Baye's Theorem

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If $E_1, E_2, \ldots E_n$ are 'n' mutually exclusive events of a sample spaces.
If 'A' is an arbitrary event which is conditional to all E_i for $i=1$ to n

then (i) $P(A) = \sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)$

(ii) $P(E_i/A) = \dfrac{P(A/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)}$

<u>proof:-</u> $E_1, E_2 \ldots E_n$ are 'n' mutually exclusive events.

$\therefore S = E_1 \cup E_2 \cup \cdots \cup E_n \longrightarrow (1)$

and $E_i \cap E_j = \phi$ ($\forall$ i and j)

we know that

$A = A \cap S$ here A is conditional event $\forall E_i$

$A = A \cap [E_1 \cup E_2 \cup \cdots \cup E_n]$

$A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cdots \cup (A \cap E_n) \longrightarrow (1)$

Taking probability on both sides wrt -

$P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n)]$

$P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$

$= \sum_{i=1}^{n} P(A \cap E_i) \longrightarrow (2)$

$E_1, \ldots E_n$
$A \cap E_1, A \cap E_2 \ldots A \cap E_n$
A, B
$P(A \cup B) = P(A) + P(B)$

From the definition of conditional probability

$P(A/E_i) = \dfrac{P(A \cap E_i)}{P(E_i)}$

$\Rightarrow P(A \cap E_i) = P(A/E_i) \cdot P(E_i) \longrightarrow (3)$

$$\Rightarrow P(A \cap E_i) = P(A/E_i) \cdot P(E_i) \longrightarrow (3)$$

Sub (3) in (2),

$$\boxed{P(A) = \sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)} \longrightarrow (a)$$

(ii) The conditional probability to the event E_i with the conditional event is

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i \cap A)}{\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)} \longrightarrow (b)$$

but $P(A/E_i) = \dfrac{P(A \cap E_i)}{P(E_i)} \Rightarrow P(A \cap E_i) = P(A/E_i) \cdot P(E_i) \longrightarrow (c)$

Sub (c) in (b)

$$\therefore (b) \Rightarrow P(E_i/A) = \frac{P(A/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)}$$ ✓

(1) In a certain college 25% of boys and 10% girls are studying mathematics. The girls constitute 60% of the student body. (a) what is the probability that mathematics being studied (b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl? (c) a boy?

Sol:-

$$P(G) = 60\% = \frac{60}{100} = \frac{3}{5}$$

$$P(B) = 40\% = \frac{40}{100} = \frac{2}{5}$$

Probability that mathematics is studied given that the student is a boy $= P(M/B) = 25\% = \frac{25}{100} = \frac{1}{4}$

$$P(M/G) = 10\% = \frac{10}{100} = \frac{1}{10}$$

(a) What is the probability that mathematics being studied?

$$P(M) = P(G) \cdot P(M/G) + P(B) \cdot P(M/B) ✓$$

$$= \frac{3}{5} \cdot \frac{1}{10} + \frac{2}{5} \cdot \frac{1}{4} = \frac{4}{5}.$$

(b) By Bayes theorem, probability of maths student is a girl.

$$P(G/M) = \frac{P(G) \cdot P(M/G)}{P(M)} = \frac{\frac{3}{5} \cdot \frac{1}{10}}{4/5} = \frac{3}{8}$$

(c) probability of a math student is a boy.

$$P(B/M) = \frac{P(B) \cdot P(M/B)}{P(M)} = \frac{\frac{2}{5} \cdot \frac{1}{4}}{4/5} = 5/8$$

(2) A bag 'A' contains 2 white and 3 red marbles. Bag B contains ~~of~~ 4 white and 5 red marbles. 1 marble is drawn at random from one of the bags.

(a) Find the probability that it is red.

probability that

drawn at random from one of the bags.

(a) Find the probability that it is red.

(b) If marble drawn is red find the probability that it is from bag A.

Bag A	Bag B
2W+3R	4W+5R

Sol:- Let A be the event of selecting bag A.

$$P(A) = \frac{1}{2}$$

Let B be the event of selecting bag B.

$$P(B) = \frac{1}{2}$$

Let R be the arbitrary event that drawn marble is red.

$$P(R) = P(A) \cdot P(R/A) + P(B) \cdot P(R/B)$$

$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{9} = \frac{26}{45}$$

(ii) Red marble drawn from box A is

$$P(A/R) = \frac{P(A) \cdot P(R/A)}{P(A) \cdot P(R/A) + P(B) \cdot P(R/B)}$$

$$= \frac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot 5/9} = \frac{27}{52}$$

(3) Of the three men, the chances that a politician, a business man or an academician will be appointed as a vice-chancellor (V.C) of a university are 0.5, 0.3, 0.2 respectively. Probability that research is promoted by these persons if they are appointed as VC are 0.3, 0.7, 0.8 respectively.

(i) Determine the probability that research is promoted.

(ii) If research is promoted, what is the probability that V.C is an Academician.

Sol:- Let A, B, C be the events that politician, business man, Academician

$$P(A) = 0.5, \quad P(B) = 0.3, \quad P(C) = 0.2$$

The probabilities that the research is promoted if they are appointed as VC are

$$P(R/A) = 0.3, \quad P(R/B) = 0.7, \quad P(R/C) = 0.8$$

(i) Probability of Research is promoted.

$$P(R) = P(A) \cdot P(R/A) + P(B) \cdot P(R/B) + P(C) \cdot P(R/C)$$

$$= (0.5)(0.3) + 0.3(0.7) + 0.2(0.8)$$

$$= 0.52$$

(ii) The probability that research is promoted when V.C is an academician

$$= P(C/R) = \frac{P(C) \cdot P(R/C)}{P(A) \cdot P(R|A) + P(B) \cdot P(R|B) + P(C) \cdot P(R/C)}$$

$$= \frac{(0.2)(0.8)}{0.52} = \frac{4}{13}$$