Problem Set 2

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1 Problem 1

Assume that markets are complete. Since markets are complete and there are no additional externalities, the competitive equilibrium for this economy will be pareto efficient. We find the allocation] with the planners problem and then back out the discount bond price. We solve

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$$\max \sum_{t=0}^{\infty} \beta^t \left[\pi(e) \frac{c(e)^{1-\alpha} - 1}{1-\alpha} + (1-\pi(e)) \frac{c(u)^{1-\alpha} - 1}{1-\alpha} \right]$$
$$\text{s.t } \pi(e)c(e) + (1-\pi(e))c(u) \leq \pi(e)y(e) + (1-\pi(e))y(u)$$

First order conditions imply

$$\beta^t(1-\alpha)c(e)^{-\alpha} = \lambda_t$$
$$\beta^t(1-\alpha)c(u)^{-\alpha} = \lambda_t$$
$$\implies c(e) = c(u).$$

To find the invariant distribution , we solve

$$\begin{bmatrix} \pi(e) \\ \pi(u) \end{bmatrix} = \begin{bmatrix} 0.97 & 0.50 \\ 0.03 & 0.50 \end{bmatrix} \begin{bmatrix} \pi(e) \\ \pi(u) \end{bmatrix}$$

along with the restriction $\pi(e) + \pi(e) = 1$ to obtain $\pi(e) \approx 0.9434$ and $\pi(u) \approx 0.0566$.

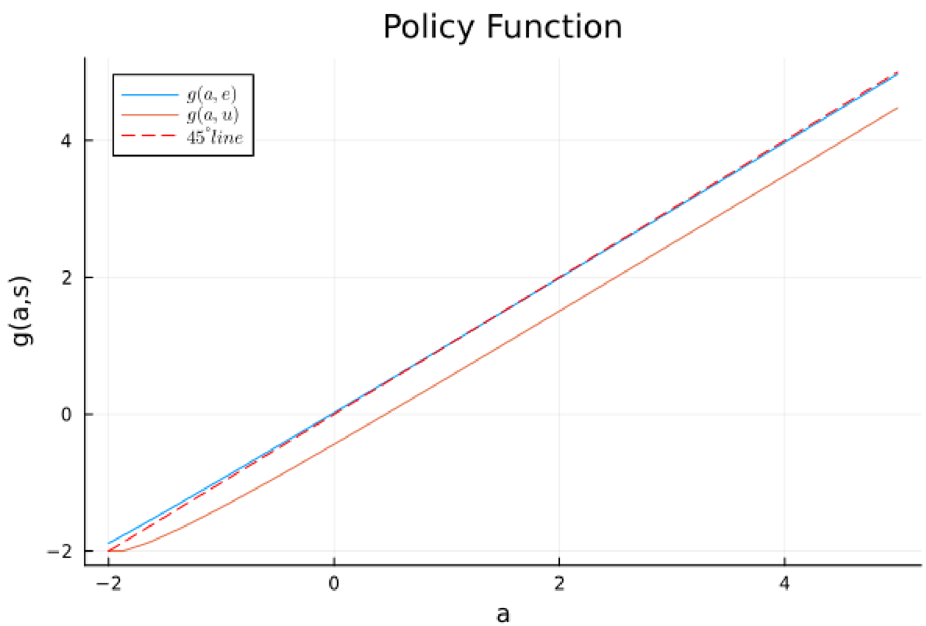
To find the price of the arrow security q_t , we simply find the stochastic discount factor

$$\beta^t u'(c(s_t))q_t = \beta^{t+1}u'(c(s_{t+1})) \implies q_t = \beta = 0.9932,$$

where we used the first/second welfare theorem to note that allocations will be equalized across all states.

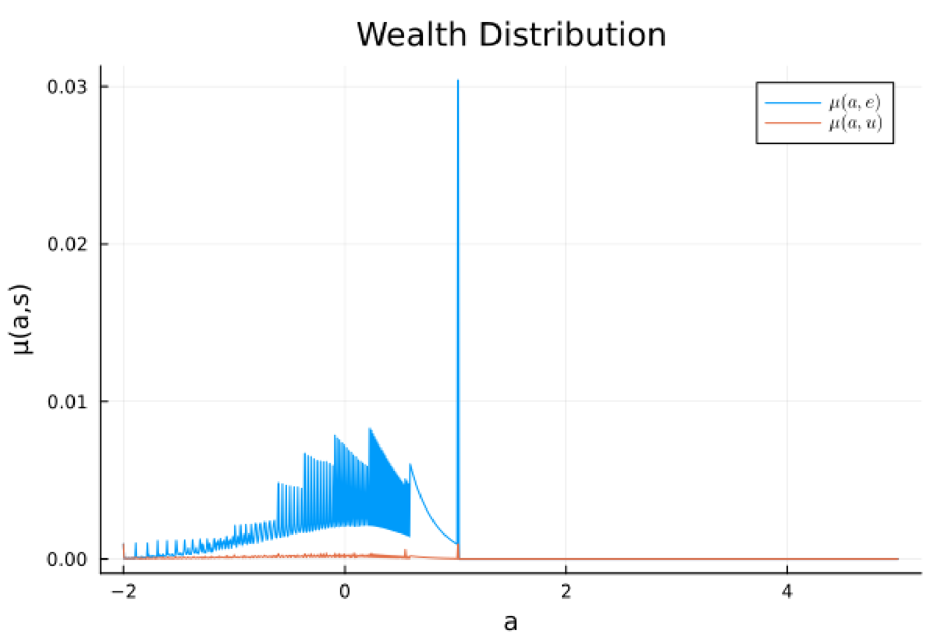
2 Problem 2

2.1 Policy Function



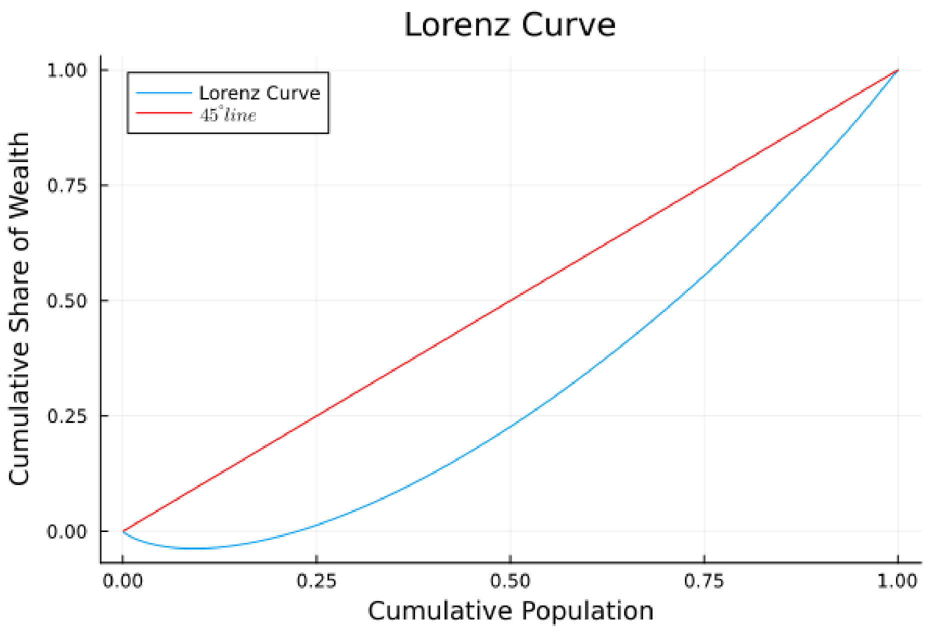
The policy function plotted above shows that $\hat{\alpha} \approx 1.28$.

2.2 Invariant Distribution and Bond Price



The wealth distribution is plotted above. The discount bond price that clears the market is $q \approx 0.9943$.

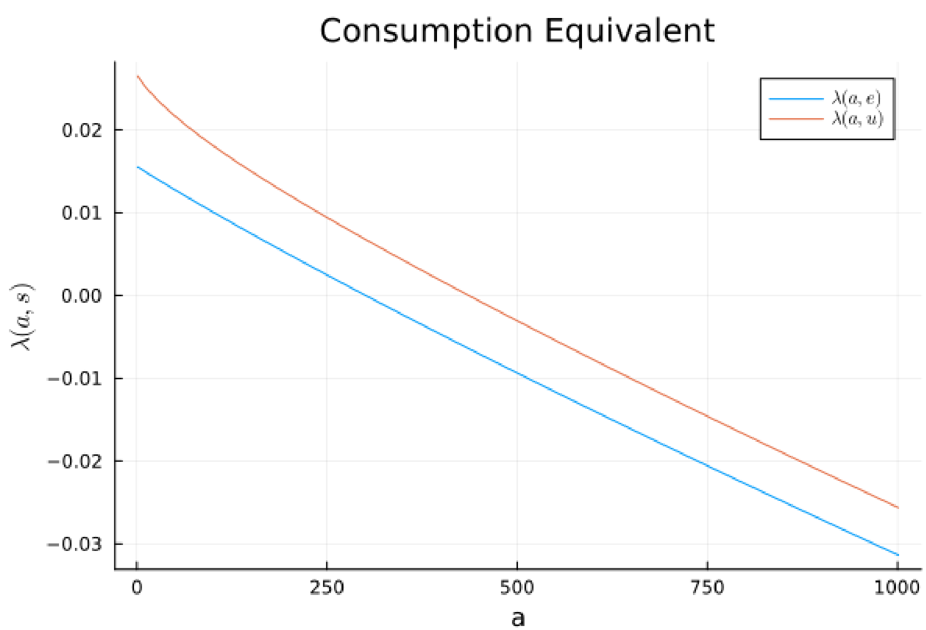
2.3 Lorenz Curve and Gini Coefficient



The lorenz curve is plotted above. The gini coefficient is ≈ 0.383.

3 Problem 3

3.1 Welfare Analysis



We plot the consumption equivalent above. Aggregate welfare with complete markets is -4.282945001004452 Aggregate welfare with incomplete markets is -4.45447969829158. The aggregate welfare gain is 0.0011564081096912394. The fractions of the population who would pay for complete markets is 0.5228655693555627.