3) Jacobian of EEF

$$\frac{d}{dt}\left({}^{W}P_{eff}\right) = \hat{\theta}_1\, {}^{W}R_{B1}\, {}^{B1}R_{B2}\, {}^{B2}P + {}^{W}R_{B1}\, \hat{\theta}_2\, {}^{B1}R_{B2}\, {}^{B2}P + \hat{\theta}_1\, {}^{W}R_{B1}\, {}^{B1}T_{B2}$$

$$= \begin{bmatrix} 0 & -\dot{\theta}_1 \\ \dot{\theta}_1 & 0 \end{bmatrix}\begin{bmatrix} \cos_1 & -\sin_1 \\ \sin_1 & \cos_1 \end{bmatrix}\begin{bmatrix} \cos_2 & -\sin_2 \\ \sin_2 & \cos_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos_1 & -\sin_1 \\ \sin_1 & \cos_1 \end{bmatrix}\begin{bmatrix} 0 & -\dot{\theta}_2 \\ \dot{\theta}_2 & 0 \end{bmatrix}\begin{bmatrix} \cos_2 & -\sin_2 \\ \sin_2 & \cos_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} +$$

$$\begin{bmatrix} 0 & -\dot{\theta}_1 \\ \dot{\theta}_1 & 0 \end{bmatrix}\begin{bmatrix} \cos_1 & -\sin_1 \\ \sin_1 & \cos_1 \end{bmatrix}\begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$aV + bW$

$[VW]\begin{bmatrix} a \\ b \end{bmatrix}$

$$= \begin{bmatrix} -\dot{\theta}_1\sin_1 & -\dot{\theta}_1\cos_1 \\ \dot{\theta}_1\cos_1 & \dot{\theta}_1 - \sin_1 \end{bmatrix}\begin{bmatrix} \cos_2 & -\sin_2 \\ \sin_2 & \cos_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos_1 & -\sin_1 \\ \sin_1 & \cos_1 \end{bmatrix}\begin{bmatrix} -\dot{\theta}_2\sin_2 & -\dot{\theta}_2\cos_2 \\ \dot{\theta}_2\cos_2 & \dot{\theta}_2 - \sin_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} +$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4.5 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} -\dot{\theta}_1\sin_1 & -\dot{\theta}_1\cos_1 \\ \dot{\theta}_1\cos_1 & \dot{\theta}_1 - \sin_1 \end{bmatrix}\begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} T_x \\ T_y \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} (-\sin_1\cos_2) + (-\cos_1\sin_2) & (\sin_1\sin_2 + -\cos_1\cos_2) \\ (\cos_1\cos_2) + (-\sin_1\sin_2) & (-\sin_2\cos_1 + -\sin_1\cos_2) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}[\dot{\theta}_1] + \begin{bmatrix} (\cos_1 - \sin_2) + (-\sin_1 \cos_2) & (-\cos_2\cos_1) + (-\sin_1 - \sin_2) \\ (-\sin_2\sin_1) + (\cos_1\cos_2) & (-\cos_2\sin_1) + (-\sin_2\cos_1) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}[\dot{\theta}_2] + \begin{bmatrix} -\sin_1 & -\cos_1 \\ \cos_1 & -\sin_1 \end{bmatrix}\begin{bmatrix} T_x \\ T_y \end{bmatrix}[\dot{\theta}_1]$$

$$= \begin{bmatrix} x\left((-\sin_1\cos_2) + (-\cos_1\sin_2)\right) \\ x\left((\cos_1\cos_2) + (-\sin_1\sin_2)\right) \end{bmatrix}\dot{\theta}_1 + \begin{bmatrix} x\left((-\cos_1\sin_2) + (-\sin_1\cos_2)\right) \\ x\left((-\sin_2\sin_1) + (\cos_1\cos_2)\right) \end{bmatrix}\dot{\theta}_2 + \begin{bmatrix} T_x(-\sin_1) \\ T_x(\cos_1) \end{bmatrix}\dot{\theta}_1$$

$$\begin{bmatrix} x\left((-\sin_1\cos_2) + (-\cos_1\sin_2)\right) + T_x(-\sin_1) & x\left((-\cos_1\sin_2) + (-\sin_1\cos_2)\right) \\ x\left((\cos_1\cos_2) + (-\sin_1\sin_2)\right) + T_x(\cos_1) & x\left((-\sin_2\sin_1) + (\cos_1\cos_2)\right) \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$