

For eqs 3 & 4 for the coefficients. Note  $h_i = x_{i+1} - x_i$ ,  $\eta_i = y_{i+1} - y_i$

$$(3) \quad 3a_i h_i^2 + 2b_i h_i + c_i = c_{i+1}$$

$$(4) \quad 6a_i h_i + 2b_i = 2b_{i+1}$$

$$g_i(x) = a_i (x - x_i)^3 + b_i (x - x_i)^2 + c_i (x - x_i) + d_i$$

To get (3), enforce  $g_i'(x_{i+1}) = g_{i+1}'(x_{i+1})$

$$g_i'(x_{i+1}) = a_i (x_{i+1} - x_i)^3 + b_i (x_{i+1} - x_i)^2 + c_i (x_{i+1} - x_i) + d_i$$

$$g_i'(x_{i+1}) = 3a_i h_i^2 + 2b_i h_i + c_i$$

$$g_{i+1}(x) = a_{i+1} (x - x_{i+1})^3 + b_{i+1} (x - x_{i+1})^2 + c_{i+1} (x - x_{i+1}) + d_{i+1}$$

$$g_{i+1}'(x) = 3a_{i+1} (x - x_{i+1})^2 + 2b_{i+1} (x - x_{i+1}) + c_{i+1}$$

$$g_{i+1}'(x_{i+1}) = 3a_{i+1} (\cancel{x_{i+1} - x_{i+1}}) + 2b_{i+1} (\cancel{x_{i+1} - x_{i+1}}) + c_{i+1}$$

$$g_{i+1}'(x_{i+1}) = c_{i+1}$$

$$\Rightarrow \boxed{3a_i h_i^2 + 2b_i h_i + c_i = c_{i+1}} \quad \checkmark$$

To get (4), enforce  $g_i''(x_{i+1}) = g_{i+1}''(x_{i+1})$

$$g_i''(x) = 6a_i (x - x_i) + 2b_i$$

$$g_i''(x_{i+1}) = 6a_i (x_{i+1} - x_i) + 2b_i = 6a_i h_i + 2b_i$$

$$g_{i+1}''(x) = 6a_{i+1} (x - x_{i+1}) + 2b_{i+1}$$

$$g_{i+1}''(x_{i+1}) = 6a_{i+1} (\cancel{x_{i+1} - x_{i+1}}) + 2b_{i+1} = 2b_{i+1}$$

$$\Rightarrow \boxed{6a_i h_i + 2b_i = 2b_{i+1}} \quad \checkmark$$