CSC418

1.

a) $l_{12} = p_1 \times p_2$

$$= \begin{bmatrix} 0 \\ 0.2828 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0.1818 \\ 0.5143 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.2315 \\ 0.1818 \\ -0.051413 \end{bmatrix}$$

$l_{23} = p_2 \times p_3$

$$= \begin{bmatrix} 0.1818 \\ 0.5143 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0.0952 \\ 0.6734 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.1591 \\ -0.0866 \\ 0.0734 \end{bmatrix}$$

$l_{34} = P_3 \times P_4$

$$= \begin{bmatrix} 0.0952 \\ 0.6734 \\ 1 \end{bmatrix} \times \begin{bmatrix} -0.1053 \\ 0.4466 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2268 \\ -0.2005 \\ 0.11342 \end{bmatrix}$$

$l_{41} = P_4 \times P_1$

$$= \begin{bmatrix} -0.1053 \\ 0.4466 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0.2828 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1638 \\ 0.1053 \\ -0.02978 \end{bmatrix}$$

Vanishing points are:

$l_{12} \times l_{34}$

$$= \begin{bmatrix} -0.2315 \\ 0.1818 \\ -0.0514 \end{bmatrix} \times \begin{bmatrix} 0.2268 \\ -0.2005 \\ 0.11342 \end{bmatrix}$$

$$\cong \begin{bmatrix} 1.9894 \\ 2.8161 \\ 1 \end{bmatrix}$$

$l_{23} \times l_{41}$

$$= \begin{bmatrix} -0.1591 \\ -0.0866 \\ 0.0734 \end{bmatrix} \times \begin{bmatrix} 0.1638 \\ 0.1053 \\ -0.02978 \end{bmatrix}$$

$$\cong \begin{bmatrix} 2.006 \\ -2.84 \\ 1 \end{bmatrix}$$

b) A line in 3d points is represented as

$$X(\lambda) = A + \lambda D$$

Using $x = fX/Z$ (focal length), the vanishing point of its image is

$$v = \lim_{\lambda \to \pm\infty} x(\lambda)$$

$$= f \frac{A + \lambda D}{A_z + \lambda D_z}$$

$$= f \frac{D}{D_z} = f \begin{pmatrix} D_x / D_z \\ D_y / D_z \\ 1 \end{pmatrix}$$

$\therefore$ V only depends on direction D and not on A.

$\therefore$ Parallel lines have same vanishing point

c) NO.

Because if they are, vanishing points v_1 and v_2 would 90° and

$$v_1 \cdot v_2 = 0$$

but $v_1 \cdot v_2 = \begin{bmatrix} 1.9894 \\ 2.816 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2.006 \\ -2.84 \\ 1 \end{bmatrix}$

$= -3.99 \neq 0$

D) The distance from origin to v_1 is

$$= \sqrt{1.989^2 + 2.816^2} = 3.448$$

Distance from origin to v_2 is

$$= \sqrt{2.008^2 + (-2.84)^2} = 3.478$$

Distance from v_1 to v_2

$$= \sqrt{(2.008 - 1.989)^2 + (-2.84 - 2.816)^2}$$

$$= 5.657$$

By Pythagoreans

$$3.478^2 = f^2 + (5.657 - d)^2$$

$$\therefore \quad 12.1 = 11.88 - d^2 + 32 + d^2 - 11.314d$$

$$\therefore \quad d = 2.8$$

Also $\quad 3.448^2 = d^2 + f^2$

$$f^2 = 11.88 - (2.8)^2$$

$$\therefore \boxed{f = 1.999 \cong 2}$$

e) Normal is same as normal intersecting the camera and vanishing line.

$$\therefore \; V_1 \times V_2 = \begin{bmatrix} 1.989 \\ 2.816 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2.008 \\ -2.84 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5.657 \\ 0.016 \\ -11.29 \end{bmatrix}$$

2. $\quad x(t) = \sin(2\pi t) + 2t$

$y(t) = t^2 \qquad\qquad 0 \leq t \leq 5$

For parametric curve

$x(t) = x\cos(\theta)$

$z(t) = x\sin(\theta)$

$$\therefore \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} (2t + \sin(2\pi t))\cos\theta \\ t^2 \\ (2t + \sin(2\pi t))\sin\theta \end{pmatrix}$$

3.

a) Points where curves intersect:

$\sqrt{2}\,z_1(t) = z_2(u) \qquad$ We are told to use only +ve values for

$\therefore \sqrt{u^2 - 1} = 2\sqrt{2}$

$\therefore u^2 - 1 = 8$

$\therefore u^2 = 9$

$\therefore u = \pm 3$

Scanned by CamScanner

and $y_1(t) = 0$

$\therefore 3\cos(2\pi t) = 0$

$\therefore t = \ldots \frac{1}{4}, \frac{3}{4} \ldots$

We are told to limit t to $0 < t < 1$

$\therefore$ for $u = \pm 3$ or $t = \frac{1}{4}, \frac{3}{4}$

We get intersection points:

$\underline{(3, 0, 2\sqrt{2})}$ and $(-3, 0, 2\sqrt{2})$

b) tangents of $f_1(t)$

$$= \left(\frac{d(3\sin(2\pi t))}{dt}, \frac{d(3\cos(2\pi t))}{dt}, \frac{d(2\sqrt{2})}{dt} \right)$$

$$= \left(6\pi\cos(2\pi t), -6\pi\sin(2\pi t), 0 \right)$$

For $t = \frac{1}{4}$, $f_1'(t) = \left(6\pi\cos\left(2\pi \times \frac{1}{4}\right), -6\pi\sin\left(2\pi \times \frac{1}{4}\right), 0 \right)$

$$= \underline{\left(0, -6\pi, 0 \right)}$$

tangents of $f_2(t)$

$$= \left(\frac{du}{du}, \frac{d0}{du}, \frac{d(\sqrt{u^2-1})}{du} \right)$$

$$= \left(1, 0, \frac{1}{2}(u^2-1)^{-\frac{1}{2}} \times 2u \right)$$

$$= \left(1, 0, \frac{u}{\sqrt{u^2-1}} \right)$$

For $u = 3$, point is $\underline{\left(1, 0, \frac{3}{2\sqrt{2}} \right)}$

c) $f_1'(t)$ at $t=\frac{1}{2}$ is $(0, -6\pi, 0)$
$f_2'(u)$ at $u=3$ is $\left(1, 0, \frac{3}{2\sqrt{2}}\right)$

$$f_1'\left(\tfrac{1}{2}\right) \times f_2'(3) = \begin{pmatrix} i & j & k \\ 0 & -6\pi & 0 \\ 1 & 0 & \frac{3}{2\sqrt{2}} \end{pmatrix}$$

$$= i\left(\frac{-\overset{3}{\cancel{6}}\cdot 3\pi}{\cancel{2}\sqrt{2}}\right) - j(0-0) + k(0+6\pi)$$

$$= \left(\frac{-9\pi}{2\sqrt{2}}, 0, 6\pi\right)$$

$$f_2'(3) \times f_1'\left(\tfrac{1}{2}\right) = \begin{pmatrix} i & j & k \\ 1 & 0 & \frac{3}{2\sqrt{2}} \\ 0 & -6\pi & 0 \end{pmatrix}$$

$$= i\left(0 + \frac{9\pi}{\sqrt{2}}\right) - j(0-0) + k(-6\pi - 0)$$

$$= \left(\frac{9\pi}{\sqrt{2}}, 0, -6\pi\right)$$

$\therefore \left(\frac{-9\pi}{\sqrt{2}}, 0, 6\pi\right)$ is th unit vector that faces out.

The magnitude $= \sqrt{\left(\frac{-9\pi}{\sqrt{2}}\right)^2 + 0^2 + (6\pi)^2}$

$$= \sqrt{\frac{153\pi^2}{2}}$$

$\therefore$ Normalized form: $\left(\dfrac{\frac{-9\pi}{\sqrt{2}}}{\sqrt{\frac{153\pi^2}{2}}}, \dfrac{0}{\sqrt{\frac{153\pi^2}{2}}}, \dfrac{6\pi}{\sqrt{\frac{153\pi^2}{2}}}\right)$

$$= (-0.727, 0, 0.686)$$

d) Normal of surface

$$g'(x,y,z) = \left(\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}\right)$$
$$= (2x, 2y, -2z)$$

e) Normal at $(3, 0, 2\sqrt{2})$

$$= (2\times 3, 0, -2(2\sqrt{2}))$$
$$= (6, 0, -4\sqrt{2})$$

magnitude $= \sqrt{6^2 + 0^2 + (-4\sqrt{2})^2}$
$= \sqrt{36+32}$
$= 2\sqrt{17}$

$\therefore$ Normalized point

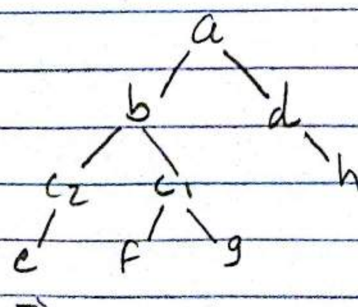
$$= \left(\frac{6}{2\sqrt{17}}, 0, \frac{-4^2\sqrt{2}}{2\sqrt{17}}\right)$$
$$= (0.727, 0, -0.686)$$

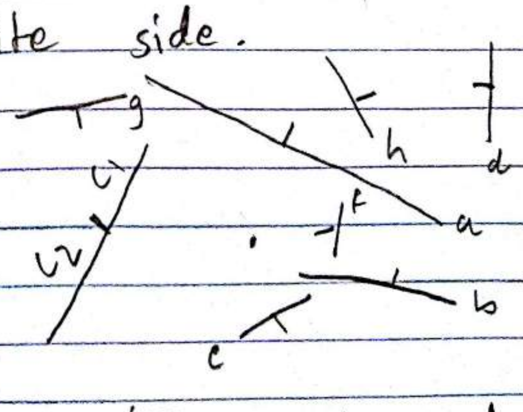
It is Normal in opposit direction compared to c).

4.

a) Yes, It is possible to exclude h, because it is completely obstructed and its normal is facing the opposite side.

b)





c) $\vec{e}$ is inside i:
- We draw evrything outside i
- draw i
- draw everything inside i

Traversal order:
$h - d - a - g - c_1 - f - b - c_2 - e$