Advanced Signal Processing - Assignment 2

Kristoffer Plagborg Bak Sørensen

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Introduction

This report covers the topics of adaptive filtering and the LMS algorithm. The report is structured as follows: First, a brief overview of the theory behind the adaptive filtering is presented Then, the algorithms LMS, NMLS and RLS are presented. These algorihms are implemented in python and tested and compared on a sampled electrocardiogram (ECG) signal that contains colored noise from the powerline. Finally, the results are presented and discussed.

Theory

Adaptive filters are FIR filters that are able to reconfigure themself based on the input signal. This is done by changing the filter coefficients. The filter coefficients are changed by an online quadratic programming algorithm that uses a known desired signal, $d[n]$, and the input signal, $x[n]$, to update the filter coefficients. The filter coefficients are updated in such a way that the filter output, $y[n]$, is as close as possible to the desired signal. This is done by minimizing the mean squared error (MSE) between the desired signal and the filter output. Adaptive filters differentiate themselves based on how they compute the update of the filter coefficients. This leads to different characteristics in terms of computational complexity, convergence speed and robustness to noise. The three most common algorithms are the LMS, NMLS and RLS algorithms. The LMS algorithm is the simplest of the three and is the most computationally efficient. The NMLS algorithm is a more advanced version of the LMS algorithm that is able to adapt to a changing input signal. The RLS algorithm is the most robust of the three and is able to adapt to a changing input signal and noise. The RLS algorithm is also the most computationally expensive of the three.

The general structure of an adaptive filter is illustrated in Figure 1.

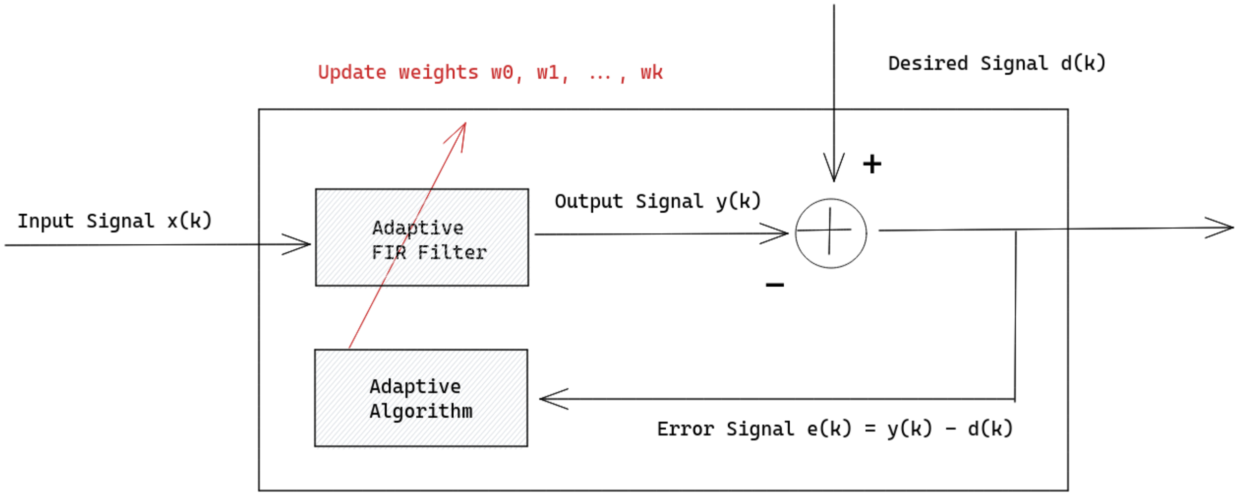

Figure 1: General block diagram for an adaptive filter algorithm

```python
def adaptive_filter(input_signal, desired_signal, n_taps, step_size):
    # Initialize filter coefficients
    # ...

    # loop over all samples


    output_signal = np.zeros(len(input_signal))
    for i in range(len(input_signal)):
        output_signal[i] = np.dot(filter_coefficients, input_signal[i])
        error = desired_signal[i] - output_signal[i]
        filter_coefficients = filter_coefficients + step_size * error * input_signal[i]
    return output_signal



# filter

# error

# update weights
```

LMS (Least Mean Squares) Algorithm

$y(k) = w^T \cdot x(k)$

$e(k) = d(k) - y(k)$

$\Delta w(k) = \mu e(k) x(k)$

$w(k+1) = w(k) + \Delta w(k)$

suggest 2 taps, but with only 2 some noice is left, so 3 or more taps is better. show zoomed in graph of a ECG peak, and demomstrates this.

5 is the smoothest

gradient descent

Adaptive Line Enhancer

use a delayed version of the input signal as the input signal and the input signal as the desired signal.

NMLS (Normalized LMS) Algorithm

$y(k) = w^T \cdot x(k)$

$e(k) = d(k) - y(k)$

$\Delta w(k) = \mu e(k) x(k) / \|x(k)\|^2$

$w(k+1) = w(k) + \Delta w(k)$

RLS (Recursive Least Squares) Algorithm

Incorporates a forgetting factor, λ, that makes the algorithm more robust to noise. The forgetting factor is a value between 0 and 1 that determines how much the algorithm should forget about previous samples. A value of 1 means that the algorithm should not forget anything, while a value of 0 means that the algorithm should forget everything. The forgetting factor is used to calculate the inverse correlation matrix, P^{-1}, which is used to calculate the filter coefficients. The inverse correlation matrix is updated by the following equation:

$y(k) = w^T \cdot x(k)$

$e(k) = d(k) - y(k)$

$P(k) = \frac{1}{\lambda} P(k-1) - \frac{P(k-1)x(k)x(k)^T P(k-1)}{\lambda + x(k)^T P(k-1) x(k)}$

$\Delta w(k) = P(k) x(k) e(k)$

Notch Filter

A notch filter is FIR filter that is used to remove a specific frequency w_n from a signal. The filter is designed to have a frequency response that is 0 at the frequency that is to be removed. It can be thought of as a combination of a low pass filter and a high pass filter, where the low pass filter has a cutoff frequency w_l that is lower than the frequency that is to be removed and the high pass filter has a cutoff frequency w_h that is higher than the frequency that is to be removed i.e. $w_l < w_n < w_h$.

Problem Statement

> An ECG signal is sampled at a sampling frequency $Fs = 500$Hz. The signals contains colored noise at a frequency of circa 50Hz, which is the frequency of the powerline the equipment is connected to. The frequency of the powergrid is not completely constant over time, which means that the noise is not guaranteed to be stationary. An adaptive filter can be used to filter out the noise. The filter should be able to adapt to the changing noise.

The sampled ECG signal is plotted in Figure 2. The ECG signal is not discernable from the noise.

To use an adaptive algorithm to filter out the noise from the ECG signal, a desired signal of the noise is needed. The desired signal is modeled as a sine wave with frequency f_{noise}

$$d = cos(2\pi \cdot f_{noise}/Fs \cdot t)$$

The powerline frequency f_{noise} is estimated using the power spectral density (PSD) of the sampled signal. The PSD is calculated using Welch's method. A plot of the PSD is shown in Figure 3. Most of the signals

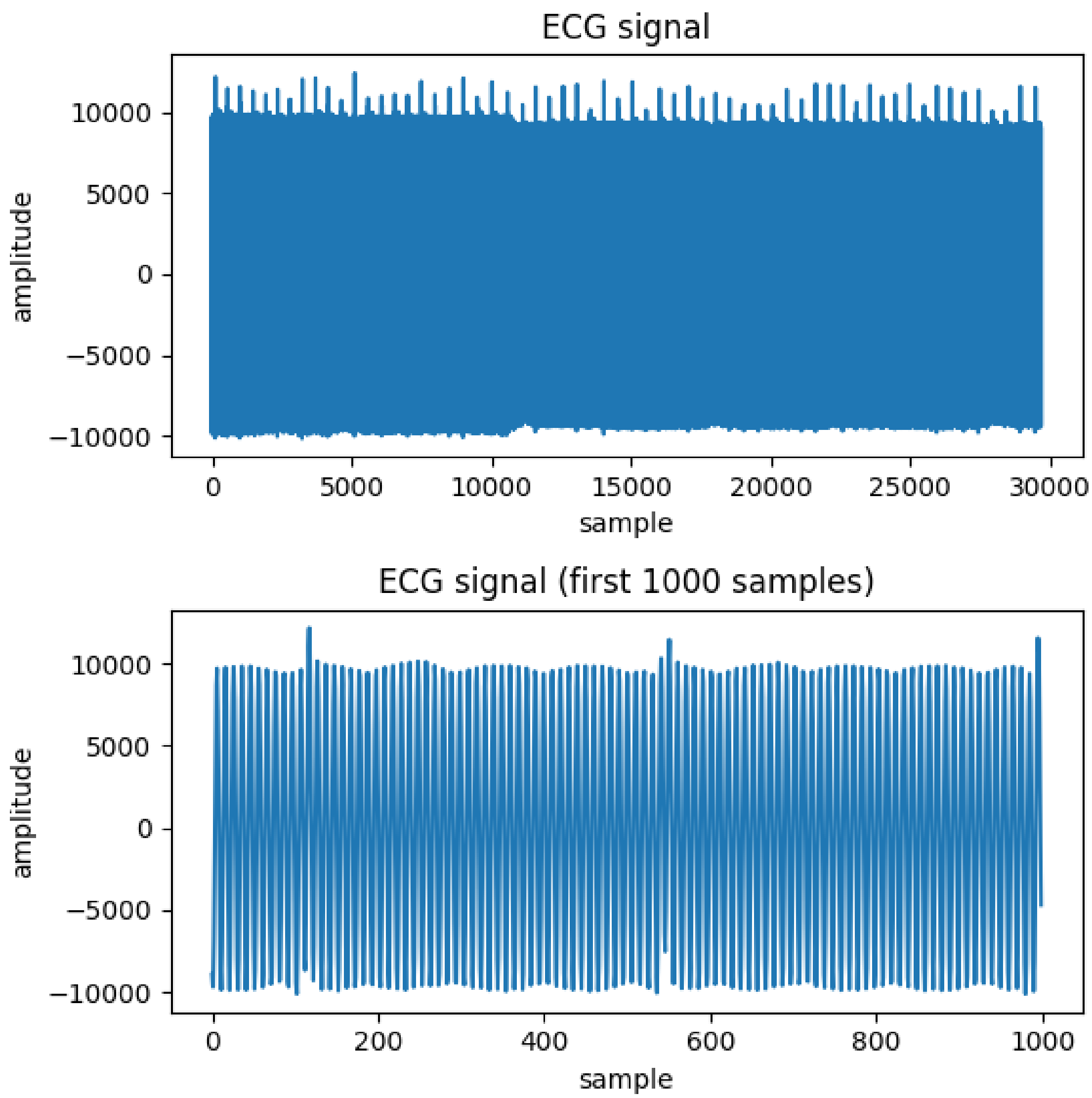

Figure 2: Sampled ECG signal with colored noise

energy is contained in the noise component, so the single peak at 49.56Hz corresponds to the frequency of the noise f_{noise}.

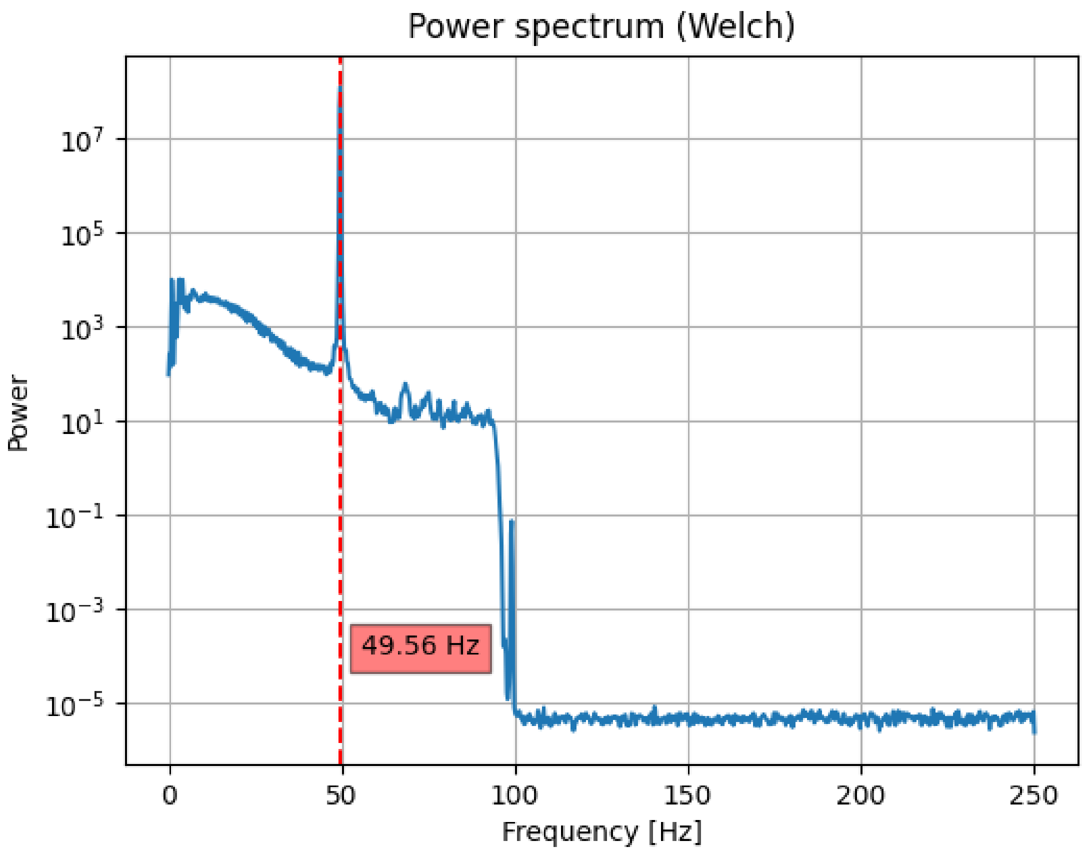

Figure 3: Power spectral density of the ECG signal

How many filter coeffcients are needed in the adaptive filter?

Since the noise in the signal is colored at a single frequency f_{noise} that varies slightly over time, it should be sufficient to have **2** taps in the FIR filter. With 1 tap only the gain of the signal can be affected, as the filters system equation will only contain a scalar $H(z) = a$.With 2 taps both the gain and phase of the signal can be affected, as H(z) = a + bz^-1, which together is enough to attenuate the noise frequency f_{noise}.

Select an appropriate value for the step-size μ.

Equation 6.74 from the book, gives an analytical inequality that describes the interval for the LMS algorithm, given the parameter mu, where the algorithm remains stable. The inequality is given by

$$0 < \mu < \frac{1}{3\text{tr}[R]}$$

where R is the autocorrelation matrix of the input signal $x(k)$.

Using python the upper bound of the inequaity was found to be 0.31. The step-size μ was set to 0.3.

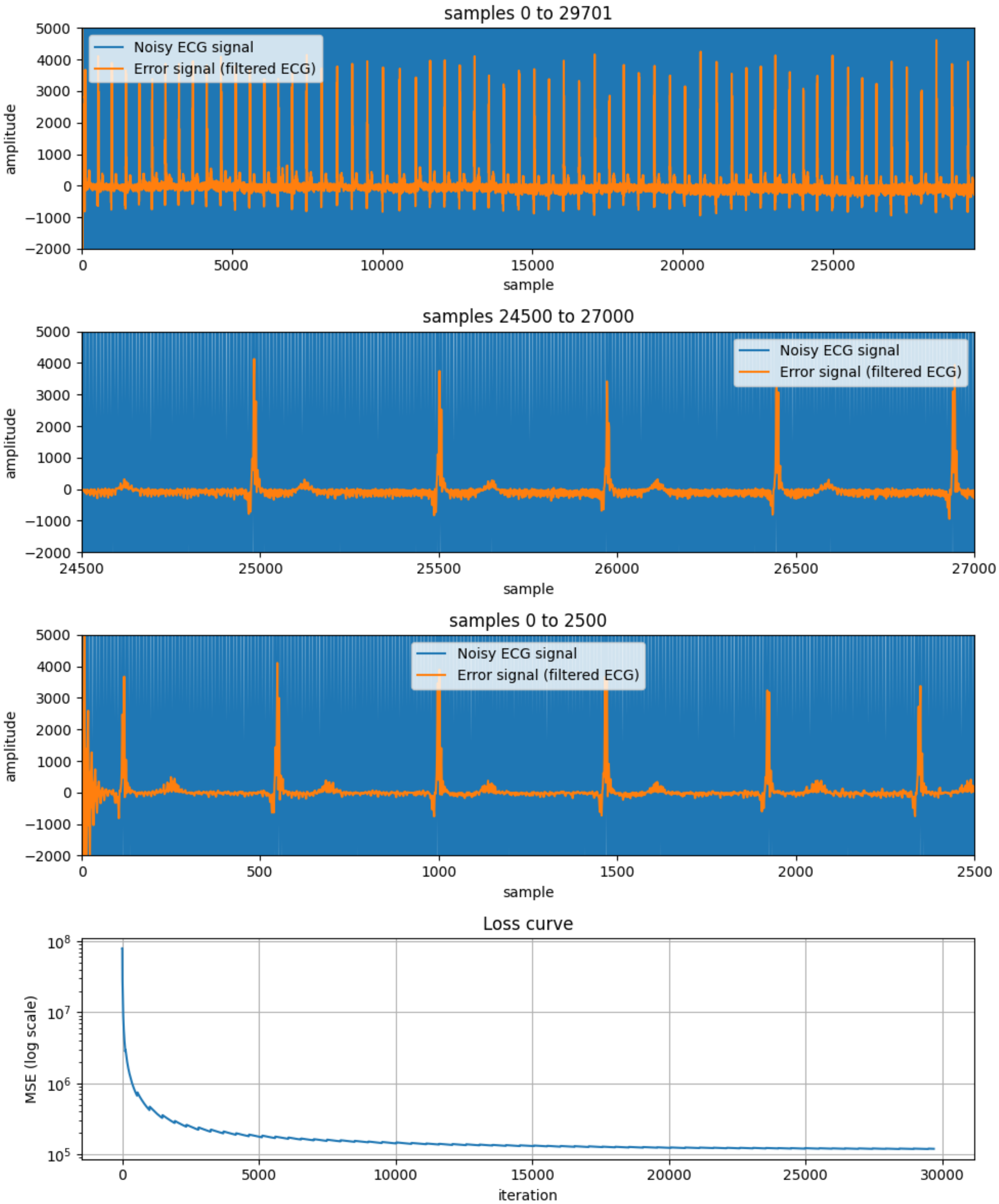

Figure 4: ECG signal with LMS filter

Using LMS with 2 taps, and $\mu = 0.3$.

Hypothesis

assume that 2 taps is sufficient to remove the noise, as the noise is colored and not white, and only contain one frequency component.

Results

Computational Complexity of the Algorithms

Algorithm	Computational Complexity
LMS	
NMLS	
RLS	

Conclusion

footnotes