MATRIX INVERSION USING RECURRENT NEURAL NETWORK

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Introduction

- Matrix inversion has been widely used in a various fields such as robotics, controls, and signal processing.

- For many large-scale problems, the orders of the matrices are very large .

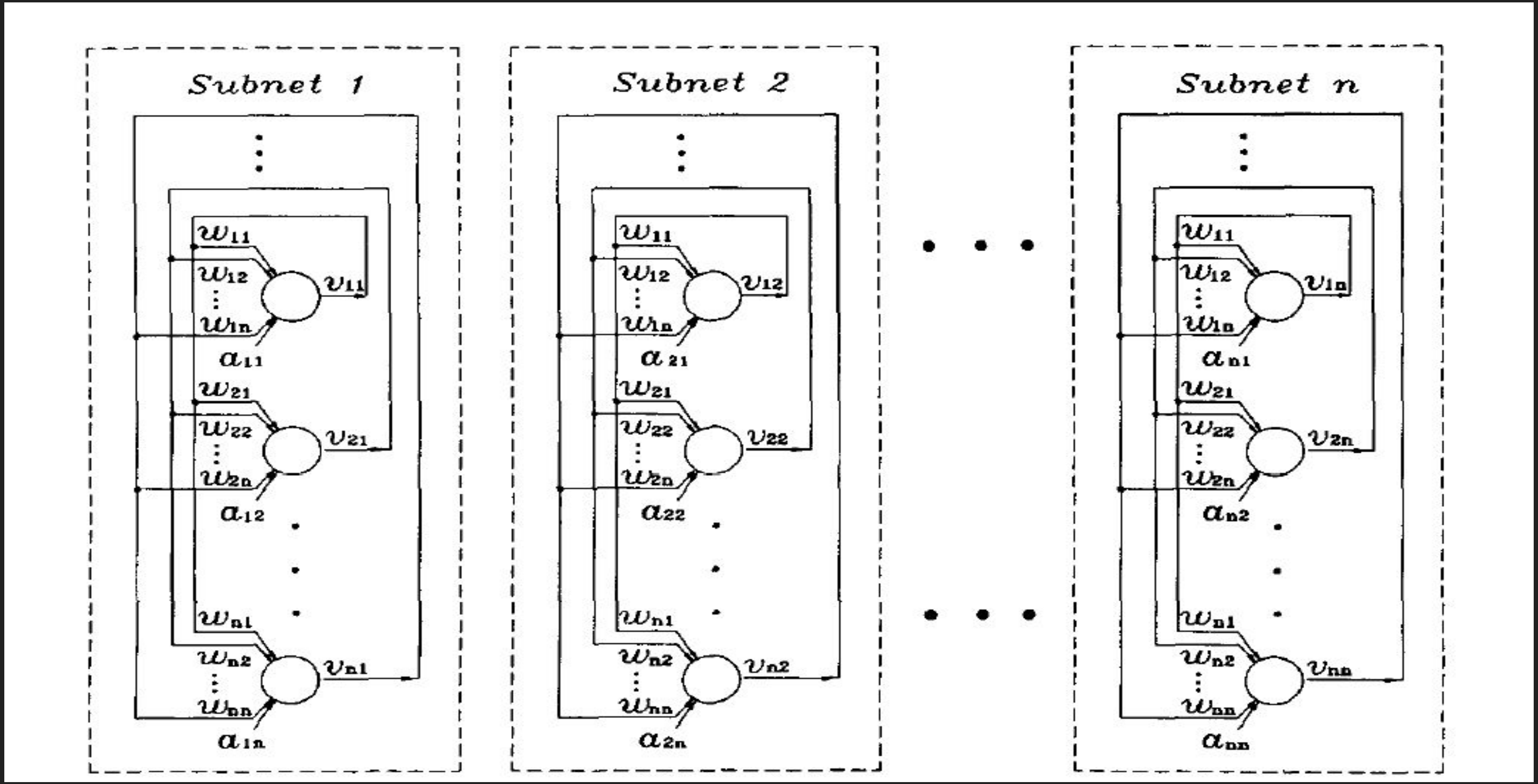
- Large-scale inverse matrices often need to be computed in real-time for monitoring and controlling dynamic systems.

- Inverse of Matrix of higher order using existing algorithms in real-time is usually not much efficient (for Hardware implementation) due to the nature of the sequential processing.For such applications, parallel distributed processing is more desirable.

Hopfield Model

- Provides Multiple-loop-feedback system with delays.
- Architecture of Hopfield follows noiseless,dynamical, Additive-model of a neuron.
- Uses nonlinear activation functions, generally a monotonically increasing in both standard and inverse input-output relations. (e.g., sigmoid)
- The Energy function is a Lyapunov function which is stable in accordance with *Theorem 1*.

Network Configuration



Architecture of the proposed recurrent neural network

Output State Equation

$$\frac{dV(t)}{dt} = -\eta A^{\top} A V(t) + \eta A^{\top}$$

Where,

- A is the input Matrix.
- V is the output Matrix.
- η is the positive scaling parameter.

At the equilibrium state V , $\frac{dV}{dt} = 0$

Thus, $A^T AV = A^T.$

Since A^TA is positive definite, $(A^TA)^{-1}$ exists.

Therefore, $\mathbf{V = (A^TA)^{-1}A^T = A^{-1}(A^T)^{-1}A^T = A^{-1}\,I = A^{-1}}$
(where I is the identity matrix).

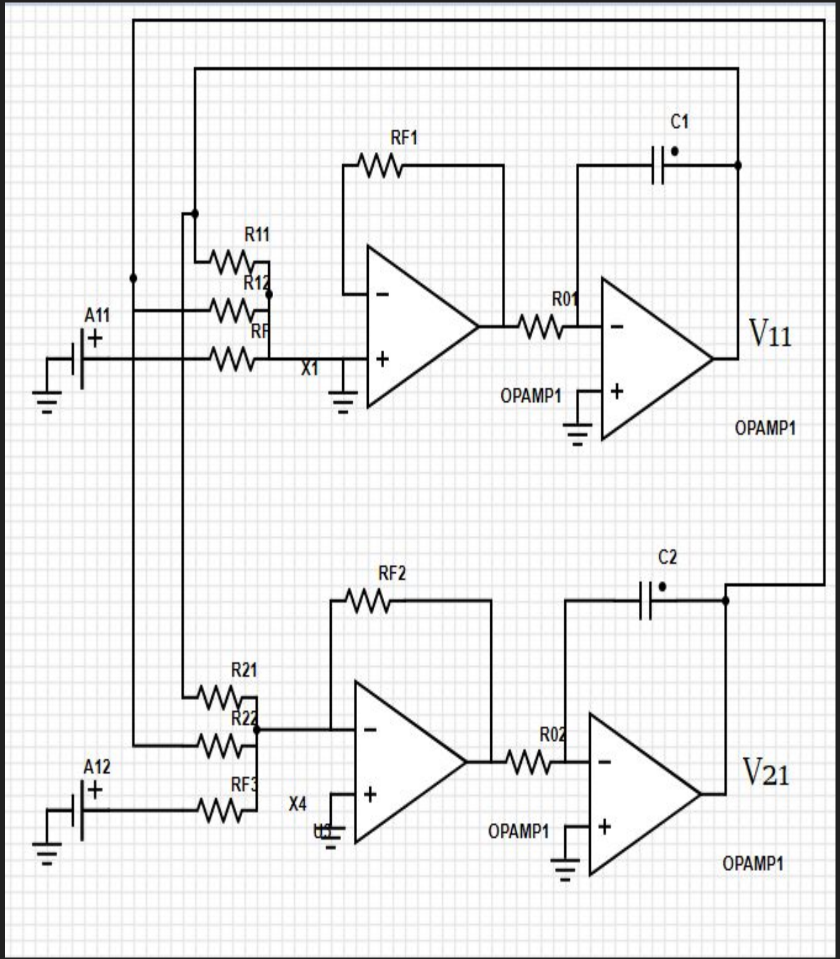
Deriving the State Equation defining the system

To Analyze the Recurrent Neural Network, We model it as an electronic circuit with :

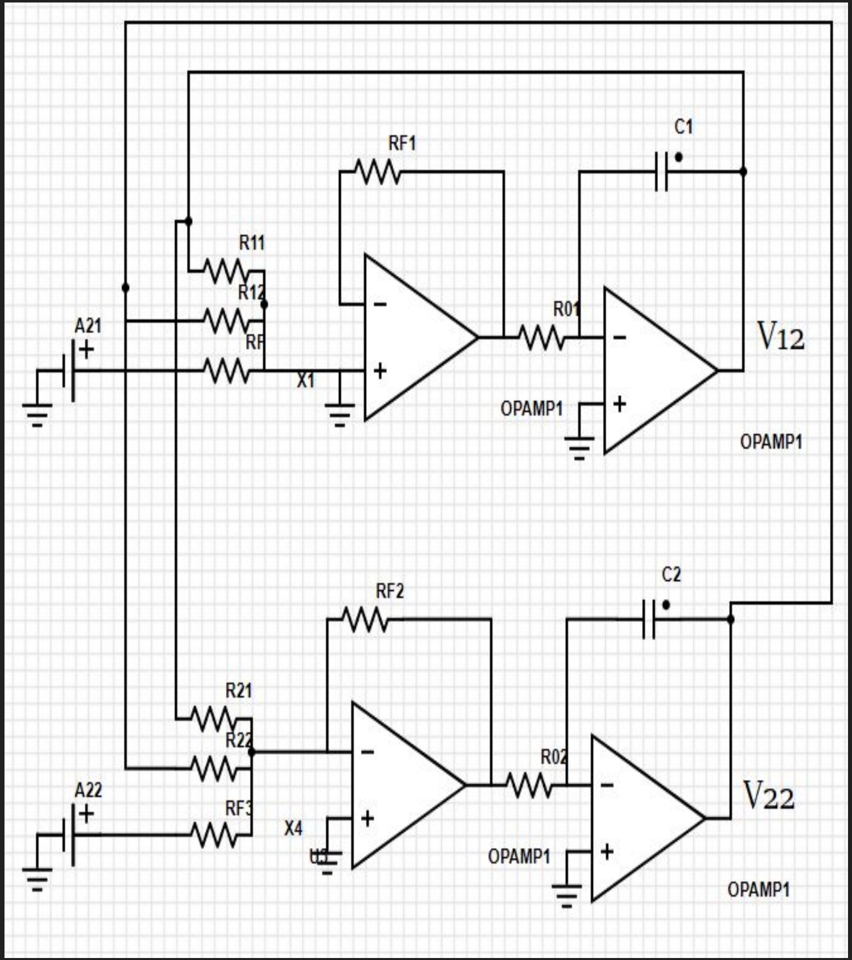
- Op-Amp adder as summing junctions.
- Op-Amp feedback gain as weights.
- Integrators as delay elements.

Analog Realization

Subnet 1



Subnet 2



State equation for V_{11}

Output of Op-Amp Adder :

$$\boxed{V_0 = -\left(V_{11} \cdot \frac{R_f}{R_{11}}\right) - \left(V_{12} \cdot \frac{R_f}{R_{12}}\right) - A_{11}}$$

Output of Integrator :

$$V_{11} = V_0 \cdot \left(\frac{-\frac{1}{CS}}{R_0}\right)$$

$$\boxed{V_{11} = \left[\left(V_{11} \cdot \frac{R_f}{R_{11}}\right) + \left(V_{12} \cdot \frac{R_f}{R_{12}}\right) + A_{11}\right] \frac{1}{R_0 C S}}$$

Let $$\frac{1}{R_0 C} = \eta$$

On converting to time domain,

$$\boxed{\frac{dV_{11}}{dt} = \eta \begin{bmatrix} \frac{R_f}{R_{11}} & \frac{R_f}{R_{12}} \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} + \eta A_{11}}$$

Similarly, obtaining state equations for **V_{12}** , **V_{21}** , **V_{22}** and writing in Matrix form, we get

$$\frac{d}{dt}\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} = \eta \begin{bmatrix} \frac{R_f}{R_{11}} & \frac{R_f}{R_{12}} \\ \frac{R_f}{R_{21}} & \frac{R_f}{R_{22}} \end{bmatrix} \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} + \eta \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$$

$$\boxed{\frac{dV}{dt} = \eta W V + \eta A^{\top}}$$

Lyapunov's Theorems

THEOREM 1 : The equilibrium state $\overline{x}$ is stable if in a small neighbourhood of $\overline{x}$, there exist a positive definite function V(x) such that its derivative with respect to time is negative semi-definite in that region.

THEOREM 2 : The equilibrium state $\overline{x}$ is asymptotically stable if in a small neighbourhood of $\overline{x}$, there exist a positive definite function V(x) such that its derivative with respect to time is negative definite in that region.

Lyapunov's Theorems

According to *theorem 1*, the equilibrium state $\overline{x}$ is stable, if

$$\frac{dV(x)}{dt} \leq 0$$

According to *theorem 2* ,the equilibrium state is asymptotically stable, if

$$\frac{dV(x)}{dt} < 0$$

Equating $\frac{dV}{dt} = 0$

We obtain:

$$\eta W V = -\eta A^T$$

W should be chosen such that we should get inverse at the output.
Hence, equating V to A^{-1} we get ,

$$W A^{-1} = -A^T$$

Therefore , $W = -A^T A$

If A is non-singular, then the equilibrium state of the recurrent neural network is asymptotically stable. Acc. to theorem 2, $A^T A$ is positive definite.

If A is non-singular, then the equilibrium state matrix of the recurrent neural network represents the inverse of matrix.

MATLAB implementation

- The implementation was a continuous time form on the Op-Amp Analog circuit.

- Discretizing it will be helpful to implement the same on currently trending Digital Technology such as FPGA.

Advantages

- Hardware friendly Algorithm.
- Faster computation of Inverse on hardware.
- Computational time is **O**(n^3/2).

Thank You