c. For the final eqn all in terms of b coeffs of the cubic spline

Final eqn is $\frac{1}{3}h_i b_i + \frac{2}{3}(h_i + h_{i+1})b_{i+1} + \frac{1}{3}h_{i+1}b_{i+2} = \frac{\eta_{i+1}}{h_{i+1}} - \frac{\eta_i}{h_i}$

From video 3, we have

① $d_i = y_i$ $\qquad\qquad h_i = x_{i+1} - x_i$

② $a_i h_i^3 + b_i h_i^2 + c_i h_i = \eta_i$ $\qquad\qquad \eta_i = y_{i+1} - y_i$

③ $3a_i h_i + b_i = b_{i+1}$ ← same as eqn 4 from previous problem, but divide by 2.

④ $3a_i h_i^2 + 2b_i h_i + c_i = c_{i+1}$

From ③, get a_i

$3a_i h_i = b_{i+1} - b_i$

$a_i = \frac{1}{3h_i}(b_{i+1} - b_i)$

From ②, get c_i (also plug in a_i)

$\frac{1}{3\cancel{h_i}}(b_{i+1} - b_i)h_i^{\cancel{3}\,2} + b_i h_i^2 + c_i h_i = \eta_i$

$\frac{1}{3}(b_{i+1} - b_i)h_i^2 + b_i h_i^2 + c_i h_i = \eta_i$

$\frac{\left[\frac{1}{3}(b_{i+1} - b_i) + b_i\right]h_i^{\cancel{2}}}{\cancel{h_i}} + \frac{c_i h_i}{h_i} = \frac{\eta_i}{h_i}$

$\left(\frac{1}{3}b_{i+1} - \frac{1}{3}b_i + b_i\right)h_i + c_i = \frac{\eta_i}{h_i}$ $\qquad -\frac{1}{3} + 1 = -\frac{1}{3} + \frac{3}{3} = \frac{2}{3}$

$\left(\frac{1}{3}b_{i+1} + \frac{2}{3}b_i\right)h_i + c_i = \frac{\eta_i}{h_i}$

$c_i = \frac{\eta_i}{h_i} - \frac{1}{3}h_i(b_{i+1} + 2b_i)$

Combine everything in ④

$\cancel{3}\frac{1}{\cancel{3}\cancel{h_i}}(b_{i+1} - b_i)h_i^{\cancel{2}} + 2b_i h_i + \frac{\eta_i}{h_i} - \frac{1}{3}h_i(b_{i+1} + 2b_i) = \frac{\eta_{i+1}}{h_{i+1}} - \frac{1}{3}h_{i+1}(b_{i+2} + 2b_{i+1})$

$(b_{i+1} - b_i)h_i + 2b_i h_i - \frac{1}{3}h_i(b_{i+1} + 2b_i) + \frac{1}{3}h_{i+1}(b_{i+2} + 2b_{i+1}) = \frac{\eta_{i+1}}{h_{i+1}} - \frac{\eta_i}{h_i}$

$\underline{b_{i+1}h_i} - \underline{b_i h_i} + \underline{2b_i h_i} - \underline{\frac{1}{3}h_i b_{i+1}} - \underline{\frac{2}{3}h_i b_i} + \frac{1}{3}h_{i+1}b_{i+2} + \underline{\frac{2}{3}h_{i+1}b_{i+1}} = \frac{\eta_{i+1}}{h_{i+1}} - \frac{\eta_i}{h_i}$

$b_i\left(-h_i + 2h_i - \frac{2}{3}h_i\right) + b_{i+1}\left(h_i - \frac{1}{3}h_i + \frac{2}{3}h_{i+1}\right) + \frac{1}{3}h_{i+1}b_{i+2} = \frac{\eta_{i+1}}{h_{i+1}} - \frac{\eta_i}{h_i}$

$$\boxed{\frac{1}{3}b_i h_i + \frac{2}{3}b_{i+1}(h_i + h_{i+1}) + \frac{1}{3}h_{i+1}b_{i+2} = \frac{\eta_{i+1}}{h_{i+1}} - \frac{\eta_i}{h_i}}$$ ✓