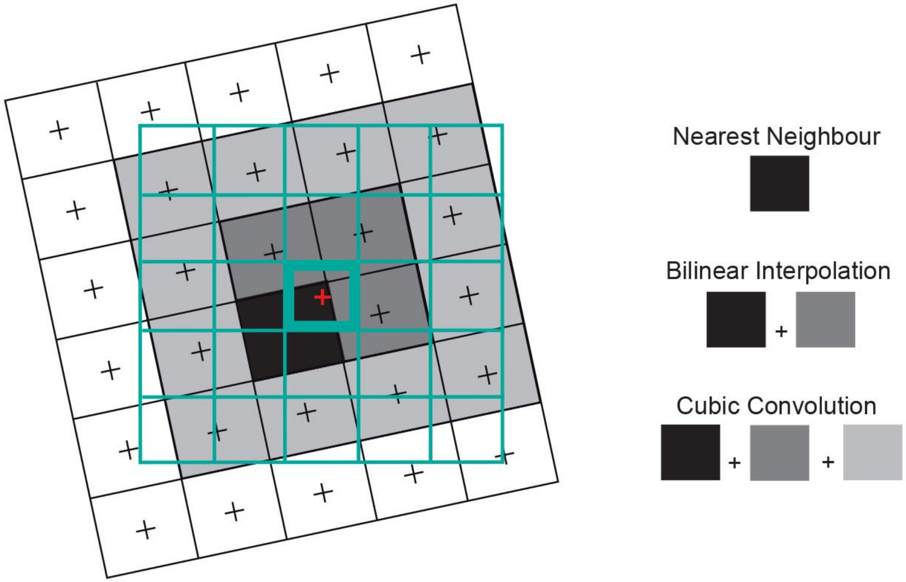
a) Zadržavanje izvornih dimenzija uz odsijecanje „viška"

b) Proširenje dimenzija tako da cijela transformirana slika stane

- Nedefinirani pikseli odredišne slike (oni kojih nema u izvorišnoj) ostaju crni

# Interpolacija

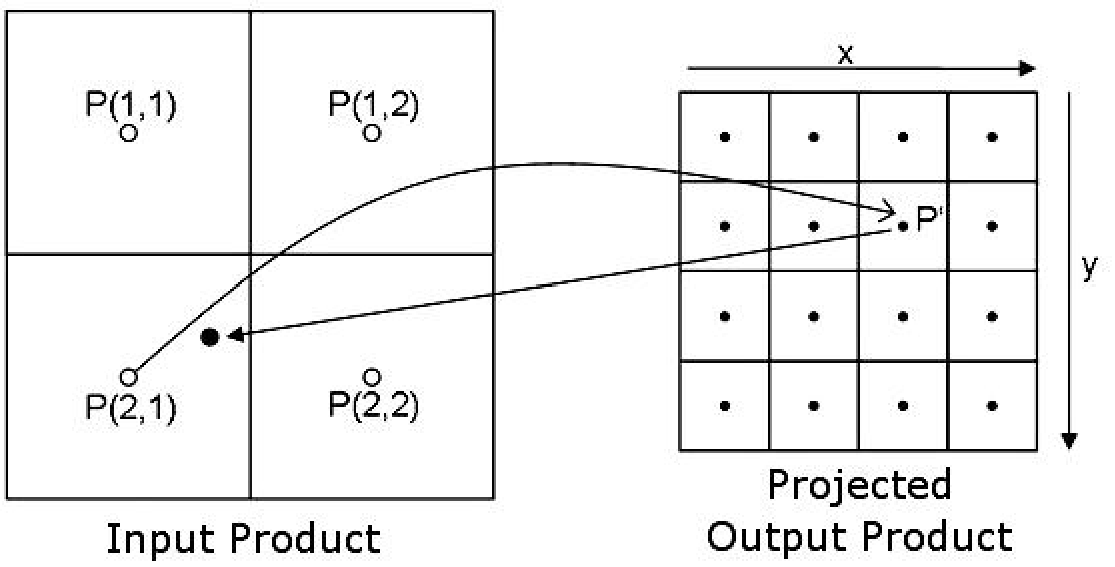
- Procjena nepoznatih vrijednosti korištenjem poznatih vrijednosti u susjedstvu
  a) Interpolacija najbližim susjedom
  b) Bilinearna interpolacija
  c) Bikubična interpolacija




https://ltb.itc.utwente.nl/498/concept/81586

# Interpolacija

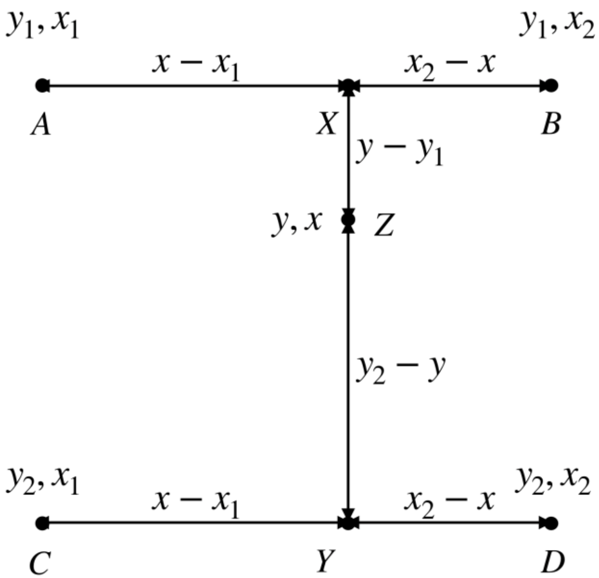
- Interpolacija najbližim susjedom
  (engl. nearest neighbour interpolation)




$$I(x + \Delta x, y + \Delta y) = I(round(x + \Delta x), round(y + \Delta y))$$

# Interpolacija

- Bilinearna interpolacija (engl. bilinear interpolation)




- $I(x, y1) = \frac{x2-x}{x2-x1} I(x1, y1) + \frac{x-x1}{x2-x1} I(x2, y1)$
- $I(x, y2) = \frac{x2-x}{x2-x1} I(x1, y2) + \frac{x-x1}{x2-x1} I(x2, y2)$
- $I(x, y) = \frac{y2-y}{y2-y1} I(x, y1) + \frac{y-y1}{y2-y1} I(x, y2)$

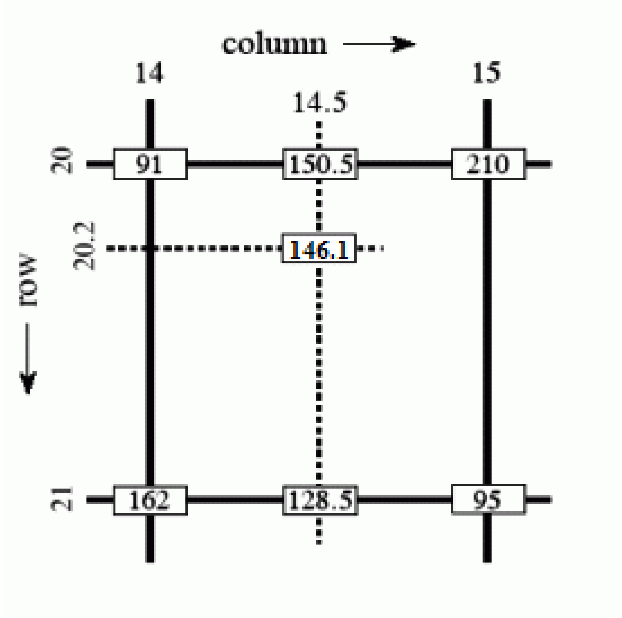
# Interpolacija

- Bilinearna interpolacija (engl. bilinear interpolation)

- Kombiniranjem prethodnih triju jednadžbi dobivamo:

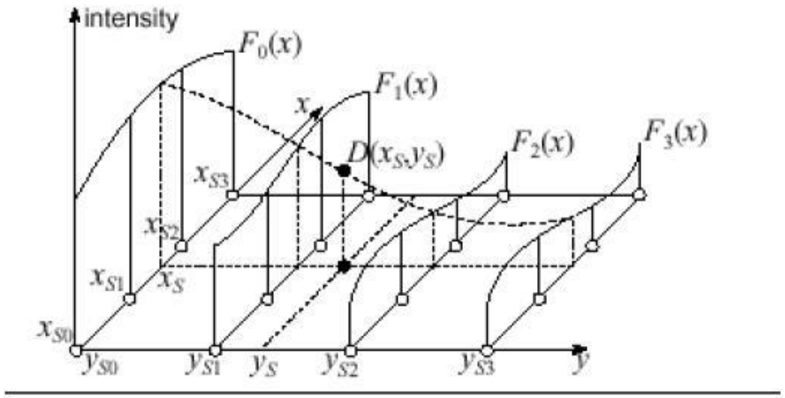
$$I(x,y) = \frac{y2-y}{y2-y1}\frac{x2-x}{x2-x1}I(x1,y1) + \frac{y2-y}{y2-y1}\frac{x-x1}{x2-x1}I(x2,y1) + \frac{y-y1}{y2-y1}\frac{x2-x}{x2-x1}I(x1,y2) + \frac{y-y1}{y2-y1}\frac{x-x1}{x2-x1}I(x2,y2)$$

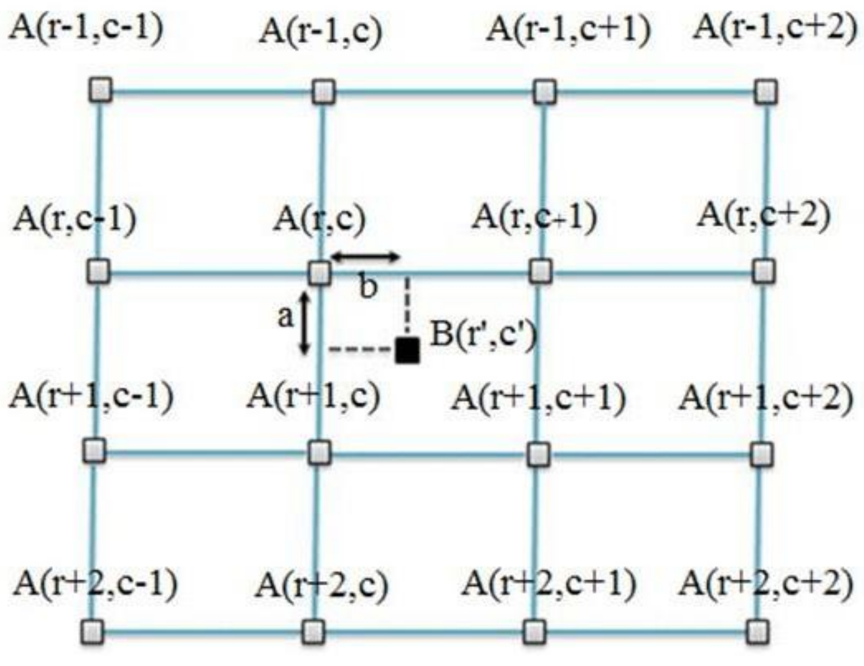
Primjer za sivu sliku:



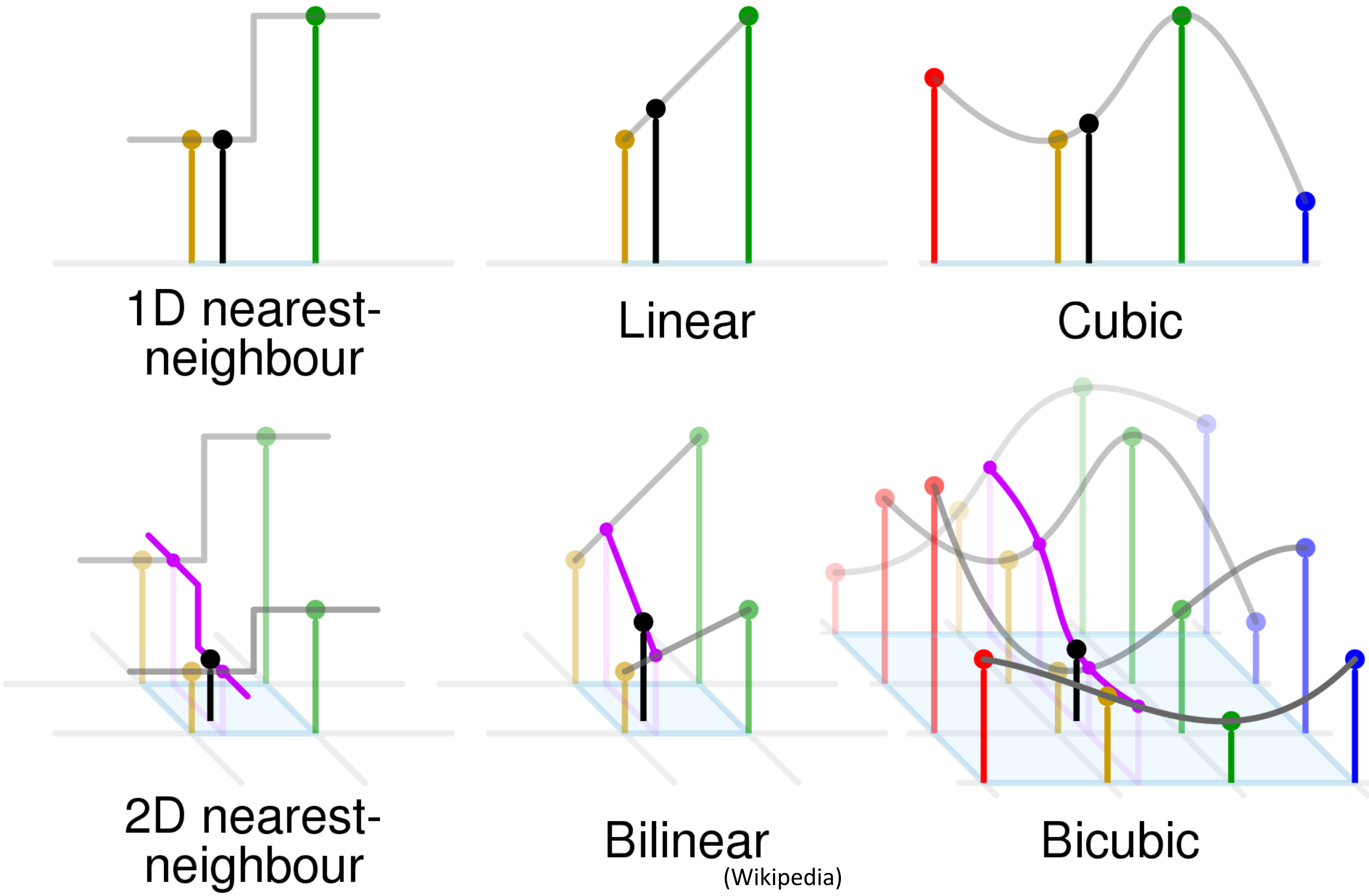
# Interpolacija

- Bikubična interpolacija (engl. bicubic interpolation)
- Koristi 16 (4x4) susjednih piksela
- Za svaki od 4 retka kubična krivulja kroz 4 točke: $I(x) = ax^3 + bx^2 + \ cx + d$
- Odredimo $I(x, yi)$ za svaki stupac (yi) –> nove 4 točke -> kubična krivulja kroz njih
- Očitamo $I(x, y)$ za tražene koordinate



# Interpolacija




(Wikipedia)

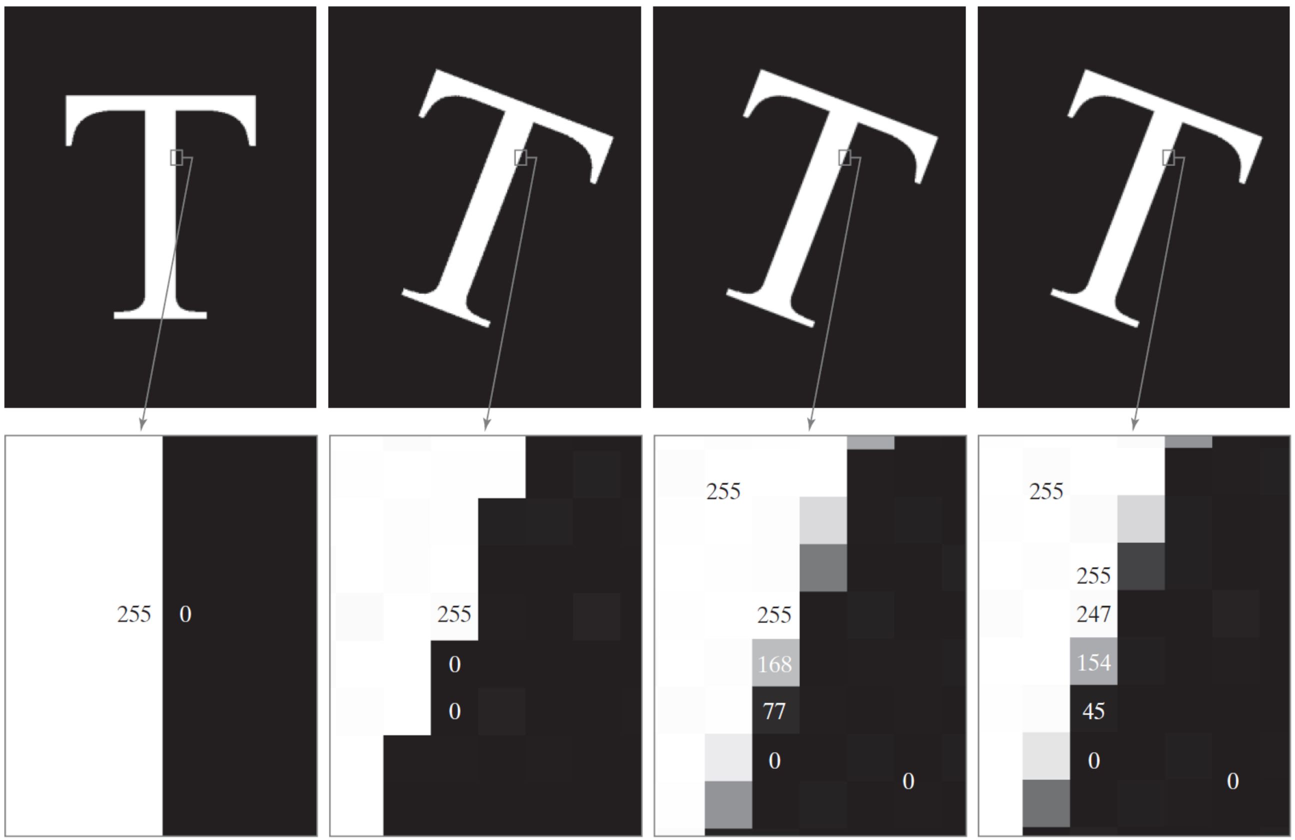
# Interpolacija – primjer 1

(Gonzales & Woods, 2018)

Redom, slijeva na desno:

- Interpolacija najbližim susjedom
- Bilinearna interpolacija
- Bikubična interpolacija

# Interpolacija – primjer 2




Redom, slijeva na desno:

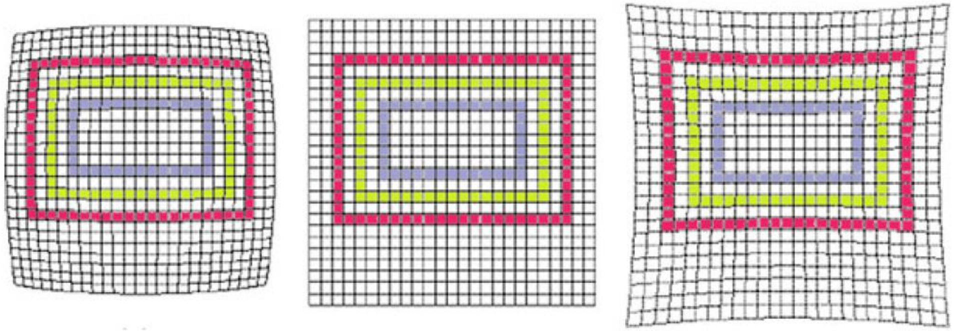
- Originalna slika
- Tri rotirane slike
  - Interpolacija najbližim susjedom
  - Bilinearna interpolacija
  - Bikubična interpolacija

(Gonzales & Woods, 2018)

# Radijalna distorzija

- Leće uzrokuju radijalnu distorziju slike
- Osobito vidljivo kod širokokutnih („fish eye”) leća
- Bačvasta (engl. barrel) ili jastučasta (engl. pincushion)

(Szeliski)



(Klette 2014)

# Radijalna distorzija - korekcija

- Označimo nekorigiranu (distortiranu) točku:

$$p_d = (x_d, y_d)$$

- Korigirana točka:

$$p_u = (x_u, y_u)$$

$$x_u = c_x + (x_d - c_x)(1 + \kappa_1 r_d^2 + \kappa_2 r_d^4)$$

$$y_u = c_x + (y_d - c_y)(1 + \kappa_1 r_d^2 + \kappa_2 r_d^4)$$

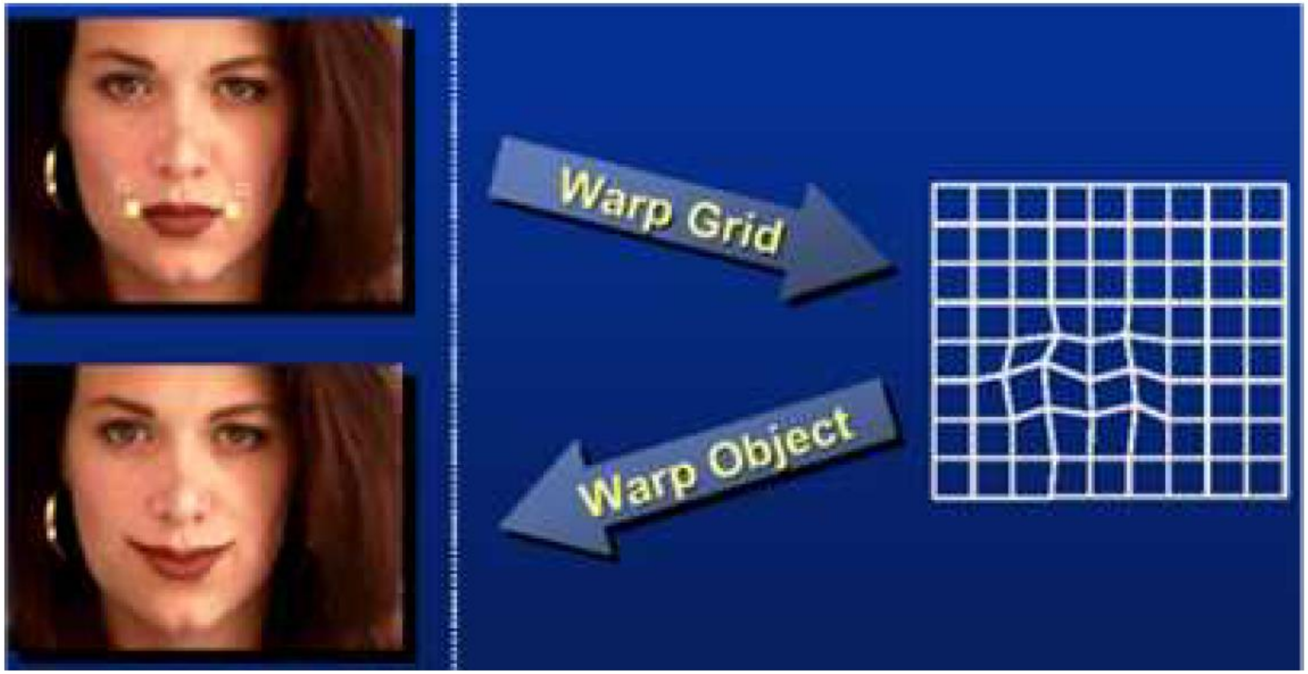
gdje je:

$$\left(c_x, c_y\right) \, središte \; slike$$

$$r_d = \sqrt{(x_d - c_x)^2 + (y_d - c_y)^2}$$

# Deformacijska polja

- Neparametarske transformacije
- Za svaki odredišni piksel zadan vektor pomaka prema izvorišnom pikselu




(Szeliski 2018)